Random Sensory Networks: A Delay Analysis

Cedric Florens, Masoud Sharif, and Robert J. McEliece, *Fellow, IEEE*

***Abstract*—A fundamental function performed by a sensory network is the retrieval of data gathered collectively by sensor nodes. The metrics that measure the efficiency of this data collection process are time and energy. In this paper, we study via simple discrete mathematical models, the statistics of the data collection time in sensory networks. Specifically, we analyze the average minimum delay in collecting randomly located/distributed sensors data for networks of various topologies when the number of nodes becomes large. Furthermore, we analyze the impact of various parameters such as size of packet, transmission range, and channel erasure probability on the optimal time performance. Our analysis applies to directional antenna systems as well as omnidirectional ones. This paper focuses on directional antenna systems and briefly presents results on omnidirectional antenna systems. Finally, a simple comparative analysis shows the respective advantages of the two systems.**

***Index Terms*—Broadcasting, data collection, delay, directional antenna, sensor networks.**

I. Introduction

RECENT technological advances in the very large scale integration (VLSI) field have contributed much to the development of microsensor systems. These combine various sensors, signal processing, data storage, wireless communication capabilities, and energy sources on a single chip. Such computational devices are referred to as sensor nodes and a collection of sensor nodes, possibly distributed over a wide area, connected through the wireless medium, form a sensory network. Applications for such networks are numerous and include environmental monitoring (seismic, meteorological) and military surveillance [1]. Sensory networks belong to the family of wireless *ad hoc* networks and as such lack an infrastructure present in traditional wireless networks such as cellular networks. In the very near future it is expected that these sensor networks autonomously extract information about their surroundings, performing basic collective processing, and transmit the collected data to the end user for further processing and analysis. It should be noted that in a sensory network while each node may be mobile, it is typically the case that once the target site of the particular sensing application is reached a semi-permanent stationary configuration is adopted for the purpose of gathering information.

In the field of general *ad hoc* networks and particularly sensory networks, research efforts focusing on design issues of the network communication architecture have been widespread. A detailed investigation of current protocol and algorithm proposals in the physical, data link, network, transport, and application layers are discussed for example in [2]. Technical issues and applications requirements to be dealt with by these protocols are multiple and often specific to the class of sensory networks. Among those, efficient management of energy budget is of paramount importance to the lifetime of the networks. Furthermore, depending on the application under consideration a tradeoff between data collection delay and energy consumption has to be achieved. Finally, the throughput of a sensory network is an important characteristic measure which is closely related to the delay of the data collection process. Theoretical results regarding capacity of general static *ad hoc* networks has appeared in [3]. Also relevant to our research is the so-called packet routing problem which consists in moving packets of data from one location to another as quickly as possible in a network and has been studied in conjunction with wireline and wireless network models (see, for example, [4]–[7]). In [8], the authors studied the problem in sensory networks of collecting sensors data at the network base station. They describe optimal strategies to perform data collection under various assumptions and derive corresponding time performances with respect to a simple discrete mathematical model for a sensor network. In this model, the amount of data accumulated at each sensor node (characterized by a number of unit data packets) after some given observation period is assumed finite and determined. In typical scenarios, however, the exact amount of data accumulated at each sensor node is unknown.

In this paper, we model the number of data packets as a random variable and analyze the delay (which is now a random variable) in collecting sensor data at the base station. More specifically, we derive the distribution and the expected value of the delay for a line network using the optimal scheduling. Furthermore, we look into the effect of various parameters including size of packet, transmission range, and channel erasure probability. We also propose a simple scheduling and anlayze its delay performance. Finally, we extend our result to more general topologies such as multiline netwroks.

This paper is organized as follows: In Section II, we present our sensory network model and recap results from [8] that will be used in the remainder of the paper. We present results on a line network in Section III. In Section IV, we expose our derivation regarding multiline networks. Finally, we give comparative results between directional and omnidirectional antenna systems in Section VI.

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C. Florens and R. J. McEliece are is with the Department of Electrical Engineering, California Institute of Technology, Pasadena, CA 91125 USA (e-mail: florens@systems.caltech.edu; rjm@systems.caltech.edu).

M. Sharif is with the Department of Electrical and Computer Engineering, Boston University, Boston, MA 02215 USA (e-mail: sharif@bu.edu).

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II. Model and Problem Statement

In this section, we describe the sensor network model on which the subsequent analysis is based and formulate our problem within the framework of this model. Furthermore, we briefly review results in [8] that are relevant to this study. As noted in the Introduction, in most sensing applications sensor nodes adopt a stationary configuration while information is being gathered. Correspondingly, our models will be static. In stationary state, after the nodes have organized themselves into a network, we distinguish between two phases of operation. In the first phase or observation phase, area monitoring results in an accumulation of data at each sensor node. In the second phase or data transfer, the collected data is transmitted to some processing center located within the sensor network (we refer to this node as the base station (BS) of the sensor network). In this paper, we investigate the efficiency limits with respect to time of such data transfers.

We define a sensor network as a collection of n identical nodes $\{N_1,\ldots,N_n\}$ and a BS. Each node N_i is associated with an integer ν_i that represents the number of data packets stored at this node at the end of the observation phase. The BS is also denoted by N_0. In this paper, we first study the data collection problem in line networks and then generalize the results to multiline networks and tree topologies. Here, a line network with n nodes refers to the case where the BS is at one side of the network and N_1 and N_n being the closest and farthest nodes to the BS, respectively.

We assume that transmission time is slotted with nodes are synchronized. Each transmission hop consumes one time slot (TS). In this paper, we assume that nodes are in a half-duplex mode, i.e., a node can only transmit or receive (and not both) one data packet per time slot. All the nodes including the BS have a common transmission range r. Inspired by the interference model of [3], we assume all nodes within distance $r(1+\delta)$ of a transmitting node with omnidirectional antenna cannot transmit or receive any other packet (otherwise collision happens) for some $\delta > 0$. This implies that simultaneous transmissions within the interference range is avoided. However, multiple transmissions may occur within the network in one TS under this interference model by virtue of spatial separation exploiting the path loss or directional antennas. In fact, directional antennas can be exploited to cause interference only in the transmitting direction.

Such a network may be represented by a weighted rooted graph $\{V,E,\boldsymbol{\nu_n}\}$ where $V=\{N_0,\ldots,N_n\}$, E denotes the set of links and $\boldsymbol{\nu_n}=(\nu_1,\ldots,\nu_n)$. In this graph model, the root represents the BS (N_0) and an edge represents an existing wireless connection between two sensor nodes, or a sensor node and the BS. In this paper, we mainly deal with the case of systems with directional antennas and single-hop transmissions such that $d \le r$ and $r(1+\delta) \le 2d$, and then generalize the results to the case where nodes equipped with omnidirectional antennas and can have multihop transmissions.

The *data collection problem* in a given sensory network is defined as the problem of routing all the data collected by the sensor nodes to the BS as efficiently as possible with respect to time and energy. The *data distribution problem*, on the other hand, is the problem of routing data to sensor nodes in a timely and energy-efficient manner. In the following work, we shall focus on the time efficiency alone of the data collection (or distribution) task.

In [9], an *optimal* strategy is proposed to minimize the data collection time when the transmission range is a single hop. Moreover, it is proved that for a one-sided line network of length[1] n in which the ith node has ν_i packets and is equipped with directional antennas, the minimum collection time of the packets at the BS, achieved by the proposed optimal strategy and denoted by $T_{\min}(\boldsymbol{\nu_n})$, is

$$T_{\min}(\boldsymbol{\nu_n}) = \max_{1\le i\le n-1}\left(i-1+\nu_i+2\sum_{j\ge i+1}^{n}\nu_j\right) \qquad (1)$$

where $\boldsymbol{\nu_n}=(\nu_1,\ldots,\nu_n)$ and it is assumed that each single-hop transmission only causes interference to the neighbor node of the destination (in the direction of the transmission). The optimal distribution algorithm is a greedy scheduling in which BS sends the packets intended for the farthest node first and then second farthest node, and so on. The factor of two in the summation of (1) is due to the interference constraint as the link between N_0 and N_1 and the link between N_1 and N_2 cannot be used simultaneously and therefore it takes two TS to send each packet to/from the BS to N_1.

Furthermore, it is proved that the distribution and collection problems are essentially the same and that the minimum data distribution time is the same as the minimum data collection time. It is worth mentioning that the optimal scheduling strategy of [9] assumes the full knowledge of all ν_i's and controls the scheduling of all transmissions (and therefore, collection and distribution problems become equivalent). Although these assumptions may be too restrictive in practice, the performance of this optimal algorithm can serve as the benchmark for performance comparison of different algorithms with less coordinations and less side information about the number of packets at all nodes (see Section III-E).

We illustrate the optimal schedules on the following example where $V=\{0,1,2,3,4,5,6,7\}, E=\{(i,i+1), 0\le i\le 6\}$, $\boldsymbol{\nu}=(2,0,0,0,3,0,1), d<r<2d, (1+\delta)r<2d$. In this example, there are six packets to be collected (or distributed) as shown in Fig. 1. The schedules of transmissions are drawn below the network for the distribution and collection tasks, respectively. Arrows represent a single data packet transmission from a node to its neighbor. Either way it is performed in 11 TS. In the distribution case, the BS strategy is as follows: send first data packets destined for the furthest node, then data packets for the second furthest one, and so on, as fast as possible while respecting the channel reuse constraints. Nodes between the BS and its destinations are required to forward packets as soon as they arrive (that is, in the TS following their arrival). For example, at TS 1, the packet destined for node 7 is transmitted by the BS to node 1, at TS 2 from node 1 to node 2, and so on, and arrives at TS 7. Note that the collection schedule is obtained from the distribution schedule by simple symmetry as

[1]By a line network, we mean one-sided line network. A line network where the BS is in the middle of the line, can be seen as a two-line network. Results for the case where we have more than one line is discussed in Section IV.

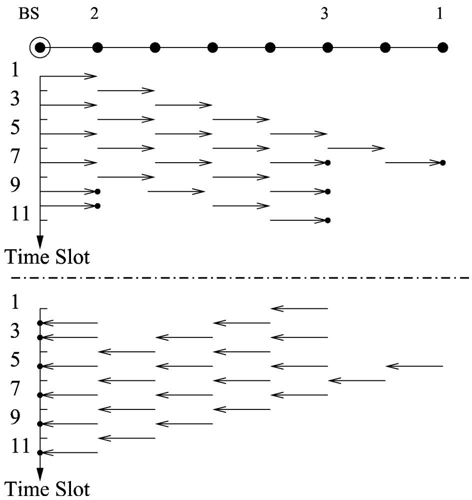

Fig. 1. eight-node line network ($\nu_1 = 2, \nu_2 = \nu_3 = \nu_4 = \nu_6 = 0, \nu_5 = 3$, $\nu_7 = 1$) followed by optimal transmission schedules for the distribution (upper schedule) and collection (lower schedule) problems. They are symmetric of one another. The job is performed in 11 TS.

shown in the next figure and *vice versa*. Such a procedure is always possible and in the rest of this paper, results will apply to either problems (i.e., collection/distribution), unless otherwise specified.

III. Random Line Networks

In this section, we characterize the delay in collecting random amount of data randomly spread over a sensor network after the observation phase. More specifically, for a one-sided line network, we first derive a recursion to compute the probability distribution function of $T_{\min}(\boldsymbol{\nu_n})$ and also we asymptotically analyze the average of $T_{\min}(\boldsymbol{\nu_n})$ when n is sufficiently large. Throughout this paper, we assume the number of packets at nodes are identically and independently distributed.

We further look into the delay when each node is allowed to transmit over $h > 1$ hops and also the effect of packet splitting on the delay in Subsections III-C and III-D. In Section III-E, we also propose a simple scheme that does not use the knowledge of the number of packets at other nodes and achieves the same scaling law for the average delay. Finally, in the last subsection, we consider the effect of error in the channel on the delay.

A. The Distribution of the Delay

In this subsection, we derive, by means of a recursion, the cumulative distribution function (CDF) of $T_{\min}(\boldsymbol{\nu_n})$ for a line network. Let us assume that ν_i corresponds to the number of packets at node i for $i = 1, \ldots, n$ and also ν_i's areindependent and identically distributed (i.i.d.) random variables chosen from the set $S_m = \{0, 1, \ldots, m-1\}$. It is clear that CDF of the minimum delay for a network with one node is just the distribution of the number of packets at node 1 as these packets can be transmitted one at a time to the BS. In the following theorem, we compute the CDF of the minimum delay for a network with n nodes using that of a network with $n-1$ nodes. This would enable us to compute the CDF for a network of general size numerically using a simple recursion.

Theorem 1: Let $F_n(t)$ be the CDF of the minimum delay $T_{\min}(\boldsymbol{\nu_n})$, i.e., $F_n(t) = \Pr\{T_{\min}(\boldsymbol{\nu_n}) \le t\}$. Then $F_n(t)$ satisfies the following recursion:

$$F_n(t) = \sum_{i=1}^{m-1} \Pr(\nu_n = i) F_{n-1}(t-2i) \mathbf{1}_{t \ge n+2(i-1)} + \Pr(\nu_n = 0) F_{n-1}(t), \quad \text{for } n \ge 2 \quad (2)$$

where

$$\mathbf{1}_{t \ge t_0} = \begin{cases} 1, & \text{if } t \ge t_0 \\ 0, & \text{otherwise} \end{cases} \quad \text{and}$$

$$F_1(t) = \begin{cases} \sum_{i=0}^{t} \Pr(\nu_1 = i), & \text{if } t < m-1 \\ 1, & \text{otherwise.} \end{cases}$$

Proof: In the proof, we consider the (equivalent) data distribution problem. We may write $F_n(t)$ by conditioning on $\nu_n = i$ for $i = 0, \ldots, m-1$ as

$$F_n(t) = \sum_{i=0}^{m-1} \Pr\{T_{\min}(\boldsymbol{\nu_n}) \le t \,|\, \nu_n = i\} \Pr(\nu_n = i). \quad (3)$$

To compute the conditional probability in (3), we use (1) and the fact that $T_{\min}(\boldsymbol{\nu_n}) \ge k - 1 + \nu_k + 2\sum_{j=k+1}^{n} \nu_j$ for all $k = 1, \ldots, n-1$. Therefore, replacing $k = n-1$ and assuming $\nu_n = i$, we get

$$T_{\min}(\boldsymbol{\nu_n}) \ge n - 2 + \nu_{n-1} + 2\nu_n \ge n + 2(i-1) \quad (4)$$

which basically implies that it takes at least $n + 2(i-1)$ TS to send i packets to the n's node. Thus, if $t < n + 2(i-1)$, then $\Pr\{T_{\min}(\boldsymbol{\nu_n}) \le t \,|\, \nu_n = i\} = 0$. Using the definition of the function $\mathbf{1}_{t \ge t_0}$, for any $i \ge 1$, and using the optimal greedy scheduling explained in Section II that first sends out all the i packets for the farthest node, we may then write the conditional probability as

$$\Pr\{T_{\min}(\boldsymbol{\nu_n}) \le t \,|\, \nu_n = i\} = \Pr\{T_{\min}(\boldsymbol{\nu_{n-1}}) \le t - 2i\} \mathbf{1}_{t \ge n+2(i-1)} \quad (5)$$

Replacing (5) in (3), we get

$$F_n(t) = F_{n-1}(t) \Pr(\nu_n = 0) + \sum_{i \ge 1}^{m-1} \Pr\{T_{\min}(\boldsymbol{\nu_{n-1}}) \le t - 2i)\} \cdot \mathbf{1}_{t \ge n+2(i-1)} \Pr(\nu_n = i)$$

which leads to (2). $\square$

We can use the result of Theorem 1 to compute the CDF of $T_{\min}(\boldsymbol{\nu_n})$. This is illustrated in Figs. 2 and 3. Fig. 2 shows the distribution of the delay $T_{\min}(\boldsymbol{\nu_n})$ for 40-sensor node line networks in which each node carries either 0 or 1 packet with probability $1/2$. Fig. 3 shows the distribution of the delay $T_{\min}(\boldsymbol{\nu_n})$ for 40-sensor node line networks in which each node carries either 0 or 1 packet with probability 0.8 and 0.2, respectively.

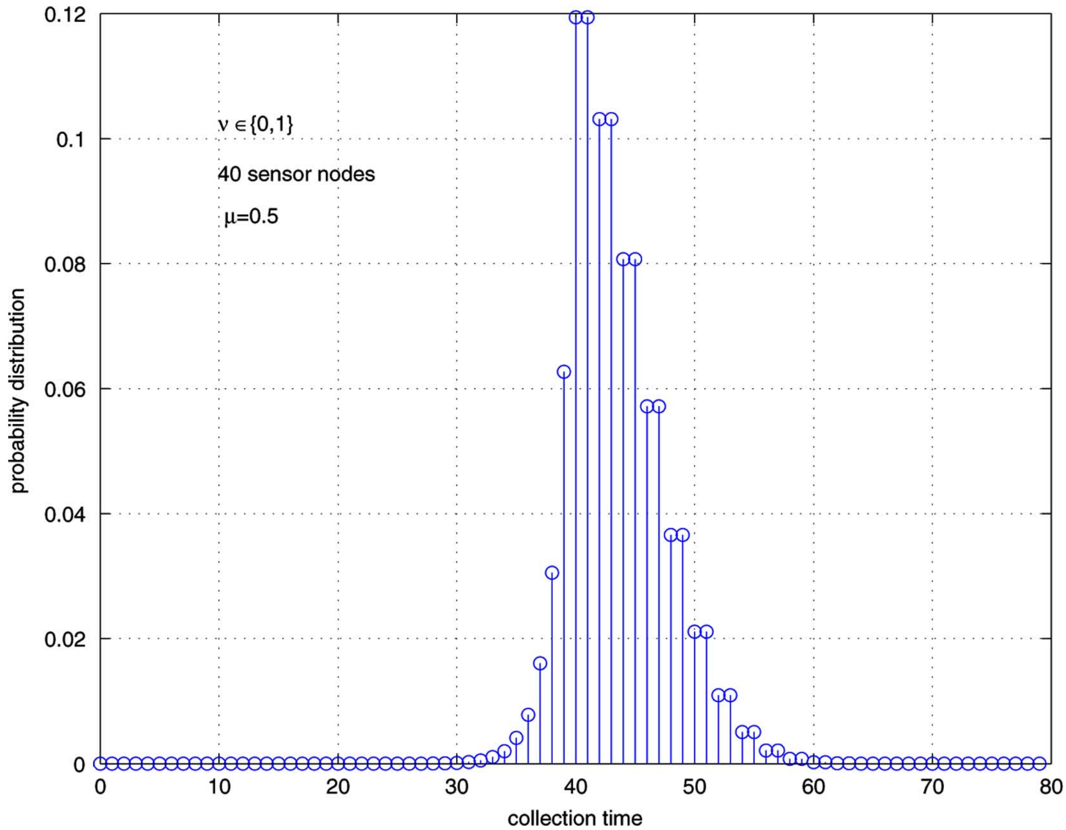

Fig. 2. Distribution of data collection time in 40-node line network. Each node in the particular network considered carries 0 or 1 data packet with probability $1/2$.

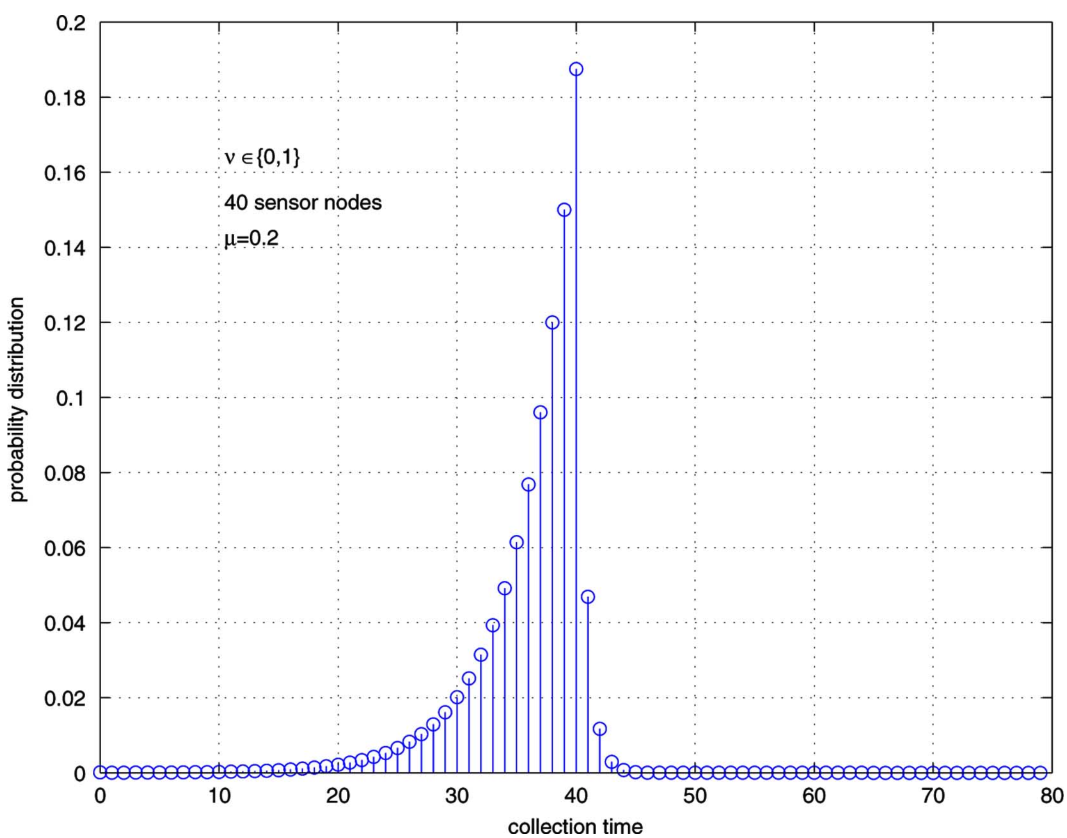

Fig. 3. Distribution of data collection time in 40-node line network. Each node in the considered network carries 0 or 1 data packet with probability 0.8 and 0.2 respectively.

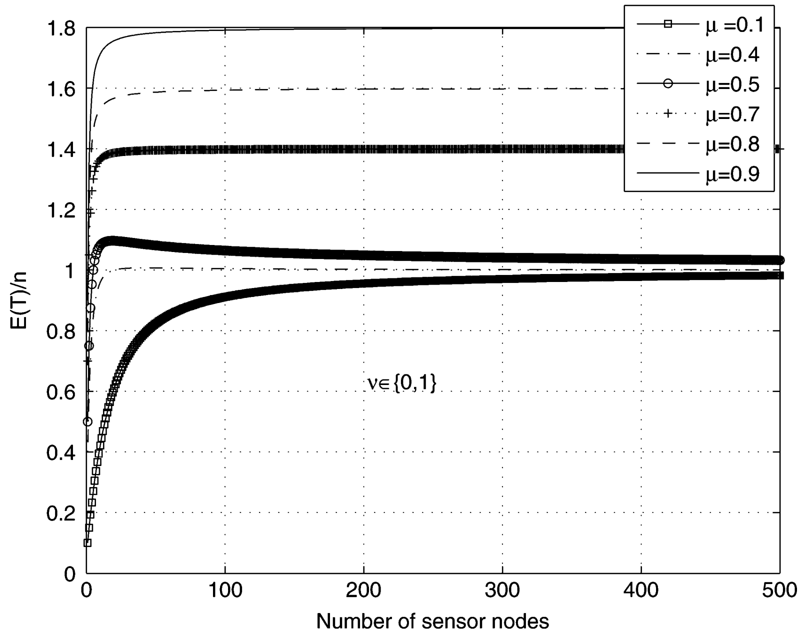

Fig. 4. Average collection time as a function of average number of packets per node and number of nodes in line network. Nodes carry 0 or 1 data packet with probability $1-\mu$ and μ, respectively.

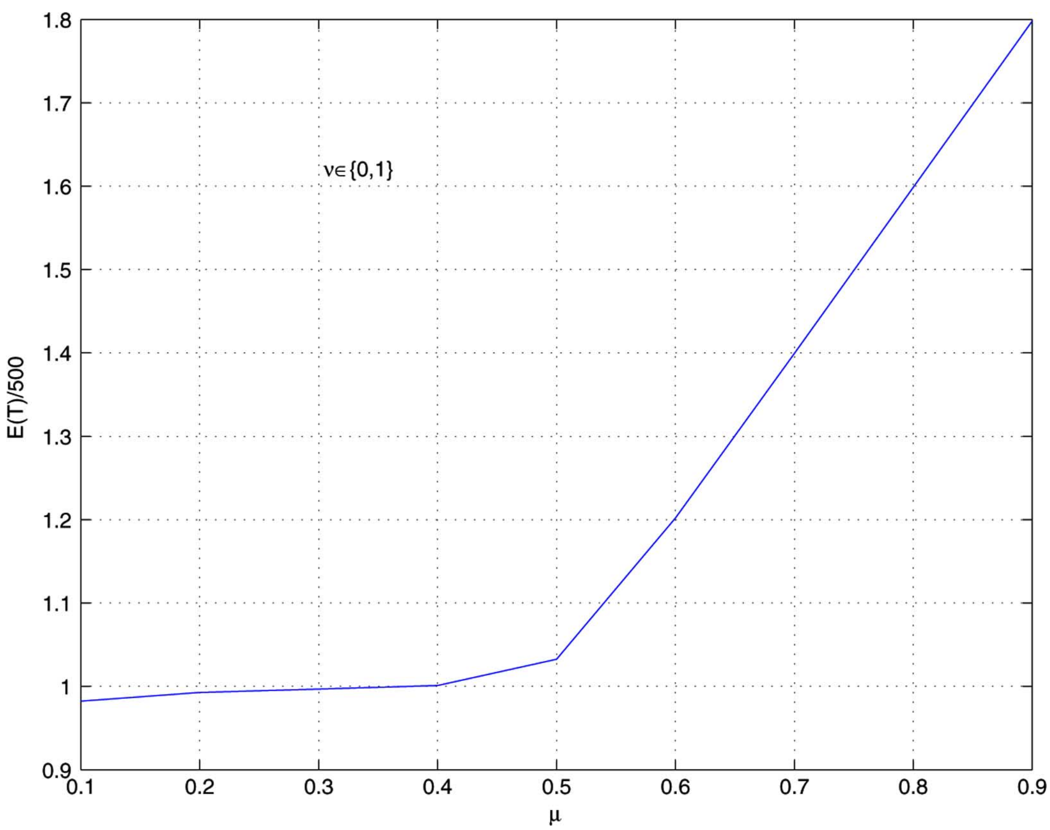

Fig. 5. Average collection time as a function of average number of packets per node in 500-node line network. Nodes carry 0 or 1 data packet with probability $1-\mu$ and μ, respectively.

Remark: It is worth noting that the result of Theorem 1 holds for any distribution of the data packets. In particular, the ν_i's need not be i.i.d., however, in this paper we deal with the case that ν_i's are i.i.d.

Interestingly, if we plot the expected value of $T_{\min}$ as in Figs. 4 and 5, we observe that the average delay scales linearly with the number of nodes n and the linear factor depends on the average number of packets per node μ. In the next section, we analyze the average delay and prove the observation rigorously.

B. Asymptotic Analysis of the Average Delay

In this subsection, we study the asymptotic behavior of the minimum average delay in collecting data from a line network as the number of nodes becomes large.

Theorem 2: Let ν_i's be i.i.d. random variables $\nu_i \in S_m$ with mean μ, variance σ^2 where μ, σ^2, m are all constants independent of n. We have

$$\lim_{n\to\infty} \frac{\mathbb{E}\{T_{\min}\}}{n} = \begin{cases} 2\mu, & \text{if } \mu \geq 1/2 \\ 1, & \text{if } \mu \leq 1/2. \end{cases} \tag{6}$$

Proof: We consider the case $\mu \geq 1/2$ first: Let us define $\nu_i' = \nu_i - \mu$. Using (1), we get

$$\begin{aligned}
&\mathbb{E}\{T_{\min}(\boldsymbol{\nu}_{\mathbf{n}})\} \\
&= 2\mu n + \mathbb{E}\left\{\max_{1\leq i\leq n-1}\left(i(1-2\mu)+\nu_i' + 2\sum_{i+1}^{n}\nu_j'\right)\right\} + \mu - 1 \\
&\leq 2\mu n + 2\mu - 1 + 2\mathbb{E}\left\{\max_{1\leq i\leq n}\sum_{j\geq i}^{n}\nu_j'\right\} \\
&= 2\mu n + 2\mu - 1 + 2\mathbb{E}\left\{\max_{1\leq i\leq n}\sum_{j=1}^{n+1-i}\nu_{n-j+1}'\right\} \qquad (7)
\end{aligned}$$

where the inequality follows by upper-bounding $i(1-2\mu)$ by zero as $\mu \geq \frac{1}{2}$ and replacing ν_i' by $2\nu_i' + \mu$ as $\nu_i' + \mu \geq 0, 1 \leq i \leq n-1$. In order to find a bound for $\mathbb{E}(\max_{1\leq i\leq n}\sum_{j\geq i}^{n}\nu_j')$, we first state the following lemma which based on a result of Erdos and Kac [10] where a convergence theorem for the distribution of the maximum of partial sums was proven.

Lemma 3: Let $S_i = \sum_{j=n-i}^{n}\nu_j'$ for $i = 0, \ldots, n-1$, then for any λ and $a > 1$

$$\Pr\left\{\max_{0\leq i\leq n-1} S_i \geq \lambda\sigma\sqrt{n}\right\} \leq \frac{a}{a-1}\Pr\left\{\sum_{j=1}^{n}\nu_j' \geq (\lambda - \sqrt{a})\sigma\sqrt{n}\right\} \tag{8}$$

where $\nu_i' = \nu_i - \mu$ and ν_i is as defined in Theorem 2.

Proof: We first define the event E_i as the event that the partial sum exceeds the threshold $\lambda\sigma\sqrt{n}$ only after the $n-i$th term added to the summation as

$$E_i = \left\{\max_{0\leq j<i} S_j \leq \lambda\sigma\sqrt{n} \leq S_i\right\}, \quad i = 0, \ldots, n-1 \tag{9}$$

which is inspired by [10]. Events E_i's are disjoint and their union is equal to the event that $\max_{0\leq i\leq n-1} S_i \geq \lambda\sigma\sqrt{n}$. Therefore, we can state the following using the union bound:

$$\begin{aligned}
&\Pr\left\{\max_{0\leq i\leq n-1} S_i \geq \lambda\sigma\sqrt{n}\right\} \\
&= \Pr\{(\cup E_i) \cap \{S_{n-1} > (\lambda - \sqrt{a})\sigma\sqrt{n}\}\} \\
&\quad + \Pr\{(\cup E_i) \cap \{S_{n-1} < (\lambda - \sqrt{a})\sigma\sqrt{n}\}\} \\
&\leq \Pr\{S_{n-1} > (\lambda - \sqrt{a})\sigma\sqrt{n}\} \\
&\quad + \sum_{i=0}^{n-1}\Pr\{E_i \cap (S_{n-1} \leq (\lambda - \sqrt{a})\sigma\sqrt{n})\}. \qquad (10)
\end{aligned}$$

To evaluate the second term on the right-hand side (RHS) of (10), we note that $S_i \geq \lambda\sigma\sqrt{n}$ and $S_{n-1} \leq (\lambda - \sqrt{a})\sigma\sqrt{n}$ imply $S_i - S_{n-1} \geq \sqrt{a}\sigma\sqrt{n}$. Then using the fact that $S_i - S_{n-1}$ is independent of S_j for $j \leq i$, we may write

$$\begin{aligned}
&\sum_{i=1}^{n}\Pr\{E_i \cap (S_{n-1} \leq (\lambda - \sqrt{a})\sigma\sqrt{n})\} \\
&\leq \sum_{i=0}^{n-1}\Pr(E_i)\Pr(S_i - S_{n-1} \geq \sqrt{a}\sigma\sqrt{n}) \\
&\leq \sum_{i=0}^{n-1}\Pr(E_i)\frac{E\{(S_i - S_{n-1})^2\}}{a\sigma^2 n} \\
&= \sum_{i=0}^{n-1}\Pr(E_i)\frac{(n-i)\sigma^2}{a\sigma^2 n} \\
&\leq \frac{1}{a}\sum_{i=0}^{n-1}\Pr(E_i) \\
&\leq \frac{1}{a}\Pr\left(\max_{0\leq i\leq n-1} S_i \geq \lambda\sigma\sqrt{n}\right) \qquad (11)
\end{aligned}$$

where in the second inequality we used Chebychev's inequality and the last inequality follows form the definition of the events E_i and noting that

$$\sum_{i=0}^{n-1}\Pr(E_i) = \Pr\left(\cup_{i=1}^{n} E_i\right) = \Pr\left(\max_{0\leq i\leq n-1} S_i \geq \lambda\sigma\sqrt{n}\right)$$

since the events E_i are disjoint events. Therefore, Lemma 3 follows from (11) and (10). □

Now we can use Chebychev's inequality to evaluate the RHS of Lemma 3 as follows:

$$\Pr\left\{S_{n-1} = \sum_{i=1}^{n}\nu_i' \geq (\lambda - \sqrt{a})\sigma\sqrt{n}\right\} \leq \frac{n\sigma^2}{(\lambda - \sqrt{a})^2\sigma^2 n} \leq \frac{1}{(\lambda - \sqrt{a})^2}.$$

Therefore, substituting $\lambda = \log n$ we get

$$\Pr\left(\max_{1\leq i\leq n}\sum_{j=i}^{n}\nu_j' \geq \sigma\log n\sqrt{n}\right) = O\left(\frac{1}{\log^2 n}\right). \tag{12}$$

Equation (12) implies that, with high probability, $\max_{1\leq i\leq n}\sum_{j\geq i}\nu_j'$ is less than $\sigma\log n\sqrt{n}$. Therefore, we may write

$$\begin{aligned}
&\mathbb{E}\left\{\max_{1\leq i\leq n-1}\sum_{j=i}^{n}\nu_j'\right\} \\
&\leq \sigma\log n\sqrt{n}\Pr\left\{\max_{1\leq i\leq n-1}\sum_{j=i}^{n}\nu_j' < \sigma\log n\sqrt{n}\right\} \\
&\quad + (m-1-\mu)n\Pr\left\{\max_{1\leq i\leq n-1}\sum_{j\geq i}^{n}\nu_j' > \sigma\log n\sqrt{n}\right\} \\
&= \sigma\log n\sqrt{n} + O\left(\frac{n}{\log^2 n}\right) \qquad (13)
\end{aligned}$$

which follows from the fact that $\nu_i' \leq m - 1 - \mu$.

We now derive a lower bound on $\mathbb{E}(T_{\min}(\boldsymbol{\nu_n}))$: From (1), we get $T_{\min}(\boldsymbol{\nu_n}) \geq \nu_1 + 2\sum_{j\geq 2}^n \nu_j$. Taking the expectation of both sides, we get

$$\mathbb{E}(T_{\min}(\boldsymbol{\nu_n})) \geq 2\mu n - \mu. \tag{14}$$

Considering (14) and the upper bound derived in (13), we deduce that

$$\begin{aligned} 2\mu n - \mu &\leq \mathbb{E}(T(\boldsymbol{\nu}_\mathbf{n})) \\ &\leq 2\mu n + 2\mu - 1 + 2\sigma \log n\sqrt{n} + O\left(\frac{n}{\log^2 n}\right) \end{aligned}$$

which leads to (6) for $\mu \geq 1/2$.

Next we consider the case $\mu \leq 1/2$: Let us define $\nu_i' = \nu_i - 1/2$. Using (1), we get

$$\begin{aligned} T_{\min}(\boldsymbol{\nu}_\mathbf{n}) &= \max_{1\leq i\leq n-1}\left(n - \frac{1}{2} + \nu_i' + 2\sum_{i+1}^{n}\nu_j'\right) \\ &\leq n - \frac{1}{2} + 2\max_{1\leq i\leq n-1}\sum_{i}^{n}\nu_j'. \end{aligned}$$

Taking the expectation of both sides and using inequality (13) we get

$$\mathbb{E}(T_{\min}(\boldsymbol{\nu_n})) \leq n + 2\sigma \log n\sqrt{n} + O\left(\frac{n}{\log^2 n}\right). \tag{15}$$

On the other hand, it is clear that if there is any packet in distance r, it takes at least r TS to be collected. Furthermore, the probability that there are no packets in the last $\log n$ nodes of the line network is $1 - (\Pr(\nu_i = 0))^{\log n}$. Therefore, noting that $\Pr(\nu_i = 0)$ is a fixed number, we may write

$$\begin{aligned} \mathbb{E}(T_{\min}(\boldsymbol{\nu_n})) &\geq (n - \log n)(1 - (\Pr(\nu_i = 0))^{\log n}) \\ &= n - O(\log n), \end{aligned} \tag{16}$$

which leads to (6) for $\mu \leq 1/2$. □

Remark: Theorem 2 can be easily generalized to the case that ν_i's are independent and have mean $\mu_i \geq \frac{1}{2}$ and variance σ_i^2 and $\nu_i \leq m-1$ where m is a constant. In fact, we can assume m is also going to infinity as well. Considering (13), the theorem goes through as long as $m = o(n)$.

Fig. 4 shows the ratio of the average delay to the number of sensor nodes, i.e., $\mathbb{E}(T_{\min}(\boldsymbol{\nu_n}))$, for a line network where each sensor node carries 0 or 1 data packet with probabilities $1-\mu$ and μ, respectively, as a function of the number of sensor nodes n in the network and the average number of packets per node μ. Fig. 5 shows the ratio of the average delay to the number of sensor nodes in a line network (where again each node carries either 0 or 1 packet with probabilities $1-\mu$ and μ, respectively) for a fixed number of sensor nodes (500) as a function of the average number of packets per node μ.

C. Multihop Case

In this subsection, we consider the problem of scheduling when each node is allowed to use up to h hops. In our model, any hop size (from one up to h) will cause interference for all the nodes that we hop over and also for the neighbor node of the destination that is not within the transmitter and receiver nodes. Therefore, these nodes cannot have a simultaneous transmission/reception of other packets. Of course, having the freedom of using up to h hops and a longer transmission range, leads to faster data collection compared to the case where $h = 1$. This is quantified in the following theorem, where the minimum data collection time $T_{\min}(h, \boldsymbol{\nu_n})$ is expressed as a function of the transmission range h (hops). This theorem is basically a generalization of the result of (1) where $h = 1$.

Theorem 4: For a one-sided line network of length n in which the ith node has ν_i packets and is equipped with directional antennas, the minimum collection time of the packets at the BS as a function of the transmission range h in hops is

$$T_{\min}(h, \boldsymbol{\nu_n}) = \max(S', S_{h+1}, S_{h+2}, \ldots, S_{n-h}) \tag{17}$$

where

$$S_i = \sum_{j>i+h}^{n}\nu_j + \left\lfloor\frac{\sum_{j>i+h}^{n}\nu_j - 1 + (i \bmod h)}{h}\right\rfloor + \left\lfloor\frac{i}{h}\right\rfloor + 1, \quad 0 \leq i \leq n-h$$

$$S' = S_0 + \max\left(\sum_{j=1}^{l}\nu_j - 1, 0\right) + \sum_{j=l+1}^{h}\nu_j \tag{18}$$

where l is the unique solution to $l + n_0 = 0 \bmod h$ such that $0 \leq l \leq h-1$.

Remark: Note that when $h = 1$, (17) reduces to the familiar equation (1) proved in [8].

Proof: This theorem was proven in [8] when $h = 1$. Here we only outline the generalization. The proof has two parts. First we need to show that the RHS of (17) is a lower bound for the collection time. Second, we prove it is an upper bound as well by exhibiting a schedule with this time performance.

In order to show that the RHS is a lower bound, we first consider the h nodes $i, 1 \leq i \leq h$ closest to the BS. They need to forward $n_h = \sum_{j>h}\nu_j$ packets. If $n_h \leq h$, this can be done in $n_h + 1$ TS or more. This takes exactly $n_h + 1$ TS if all packets to be distributed are located at node $h+1$ and more otherwise. If $h+1 \leq n_h \leq 2h$, this can be done in $n_h + 2$ TS or more. So, in general, it takes at least $n_h + \lfloor\frac{n_h - 1}{h}\rfloor + 1$ TS. More generally if $n_{i,h}$ denotes the number of packets to be forwarded by the h nodes $j, i+1 \leq j \leq i+h$, it can be shown that it takes at least $n_{i,h} + \lfloor\frac{n_{i,h} + (n_{i,h} \bmod h) - 1}{h}\rfloor + \lfloor\frac{i}{h}\rfloor + 1$ TS to do so. Therefore, the maximum of the previous expression over i gives a lower bound for the data collection time performance. We are not done though. Indeed, this lower bound is not achievable when there are packets to be distributed at distance i where $i, 1 \leq i \leq h$. An additional lower bound may be derived to handle this case by reconsidering the first h nodes. They must not only forward $\sum_{j>h}\nu_j$ packets, but also receive $\sum_{j\leq h}\nu_j$ packets. The lower bound S_0 may be adjusted (to S') to take this fact into account.

A possible (optimal) schedule for the distribution problem is as follows: It consists of transmitting data packets first to the furthest node, then to the second furthest node, and so on, as fast as possible until all packets at distance greater than h have been served. Packets at distance $i, 1 \leq i \leq h$ are served in the

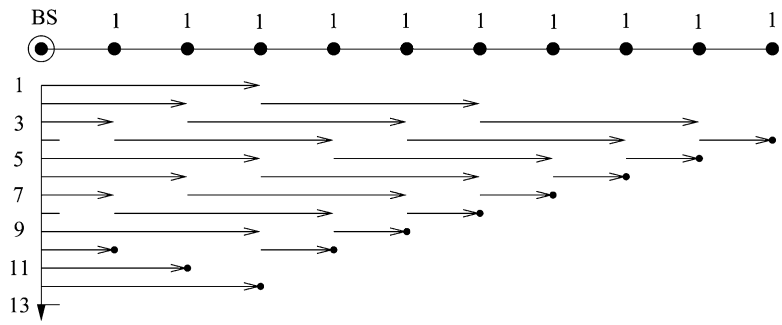

Fig. 6. Minimum length data distribution schedule of a 10-node line network when maximum transmission range is three hops.

reversed order, i.e., from closest to the BS to furthest. To prove this is indeed optimal, we compute its time performance and show it achieves the lower bound previously exhibited. This is similar to what was done in [8] and is left out here for the sake of brevity. □

In order to gain insight into the result of Theorem 4, we give a simple illustrative example before obtaining the asymptotic behavior of the expected minimum delay as n approaches infinity in the next theorem. Theorem 5, in fact, quantifies the dependency between the minimum collection time and the transmission range.

Example: We consider a line network of length n, where each node carries exactly one data packet and has a transmission range of $h \leq n$ hops. Direct application of Theorem 4 gives the minimum collection time as

$$T_{\min} = n + \left\lfloor \frac{n}{h} \right\rfloor - 1 \tag{19}$$

Fig. 6 shows an instance of this network: $n = 10$ and $h = 3$. Hence the data collection time is 12 TS. The associated distribution schedule accompanies the figure.

Theorem 5: Let h be the transmission range, let ν_i's be i.i.d. random variables $\nu_i \in \{0, 1, \ldots, m-1\}$ with mean μ and variance σ^2 where h, m, μ, σ^2 are constants independent of n.

$$\lim_{n\to\infty} \frac{\mathbb{E}\{T_{\min}(h, \boldsymbol{\nu_n})\}}{n} = \begin{cases} (1+\frac{1}{h})\mu, & \text{if } \mu \geq \frac{1}{h+1} \\ \frac{1}{h}, & \text{if } \mu \leq \frac{1}{h+1}. \end{cases} \tag{20}$$

Proof: The theorem follows by using the same machinery as in the proof of Theorem 2 and we omit the proof for the sake of brevity. □

We can now evaluate the gain in increasing the transmission range of a sensor node. Theorem 5 shows that a maximum gain of 2 on the collection time may be obtained by increasing the transmission range (in the limit when h approaches infinity) from $h = 1$. One should note, however, that this gain necessitates a significant amount of energy, in fact in the order of $O(\sum_i i^2 \nu_i) = O(n^3)$ if the energy expanded is taken to be proportional to the square of the distance traveled by a packet, whereas the minimum energy expanded (case $h = 1$) is of the order $O(\sum_i i\nu_i) = O(n^2)$.

D. Packet Splitting to Improve the Average Delay

As (6) implies, if the network is under-loaded (i.e., $\mu \leq \frac{1}{2}$), the ratio of the expected collection time to the expected number of packets in the network is $\frac{1}{\mu}$ and is rather high. One approach to decrease this ratio for small μ is to artificially increase the expected number of packets at each node by splitting each packet into k packets with length $\frac{1}{k}$ times of the original one. Clearly, this increases μ by a factor of k, and therefore, can potentially decrease the delay. It is also worth noting that the time needed for sending the smaller size packets is $\frac{1}{k}$ of the time to send of the original packets. We should further remark that splitting packets needs to be handled with care in practice as there is generally a constant overhead associated with each packet that limits the gain in packet splitting. In this paper, however, we do not deal with this tradeoff.

In this subsection, we examine the potential gain obtained by splitting data packets into subpackets. As a first step, we prove that the delay is a decreasing function of k in the next theorem.

Theorem 6: Given a line network $\boldsymbol{\nu_n}$ there is a gain $k \geq G(\boldsymbol{\nu_n}, k) \geq 1$ in splitting the data packets into k subpackets. Furthermore, $G(\boldsymbol{\nu_n}, k)$ is a nondecreasing function of k and the maximum achievable gain is

$$\begin{aligned} G_{\max}(\boldsymbol{\nu_n}) &= \lim_k G(\boldsymbol{\nu_n}, k) \\ &= \frac{\max_{1\leq i\leq n-1}\left(i - 1 + \nu_i + 2\sum_{j\geq i+1}^{n} \nu_j\right)}{\nu_1 + 2\sum_{j>1}^{n} \nu_j}. \end{aligned} \tag{21}$$

Proof: In general, if each packet is split into k subitems, the gain $G(\boldsymbol{\nu_n}, k)$ satisfies

$$\begin{aligned} G(\boldsymbol{\nu_n}, k) &= \frac{k \max_{1\leq i\leq n}(i - 1 + \nu_i + 2\sum_{j>i}^{n} \nu_j)}{\max_{1\leq i\leq n}(i - 1 + k\nu_i + 2k\sum_{j\geq i+1}^{n} \nu_j)} \\ &= \frac{\max_{1\leq i\leq n}(i - 1 + \nu_i + 2\sum_{j>i}^{n} \nu_j)}{\max_{1\leq i\leq n}(\frac{i-1}{k} + \nu_i + 2\sum_{j\geq i+1}^{n} \nu_j)} \end{aligned} \tag{22}$$

It is easy to check that $1 \leq G(\boldsymbol{\nu_n}, k) \leq k$. Furthermore, $G(\boldsymbol{\nu_n}, k)$ is a nondecreasing function of k as the denominator decreases as k increases. The limit when k goes to infinity can be also computed by noticing that the denominator of (22) for large k simplifies to $\nu_1 + 2\sum_{j>1}^{n} \nu_j$ as ν_i's are nonnegative. This completes the proof of (21). □

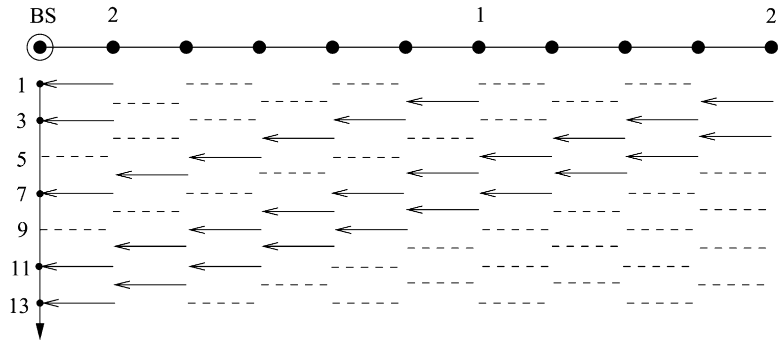

Fig. 7. Suboptimal data collection strategy described in Section III-E.

Next we derive the average collection time in random sensor network in the limit when n goes to infinity and when packets have been split into k subpackets.

Theorem 7: Let ν_i's be i.i.d. random variables $\nu_i \in S_m$ with mean μ, variance σ^2 where μ, σ^2, m are all constants independent of n. If each packet is split into k subpackets we have

$$\lim_{n\to\infty} \frac{\mathbb{E}\{T_{\min}\}}{n} = \begin{cases} 2\mu, & \text{if } \mu \geq 1/2k \\ 1/k, & \text{if } \mu \leq 1/2k. \end{cases} \tag{23}$$

Proof: The proof falls along the same line as the proof of Theorem 2 substituting ν_i with $k\nu_i$, for all $i, 1 \leq i \leq n$ and noting that the smaller size packets are transmitted k times faster. □

The limit in (23) should be compared to the data collection in the case where packets are not split as shown in (6). We conclude that in the asymptotic case, data splitting results in gain in the collection time for networks with low data load, i.e., $\mu \leq \frac{1}{2}$. It is also worth noting that (23) and (6) imply that if $k \geq \frac{1}{\mu}$ there is no gain in further increasing k; the expected delay remains the same as k further increases. For example, if $\mu = \frac{1}{5}$, the expected delay behaves like n, $\frac{1}{2}n$, and $\frac{2}{5}n$ for $k=1, k=2$, and $k \geq 3$, respectively. In other words, increasing k beyond $\frac{1}{2\mu}$ does not lead to any improvement on the scaling law of the average delay.

E. A Simple Suboptimal Strategy

It is important to note that the minimum collection time in (1) is achieved under the assumption that each sensor node has a perfect knowledge of the network topology and data packets locations. A more practical strategy that does not require knowledge of the packets' locations and therefore can be run in a distributed fashion is as follows: nodes at odd (resp., even) distance from the BS transmit to their closest neighbors toward the BS at odd (resp., even) TS. We refer to this scheduling as Strategy 1. It is illustrated in Fig. 7.

The following theorem compares the performance of this strategy to the minimal collection time derived in (1).

Theorem 8: For a one-sided line network of length n in which the ith node has ν_i packets and is equipped with directional antennas, the collection time of the packets at the BS under simple scheduling strategy, denoted by $T(\boldsymbol{\nu_n})$, is

$$T(\boldsymbol{\nu_n}) = \max_{1\leq i\leq n} \left(i - 2 + 2\sum_{j\geq i-1}^{n} \nu_j \right). \tag{24}$$

This further assumes that the closest, third closest,... edges to the BS are activated at TS $1, 3, \ldots$ whereas the second closest, fourth closest,... edges are activated at TS $2, 4, \ldots$. In the opposite case the data collection time is

$$T(\boldsymbol{\nu_n}) = \max_{1\leq i\leq n} \left(i - 1 + 2\sum_{j\geq i}^{n} \nu_j \right). \tag{25}$$

Proof: In the remainder of this paper, we refer to the closest edge to the BS as edge 1, second closest as edge 2, and so on. Assume nodes $1, 3, 5\ldots$ can only transmit at TS $1, 3, 5, \ldots$ and receive at TS $2, 4, 6\ldots$. The BS may receive at most one packet/TS at TS $1, 3, 5, \ldots$. Either it is busy at all $\text{TS} \geq 1$, or it is busy at all those $\text{TS} \geq 3$, or at all $\text{TS} \geq 5$, etc. In general, if the BS is busy at all $\text{TS} \geq i$ and the packet received at TSi comes from node i or $i-1$, the data collection time is $i-2+2\sum_{j\geq i-1}^{n} \nu_j$ TS. This completes the proof for (24). Equation (25) follows similarly. □

The aforementioned absence of knowledge (packets location) translates into a delay cost $T(\boldsymbol{\nu_n}) - T_{\min}(\boldsymbol{\nu_n}) \geq 0$. More generally, we have the following relationship between $T(\boldsymbol{\nu_n})$ and $T_{\min}(\boldsymbol{\nu_n})$, which follows from (1) and (25):

$$T_{\min}(\boldsymbol{\nu_n}) \leq T(\boldsymbol{\nu_n}) \leq 2T_{\min}(\boldsymbol{\nu_n}) - 1. \tag{26}$$

The worst performance of this simple strategy relative to the optimal strategy occurs when n packets are located at distance 1 from the BS (indeed, $T_{\min} = n$ and $T = 2n-1$ then). However, on average, achieving the upper bound in (26) is unlikely and we have the following asymptotic comparative result, according to which the simple scheduling strategy is asymptotically optimal with respect to time.

Theorem 9: Let ν_i's be i.i.d. random variables $\nu_i \in \{0, 1, \ldots, m-1\}$ with mean μ and variance σ^2 where μ, σ^2, m are constants independent of n.

$$\lim_{n\to\infty} \frac{\mathbb{E}\{T(\boldsymbol{\nu_n})\}}{n} = \begin{cases} 2\mu, & \text{if } \mu \geq 1/2 \\ 1, & \text{if } \mu \leq 1/2. \end{cases} \tag{27}$$

That is, $\lim_{n\to\infty} \frac{\mathbb{E}\{T(\boldsymbol{\nu_n}) - T_{\min}(\boldsymbol{\nu_n})\}}{n} = 0$.

Proof: This proof is similar to the proof of Theorem 2. □

F. Imperfect Channel

In this final subsection, we introduce noise in the channel. Specifically, we model the channel as an erasure channel with

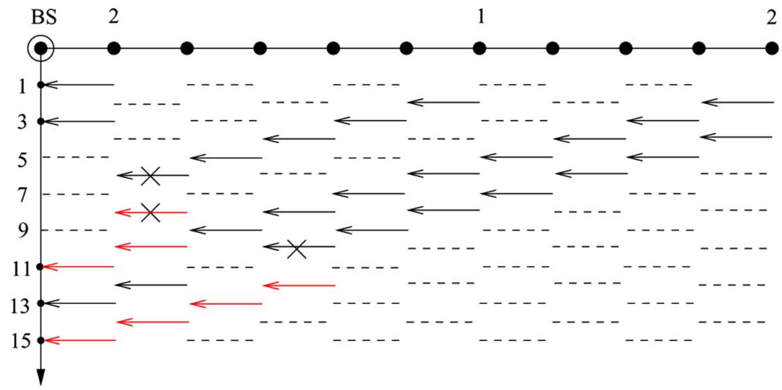

Fig. 8. Suboptimal data collection strategy described in Section III-E. Erasure channel. An erased packet is marked with a cross.

erasure probability p and measure the time performance degradation as a function of p. We assume that a node is instantaneously informed that a packet has not reached its (intermediate) destination and immediately retransmits the erased packet at the next available TS (that is 2 TS later). For reasons discussed at the biginning of this section, we focus on the simple scheduling strategy introduced in Section III-E. Fig. 8 illustrates the process. This is the same network as shown in Fig. 7 but it is now affected by three erasures (each shown by a crossed arrow). The new transmission time is 15 TS, an increase of 2 TS.

Theorem 10: Given a probability p of packet erasure, the average data collection time $T(p, \boldsymbol{\nu_n})$ in a line network $\boldsymbol{\nu_n}$ when the simple scheduling strategy is used is as shown in (28) at the bottom of the page.

Proof: The collection time may be expressed as an average of collection times. The probability that the entire collection process is not affected by any error is $(1-p)^{\sum_{i=1}^n i\nu_i}$. In that case, the collection time is $T(\boldsymbol{\nu_n})$. The probability that the collection process is affected by exactly k errors is $(1-p)^{\sum_{i=1}^n i\nu_i}p^k$. Notice that a packet erasure along a specific edge increases the collection time from $T(\boldsymbol{\nu_n})$ to $T(\boldsymbol{\nu_n}+\boldsymbol{e_i})$, where $\boldsymbol{e_i}$ is the vector of length n whose ith component is 1 and other components are 0 and where i is the source node for the packet. For a given source node there are $\binom{i\nu_i+e_i-1}{i\nu_i-1}$ choices of e_i erasures. One needs to consider all the possible schedules with exactly k erasures. This can be done by solving the equation $\sum_i e_i\chi(\nu_i>0)=k$. □

In order to see the impact of the erasure probability on the data collection time the ratio $T(p)/T(0)$ is plotted for increasing values of p for a specific line network ($\boldsymbol{\nu}=(0,2,0,0,0,0,0,1,1,1)$) in Fig. 9. It shows a degradation of 50% for an erasure probability $p=0.1$. However, the insight provided by Theorem 10 is limited. In the following theorem, instead of considering the expected delay for a specific network, we consider a random line network and obtain an upper bound for the expected delay as a function of the packet erasure probability.

Theorem 11: Let ν_i's be i.i.d. random variables $\nu_i \in \{0,1,\ldots,m-1\}$ with mean μ and variance σ^2 where μ, σ^2, m are constants independent of n then

$$1 \le \frac{\mathbb{E}(T(p,\boldsymbol{\nu_n}))}{\mathbb{E}(T(0,\boldsymbol{\nu_n}))} \le 1 + O(np). \tag{29}$$

Proof: In order to find an upper bound for the expected delay, we may use any strategy in scheduling. Here, we assume that whenever an erasure occurs, the transmitting node retransmits the packet until it gets through and all the other nodes remain silent at that period. Denoting α_i for $i=1,\ldots,\sum i\nu_i$ as the number of extra time slots needed to transmit the packet at the ith transmission, we may write

$$T(p,\boldsymbol{\nu_n}) \le \sum_{j=1}^{\sum_{i=1}^n ip_i} \alpha_j + T(0,\boldsymbol{\nu_n}) \tag{30}$$

where α_i has geometric distribution, i.e.,

$$\Pr(\alpha_i) = p^{i-1}(1-p) \Rightarrow \mathbb{E}(\alpha_i) = \frac{p}{1-p}. \tag{31}$$

Taking expectation of both sides of (30), we obtain\

$$\frac{\mathbb{E}(T(p))}{\mathbb{E}(T(0))} \le \frac{p\sum_{i=1}^n ip_i}{(1-p)\mathbb{E}(T(0))} + 1 \tag{32}$$

which completes the proof of our theorem. □

In particular, Theorem 11 implies that for networks of large size, a probability of erasure p of order $o(\frac{1}{n})$ does not significantly affect the time performance of the data collection process.

IV. Random Multiline Networks

In this section, we consider a more general network, i.e., a network consisting of $L \ge 2$ lines. Here we assume these lines

$$T(p,\boldsymbol{\nu_n}) = (1-p)^{\sum_{i=1}^n i\nu_i} \sum_{k\ge 0} p^k \sum_{\sum_i e_i\chi(\nu_i>0)=k} \prod_{i\ge 1}^{n} \binom{i\nu_i+e_i-1}{i\nu_i-1} T(\boldsymbol{\nu_n}+\boldsymbol{e_i}). \tag{28}$$

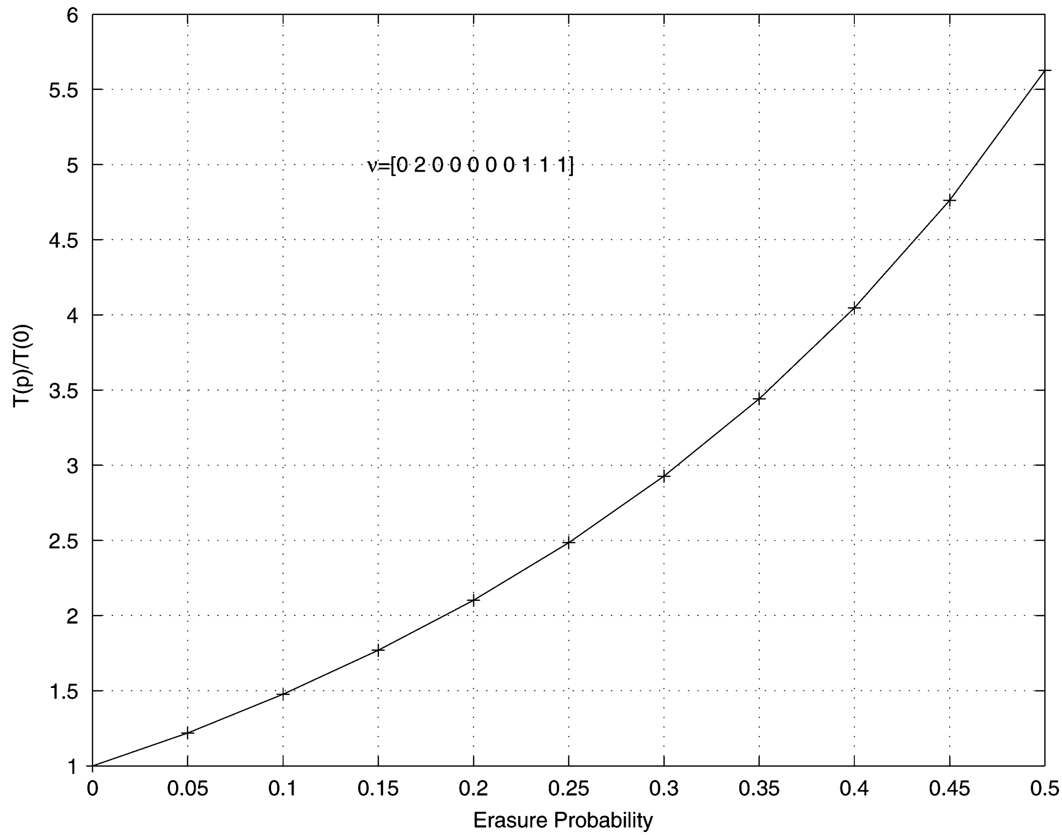

Fig. 9. Ratio $\frac{T(p)}{T(0)}$ as a function of p for specific line network.

are separated in space and therefore there is no crossover interference between the transmissions in different lines.[2] However, only one of the links between BS and the first node of different lines can be used at a time. For simplicity, we assume each line has n_0 nodes. This is illustrated in Fig. 10. Furthermore, each node carries $\nu \in S_m$ packets with probability distribution $(p_0, p_1, \ldots, p_{m-1})$. We will later argue that the results for the more general case follows along the same line of this simple case.

It is quite easy to state a lower bound for the average delay. Assuming ν_i's are i.i.d., and denoting $T_{\min}^{L,n_0}$ as the minimum data collection time for a multiline network with $L \geq 2$ lines of length n_0, we have

$$\mathbb{E}\left(T_{\min}^{L,n_0}\right) \geq n_0 L \mathbb{E}(\nu_i) \tag{33}$$

which follows by taking the expectation of both sides of the inequality $T_{\min}^{L,n_0} \geq$ (number of packets in network).

In what follows, we shall prove that as L increases, the expected collection time converges toward this lower bound. To prove our asymptotic result, we describe a suboptimal procedure to collect the data at the BS: we may divide the network into two subnetworks $\mathcal{S}_1$ consisting of odd lines and $\mathcal{S}_2$ consisting of even lines. For $l \in \mathcal{S}_2$, nodes at even distance from the BS transmit toward the BS at even time slots and nodes at odd distance from the BS transmit toward the BS at odd time slots. If $l \in \mathcal{S}_1$, the opposite happens, i.e., nodes at even distance transmit toward the BS at odd time slots and *vice versa*. However, if at a given TS multiple nodes at distance 1 from the BS carry data packets, only one packet (randomly chosen from all available packets) gets transmitted to the BS (since this BS can only receive one packet at a time). Remaining packets are stored for later transmission. This strategy is followed until all packets in the network have reached the BS or a node at distance one from the BS. At this point, packets at distance one from the BS are simply transmitted to the BS in turn so that the BS does not become idle until all packets have been collected.

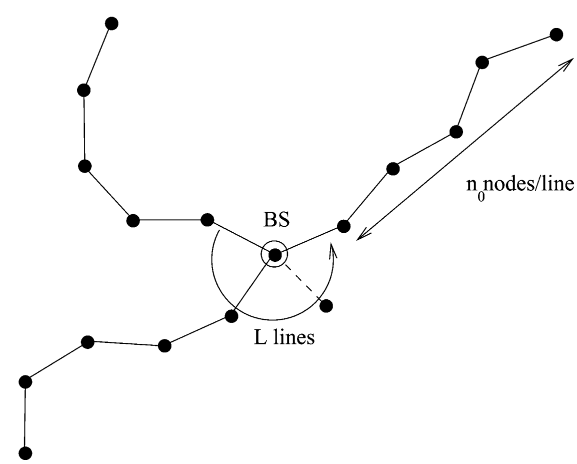

Fig. 10. Multiline network.

[2]It should be mentioned that as the number of lines grows, this assumption may not be valid. However, in this paper, we only deal with the case where there is no cross interference for simplicity.

With this scheduling and assuming each node carries at most $m-1$ data packets it is clear that after $(m-1)(2n_0-3)$ TS (assuming that $\nu_i \in S_2$), all the packets are within distance one of the BS (since it is true in the worst case where each node carries exactly $m-1$ packets). Therefore, we may think of data collection as two separate phases: first collecting all the packets to the nodes with distance one of the BS which at most takes $(m-1)(2n_0-3)$ TS, and in the second phase, the nodes within distance one of BS are the only nodes with packets and they try to send their packets to BS.

Theorem 12: Consider a multiline network with $L \geq 2$ lines of length n_0, and ν_i's are i.i.d. chosen from $\{0, 1, \ldots, m-1\}$ with an arbitrary distribution. Let $\forall k, 0 \leq k \leq m-1, \Pr(\nu_i = k) = p_k$, where $p_{m-1} \neq 0$. Further assume that $\mathbb{E}(\nu_i) = \mu$ Then

$$n_0 L\mu \leq \mathbb{E}\left(T_{\min}^{L,n_0}\right) \leq n_0 L\mu + O\left(\frac{1}{L}\right) + (m-1)(2n_0-3)(1-p_{m-1})^{L/2}. \quad (34)$$

In particular

i) if $L > (2+O(1))\log_\alpha n_o$,

$$\lim_{n_o\to\infty} \mathbb{E}\left(T_{\min}^{L,n_0}\right) - n_o L\mu = 0. \quad (35)$$

ii) if $2\log_\alpha n_o > L$ and

$$\lim_{n_o\to\infty} L = +\infty, \quad \lim_{n_o\to\infty} \frac{\mathbb{E}\left(T_{\min}^{L,n_0}\right)}{n_o L\mu} = 1 \quad (36)$$

iii) if $L = c, n_o L\mu \leq \mathbb{E}\left(T_{\min}^{L,n_0}\right) \leq n_o L\mu(1+\epsilon)$ (37)

where $\alpha = \frac{1}{1-p_{m-1}}$, c and ϵ are constants independent of n_o.

Proof: The lower bound follows from (33) and noting that $\mathbb{E}(\nu_i) = \mu$. To prove the upper bound, we use the suboptimal scheduling described before to collect the data packets. We also define the random variable $e_i \in \{0,1\}$, for $i = 1, \ldots, (m-1)(2n_0-3)$, such that $e_i = 0$ if the BS is busy at TS i, and $e_i = 1$ if it is not.

Considering the steps in collecting packets in the network with our scheduling, if the total number of packets is greater than $(m-1)(2n_0-3)$, then the time needed to collect the data packets is equal to the total number of packets in the network (denoted by η) plus the number of times that the BS was not busy during $1 \leq t \leq (m-1)(2n_0-3)$ which is equal to $\sum_{i=1}^{(m-1)(2n_0-3)} e_i$. Therefore, we can write an upper bound for the delay as

$$T_{\min}^{L,n_0} \leq \max\{\eta, (m-1)(2n_0-3)\} + \sum_{i=1}^{(m-1)(2n_0-3)} e_i. \quad (38)$$

To find an upper bound for the expected delay, we have to find $\Pr(e_i = 1)$ and $\Pr(\eta \leq (m-1)(2n_0-3))$. It is clear that for $k = 2, \ldots, n_0-2$, to find an upper bound for the expected delay, we find $\Pr(e_i = 1)$ and $\Pr(\eta \leq (m-1)(2n_0-3)$. It is clear that

$$\begin{aligned}
&\Pr(e_{2k} = 0) \\
&\quad \geq \Pr(\text{ having at least } m-1 \text{ packets at dist. } k) \\
&\quad \geq \Pr(\text{at least one node at dist.} k \text{ has } m-1 \text{ packets}) \\
&\quad = 1-(1-p_{m-1})^{L/2}. \quad (39)
\end{aligned}$$

A similar expression can be written for $\Pr(e_{2k+1} = 0)$. Furthermore, using Chebychev's inequality and noting that η is the total number of packets in the network, i.e., $\eta = \sum_{i=1}^{n_0 L} \nu_i$, we may write

$$\Pr((m-1)(2n_0-3) \leq \eta) \geq 1 - O\left(\frac{1}{n_0 L}\right) \quad (40)$$

which implies that $\Pr(\eta \leq (m-1)(2n_0-3)) \leq O(\frac{1}{n_0 L})$. Now we can take the expectation from both sides of (38) to get

$$\begin{aligned}
\mathbb{E}\left(T_{\min}^{L,n_0}\right) &\leq \mathbb{E}(\eta) + (m-1)(2n_0-3) \\
&\quad \times \Pr(\eta \leq (m-1)(2n_0-3)) \\
&\quad + \sum_{i=1}^{(m-1)(2n_0-3)} \Pr(e_i = 1) \\
&\leq n_0 L\mu + O\left(\frac{1}{L}\right) \\
&\quad + (m-1)(2n_0-3)(1-p_{m-1})^{L/2} \quad (41)
\end{aligned}$$

that completes the proof for the first part. □

Theorem 15 shows that the difference of the expected delay and $n_0 L/2$ is converging to zero as $L \to \infty$ and n_0 grows slower than 2^{2L} (or, equivalently, L grows faster than $O(\log n)$). It is a reasonable hypothesis in general. Indeed, as the number of sensor nodes per unit of observation area increases, noting that L is the number of sensors within reach of the BS, it can be shown that L scales like $\log n + c(n)$ where $c(n) \to \infty$ [11]. Therefore, fixing the area of the network, having n go to infinity, and noting that $n_0 = n/L$, the aforementioned condition is satisfied.

In the more general case, where the number of sensors per line is n_0^l for $l = 1, \ldots, L$ (instead of n_0 for all l's) the lower bounds on the expected delay becomes $\mathbb{E}(T_{\min}^{L,n_0}) \geq \mu \sum_{l=1}^{L} n_0^l$. We can further find an upper bound by replacing n_0 by $\max n_0^l$ in (38) and noting that $\mathbb{E}(\eta)$ is equal to the lower bound. The result follows in a similar fashion. Therefore, as long as $(\max n_0^l)m = o(\frac{1}{1-p_{m-1}})^L$ and L grows to infinity, the expected delay converges to $E\{\eta\}$. In Fig. 11, the data collection time for multiline networks is plotted as the function of the number of lines for various average number of packets per node (and a fixed number of nodes per line, $n_0 = 25$) using Monte Carlo simulation. Each instance of a random network has L lines of n_0 nodes. Each node caries either 0 or 1 packet with probability $1-\mu$ and μ, respectively. The exact collection time for a particular instance is known and given in [8] and this is averaged over multiple instances (20000) to yield Fig. 11.

A. Delay Analysis for More General Topologies

Insightful results about the delay in collecting data from sensory networks forming more general topologies may be inferred from Sections III and IV. In this section we discuss the implications of previous results for networks of more general topologies.

Clearly for a sensor network of any topology, the expected minimum collection delay satisfies: $\mathbb{E}(T) \geq n\mathbb{E}(\nu_i)$ where n

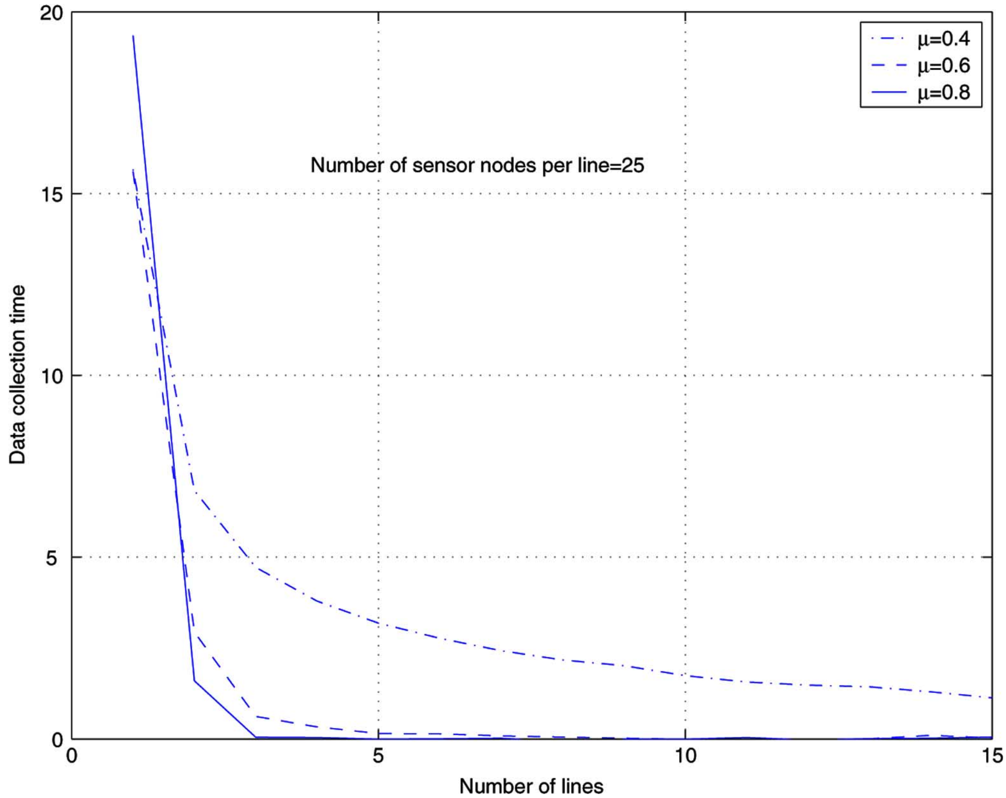

Fig. 11. Average collection time as a function of average number of packets per node and number of lines in multiline network (25 nodes per line). Nodes carry 0 or 1 data packet with probability $1-\mu$ and μ, respectively.

is the number of sensor nodes in the network. However, in the particular case where only a single path exists from the sensors to the BS (i.e., the degree of the BS is one), this lower bound is not tight and may be improvede to $2n\mathbb{E}(\nu_i)$ using Theorem 1.

If the degree of the BS is 1, It is shown in [9] that the network may be thought of as a line network—for analysis purposes—by combining nodes at the same distance from the BS without impeding the time performance of a particular data collection strategy. In the resulting "linearized" network the number of data packets at a given distance from the BS is the sum of the packets at that distance in the original network. Consequently, results in Section III-A may be applied to this type of networks to derive the exact delay distribution. Results in Section III-B hold as well (in the case where $\nu_i \in S_2$ and $\Pr(\nu_i = 0) = 1/2$ the number of packets at distance i in the linearized network is $\mathrm{Bin}(m_i, 1/2)$ and our claim is a matter of extending Lemma 3 to binomial distributions of packets). That is the delay is $2n\mathbb{E}(\nu_i)$ asymptotically in the first order.

If the degree of the BS is greater than 1, it is straightforward to extend the previous results on multiline networks to tree topologies (given what was said earlier, a tree may be thought of as a multiline network). On general topologies it holds that the average collection delay converges toward the average number of packets in the network when the number of sensors is large. The proof is based on Section IV by extracting a shortest path spanning tree of the considered network and noticing that the maximum distance of a sensor to the BS (the distance being the length in number of hops of a shortest path to the BS) grows slower than 2^{2L} where L is the number of sensors within reach of the BS.

V. Comparison of Omnidirectional/Directional Systems

The previous analysis of directional antenna systems may be extended to omnidirectional systems. In these systems, nodes are equipped with omnidirectional antennas generating interference for all surrounding nodes. In particular, in a line network this implies that a packet transmission to the left (or right) neighbor creates interference at both the left and right neighbors. This in turns increases the length of the optimum data collection schedule (when compared to directional systems). In fact, it was shown in [9] that the minimum data collection time $T_{o,\min}(\boldsymbol{\nu_n})$ over a line network of length n equipped with omnidirectional antennas in which the ith node has ν_i packets becomes

$$T_{o,\min}(\boldsymbol{\nu_n}) = \max_{1\le i\le n-2}\left(i - 1 + \nu_i + 2\nu_{i+1} + 3\sum_{j\ge i+2}^{n}\nu_j\right) \tag{42}$$

where $\boldsymbol{\nu_n} = (\nu_1,\ldots,\nu_n)$. It was shown in [8] that this represents a maximum increases of 50% over the data collection time achieved by a directional antenna system for the same considered line network. In the example of section of Fig. 1, the minimum data collection time becomes 14 TS, a 40% increase.

In the following subsections, we presents results for the delay analysis for the network equipped with ominidirectional antennas. Results are analogous to the results stated in Section III and we omit the proofs for the sake of brevity.

A. Delay Distribution

In this subsection, we derive, by means of a recursion,CDF of $T_o(\boldsymbol{\nu_n})$ for a line network. Let us assume that ν_i's are i.i.d. random variables chosen from the set $S_m = \{0, 1, \ldots, m-1\}$.

Theorem 13: Let $F_n(t)$ be the CDF of the minimum delay $T_o(\boldsymbol{\nu_n})$, i.e., $F_n(t) = \Pr\{T_o(\boldsymbol{\nu_n}) \le t\}$. Then $F_n(t)$ satisfies the following recursion:

$$F_n(t) = \sum_{i=1}^{m-1} \Pr(\nu_n = i) F_{n-1}(t - 3i) \mathbf{1}_{t \ge n+3(i-1)} + \Pr(\nu_n = 0) F_{n-1}(t), \quad \forall n \ge 3 \qquad (43)$$

where

$$\mathbf{1}_{t \ge t_0} = \begin{cases} 1, & \text{if } t \ge t_0 \\ 0, & \text{otherwise} \end{cases}$$

and

$$F_1(t) = \begin{cases} \sum_{i=0}^{t} \Pr(\nu_1 = i), & \text{if } t < m-1 \\ 1, & \text{otherwise} \end{cases}$$

$$F_2(t) = \sum_{i=1}^{m-1} \Pr(\nu_2 = i) F_1(t - 2i) \mathbf{1}_{t \ge 2i} + \Pr(\nu_2 = 0) F_1(t).$$

B. Asymptotic Analysis of the Average Delay

In this subsection, we study the asymptotic behavior of the minimum average delay in collecting data from a line network as the number of nodes becomes large.

Theorem 14: Let ν_i's be i.i.d. random variables $\nu_i \in S_m$ with mean μ, variance σ^2 where μ, σ^2, m are all constants independent of n. We have

$$\lim_{n \to \infty} \frac{\mathbb{E}\{T_o\}}{n} = \begin{cases} 3\mu, & \text{if } \mu \ge 1/3 \\ 1, & \text{if } \mu \le 1/3. \end{cases} \qquad (44)$$

C. Multiline/Omnidirectional Case

Theorem 15: Consider a multiline network with L lines of length n_0, and ν_i's are i.i.d. chosen from $\mathcal{S}_m$ such that $\forall k, 0 \le k \le m-1, \Pr(\nu_i = k) = p_k$ where $p_{m-1} \ne 0$. Further assume that $\mathbb{E}(\nu_i) = \mu$ Then

$$n_0 L\mu \le \mathbb{E}(T) \le n_0 L\mu + O\left(\frac{1}{L}\right) + (3n_0(m-1) - 2)(1 - p_{m-1})^{L/3}. \qquad (45)$$

In particular

i) if $L > (3 + O(1)) \log_\alpha n_o$
$$\lim_{n_o \to \infty} \mathbb{E}(T) - n_o L\mu = 0 \qquad (46)$$

ii) if $3 \log_\alpha n_o > L$ and
$$\lim_{n_o \to \infty} L = +\infty, \lim_{n_o \to \infty} \frac{\mathbb{E}(T)}{n_o L\mu} = 1 \qquad (47)$$

iii) if $L = \text{cte}$, $n_o L\mu \le \mathbb{E}(T) \le n_o L\mu(1 + \epsilon) \qquad (48)$

where $\alpha = \frac{1}{1 - p_{m-1}}$ and ϵ is a constant independent of n_o when L is fixed.

VI. Conclusion

This work is concerned with characterizing the statistics of the minimum delay in collecting (or distributing) data packets in sensory networks at the BS. We study the statistics of the minimum delay achieved by the optimal (greedy) algorithm proposed in [9] for different topologies including line and multiline networks with directional or omnidirectional antennas. For a line network, we obtain the distribution of the minimum delay in collecting the packets using a recursion. Under the assumption that the number of data packets accumulated by a sensor node is identically and independently distributed across different nodes, we further analyze the asymptotic behavior of the average minimum delay for a large number of nodes and show that it converges to twice the average number of packets in the network when the average number of packets per node is greater than $\frac{1}{2}$. We show that a simple time division scheduling of nodes can also achieve the same scaling law for the average minimum delay without requiring the knowledge of the number of packets in each node. For a multiline network, we show that the average minimum delay converges to the expected number of packets in the network for large number of nodes. We further study the impact of packet size, transmission range, and channel erasure probability on the delay performance of optimal collection/distribution of packets.

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Cedric Florens (S'02) received the Diplme d'Ingnieur degree from the Ecole Suprieure d'Ingnieurs en Electronique et Electrotechnique, Paris, France, and the M.S. and Ph.D. degrees in electrical engineering from the California Institute of Technology, Pasadena, in 1999 and 2005, respectively.

From 1999 to 2000, he was with the Deep Space Concepts Communication Group, Jet Propulsion Laboratory, California Institute of Technology. From 2000 to 2001, he was employed by QPlus Networks, Long Beach, CA, an optical telecommunications startup.

Masoud Sharif received the Ph.D. degree in electrical engineering (2005) from the California Institute of Technology, Pasadena.

In 2005, he was a Postdoctoral Scholar in the Department of Electrical Engineering at Caltech. Since January 2006, he has been an Assistant Professor at Boston University, Boston, MA.

He is a member of the Center for Information and Systems Engineering at Boston University. His research interests include *ad hoc* and sensor networks, multiple-user multiple-antenna communication channels, cross-layer design for wireless networks, and multiuser information theory. His recent research has focused on collaborative communication scheme in *ad hoc* and sensor networks and the capacity of multiple antenna broadcast channels.

Dr. Sharif was awarded the C. H. Wilts Prize in 2006 for best doctoral dissertation in electrical engineering at Caltech.

Robert J. McEliece (M'70–SM'81–F'84) was born in Washington, DC, in 1942. He received the B.S. and Ph.D. degrees in mathematics from the California Institute of Technology, Pasadena, in 1964 and 1967, respectively, and attended Trinity College, Cambridge University, Cambridge, U.K., during 1964–1965.

From 1963 to 1978, he was with the Jet Propulsion Laboratory, California Institute of Technology, where he was Supervisor of the Information Processing Group, Communications Research Section, from 1971 to 1978. From 1978 to 1982, he was a Professor of Mathematics and a Research Professor at the Coordinated Science Laboratory, University of Illinois, Urbana-Champaign. Since 1982, he has been on the faculty at the California Institute of Technology, where he is now the Allen E. Puckett Professor of Electrical Engineering. From 1990 to 1999, he served as Executive Officer for Electrical Engineering at the California Institute of Technology. He has been a Consultant in the Communications Research Section of the Jet Propulsion Laboratory since 1978. His research interests include deep-space communication, communication networks, coding theory, and discrete mathematics.

Dr. McEliece is a member of the National Academy of Engineering.