Incremental Redundancy Cooperative Coding for Wireless Networks: Cooperative Diversity, Coding, and Transmission Energy Gain

Ruoheng Liu, Predrag Spasojević, and Emina Soljanin

Abstract

We study an *incremental redundancy* (IR) cooperative coding scheme for wireless networks. To exploit the spatial diversity benefit we propose a cluster-based collaborating strategy for a quasi-static Rayleigh fading channel model and based on a network geometric distance profile. Our scheme enhances the network performance by embedding an IR cooperative coding scheme into an existing noncooperative route. More precisely, for each hop, we form a collaborating cluster of $M-1$ nodes between the (hop) sender and the (hop) destination. The transmitted message is encoded using a mother code and partitioned into M blocks corresponding to the each of M slots. In the first slot, the (hop) sender broadcasts its information by transmitting the first block, and its helpers attempt to relay this message. In the remaining slots, the each of left-over $M-1$ blocks is sent either through a helper which has successfully decoded the message or directly by the (hop) sender where a dynamic schedule is based on the ACK-based feedback from the cluster. By employing powerful *good codes* (e.g., turbo codes, LDPC codes, and raptor codes) whose performance is characterized by a threshold behavior, our approach improves the reliability of a multi-hop routing through not only cooperation diversity benefit but also a coding advantage. The study of the diversity and the coding gain of the proposed scheme is based on a *simple threshold* bound on the frame-error rate (FER) of maximum likelihood decoding. A average FER upper bound and its asymptotic (in large SNR) version are derived as a function of the average fading channel SNRs and the code threshold. Based on asymptotic bounds, we investigate both the diversity, coding, and transmission energy gain in the high and moderate SNR regime for three different scenarios: cooperative transmission, cooperative reception, and cluster hopping. Furthermore, given a geometric distance profile of the network, these bounds guide to the design of the collaborating cluster and the IR cooperation scheme.

Index Terms

Fading channel, diversity techniques, user cooperation, incremental redundancy, turbo codes.

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I. INTRODUCTION

To overcome fading, wireless networks employ various diversity techniques, e.g., channel interleavers, multiple antennas, frequency hopping, etc. In [1], Sendonaris, Erkip, and Aazhang have proposed the so-called *user-cooperation diversity* where users partner in sharing their antennas and other resources to create a virtual array through distributed transmission and signal processing. Only limited time and frequency diversity is available when the propagation environment changes slowly relative to the signaling rate, the receiver has a strict decoding delay constraint (as in speech and video transmission), and the bandwidth of the channel input is considerably less than the coherence bandwidth. Hence, since signal experiences a quasi-static frequency-flat fading, introducing spacial diversity through cooperation is especially beneficial. An information-theoretic analysis of several cooperative protocols achieving diversity gain through *repetition-coding* and *space-time-coding* based schemes have been reported in [2], [3]. More recently, practical cooperative coding design has became a topic of active research. Coding strategies for two-user collaborating transmission based on rate compatible punctured convolutional codes have been considered in [4], [5]. Cooperation schemes employing distributed turbo codes have been studied in [6]–[8].

In this paper, we study an *incremental redundancy* (IR) cooperative coding scheme for wireless networks. To exploit the spatial diversity benefit we propose a cluster-based collaborating strategy for a quasi-static Rayleigh fading channel model and based on a network geometric distance profile. The cluster-based collaborating strategy enhances the network performance by embedding an IR cooperative coding into an existing noncooperative route. More precisely, for each of the noncooperative hops, we form a collaborating cluster of $M-1$ nodes between the (hop) sender and the hop destination. The transmitted message is encoded using a mother code which is partitioned into M blocks each assigned to one of M transmission slots. In the first slot, the (hop) sender broadcasts own information by transmitting the first block, and its helpers attempt to relay the message. In the remaining slots, the each of left-over $M-1$ blocks is sent either through a helper which has successfully decoded the message or directly by the (hop) sender based on an acknowledgement (ACK) driven a dynamic schedule. By employing the powerful *good codes* [9], [10] (e.g., turbo, LDPC, and raptor codes) whose performance is characterized by a threshold behavior, our approach improves the reliability of multi-hop routing through not only cooperation diversity benefit but also a coding advantage.

To investigate benefits of the proposed IR cooperative coding scheme, we evaluate its frame error rate (FER) performance for a quasi-static frequency-flat Rayleigh fading channel by studying the threshold behavior of good codes [10]. We base the analysis on a *simple code threshold* for transmission of good codes over an AWGN channel. This threshold ensures (asymptotically in the codeword length) reliable performance under *maximum likelihood* (ML) decoding. The FER performance bound based on the simple threshold has the same form as the *union Bhattacharyya* (UB) code threshold bound [11], [12], but the former bound is tighter by almost 1 *dB* for an example of a rate $1/7$ turbo code on an AWGN channel. Based on this threshold and the channel outage concept, we derive a tight average FER upper-bound which predicts well fading channel simulation results for the proposed scheme. A corresponding closed-form FER upper-bounds asymptotic in large SNRs and obtained for three different scenarios: cooperative transmission, moderate reception, and cluster hopping. These bounds allow for illustrating cooperative diversity and coding gains in the high SNR regime. Finally, we express FER bounds in terms of the distance profile and discuss how to design collaborating clusters to achieve cooperation benefits including energy efficiency gain.

The paper is organized as follows: We describe the system model and an IR cooperative coding scheme in Sec. II. We derive an upper bound on the scheme FER in Sec. III, and its asymptotic versions for different cooperation scenarios in Sec. IV. Simulation results and the collaborating cluster design are discussed in Sec. V.

II. System Model

In this section, we describe a collaborating cluster, an IR cooperative coding scheme, and the fading channel model which incorporates the geometric distance profile of the network.

A. Collaborating Cluster

We assume that, initially, a suitable non-cooperative route used for delivering packets from the source to the sink is established. As shown in Fig. 1, for each of the noncooperative hops, a group of $M-1$ nodes forms a *collaborating cluster* $\mathcal{S}$ which is between the (hop) sender and the (hop) destination. The

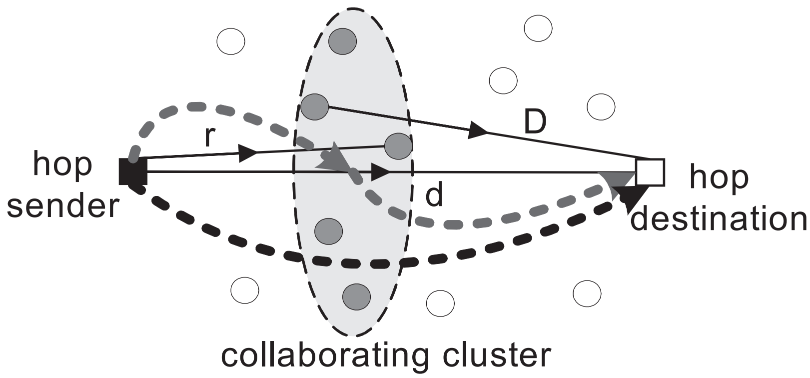


Fig. 1. Cooperative routing in wireless network with a geometric distance profile

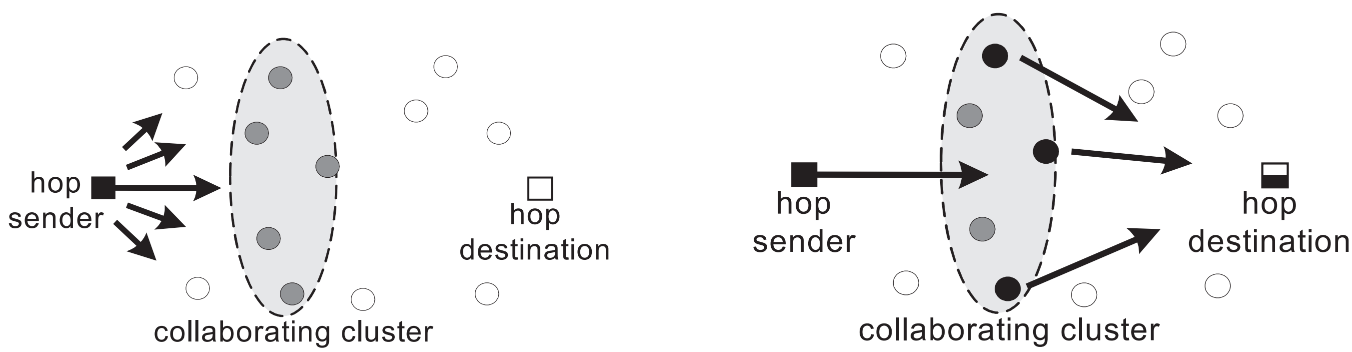
(hop) sender serves as the regional broadcast node, and each cluster member, termed helper, attempts to relay the package to the (hop) destination. We further assume that the communication bandwidth and time slots are both introduced for spatial reuse across noncooperative hops in a proper manner. Therefore, since we can neglect inter-hop interference, it is sufficient to study the cooperative scheme corresponding to a single noncooperative hop.

B. IR Cooperative Coding Scheme

In order to establish IR coding collaboration in a single noncooperative hop, we assume that the (hop) sender and cluster members have acquired a common time reference and are operating in a single radio frequency band W. The *medium access control* (MAC) within a noncooperative hop is based on a time-division scheme. The one hop transmission interval T is partitioned into M non-overlapping *slots* of duration $\tau_0 T, \ldots, \tau_{M-1} T$, where $\sum_{j=0}^{M-1} \tau_j = 1$. The cluster can transmit close to $N = \lfloor WT \rfloor$ symbols during the transmission interval.

Each node in the wireless network has an encoder, a decoder, and a mapping device. The (hop) sender encodes the information and obtains a mother codeword $\mathcal{C}$ of length N and rate R. The mapping device partitions the codeword $\mathcal{C}$ into M blocks of relative lengths τ_j, $j = 0, \ldots, M-1$. For analysis simplicity, we employ a probabilistic mapping device [12] by which the bits of $\mathcal{C}$ are assigned randomly to one of M blocks with the *assignment rate* τ_j.

Let $\mathcal{C}_j$ denote the punctured codeword obtained from $\mathcal{C}$ by choosing the j-th block. For a given good (mother) code ensemble $[\mathcal{C}]$ and the (random) assignment rate τ_j, the punctured code ensemble $[\mathcal{C}_j]$ exhibits a SNR threshold behavior for a general Gaussian (with or without fading) channel, i.e., if the received SNR is larger than the punctured code threshold, the average ML decoding word error probability of $[\mathcal{C}_j]$ decays to zero as the codeword length $N \to \infty$.[1] In order to predict the error performance accurately, we have introduced a tight punctured code threshold $\chi^{[\mathcal{C}]}(\tau_j)$ (see detail in Sec. III-A), which can be calculated based on the mother code threshold and shows a good match with the simulation result under iterative decoding. Hence, this code threshold behavior helps us dynamically schedule the cooperation in the following two stages. For notation convenience, let Node 0 denote the (hop) sender, Node m, for $m = 1, \ldots, M-1$, denote the cluster members, and Node M denote the (hop) destination (i.e., the next-hop (hop) sender).



a. Slot 1: (hop) sender broadcasts message   b. Slot 2–Slot M: reliable nodes relay information

Fig. 2. Cooperation coding scheme (In the first slot, (hop) sender broadcasts its information. In other slots, each cluster member decides whether to relay decoded information by sending additional parity bits to form a more powerful code.)

1) Broadcast Stage: As shown in Fig. 2.a, the (hop) sender broadcasts its information by transmitting $\mathcal{C}_0$ during Slot 0. Each helper listens and attempts to decode this message. More precisely, helpers estimate the instantaneous received SNR from the incoming signal early during the broadcast slot. Let $\mathcal{F} \subseteq \mathcal{S}$ denote the *reliable* set of cluster members whose received instantaneous SNR is larger than the punctured code threshold $\chi^{[\mathcal{C}]}(\tau_0)$. This implies that Node $k \in \mathcal{F}$ can guarantee to decode $\mathcal{C}_0$ successfully. Next, each *reliable* node sends an ACK back to the (hop) sender over a fast and error-free feedback channel. We note that in [3], [15], authors also define the cooperation set as the set of relays that can decode the message successfully. Different from prior approaches, our scheme allows for a *fast ACK* message sent

[1]For typical practical systems, the codeword length is fairly large [13], e.g., in CDMA2000 standard [14], the encoder allows for a variable input length up to $K \simeq 20730$.

back the (hop) sender from the reliable nodes *without* a need for prior decoding, and, thus, allowing for a streamlined scheduling of cooperation retransmissions. At the end of the first slot, only reliable nodes decode the received signal.

2) Forwarding stage: Fig. 2.b illustrates the transmission interval from Slot 1 to Slot $M-1$ (here, dark solid circles represent reliable nodes). Node k, for $k \in \mathcal{F}$, re-encodes and punctures received information in the same manner as done by the (hop) sender and, consequently, relays the k-th block $\mathcal{C}_k$ to the destination in Slot k. The (hop) sender transmits the left-over blocks in the remaining transmission slots based on received ACKs. The signal received at the destination corresponds to an IR scheme with a fixed number of retransmissions, where a retransmission experiences a different channel quality in the case that the corresponding sender-to-heper instantaneous SNR is sufficiently good.

C. Channel Model

We consider the quasi-static frequency-flat Rayleigh fading channel model [16], where the fading coefficient is random, but invariant during the transmission interval T. The discrete-time channel model is

$$y_{i,j} = d_{i,j}^{-L/2}\, a_{i,j}\, x_i + z_{i,j} \quad \text{for } i \in \{0, \ldots, M-1\} \text{ and } j \in \{1, \ldots, M\}$$

where x_i is the signal transmitted by Node i, L is the path loss exponent, $d_{i,j}$, $a_{i,j}$, and $z_{i,j}$ are the distance, fading coefficient, and background noise between Nodes i, j, respectively, and $y_{i,j}$ is the signal at Node j received from Node i. We assume that $E \triangleq |x_i|^2$ is the transmitted symbol energy which is identical for all cluster nodes, $z_{i,j}$ is modeled as the mutually independent additive Gaussian noise $\mathcal{N}(0, 1/2)$, and $\nu_{i,j} = |a_{i,j}|^2$ is the exponentially distributed "channel power" with mean 1. Then, the average and instantaneous SNRs of the signal at Node j received from Node i can respectively be expressed as

$$\mathsf{SNR}_{i,j} \triangleq E \cdot d_{i,j}^{-L}, \quad \text{and} \quad \theta_{i,j} \triangleq \nu_{i,j} \cdot \mathsf{SNR}_{i,j}.$$

We further assume that decoding is done with the knowledge of the fading coefficients.

III. Noise Threshold Based Performance Analysis

The decoding is performed at the destination upon completion of M transmission slots. Proposed cooperative coding scheme implies that the received signal is always the mother codeword $\mathcal{C}$ modified by the fading channel. Moreover, all communication links experience an independent quasi-static Rayleigh fading channel, thus, the codeword $\mathcal{C}$ is equivalently transmitted over M parallel channels corresponding to each of the M slots, and experiences $|\mathcal{F}|+1$ independent channel gains. This requires studying the performance of codes transmitted over a block fading channel [16].

A. Threshold Behavior of Good Codes for an AWGN Channel

The basis for our analysis is the threshold behavior of good codes. In [11], Jin and McEliece have defined the UB code threshold of a good code ensemble $[\mathcal{C}]$ as

$$c_0^{[\mathcal{C}]} \triangleq \sup_{0<\delta\leq 1} \limsup_{N\to\infty} \frac{\ln \overline{A}_{\lfloor \delta N \rfloor}^{[\mathcal{C}]}}{\delta} \tag{1}$$

where $\overline{A}_{\lfloor \delta N \rfloor}^{[\mathcal{C}]}$ is average weight enumerator (AWE) of the code ensemble, and δ is the normalized Hamming weights. The authors in [11] have derived the following coding theorem for a good code ensemble.

Theorem 1 [11, Theorem 8.1] Let a good code ensemble $[\mathcal{C}]$ with a finite UB code threshold $c_0^{[\mathcal{C}]}$. For any binary-input memoryless channel whose Bhattacharyya noise parameter[2] $\gamma < \exp(-c_0^{[\mathcal{C}]})$, the average ML decoding word error probability $P_W^{[\mathcal{C}]}(\gamma) \xrightarrow{N} 0$. ■

This result is based on the classical union bound. Hence, the threshold $c_0^{[\mathcal{C}]}$ is loose.

To obtain a tight code threshold, we partition the normalized Hamming weights into two subsets, $U(P) = \{\delta : 0 < \delta \leq P \text{ or } 1-P < \delta \leq 1\}$ and $U^c(P) = \{\delta : P < \delta \leq 1-P\},$ for $P \in (0, 0.5]$, and define a simple code threshold by optimizing the weight partition parameter P as follows

$$c_\star^{[\mathcal{C}]} \triangleq \min_{0<P\leq 0.5} \left\{ c_P^{[\mathcal{C}]} :\ 1-\exp(-c_P^{[\mathcal{C}]}) \geq R + \xi_P^{[\mathcal{C}]} \right\} \tag{2}$$

where

$$c_P^{[\mathcal{C}]} \triangleq \limsup_{N\to\infty} \max_{\delta\in U(P)} \frac{\ln \overline{A}_{\lfloor \delta N \rfloor}^{[\mathcal{C}]}}{\lfloor \delta N \rfloor} \quad \text{and} \quad \xi_P^{[\mathcal{C}]} \triangleq \limsup_{N\to\infty} \max_{\delta\in U^c(P)} \frac{1}{N}\left(\ln \overline{A}_{\lfloor \delta N \rfloor}^{[\mathcal{C}]} - \ln \overline{A}_{\lfloor \delta N \rfloor}^{[\mathcal{RB}]} \right) \tag{3}$$

denote the restriction code UB noise threshold and SF distance corresponding to the weight subsets $U(P)$ and $U^c(P)$, respectively, and $[\mathcal{RB}]$ denotes the ensemble of random binary codes. Based on $c_\star^{[\mathcal{C}]}$, we have the following coding theorem for parallel AWGN channels.

[2]For a binary input memoryless channel with output alphabet $\mathcal{Y}$ and transition probabilities $\mathrm{P}(y|0)$ and $\mathrm{P}(y|1)$, $y \in \mathcal{Y}$, the *Bhattacharyya noise parameter* defined as $\gamma = \sum_{y\in\mathcal{Y}} \sqrt{\mathrm{P}(y|0)\mathrm{P}(y|1)}$. In particular, for an AWGN channel with the received SNR θ, the Bhattacharyya noise parameter is $\exp(-\theta)$ (see [17] for more detail).

Theorem 2 Let the symbols of a good binary code ensemble $[\mathcal{C}]$ be randomly assigned to Q parallel binary-input AWGN channels where the set of assignment rates is $\{\tau_0, \ldots, \tau_{Q-1}\}$ and the set of Bhattacharyya noise parameters is $\{\gamma_0, \ldots, \gamma_{Q-1}\}$. If

$$\overline{\gamma} < \exp(-c_\star^{[\mathcal{C}]})$$

where $\overline{\gamma} \triangleq \sum_{j=0}^{Q-1} \tau_j \gamma_j$ is the average Bhattacharyya noise parameter, then the average ML decoding word error probability $P_W^{[\mathcal{C}]}(\overline{\gamma}) \xrightarrow{N} 0$.

Proof: The proof is in the Appendix B. ■

Example 1 (Simple threshold for turbo codes) Here, we study the UB and simple code thresholds of a $R = 1/7$ turbo code. The turbo encoder consists of $J = 3$ recursive convolutional encoders with two random interleavers. The component code transfer functions are $G_1 = (1,\, 13/15,\, 17/15)$ and $G_2 = G_3 =$

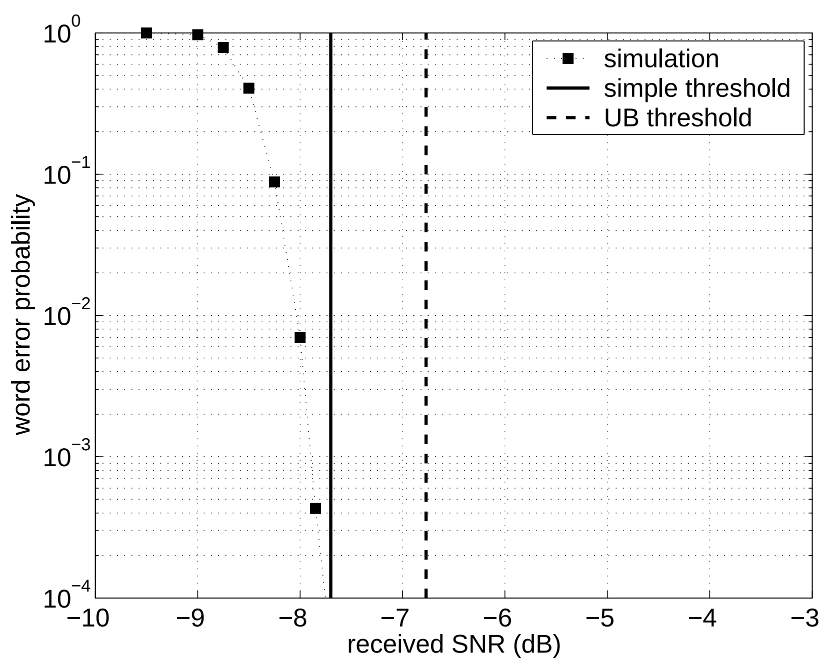


Fig. 3. UB and simple code thresholds for an AWGN channel (turbo code of $R = 1/7$ and $N = 5376$)

$(13/15,\, 17/15)$. We compute the AWE based on the technique of [18] for $N = 5376$. By applying (1) and (2), we calculate the UB threshold $c_0^{[\mathcal{TC}]} \doteq 0.21$ and the simple threshold $c_\star^{[\mathcal{TC}]} \doteq 0.17$. As shown in Fig. 3, we compare the UB and simple threshold with simulation results under iterative decoding when the turbo codes are transmitted over an AWGN channel. Fig. 3 illustrates that the simple threshold predicts the cliff of the simulated word error probability accurately, and that the gap between $c_0^{[\mathcal{TC}]}$ and $c_\star^{[\mathcal{TC}]}$ is almost 1 dB. ■

For punctured codes, we may assume that punctured bits are sent to a dummy memoryless component channel whose output is independent of the input, i.e., $\gamma_D = 1$, whereas, non-punctured bits are transmitted over the real channel with the Bhattacharyya noise parameters γ. By assuming the puncturing rate $1-\tau$, the average Bhattacharyya noise parameters is $\overline{\gamma} = \gamma \cdot \tau + 1 \cdot (1-\tau)$. Hence, Theorem 2 implies the following result for a (randomly) punctured code ensemble. If an ensemble of good codes are randomly punctured at a rate $1-\tau$ then there exists a punctured code threshold

$$\chi^{[\mathcal{C}]}(\tau) = \ln \frac{\tau}{\exp(-c_\star^{[\mathcal{C}]}) - (1-\tau)} \quad \text{for } \tau > 1 - \exp(-c_\star^{[\mathcal{C}]}) \tag{4}$$

such that, if the Bhattacharyya distance $-\ln\gamma$ (i.e., the received SNR at an AWGN channel) is larger than $\chi^{[\mathcal{C}]}(\tau)$, the average ML decoding word error probability approaches zero as $N \to \infty$.

B. Code Outage for a Block Fading Gaussian Channel

In the case of a Q-block fading Gaussian channel, the fading coefficient is essentially invariant during a single block and different from one block to another. For the j-th block, the Bhattacharyya noise parameter γ_j is a function of the average received SNR SNR_j and the channel power ν_j of the corresponding block, i.e., $\gamma_j = \exp(-\nu_j \cdot \mathsf{SNR}_j)$. Hence, the average Bhattacharyya noise parameter over Q blocks

$$\overline{\gamma}(\boldsymbol{\nu}) = \sum_{j=0}^{Q-1} \tau_j \gamma_j = \sum_{j=0}^{Q-1} \tau_j \exp(-\nu_j \cdot \mathsf{SNR}_j)$$

is a function of the random vector $\boldsymbol{\nu} \triangleq \{\nu_1, \ldots, \nu_Q\}$ and, thus, for a given good code, there is a non-negligible probability that the effective Bhattacharyya noise distance $-\ln\overline{\gamma}(\boldsymbol{\nu})$ is less then the code threshold $c_\star^{[\mathcal{C}]}$, termed *code outage* probability. Thus, the error probability is a function of both the fading distribution and the noise threshold of the code. More precisely, the average ML decoding word error probability for a good code ensemble $[\mathcal{C}]$ transmitted over a Q-block fading channel can be bounded as follows:

$$\begin{aligned} \overline{P}_W^{[\mathcal{C}]}(\overline{\gamma}) \triangleq \mathrm{E}\big[P_W^{[\mathcal{C}]}(\overline{\gamma})\big] &= \mathrm{P}\{\mathrm{error}(N),\, -\ln\overline{\gamma}(\boldsymbol{\nu}) \le c_\star^{[\mathcal{C}]}\} + \mathrm{P}\{\mathrm{error}(N),\, -\ln\overline{\gamma}(\boldsymbol{\nu}) > c_\star^{[\mathcal{C}]}\} \\ &\le \mathrm{P}\{-\ln\overline{\gamma}(\boldsymbol{\nu}) \le c_\star^{[\mathcal{C}]}\} + o(1). \end{aligned} \tag{5}$$

The second term approaches zero with the code length N. We will omit $o(1)$ in the further analysis.

C. IR Cooperative Turbo Coding Performance

Now we study the FER performance of the IR cooperative coding scheme for a quasi-static frequency-flat Rayleigh-fading channel.

In Slot 0, the (hop) sender (Node 0) broadcasts its information by sending the punctured codeword $\mathcal{C}_0$. The channel powers $\nu_{0,j}$, $j = 1, \ldots, M-1$, are i.i.d. exponential random variables invariant during each transmission period. Thus, the reliable set $\mathcal{F}$ is randomly distributed over the *collection* of 2^{M-1} subsets of $\mathcal{S}$ with the probability

$$\begin{aligned} \mathrm{P}(\mathcal{F}) &= \prod_{j\in\mathcal{F}} \mathrm{P}\{\theta_{0,j} > \chi^{[\mathcal{C}]}(\tau_0)\} \prod_{j\in\mathcal{F}^c} \mathrm{P}\{\theta_{0,j} \le \chi^{[\mathcal{C}]}(\tau_0)\} \\ &= \prod_{j\in\mathcal{F}} \exp\bigl[-\chi^{[\mathcal{C}]}(\tau_0)\mathsf{SNR}_{0,j}^{-1}\bigr] \prod_{j\in\mathcal{F}^c} \bigl\{1 - \exp\bigl[-\chi^{[\mathcal{C}]}(\tau_0)\mathsf{SNR}_{0,j}^{-1}\bigr]\bigr\}, \end{aligned} \tag{6}$$

where $\mathcal{F}^c \triangleq \mathcal{S} \setminus \mathcal{F}$.

For a given $\mathcal{F}$, the IR cooperation scheme allows the mother codeword to be transmitted to the destination (Node M) over M parallel channels (slots) associated with $|\mathcal{F}| + 1$ independent quasi-static fading gains. Hence, the Bhattacharyya noise parameter of Channel i is

$$\gamma_i = \begin{cases} \exp(-\theta_{i,M}) & i \in \mathcal{F}, \\ \exp(-\theta_{0,M}) & i \in \mathcal{F}^c \cup \{0\}. \end{cases} \tag{7}$$

Consequently, the Bhattacharyya noise parameter averaged over M parallel channels is now

$$\begin{aligned} \overline{\gamma}(\boldsymbol{\nu}, \mathcal{F}) &= \bigl(1 - \sum_{i\in\mathcal{F}} \tau_i\bigr) \exp(-\theta_{0,M}) + \sum_{i\in\mathcal{F}} \tau_i \exp(-\theta_{i,M}) \\ &= \bigl(1 - \sum_{i\in\mathcal{F}} \tau_i\bigr) \exp(-\nu_{0,M}\mathsf{SNR}_{0,M}) + \sum_{i\in\mathcal{F}} \tau_i \exp(-\nu_{i,M}\mathsf{SNR}_{i,M}), \end{aligned} \tag{8}$$

where $\boldsymbol{\nu} = \{\nu_{0,M}, \nu_{1,M} \ldots, \nu_{M-1,M}\}$ is a random vector M-tuple with the exponential distribution. Inequality (5) implies that the conditional average word error probability given a reliable set $\mathcal{F}$ can be bounded as follows:

$$\overline{P}_W^{[\mathcal{C}]}\bigl(\overline{\gamma} \mid \mathcal{F}\bigr) \le \mathrm{P}\{-\ln \overline{\gamma}(\boldsymbol{\nu}, \mathcal{F}) \le c_\star^{[\mathcal{C}]}\} = \int_{\mathcal{A}} \prod_{i=0}^{M-1} e^{-\nu_{i,M}}\, d\boldsymbol{\nu} \triangleq \mathcal{G}(M, \mathcal{F}, \mathbf{SNR}) \tag{9}$$

where $\mathcal{A} \triangleq \bigl\{\boldsymbol{\nu} :\ \overline{\gamma}(\boldsymbol{\nu}, \mathcal{F}) \ge \exp(-c_\star^{[\mathcal{C}]})\bigr\}$ and $\mathbf{SNR} = \{\mathsf{SNR}_{0,M}, \ldots, \mathsf{SNR}_{M-1,M}\}$. Thus, the ML decoding FER for cooperative turbo coding scheme averaged over all possible reliable sets can be bounded as

$$\mathsf{FER}^{(M)} = \sum_{\text{all possible } \mathcal{F}} \mathrm{P}(\mathcal{F}) \overline{P}_W^{[\mathcal{C}]}\bigl(\overline{\gamma} \mid \mathcal{F}\bigr) \le \sum_{\text{all possible } \mathcal{F}} \mathrm{P}(\mathcal{F}) \cdot \mathcal{G}(M, \mathcal{F}, \mathbf{SNR}). \tag{10}$$

Example 2 ($M=1$) The case $M=1$ is equivalent to the traditional non-cooperation transmission, and the signal-hop FER is bounded as

$$\mathsf{FER}^{(1)} \leq \mathcal{G}(1,\emptyset,\mathsf{SNR}_{0,1}) = \mathrm{P}\{\nu_{0,1}\mathsf{SNR}_{0,1} \leq c_\star^{[\mathcal{C}]}\} = 1-\exp(-c_\star^{[\mathcal{C}]}\mathsf{SNR}_{0,1}). \tag{11}$$

Example 3 ($M=2$) For the two-node cooperation case, the scheme FER can be written as

$$\begin{aligned}\mathsf{FER}^{(2)} &\leq \mathrm{P}(\mathcal{F}=\emptyset)\overline{P}_W^{[\mathcal{C}]}\big(\overline{\gamma}\mid\emptyset\big)+\mathrm{P}(\mathcal{F}=\{1\})\overline{P}_W^{[\mathcal{C}]}\big(\overline{\gamma}\mid\{1\}\big)\\ &= \Big[1-e^{-\chi^{[\mathcal{C}]}(\tau)\mathsf{SNR}_{0,1}^{-1}}\Big]\,\mathcal{G}(2,\emptyset,\mathbf{SNR})+e^{-\chi^{[\mathcal{C}]}(\tau)\mathsf{SNR}_{0,1}^{-1}}\,\mathcal{G}(2,\{1\},\mathbf{SNR}),\end{aligned} \tag{12}$$

where (assuming $\tau_0,\tau_1 \leq \exp(-c_\star^{[\mathcal{C}]})$)

$$\mathcal{G}(2,\emptyset,\mathbf{SNR}) = 1-\exp(-c_\star^{[\mathcal{C}]}\mathsf{SNR}_{0,2}^{-1}) \tag{13}$$

$$\mathcal{G}(2,\{1\},\mathbf{SNR}) = 1-\omega-\int_\omega^1\left[\frac{\exp(-c_\star^{[\mathcal{C}]})-\tau_1 x^{\mathsf{SNR}_{0,2}}}{\tau_0}\right]^{1/\mathsf{SNR}_{1,2}}dx, \tag{14}$$

and $\omega=\exp\big[-\chi^{[\mathcal{C}]}(\tau_0)\cdot\mathsf{SNR}_{0,2}^{-1}\big]$.

In general, (10) cannot be calculated in a closed form and one needs to resort to numerical integration methods.

IV. Asymptotic Analysis

In this section we consider several different cooperation scenarios and derive asymptotic (in large SNR) FER bounds, which have a closed form. For simplicity we guarantee that each punctured code can be self-decodable by assuming

$$\tau_0,\ldots,\tau_{M-1} > 1-\exp(c_\star^{[\mathcal{C}]}). \tag{15}$$

Next, we refer to $r=\max[d_{0,1},\cdots,d_{0,M-1}]$ as the sender-to-cluster distance, $d=d_{0,M}$ as the (non-cooperation) hop distance, and $D=\max[d_{1,M},\cdots,d_{M-1,M}]$ as the cluster-to-destination distance as shown in Fig 1. Similarly, we define the sender-to-cluster SNR, the (non-cooperation) single-hop SNR, and the cluster-to-destination SNR as

$$\rho \triangleq E\cdot r^{-L},\quad \eta = E\cdot d^{-L},\quad \text{and}\quad \lambda \triangleq E\cdot D^{-L}. \tag{16}$$

Our analysis is based on the following theorem

Theorem 3 Let $\phi_1(\lambda_1), \ldots, \phi_U(\lambda_U)$ be U independent random variables with the property that:

$$\lim_{\lambda_m \to \infty} \lambda_m \, \mathrm{P}\big[\phi_m(\lambda_m) > c\big] = -\ln c,$$
$$\text{and} \quad 0 \le \phi_m(\lambda_m) \le 1 \quad \text{for} \quad m = 1, \ldots, U.$$

Then, for $\tau, \ldots, \tau_U > 1 - c$ and $\sum_{m=1}^{U} \tau_m = 1$,

$$\limsup_{\{\lambda_1, \ldots, \lambda_U\} \to \infty} \prod_{m=1}^{U} \lambda_m \mathrm{P}\left[\sum_{m=1}^{U} \tau_m \phi_m(\lambda_m) > c\right] \le \frac{1}{U!} \prod_{m=1}^{U} \ln \frac{\tau_m}{c - (1 - \tau_m)}, \tag{17}$$

where $\{\lambda_1, \ldots, \lambda_U\} \to \infty$ means $\lambda_1 \to \infty, \ldots, \lambda_U \to \infty$.

Proof: The proof is in the Appendix C. ■

A. Cooperative Transmission

In the *cooperative transmission* scenario we assume that $M - 1$ cluster members are very close to the (hop) sender such that $r \to 0$. For this setting, we call the within-cluster channel as *perfect* and $\mathrm{P}(\mathcal{F} = \{1, \ldots, M-1\}) = 1$. Thus, the cooperation scheme FER can be written as

$$\begin{aligned}
\mathsf{FER}_{\mathrm{T}}^{(M)} &= \overline{P}_W^{[\mathcal{C}]}\big(\overline{\gamma} \mid \mathcal{F} = \{1, \ldots, M-1\}\big) \\
&\le \mathrm{P}\{-\ln \overline{\gamma}(\boldsymbol{\nu}, \{1, \ldots, M-1\}) \le c_\star^{[\mathcal{C}]}\} \\
&= \mathcal{G}(M, \mathcal{F} = \{1, \ldots, M-1\}, \mathbf{SNR} = \{E d_{i,M}^{-L}\})
\end{aligned} \tag{18}$$

Now, let's consider the large SNR case. Note that

$$\begin{aligned}
&\lim_{\mathsf{SNR}_{i,M} \to \infty} \mathsf{SNR}_{i,M} \, \mathrm{P}\Big[\exp(-\nu_{i,M} \mathsf{SNR}_{i,M}) \ge \exp(-c_\star^{[\mathcal{C}]})\Big] \\
=& \lim_{\mathsf{SNR}_{i,M} \to \infty} \frac{1 - \exp(-c_\star^{[\mathcal{C}]} \mathsf{SNR}_{i,M}^{-1})}{\mathsf{SNR}_{i,M}^{-1}} \\
=& c_\star^{[\mathcal{C}]}.
\end{aligned} \tag{19}$$

Thus, (4), (9), (15), and Theorem 3 imply

$$\limsup_{\mathbf{SNR} \to \infty} \prod_{i=0}^{M-1} \mathsf{SNR}_{i,M} \cdot \mathcal{G}(M, \{1, \ldots, M-1\}, \mathbf{SNR}) \le \frac{1}{M!} \prod_{i=0}^{M-1} \chi^{[\mathcal{C}]}(\tau_i). \tag{20}$$

For large enough E, we can rewrite (18) as

$$\begin{aligned}\mathsf{FER}_{\mathrm{T}}^{(M)} &\leq_E \frac{1}{M!}\prod_{i=0}^{M-1}\chi^{[\mathcal{C}]}(\tau_i)\mathsf{SNR}_{i,M}^{-1} &(21)\\ &= \frac{1}{E^M M!}\prod_{i=0}^{M-1}\chi^{[\mathcal{C}]}(\tau_i)d_{i,M}^{L} &(22)\\ &= \frac{d^{ML}}{E^M M!}\prod_{i=0}^{M-1}\chi^{[\mathcal{C}]}(\tau_i) &(23)\end{aligned}$$

where $\leq_E$ means that the inequality holds for sufficiently large E, and the last step is based on the triangle inequality $d - r \leq d_{i,M} \leq d + r$ and $r \to 0$.

B. Cooperative Reception

In the *cooperative reception* scenario we assume that $M - 1$ cluster members are very close to the destination such that $D \to 0$. Hence, (15) implies that

$$\mathcal{G}(M, \mathcal{F}, \mathbf{SNR}) = 0 \quad \text{for } \mathcal{F} \neq \emptyset. \tag{24}$$

Therefore, we can bound the cooperation scheme FER as

$$\begin{aligned}\mathsf{FER}_{\mathrm{R}}^{(M)} &= \mathrm{P}(\mathcal{F} = \emptyset)\overline{P}_W^{[\mathcal{C}]}\big(\overline{\gamma} \mid \mathcal{F} = \emptyset\big)\\ &\leq \mathrm{P}(\mathcal{F} = \emptyset)\mathcal{G}(M, \mathcal{F} = \emptyset, \mathbf{SNR})\\ &= \prod_{j=1}^{M-1}\left\{1 - \exp\left[-\chi^{[\mathcal{C}]}(\tau_0)\mathsf{SNR}_{0,j}^{-1}\right]\right\}\left[1 - \exp(-c_\star^{[\mathcal{C}]}\eta^{-1})\right] &(25)\\ &= \prod_{j=1}^{M-1}\left\{1 - \exp\left[-\chi^{[\mathcal{C}]}(\tau_0)E^{-1}d_{0,j}^{L}\right]\right\}\left[1 - \exp(-c_\star^{[\mathcal{C}]}E^{-1}d^{L})\right] &(26)\end{aligned}$$

Again, we focus on the large SNR case. Note that

$$\lim_{\mathsf{SNR}\to\infty}\mathsf{SNR}\cdot\left[1 - \exp(-a\mathsf{SNR}^{-1})\right] = a \text{ for } a > 0. \tag{27}$$

Thus, for large enough E, we can rewrite (26) as

$$\begin{aligned}\mathsf{FER}_{\mathrm{R}}^{(M)} &\leq_E \left[\chi^{[\mathcal{C}]}(\tau_0)\right]^{M-1}c_\star^{[\mathcal{C}]}\cdot\prod_{j=1}^{M}\mathsf{SNR}_{0,j}^{-1} &(28)\\ &= \frac{\left[\chi^{[\mathcal{C}]}(\tau_0)\right]^{M-1}c_\star^{[\mathcal{C}]}}{E^M}\cdot\prod_{j=1}^{M}d_{0,j}^{L}\\ &= \frac{\left[\chi^{[\mathcal{C}]}(\tau_0)\right]^{M-1}c_\star^{[\mathcal{C}]}}{E^M}\cdot d^{ML} &(29)\end{aligned}$$

where the last step is due to the geometric property $d - D \leq d_{0,j} \leq d + D$ and $r \to 0$.

C. Cooperative Hopping

Here, we assume that $d_{0,1} = \cdots = d_{0,M-1} = r > 0$ and $d_{1,M} = \cdots = d_{M-1,M} = D > 0$. Hence, the

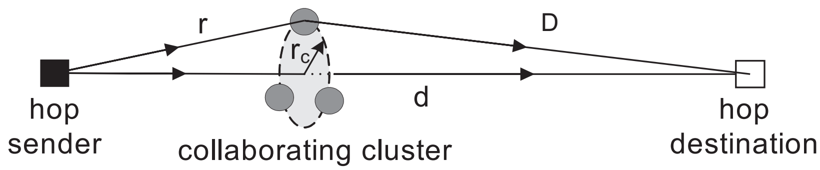


Fig. 4. geometric parameters for cooperative hopping

distances d, r, and D satisfy the triangle inequality as shown in Fig. 4, and, thus, in the high SNR regime we have

$$\begin{aligned}
&\lim_{\mathsf{SNR}\to\infty} \prod_{j\in\mathcal{F}^c\setminus\{0\}} \mathsf{SNR}_{0,j}\,\mathrm{P}(\mathcal{F}) \\
&= \lim_{\rho\to\infty} \rho^{M-(|\mathcal{F}|+1)}\,\mathrm{P}(\mathcal{F}) \\
&= \lim_{\rho\to\infty} \left\{ \exp\left[-\chi^{[\mathcal{C}]}(\tau_0)\rho^{-1}\right] \right\}^{|\mathcal{F}|} \left\{ \frac{1-\exp(-\chi^{[\mathcal{C}]}(\tau_0)\rho^{-1})}{\rho^{-1}} \right\}^{M-(|\mathcal{F}|+1)} \\
&= [\chi^{[\mathcal{C}]}(\tau_0)]^{M-(|\mathcal{F}|+1)}. \qquad (30)
\end{aligned}$$

Moreover, (4), (9), (15), (19) and Theorem 3 imply

$$\begin{aligned}
&\limsup_{\mathsf{SNR}\to\infty} \prod_{i\in\mathcal{F}\cup\{0\}} \mathsf{SNR}_{i,M}\cdot\mathcal{G}(M,\mathcal{F},\mathbf{SNR}) \\
&= \limsup_{\lambda,\,\eta\to\infty} (\lambda^{|\mathcal{F}|}\,\eta)\cdot\mathcal{G}(M,\mathcal{F},\mathbf{SNR}) \\
&\le \frac{\chi^{[\mathcal{C}]}(1-\sum_{i\in\mathcal{F}}\tau_i)}{(|\mathcal{F}|+1)!}\prod_{i\in\mathcal{F}}\chi^{[\mathcal{C}]}(\tau_i). \qquad (31)
\end{aligned}$$

Hence, for large enough E, we can rewrite (26) as

$$\begin{aligned}
\mathsf{FER}^{(M)} &\le_E \sum_{\mathcal{F}} \left\{ \frac{[\chi^{[\mathcal{C}]}(\tau_0)]^{M-(|\mathcal{F}|+1)}\chi^{[\mathcal{C}]}(1-\sum_{i\in\mathcal{F}}\tau_i)}{(|\mathcal{F}|+1)!} \prod_{i\in\mathcal{F}}\chi^{[\mathcal{C}]}(\tau_i) \right\} (\rho^{|\mathcal{F}|+1-M}\lambda^{-|\mathcal{F}|}\,\eta^{-1}) \qquad (32) \\
&= \underbrace{\sum_{\mathcal{F}} \left\{ \frac{[\chi^{[\mathcal{C}]}(\tau_0)]^{M-(|\mathcal{F}|+1)}\chi^{[\mathcal{C}]}(1-\sum_{i\in\mathcal{F}}\tau_i)}{(|\mathcal{F}|+1)!} \prod_{i\in\mathcal{F}}\chi^{[\mathcal{C}]}(\tau_i) \right\}}_{\text{coding advantage}} \cdot \underbrace{(r^{M-|\mathcal{F}|-1}D^{|\mathcal{F}|}\,d)^L}_{\text{geometric distance profile}} \cdot E^{-M}. \qquad (33)
\end{aligned}$$

D. Diversity Gain

Following [19], the *diversity gain* is defined as

$$\mathsf{div} \triangleq \lim_{\mathsf{SNR}\to\infty} \frac{-\log \mathsf{FER}}{\log \mathsf{SNR}}. \tag{34}$$

Since our collaborating model is a distributed *multiple-input single-output* (MISO) system, the maximum achievable diversity gain is M. Equations (21), (28), and (32) illustrate that all of the three discussed scenarios: cooperative transmission, moderate reception, and cooperative hopping can achieve the *full* diversity gain, i.e., $\mathsf{div} = M$, in large SNR regime.

E. Cooperative Coding Gain

For small $c_\star^{[\mathcal{C}]}$, we can build the following simple relationship between the punctured code threshold and the mother code threshold $c_\star^{[\mathcal{C}]}$. Equation (4) implies

$$\lim_{c_\star^{[\mathcal{C}]}\to 0} \frac{\chi^{[\mathcal{C}]}(\tau)}{c_\star^{[\mathcal{C}]}} = \lim_{c_\star^{[\mathcal{C}]}\to 0} (c_\star^{[\mathcal{C}]})^{-1} \ln \frac{\tau}{\exp(-c_\star^{[\mathcal{C}]}) - (1-\tau)} = \frac{1}{\tau}. \tag{35}$$

Thus, we can rewrite (33) as

$$\mathsf{FER}^{(M)} \leq_{E, c_\star^{[\mathcal{C}]}} \sum_{\mathcal{F}} \left\{ \frac{\left(c_\star^{[\mathcal{C}]}\right)^M}{\tau_0^{M-(|\mathcal{F}|+1)}(1-\sum_{i\in\mathcal{F}}\tau_i)(\prod_{i\in\mathcal{F}}\tau_i)(|\mathcal{F}|+1)!} \right\} \cdot (r^{M-|\mathcal{F}|-1} D^{|\mathcal{F}|}\, d)^L \cdot E^{-M}. \tag{36}$$

where $\leq_{E,\, c_\star^{[\mathcal{C}]}}$ means that the inequality holds for sufficiently large E and small $c_\star^{[\mathcal{C}]}$.

Example 4 ($M = 1$ limiting case)

$$\mathsf{FER}^{(M=1)} \leq_E \frac{c_\star^{[\mathcal{C}]}}{\eta} = \frac{c_\star^{[\mathcal{C}]}}{E} d^L. \tag{37}$$

Example 5 ($M = 2$ limiting case)

$$\mathsf{FER}^{(M=2)} \leq_{E,\, c_\star^{[\mathcal{C}]}} \frac{\left(c_\star^{[\mathcal{C}]}\right)^2}{\tau_0\,\eta\,\rho} + \frac{\left(c_\star^{[\mathcal{C}]}\right)^2}{2\tau_0\tau_1\,\eta\,\lambda} = \left(\frac{c_\star^{[\mathcal{C}]}}{E}\right)^2 \left[\frac{(d\,r)^L}{\tau_0} + \frac{(d\,D)^L}{2\tau_0\tau_1}\right]. \tag{38}$$

Similarly, (23) and (29) can be rewritten as

$$\mathsf{FER}_{\mathrm{T}}^{(M)} \leq_{E,\, c_\star^{[\mathcal{C}]}} \frac{1}{M! \prod_{i=0}^{M-1} \tau_i} \left(\frac{c_\star^{[\mathcal{C}]} d^L}{E}\right)^M \tag{39}$$

$$\mathsf{FER}_{\mathrm{R}}^{(M)} \leq_{E,\, c_\star^{[\mathcal{C}]}} \tau_0^{-(M-1)} \left(\frac{c_\star^{[\mathcal{C}]} d^L}{E}\right)^M \tag{40}$$

In [20], the author defines the *cooperative coding gain* as

$$\mathsf{cop} \triangleq \lim_{\mathsf{SNR}\to\infty} \frac{\mathsf{FER}^{-1/\mathsf{div}}}{\mathsf{SNR}}. \tag{41}$$

Let $\eta = \mathsf{SNR}$, the coding gain of cooperative transmission, moderate reception, and cooperative hopping schemes satisfy

$$\lim_{c_\star^{[\mathcal{C}]}\to 0} c_\star^{[\mathcal{C}]} \cdot \mathsf{cop}_{\mathrm{T}}^{(M)} \geq \left(M! \prod_{i=0}^{M-1} \tau_i \right)^{1/M}, \tag{42}$$

$$\lim_{c_\star^{[\mathcal{C}]}\to 0} c_\star^{[\mathcal{C}]} \cdot \mathsf{cop}_{\mathrm{R}}^{(M)} \geq \tau_0^{(M-1)/M}, \tag{43}$$

$$\lim_{c_\star^{[\mathcal{C}]}\to 0} c_\star^{[\mathcal{C}]} \cdot \mathsf{cop}^{(M)} \geq \left\{ \sum_{\mathcal{F}} \frac{\left[(r/d)^{M-|\mathcal{F}|-1}(D/d)^{|\mathcal{F}|}\right]^L}{\tau_0^{M-(|\mathcal{F}|+1)}(1-\sum_{i\in\mathcal{F}}\tau_i)(\prod_{i\in\mathcal{F}}\tau_i)(|\mathcal{F}|+1)!} \right\}^{-1/M}. \tag{44}$$

Inequality (44) illustrates that, in general, the cooperative coding gain is a function of τ_i (a parameter of the cooperation scheme) and the geometric distance profile of the network.

V. Simulations and Discussions

A. IR Cooperative Turbo Coding

In this section, we study error performance of the cooperation scheme based on the mother turbo code described in Example 1. FER simulations consider binary antipodal signaling and an independent flat quasi-static Rayleigh fading. Each receiver has perfect channel state information and employs coherent detection. All receivers employ the multiple turbo decoder associated with a triangle iterative decoding algorithm [21].

Here we consider a $M = 5$ collaborating cluster and assume $\mathsf{SNR}_{0,1} = \cdots = \mathsf{SNR}_{0,4} = \rho$ and $\eta = \mathsf{SNR}_{0,5} = \cdots = \mathsf{SNR}_{4,5} = \lambda$. Thus, the FER performance of cooperative turbo codes is a function of both cluster-to-destination SNR ρ and sender-to-cluster SNR λ. Fig. 5 depicts the FER by fixing $\rho = -15,\ 0,\ 15$ dB and changing λ from -2 to 16 dB. On the other hand, in Fig. 6, we fix $\lambda = 2, 6$ dB and study the FER performance vs. sender-to-cluster SNR ρ. For these two cases, we compare the simulation result with the analytic upper bound (10) and the asymptotic bound (36). We observe that the upper bound (10) accurately predicts the coding performance and the asymptotic bound (36) converges to bound in (10) for medium and high SNR. This observation enables us to estimate the FER performance

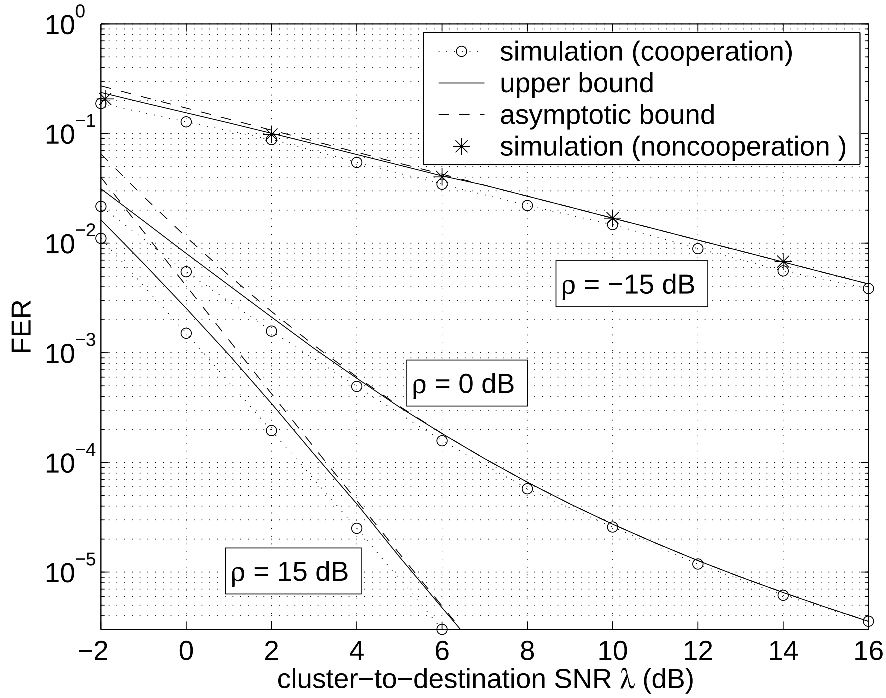


Fig. 5. FER vs. cluster-to-destination SNR ($M = 5$, within-cluster SNR $\rho = -15, 0, 15$, and mother turbo code of rate $R = 1/7$ and length $N = 5376$)

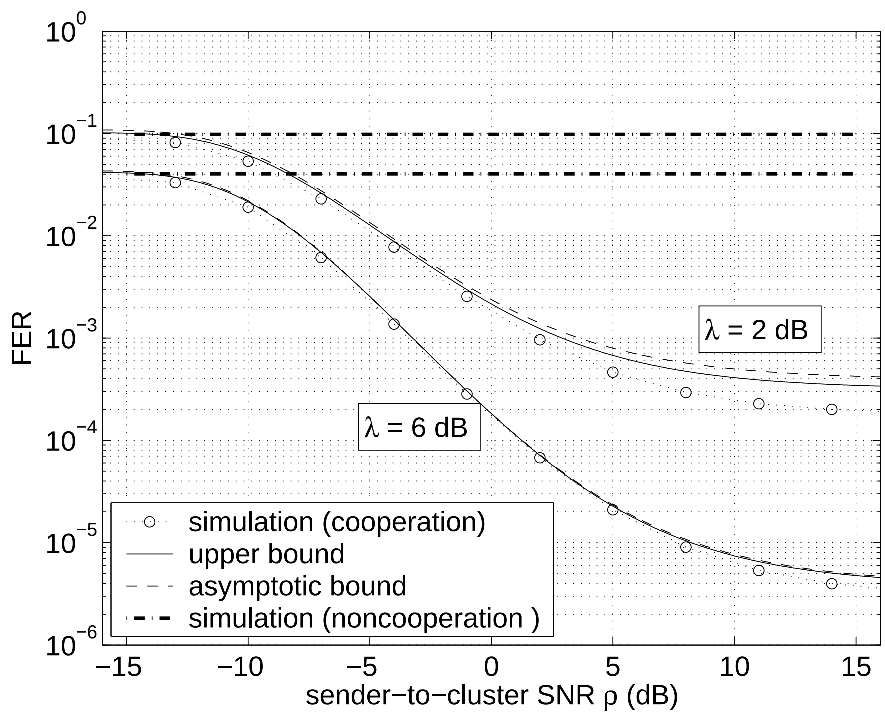


Fig. 6. FER performance vs. sender-to-cluster SNR ($M = 5$, cluster-to-destination SNR $\lambda = 2, 6$, and mother turbo code of rate $R = 1/7$ and length $N = 5376$)

as a function of ρ and λ by combining (10) and (36) in Fig. 7, where we use the bound (10) for low SNR and employ bound (36) to simplify the computation for medium and high SNR.

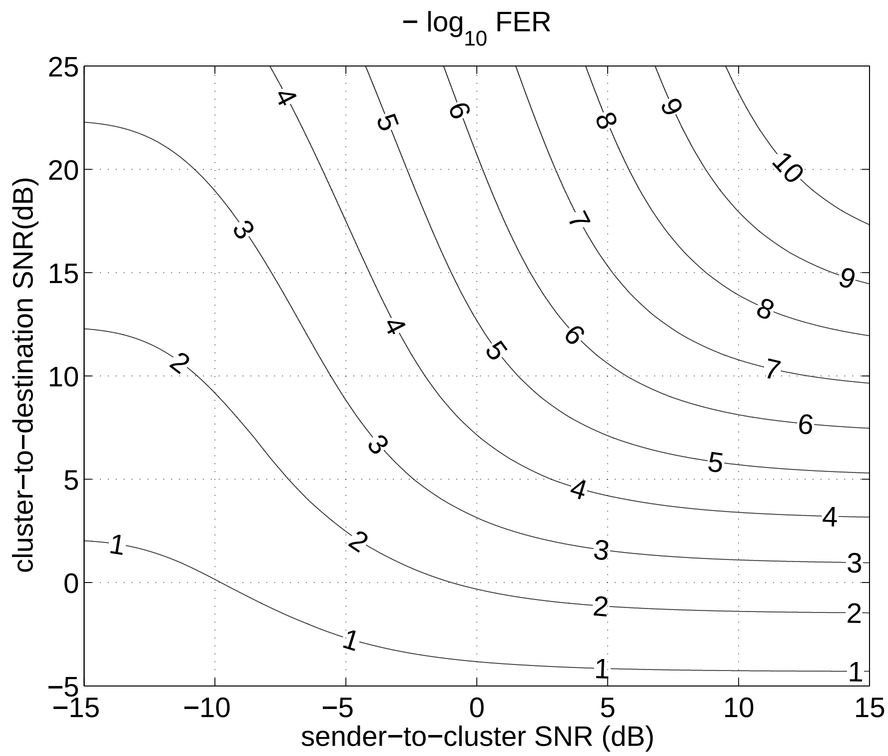


Fig. 7. FER performance as a function of within-cluster and cluster-to-destination SNRs ($M = 5$ and mother turbo code of rate $R = 1/7$)

B. Collaborating Cluster Size M

In this subsection we study the effect of the collaborating cluster size M on the performance of cooperative transmission. For most practical wireless networks, we assume that nodes have limited battery energy. Hence, achieving high transmission energy efficiency is more important that maximizing diversity gain. Our approach is to assume that the allowable FER is ϵ, which guarantees the *quality of service* (QoS), and to determine the ϵ-achievable transmission energy by applying the bounds described in Section IV. The closed form bound predict accurately the error performance for medium and high SNR.

Let $\tau_0 = \cdots = \tau_{M-1} = 1/M$, now, (39) implies

$$\mathsf{FER}_{\mathrm{T}}^{(M)} \leq_{E,\, c_\star^{[\mathcal{C}]}} \frac{1}{M!} \left(\frac{M \cdot c_\star^{[\mathcal{C}]} d^L}{E^{(M)}} \right)^M . \tag{45}$$

To satisfy the QoS requirement, we require[3]

$$\frac{1}{M!}\left(\frac{M\cdot c_{\star}^{[\mathcal{C}]}d^{L}}{E^{(M)}}\right)^{M}=\epsilon. \tag{46}$$

With the help of Stirling's approximation [22] and by (16), the transmission energy is

$$E^{(M)}\doteq\frac{c_{\star}^{[\mathcal{C}]}\cdot ed^{L}}{(\epsilon\sqrt{2\pi M})^{1/M}}. \tag{47}$$

To illustrate how much energy can be saved with cooperative transmission, we consider non-cooperation transmission and turbo codes transmitted over a fully interleaved fading channel cases as benchmarks. For non-cooperation transmission (i.e, $M=1$), without Stirling's approximation, ϵ-achievable energy is given by

$$E^{(1)}=\frac{c_{\star}^{[\mathcal{C}]}\cdot d^{L}}{\epsilon}. \tag{48}$$

For a fully interleaved fading channel, we can equivalently consider the error performance analysis for a M-block fading channel with $M\to\infty$. In this case, the strong law of large numbers implies that

$$\begin{aligned}\overline{\gamma}&\triangleq\frac{1}{M}\sum_{j=1}^{M}\gamma_{j}\\&=\mathrm{E}(\gamma)\qquad w.p.\ 1\\&=\int_{0}^{\infty}e^{-\eta\nu}e^{-\nu}\,d\nu=\frac{1}{1+\eta}\end{aligned} \tag{49}$$

which is exactly the Bhattacharyya noise parameter for a fully interleaved fading channel [11]. Theorem 2 illustrates that, the word error probability $P_{W}^{[\mathcal{C}]}(\gamma)\xrightarrow{N}0$ if

$$\overline{\gamma}=\frac{1}{1+\eta}<\exp(-c_{\star}^{[\mathcal{C}]}).$$

Thus, in this case, the reliable transmission energy is

$$E^{(\infty)}=\left[\exp(c_{\star}^{[\mathcal{C}]})-1\right]\cdot d^{L}<c_{\star}^{[\mathcal{C}]}\cdot d^{L} \tag{50}$$

[3]Strictly speaking, the bounds (36), (39), and (40) are based on the large SNR assumption. However, through simulations, we observe that the asymptotic bounds also works well for the medium SNR. On the other hand, these asymptotic bounds can be expressed in a closed form, whereas, the calculation for the bound (10) must be resorted to numerical integration method. Thus, here and hereafter, we use these asymptotic bound to estimate the FER performance.

TABLE I

ENERGY SAVING VS. M

M	2	3	4	5	∞
Estimated $U^{(M)}$ (dB)	8.4	11.1	12.4	13.2	< 20

which indicates the limiting case in the sense of $M \to \infty$. Let $U^{(M)} \triangleq E^{(1)}/E^{(M)}$ denote the transmission energy saving. Equations (47) and (48) lead to

$$U^{(M)} \doteq \frac{(2\pi M)^{1/2M}}{e \cdot \epsilon^{1-1/M}} \tag{51}$$

which illustrates the fact that the energy saving is a function of only M and ϵ, and does not depend on the good code. Furthermore, $U^{(M)}$ increases with M, however, the increase is very slow for large M. For example, let $\epsilon = 0.01$ and consider the turbo codes described in Example 1. We numerically compute the estimated energy saving from (51) in Table I and compare it with the fully interleaved fading channel saving $U^{(\infty)} = E^{(1)}/E^{(\infty)} < 1/\epsilon$. Fig. 8 illustrates the simulated FER performance versus transmission energy E as well as the upper bound (10) and its asymptotic version (36) for $d = 1$ and $r \to 0$. Furthermore,

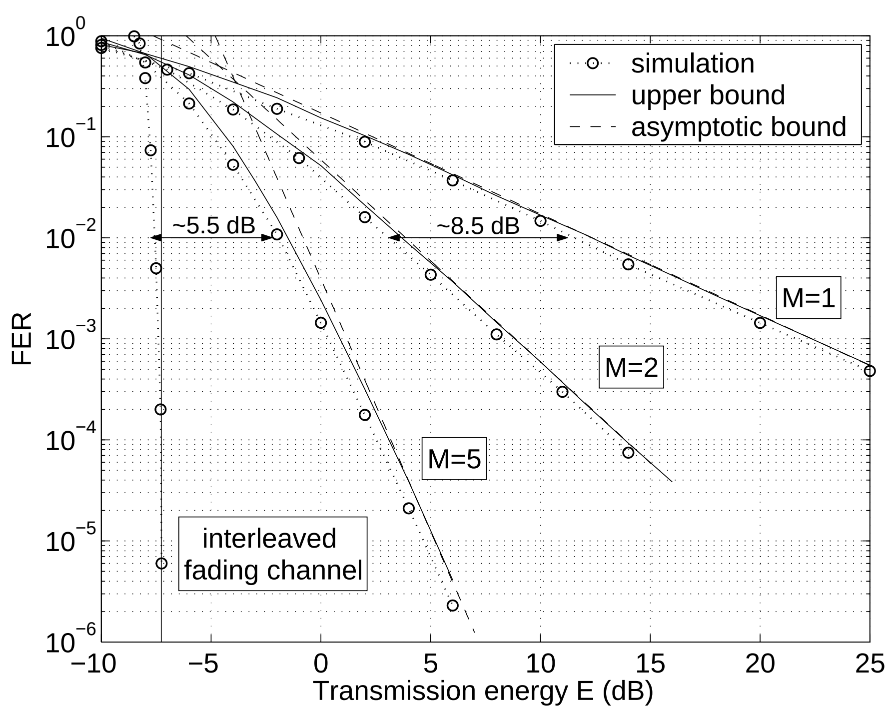


Fig. 8. FER vs. transmission energy for perfect within-cluster channel (D=1, $\mathrm{P}(|\mathcal{F}| = M-1) = 1$, and mother turbo code of rate $R = 1/7$ and length $N = 5376$)

in Fig. 8, we compare the FER performance of cooperative transmissions vs. transmission over fully a interleaved fading channel. in the latter case the error performance exhibits a threshold behavior and the reliable transmission energy is described in (50). Both Table I and Fig. 8 illustrate the fact that, although the cumulative energy saving significantly increases with M, the rate of increase drops quickly.

C. Normalized Cluster Distances

Here, we assume $d_{0,1},\ldots,d_{0,M-1}=r$, $d_{1,M}=\cdots=d_{M-1,M}=D$, $d\approx r+D$ and $\tau_0=\cdots=\tau_{M-1}=1/M$. We move the collaborating cluster from the hop sender towards the hop destination, and evaluate the energy saving relative a non-cooperative hop as a function of the normalized cluster distance r/d . Again, we consider the ϵ-achievable energy based on the bound (36). This bound implies that

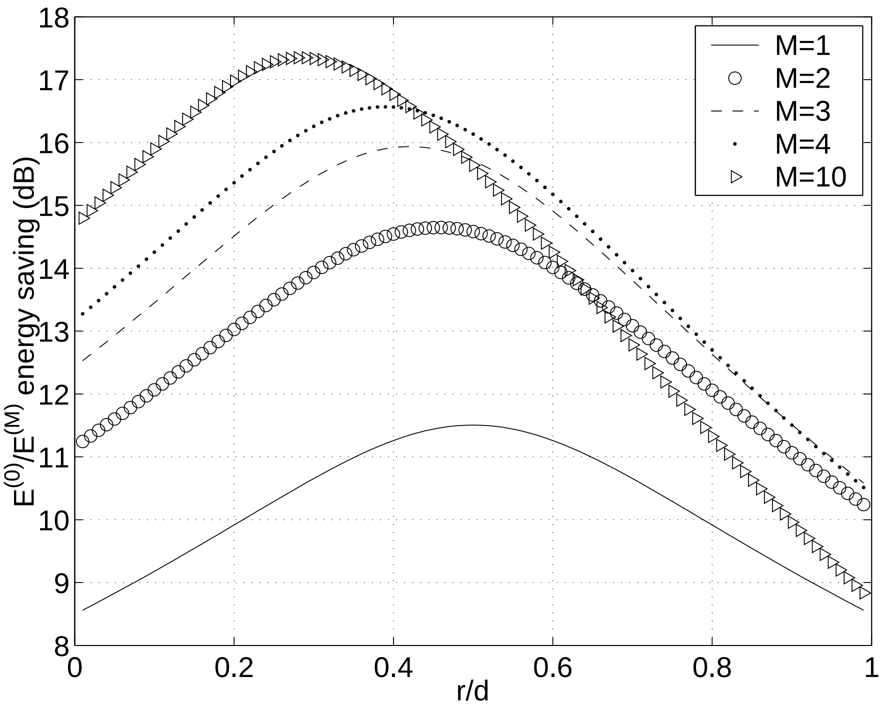


Fig. 9. ϵ-achievable transmission energy saving vs. normalized cluster distance r/d (the required FER $\epsilon=0.01$, path loss exponent $L=3$)

$$\sum_{k=0}^{M-1}\binom{M-1}{k}\frac{\left(Mc_\star^{[\mathcal{C}]}\right)^M}{(M-k)(k+1)!}\cdot\left[(r/d)^{M-k-1}(D/d)^k\right]^L\cdot(E^{(M)}d^{-L})^{-M}=\epsilon, \tag{52}$$

and

$$U^M=\sqrt{\frac{E^{(1)}}{E^{(M)}}}=M^{-1}\left\{\epsilon^{M-1}\sum_{k=0}^{M-1}\binom{M-1}{k}\frac{1}{(M-k)(k+1)!}\left[(r/d)^{M-k-1}(1-r/d)^k\right]^L\right\}^{-1/M} \tag{53}$$

The energy saving is depicted in Fig. 9 for $M = 2, \ldots, 5$ and $0 < r/d < 1$, where we assume $\epsilon = 0.01$, path loss exponent $L = 3$, and a rate $R = 1/7$ mother turbo code described in Example 1.

APPENDIX

A. Modified Shulman-Feder Reliable Channel Region

In this appendix, we shall state for reference the MSF reliable channel regions for good binary codes transmitted over parallel channels [3].

Theorem 4 (cf. [3, Theorem 5]) Let's consider a good code ensemble $[\mathcal{C}]$ of rate R transmitted over Q binary-input symmetric-output parallel channels with a set of mutual information $\{I_q\}$, Bhattacharyya noise parameters $\{\gamma_q\}$, and assignment rate $\{\tau_q\}$, if

$$\overline{\gamma} < \exp\left(-c_P^{[\mathcal{C}]}\right) \quad \text{and} \quad \overline{I} > R + \xi_P^{[\mathcal{C}]} \tag{54}$$

where

$$\overline{I} = \sum_{q=1}^{Q} \tau_q I_q \quad \text{and} \quad \overline{\gamma} = \sum_{q=1}^{Q} \tau_q I_q$$

denote the average mutual information and Bhattacharyya noise parameter of the Q parallel channels, then, the average ML decoding word error probability decays to zero as the codeword length approaches infinity.

B. Proof of Theorem 2

Proposition 1

$$\ln(1+x) < \frac{2x}{1+x} \quad \text{for } x < 1 \quad \text{and} \quad \ln(1+x) > \frac{2x}{1+x} \quad \text{for } x \geq 1. \tag{55}$$

Proof: [Proposition 1] Consider

$$g(x) = \ln(1+x) - \frac{2x}{1+x}.$$

We have $g(x) = 0$ for $x = 1$, and

$$g'(x) = \frac{x-1}{(1+x)^2}. \tag{56}$$

Therefore, (55) holds. ∎

Lemma 1 For a binary-input AWGN channel with received SNR λ, the Bhattacharyya noise parameter γ and channel mutual information I satisfy

$$\gamma(\lambda) \geq 1 - I(\lambda) \tag{57}$$

where

$$\gamma(\lambda) = e^{-\lambda}$$
$$I(\lambda) = 1 - \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} e^{-(y-\sqrt{\lambda})^2} \log_2\Big(1 + e^{-4y\sqrt{\lambda}}\Big)\, dy \tag{58}$$

and the equality holds for $\lambda = 0$ or $\lambda \to \infty$.

Proof: [Lemma 1] It is easy to verify that the equality in (57) holds for $\lambda \to \infty$, since $\gamma(\lambda) \to 0$ and $I(\lambda) \to 1$ as $\lambda \to \infty$. Now, we only consider finite λ, i.e., $\gamma(\lambda) > 0$. Let $a = \sqrt{\lambda}$, and

$$h(a) = \frac{1 - I(a^2)}{\gamma(a^2)}. \tag{59}$$

Note that

$$h(a) = 1, \quad \text{for } a = 0. \tag{60}$$

Hence, it will be sufficient to prove that $h(a)$ is a decreasing function over $(0, \infty)$. following (58), $h(a)$ can be rewritten as

$$h(a) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} e^{-y^2+2ay} \log_2\Big(1 + e^{-4ay}\Big)\, dy \tag{61}$$

with

$$h'(a) = \frac{2\log_2 e}{\sqrt{\pi}} \int_{-\infty}^{+\infty} e^{-y^2+2ay} y \cdot \left[\ln\Big(1 + e^{-4ay}\Big) - \frac{2e^{-4ay}}{1 + e^{-4ay}}\right] dy. \tag{62}$$

Proposition 1 implies

$$y \cdot \left[\ln\Big(1 + e^{-4ay}\Big) - \frac{2e^{-4ay}}{1 + e^{-4ay}}\right] \leq 0 \quad \forall\ y \text{ and } a > 0$$

and yields $h'(a) \leq 0$ for $a > 0$. Combining with (60), we have $h(a) \leq 1$ for $a \geq 0$. Thus, we have the desired result (57). ■

Lemma 2 For a good code ensemble with rate R, the UB code threshold $c_0^{[\mathcal{C}]}$ satisfies

$$1 - \exp(-c_0^{[\mathcal{C}]}) \geq R \tag{63}$$

where $c_0^{[\mathcal{C}]}$ is defined in (2).

Proof: [Lemma 2] Proof by contradiction. We assume that (63) does not hold. Then, there exists a positive ϵ_0 such that

$$R > 1 - \exp(-c_0^{[\mathcal{C}]}) + \epsilon_0. \tag{64}$$

Let's consider a binary erasure channel (BEC) with erasure probability $p = \exp(-c_0^{[\mathcal{C}]}) - \epsilon_0$. Then, the channel capacity and Bhattacharyya noise parameter are

$$C(p) \triangleq 1 - p = 1 - \exp(-c_0^{[\mathcal{C}]}) + \epsilon_0 \quad \text{and} \quad \gamma(p) \triangleq p = \exp(-c_0^{[\mathcal{C}]}) - \epsilon_0$$

Since $\gamma(p) < \exp(-c_0^{[\mathcal{C}]})$, the decoding error probability

$$\lim_{N\to\infty} P_W^{[\mathcal{C}]}(\gamma) = 0. \tag{65}$$

By Shannon's channel coding theorem, (65) implies

$$C(p) = 1 - \exp(-c_0^{[\mathcal{C}]}) + \epsilon_0 \geq R,$$

which contradicts (64). ■

Now, we consider the proof of Theorem 2.

Proof: [Theorem 2] First, we prove the existence of $c_\star^{[\mathcal{C}]}$, i.e., that the set

$$\Omega \triangleq \left\{P:\ 1 - \exp(-c_P^{[\mathcal{C}]}) \geq R + \xi_P^{[\mathcal{C}]}\right\}$$

is not empty. Let's consider a particular weight partition P_0 such that $\Phi_{P_0}^{+} = \Phi$ and $\Phi_{P_0}^{-} = \emptyset$. Equation (1) and (3) imply $c_{P_0}^{[\mathcal{C}]} = c_0^{[\mathcal{C}]}$ and $\xi_{P_0}^{[\mathcal{C}]} = 0$. Moreover, by Lemma 2 lead to

$$1 - \exp(-c_{P_0}^{[\mathcal{C}]}) \geq R + \xi_{P_0}^{[\mathcal{C}]}.$$

Hence, the set $\Omega \neq \emptyset$.

Next, we prove, for parallel AWGN channels, that $P_W^{[\mathcal{C}]}(\overline{\gamma}) \xrightarrow{N} 0$ if

$$-\ln \overline{\gamma} > c_P^{[\mathcal{C}]} \quad \text{for any } P \in \Omega. \tag{66}$$

Note that, when (66) holds, we have

$$1 - \overline{\gamma} \geq 1 - \exp(-c_P^{[\mathcal{C}]}) \geq R + \xi_P^{[\mathcal{C}]} \quad \text{for any } P \in \Omega.$$

On the other hand, Lemma 1 implies

$$\overline{I} = \sum_{q=1}^{Q} \tau_q I_q \geq \sum_{q=1}^{Q} (1 - \tau_q \gamma_q) = 1 - \overline{\gamma}.$$

Therefore,

$$\overline{I} > R + \xi_P^{[\mathcal{C}]} \quad \text{for any } P \in \Omega.$$

By combining (66) and Theorem 4, we have the desired result $P_W^{[\mathcal{C}]}(\overline{\gamma}) \xrightarrow{N} 0$. ■

C. Proof of Theorem 3

Lemma 3 Let $\phi(\lambda_0)$ and $\xi(\lambda_1)$ be two independent random variable with the property that:

$$\lim_{\lambda_0\to\infty} \lambda_0\,\mathrm{P}[\phi(\lambda_0) > c] = f(c), \quad \limsup_{\lambda_1,\to\infty} \lambda_1\,\mathrm{P}[\xi(\lambda_1) > c] \le h(c), \quad \text{and} \quad 0 \le \phi(\lambda_0),\ \xi(\lambda_1) \le 1 \tag{67}$$

where $f(c)$ and $h(c)$ are monotone decreasing and integrable, and $f'(c)$ is integrable. Then, for $0 < 1-c < \tau,\, 1-\tau$,

$$\limsup_{\lambda_0,\lambda_1\to\infty} \lambda_0\lambda_1\,\mathrm{P}[\tau\phi(\lambda_0) + (1-\tau)\xi(\lambda_1) > c] \le -\int_q^1 h\Big(\frac{c-\tau z}{1-\tau}\Big) f'(z)dz \tag{68}$$

where $q = [c-(1-\tau)]/\tau$.

Proof: [Lemma 3] The outline of the proof is as follows: first, we let $\Phi = \{z_0, \dots, z_L\}$ for some finite L be any partition of the interval $[k(c),\, 1]$ with $z_0 = k(c)$ and $z_L = 1$. Next we obtain an outer bound on the event $\tau\phi(\lambda_0) + (1-\tau)\xi(\lambda_1) > c$ as

$$\{\tau\phi(\lambda_0) + (1-\tau)\xi(\lambda_1) > c\} \supseteq \bigcup_{i=1}^{L} \{z_{i-1} \le \phi(\lambda_0) < z_i\} \cap \Big\{\xi(\lambda_1) > \frac{c-\tau z_i}{1-\tau}\Big\} \tag{69}$$

Since $\phi(\lambda_0)$ and $\xi(\lambda_1)$ are independent, the upper bound becomes

$$\mathrm{P}\left[z_{i-1} \le \phi(\lambda_0) < z_i,\ \xi(\lambda_1) > \frac{c-\tau z_i}{1-\tau}\right] = \Big\{\mathrm{P}[\phi(\lambda_0) > z_{i-1}] - \mathrm{P}[\phi(\lambda_0) > z_i]\Big\}\,\mathrm{P}\left[\xi(\lambda_1) > \frac{c-\tau z_i}{1-\tau}\right] \tag{70}$$

Thus,

$$\begin{aligned}\limsup_{\lambda_0,\lambda_1\to\infty} \lambda_0\lambda_1\,\mathrm{P}[\tau\phi(\lambda_0) + (1-\tau)\xi(\lambda_1) > c] &\le \sum_{i=1}^{L} \limsup_{\lambda_0,\lambda_1\to\infty} \lambda_0\lambda_1\,\mathrm{P}\left[z_{i-1} \le \phi(\lambda_0) < z_i,\ \xi(\lambda_1) > \frac{c-\tau z_i}{1-\tau}\right] \\ &= \sum_{i=1}^{L} \big[f(z_{i-1}) - f(z_i)\big] h\Big(\frac{c-\tau z_i}{1-\tau}\Big). \end{aligned} \tag{71}$$

Note that (71) holds for all partitions Φ of the interval $[k(c),\, 1]$; and $f(c)$, $f'(c)$, and $h(c)$ are all integrable, the supremum of the right-hand side of (71) becomes the integral in (68). ■

Now, we consider the proof of Theorem 3.

Proof: [Theorem 3] Following the induction method, first we check the $U = 1$ case. Note that $\tau = 1$ in this case, thus, (17) is satisfied for $U = 1$.

Next we assume that (17) holds for $U = j-1$ and consider $U = j$. Let

$$\tau'_m = \frac{\tau_m}{1-\tau_j} \quad \text{for} \quad m = 1, \dots, j-1,$$
$$f(c) = -\ln c, \quad \text{and} \quad h(c) = \frac{1}{(j-1)!} \prod_{m=1}^{j-1} \ln \frac{\tau'_m}{c-(1-\tau'_m)}$$

Since $\tau'_m < 1-c$, $\sum_{m=1}^{j-1} \tau_m = 1$, and the induction hypothesis, we have

$$\limsup_{\{\lambda_1,\dots,\lambda_{j-1}\}\to\infty} \prod_{m=1}^{j-1} \lambda_m \cdot \mathrm{P}\left[\sum_{m=1}^{j-1} \tau'_m \phi_m(\lambda_m) > c\right] \le h(c). \tag{72}$$

Note that $f(c)$ and $h(c)$ are monotone decreasing and integrable, and $f'(c)$ is integrable. Then, by Lemma 3,

$$\begin{aligned}\limsup_{\{\lambda_1,\ldots,\lambda_j\}\to\infty} \prod_{m=1}^{j} \lambda_m \, \mathrm{P}\left[\sum_{m=1}^{j} \tau_m \phi_m(\lambda_m) > c\right] &= \limsup_{\{\lambda_1,\ldots,\lambda_j\}\to\infty} \prod_{m=1}^{j} \lambda_m \, \mathrm{P}\left[\tau_j\phi_j(\lambda_j) + (1-\tau_j)\sum_{m=1}^{j-1} \tau'_m \phi_m(\lambda_m) > c\right] \\ &\leq -\int_q^1 h\Big(\frac{c-\tau z}{1-\tau}\Big) f'(z)dz \\ &= \frac{1}{(j-1)!}\int_q^1 \frac{1}{z}\prod_{m=1}^{j-1} \ln \frac{\tau_m}{c - z\tau_j - (1-\tau_j-\tau_m)} dz \end{aligned} \tag{73}$$

where $q = [c-(1-\tau_j)]/\tau_j$. Since $-\ln z$ is convex, Jensen's inequality implies that

$$\begin{aligned}\ln \frac{\tau_m}{c - z\tau_j - (1-\tau_j-\tau_m)} &= -\ln\Big[\frac{z\tau_j - c + (1-\tau_j)}{1-c}\cdot\frac{\tau_m - (1-c)}{\tau_m} + \frac{\tau_j - z\tau_j}{1-c}\cdot 1\Big] \\ &\leq -\frac{z\tau_j - c + (1-\tau_j)}{1-c}\ln\frac{\tau_m-(1-c)}{\tau_m} - \frac{\tau_j - z\tau_j}{1-c}\ln 1 \\ &= \frac{z-q}{1-q}\ln\frac{\tau_m}{c-(1-\tau_m)} \qquad \text{for} \quad q \leq z \leq 1, \; 1 \leq m \leq j-1. \end{aligned} \tag{74}$$

Hence,

$$\int_q^1 \frac{1}{z}\prod_{m=1}^{j-1}\ln\frac{\tau_m}{c - z\tau_j - (1-\tau_j-\tau_m)}dz \leq \left[\prod_{m=1}^{j-1}\ln\frac{\tau_m}{c-(1-\tau_m)}\right]\int_q^1 \frac{1}{z}\left[\frac{z-q}{1-q}\right]^{j-1} dz. \tag{75}$$

Note that $1/z$ and $[(z-q)/(1-q)]^{j-1}$ are, respectively, monotonically decreasing and increasing in z. Chebyshev integral inequality [23] implies

$$\int_q^1 \frac{1}{z}\left[\frac{z-q}{1-q}\right]^{j-1} dz \leq \frac{1}{1-q}\int_q^1 \frac{1}{z}\,dz \int_q^1 \left[\frac{z-q}{1-q}\right]^{j-1} dz = \frac{\ln q}{j}. \tag{76}$$

Finally, by combining (73), (75), and (76), we have the desired result

$$\limsup_{\{\lambda_1,\ldots,\lambda_j\}\to\infty} \prod_{m=1}^{j} \lambda_m \, \mathrm{P}\left[\sum_{m=1}^{j} \tau_m \phi_m(\lambda_m) > c\right] \leq \frac{1}{j!}\left[\prod_{m=1}^{j}\ln\frac{\tau_m}{c-(1-\tau_m)}\right]. \tag{77}$$

Since both the base case and the inductive step satisfy (17), we conclude that (17) holds for all U. ■

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