

# Key recycling in authentication

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## Abstract

In their seminal work on authentication, Wegman and Carter propose that to authenticate multiple messages, it is sufficient to reuse the same hash function as long as each tag is encrypted with a one-time pad. They argue that because the one-time pad is perfectly hiding, the hash function used remains completely unknown to the adversary.

Since their proof is not composable, we revisit it using a composable security framework. It turns out that the above argument is insufficient: if the adversary learns whether a corrupted message was accepted or rejected, information about the hash function is leaked, and after a bounded finite amount of rounds it is completely known. We show however that this leak is very small: Wegman and Carter's protocol is still  $\varepsilon$ -secure, if  $\varepsilon$ -almost strongly universal<sub>2</sub> hash functions are used. This implies that the secret key corresponding to the choice of hash function can be reused in the next round of authentication without any additional error than this  $\varepsilon$ .

We also show that if the players have a mild form of synchronization, namely that the receiver knows when a message should be received, the key can be recycled for any arbitrary task, not only new rounds of authentication.

## 1 Introduction

If a player, say, Bob, receives a message  $x$  that claims to come from Alice, he might wish to know if this is true, or if the message was generated or modified by some adversary. This task is called *authentication*, and in their seminal work [1], Wegman and Carter showed that it can be achieved with in-

formation-theoretic security by appending a tag  $t$  to the message (often called a message authentication code or MAC), where  $t = h_k(x)$ ,  $\mathcal{H} = \{h_k\}_{k \in \mathcal{K}}$  is a family of almost strongly universal<sub>2</sub> (ASU<sub>2</sub>) hash functions,<sup>1</sup> and  $k$  is a secret key shared by Alice and Bob.

Wegman and Carter [1] propose a scheme to use fewer bits of key when multiple messages are to be authenticated: each tag should be encrypted with a fresh one-time pad (OTP), but the same hash function can be used each time. Alice thus appends the tag  $t_i = h_{k_1}(x_i) \oplus k_2^i$  to her  $i^{\text{th}}$  message  $x_i$ , where  $k_1$  is used for all messages and  $k_2^i$  is a fresh key used only in this round.

To prove the security of this scheme, Wegman and Carter show that given any number of message-tag pairs  $(x_1, t_1), (x_2, t_2), \dots$ , the secret key  $k_1$  is still perfectly uniform. They then argue that the probability of an adversary successfully corrupting any new message is the same in every round, and guaranteed to be small by the properties of the ASU<sub>2</sub> hash functions.

However, proving that a protocol is secure in a stand-alone model does not necessarily guarantee that it is still secure when combined with other protocols, not even when combined with itself like Wegman and Carter's scheme. A lot of research has gone into compositability of cryptographic tasks in recent years. A general framework for proving composable security was developed by Canetti [3–5], and dubbed *Universally Composable (UC) security*. Independently, Backes, Pfitzmann and Waidner [6–9] introduced the equivalent notion of *Reactive Simulatability*. These security notions have

<sup>1</sup>ASU<sub>2</sub> hashing was only formally defined later by Stinson [2]. A family of functions is said to be ASU<sub>2</sub> if any two different messages are almost uniformly mapped to all pairs of tags. An exact definition is given in Definition 1 on page 4.

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been extended to the quantum setting by Ben-Or and Mayers [10] and Unruh [11, 12]. More recently, Maurer and Renner proposed a new composable security framework [13], *Abstract Cryptography (AC)*, which both generalizes and simplifies previous frameworks. It defines composability of abstract systems, without specifying the underlying computational model, and is thus equally valid for classical and quantum security.<sup>2</sup> We use AC in this work, because of the extra clarity it provides. The same results could however be obtained from other composable security frameworks.

An essential application of information-theoretic authentication is in quantum key distribution (QKD) protocols.<sup>3</sup> Every (classical) message exchanged between the two parties generating the key needs to be authenticated with information-theoretic security in order to guarantee that the composed protocol remains secure in the presence of a computationally unbounded adversary. Recycling the hash function is a practical way to save a large part of the secret key consumed in each round. Ben-Or et al. [15] and Müller-Quade and Renner [16] discuss the composability of QKD. More recently, Portmann and Renner [17] provide an extensive review of composable security with many examples from QKD. In particular, they show that if the authentication and QKD schemes are proven to be composable, and if a short initial key is available, a continuous key stream can be generated with arbitrarily small error. But since Wegman and Carter’s security proof does not fit in any composable security framework, it raises the question of whether this QKD application is still secure.

## 1.1 Related work

Many works, e.g., [18–21], reuse Wegman and Carter’s authentication scheme with key recycling, and they all sketch the security using the same non-composable argument as Wegman and Carter. Composable security for key recycling in authentication has been studied in the case of quantum messages by Hayden, Leung and Mayers [22],<sup>4</sup> but

<sup>2</sup>This unified treatment of classical and quantum cryptography allows for a seamless composition of the two, see Remark 3 on page 8.

<sup>3</sup>We refer to a review article such as [14] for a general overview of QKD.

<sup>4</sup>In the quantum case, recycling is possible due to the no-cloning theorem. If the message is successfully authenti-

to the best of our knowledge has not been treated when the messages are classical.<sup>5</sup> Computationally secure variants of Wegman and Carter’s scheme have been proposed [23, 24], but not analysed in a composable framework either.<sup>6</sup>

Some works [26, 27] have pointed out that information-theoretic authentication might not be composable with QKD. They attempt to study this problem by analyzing the security of authentication when the secret keys used are not perfect. In particular, Abidin and Larsson [27] suggest that when QKD and authentication with key recycling are combined recursively, and the (imperfect) secret key resulting from QKD is fed back into the next round of authentication and QKD, the total error could increase exponentially in the number of rounds.

## 1.2 New results

We therefore prove that Wegman and Carter’s authentication scheme with key recycling [1] is secure using the AC framework of Maurer and Renner [13]. Since this framework defines composable security independently from the computational model of the underlying system, any (classical) protocol proven to be secure is immediately composable with quantum protocols. The authentication scheme with key recycling studied in this work can thus be used by a QKD protocol.

We show that simply by learning if a message was accepted, the recycled hash function is gradually leaked to the adversary, even when the key used for the OTP is perfect. This leakage is however very small: we prove that this scheme is indeed  $\varepsilon$ -secure if the hash functions used are  $\varepsilon$ -ASU<sub>2</sub>. Since for any good ASU<sub>2</sub> hash function construction  $\varepsilon$  decreases exponentially fast in the size of the family,  $\log |\mathcal{H}|$ , it can easily be made arbitrarily small. As a consequence, the doubts of [26, 27] are unfounded

ated, the eavesdropper cannot have any information about it or the corresponding cipher, and therefore has no information about the secret key used either.

<sup>5</sup>Standard information-theoretic authentication — in which a different hash function is used for every new message — has not been explicitly proven to be composable either. But as we note in the Appendix, its security reduces to the stand-alone security criterion used in the literature, and is therefore immediate from Stinson’s work [2].

<sup>6</sup>Though the composability of public-key authentication constructed from digital signatures has been extensively studied [25].

for all ranges of the parameter  $\varepsilon$ . In fact, the hash functions used are slightly weaker than  $\text{ASU}_2$  hashing, namely almost XOR universal<sub>2</sub> ( $\text{AXU}_2$ ) hash functions.<sup>7</sup>

In a composable framework, a protocol is said to be  $\varepsilon$ -secure, if every use of this protocol increases the overall error by at most  $\varepsilon$ .<sup>8</sup> An immediate implication of our result and the composability of the AC framework is that if  $n$  messages are authenticated this way in each round of an  $\varepsilon'$ -secure QKD protocol, which is run  $r$  times, recycling the same hash function throughout all the runs, the concatenation of all generated keys — all bits of key that have not be used by other rounds of QKD — has distance at most  $rn\varepsilon + r\varepsilon'$  from uniform.

We analyze Wegman and Carter’s authentication scheme in two settings. In the first, the sender and receiver, Alice and Bob, have a mild form of synchronization: Bob knows that a message has been sent,<sup>9</sup> and rejects any message that is delayed for too long, even if it passes the authentication. The reason for this is that, were Alice to reuse the key in an application which leaks it to Eve — e.g., encrypting a message known to Eve — Eve could then use this information about the key to modify the original message and only then deliver it to Bob, breaking the authentication scheme. So Alice only recycles her key *after* the timeout has occurred.<sup>10</sup>

The second setting is a completely asynchronous network. Here, the key cannot be reused arbitrarily. However, we show that it is still safe to recycle it for further rounds of authentication. This is because obtaining extra message-tag pairs give no information to Eve about the recycled key. So the delayed attack outlined above is useless.

<sup>7</sup>A family of functions is said to be  $\text{AXU}_2$  if the bitwise XOR of the hash of any two different messages is almost uniform over the choice of hash function. See Definition 2 on page 4 for an exact definition.

<sup>8</sup>More precisely, a protocol is  $\varepsilon$ -secure if the resulting real system is  $\varepsilon$ -close to an ideal system (see Definition 4 on page 8). By a triangle inequality type argument, the distance of a composed protocol from ideal is the sum of the distances of the individual protocols [17].

<sup>9</sup>A similar notion of synchronization was used by Maurer [28] to authenticate a long message given an authentic channel for a short message, but no shared key.

<sup>10</sup>Alternatively, Alice could recycle her key before the timeout if she receives a confirmation of reception from Bob. But this requires extra authenticated communication, see the discussion in Section 4.

### 1.3 Key consumption

Finding an authentication scheme which allows part of the key to be recycled is quite trivial: if only part of the key is actually used by the protocol, the remaining key bits can be “reused”. Alternatively, the classical message could be authenticated using the quantum scheme of Hayden et al. [22].<sup>11</sup> The former obviously does not allow any key to be saved, the latter is impossible to implement (with today’s technology).

Wegman and Carter’s key recycling scheme is of interest, because it results in a net gain in secret key bits consumed and is efficiently implementable (if the chosen hash functions are efficiently implementable). It is already in use in QKD systems, for example in [29]. There, the authors chose to use an  $\text{ASU}_2$  hash function construction of Bierbrauer et al. [30], which is efficiently implementable in hardware. This  $\text{ASU}_2$  family — which has size  $\log |\mathcal{H}| \approx 2 \log \log |\mathcal{X}| + 3 \log |\mathcal{T}|$ , where  $\mathcal{X}$  is the message alphabet and  $\mathcal{T}$  the tag alphabet — is equivalent to an  $\text{AXU}_2$  family with an additional OTP, i.e., it already has the form  $h_{k_1, k_2}(x) = h'_{k_1}(x) \oplus k_2$ . It can thus be used in a scheme with key recycling without needing any extra key bits to encrypt the tag for the first message. All subsequent messages then only need  $\log |\mathcal{T}|$  bits of fresh key, at least a 2/3 gain over using a new hash function each time.

### 1.4 Structure of this paper

We start in Section 2 with some brief technical preliminaries. In Section 3 we introduce Abstract Cryptography, and model standard authentication (without key recycling) in the AC framework to illustrate its basic principles. In Section 4 we define the task of recycling part of the key for arbitrary use. We show that this is achieved by Wegman and Carter’s scheme with a mild form of synchronization between sender and receiver. In Section 5 we recursively use the authentication scheme  $n$  times, but in a fully asynchronous network, and prove that it is secure with error  $n\varepsilon$ . And finally in Section 6 we take a closer look at the secret key which is leaked to the adversary, and show that an optimal attack over  $n$  rounds of authentication which takes

<sup>11</sup>Although this would only allow the key to be recycled if the authentication is successful.

advantage of this key leakage allows the adversary to break the scheme with probability exactly  $n\varepsilon$ .

## 2 Technical preliminaries

### 2.1 Notation

In the AC framework that we introduce in the next section, the real and ideal systems<sup>12</sup> are modeled as (black) boxes with ports that take inputs and produce outputs. We use upper case letters,  $X$ ,  $Y$ —to label these ports, lower case letters  $x$ ,  $y$ —to denote the values that are in- or output, and a calligraphic font for the alphabets of permitted values  $\mathcal{X}$ ,  $\mathcal{Y}$ . To describe probability distributions over these values we write  $P_X$ ,  $Q_X$ , where the subscript denotes the port. Different distributions over the same port are differentiated by the main upper case letter, e.g.,  $P$ ,  $Q$ . For example, in the real setting  $x$  might be output at port  $X$  with probability  $P_X(x)$  and in the ideal setting with probability  $Q_X(x)$ . To denote the probability of  $y$  being output at port  $Y$  given that  $x$  was input at port  $X$ , we use standard notation and write  $P_{Y|X}(y|x)$ .

### 2.2 Statistical distance

To measure the distance between two settings with distributions  $P_X$  and  $Q_X$ , we use the *statistical* or *total variation* distance, which is given by

$$\begin{aligned} & \frac{1}{2} \sum_{x \in \mathcal{X}} |P_X(x) - Q_X(x)| \\ &= \max_{\mathcal{X}' \subseteq \mathcal{X}} \sum_{x \in \mathcal{X}'} (P_X(x) - Q_X(x)) \\ &= \sum_{x: P_X(x) > Q_X(x)} (P_X(x) - Q_X(x)). \quad (1) \end{aligned}$$

In this paper we use many times this last equality to bound the statistical distance. We find the subset  $\mathcal{X}' \subseteq \mathcal{X}$  for which  $P_X(x) \geq Q_X(x)$ , and then only need to evaluate the distributions  $P_X$  and  $Q_X$  on this subset.

<sup>12</sup>Composable security frameworks define security as the distance between the real system and some ideal system that performs the task in an perfect way.

### 2.3 Universal hashing

Standard authentication is performed by sending the message along with a hash of it, where the hash function is taken from an  $\text{ASU}_2$  family, defined as follows.

**Definition 1** (strongly universal<sub>2</sub> hash function<sup>13</sup> [2]). A family of hash functions  $\{h_k : \mathcal{X} \rightarrow \mathcal{T}\}_{k \in \mathcal{K}}$  is said to be  $\varepsilon$ -almost strongly universal<sub>2</sub> ( $\varepsilon$ - $\text{ASU}_2$ ) if for  $k$  chosen uniformly at random and all  $x_1, x_2 \in \mathcal{X}$  with  $x_1 \neq x_2$  and all  $t_1, t_2 \in \mathcal{T}$ ,

$$\Pr_k[h_k(x_1) = t_1 \text{ and } h_k(x_2) = t_2] \leq \frac{\varepsilon}{|\mathcal{T}|}. \quad (2)$$

In the case of key recycling, we use a weaker family of hash functions, but then encrypt the tag with a OTP. These hash functions have been dubbed  $\varepsilon$ -almost XOR universal<sub>2</sub> by Rogaway [20],  $\varepsilon$ -otp secure by Krawczyk [18, 19], and  $\varepsilon$ - $\Delta$  universal by Stinson [31].<sup>14</sup>

**Definition 2** (XOR universal<sub>2</sub> hash function [20]). A family of hash functions  $\{h_k : \mathcal{X} \rightarrow \mathcal{T}\}_{k \in \mathcal{K}}$  for  $\mathcal{T} = \{0, 1\}^m$  is said to be  $\varepsilon$ -almost XOR universal<sub>2</sub> ( $\varepsilon$ - $\text{AXU}_2$ ) if for  $k$  chosen uniformly at random and all  $x_1, x_2 \in \mathcal{X}$  with  $x_1 \neq x_2$  and all  $t \in \mathcal{T}$ ,

$$\Pr_k[h_k(x_1) \oplus h_k(x_2) = t] \leq \varepsilon. \quad (3)$$

It is immediate from this definition that the hash function  $g_{k_1, k_2}(x) := h_{k_1}(x) \oplus k_2$  is  $\varepsilon$ - $\text{ASU}_2$ , i.e., for all  $x_1, x_2 \in \mathcal{X}$  with  $x_1 \neq x_2$  and all  $t_1, t_2 \in \mathcal{T}$ ,

$$\Pr_{k_1, k_2}[g_{k_1, k_2}(x_1) = t_1 \text{ and } g_{k_1, k_2}(x_2) = t_2] \leq \frac{\varepsilon}{|\mathcal{T}|}.$$

We derive two more useful statements about hash functions of the form  $g_{k_1, k_2}(x) = h_{k_1}(x) \oplus k_2$ , where  $h_{k_1}$  is  $\varepsilon$ - $\text{AXU}_2$ . The first is that the tag for one message is uniformly distributed and independent from the key  $k_1$ : since XORing a uniform string  $k_2$  to any value yields a uniform string we have for all  $x$ ,  $t$  and  $k_1$ ,

$$\Pr_{k_2}[g_{k_1, k_2}(x) = t] = \frac{1}{|\mathcal{T}|}. \quad (4)$$

<sup>13</sup>The more common definition of strongly universal<sub>2</sub> hashing [2, 21, 30, 31] has an extra condition, namely that for all  $x \in \mathcal{X}$  and  $t \in \mathcal{T}$ ,  $\Pr[h_k(x) = t] = \frac{1}{|\mathcal{T}|}$ . This is however not a necessary condition to prove the security of authentication, so we omit it.

<sup>14</sup>Stinson [31] generalizes this notion to any additive abelian group  $\mathcal{T}$  instead of only bit strings.

The second is a conditional form of the strongly universal<sub>2</sub> property of these hash functions: by combining the two equations above we immediately get

$$\Pr_{k_1, k_2} [g_{k_1, k_2}(x_2) = t_2 | g_{k_1, k_2}(x_1) = t_1] \leq \varepsilon. \quad (5)$$

These probabilities of hashing to a certain value can be rewritten as conditional probabilities of the in- and outputs of an authentication system in the following way. If a message  $x$  is input into an authentication system which outputs a tag  $t = h_k(x)$ , the probability distribution of the tag is given by

$$P_{T|X}(t|x) = \Pr_k[h_k(x) = t].$$

If an adversary obtains a valid message and tag  $x||t$  and chooses some  $x'||t'$  to input into a verification system which outputs a decision  $y \in \{\text{acc}, \text{rej}\}$ , the probability of this corrupted message of being accepted is

$$\begin{aligned} P_{Y|XTX'T'}(\text{acc}|x, t, x', t') \\ = \Pr_k[h_k(x') = t' | h_k(x) = t]. \end{aligned}$$

### 3 Abstract cryptography

To model security we use Maurer and Renner’s [13] Abstract Cryptography framework. In this section we give an introduction to the special case needed in this work, namely information-theoretic security of classical systems with three parties, an honest Alice and Bob and dishonest Eve. For more extensive introductions to the AC framework in the three party setting we refer to [32] and [17]. The first reference treats only the classical case, the second is also valid for quantum systems.

The AC security definition<sup>15</sup> applies to abstract systems, which can be instantiated with different models of computation. In particular, it is equally valid for classical and quantum systems [17, 33], and any protocol proven to be information-theoretic “classically secure” is immediately “quantum secure”. An equivalent to Unruh’s lifting lemma [12] — which proves that classical UC security of a classical scheme implies quantum UC security — is unnecessary, since this is immediate from the model.<sup>16</sup>

<sup>15</sup>Definition 4 on page 8.

<sup>16</sup>See Remark 3 on page 8 for further explanation.

### 3.1 Overview

The traditional approach to defining security can be seen as *bottom-up*. One first defines (at a low level) a computational model (e.g., a Turing machine or a circuit). Based on this, the concept of an algorithm for the model and a communication model (e.g., based on tapes) are defined. After this, notions of complexity, efficiency, and finally the security of a cryptosystem can be defined. The AC framework uses a *top-down* approach: in order to state definitions and develop a theory, one starts from the other end, the highest possible level of abstraction — the composition of abstract systems — and proceeds downwards, introducing in each new lower level only the minimal necessary specializations. One may give the analogous example of group theory, which is used to describe matrix multiplication. In the bottom-up approach, one would start explaining how matrices are multiplied, and then based on this find properties of the matrix multiplication. In contrast to this, the AC approach would correspond to first defining the (abstract) multiplication group and prove theorems already on this level. The matrix multiplication would then be introduced as a special case of the multiplicative group.

On a high level of abstraction, a cryptographic protocol,  $\pi$ , can be seen as constructing some resource  $\mathcal{S}$  from other resources  $\mathcal{R}$ . For example, information-theoretic authentication constructs an *authentic channel* resource from an *insecure channel* resource and a *secret key* resource. The resource constructed can be used by other protocols, e.g., a QKD protocol uses an *insecure quantum channel* resource and an *authentic (classical) channel* to construct a *secret key* resource. In general however, these resources are not constructed perfectly, there is some small probability,  $\varepsilon$ , that the adversary can break the scheme, e.g., corrupt a message or learn a secret key. If  $\pi$  constructs  $\mathcal{S}$  out of  $\mathcal{R}$  within  $\varepsilon$ , we write

$$\mathcal{R} \xrightarrow{\pi, \varepsilon} \mathcal{S}. \quad (6)$$

For a cryptographic construction to be usable in an arbitrary context, it needs to be composable, i.e., the following conditions must be fulfilled:

$$\begin{aligned} \mathcal{R} \xrightarrow{\pi, \varepsilon} \mathcal{S} \text{ and } \mathcal{S} \xrightarrow{\pi', \varepsilon'} \mathcal{T} &\implies \mathcal{R} \xrightarrow{\pi' \circ \pi, \varepsilon + \varepsilon'} \mathcal{T}, \\ \mathcal{R} \xrightarrow{\pi, \varepsilon} \mathcal{S} \text{ and } \mathcal{R}' \xrightarrow{\pi', \varepsilon'} \mathcal{S}' &\implies \mathcal{R} || \mathcal{R}' \xrightarrow{\pi || \pi', \varepsilon + \varepsilon'} \mathcal{S} || \mathcal{S}', \end{aligned}$$

where  $\mathcal{R} \parallel \mathcal{R}'$  is a parallel composition of resources, and  $\pi' \circ \pi$  and  $\pi | \pi'$  are sequential and parallel composition of protocols. In Section 3.3 we give a definition for Eq. (6) which satisfies these composable conditions. Intuitively, the resource  $\mathcal{R}$  along with the protocol  $\pi$  are part of the *real* world, and the resource  $\mathcal{S}$  is the *ideal* resource we want to build. Eq. (6) is then satisfied if an adversary could, in an ideal world where the ideal resource is available, achieve anything that she could achieve in the real world. This argument involves, as a thought experiment, simulator systems which transform the ideal resource into the real world system consisting of the real resource and the protocol. But before stating the security definition, we first define in Section 3.2 the elements present in Eq. (6), namely resources  $\mathcal{R}$ ,  $\mathcal{S}$ , a protocol  $\pi$ , and a pseudo-metric<sup>17</sup> on the space of resources to quantify the failure  $\varepsilon$ .

### 3.2 Resources, converters and distinguishers

On the highest level of abstraction, a *system* is an abstract object with interfaces that interacts with its environment and with other systems. Two systems can be composed into a single system by connecting one interface of each system. In order to define Eq. (6), it is sufficient to introduce (abstract) systems that can be composed into larger systems and define an appropriate pseudo-metric. These systems can then be instantiated at a lower level with any formalism that satisfies the abstract (composition and metric) properties required by the abstract systems, e.g., random systems [34, 35] in the classical case, or, if the underlying computational model is quantum, as a sequence of completely positive, trace-preserving maps with internal memory (e.g., quantum strategies [36] and combs [37]). It is however not necessary to define — or even consider — these lower levels to define cryptographic security.

Since the concrete systems used in this work are all very intuitive, we will not provide a generic

<sup>17</sup>A function  $d : \Omega \times \Omega \rightarrow \mathbb{R}_+$  is a pseudo-metric on  $\Omega$  if for all  $a, b, c \in \Omega$  the three following conditions hold:

$$\begin{aligned} d(a, a) &= 0, \\ d(a, b) &= d(b, a), \\ d(a, b) &\leq d(a, c) + d(c, b). \end{aligned}$$

If additionally  $d(a, b) = 0 \implies a = b$ , then  $d$  is a metric.

(mathematical) definition of these lower levels. Instead we define each concrete system individually when needed.

**Resource.** An  $\mathcal{I}$ -resource is an (abstract) system with interfaces specified by a set  $\mathcal{I}$  (e.g.,  $\mathcal{I} = \{A, B, E\}$ ). Each interface  $i \in \mathcal{I}$  is accessible to a user  $i$  and provides her or him with certain functionalities. Resources are equipped with a parallel composition operator,  $\parallel$ , that maps two resources to another resource.

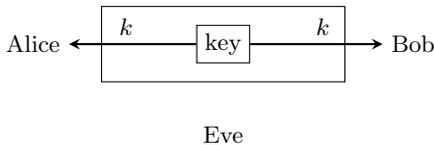
**Converter.** To transform one resource into another, we use *converters*. These are (abstract) systems with two interfaces, an *inside* interface and an *outside* interface. The inside interface connects to an interface of a resource, and the outside interface becomes the new interface of the constructed resource. We write either  $\alpha_i \mathcal{R}$  or  $\mathcal{R} \alpha_i$  to denote the new resource with the converter  $\alpha_i$  connected at the interface  $i$ ,<sup>18</sup> and  $\alpha \mathcal{R}$  or  $\mathcal{R} \alpha$  for a set of converters  $\alpha = \{\alpha_i\}_i$ , for which it is clear to which interface they connect. Converters are equipped with parallel and sequential composition operators,  $|$  and  $\circ$ , that map two converters to another converter. A protocol  $\pi = \{\pi_i\}_i$  is a set of converters  $\pi_i$ , indexed by a subset of interfaces  $i \in \mathcal{I}$ .

**Example.** To illustrate these notions we model standard information-theoretic authentication. To provide an example it is necessary to make these systems concrete. We only do this informally in the following. By using the language of random systems [34, 35] (or any other appropriate formalism), this may be made mathematically precise.

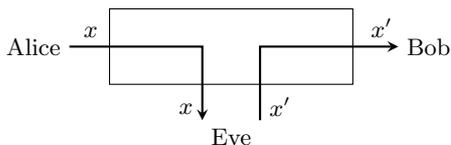
Let us first consider the resources used in the construction of an authentic channel. A *secret key resource*,  $\mathcal{K}$ , can be seen as a box that outputs a secret key at Alice's and Bob's interfaces, but does not provide any functionality at Eve's interface. This is illustrated in Figure 1.

The authentication protocol is run over an *insecure channel*,  $\mathcal{C}$ , completely under the control of Eve. One way of modeling such a channel is to allow Eve to intercept the message sent from Alice to

<sup>18</sup>There is no mathematical difference between  $\alpha_i \mathcal{R}$  and  $\mathcal{R} \alpha_i$ . It sometimes simplifies the notation to have the converters for some players written on the right of the resource and the ones for other players on the left, instead of all on the same side, hence the two notations.



**Figure 1** – A *secret key resource*  $\mathcal{K}$  that always gives a key  $k$  to Alice and Bob, and nothing to Eve.

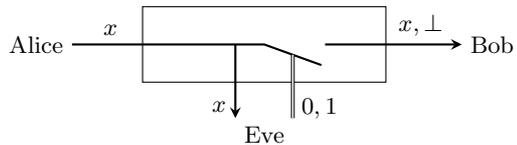


**Figure 2** – An *insecure channel*  $\mathcal{C}$  from Alice to Bob. Eve obtains Alice’s message  $x$ , and can choose what Bob receives, e.g.,  $x'$ .

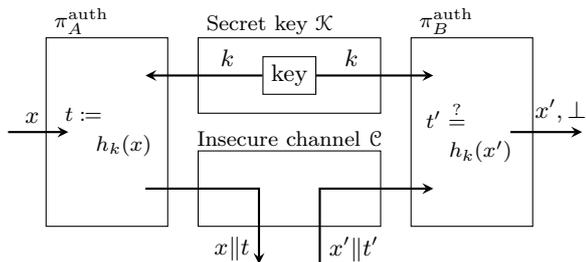
Bob, and replace it with any message of her choice. This is depicted in Figure 2.

The channel we wish to construct from a secret key and insecure channel, is an *authentic channel resource*,  $\mathcal{A}$ . Ideally, we would like the channel to always deliver the correct message to the receiver. This is however impossible to construct from an insecure channel, since Eve can always jumble the communication between Alice and Bob. What can be constructed, is a channel which guarantees that Bob does not receive a corrupted message. He either receives the correct message sent by Alice, or an error, which symbolizes an attempt by Eve to change or block the message. This can be modelled by giving Eve’s idealized interface two controls: the first provides her with Alice’s message, the second allows her to decide if Alice’s message should be delivered to Bob or if he gets an error flag instead. We illustrate this in Figure 3. Note that Eve also has the option of not inputting anything. In a completely asynchronous model, Bob would then not receiving anything. If the model allows him to know that a message was sent, he would get an error flag  $\perp$  after the timeout.

For Alice to send an authenticated message to Bob, her part of the protocol,  $\pi_A^{\text{auth}}$ , is a converter, which gets a key  $k$  at the inner interface from an ideal key resource, a message  $x$  at the outer interface from Alice, computes a tag  $t = h_k(x)$  and sends the concatenation of the message and tag,  $x||t$  through its inside interface down the insecure



**Figure 3** – An *authentic channel*  $\mathcal{A}$  from Alice to Bob. Eve obtains Alice’s message  $x$ , and can choose whether Bob receives the original message or an error  $\perp$ .



**Figure 4** – The *real authentication system*. Alice has access to the left interface, Bob to the right interface and Eve to the lower interface. The converters  $(\pi_A^{\text{auth}}, \pi_B^{\text{auth}})$  of the authentication protocol are connected to the resource  $\mathcal{K}||\mathcal{C}$  consisting of a secret key and an insecure channel.  $\pi_A^{\text{auth}}$  generates a tag  $t$  for the message  $x$  using the secret key  $k$ .  $\pi_B^{\text{auth}}$  checks if the received message and tag,  $x'||t'$ , match.

channel. Bob’s part of the protocol,  $\pi_B^{\text{auth}}$ , gets the same key as Alice,  $k$ , from the ideal key resource at its inner interface, a message  $x'||t'$  from the channel at its inner interface, and outputs at its outer interface either  $x'$  if  $t' = h_k(x')$ , or an error  $\perp$  otherwise. This is depicted in Figure 4.

**Distinguisher.** To measure how close two resources are, we define a pseudo-metric<sup>19</sup> on the space of resources. We do this with the help of a *distinguisher*. For  $n$ -interface resources a distinguisher  $\mathcal{D}$  is a system with  $n + 1$  interfaces, where  $n$  interfaces connect to the interfaces of a resource  $\mathcal{R}$  and the other (outside) interface outputs a bit. For a class of distinguishers  $\mathbb{D}$ , the induced pseudo-metric, the distinguishing advantage, is

$$d(\mathcal{R}, \mathcal{S}) := \max_{\mathcal{D} \in \mathbb{D}} \Pr[\mathcal{D}\mathcal{R} = 1] - \Pr[\mathcal{D}\mathcal{S} = 1],$$

<sup>19</sup>Recall Footnote 17 on page 6.

where  $\mathcal{DR}$  is the binary random variable corresponding to  $\mathcal{D}$  connected to  $\mathcal{R}$ . In this work we study information-theoretic security, and therefore the only class of distinguishers that we consider is the set of all distinguishers. If  $d(\mathcal{R}, \mathcal{S}) \leq \varepsilon$ , we say that the two resources are  $\varepsilon$ -close and sometimes write  $\mathcal{R} \approx_\varepsilon \mathcal{S}$ ; or  $\mathcal{R} = \mathcal{S}$  if  $\varepsilon = 0$ .

In Section 3.4 we introduce some of the properties of the distinguishing advantage. We also show how it can be related to the statistical distance between the probability distributions of the underlying real and ideal systems.

*Remark 3.* If two *classical* systems  $\mathcal{R}$  and  $\mathcal{S}$  are indistinguishable for the set of all classical distinguishers, then they are also indistinguishable for the set of all quantum distinguishers, since classical computers (or distinguishers) can simulate quantum ones. A secure classical protocol is then secure regardless of whether it is later composed with quantum systems. This is immediate from the framework without needing to define quantum security.<sup>20</sup>

### 3.3 Security definition

We can now define the security of a protocol in the three party setting with honest Alice and Bob, and dishonest Eve.

**Definition 4** (Composable security [13]). Let  $\mathcal{R}$  and  $\mathcal{S}$  be resources with interfaces  $\mathcal{I} = \{A, B, E\}$ . We say that a protocol  $\pi = (\pi_A, \pi_B)$  (securely) constructs  $\mathcal{S}$  out of  $\mathcal{R}$  within  $\varepsilon$ , and write  $\mathcal{R} \xrightarrow{\pi, \varepsilon} \mathcal{S}$ , if the two following conditions hold:

- i) For converters  $\sharp_E$  and  $\flat_E$  which emulate an honest behavior at Eve's interfaces,

$$d(\pi\mathcal{R}\sharp_E, \mathcal{S}\flat_E) \leq \varepsilon.$$

- ii) There exists a converter  $\sigma_E$  — which we call simulator — such that

$$d(\pi\mathcal{R}, \mathcal{S}\sigma_E) \leq \varepsilon.$$

If it is clear from the context what resources  $\mathcal{R}$  and  $\mathcal{S}$  are meant, we simply say that  $\pi$  is  $\varepsilon$ -secure.

<sup>20</sup>In the case of computational security, the class of classical distinguishers efficiently implementable on a quantum computer guarantees security under composition with quantum systems.

The first of these two conditions can be seen as capturing correctness of the protocol in the case where no adversary is present, i.e., when the distinguisher does not access the adversarial controls covered by  $\sharp_E$  and  $\flat_E$ . If however the opponent is active, the converters  $\sharp_E, \flat_E$  are removed and the distinguisher has full access to Eve's interfaces. This is captured by the second condition in Definition 4. In the case of authentication, it is not hard to see that  $\pi^{\text{auth}}(\mathcal{K}|\mathcal{C})\sharp_E = \mathcal{A}\flat_E$ . If the converter  $\sharp_E$  faithfully transmits the string  $x||t$  to Bob, the system  $\pi^{\text{auth}}(\mathcal{K}|\mathcal{C})\sharp_E$  is a simple channel that transmits Alice's message  $x$  to Bob. If the converter  $\flat_E$  at Eve's interface of the ideal authentic channel  $\mathcal{A}$  inputs the bit allowing Alice's message through, we then also have a channel that transmits Alice's message  $x$  to Bob.

To show that condition (ii) is fulfilled (for some well chosen family of hash functions), we need to find a simulator  $\sigma_E^{\text{auth}}$  that can recreate the real  $E$ -interface while accessing just the idealized one. An obvious choice for the simulator is to first generate its own key  $k$  and output  $x||h_k(x)$ . Then upon receiving  $x'||t'$ , it checks if  $x'||t' = x||h_k(x)$  and sets the control bit of the ideal authentic channel  $\mathcal{A}$  accordingly. If the distinguisher chooses to provide the inputs in reversed order, first input  $x'||t'$  at the  $E$ -interface and then choose a message  $x$  that it inputs at the  $A$ -interface, the simulator always sets the control bit on the authentic channel to output an error  $\perp$  at the  $B$ -interface and does not choose the key  $k$  uniformly at random from the entire set, but only from the subset of keys such that  $h_k(x') \neq t'$ . We illustrate this simulator in Figure 5.

### 3.4 Distinguishing advantage

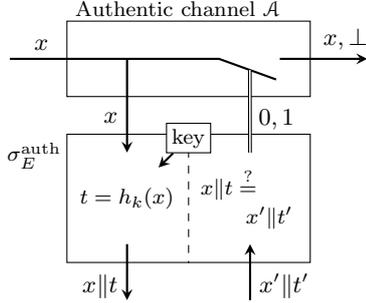
Because it is a pseudo-metric, the distinguishing advantage respects the triangle inequality. For any resources  $\mathcal{R}, \mathcal{S}, \mathcal{T}$ ,

$$d(\mathcal{R}, \mathcal{S}) \leq d(\mathcal{R}, \mathcal{T}) + d(\mathcal{T}, \mathcal{S}). \quad (7)$$

The distinguishing advantage is also non-increasing under composition with any other (abstract) system. For any systems  $\mathcal{R}, \mathcal{S}, \mathcal{T}$ ,

$$d(\mathcal{R}\mathcal{T}, \mathcal{S}\mathcal{T}) \leq d(\mathcal{R}, \mathcal{S}). \quad (8)$$

This holds because the distinguisher can run  $\mathcal{T}$  internally, so the maximization over all distinguishers



**Figure 5** – The *ideal authentication system*. Alice has access to the left interface, Bob to the right interface and Eve to the lower interface. The simulator  $\sigma_E^{\text{auth}}$  is plugged into the  $E$ -interface of the ideal authentic channel  $\mathcal{A}$ . It generates its own tag  $t = h_k(x)$  to simulate the message  $x||t$  on the insecure channel, and notifies the ideal authentic channel to output an error  $\perp$  if any part of  $x||t$  got changed.

already includes the composition with  $\mathcal{J}$ .<sup>21</sup>

Resources can be seen as interactive black boxes. Each interface has various ports which either take inputs or produce outputs. A distinguisher holding a resource can input the value of its choosing at any port (as long as the value is from the permitted alphabet for that port), and store the outputs it receives. For example, if it holds the real authentication system of Figure 4, it can input some message  $x$  at the  $A$ -interface and receives  $x||t$  at the  $E$ -interface. This in- and output pair is described by a joint probability distribution  $P_{XT}$  where  $X$  is the label of the input port on the  $A$ -interface, and  $T$  denotes the port outputting  $t$  on the  $E$ -interface.<sup>22</sup> After having interacted with all the ports, the distinguisher can make a guess as to whether it is holding the real or the ideal system. Its distinguishing advantage is given by the statistical distance (Eq. (1)) between the probability distributions of the corresponding in- and outputs. By maximizing the statistical distance over all permutations of the input ports and all choices of inputs we get the distinguishing advantage between the two resources.

<sup>21</sup>This also holds in the case of computational security, since the class of efficiently implementable distinguishers is closed under composition with efficiently implementable systems  $\mathcal{J}$ .

<sup>22</sup> $x$  is also output at the  $E$ -interface, but since it is always identical to the input at port  $X$ , we avoid writing it a second time.

For example, the real and ideal authentication systems of Figures 4 and 5 take two inputs, a message  $x$  at the  $X$  port of the  $A$ -interface and a string  $x' || t'$  at the  $X'T'$  port of the  $E$ -interface, so there are also two orderings in which the distinguisher can interact with the system. In the standard authentication literature these two possible orders are referred to as *substitution attack* (when the distinguisher first obtains  $x||t$  and modifies it to create  $x' || t'$ ) and *impersonation attack* (when the distinguisher directly generates  $x' || t'$  without having seen a valid message-tag pair).

In the case of a substitution attack, let the distinguisher choose some input  $x$  with probability  $P_X(x)$ . It then receives  $x||t$  where  $P_{T|X}(t|x)$  is defined by the scheme used, and is identical in the real and ideal situations. It chooses some  $x' || t'$  with probability  $P_{X'T'|XT}(x', t'|x, t)$ , and receives a final output  $y$  which can take two values,  $x'$  or  $\perp$ . The probability of taking either of these values is however different in the real and ideal settings. In the real one,  $y = x'$  if  $h_k(x') = t'$ , and in the ideal one,  $y = x'$  if  $x' || t' = x||t$ . Let  $P_{Y|XTX'T'}$  and  $Q_{Y|XTX'T'}$  be the probability distributions of  $y$  in these two settings. The real world is then completely described by  $P_{XTX'T'Y}$  and the ideal world by  $Q_{XTX'T'Y}$ , where  $Q_{XTX'T'} = P_{XTX'T'}$ . The distinguisher can now pick a subset of values  $\mathcal{D} \subseteq \mathcal{X} \times \mathcal{T} \times \mathcal{X} \times \mathcal{T} \times (\mathcal{X} \cup \{\perp\})$ , for which it will guess that it is holding the real system, i.e., for all  $(x, t, x', t', y) \in \mathcal{D}$  it outputs 1 and for all others it outputs 0. Its advantage is then

$$\begin{aligned} & \max_{\mathcal{D}} \sum_{(x,t,x',t',y) \in \mathcal{D}} (P_{XTX'T'Y}(x, t, x', t', y) \\ & \quad - Q_{XTX'T'Y}(x, t, x', t', y)) \\ & = \frac{1}{2} \sum_{x,t,x',t',y} |P_{XTX'T'Y}(x, t, x', t', y) \\ & \quad - Q_{XTX'T'Y}(x, t, x', t', y)|, \quad (9) \end{aligned}$$

the statistical distance between  $P_{XTX'T'Y}$  and  $Q_{XTX'T'Y}$ .

In the case of an impersonation attack, the distinguisher will gain nothing from inputting  $x$ , after having input  $x' || t'$ : if it was lucky and the real system accepted  $x' || t'$ , it can already distinguish them with advantage 1, and if  $x' || t'$  was rejected, both systems will generate a  $x||t$  with the same distribution since the simulator picks  $k$  from all keys such

that  $h_k(x') \neq t'$ . We can thus consider only the ports  $X'T'$  and  $Y$ . The advantage is then

$$\frac{1}{2} \sum_{x', t', y} |P_{X'T'Y}(x', t', y) - Q_{X'T'Y}(x', t', y)|. \quad (10)$$

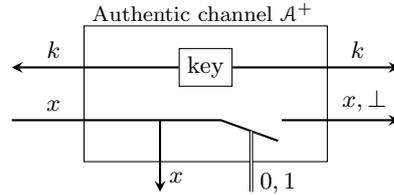
In Appendix A we provide a proof that Eqs. (9) and (10) are both bounded by  $\varepsilon$  for all  $P_X, P_{X'T'|XT}$  and  $P_{X'T'}$ , respectively, if  $\varepsilon$ -ASU<sub>2</sub> hash functions are used by the protocol.

## 4 Arbitrary key recycling with synchronization

The authentication protocol depicted in Figure 4 requires a new hash function and therefore a (completely) new secret key  $k$  for every new message. We study in this section a scheme which allows part of the secret key to be reused. Intuitively, this is possible because the recycled part of the key is almost uniform conditioned on all the inputs, outputs and the adversary's information produced throughout the protocol.

It is however vital that the key be recycled only *after* the receiver, Bob, either obtains the message or decides not to accept anymore an incoming message. Otherwise Eve could first obtain the recycled key from whatever protocol uses it next and then modify the authenticated message. In this section we thus assume a mild form of synchronization between sender and receiver: an incoming message will not be accepted by Bob after the key has been recycled by Alice.

For practicality we model this as a timeout, i.e., after a predefined amount of time has passed the sender can recycle the key and the receiver will not accept a message arriving late. It can however be achieved by other means, e.g., Bob sends an authenticated acknowledgment of receptions. This occurs naturally if the messages alternate, one from Alice to Bob and the next from Bob to Alice. If Alice successfully authenticates an incoming message, this guarantees — up to an error  $\varepsilon$  — that her previous message was received by Bob, who otherwise would not have sent the next message. So she can already recycle the key, even if the timeout has not occurred. Likewise Bob can recycle the key used for the messages in the other direction as soon as he receives an authenticated response from Alice.



**Figure 6** – An *authentic channel with key recycling*  $\mathcal{A}^+$ . Alice has access to the left interface, Bob to the right interface and Eve to the lower interface. Eve obtains Alice's message  $x$ , and can choose whether Bob receives the original message or an error  $\perp$ . After providing Bob with the error or message, the resource generates a new key  $k$ , which is given to both Alice and Bob.

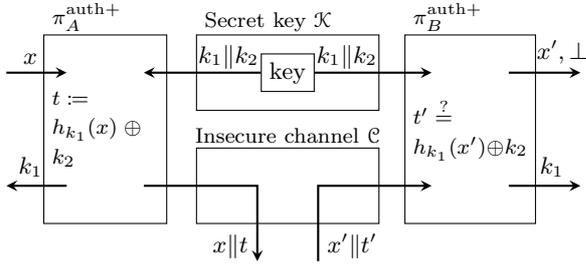
### 4.1 Ideal resource

An authentic channel with key recycling,  $\mathcal{A}^+$ , does not only guarantee that the message delivered has not been tampered with, it also provides the users with a new (recycled) key that they can use for any arbitrary application requiring a shared secret key. It can be seen as the combination of an authentic channel (Figure 3) and a secret key resource (Figure 1), which we illustrate in Figure 6.

This ideal resource must follow the same timing rules as the real system. As Bob can recycle the key as soon as he receives the message from Alice, the resource  $\mathcal{A}^+$  outputs the new key  $k$  at the  $B$ -interface at the same time as the message  $x$  or the decision to reject it,  $\perp$ . However, the new key  $k$  is only output at the  $A$ -interface when the timeout occurs. In the real system, Bob's protocol can output an error  $\perp$  if no message has arrived when the time is up, since no late message is accepted. The ideal resource  $\mathcal{A}^+$  thus also outputs  $\perp$  in this case.

### 4.2 Protocol

Like for standard authentication, we construct the ideal resource from a secret key and an insecure channel. As stated in the introduction, the main idea is to encrypt the tag  $t$  appended to the message in standard authentication with a OTP, and then reuse the same hash function. For this, Alice's protocol  $\pi_A^{\text{auth}^+}$  splits the shared secret key in two parts  $k = k_1 \| k_2$ , and uses them to generate a tag  $t := h_{k_1}(x) \oplus k_2$  for the message  $x$ , where  $\{h_k\}_k$  is a family of  $\varepsilon$ -AXU hash functions. The string  $x \| t$  is



**Figure 7** – The *real authentication with key recycling* system. Alice has access to the left interface, Bob to the right interface and Eve to the lower interface.  $\pi_A^{\text{auth}+}$  first receives a message  $x$  and key  $k_1||k_2$ , and sends  $x||t$  to Bob.  $\pi_B^{\text{auth}+}$  checks whether  $t' = h_{k_1}(x') \oplus k_2$  and outputs the result  $x'$  or  $\perp$ . Both converters recycle  $k_1$ , Bob's immediately and Alice's after the timeout.

sent to Bob on the insecure channel  $\mathcal{C}$ . After a predefined amount of time has elapsed, Alice outputs the key  $k_1$  for recycling.

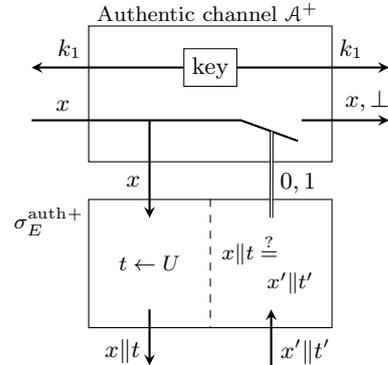
Bob's protocol  $\pi_B^{\text{auth}+}$ , upon receiving  $x'||t'$  on the insecure channel, checks whether  $t' = h_{k_1}(x') \oplus k_2$ , and if so, it accepts and outputs the message  $x'$  at its outer interface. Otherwise, it outputs an error symbol  $\perp$ . If no message has been received before the timeout, it outputs an error and does not accept incoming messages any more. The key  $k_1$  is also output for recycling as soon as accepted or rejected, or a timeout occurred. The entire protocol is depicted in Figure 7.

If Eve does not intervene, security is modeled by placing a converter  $\sharp_E$  over the  $E$ -interface of the system, which connects the out- and in-ports of the insecure channel and emulates an honest behavior by forwarding  $x||t$  to Bob.

### 4.3 Security

To prove that the protocol depicted in Figure 7 constructs the ideal resource of Figure 6, we need to find a simulator  $\sigma_E^{\text{auth}+}$  that when plugged into the idealized  $E$ -interface of the authentication channel with key recycling recreates the real interface of Figure 7. We do this with the simulator from Figure 8.

Similar to that of Figure 5, the simulator  $\sigma_E^{\text{auth}+}$  generates a tag  $t$  uniformly at random, which it outputs with the message  $x$ . Then upon receiving  $x'||t'$ ,



**Figure 8** – The *ideal authentication with key recycling* system. Alice has access to the left interface, Bob to the right interface and Eve to the lower interface. Upon receiving a message  $x$  from the ideal channel, the simulator  $\sigma_E^{\text{auth}+}$  picks a tag  $t$  uniformly at random and outputs  $x||t$  at the  $E$ -interface. When it receives  $x'||t'$ , it checks if this is equal to  $x||t$ , and sets the control bit on the ideal channel accordingly.

it checks if the message has been changed and notifies the ideal resource  $\mathcal{A}^+$  of this. In the case of an impersonation attack, in which first  $x'||t'$  is input at the  $E$ -interface and then  $x$  at the  $A$ -interface, the simulator still generates the tag  $t$  uniformly at random.

**Theorem 5.** *The protocol  $(\pi_A^{\text{auth}+}, \pi_B^{\text{auth}+})$  described above is  $\varepsilon$ -secure, i.e.,*

$$\mathcal{K}||\mathcal{C} \xrightarrow{\pi^{\text{auth}+}, \varepsilon} \mathcal{A}^+.$$

As described in Section 3.4, the real and ideal systems depicted in Figures 7 and 8 can be seen as interactive black boxes. The distinguisher is given one of the two, can input the values of its choosing, and based on the outputs, it must guess which of the two it is holding. Similar to the discussion in Section 3.4, for every ordering of the inputs and every (probabilistic) choice of inputs we derive a probability distribution of the in- and outputs for the real and ideal systems, and need to bound the statistical distance between the two. To do this, we show that the probability of an event which includes a rejected corrupted message is always larger in the ideal case. Since the probability of an event which includes an accepted corrupted message is always larger in the real case (it never happens in

the ideal case), we have from Eq. (1) that the statistical distance reduces to the difference between the distributions in the case of accepted corrupted messages. But this is simply the probability of accepting a corrupted message in the real setting.

*Proof.* In the case where no adversary is present, we trivially have

$$\pi^{\text{auth}+}(\mathcal{K}||\mathcal{C})\#_E = \mathcal{A}^+ \flat_E,$$

since both systems are equivalent to a channel that faithfully transmits a message from Alice to Bob. So we only need to analyze the case of an active adversary.

Each box in Figures 7 and 8 takes two inputs— a message  $x$  at the  $A$ -interface, and possibly corrupted message  $x'|t'$  at the  $E$ -interface. So the distinguisher can choose to start by first providing either some  $x$  or some  $x'|t'$  to the system it is holding.

We first analyze the latter, the impersonation attack. Note that after choosing the input  $x'|t'$  and receiving Bob's output and the recycled key at the  $B$ -interface, the distinguisher can still input a value  $x$  at the  $A$ -interface and receive the corresponding tag  $t$ . Let  $P_{X'T'YKXT}$  and  $Q_{X'T'YKXT}$  describe the distributions of the real and ideal systems. We show in the following that for all  $x', t', k_1, x, t$ ,

$$\begin{aligned} P_{X'T'YKXT}(x', t', \perp, k_1, x, t) \\ \leq Q_{X'T'YKXT}(x', t', \perp, k_1, x, t). \end{aligned} \quad (11)$$

Since  $Q_{X'T'YKXT}(x', t', x', k_1, x, t) = 0$ , the statistical distance is then given by

$$\begin{aligned} \sum_{x', t', k_1, x, t} P_{X'T'YKXT}(x', t', x', k_1, x, t) \\ &= \sum_{x', t'} P_{X'T'Y}(x', t', x') \\ &= \sum_{x', t'} P_{X'T'}(x', t') P_{Y|X'T'}(x'|x', t') \\ &= \sum_{x', t'} P_{X'T'}(x', t') 2^{-|k_2|} = 2^{-|k_2|}, \end{aligned}$$

where to reach the last line we used Eq. (4), namely

$$P_{Y|X'T'}(x'|x', t') = \Pr_{k_1, k_2} [h_{k_1}(x') \oplus k_2 = t'] = \frac{1}{2^{|k_2|}}.$$

It now remains to show that Eq. (11) holds. In the ideal case the tag  $t$  and key  $k_1$  are always generated uniformly at random and  $\perp$  is output of the  $Y$  port with probability 1. So

$$\begin{aligned} Q_{X'T'YKXT}(x', t', \perp, k_1, x, t) \\ &= Q_{X'T'}(x', t') Q_{Y|X'T'}(\perp|x', t') \\ &\quad Q_{K|X'T'Y}(k_1|x', t', \perp) Q_{X|X'T'YK}(x|x', t', \perp, k_1) \\ &\quad Q_{T|X'T'YKX}(t|x', t', \perp, k_1, x) \\ &= P_{X'T'}(x', t') P_{X|X'T'YK}(x|x', t', \perp, k_1) 2^{-|k_1|-|k_2|}. \end{aligned}$$

In the real case, the probability of accepting  $x'$  is exactly  $2^{-|k_2|}$ , so the probability of detecting the impersonation is

$$P_{Y|X'T'}(\perp|x', t') = 1 - 2^{-|k_2|}.$$

As shown in Eq. (4), the probability of detecting the impersonation is actually independent of the value of the recycled key, hence

$$P_{K|X'T'Y}(k_1|x', t', \perp) = 2^{-|k_1|}.$$

Finally, knowledge of an invalid pair  $(x', t')$  and the recycled key  $k_1$  excludes exactly one possible value for  $t$ , the others are all equally likely. So

$$P_{T|X'T'YKX}(t|x', t', \perp, k_1, x) \leq \frac{1}{2^{|k_2|} - 1}.$$

Putting this together,

$$\begin{aligned} P_{X'T'YKXT}(x', t', \perp, k_1, x, t) \\ &= P_{X'T'}(x', t') P_{Y|X'T'}(\perp|x', t') \\ &\quad P_{K|X'T'Y}(k_1|x', t', \perp) P_{X|X'T'YK}(x|x', t', \perp, k_1) \\ &\quad P_{T|X'T'YKX}(t|x', t', \perp, k_1, x) \\ &\leq P_{X'T'}(x', t') P_{X|X'T'YK}(x|x', t', \perp, k_1) 2^{-|k_1|-|k_2|} \\ &= Q_{X'T'YKXT}(x', t', \perp, k_1, x, t). \end{aligned}$$

We now consider the substitution attack. In both the real and ideal cases, the distinguisher chooses some  $x$ , obtains  $x||t$  and then has to pick some  $x'|t'$ . Note that if the distinguisher chooses  $x' = x$ , both systems behave identically: they reject  $x'$  if  $t' \neq t$ , accept it if  $t' = t$ , and output a recycled key which is uniform and independent from  $(x, t, x', t')$  (see Eq. (4) for the independence of  $k_1$ ). So it is sufficient to consider strategies for which  $x' \neq x$ . Thus, w.l.o.g. we assume that the distinguisher chooses values for the inputs at ports  $X$  and  $X'$  such that

for all  $x$ ,  $P_{XX'}(x, x) = Q_{XX'}(x, x) = 0$ . In the following, we show that for all  $x, t, x', t', k_1$

$$\begin{aligned} P_{XTX'T'YK}(x, t, x', t', \perp, k_1) \\ \leq Q_{XTX'T'YK}(x, t, x', t', \perp, k_1). \end{aligned} \quad (12)$$

Since  $Q_{XTX'T'YK}(x, t, x', t', x', k_1) = 0$ , the statistical distance is then given by

$$\begin{aligned} \sum_{x, t, x', t', k_1} P_{XTX'T'YK}(x, t, x', t', x', k_1) \\ = \sum_{x, t, x', t'} P_{XTX'T'Y}(x, t, x', t', x') \\ = \sum_{x, t, x', t'} P_{XTX'T'}(x, t, x', t') \\ P_{Y|XTX'T'}(x' | x, t, x', t') \\ \leq \sum_{x, t, x', t'} P_{XTX'T'}(x, t, x', t') \varepsilon = \varepsilon, \end{aligned}$$

where to reach the last line we used

$$\begin{aligned} P_{Y|XTX'T'}(x' | x, t, x', t') = \\ \Pr_{k_1, k_2} [h_{k_1}(x') \oplus k_2 = t' | h_{k_1}(x) \oplus k_2 = t] \end{aligned}$$

and Eq. (5).

It now remains to prove that Eq. (12) holds. Note that

$$\begin{aligned} Q_{XTX'T'YK}(x, t, x', t', \perp, k_1) \\ = P_{XTX'T'}(x, t, x', t') 2^{-|k_1|}, \end{aligned}$$

so it is sufficient to show that for all  $x, t, x', t', k_1$ ,

$$P_{YK|XTX'T'}(\perp, k_1 | x, t, x', t') \leq 2^{-|k_1|}.$$

This is however immediate from

$$\begin{aligned} P_{YK|XTX'T'}(\perp, k_1 | x, t, x', t') \\ = P_{K|XTX'T'}(k_1 | x, t, x', t') \\ P_{Y|XTX'T'K}(\perp | x, t, x', t', k_1) \\ \leq P_{K|XTX'T'}(k_1 | x, t, x', t') \\ = P_{K|XT}(k_1 | x, t) = 2^{-|k_1|}, \end{aligned}$$

where in the last line we used the fact that knowing one valid message-tag pair  $(x, t)$  provides no information about  $k_1$ , already expressed in Eq. (4).  $\square$

## 5 Key recycling for further rounds of authentication

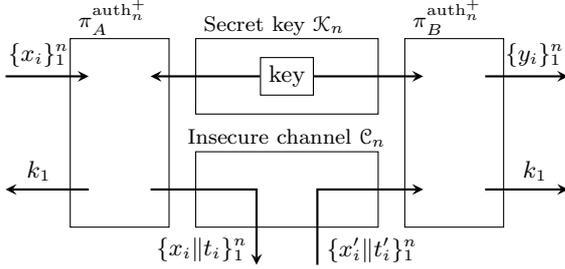
Since the key recycled in the scheme analyzed in Section 4 can be used for arbitrary applications, it can be used in particular for authenticating another message. By the composition of the security definition, running  $n$  times that protocol for authenticating  $n$  messages is  $n\varepsilon$ -secure and uses a key of total length  $|k_1| + n|k_2|$ . This requires synchronization between every message, e.g., if the timeout is set to  $s$  seconds and all messages go from Alice to Bob with no other communication, then only one message could be sent every  $s$  seconds.

It is however possible to authenticate multiple messages without any form of synchronization. This is because revealing new message-tag pairs does not leak any information about the key  $k_1$  to the adversary, since the tags are one-time padded by  $k_2$ . Hence the next rounds of authentication can be started before Bob receives the message from the first round. It is only necessary to be synchronized after all the messages have been delivered, so that  $k_1$  may then be put back in the pool of keys for arbitrary future use.

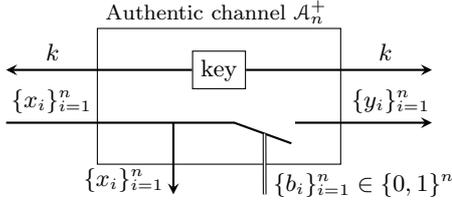
If the key  $k_1$  is deleted after  $n$  rounds of authentication instead of being recycled, then no synchronization is needed at all. The security proof for the protocol in a completely asynchronous network follows as a corollary from security with synchronization, in which  $k_1$  is deleted after  $n$  rounds.

### 5.1 Real and ideal systems

We thus define the protocol  $\pi^{\text{auth}_n^+}$  to authenticate  $n$  messages by using one key  $k_1$ ,  $n$  keys  $\{k_2^i\}_{i=1}^n$  — we denote by  $\mathcal{K}_n$  the resource generating these keys — and a multiple use insecure channel  $\mathcal{C}_n$ . The tag for the  $i^{\text{th}}$  message  $x_i$  is given by  $t_i = h_{k_1}(x_i) \oplus k_2^i$ , where  $\{h_k\}_{k \in \mathcal{K}}$  is a  $\varepsilon$ -AXU<sub>2</sub> family of hash functions. As in the previous section, we assume a mild form of synchronization which allows the key  $k_1$  to be recycled by Alice at the end of the protocol, when all  $n$  messages have either been received or Bob has decided not to accept any more incoming messages. Bob's protocol can however recycle  $k_1$  as soon as all messages have been received. If some messages are still missing after the timeout, Bob's protocol outputs an error for each of these



**Figure 9** – The *real multiple authentication with key recycling* system. Alice has access to the left interface, Bob to the right interface and Eve to the lower interface. The  $i^{\text{th}}$  message  $x_i$  is appended with a tag  $t_i = h_{k_1}(x_i) \oplus k_2^i$ , where a different  $k_2^i$  is used for every message but the same  $k_1$  is reused throughout. Bob’s protocol either accepts the received message or outputs an error,  $y_i \in \{x'_i, \perp\}$ .



**Figure 10** – A *multiple authentic channel with key recycling*  $\mathcal{A}_n^+$ . Alice has access to the left interface, Bob to the right interface and Eve to the lower interface. Alice can send  $n$  messages  $\{x_i\}_{i=1}^n$ . Eve obtains these messages, and can choose for each one whether Bob receives it or an error  $\perp$ . After having provided Bob with either  $y_i \in \{x_i, \perp\}$  for every message, the resource generates a new key  $k$ , which is given to both players.

messages. This is depicted in Figure 9.

The resource constructed by this protocol is illustrated in Figure 10. It corresponds to a simple modification of the resource for one message (Figure 6), in which multiple messages can now be input. For each message, the adversary can provide one bit  $b_i \in \{0, 1\}$  that either lets the original message through or produces an error at Bob’s interface. Once all bits  $\{b_i\}_{i=1}^n$  have been input, the resource  $\mathcal{A}_n^+$  outputs a new key  $k$  at Bob’s interface, and waits for the timeout to occur to output  $k$  at Alice’s interface as well.

Note that neither of the resources from Figures 9 and 10 expect all messages  $\{x_i\}_{i=1}^n$  to be input si-

multaneously, they can be transmitted one at a time. These messages are processed in the order in which they are received, i.e., the first message input at the  $A$ -interface is labeled  $x_1$ , the second is labeled  $x_2$ , etc. Likewise, the  $i^{\text{th}}$  message-tag pair input at the  $E$ -interface is labeled  $x'_i || t'_i$  and its authenticity is verified with the keys  $k_1$  and  $k_2^i$ .

## 5.2 Security

To prove that  $\pi_n^{\text{auth}_n^+}$  constructs the ideal resource  $\mathcal{A}_n^+$  we use as simulator  $n$  independent copies of the simulator  $\sigma_E^{\text{auth}_n^+}$  from Figure 8. We show that with this simulator, the ideal and real systems are  $n\varepsilon$ -close, namely

$$\pi_n^{\text{auth}_n^+}(\mathcal{K}_n || \mathcal{C}_n) \approx_{n\varepsilon} \mathcal{A}_n^+ \sigma_E^{\text{auth}_n^+}.$$

To do this, we rewrite the real and ideal systems as sequences of boxes which we denote  $\mathcal{R}$  and  $\mathcal{J}$ , behaving similarly to one round of the real and ideal systems, respectively. We then show by induction that  $n$  copies of  $\mathcal{R}$  and one key generation system  $\mathcal{K}$  are  $n\varepsilon$ -close to  $n$  copies of  $\mathcal{J}$  and one  $\mathcal{K}$ .

**Theorem 6.** *The protocol for authenticating  $n$  messages with the same hash function,  $\pi_n^{\text{auth}_n^+}$ , is  $n\varepsilon$ -secure, i.e.,*

$$\mathcal{K}_n || \mathcal{C}_n \xrightarrow{\pi_n^{\text{auth}_n^+, n\varepsilon}} \mathcal{A}_n^+.$$

*Proof.* As in the case of one round of authentication from Section 4, if Eve is inactive—which we model as a converters  $\sharp_E, b_E$  plugged into the  $E$ -interface and allowing Alice’s messages through—the real and ideal systems are both equivalent to a channel that perfectly transmits  $n$  messages. They are thus indistinguishable.

In the case of an active adversary, let  $\sigma_E^{\text{auth}_n^+}$  denote the  $n$  copies of the simulator for the one message case, which we use as simulator in this proof. Consider the real and ideal systems, namely  $\pi_n^{\text{auth}_n^+}(\mathcal{K}_n || \mathcal{C}_n)$  and  $\mathcal{A}_n^+ \sigma_E^{\text{auth}_n^+}$ . Both of them output at Alice’s interface a key  $k_1$  which is a copy of the key output at Bob’s interface; both of them output the key at the same time, when the timeout occurs. Receiving this key therefore cannot help the distinguisher, so we can ignore it.

Now consider two boxes. The first,  $\mathcal{K}$ , simply produces a uniformly distributed key  $k_1$ . The second,  $\mathcal{R}$ , behaves similarly to the composed systems

of Alice, Bob and the channels for one round of authentication. It takes a key  $k_1$  and message  $x$  as input. If it receives  $k_1$  before  $x$ , it stores the key, generates a random  $k_2$  and waits for  $x$ . Upon receiving  $x$ , it outputs  $x\|t$  with  $t = h_{k_1}(x) \oplus k_2$ . If it first receives the message  $x$  before  $k_1$ , it outputs  $x\|t$  where  $t$  is a uniformly random tag of the correct length. Upon receiving  $k_1$  later, it retroactively computes what key  $k_2$  would have resulted in the tag  $t$ , namely  $k_2 = t \oplus h_{k_1}(x)$ . Upon receiving  $x'\|t'$  it checks if  $t' = h_{k_1}(x') \oplus k_2$  and outputs either  $x'$  or  $\perp$  as well as  $k_1$ . If no message  $x'\|t'$  is input by the timeout, it outputs  $\perp$  and  $k_1$ .

A sequence of  $n$  such boxes—depicted on the left in Figure 11—is identical to the real system without Alice’s recycled key. Both systems output exactly the same messages with the same probabilities. What is more, every system  $\mathcal{R}$  receives  $k_1$  before  $x'\|t'$ , because the messages are authenticated in order, i.e., the  $i^{\text{th}}$  pair is sent to the  $i^{\text{th}}$  box at which point  $k_1$  is transmitted to the box  $i + 1$ . It can thus always check whether  $t' = h_{k_1}(x') \oplus k_2$ .

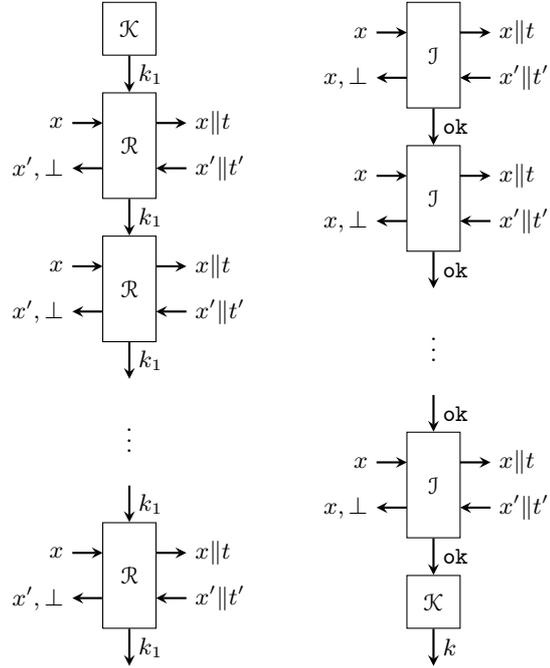
The ideal system can similarly be rewritten as a sequence of boxes  $\mathcal{J}$ , with one final box  $\mathcal{K}$  that generates a new key. After receiving some message  $x$ , each box  $\mathcal{J}$  outputs a uniform tag  $t$  appended to the message. Upon receiving some  $x'\|t'$ , each box outputs  $\perp$  if no  $x$  had been received or if  $(x'\|t') \neq (x\|t)$ , and  $x$  otherwise. It then notifies the next box that it is finished. The final box notifies the key generating box  $\mathcal{K}$ , which outputs a new key. If the timeout occurs, all boxes that have not produced an output  $y \in \{x, \perp\}$  yet, output an error  $\perp$  and the key generation box outputs a new key.

A sequence of  $n$  such boxes—depicted on the right in Figure 11—is identical to the ideal system without Alice’s recycled key. Both systems output exactly the same messages with the same probabilities. So the distance between the real and ideal systems of multiple message authentication with key recycling is equal to the distance between these recursive systems depicted in Figure 11,

$$\begin{aligned} d\left(\pi_n^{\text{auth}^+}(\mathcal{K}_n\|\mathcal{C}_n), \mathcal{A}_n^+ \sigma_E^{\text{auth}^+}\right) \\ = d(\mathcal{K}\mathcal{R}_1 \cdots \mathcal{R}_n, \mathcal{J}_1 \cdots \mathcal{J}_n \mathcal{K}), \end{aligned}$$

where we have numbered the boxes with subscripts.

We prove by induction that these systems can be distinguished with advantage at most  $n\varepsilon$ . The case



**Figure 11** – The real and ideal authentication systems rearranged as a sequence of boxes  $\mathcal{R}$  and  $\mathcal{J}$ .  $\mathcal{J}$  simply generates a uniform tag  $t$  independently from the message  $x$ , and outputs an error  $\perp$  if  $(x\|t) \neq (x'\|t')$  or if no message  $x$  has been provided. The box  $\mathcal{R}$  transmits  $k_1$  to the next box only after having output  $y \in \{x, \perp\}$ . If it receives  $x$  before having gotten  $k_1$  from the previous box, it outputs a uniformly random  $t$  and retroactively computes  $k_2 = h_{k_1}(x) \oplus t$  when provided with  $k_1$ .

of  $n = 1$  is proven by Theorem 5. If this holds for  $n - 1$ , we have

$$d(\mathcal{K}\mathcal{R}_1 \cdots \mathcal{R}_{n-1}, \mathcal{J}_1 \cdots \mathcal{J}_{n-1} \mathcal{K}) \leq (n - 1)\varepsilon.$$

Because the distinguishing advantage respects the triangle inequality (Eq. (7)) and is non-increasing under composition with other systems (Eq. (8)), we have

$$\begin{aligned} d(\mathcal{K}\mathcal{R}_1 \cdots \mathcal{R}_n, \mathcal{J}_1 \cdots \mathcal{J}_n \mathcal{K}) \\ \leq d(\mathcal{K}\mathcal{R}_1 \cdots \mathcal{R}_n, \mathcal{J}_1 \mathcal{K} \mathcal{R}_2 \cdots \mathcal{R}_n) \\ \quad + d(\mathcal{J}_1 \mathcal{K} \mathcal{R}_2 \cdots \mathcal{R}_n, \mathcal{J}_1 \cdots \mathcal{J}_n \mathcal{K}) \\ \leq d(\mathcal{K}\mathcal{R}_1, \mathcal{J}_1 \mathcal{K}) + d(\mathcal{K}\mathcal{R}_2 \cdots \mathcal{R}_n, \mathcal{J}_2 \cdots \mathcal{J}_n \mathcal{K}) \\ \leq \varepsilon + (n - 1)\varepsilon. \quad \square \end{aligned}$$

The protocol  $\pi_n^{\text{auth}^+}$  still uses synchronization for Alice to know that it is safe to recycle the key  $k_1$

for arbitrary use after the  $n^{\text{th}}$  message has been sent. Wegman and Carter’s original authentication scheme does not do this, it only recycles the key for further rounds of authentication. This can be seen as destroying the key when there are no more messages to be authenticated. It can thus be modeled with the real and ideal systems of  $\pi^{\text{auth}_n^+}$ , but with the final recycled key removed, which simultaneously removes all need for synchronization. This can only decrease the distance between the real and ideal systems. Note that if we remove all form of synchronization, Bob cannot know that messages have been sent, he will therefore not output some error  $\perp$  if he receives nothing, but simply wait and do nothing.

**Corollary 7.** *Wegman and Carter’s scheme for authenticating  $n$  messages is  $n\varepsilon$ -secure in a completely asynchronous network.*

## 6 Secret key leakage

In this section we look at attacks on Wegman and Carter’s authentication scheme. We show that in the special case where a  $\varepsilon$ -AXU<sub>2</sub> hash function with  $\varepsilon = \frac{1}{|\mathcal{T}|}$  is recycled for  $n$  rounds, there exists an attack which meets the upper bound on the failure probability of Corollary 7. This means that the attack successfully corrupts at least one of the  $n$  messages with probability  $n\varepsilon$ .

For the adversary to obtain this total success probability, the success probability in each round must increase, as the following calculation shows. Let us define  $\{F_i\}_i$  to be a sequence of random variables taking the value 1 if the adversary successfully corrupts a message in any of the first  $i$  rounds, and 0 otherwise. We then have for any  $0 \leq i \leq 1/\varepsilon - 1$ ,

$$\begin{aligned} P_{F_{i+1}|F_i}(1|0) &= \frac{P_{F_{i+1}}(1) - P_{F_{i+1}F_i}(1,1)}{P_{F_i}(0)} \\ &= \frac{(i+1)\varepsilon - i\varepsilon}{1 - i\varepsilon} = \frac{\varepsilon}{1 - i\varepsilon}. \end{aligned}$$

$P_{F_{i+1}|F_i}(1|0)$  is strictly increasing in  $i$ , i.e., the success probability for the adversary increases in every round. This is because — as we show in Theorem 8 here below — some information about the hash function is leaked in every round, even if the key used for the OTP is perfectly uniform. The entropy of the

hash function gradually decreases, until the adversary has enough information to successfully corrupt a new message with probability 1.

This result contrasts strongly with the non-composable analysis found in [1]. There, the adversary simply collects the pairs of messages and tags  $(x_1, t_1), (x_2, t_2), \dots$ , and attempts to corrupt a message in each round, independently from the attempts in previous rounds. In this case, due to the hiding property of the OTP, the distribution of the hash function always remains perfectly uniform given these message-tag pairs.

**Theorem 8.** *If the same hash function from a family of  $\frac{1}{|\mathcal{T}|}$ -almost XOR universal<sub>2</sub> functions is recycled in  $n$  rounds of authentication, for any  $1 \leq n \leq |\mathcal{T}|$ , there exists an attack that allows the adversary to successfully corrupt one of the first  $n$  messages with probability at least  $n/|\mathcal{T}|$ . Furthermore, after  $n$  rounds, the entropy of the recycled key  $K$  is bounded by*

$$H(K|Z_n) \leq \log \frac{|\mathcal{K}|}{|\mathcal{T}|} + \left(1 - \frac{n}{|\mathcal{T}|}\right) \log(|\mathcal{T}| - n),$$

where  $Z_n$  consists of all the inputs and outputs of the protocol — the messages, tags, and accept or reject results — from these  $n$  rounds.

*Proof.* This attack assumes that the adversary, Eve, gets to choose which messages are sent, and learns whether a corrupted message was accepted or not. Eve chooses to always send the same message  $x$  during these  $n$  rounds, and always substitutes the same message  $x' \neq x$  for  $x$  in each round as well. To be successful, she needs to guess correctly the value  $c = h_{k_1}(x) \oplus h_{k_1}(x')$ , since  $t' = t \oplus c$ , where  $t$  is the tag that comes with  $x$  and  $t'$  is the correct tag for  $x'$ . Since the family of hash functions is  $\frac{1}{|\mathcal{T}|}$ -AXU<sub>2</sub>, the distribution of  $c$  is uniform.<sup>23</sup> Eve therefore makes a list of the  $|\mathcal{T}|$  possible values for  $c$ , and in each round eliminates one from her list.

In the first round she receives  $(x, t_1)$  from the insecure channel, picks a  $c_1$  from her list and sends  $(x', t_1 \oplus c_1)$  back on the channel. The legitimate player accepts the message received from the adversary only if  $c_1 = h_{k_1}(x) \oplus h_{k_1}(x')$ , which happens with probability  $p_1 = |\mathcal{T}|^{-1}$ .

<sup>23</sup>For any family of functions,  $\sum_c \Pr_k[h_k(x_1) \oplus h_k(x_2) = c] = 1$  for all  $x_1 \neq x_2$ . So being  $\frac{1}{|\mathcal{T}|}$ -AXU<sub>2</sub> means that  $\Pr_k[h_k(x_1) \oplus h_k(x_2) = c] = \frac{1}{|\mathcal{T}|}$ .

If she is unsuccessful at corrupting the message, she can cross  $c_1$  off her list. In the second round she then receives  $(x, t_2)$ , picks a new  $c_2 \neq c_1$ , and sends  $(x', t_2 \oplus c_2)$ . This time her success probability is  $p_2 = (|\mathcal{T}| - 1)^{-1}$ , since she only has  $|\mathcal{T}| - 1$  elements  $c$  left on the list which are all equally probable.

If we repeat this for each round, the success probability in the  $i^{\text{th}}$  round given that the previous  $i - 1$  were unsuccessful is  $p_i = (|\mathcal{T}| - i + 1)^{-1}$ . We now prove by induction that the probability of successfully corrupting at least one message with this strategy is exactly  $i/|\mathcal{T}|$ . Let  $F_i$  be a random variable taking the value 1 if the adversary successfully corrupts a message in any of the first  $i$  rounds, and 0 otherwise. We have  $P_{F_1}(1) = p_1 = 1/|\mathcal{T}|$ . And if  $P_{F_{i-1}}(1) = (i - 1)/|\mathcal{T}|$ , then

$$\begin{aligned} P_{F_i}(1) &= P_{F_{i-1}}(1) + P_{F_{i-1}}(0)p_i \\ &= \frac{i-1}{|\mathcal{T}|} + \left(1 - \frac{i-1}{|\mathcal{T}|}\right) \frac{1}{|\mathcal{T}| - i + 1} = \frac{i}{|\mathcal{T}|}. \end{aligned}$$

Let  $z_0$  represent any value of  $Z_n$  in which Eve fails to corrupt any message, and  $z_1$  be the case where she does trick the legitimate players. If she is successful, she immediately learns the correct value  $c$ , and thus

$$H(K|Z_n = z_1) = \log \frac{|\mathcal{K}|}{|\mathcal{T}|}.$$

If Eve is not successful, she has still managed to cross  $n$  values for  $c$  off her list, so

$$H(K|Z_n = z_0) = \log \frac{|\mathcal{K}|}{|\mathcal{T}|} (|\mathcal{T}| - n).$$

Combining the two equations above with the corresponding probabilities, we get

$$\begin{aligned} H(K|Z_n) &= \left(1 - \frac{n}{|\mathcal{T}|}\right) \log \frac{|\mathcal{K}|}{|\mathcal{T}|} (|\mathcal{T}| - n) \\ &\quad + \frac{n}{|\mathcal{T}|} \log \frac{|\mathcal{K}|}{|\mathcal{T}|}. \quad \square \end{aligned}$$

## Appendices

### A Security proof for standard authentication

We showed in Section 3 that the security of standard authentication reduces to proving that

Eqs. (9) and (10) are bounded by  $\varepsilon$  for all choices of input distributions for  $x, x', t'$ , which we rewrite here:

$$\frac{1}{2} \sum_{x,t,x',t',y} |P_{XTX'T'Y}(x, t, x', t', y) - Q_{XTX'T'Y}(x, t, x', t', y)| \leq \varepsilon, \quad (13)$$

$$\frac{1}{2} \sum_{x',t',y} |P_{X'T'Y}(x', t', y) - Q_{X'T'Y}(x', t', y)| \leq \varepsilon. \quad (14)$$

In fact, these two equations are strictly weaker than the security conditions derived by Wegman and Carter [1] and Stinson [2], who maximize over all  $x, t, x', t'$  instead of over  $x, x', t'$ . It is therefore immediate that previous work on information-theoretic authentication (without key recycling) is composable. We have however given a simplification of the more common  $\varepsilon$ -ASU<sub>2</sub> hashing definition, which does not include the extra requirement that  $\Pr[h_k(x) = t] = \frac{1}{|\mathcal{T}|}$  (see Footnote 13 on page 4), without which Stinson's proof [2] does not apply. We therefore provide a new proof of security here.

**Lemma 9.** *The standard authentication protocol,  $\pi^{\text{auth}}$ , is  $\varepsilon$ -secure.*

This lemma uses the same proof technique as Theorem 5: we reduce the statistical distance to the sum over all events which are more probable in the real case, namely accepting a corrupted message.

*Proof.* We start with impersonation attacks, namely Eq. (14). From the definition of ASU<sub>2</sub> hashing, we have for any  $x', t'$  and  $x \neq x'$ ,

$$\begin{aligned} \Pr_k[h_k(x') = t'] &= \sum_t [h_k(x) = t \text{ and } h_k(x') = t'] \\ &\leq \varepsilon. \end{aligned}$$

And since for all  $x', t'$  the ideal system always rejects the corrupted message,  $Q_{Y|X'T'}(\perp|x', t') = 1$ , and the maximization of Eq. (14) over  $P_{X'T'}$  reduces to

$$\max_{x',t'} P_{Y|X'T'}(x'|x', t') = \max_{x',t'} \Pr_k[h_k(x') = t'] \leq \varepsilon.$$

For the substitution attack we need to show that Eq. (13) is satisfied for all  $P_X = Q_X$  and  $P_{X'T'|XT} = Q_{X'T'|XT}$ . If the distinguisher chooses  $x' = x$ , both the real and ideal systems behave identically—they both accept  $x$  if  $t' = t$  and produce an error otherwise. We can therefore only consider the distributions with  $P_{XX'}(x, x) = 0$ . In this case, the simulator in the ideal setting always outputs an error  $\perp$ , i.e.,  $Q_Y(\perp) = 1$ . Then from Eq. (1), Eq. (13) reduces to

$$\sum_{x,t,x',t'} P_{XTX'T'Y}(x, t, x', t', x') \leq \varepsilon.$$

Combining,

$$\begin{aligned} P_{XTX'T'}(x, t, x', t') &= P_X(x)P_{T|X}(t|x)P_{X'T'|XT}(x', t'|x, t), \\ P_{T|X}(t|x) &= \Pr_k[h_k(x) = t], \\ P_{Y|XTX'T'}(x'|x, t, x', t') &= \Pr_k[h_k(x') = t'|h_k(x) = t], \end{aligned}$$

we get

$$\begin{aligned} &\sum_{x,t,x',t'} P_{XTX'T'Y}(x, t, x', t', x') \\ &= \sum_{x,t,x',t'} P_X(x)P_{X'T'|XT}(x', t'|x, t) \\ &\quad \Pr_k[h_k(x) = t \text{ and } h_k(x') = t'] \\ &\leq \sum_{x,t,x',t'} P_X(x)P_{X'T'|XT}(x', t'|x, t) \frac{\varepsilon}{|\mathcal{T}|} \\ &= \sum_t \frac{\varepsilon}{|\mathcal{T}|} = \varepsilon. \quad \square \end{aligned}$$

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