Spatial Throughput of Mobile Ad Hoc Networks Powered by Energy Harvesting

Kaibin Huang

as kinetic activities or electromagnetic radiation will enable wireless networks to be self sustaining. In this paper, the spatial throughput of a mobile ad hoc network powered by energy harvesting is analyzed using a stochastic-geometry model. In this model, transmitters are distributed as a Poisson point process and energy arrives at each transmitter randomly with a uniform average rate called the energy arrival rate. Upon harvesting sufficient energy, each transmitter transmits with fixed power to an intended receiver under an outage-probability constraint for a target signal-to-interference-and-noise ratio. It is assumed that transmitters store energy in batteries with infinite capacity. By applying the random-walk theory, the probability that a transmitter transmits, called the transmission probability, is proved to be equal to the smaller of one and the ratio between the energy-arrival rate and transmission power. This result and tools from stochastic geometry are applied to maximize the network throughput for a given energy-arrival rate by optimizing transmission power. The maximum network throughput is shown to be proportional to the optimal transmission probability, which is equal to one if the transmitter density is below a derived function of the energy-arrival rate or otherwise is smaller than one and solves a given polynomial equation. Last, the limits of the maximum network throughput are obtained for the extreme cases of high energy-arrival rates and sparse/dense networks.

Abstract-Designing mobiles to harvest ambient energy such

Index Terms—Energy harvesting, mobile ad hoc networks, throughput, power control, stochastic processes, mobile communication

I. INTRODUCTION

Recent years have seen increasing popularity of mobile devices such as sensors and smart phones, giving rise to two design issues among others. First, the power consumption of mobile-device networks makes an escalating contribution to global warming. Second, conventional batteries that power mobile devices periodically interrupt their operation due to finite battery lives; battery recharging or replacement is inconvenient or even impossible in certain cases. These issues provide strong motivation for powering mobile devices by harvesting ambient energy such as solar energy, vibration, kinetic activities and electromagnetic radiation [1]. The capacity of mobile-device networks powered by energy harvesting remains largely unknown, which is addressed in this paper.

This paper considers a mobile ad hoc network (MANET) where transmitters are modeled as a homogeneous Poisson

point process (PPP). Energy arrives randomly at a transmitter with a fixed average rate, called the *energy-arrival rate*. The energy-arrival process is modeled as an independent and identically distributed (i.i.d.) sequence of random variables and different processes are assumed independent. Each transmitter deploys an energy harvester that stores arriving energy in a rechargeable battery. Upon harvesting sufficient energy, a transmitter transmits with fixed power to an intended receiver under an outage-probability constraint for a target signalto-interference-and-noise ratio (SINR). Based on the above model, the network spatial throughput is maximized by optimizing transmission power for a given energy-arrival rate.

A. Prior Work and Motivation

The fluctuation in harvested energy due to random energy arrivals requires redesigning existing transmission algorithms for wireless communication systems. Assuming infinitely backlogged data, existing work focuses on adapting transmission power to channel states and the temporal profile of energy arrivals to maximize the system throughput [2]-[5]. For single-user systems, the optimal power-control policies are shown to be variations of the classic water-filling policy such that the causality of energy arrivals and finite battery capacity are accounted for [2], [3]. Adaptive transmission for broadcast channels with energy harvesting has been also investigated [4], [5]. In [4], the optimal power-control for a two-user singleantenna broadcast channel is shown to attempt to allocate a fix amount of harvested energy to the user with the better channel before giving the remaining energy to the other user. A two-user multiple-input-multiple-output broadcast channel is considered in [5] where one user receives data and the other scavenges transmission energy, and the precoder at the base station is designed to optimize the tradeoff between the data rate and the rate of harvested energy.

In wireless communication systems with both bursty dataand-energy arrivals, buffering energy and data creates two corresponding queues at each transmitter. Jointly controlling these two coupled queues is more challenging than controlling only the data queue in traditional systems with reliable power supplies [6]. The algorithms for optimally controlling the energy-and-data queues have been proposed for single-user systems [7] and downlink systems [8] to minimize the packet transmission delay, for interference channels to minimize queueing delay [9], and for downlink systems to maximize the system throughput [10]. These algorithms share a common objective of optimizing a particular performance metric for given average harvested power. The objective is aligned with

K. Huang is with the Hong Kong Polytechnic University, Hong Kong. Email: huangkb@ieee.org. This paper has been presented in part at Asilomar Conf. on Signals, Systems, and Computers 2011 and at IEEE Intl. Conf. on Communications (ICC) 2013. Copyright © 2012 IEEE. Personal use of this material is permitted. However, permission to use this material for any other purposes must be obtained from the IEEE by sending a request to pubspermissions@ieee.org.

that for designing the traditional energy-efficient systems with only data queues, namely minimizing the average transmission power under a performance constraint such as fixed packettransmission delay for single-user systems [11], [12], allowed queueing delay for downlink systems [13], and given traffic in wireless networks [14].

Wireless networks with energy harvesting have been studied [15]–[17]. For a wireless sensor network with energy harvesting and based on a simple channel model that omits channel noise and path loss, the probability that a sensor successfully transmits a data packet to a fusion center is analyzed in [15] for different multiple-access protocols including time-division multiple access and Aloha like random access. Managing traffic load in time and space is important for wireless sensor networks to be self sustaining through energy harvesting. Therefore, distributive strategies are proposed in [16] for adapting traffic load to the spatial-and-temporal energy profile and evaluated using a network prototype. For a two-user interference network with energy harvesting, the data-andenergy arrivals are modeled as Bernoulli processes and the stability region is characterized such that it comprises all datarate pairs under the constraint of finite data-queue lengths [17]. In view of prior work, there are few results that quantify the tradeoff between the network throughput and the energyarrival rate though such results specify the fundamental limit of the network performance. This tradeoff is investigated in the sequel using a stochastic-geometry approach.

Stochastic geometry provides a set of powerful mathematical tools for modeling and designing wireless networks [18]. MANETs based on random access and carrier-sensing multiple access have been modeled using the PPPs [19], [20] and Matern hard-core processes [21], respectively. Cellular networks have been shown to be suitably modeled using the Poisson Voronoi tessellation [22]. Models of coexisting networks can be constructed by superimposing multiple point processes [23], [24]. Stochastic-geometry models of wireless networks have been employed to quantify the networkperformance gains due to physical-layer techniques such as opportunistic transmission [25], bandwidth partitioning [26], successive interference cancellation [27], and multi-antenna techniques [28]-[32]. The performance metric typically considered in the literature is the network spatial throughput under a constraint on the outage probability for a target SINR, which is also adopted in this paper. Using this metric, most prior work focuses on deriving the outage probability using techniques such as the Laplace transform [19], [22] and probabilistic inequalities [20], [25]. This paper considers a MANET with Poisson distributed transmitters similar to the existing literature (see e.g., [20]). However, the transmitters in the current network model are powered by energy harvesting instead of reliable power supplies as in prior work. The consideration of energy harvesting introduces several new design issues including the aforementioned tradeoff between the network throughput and energy-arrival rate, the corresponding optimization of transmission power, and the effect of finite energy storage, which are investigated in the sequel.

2

B. Contributions and Organization

For exposition, a few definitions and notations are provided as follows. Time is slotted. Define the transmission probability ρ as the probability that a transmitter transmits and the *network interference temperature* as the maximum active transmitter density under the outage-probability constraint. Let λ_0 denote the transmission power, and Z a nonnegative random variable representing the amount of energy harvested by a typical harvester in an arbitrary slot.¹ Note that $E[Z] = \lambda_e$ and the density of active transmitters is equal to $\rho\lambda_0$.

The main contributions of this paper are summarized as follows.

- Assume infinite battery capacity. Using the law of large numbers and random-walk theory, it is proved that ρ is equal to the smaller of λ_e/P and one. It is worth mentioning that the tractable analysis relies on assuming the sub-optimal fixed-power transmission. To the best of the author's knowledge, the aforementioned result is unknown from existing work that mostly focuses on designing the optimal adaptive-transmission algorithms [2]–[5], [7]–[10].
- 2) Consider the case of finite battery capacity. Bounds on ρ are derived, which converge to the results stated above as the battery capacity increases. Moreover, two special cases are considered. If Z is bounded and no larger than P, it is shown that ρ is equal to λ_e/P so long as the battery capacity is larger than 2P. If Z is a discrete random variable, ρ is analyzed using Markov-chain theory.
- 3) Assume infinite battery capacity. By applying derived results on transmission probability and tools from stochastic geometry, the network throughput is maximized by optimizing P for given λ_e. Consider the condition that λ₀ is smaller than the network interference temperature evaluated for equal P and λ_e. If this condition holds, the maximum throughput R^{*} is shown to be

$$R^* = \lambda_0 \log_2(1+\theta)$$

where θ is the target SINR. If the aforementioned condition is not satisfied,

$$R^* = \frac{\lambda_0 \lambda_e}{P^*} \log_2(1+\theta)$$

where the optimal transmission power P^* is larger than λ_e and solves a derived polynomial equation.

 Furthermore, the limits of the maximum network throughput are obtained for the extreme cases of high energy-arrival rates (λ_e → ∞) and dense networks (λ₀ → ∞). Specifically.

$$\lim_{\lambda_e \to \infty} R^*(\lambda_e) = \min\left(\lambda_0, \frac{\mu_{\epsilon}}{\theta^{\frac{2}{\alpha}}}\right) \log_2(1+\theta)$$
$$\lim_{\lambda_0 \to \infty} R^*(\lambda_0) = \frac{\mu_{\epsilon}}{\theta^{\frac{2}{\alpha}}} \log_2(1+\theta)$$

¹A typical point is selected from a spatial point process by uniform sampling.



Fig. 1. Single-antenna transmitters in the MANET are modeled as a homogeneous PPP in the horizontal plane. Each transmitter is powered by an energy harvester and transmits to an intended receiver at an unit distance.

where μ_{ϵ} is a positive constant determined by the maximum outage probability.

The remainder of this paper is organized as follows. The network model and performance metric are described in Section II. The transmission probability is analyzed in Section III. The results are applied to maximize the network throughput in Section IV. Numerical results are presented in Section V followed by concluding remarks in Section VI.

II. MODEL AND METRIC

A. Network Model

As illustrated in Fig. 1, the transmitters $\{T\}$ of the MANET are distributed in the Euclidean plane \mathbb{R}^2 following a homogeneous PPP Φ with density λ_0 , where T denotes the coordinates of a transmitter. Each transmitter is associated with an intended receiver located at a unit distance, which is assumed to simplify the expression for the received signal power by omitting the data-link path loss. The signal transmitted by Twith power P is received by a receiver located at X with power equal to $P|X - T|^{-\alpha}$ with $\alpha > 2$ being the path-loss exponent. In other words, propagation is characterized by path loss while fading is omitted to simplify notation.²

Time is partitioned into slots of unit duration with t denoting the slot index. The amount of energy harvested by the typical harvester in the t-th slot is represented by the nonnegative random variable Z_t .

Assumption 1. The energy-arrival process $\{Z_t\} \subset \mathbb{R}^+$ is an i.i.d. sequence and independent of other energy-arrival processes. Moreover, the *cumulant generating function* of the random variable $(Z_t - \beta)$ with β being a given constant, namely $\ln \mathsf{E} \left[e^{r(Z_t - \beta)} \right]$, has a root $r^*(\beta)$ such that $r^*(\beta) > 0$ if $\beta > \lambda_e$ and $r^*(\beta) < 0$ if $\beta < \lambda_e$.

This assumption allows the use of results on the large deviation of random walks in the subsequent analysis [33].

Let *B* denote the battery capacity identical for all harvesters. Moreover, the typical transmitter and the battery level of the corresponding (typical) harvester are represented by T_0 and S_t , respectively. A transmitter transmits one data packet with fixed power *P* whenever the corresponding battery level exceeds *P*. As a result, S_t evolves as

$$S_t = \min(S_{t-1} + Z_t - PI(S_{t-1} \ge P), B), \quad t = 1, 2, \cdots (1)$$

where $S_0 = 0$ and the indicator function I(A) for an event A is equal to one if A occurs or else is zero. The battery-level evolutions in prior work are similar to that in (1) but with fixed power P replaced with power adapted to factors such as the channel state and battery level [2], [9], [10].

B. Performance Metric

Assume infinitely backlogged and packetized data. The transmission probability ρ can be written as

$$o = \lim_{n \to \infty} \frac{1}{n} \sum_{t=1}^{n} \mathsf{E}[I(S_t \ge P)].$$
 (2)

According to Coloring Theorem [34], the process of active transmitters, denoted as II, is a PPP with density $\lambda_t = \rho \lambda_0$. Data is encoded at a fixed rate $\log_2(1 + \theta)$ bit/s/Hz with $\theta > 0$ being the target SINR. Correct decoding of a data packet requires the received SINR to be no smaller than θ or else an outage event occurs. The outage probability P_{out} is defined as $P_{\text{out}} = \Pr(\text{SINR} < \theta)$ where SINR represents the received SINR at the receiver for T_0 . It is assumed that the receiver for T_0 is located at the origin, which does not compromise the generality based on Slyvnyak's Theorem [35], and that noise has unit variance. Based on these assumptions, P_{out} can be written as

$$P_{\mathsf{out}} = \Pr\left(\frac{P}{\sum_{T \in \Pi \setminus \{T_0\}} P|T|^{-\alpha} + 1} < \theta\right)$$
(3)

$$= \Pr\left(\sum_{T \in \Pi \setminus \{T_0\}} |T|^{-\alpha} > \frac{1}{\theta} - \frac{1}{P}\right)$$
(4)

$$= \Pr\left(\sum_{T \in \Pi} |T|^{-\alpha} > \frac{1}{\theta} - \frac{1}{P}\right)$$
(5)

where the summation in (3) represents the interference power and (5) uses Slyvnyak's Theorem. It is worth mentioning that the probabilities in (3) and (4) are *palm measures* [35] but that in (5) is not. To ensure the quality-of-service, an outageprobability constraint is applied such that $P_{\text{out}} \leq \epsilon$ with $0 < \epsilon \ll 1$. The performance metric is the spatial networkthroughput density R (bit/s/Hz/unit-area) that is referred to simply as the network throughput and defined as

$$R = \lambda_t \log_2(1+\theta) \tag{6}$$

$$=\lambda_0 \rho \log_2(1+\theta) \tag{7}$$

where ρ is controlled by adjusting P such that the outageprobability constraint is satisfied. To be precise, R should be scaled by the success probability $(1 - P_{out})$ but this factor is close to one given $\epsilon \ll 1$ and thus omitted for ease of notation.

²The consideration of random transmission distances and fading has no effect on the main results except that the parameter μ_{ϵ} defined in (34) has to be redefined by including additional random variables.

III. TRANSMISSION PROBABILITY

A. Infinite Battery Capacity

Deriving transmission probability requires analyzing the distribution of battery levels at energy harvesters. By substituting $B \rightarrow \infty$ into (1), the battery level at the typical energy harvester with infinite battery capacity evolves as

$$S_t = S_{t-1} + Z_t - PI(S_{t-1} \ge P).$$
(8)

The distribution of S_t can be related to the threshold-crossing probability for a random walk as follows. Denote the instants when the battery level crosses the threshold P from below as t_1, t_2, \cdots , namely that $S_{t_n-1} < P$ and $S_{t_n} \ge P$ for n = $1, 2, \cdots$. These time instants are grouped into the set $\mathcal{T} =$ $\{t_1, t_2, \cdots\}$. Moreover, define the random variable $\overline{Z}_t = Z_t -$ P and two random processes $\{G_t\}$ and $\{G'_t\}$ as

$$G_t = \max(G_{t-1} + \bar{Z}_t, 0)$$
(9)

$$G'_{t} = \begin{cases} S_{t}, & t \in \mathcal{T} \\ G'_{t-1}, & t \notin \mathcal{T} \end{cases}$$
(10)

with $G_0 = 0$ and $G'_0 = P$. Based on (9), the probability that $\{G_t\}$ crosses a threshold x > 0 in the *t*-th slot can be written as

$$\Pr(G_t > x) = \Pr\left(\max\left(\bar{Z}_t, \bar{Z}_t + \bar{Z}_{t-1}, \cdots, \sum_{n=1}^t \bar{Z}_n\right) > x\right).$$
(11)

Consider the random walk $\{\sum_{n=1}^{m} \bar{Z}_{t-m+1}\}\$ starting in the *t*-th slot and progressing backwards. The probability in (11) can be interpreted as the probability that the said random walk with a negative drift ever crosses the threshold *x* by the *t*-th step. Applying Kingman bound on the threshold-crossing probability for a random walk [33, p234] gives that for all t > 0,

$$\Pr(G_t > x) \le e^{-r^*(P)x}, \qquad \lambda_e < P \tag{12}$$

where $r^*(P)$ as defined in Assumption 1 with $\beta = P$ is the positive root of the cumulant generating function of \overline{Z}_t .

Lemma 1. Given infinite battery capacity, the battery level S_t satisfies

$$S_t \le G_t + G'_t.$$

The proof of Lemma 1 is provided in Appendix A. Using (12) and Lemma 1, the threshold-crossing probability for the battery level can be shown to be bounded as follows.

Lemma 2. Given infinite battery capacity and $\lambda_e < P$, the distribution of the battery level S_t satisfies

$$\Pr(S_t > x) \le 2e^{-\frac{1}{2}r^*(P)(x-2P)}$$
(13)

with $r^*(P) > 0$.

The proof of Lemma 2 is given in Appendix B. Define the *energy-overshoot function* $D_t : \mathbb{R}^+ \to \mathbb{R}^+$ as the expected amount of energy stored in the typical harvester in excess of a threshold x > 0 in the *t*-th slot:

$$D_t(x) = \int_x^\infty (y - x) f_s(y, t) dy \tag{14}$$

where $f_s(y,t)$ represents the probability density function of S_t . The function $D_t(x)$ can be bounded as shown in Lemma 3, which is proved in Appendix C using Lemma 2.

Lemma 3. Given infinite battery capacity and $\lambda_e < P$, the energy-overshoot function $D_t(x)$ satisfies

$$D_t(x) \le \frac{4}{r^*(P)} e^{-\frac{1}{2}r^*(P)(x-2P)}, \quad \forall t \ge 0$$

with $r^*(P) > 0$.

Using Lemma 3, the main result of this section is readily obtained as shown below.

Theorem 1. Given infinite battery capacity, the transmission probability is

$$\rho = \min\left(1, \frac{\lambda_e}{P}\right).$$

Proof: First, consider the case of $\lambda_e > P$. Replacing the indicator function in (8) with one yields a lower bound on S_t , namely that $S_t \ge \sum_{m=1}^t \overline{Z}_m$. As a result, ρ given in (2) can be lower bounded as

$$\rho \ge \lim_{n \to \infty} \frac{1}{n} \sum_{t=1}^{n} \mathsf{E} \left[I \left(\sum_{m=1}^{t} \bar{Z}_m \ge P \right) \right]$$
$$= \lim_{n \to \infty} \frac{1}{n} \sum_{t=1}^{n} \Pr \left(\frac{1}{t} \sum_{m=1}^{t} (Z_m - \lambda_e) \ge P - \lambda_e + \frac{P}{t} \right). \quad (15)$$

Using $E[Z_n] = \lambda_e$ and applying the weak law of large numbers [33], for given $\tau > 0$ and $\delta > 0$, there exists k > 0 such that for all $t \ge k$,

$$\Pr\left(\frac{1}{t}\sum_{m=1}^{t}(Z_m - \lambda_e) \ge -\tau\right) \ge 1 - \delta.$$
 (16)

Since $\lambda_e > P$, it follows that given $\delta > 0$, there exist $\tau > 0$ and k' > 0 such that $\tau < (\lambda_e - P - \frac{P}{t})$ for all $t \ge k'$. Using this fact and substituting (16) into (15),

$$\rho \ge \lim_{n \to \infty} \frac{1}{n} \sum_{t=\max(k,k')}^{n} \Pr\left(\frac{1}{t} \sum_{m=1}^{t} (Z_m - \lambda_e) \ge -\tau\right)$$
$$\ge \lim_{n \to \infty} \frac{1}{n} \sum_{t=\max(k,k')}^{n} (1-\delta)$$
$$= \lim_{n \to \infty} \frac{[n - \max(k,k')](1-\delta)}{n}$$
$$= 1 - \delta.$$
(17)

As δ is arbitrary and $\rho \leq 1$, the desired result for the case of $\lambda_e > P$ follows from (17) and letting $\delta \to 0$.

Next, consider the case of $\lambda_e < P$. The expected total amounts of harvested and transmitted energy by the *t*-th slot differ by the battery level in the *t*-th slot, namely

$$\sum_{m=1}^{t} \mathsf{E}[Z_m] = P \sum_{m=1}^{t} \mathsf{E}[I(S_{m-1} > P)] + \mathsf{E}[S_t].$$
(18)

Since $\mathsf{E}[S_t] \ge 0$,

$$\lambda_e \ge P \lim_{t \to \infty} \frac{1}{t} \sum_{m=1}^{\iota} \mathsf{E}[I(S_{m-1} > P)]$$
$$= P\rho \tag{19}$$

where (19) is obtained using the definition of ρ in (2). It follows that

$$\rho \le \frac{\lambda_e}{P}, \qquad \lambda_e < P.$$
(20)

Note that $\mathsf{E}[S_t] \leq x + D_t(x)$ with x > 0. Using Lemma 3, for given $\delta > 0$, there exists x > 0 such that $\mathsf{E}[S_t] \leq x + \delta$. Combining this fact and (18) yields

$$\lambda_e = P \lim_{t \to \infty} \frac{1}{t} \sum_{m=1}^t \mathsf{E}[I(S_{t-1} > P)] + \lim_{t \to \infty} \frac{\mathsf{E}[S_t]}{t}$$
$$\leq P \lim_{t \to \infty} \frac{1}{t} \sum_{m=1}^t \mathsf{E}[I(S_{t-1} > P)] + \lim_{t \to \infty} \frac{x+\delta}{t}$$
$$= P\rho.$$

As a result,

$$\rho \ge \frac{\lambda_e}{P}, \qquad \lambda_e < P.$$
(21)

The desired result for the case of $\lambda_e < P$ is proved by combining (20) and (21).

Last, the desired result for the boundary case of $\lambda_e = P$ is proved by using the results proved above for $\lambda_e \neq P$ and letting $P \rightarrow \lambda_e$ from either the right or the left, completing the proof.

Remark 1. According to Theorem 1, if $P < \lambda_e$, each transmitter transmits continuously in the steady state and is free of interruption caused by energy shortage. However, continuous transmissions are at the cost that the fraction of harvested energy at the rate of $(\lambda_e - P)$ is never used for transmission and hence wasted. Next, if $P > \lambda_e$, there exists nonzero probability that the battery level of a transmitter is below P and hence transmission can be interrupted. Nevertheless, all harvested energy will be eventually used for transmission.

B. Finite Battery Capacity

The dynamics of the battery level S_t at the typical harvester are characterized in (1). Let $\tilde{D}_t(x)$ denote the energyovershoot function for the case of finite-battery capacity, which is defined similarly as $D_t(x)$ in the preceding section. Given finite battery capacity, battery overflow can occur such that the battery saturates and arriving energy has to be discarded, where the expected amount of discarded energy is measured by $\tilde{D}_t(B)$. For the case of $\lambda_e < P$, $\tilde{D}_t(B)$ can be bounded by the same upper bound on $D_t(x)$ (see Lemma 3) as shown below.

Lemma 4. Given finite battery capacity and $\lambda_e < P$, the energy-overshoot function $\tilde{D}_t(B)$ satisfies

$$\tilde{D}_t(B) \le \frac{4}{r^*(P)} e^{-\frac{1}{2}r^*(P)(B-2P)}$$

with $r^*(P) > 0$.

Lemma 4 is proved in Appendix D. Next, the tail probability of S_t can be bounded for the case of $\lambda_e > P$ as shown in the following lemma, which is proved in Appendix E.

Lemma 5. Given finite battery capacity and $\lambda_e > P$, the distribution of the battery level S_t satisfies

$$\lim_{n \to \infty} \frac{1}{n} \sum_{t=1}^{n} \Pr(S_t < x) \le e^{r^*(P)(B - x)}$$

with $r^*(P) < 0$ and $x \in [0, B]$.

Using Lemma 4 and 5, bounds on the transmission probability are obtained as follows.

Proposition 1. Given finite battery capacity, the transmission probability ρ satisfies the following.

1) If
$$\lambda_e < P$$

$$\frac{\lambda_e}{P} \left[1 - \frac{4}{\lambda_e r^*(P)} e^{-\frac{1}{2}r^*(P)(B-2P)} \right] \le \rho \le \frac{\lambda_e}{P} \quad (22)$$
with $r^*(P) > 0$.
2) If $\lambda_e > P$
 $1 - e^{r^*(P)(B-P)} \le \rho \le 1 \quad (23)$

with
$$r^*(P) < 0$$
.
3) If $\lambda_e = P$

$$\max_{x>0} \left\{ \frac{\lambda_e}{\lambda_e + x} \left[1 - \frac{4e^{-\frac{1}{2}r^*(\lambda_e + x)(B - 2\lambda_e - 2x)}}{\lambda_e r^*(\lambda_e + x)} \right] \right\} \quad (24)$$

$$\leq \rho \leq 1$$

with $r^*(\lambda_e + x) > 0$ given x > 0.

Proof: Consider the case of $\lambda_e < P$. By accounting for the discarded energy due to battery overflow, the expected total amounts of transmitted and harvested energy by the *t*-th slot is related by modifying (18) as

$$\sum_{m=1}^{t} \mathsf{E}[Z_m] = P \sum_{m=1}^{t} \mathsf{E}[I(S_{m-1} > P)] + \mathsf{E}[S_t] + \sum_{m=1}^{t} \tilde{D}_m(B)$$

where the last term gives the expected amount of discarded energy. Applying Lemma 4 and $S_t \leq B$ yields

$$\begin{split} \lambda_e &\leq P \lim_{t \to \infty} \frac{1}{t} \sum_{m=1}^t \mathsf{E}[I(S_{m-1} > P)] + \lim_{t \to \infty} \frac{B}{t} + \\ & \frac{4}{r^*(P)} e^{-\frac{1}{2}r^*(P)(B-2P)} \\ & \leq P\rho + \frac{4}{r^*(P)} e^{-\frac{1}{2}r^*(P)(B-2P)} \end{split}$$

and the first inequality in (22) follows. The second inequality is proved using Theorem 1 and the fact that limiting the battery capacity reduces ρ .

Next, consider the case of $\lambda_e > P$. The definition of ρ in (2) can be rewritten as

$$\rho = 1 - \lim_{n \to \infty} \frac{1}{n} \sum_{t=1}^{n} \Pr(S_t < P).$$
(25)

The last term can be upper bounded using Lemma 5, yielding the first inequality in (23). The second inequality is trivial.

Last, consider the case of $\lambda_e = P$. Let $\rho'(x)$ denote the transmission probability for the virtual scenario where all

transmissions use the power (P + x) with x > 0. It can be proved similarly as the first inequality in (22) that

$$\rho'(x) \ge \frac{\lambda_e}{\lambda_e + x} \left[1 - \frac{4e^{-\frac{1}{2}r^*(\lambda_e + x)(B - 2\lambda_e - 2x)}}{\lambda_e r^*(\lambda_e + x)} \right]$$
(26)

with x > 0. Note that $\rho'(x)$ also gives the transmission probability for a virtual transmission strategy that removes x unit of energy from the battery of a harvester following every instance of transmission. Since removing energy from the battery reduces the transmission probability, $\rho \ge \rho'(x)$ holds. Combining this inequality and (26) yields

$$\rho \geq \frac{\lambda_e}{\lambda_e + x} \left[1 - \frac{4}{\lambda_e r^* (\lambda_e + x)} e^{-\frac{1}{2} r^* (\lambda_e + x)(B - 2\lambda_e - 2x)} \right]$$

for all x > 0. The first inequality in (23) follows. The second inequality is trivial, completing the proof.

Remark 2. For a sanity check, it can be observed from Proposition 1 that as $B \to \infty$, ρ converges to its counterpart for the case of infinite battery capacity as stated in Theorem 1.

Remark 3. By comparing Propostion 1 with Theorem 1, it is observed that the degradation of ρ due to finite battery capacity decreases exponentially with increasing *B*. Hence the effect of finite battery capacity on ρ is expected to diminish rapidly as *B* increases, which is confirmed by simulation results in the sequel.

Remark 4. The battery-level process for the case of $\lambda_e = P$ is related to a random walk with a zero drift for which $r^*(\lambda_e)$ is not defined and the threshold-crossing probability does not have an exponential upper bound [33]. This causes the difficulty in deriving a lower bound on ρ simpler than that in (24). Moreover, the maximization of the lower bound in (24) cannot be solved analytically. One should not expect that the bound is maximized as $x \to 0$ because the function $r^*(\lambda_e + x)$ can be a monotone increasing function of x. For instance, given that Z_t follows the exponential distribution with unit mean, it is obtained that

$$r^*(1+x) = \frac{W_0\left(-(1+x)e^{-(1+x)}\right)}{1+x} + 1$$

where W_0 denotes the 0-th branch of the Lambert W function. The function can be plotted and observed to be monotone increasing for $x \ge 0$.

In general, exact analysis of ρ for the case of finite battery capacity is challenging except for some special cases, two of which are discussed as follows.

1) Special case: bounded energy arrivals: Consider the case that Z_t has bounded support and $Z_t \in [0, z_{\max}]$.

Proposition 2. Consider bounded energy arrivals. If $z_{\text{max}} \leq P$ and the battery capacity B > 2P, the probability for batteryoverflow is zero and the transmission probability is

$$\rho = \frac{\lambda_e}{P} \tag{27}$$

Proof: By expanding (1),

$$S_t = \begin{cases} \min(S_{t-1} + Z_t - P, B), & S_{t-1} \ge P\\ \min(S_{t-1} + Z_t, B), & S_{t-1} < P. \end{cases}$$
(28)

Given B > 2P and $Z_t \leq P$, it follows from (28) that $S_{t-1} \leq 2P$ ensures $S_t \leq 2P$. Since $S_0 = 0$, this result implies zero battery-overflow probability. Consequently, the transmission probability is identical to that for the case of infinite battery capacity in Theorem 1, proving the desired result in (27).

2) Special case: discrete energy arrivals: Assume that Z_t is a discrete random variable and takes on values from $\{0, 1, 2, \dots\}$, and that P and B are positive integers with $B \ge P$. Under these assumptions, the distribution of battery levels can be analyzed using Markov-chain theory. Since S_t is independent of $\{S_n\}_{n=1}^{t-2}$ given S_{t-1} , $\{S_t\}$ satisfies the Markov property and is hence a Markov chain. Given battery capacity B, the Markov chain $\{S_t\}$ has the state space $\{0, 1, \dots, B\}$. Define the transition probability p_{mn} as $p_{mn} = \Pr(S_t = n \mid S_{t-1} = m)$. If n < B, the battery level is below its limit and the transition probability is given as

$$p_{mn} = \Pr(Z_t = n - m + PI(m \ge P)), \quad n < B$$
 (29)

where the indicator function I specifies if energy of P units is consumed for transmission in the current slot depending on if m reaches P. If n = B, the transmission from state m to n includes all events that the energy arrival in the current slot causes battery saturation, namely $Z_t = \{B - m, B - m +$ $1, \dots\}$ if m < P and $Z_t = \{B - m + P, B - m + P + 1, \dots\}$ if $m \ge P$. It follows that

$$p_{mn} = \sum_{k=B-m}^{\infty} \Pr\left(Z_t = k + PI(m \ge P)\right), \quad n = B.$$
 (30)

Combining (29) and (30) yields that

$$p_{mn} = \begin{cases} \Pr(Z_t = n - m), & m < P, n < B\\ \Pr(Z_t = n - m + P), & m \ge P, n < B\\ \sum_{k=B-m}^{\infty} \Pr(Z_t = k), & m < P, n = B\\ \sum_{k=B-m}^{\infty} \Pr(Z_t = k + P), & m \ge P, n = B. \end{cases}$$
(31)

Let π_m denote the steady-state probability of state m of the Markov chain $\{S_t\}$. Moreover, let **P** represent the transitionprobability matrix with the (m, n)-th element given by p_{mn} and π the steady-state-probability row vector with the mth element given by π_m . Applying Perron-Frobenius theorem [33],

$$\pi \mathbf{P} = \boldsymbol{\pi}.\tag{32}$$

The stationary probabilities $\{\pi_m\}$ can be computed by solving (32) under the constraint $\sum_{m=0}^{B} \pi_m = 1$. Given π , the transmission probability is obtained as $\rho = \sum_{m=P}^{B} \pi_m$.

The stationary probabilities $\{\pi_m\}$ can be derived in closed form for the simple case of binary energy arrivals, namely that $Z_t \in \{0, 1\}$. Then energy-arrival rate is $\lambda_e = \mathsf{E}[Z_t] =$

where $\lambda_e \leq P$.



Fig. 2. A Markov chain modeling the battery level at the typical transmitter for the special case of binary energy arrivals and P = 3.

 $\Pr(Z_t=1).$ The corresponding transition probabilities are modified from (31) as

$$p_{mn} = \begin{cases} 1 - \lambda_e, & (m = P, n = 0) \text{ or } (m < P, n = m) \\ \lambda_e, & (m = P, n = 1) \text{ or } (m < P, n = m + 1) \\ 0, & \text{otherwise.} \end{cases}$$

Based on above transition probabilities, the Markov chain $\{S_t\}$ is illustrated in Fig. 2. The stationary distribution $\{\pi_m\}$ satisfies the following equations obtained from (32) and the above expression for p_{mn} :

$$\pi_{0} = \pi_{P}(1 - \lambda_{e}) + \pi_{0}(1 - \lambda_{e})$$

$$\pi_{1} = \pi_{P}\lambda_{e} + \pi_{1}(1 - \lambda_{e}) + \pi_{0}\lambda_{e}$$

$$\pi_{m} = \pi_{m-1}\lambda_{e} + \pi_{m}(1 - \lambda_{e}), \qquad m = 2, \cdots, P - 1$$

$$\pi_{P} = \pi_{P-1}\lambda_{e}.$$
(33)

Solving the equations in (33) and $\sum_{m=0}^{P} \pi_m = 1$ gives the following proposition.

Proposition 3. Consider the case that energy arrivals are binary $(Z_t \in \{0,1\})$ and the transmission power P is a positive integer no larger than the battery capacity B. The distribution of the battery level S_t is

$$\pi_0 = \frac{1 - \lambda_e}{P}, \qquad \pi_P = \frac{\lambda_e}{P}$$
$$\pi_m = \frac{1}{P}, \qquad m = 1, \cdots, P - 1$$

The probability for battery overflow is zero and the transmission probability is $\rho = \pi_P = \lambda_e/P$.

IV. NETWORK THROUGHPUT

In this section, using results on transmission probability as derived in the preceding section, the network throughput is maximized by optimizing transmission power under the outage-probability constraint and assuming infinite battery capacity. It is straightforward though tedious to extend the network-throughput analysis to the case of finite battery capacity using related results from the last section.³ Such an extension provides few new insights and thus is omitted.

A. Maximum Network Throughput

To characterize the network throughput, transmission power P is related to the active transmitter density λ_t under the outage-probability constraint. To this end, we define a parameter μ_{ϵ} , called the *nominal node density*, as the density of a homogeneous PPP $\Lambda(\mu_{\epsilon})$ such that

$$\Pr\left(\sum_{T\in\Lambda(\mu_{\epsilon})}|T|^{-\alpha}>1\right)=\epsilon\tag{34}$$

where the summation can be interpreted as the interference power measured at a receiver located at the origin from unitpower interferers distributed as $\Lambda(\mu_{\epsilon})$. Note that μ_{ϵ} depends only on ϵ , α and the distribution of a PPP and is independent of other network parameters. Moreover, μ_{ϵ} is a strictlymonotone-increasing function of ϵ due to the fact that denser interferers result in larger outage probability for a link. The expression of μ_{ϵ} has no closed form and its value can be computed by simulation (see e.g., [36]). The relation between μ_{ϵ} and ϵ is shown in Fig. 3. The following lemma results from Mapping Theorem [34, p18].

Lemma 6. Consider the homogeneous PPP $\Lambda(\mu_{\epsilon}) = \{X\}$. The process $a\Lambda(\mu_{\epsilon}) = \{aX\}$ with a > 0 is a homogeneous PPP with density μ_{ϵ}/a^2 .

Define the admissible set \mathcal{F} as all combinations of (λ_t, P) that satisfy the outage-probability constraint:

$$\mathcal{F} = \{ (\lambda_t, P) \in \mathbb{R}^+ \times \mathbb{R}^+ \mid P_{\mathsf{out}}(\lambda_t, P) \le \epsilon \}.$$
(35)

A combination (λ_t, P) is *admissible* if it belongs to \mathcal{F} . To derive \mathcal{F} , since Π follows the same distribution as $\Lambda(\lambda_t)$, P_{out} in (5) can be rewritten as

$$P_{\mathsf{out}} = \Pr\left(\sum_{T \in \Lambda(\lambda_t)} |T|^{-\alpha} > \frac{1}{\theta} - \frac{1}{P}\right).$$

According to Lemma 6, $\Lambda(\lambda_t)$ has the same distribution as $a\Lambda(\mu_{\epsilon})$ with $a = \sqrt{\mu_{\epsilon}/\lambda_t}$. Consequently

$$P_{\mathsf{out}} = \Pr\left(\sum_{T \in a\Lambda(\mu_{\epsilon})} |T|^{-\alpha} > \frac{1}{\theta} - \frac{1}{P}\right)$$
$$= \Pr\left(\sum_{T \in \Lambda(\mu_{\epsilon})} |aT|^{-\alpha} > \frac{1}{\theta} - \frac{1}{P}\right)$$
$$= \Pr\left(\sum_{T \in \Lambda(\mu_{\epsilon})} |T|^{-\alpha} > \left(\frac{\mu_{\epsilon}}{\lambda_{t}}\right)^{\frac{\alpha}{2}} \left(\frac{1}{\theta} - \frac{1}{P}\right)\right). \quad (36)$$

Combining (34), (35) and (36) leads to the following lemma.

Lemma 7. The admissible set \mathcal{F} is given as

$$\mathcal{F} \to \left\{ (\lambda_t, P) \in \mathbb{R}^+ \times \mathbb{R}^+ \mid \lambda_t \le \zeta(P) \right\}$$
(37)

where $\zeta(P)$ represents the network interference temperature and is given as

$$\zeta(P) = \mu_{\epsilon} \left(\frac{1}{\theta} - \frac{1}{P}\right)^{\frac{2}{\alpha}}, \qquad P \ge \theta.$$
(38)

³Essentially, the network throughput for the case of finite battery capacity can be bounded by modifying the current analysis such that the transmission probability ρ is replaced with its bounds as specified in Proposition 1.



Fig. 3. The relation between the nominal node density μ_{ϵ} and the maximum outage probability ϵ

Remark 5. The network interference temperature $\zeta(P)$ specifies the maximum density of interference a link can tolerate without violating the outage-probability constraint. This quantity is analogous to the *interference temperature* in cognitiveradio systems that measures the maximum amount of additional interference for a certain frequency band without significantly degrading the reliability of communications therein [37]. On one hand, the network for large P is interferencelimited and further increasing P does not contribute any network-throughput gain. Correspondingly, it can be observed from (38) that $\zeta(P)$ saturates as $P \to \infty$:

$$\lim_{P \to \infty} \zeta(P) = \frac{\mu_{\epsilon}}{\theta^{\frac{2}{\alpha}}}.$$
(39)

On the other hand, the network for small P is noise-limited. The value of $\zeta(P)$ is not well defined if P is below the target SINR θ , for which the outage constraint cannot be satisfied even in the absence of interference.

Remark 6. The admissible set \mathcal{F} is illustrated as the shaded region in Fig. 4. Increasing μ_{ϵ} (relaxing the outage-probability constraint) enlarges \mathcal{F} and vice versa. Let λ^* denote the maximum of the admissible values for λ_t under the outage constraint. Then the boundary of \mathcal{F} corresponds to $\lambda^* = \zeta(P)$.

Remark 7. The condition $\lambda_t \leq \zeta(P)$ for the admissible set in (37) guarantees that *P* (equal to the received SNR) is no smaller than θ that is the minimum received SINR required for correct decoding.

We are ready to derive the maximum network throughput R^* and the optimal transmission power P^* . Let f denote the function that maps P to λ_t for fixed λ_0 and λ_e , which is obtained from Theorem 1 as

$$f(P) = \lambda_0 \min\left(1, \frac{\lambda_e}{P}\right)$$

The derivation of R^* can be intuitively explained using Fig. 4. It can be observed form the curve depicting the function f(P) that f(P) is fixed at λ_0 for all $P \leq \lambda_e$ and a strictly-monotone-decreasing function for $P > \lambda_e$. Let P^* correspond to the intersection between the curve (P, f(P)) and the admissible set \mathcal{F} . Then under the outage-probability constraint, f(P) is maximized at $P = P^*$, which corresponds to R^* based on the definition in (6) since it is proportional to λ_t . Note that the claim is based on the assumption $\epsilon \ll 1$ or else need not hold (see Remark 10). Depending on if P^* is larger or smaller than λ_e , P^* and R^* have different expressions as shown in Theorem 2 that states the main result of this section.

Theorem 2. Given infinite battery capacity, the maximum network throughput R^* and the optimal transmission power P^* are specified as follows.

1) If $\lambda_0 \leq \zeta(\lambda_e)$,

$$R^* = \lambda_0 \log_2(1+\theta) \tag{40}$$

and P^* is an arbitrary value in the range $\left[\frac{\theta}{1-\theta(\lambda_0/\mu_{\epsilon})^{\frac{\alpha}{2}}}, \lambda_e\right]$. 2) If $\lambda_0 > \zeta(\lambda_e) > 0$,

$$R^* = \frac{\lambda_0 \lambda_e}{P^*} \log_2(1+\theta) \tag{41}$$

where $P^* \ge \lambda_e$ and $\sqrt{P^*}$ solves the following polynomial equation:

$$x^{\alpha} - \theta x^{\alpha - 2} - \theta \left(\frac{\lambda_0 \lambda_e}{\mu_{\epsilon}}\right)^{\frac{\alpha}{2}} = 0.$$
 (42)

Proof: First, consider the case of $\lambda_0 \leq \zeta(\lambda_e)$. The throughput expression in (7) implies that

$$R^* \le \lambda_0 \log(1+\theta). \tag{43}$$



Fig. 4. The admissible set \mathcal{F} is sketched as the shaded region that contains all combinations of active transmitter density λ_t and transmission power P that satisfy the outage-probability constraint. Given infinite battery capacity and fixed transmitter density λ_0 , f(P) is a monotone-decreasing function of P as plotted with the thick lines based on Theorem 1 for the cases of $\lambda_0 > \zeta(\lambda_e) > 0$ and $\lambda_0 \le \zeta(\lambda_e)$, where the intersections are indicated by P^* and P_0 , respectively. Note that P^* and an arbitrary value in $[P_0, \lambda_e]$ give the optimal transmission power that maximizes the network throughput for the corresponding cases.

Define P_0 such that $\lambda_0 = \zeta(P_0)$ (see Fig. 4). Using the definition of ζ in (38),

$$P_0 = \frac{\theta}{1 - \theta(\lambda_0/\mu_\epsilon)^{\frac{\alpha}{2}}}$$

It can be observed from (38) that ζ is a strictly-monotoneincreasing function. As a result, since $\lambda_0 \leq \zeta(\lambda_e)$ and $\lambda_0 = \zeta(P_0)$, $P_0 \leq \lambda_e$ and thus the set $[P_0, \lambda_e]$ (the range of P^* in the theorem statement) is nonempty. Consider an arbitrary value $p \in [P_0, \lambda_e]$. Since $\lambda_0 \leq \zeta(p)$ from the monotonicity of ξ , $(\lambda_t, P) = (\lambda_0, p)$ is admissible according to Lemma 7. Furthermore, $(\lambda_t, P) = (\lambda_0, p)$ is feasible as $\rho(p) = 1$ based on Theorem 1 and $p \leq \lambda_e$. It follows that P = p maximizes R by achieving the equality in (43). This proves the desired result for the case of $\lambda_0 \leq \zeta(\lambda_e)$.

Next, consider the other case of $\lambda_0 > \zeta(\lambda_e) > 0$. Using the throughput expression in (7) and Lemma 7, the problem of maximizing the network throughput is equivalent to

$$\begin{array}{ll} \text{maximize} & \rho(P) \\ \text{subject to} & \lambda_0 \rho(P) \leq \zeta(P). \end{array} \tag{44}$$

The inequality $P^* \geq \lambda_e$ in the theorem statement can be proved by contradiction as follows. Assume that $P^* < \lambda_e$. This assumption results in $\rho(P^*) = 1$ by applying Theorem 1. Moreover, $\zeta(P^*) < \zeta(\lambda_e)$ is obtained using the aforementioned monotonicity of ζ , and hence $\lambda_0 > \zeta(P^*)$ given that $\lambda_0 > \zeta(\lambda_e)$. Combining $\lambda_0 > \zeta(P^*)$ and $\rho(P^*) = 1$ shows that the assumption of $P^* < \lambda_e$ violates the constraint in (44), proving that $P^* \geq \lambda_e$. Then applying Theorem 1 yields the desired result in (41). Last, since $\rho(P)$ and $\zeta(P)$ are strictly-monotone-decreasing and strictly-monotone-increasing functions, respectively, the solution P^* for the problem in (44) must satisfy $\lambda_0 \rho(P^*) = \zeta(P^*)$ or equivalently $\sqrt{P^*}$ solves the polynomial equation in (42), completing the proof.

Remark 8. For the case of $\lambda_0 \leq \zeta(\lambda_e)$, the network is relatively sparse and λ_e is sufficiently large such that it is optimal as well as feasible for all transmitters to transmit with probability one, resulting in the network throughput in (40). For the case of $\lambda_0 > \zeta(\lambda_e)$, the network is relatively dense and high transmission power is required for satisfying the outage-probability constraint. Consequently, not all transmitters can transmit simultaneously, corresponding to the optimal transmission probability smaller than one and the network throughput in (41).

Remark 9. With $\alpha > 2$ and the last coefficient at the righthand side being negative, the polynomial equation in (42) has at least one strictly positive solution that gives P^* for the case of $\lambda_0 > \zeta(\lambda_e)$. For the special case of $\alpha = 4$, the polynomial equation in (42) is quadratic and solving it gives P^* in closed form as shown below.

Corollary 1. Given infinite battery capacity, $\lambda_0 > \zeta(\lambda_e)$ and $\alpha = 4$, the optimal transmission power P^* is

$$P^* = \frac{\theta + \sqrt{\theta^2 + 4\theta \left(\frac{\lambda_0 \lambda_e}{\mu_\epsilon}\right)^{\frac{\alpha}{2}}}}{2}$$

Remark 10. Recall that the throughput maximized in Theorem 2 is defined in (6) based on the assumption $\epsilon \ll 1$, where the scaling factor $(1 - P_{out})$ (success probability) is omitted for simplicity since it is close to one under the outage-probability constraint. If this factor is considered, changing the value of P^* over the range $\left[\frac{\theta}{1-\theta(\lambda_0/\mu_e)^{\frac{\alpha}{2}}}, \lambda_e\right]$ [see Theorem 2 for the case of $\lambda_0 \leq \zeta(\lambda_e)$] can lead to a throughput variation no

larger than ϵR^* , which is negligible given $\epsilon \ll 1$. However, if ϵ is comparable with one or there is no outage constraint ($\epsilon = 1$), the success probability should be accounted for and the throughput redefined as

$$R = (1 - P_{\mathsf{out}})\rho\lambda_0 \log_2(1 + \theta)$$

where $P_{out} \leq \epsilon$. The results in Theorem 2 can be extended using the redefined metric by analyzing P_{out} as a function of P, which has no closed-form but can be approximated by its bounds [27].

B. Maximum Network Throughput: Extreme Cases

Consider a network with a high energy-arrival rate ($\lambda_e \rightarrow \infty$). The maximum network throughput R^* can be upper bounded as

$$R^* \le \frac{\mu_{\epsilon}}{\theta^{\frac{2}{\alpha}}} \log_2(1+\theta) \tag{45}$$

since the outage-probability constraint requires that (see Lemma 7)

$$\lambda_t \le \zeta(P) \\ \le \frac{\mu_\epsilon}{\theta_\alpha^2} \tag{46}$$

where (46) follows from (39) and that ζ is a monotoneincreasing function. Combining the two upper bounds on R^* in (43) and (45) gives

$$R^* \le \min\left(\lambda_0, \frac{\mu_{\epsilon}}{\theta^{\frac{2}{\alpha}}}\right) \log_2(1+\theta).$$
(47)

For a high energy-arrival rate, equality is achieved in (47) as shown below.

Proposition 4. Given infinite battery capacity, as the energyarrival rate $\lambda_e \rightarrow \infty$, the maximum network throughput converges as

$$\lim_{\lambda_e \to \infty} R^*(\lambda_e) = \min\left(\lambda_0, \frac{\mu_\epsilon}{\theta^{\frac{2}{\alpha}}}\right) \log_2(1+\theta).$$
(48)

Proof: First, consider the case of $\lambda_0 \leq \mu_{\epsilon} \theta^{-\frac{2}{\alpha}}$. Set $P = \lambda_e - \delta$ with $\delta > 0$. This results in $\rho = 1$ according to Theorem 1. Consequently, $\lambda_0 = \lambda_t$ and hence $\lambda_t \leq \mu_{\epsilon} \theta^{-\frac{2}{\alpha}}$ from the assumption about λ_0 . Combining this inequality and (39) yields that $\lambda_t \leq \zeta(P)$ as $P \to \infty$ along with $\lambda_e \to \infty$. It follows that as $\lambda_e \to \infty$, the combination $(\lambda_t, P) = (\lambda_0, \lambda_e - \delta)$ is admissible according to Lemma 7. This proves the equality in (47) for the current case.

Next, consider the case of $\lambda_0 > \mu_{\epsilon} \theta^{-\frac{2}{\alpha}}$. Given this strict inequality, there exists $\delta > 0$ such that

$$\mu_0 > \frac{\mu_\epsilon}{\theta^{\frac{2}{\alpha}}} - \delta.$$
(49)

Set P as

$$P = \frac{\lambda_e \lambda_0}{\mu_\epsilon \theta^{-\frac{2}{\alpha}} - \delta}.$$
 (50)

Combining (49) and (50) gives $\lambda_e < P$. Consequently, applying Theorem 1 gives

$$\rho = \frac{\mu_{\epsilon}\theta^{-\frac{2}{\alpha}} - \delta}{\lambda_0}$$

and hence

$$\lambda_t = \frac{\mu_\epsilon}{\theta^{\frac{2}{\alpha}}} - \delta. \tag{51}$$

As $P \to \infty$ along with $\lambda_e \to \infty$, it follows from (39) and (51) that there exists $\delta > 0$ such that $\lambda_t \leq \zeta(P)$. As a result, by applying Lemma 7, the combination of (λ_t, P) as specified in (50) and(51) is admissible. This leads to

$$\lim_{\lambda_e \to \infty} R^*(\lambda_e) = \left(\frac{\mu_{\epsilon}}{\theta^{\frac{2}{\alpha}}} - \delta\right) \log_2(1+\theta).$$

Letting $\delta \to 0$ proves the equality in (48) for the current case, completing the proof.

Remark 11. Given a high energy arrival rate and infinite battery capacity, in the steady state, transmitters always have sufficient energy for transmission. Therefore, the expression in (48) also specifies the maximum network throughput of a MANET with reliable power supplies instead of energy harvesting.

Remark 12. For the case of $\lambda_0 < \mu_{\epsilon} \theta^{-\frac{2}{\alpha}}$, the active transmitter density is below the network-interference temperature even though all transmitters transmit with probability one. Therefore, there is margin for further increasing active transmitter density without violating the outage-probability constraint. For this reason, the network-throughput limit in (48) for the current case is proportional to the transmitter density. However, for the case of $\lambda_0 \ge \mu_{\epsilon}/\theta^{\frac{2}{\alpha}}$, active transmitter density reaches the network-interference temperature and cannot be further increased. Consequently, the corresponding network-throughput limit in (48) is independent of the transmitter density.

Consider a sparse network $(\lambda_0 \to 0)$. It is optimal for each transmitter to transmit with probability one by setting the transmission power $P^* \in (\theta, \lambda_e]$ if $\lambda_e > \theta$ or otherwise with probability λ_e/θ by setting P^* to be equal to θ (see Theorem 1). The corresponding network throughputs are $R^* = \lambda_0 \log_2(1+\theta)$ and $R^* = \frac{\lambda_0 \lambda_e}{\theta} \log_2(1+\theta)$, respectively, which both diminish as $\lambda_0 \to 0$. Next, for a dense network $(\lambda_0 \to \infty)$, the maximum network throughput is specified as follow.

Proposition 5. Given infinite battery capacity, as the transmitter density $\lambda_0 \rightarrow \infty$, the maximum network throughput converges as

$$\lim_{\lambda_0 \to \infty} R^*(\lambda_0) = \frac{\mu_{\epsilon}}{\theta^{\frac{2}{\alpha}}} \log_2(1+\theta).$$
 (52)

Proof: Set *P* as the following function of λ_0 :

$$P(\lambda_0) = \frac{\lambda_0 \lambda_e}{\mu_\epsilon} \left(\frac{1}{\theta} - \frac{1}{\log \lambda_0}\right)^{-\frac{2}{\alpha}}$$
(53)

that is shown shortly to achieve the limit of R^* in (52). Given (53), there exists $\tau_1 > 0$ such that $P(\lambda_0) \ge \lambda_e$ for all $\lambda_0 > \tau_1$. Therefore, it follows from (7) and Theorem 1 that

$$\lim_{\lambda_0 \to \infty} R(\lambda_0, P(\lambda_0)) = \lim_{\lambda_0 \to \infty} \frac{\lambda_0 \lambda_e}{P(\lambda_0)} \log_2(1+\theta)$$
$$= \lim_{\lambda_0 \to \infty} \mu_\epsilon \left(\frac{1}{\theta} - \frac{1}{\log \lambda_0}\right)^{\frac{2}{\alpha}} \log_2(1+\theta)$$
$$= \frac{\mu_\epsilon}{\theta^{\frac{2}{\alpha}}} \log_2(1+\theta).$$
(54)



Fig. 5. Maximum network throughput versus energy-arrival rate for optimal transmission power, infinite battery capacity, and the transmitter density $\lambda_0 = \{0.02, 0.05, 0.5\}$.

Combining (45) and (54) shows that the maximum network throughput has the limit in (52) as λ_0 increases.

The remaining proof verifies that $P(\lambda_0)$ and the corresponding λ_t are admissible as $\lambda_0 \to \infty$. It follows from (53) and Theorem 1 that for all $\lambda_0 > \tau_1$, λ_t is a function of λ_0 and given as

$$\lambda_t(\lambda_0) = \mu_\epsilon \left(\frac{1}{\theta} - \frac{1}{\log \lambda_0}\right)^{\frac{2}{\alpha}}.$$
 (55)

Substituting (53) into (38) yields

$$\zeta(P(\lambda_0)) = \mu_{\epsilon} \left(\frac{1}{\theta} - \frac{\mu_{\epsilon}}{\lambda_0 \lambda_e} \left(\frac{1}{\theta} - \frac{1}{\log \lambda_0} \right)^{\frac{2}{\alpha}} \right)^{\frac{2}{\alpha}}.$$
 (56)

By comparing (55) and (56), there exists $\tau_2 > 0$ such that $\lambda_t(\lambda_0) \leq \zeta(P(\lambda_0))$ for all $\lambda_0 \geq \tau_2$. This proves the admissibility of $P(\lambda_0)$ in (53) and $\lambda_t(\lambda_0)$ in (55) as $\lambda_0 \to \infty$, completing the proof.

Remark 13. The rate of total energy harvested per unit area is $\lambda_t P^* = \lambda_0 \lambda_e$ as $\lambda_0 \to \infty$. The linear growth of the rate with increasing λ_0 is due to that the harvester density is equal to λ_0 . However, more aggressive energy harvesting in a dense network does not continuously increase the network throughput that saturates at high transmitter power as the network becomes interference limited (see Proposition 5). This issue may be resolved by using an alternative multipleaccess protocol such as frequency-hopping multiple access that reduces the density of simultaneous co-channel transmitters.

V. NUMERICAL RESULTS

The nominal node density μ_{ϵ} is fixed as 0.05 for all numerical results, corresponding to the maximum outage probability $\epsilon \approx 0.015$. The relation between μ_{ϵ} and ϵ is obtained by simulation based on the following procedure (see e.g., [36]). The summation over the PPP $\Lambda(\mu_{\epsilon})$ in (34) is approximated by the signal power measured at the origin due to unit-power transmissions by transmitters uniformly distributed in a disk. The number of transmitters follows the Poisson distribution with mean 200 and the disk radius is adjusted such that the expected transmitter density is equal to μ_{ϵ} . Based on this setup, the values of $(\epsilon, \mu_{\epsilon})$ are computed using the Monte Carlo method that yields the plot in Fig. 3. In addition, all numerical results are based on the SINR threshold $\theta = 3$ and the path-loss exponent $\alpha = 3$.

The distribution of the energy arrival process $\{Z_t\}$ is specified as follows. Let $\{V_t\}$ denote an i.i.d. sequence of random variables following the chi-squared distribution with $d \in \{1, 2, \cdots\}$ degrees of freedom (DoF) and mean equal to $d\lambda_e$. Let $\{Z_t\} = \{\frac{1}{d}V_t\}$ and hence Z_t has mean λ_e and variance $2\lambda_e^2/d$. The chosen distribution of Z_t allows its variance (randomness) to be controlled by varying d while the mean of Z_t is fixed. Note that Z_t converges to a constant λ_e in probability as $d \to \infty$ by the law of large numbers.

Infinite battery capacity is assumed for the numerical results presented in Fig. 5 and Fig. 6. In Fig. 5, the maximum network throughput R^* computed using Theorem 2 is plotted against the increasing energy-arrival rate λ_e for the transmitter density $\lambda_0 = \{0.02, 0.05, 0.5\}$. It can be observed from Fig. 5 that R^* grows as λ_e increases and saturates for large λ_e . The limits agree with those computed using Proposition 4, namely 0.04 bit/s/Hz/unit-area for $\lambda_0 = 0.02$ and 0.048 bit/s/Hz/unit-area for $\lambda_0 = \{0.05, 0.5\}$. In addition, Fig. 5 shows that in a denser network (i.e., $\lambda_e = 0.5$), R^* reaches its limit more rapidly as λ_e increases.

Fig. 6 shows the curves of R^* versus λ_0 for $\lambda_e = \{0.5, 1, 5\}$, which are obtained using Theorem 2. As λ_0 increases and regardless of the value of λ_e , R^* is observed to converge to the limit 0.048 bit/s/Hz/unit-area predicted by Proposition 5. Moreover, it is observed from Fig. 6 that larger λ_e results in faster convergence of R^* to its limit as λ_0



Fig. 6. Maximum network throughput versus transmitter density for optimal transmission power, infinite battery capacity, and the energy-arrival rate $\lambda_e = \{0.5, 1, 5\}$.



Fig. 7. A comparison between the average tail probability of the battery level, $\lim_{n\to\infty} \frac{1}{n} \sum_{t=1}^{n} \Pr(S_t > x)$, evaluated by simulation and its upper bound computed based on Lemma 2 for infinite battery capacity, the DoF of the energy-arrival process d = 4, the energy-arrival rate $\lambda_e = 2$ and the transmission power P = 4.

increases.

The average tail probability of the battery level, $\lim_{n\to\infty} \frac{1}{n} \sum_{t=1}^{n} \Pr(S_t > x)$, is evaluated by simulation and compared in Fig. 7 with its upper bound from Lemma 2 given infinite battery capacity, d = 4, $\lambda_e = 2$ and P = 4. The bound is observed to be loose but sufficient for the analysis. The similar observation and remark also hold for the upper bound on the energy-overshoot function $D_t(x)$ as given in Lemma 3 and the numerical results are omitted for brevity.

Next, consider the case of finite battery capacity. In Fig. 8, the transmission probability ρ obtained by simulation is plotted

against increasing transmission power P for finite battery capacity $B = \{1.5P, 2P, 4P, 10P\}$, d = 4, $\lambda_e = 2$, and $\lambda_0 = 0.02$. It is found that B = 10P is sufficiently large such that the values of ρ closely match those for the case of infinite battery capacity as computed using Theorem 1. As observed from Fig. 8, finite battery capacity degrades ρ significantly only when P and hence B are relatively small; as P and Bincrease, ρ rapidly approaches the counterpart for the case of infinite battery capacity (or that for B = 10P).

Fig. 9 displays the curves of ρ versus P obtained by simulation for B = 1.5P, $d = \{2, 4, 8, 16\}$, $\lambda_e = 2$ and $\lambda_0 = 0.02$.



Fig. 8. Transmission probability versus transmission power for finite battery capacity $B = \{1.5P, 2P, 4P, 10P\}$, the DoF of the energy-arrival process d = 4, the energy-arrival rate $\lambda_e = 2$, and the transmitter density $\lambda_0 = 0.02$.



Fig. 9. Transmission probability versus transmission power for both the cases of finite (B = 1.5P) and infinite battery capacity. The DoF of the energy-arrival process is $d = \{2, 4, 8, 16\}$, the energy-arrival rate $\lambda_e = 2$, and the transmitter density $\lambda_0 = 0.02$.

For comparison, the curve for the case of infinite battery capacity is also plotted. As observed from Fig. 9, reducing the randomness of the energy arrival process by increasing d leads to smaller battery-overflow probability and hence higher ρ . The effect of d on ρ diminishes as P (and hence battery capacity) increases and ρ converges to its counterpart for the case of infinite battery capacity.

Last, we investigate the effect of the DoF of the energy arrival process on the network throughput. To this end, Fig. 10 shows the curves of R^* versus λ_e obtained by simulation for B = 1.5P, $d = \{2, 4, 8, 16\}$, and $\lambda_0 = 0.02$. The curve for the case of infinite battery capacity is also plotted for comparison. It can be observed from Fig. 10 that finite battery capacity causes significant throughput loss especially for large λ_e . Such loss is smaller for larger *d* because of less randomness in harvested energy and hence smaller battery-overflow probability.

VI. CONCLUSION

The energy dynamics in a mobile ad hoc network have been characterized in terms of transmission probability. Assuming infinite battery capacity, it has been found that the transmission probability is equal to one when the energy-arrival rate exceeds transmission power or otherwise is equal to their



Fig. 10. Maximum network throughput versus energy-arrival rate for optimal transmission power and both the cases of finite (B = 1.5P) and infinite battery capacity. The DoF of the energy-arrival process is $d = \{2, 4, 8, 16\}$ and the transmitter density $\lambda_0 = 0.02$.

ratio. Moreover, for the case of finite battery capacity, bounds on the transmission probability have been obtained and exact expressions have been derived for the special cases of bounded or discrete energy arrivals. The results on transmission probability have been applied to derive the maximum network spatial throughput for a given energy-arrival rate and optimized transmission power. It has been shown that it is optimal and feasible for all transmitters to transmit with probability one if the transmitter density is below a threshold that depends on the energy-arrival rate; otherwise, each transmitter should transmit with probability smaller than one.

There are several potential directions for extending this work. Coexisting wireless networks may harvest electromagnetic (EMR) energy from each others' transmissions. Modeling and designing coexisting networks with EMR energy harvesting give rise to many new research issues ranging from algorithm design to throughput analysis. The current work focuses on ad hoc networks with random access and can be extended to other types of networks such as cellular networks or other medium-access-control protocols such as carriersensing multiple access. Last, it is interesting to investigate the effects of bursty data arrivals and more sophisticated power control on the throughput of wireless networks powered by energy harvesting.

ACKNOWLEDGEMENT

The author thanks Rui Zhang for helpful discussion that motivated this research, and the anonymous reviewers whose comments have significantly improved the quality of this paper.

APPENDIX A

Proof of Lemma 1

If $t \in \mathcal{T}$, it follows from (10) that $S_t = G'_t$ and hence the inequality in the lemma statement holds since $G_t \ge 0$. Next, consider the case of $t \notin \mathcal{T}$ and let $t_0 \in \mathcal{T}$ denote the time instant closest to but smaller than t. It follows that the random walk $\{S_t\}$ does not cross the threshold P from below in the time slots $\{t_0+1, \cdots, t\}$. Therefore, if $S_t \geq S_{t_0}$, $S_t = S_{t_0} + \sum_{m=t_0+1}^t \overline{Z}_m$. Then S_t can be upper bounded as

$$S_{t} \leq S_{t_{0}} + \max\left(0, \bar{Z}_{t}, \bar{Z}_{t} + \bar{Z}_{t-1}, \cdots, \sum_{m=t_{0}+1}^{t} \bar{Z}_{m}\right).$$
(57)

It can be obtained from (9) that

$$G_{t} = \max\left(0, \bar{Z}_{t}, \bar{Z}_{t} + \bar{Z}_{t-1}, \cdots, \sum_{m=t_{0}+2}^{t} \bar{Z}_{m}, G_{t_{0}} + \sum_{m=t_{0}+1}^{t} \bar{Z}_{m}\right).$$
(58)

Since $G_{t_0} \ge 0$ and $G'_{t_0} = S_{t_0}$ from (10), combining (57) and (58) proves the inequality in the lemma statement for the case of $t \notin \mathcal{T}$, completing the proof.

APPENDIX B Proof of Lemma 2

Using Lemma 1 and for $0 \le a \le x$,

$$\Pr(S_t > x) \leq \Pr(G_t + G'_t > x)$$

=
$$\Pr(G_t > x - G'_t \mid G'_t \ge a) \Pr(G'_t \ge a) +$$

$$\Pr(G_t > x - G'_t \mid G'_t < a) \Pr(G'_t < a)$$

<
$$\Pr(G'_t \ge a) + \Pr(G_t \ge x - a).$$
(59)

Let $t_0 \in \mathcal{T}$ specify the slot such that $G'_t = S_{t_0}$. Since $S_{t_0-1} < P$ based on the definition of \mathcal{T} , $S_{t_0} = S_{t_0-1} + Z_{t_0}$ using (8). It follows from this equality and (59) that

$$\Pr(S_t > x) \le \Pr(S_{t_0-1} + Z_{t_0} \ge a) + \Pr(G_t > x - a)$$

$$\le \Pr(Z_{t_0} \ge a - P) + \Pr(G_t > x - a)$$

$$= \Pr(\bar{Z}_{t_0} \ge a - 2P) + \Pr(G_t > x - a).$$
(60)

By bounding the first term in (60) using Chernoff bound [33] and the second using (12),

$$\Pr(S_t > x) \le \min_{r \ge 0} \mathsf{E}\left[e^{r\bar{Z}_1}\right] e^{-r(a-2P)} + e^{-r^*(P)(x-a)} \le e^{-r^*(P)(a-2P)} + e^{-r^*(P)(x-a)}$$
(61)

where (61) results from setting $r = r^*(P)$. By choosing *a* such that the exponents of the two terms in (61) are equal, the desired result follows.

APPENDIX C Proof of Lemma 3

From the definition in (14),

$$D_t(x) = \int_x^\infty y f_s(y, t) dy - x \Pr(S_t > x)$$

=
$$\int_x^\infty \Pr(S_t > y) dy$$
(62)

$$\leq \int_{x}^{\infty} 2e^{-\frac{1}{2}r^{*}(P)(y-2P)}dy$$
 (63)

where (62) and (63) are obtained using integration by parts and Lemma 2, respectively. The desired result follows from (63).

APPENDIX D PROOF OF LEMMA 4

The definitions of the random processes $\{G_t\}$ and $\{G'_t\}$ in (9) and (10) are modified for the case of finite-battery capacity. Specifically, $\{G_t\}$ is redefined as

$$G_t = \min(\max(G_{t-1} + \bar{Z}_t, 0), B - P)$$
(64)

and $\{G'\}$ is as given in (10) but with the battery-level evolution following (1). Given finite battery capacity B, the inequality $S_t \leq G'_t + G_t$ can be proved using induction as follows. This inequality holds for t = 0 since $S_0 = G_0 = 0$ and $G'_0 = P$. Assume that $S_t \leq G'_t + G_t$. Consider the case of $(t+1) \in \mathcal{T}$. It follows from the definition of $\{G'_t\}$ in (10) that $S_{t+1} = G'_{t+1}$. Therefore, $S_{t+1} \leq G'_{t+1} + G_{t+1}$ since $G_{t+1} \geq 0$ from (64). Next, consider the case of $(t+1) \notin \mathcal{T}$. Based on the evolution of $\{S_t\}$ in (1),

$$S_{t+1} = \begin{cases} \min(S_t + Z_{t+1}, B), & S_t < P\\ \min(S_t + \bar{Z}_{t+1}, B), & S_t \ge P. \end{cases}$$
(65)

Given that $(t+1) \notin \mathcal{T}$ and $S_t < P$, $S_t + Z_{t+1} \leq P$ based on the definition of \mathcal{T} . As a result,

$$\min(S_t + Z_{t+1}, B) \le G'_{t+1} + G_{t+1} \tag{66}$$

since $G_{t+1} \geq 0$ and $G_{t+1}' \geq P$ from (10). If $S_t \geq P,$ since $S_t \leq G_t' + G_t,$

$$\min(S_t + Z_{t+1}, B) \le \min(G'_t + G_t + Z_{t+1}, B)$$

$$\le G'_t + \min(G_t + \bar{Z}_{t+1}, B - P) \quad (67)$$

$$\le G'_{t+1} + G_{t+1} \quad (68)$$

where (67) applies $G'_t \ge P$, and (68) uses (64) and $G'_{t+1} = G'_t$ given that $(t+1) \notin \mathcal{T}$. Combining (65), (66) and (68) proves that $S_{t+1} \le G'_{t+1} + G_{t+1}$ if $S_t \le G'_t + G_t$. It follows that $S_t \leq G'_t + G_t$ for all $t \geq 0$. Furthermore, it can be shown by expanding (64) that $\Pr(G_t > x)$ is no larger than that for the case of infinite battery capacity. Using these results and following the same procedures as for proving Lemma 2 and 3, it can be shown that (13) also holds for the case of finite battery capacity and

$$\tilde{D}_t(x) \le \frac{4}{r^*(P)} e^{-\frac{1}{2}r^*(P)(x-2P)}, \quad \forall t \ge 1.$$

The desired result follows by setting x = B.

APPENDIX E Proof of Lemma 5

Define the random process $\{Q_t\}$ such that

$$Q_t = \min(Q_{t-1} + \bar{Z}_t, B), \qquad t = 1, 2, \cdots$$
 (69)

with $Q_0 = 0$. Comparing (69) and the evolution of S_t in (1) shows that $S_t \ge Q_t$. Therefore, given $x \in [0, B)$

$$\Pr(S_t < x) \le \Pr(Q_t < x). \tag{70}$$

By expanding (69)

$$Q_{t} = \min\left(B, B + \bar{Z}_{t}, B + \bar{Z}_{t} + \bar{Z}_{t-1}, \cdots, B + \sum_{m=2}^{t} \bar{Z}_{m}, \sum_{m=1}^{t} \bar{Z}_{m}\right).$$
(71)

For ease of notation, define

$$\tilde{Q}_t = B + \min\left(0, \bar{Z}_t, \bar{Z}_t + \bar{Z}_{t-1}, \cdots, \sum_{m=2}^t \bar{Z}_m\right).$$

Then $Q_t = \min(\tilde{Q}_t, \sum_{m=1}^t \bar{Z}_m)$. It follows that

$$\Pr(Q_t < x) = \Pr\left(\sum_{m=1}^t \bar{Z}_m < x \mid Q_t = \sum_{m=1}^t \bar{Z}_m\right) \times \\\Pr\left(Q_t = \sum_{m=1}^t \bar{Z}_m\right) + \\\Pr\left(\tilde{Q}_t < x \mid Q_t = \tilde{Q}_t\right) \Pr\left(Q_t = \tilde{Q}_t\right) \\ \leq \Pr\left(Q_t = \sum_{m=1}^t \bar{Z}_m\right) + \\\Pr\left(\tilde{Q}_t < x \mid Q_t = \tilde{Q}_t\right).$$
(72)

By inspecting (71), the event $Q_t = \sum_{m=1}^t \bar{Z}_m$ is equivalent to the one $\max(\bar{Z}_1, \bar{Z}_1 + \bar{Z}_2, \dots, \sum_{m=1}^t \bar{Z}_m) \leq B$. Then the inequality in (72) can be rewritten as

$$\Pr(Q_t < x) \le \Pr\left(\max\left(\bar{Z}_1, \bar{Z}_1 + Z_2, \cdots, \sum_{m=1}^t \bar{Z}_m\right) \le B\right) + \\\Pr\left(\tilde{Q}_t < x \mid \max\left(\bar{Z}_1, \bar{Z}_1 + Z_2, \cdots, \sum_{m=1}^t \bar{Z}_m\right) > B\right).$$

Note that removing the conditioning of the last term increases the probability. Therefore

$$\Pr(Q_t < x) \le \Pr\left(\max\left(\bar{Z}_1, \bar{Z}_1 + Z_2, \cdots, \sum_{m=1}^t \bar{Z}_m\right) \le B\right) + \Pr\left(\tilde{Q}_t < x\right)$$
$$\le \Pr\left(\sum_{m=1}^t \bar{Z}_m \le B\right) + \Pr\left(\tilde{Q}_t < x\right).$$
(73)

Applying a similar technique as for proving the result in Theorem 1 for the case of $\lambda_e > P$ shows that given $\lambda_e > P$

$$\lim_{n \to \infty} \frac{1}{n} \sum_{t=1}^{n} \Pr\left(\sum_{m=1}^{t} \bar{Z}_m \le B\right) = 0.$$
(74)

Using the definition of \tilde{Q}_t , the last term in (73) can be rewritten as

$$\Pr\left(\tilde{Q}_{t} < x\right) = \Pr\left(\min\left(0, \bar{Z}_{t}, \bar{Z}_{t} + \bar{Z}_{t-1}, \cdots, \sum_{m=2}^{t} \bar{Z}_{m}\right) \leq x - B\right)$$
$$= \Pr\left(\min\left(\bar{Z}_{t}, \bar{Z}_{t} + \bar{Z}_{t-1}, \cdots, \sum_{m=2}^{t} \bar{Z}_{m}\right) \leq x - B\right)$$

since $(x-B) \leq 0$. Applying Kingman bound in a similar way as for obtaining (11) yields

$$\Pr\left(\tilde{Q}_t < x\right) \le e^{r^*(P)(B-x)} \tag{75}$$

where $r^*(P) < 0$ according to Assumption 1 given $\lambda_e > P$. By combining (73), (74) and (75)

$$\lim_{n \to \infty} \frac{1}{n} \sum_{t=1}^{n} \Pr(Q_t < x) \le e^{r^*(P)(B-x)}.$$
 (76)

The desired result follows from (70) and (76).

REFERENCES

- J. A. Paradiso and T. Starner, "Energy scavenging for mobile and wireless electronics," *IEEE Pervasive Computing*, vol. 4, pp. 1536–1268, Jan.-Mar. 2005.
- [2] C. Ho and R. Zhang, "Optimal energy allocation for wireless communications with energy harvesting constraints," *IEEE Trans. on Signal Processing*, vol. 60, pp. 4808–4818, Sep. 2012.
- [3] O. Ozel, K. Tutuncuoglu, J. Yang, S. Ulukus, and A. Yener, "Transmission with energy harvesting nodes in fading wireless channels: Optimal policies," *IEEE Journal on Selected Areas in Comm.*, vol. 29, pp. 1732– 1743, Sep. 2011.
- [4] J. Yang, O. Ozel, and S. Ulukus, "Broadcasting with an energy harvesting rechargeable transmitter," *IEEE Trans. on Wireless Comm.*, vol. 11, pp. 571–583, Feb. 2012.
- [5] R. Zhang and C. Ho, "MIMO broadcasting for simultaneous wireless information and power transfer," *submitted to IEEE Tarns. on Comm.* (Avaiable: http://arxiv.org/abs/1105.4999).
- [6] L. Georgiadis, M. Neely, and L. Tassiulas, *Resource Allocation and Cross Layer Control in Wireless Networks*. Now Publishers Inc, 1 ed., 2006.

- [7] J. Yang and S. Ulukus, "Optimal packet scheduling in an energy harvesting communication system," *IEEE Trans. on Comm.*, vol. 60, pp. 220–230, Jan. 2012.
- [8] M. A. Antepli, E. Uysal-Biyikoglu, and H. Erkal, "Optimal packet scheduling on an energy harvesting broadcast link," *IEEE Journal on Sel. Areas in Comm.*, vol. 29, pp. 1721–1731, Aug. 2011.
- [9] H. Huang and V. K. N. Lau, "Decentralized delay optimal control for interference networks with limited renewable energy storage," *IEEE Trans. on Signal Processing*, vol. 60, pp. 2552–2561, May 2012.
- [10] M. Gatzianas, L. Georgiadis, and L. Tassiulas, "Control of wireless networks with rechargeable batteries," *IEEE Trans. on Wireless Comm.*, vol. 9, pp. 581–593, Feb. 2010.
- [11] E. Uysal-Biyikoglu, B. Prabhakar, and A. El Gamal, "Energy-efficient packet transmission over a wireless link," *IEEE/ACM Trans. on Networking*, vol. 10, pp. 487–499, Apr. 2002.
- [12] W. Chen, M. J. Neely, and U. Mitra, "Energy-efficient transmissions with individual packet delay constraints," *IEEE Trans. on Information Theory*, vol. 54, pp. 2090–2109, May 2008.
- [13] M. J. Neely, "Optimal energy and delay tradeoffs for multiuser wireless downlinks," *IEEE Trans. on Information Theory*, vol. 53, pp. 3095–3113, Sep. 2007.
- [14] M. J. Neely, "Energy optimal control for time-varying wireless networks," *IEEE Trans. on Information Theory*, vol. 52, pp. 2915–2934, Jul. 2006.
- [15] F. Iannello, O. Simeone, and U. Spagnolini, "Medium access control protocols for wireless sensor networks with energy harvesting," *IEEE Trans. on Comm.*, vol. 60, pp. 1381–1389, May 2012.
- [16] A. Kansal, J. Hsu, M. Srivastava, and V. Raghunathan, "Harvesting aware power management for sensor networks," ACM Trans. on Embedded Computing Sys., vol. 6 (Article 32), Sep. 2007.
- [17] J. Jeon and A. Ephremides, "On the stability of random multiple access with stochastic energy harvesting," in *Proc., IEEE Intl. Symposium on Information Theory*, Jul. 31 - Aug. 5 2011.
- [18] M. Haenggi, J. G. Andrews, F. Baccelli, O. Dousse, and M. Franceschetti, "Stochastic geometry and random graphs for the analysis and design of wireless networks," *IEEE Journal on Selected Areas in Comm.*, vol. 27, pp. 1029–1046, Jul. 2009.
- [19] F. Baccelli, B. Blaszczyszyn, and P. Muhlethaler, "An ALOHA protocol for multihop mobile wireless networks," *IEEE Trans. on Information Theory*, vol. 52, pp. 421–36, Feb. 2006.
- [20] S. P. Weber, X. Yang, J. G. Andrews, and G. de Veciana, "Transmission capacity of wireless ad hoc networks with outage constraints," *IEEE Trans. on Information Theory*, vol. 51, pp. 4091–4102, Dec. 2005.
- [21] H. Nguyen, F. Baccelli, and D. Kofman, "A stochastic geometry analysis of dense IEEE 802.11 networks," in *Proc., IEEE Infocom*, pp. 1199– 1207, May 2007.
- [22] J. G. Andrews, F. Baccelli, and R. K. Ganti, "A tractable approach to coverage and rate in cellular networks," *IEEE Trans. on Comm.*, vol. 59, pp. 3122–3134, Nov. 2011.
- [23] H. Dhillon, R. Ganti, F. Baccelli, and J. Andrews, "Modeling and analysis of K-tier downlink heterogeneous cellular networks," to appear in IEEE Journal on Sel. Areas in Comm. (Available: http://arxiv.org/abs/1103.2177).
- [24] K. Huang, V. K. N. Lau, and Y. Chen, "Spectrum sharing between cellular and mobile ad hoc networks: Transmission-capacity trade-off," *IEEE Journal on Selected Areas in Comm.*, vol. 27, pp. 1256–1267, Sep. 2009.
- [25] S. P. Weber, J. G. Andrews, and N. Jindal, "The effect of fading, channel inversion, and threshold scheduling on ad hoc networks," *IEEE Trans.* on *Information Theory*, vol. 53, pp. 4127–4149, Nov. 2007.
- [26] N. Jindal, J. G. Andrews, and S. P. Weber, "Bandwidth partitioning in decentralized wireless networks," *IEEE Trans. on Wireless Comm.*, vol. 7, pp. 5408–5419, Jul. 2008.
- [27] S. P. Weber, J. G. Andrews, X. Yang, and G. de Veciana, "Transmission capacity of wireless ad hoc networks with successive interference cancelation," *IEEE Trans. on Information Theory*, vol. 53, pp. 2799– 2814, Aug. 2007.
- [28] A. M. Hunter, J. G. Andrews, and S. P. Weber, "Transmission capacity of ad hoc networks with spatial diversity," *IEEE Trans. on Wireless Comm.*, vol. 7, pp. 5058–5071, Dec. 2008.
- [29] R. Vaze and R. W. Heath Jr., "Transmission capacity of ad-hoc networks with multiple antennas using transmit stream adaptation and interference cancellation," *IEEE Trans. on Information Theory*, vol. 58, pp. 780–792, Feb. 2012.
- [30] N. Jindal, J. G. Andrews, and S. Weber, "Multi-antenna communication in ad hoc networks: Achieving MIMO gains with SIMO transmission," *IEEE Trans. on Comm.*, vol. 59, pp. 529–540, Feb. 2011.

- [31] R. H. Y. Louie, M. R. McKay, and I. B. Collings, "Open-loop spatial multiplexing and diversity communications in ad hoc networks," *IEEE Trans. on Information Theory*, vol. 57, pp. 317–344, Jan. 2010.
- [32] K. Huang, J. G. Andrews, R. W. Heath, Jr., D. Guo, and R. A. Berry, "Spatial interference cancelation for mobile ad hoc networks," *IEEE Trans. on Information Theory*, vol. 58, pp. 1660–1676, Mar. 2012.
- [33] R. Gallager, Discrete Stochastic Processes. Springer, 1st ed., 1995.
- [34] J. F. C. Kingman, Poisson processes. Oxford University Press, 1993.
- [35] D. Stoyan, W. S. Kendall, and J. Mecke, Stochastic Gemoetry and its Applications. Wiley, 2nd ed., 1995.
- [36] S. P. Weber and M. Kam, "Computational complexity of outage probability simulations in mobile ad-hoc networks," in *Proc., Conf. on Information Sciences and Systems*, Mar. 2005.
- [37] S. Haykin, "Cognitive radio: Brain-empowered wireless communications," *IEEE Journal on Selected Areas in Comm.*, vol. 23, pp. 201–220, Feb. 2005.

Kaibin Huang (S'05, M'08) received the B.Eng. (first-class hons.) and the M.Eng. from the National University of Singapore in 1998 and 2000, respectively, and the Ph.D. degree from The University of Texas at Austin (UT Austin) in 2008, all in electrical engineering.

Since Jul. 2012, he has been an assistant professor in the Dept. of Applied Mathematics at The Hong Kong Polytechnic University (PolyU), Hong Kong. He had held the same position in the School of Electrical and Electronic Engineering at Yonsei University, S. Korea from Mar. 2009 to Jun. 2012 and presently is affiliated with the school as an adjunct professor. From Jun. 2008 to Feb. 2009, he was a Postdoctoral Research Fellow in the Department of Electrical and Computer Engineering at the Hong Kong University of Science and Technology. From Nov. 1999 to Jul. 2004, he was an Associate Scientist at the Institute for Infocomm Research in Singapore. He frequently serves on the technical program committees of major IEEE conferences in wireless communications. He will chair the Comm. Theory Symp. of IEEE GLOBECOM 2014 and has been the technical co-chair for IEEE CTW 2013, the track chair for IEEE Asilomar 2011, and the track co-chair for IEE VTC Spring 2013 and IEEE WCNC 2011. He is an editor for the IEEE Wireless Communications Letters and also the Journal of Communication and Networks. He is an elected member of the SPCOM Technical Committee of the IEEE Signal Processing Society. Dr. Huang received the Outstanding Teaching Award from Yonsei, Motorola Partnerships in Research Grant, the University Continuing Fellowship at UT Austin, and a Best Paper Award from IEEE GLOBECOM 2006. His research interests focus on the analysis and design of wireless networks using stochastic geometry and multi-antenna limited feedback techniques.