# Cognitive Interference Channels with Confidential Messages under Randomness Constraint 

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#### Abstract

The cognitive interference channel with confidential messages (CICC) proposed by Liang et. al. is investigated. When the security is considered in coding systems, it is well known that the sender needs to use a stochastic encoding to avoid the information about the transmitted confidential message to be leaked to an eavesdropper. For the CICC, the trade-off between the rate of the random number to realize the stochastic encoding and the communication rates is investigated, and the optimal trade-off is completely characterized.


Index Terms-Cognitive Interference Channel, Confidential Messages, Randomness Constraint, Stochastic Encoder, Superposition Coding

## I. Introduction

Cognitive radio has attracted considerable attention recently, for it can improve the spectrum efficiency of wireless networks [1]. In information theoretical study of the cognitive radio, it is usually modeled by a interference channel called cognitive interference channel (CIC), in which the cognitive transmitter can non-causally know the other transmitter's message [2], [3], [4], [5]. We consider the (CIC) model investigated by Jiang et. al. [6], Zhong et. al. [7], and Liang et. al. [8], in which one receiver needs to decode both messages. Especially as in [8], we also consider the security, i.e., the message sent by the cognitive transmitter must be kept secret from one of the receivers. We call this problem the cognitive interference channel with confidential messages (CICC). The coding system investigated in this paper is described in Fig. 1

When the security is considered, it is well known that the sender needs to use a stochastic encoder to avoid the information about the transmitted confidential message to be leaked to the eavesdropper Eve. The stochastic encoder is usually realized by preparing a dummy random number in addition to the intended messages and by encoding them to a transmitted signal by a deterministic encoder. Furthermore, random numbers are also needed to realize the coding technique called channel prefixing.

In literatures of information theoretic security (eg. [9], [10], [11]), the random number has been regarded as free resource, and the amount of the random number used in the stochastic encoding has been paid no attention. However in practice, the random number is quite precious resource. For example, generation rates of any existing true random number generators

[^0]are not as fast as communication rates of wireless networks [12]. Although the random number generator equipped in the forthcoming Intel's CPU can generate the random number as fast as 3 Gbps [13], the communication rate of the new IEEE wireless communication standard is said to be over Gbps [14]. Thus, the random number should be regarded as at least as precious as communication resources. For this purpose, we formulate the problem of the CICC by randomness constrained stochastic encoder, and completely characterize the capacity region of this new problem. We assume that the non-cognitive transmitter, Charlie, only uses a deterministic encoding. This assumption seems natural because Charlie only observes the common message, and the common message need not to be kept secret.

The present problem to consider the CICC by the randomness constrained stochastic encoder is an extension of the authors' series of works. In [15], the authors investigated the capacity region of the relay channel with confidential messages for the completely deterministic encoder, and the capacity region of the broadcast channel with confidential messages (BCC) for the completely deterministic encoder was characterized as a corollary. In [16], the authors completely characterized the capacity region of the BCC by the randomness constrained stochastic encoder. The problem formulation in this paper is the extension of that in [16] to the CIC, and more involved coding techniques are needed.

Since the security criterion employed in this paper is slightly different from that in [8], it should be remarked. In [8], the cognitive transmitter, Alice, sends two kinds of messages, the common message and the confidential message, and the level of secrecy of the confidential message was evaluated by the equivocation rate. In this paper, Alice sends three kinds of messages, the common message, the private message, and the confidential message. The role of the common message is the same as that in [8]. The private message is supposed to be decoded by one of the receiver, Bob, and we do not care whether Eve can decode the private message or not. On the other hand, the confidential message is supposed to be decoded by Bob, and it must be kept completely secret from Eve. The secrecy of the confidential message is evaluated by the socalled strong security criterion [17], [18]. As a byproduct, our direct coding theorem is stronger than that in [8], i.e., our theorem states the strong secrecy.

The reason we do not use the equivocation rate formulation is as follows. In the conventional equivocation rate formulation, if the rate of dummy randomness is not sufficient, a part of the confidential message is sacrificed to make the other part completely secret and the rate of the completely secret part corresponds to the equivocation rate. We think that the rates


Fig. 1. The coding system investigated in this paper. Alice sends common message $K_{n}$, private message $L_{n}$, and confidential message $S_{n}$ by using a deterministic function $f_{n}$ and a limited amount of dummy randomness $A_{n}$. Charlie also sends a signal $X_{2}^{n}$ which is a deterministic function of the common message $K_{n}$. The common message is supposed to be decoded by both Bob and Eve. The private message is supposed to be decoded by Bob, and we do not care whether Eve can decode the private message or not. The confidential message is supposed to be decoded by Bob, and it must be kept completely secret from Eve.
of sacrificed part and completely secret part become clearer by employing our formulation.

The rest of this paper is organized as follows. In Section III the problem formulation is explained and main results are presented. In Section III the proof of the main theorem is presented. Some technical arguments are presented in Appendices.

## II. Problem Formulation and Main Results

Let $P_{Y \mid X_{1} X_{2}}$ and $P_{Z \mid X_{1} X_{2}}$ be two channels with common input alphabets $\mathcal{X}_{1} \times \mathcal{X}_{2}$ and output alphabets $\mathcal{Y}$ and $\mathcal{Z}$ respectively. Throughout the paper, the alphabets are assumed to be finite though we do not use finiteness of the alphabet except cardinality bonds on auxiliary random variables.

Let $\mathcal{K}_{n}$ be the set of the common message, $\mathcal{L}_{n}$ be the set of the private message, and $\mathcal{S}_{n}$ be the set of the confidential message. The common message is supposed to be decoded by both Bob and Eve. The private message is supposed to be decoded by Bob, and we do not care whether Eve can decode the private message or not. The confidential message is supposed to be decoded by Bob, and it must be kept completely secret from Eve.

Typically, Alice use a stochastic encoder to make the confidential message secret from Eve, and it is practically realized by using a uniform dummy randomness on the alphabet $\mathcal{A}_{n}$. When the size $\left|\mathcal{A}_{n}\right|$ of dummy randomness is infinite, any stochastic encoder from $\mathcal{K}_{n} \times \mathcal{L}_{n} \times \mathcal{S}_{n}$ to $\mathcal{X}_{1}^{n}$ can be simulated by a deterministic encoder $f_{n}: \mathcal{K}_{n} \times \mathcal{L}_{n} \times \mathcal{S}_{n} \times \mathcal{A}_{n} \rightarrow \mathcal{X}^{n}$. But we are interested in the case with bounded size $\left|\mathcal{A}_{n}\right|$ in this paper. In this paper, we assume that Charlie only use a deterministic encoder $f_{n}^{\prime}: \mathcal{K}_{n} \rightarrow \mathcal{X}_{2}^{n}$.

Bob's decoder is defined by function $g_{n}: \mathcal{Y}^{n} \rightarrow \mathcal{K}_{n} \times \mathcal{L}_{n} \times$ $\mathcal{S}_{n}$ and the error probability is defined as

$$
\begin{align*}
& P_{\text {err }}\left(f_{n}, f_{n}^{\prime}, g_{n}\right) \\
&= \sum_{k_{n} \in \mathcal{K}_{n}} \sum_{\ell_{n} \in \mathcal{L}_{n}} \sum_{s_{n} \in \mathcal{S}_{n}} \sum_{a_{n} \in \mathcal{A}_{n}} \frac{1}{\left|\mathcal{K}_{n}\right|\left|\mathcal{L}_{n}\right|\left|\mathcal{S}_{n}\right|\left|\mathcal{A}_{n}\right|} \\
& P_{Y \mid X_{1} X_{2}}\left(y^{n} \mid f_{n}\left(k_{n}, \ell_{n}, s_{n}, a_{n}\right), f_{n}^{\prime}\left(k_{n}\right)\right) \\
& \mathbf{1}\left[g_{n}\left(y^{n}\right) \neq\left(k_{n}, \ell_{n}, s_{n}\right)\right], \tag{1}
\end{align*}
$$

where $\mathbf{1}[\cdot]$ is the indicator function. Eve's decoder is defined by function $\phi_{n}: \mathcal{Z}^{n} \rightarrow \mathcal{K}_{n}$ and the error probability $P_{\text {err }}\left(f_{n}, f_{n}^{\prime}, \phi_{n}\right)$ is defined in a similar manner as Eq. (11).

Let

$$
\begin{aligned}
& P_{\tilde{Z}^{n} \mid S_{n}}\left(z^{n} \mid s_{n}\right)= \sum_{k_{n} \in \mathcal{K}_{n}} \sum_{\ell_{n} \in \mathcal{L}_{n}} \sum_{a_{n} \in \mathcal{A}_{n}} \frac{1}{\left|\mathcal{K}_{n}\right|\left|\mathcal{L}_{n}\right|\left|\mathcal{A}_{n}\right|} \\
& P_{Z \mid X_{1} X_{2}}^{n}\left(z^{n} \mid f_{n}\left(k_{n}, \ell_{n}, s_{n}, a_{n}\right), f_{n}^{\prime}\left(k_{n}\right)\right), \\
& P_{\tilde{Z}^{n}}\left(z^{n}\right)= \sum_{s_{n} \in \mathcal{S}_{n}} \frac{1}{\left|\mathcal{S}_{n}\right|} P_{\tilde{Z}^{n} \mid S_{n}}\left(z^{n} \mid s_{n}\right)
\end{aligned}
$$

be the output distributions of the channel $P_{Z \mid X_{1} X_{2}}^{n}$. In this paper, we consider the security criterion given by

$$
\begin{aligned}
D\left(f_{n}, f_{n}^{\prime}\right) & :=D\left(P_{S_{n} \tilde{Z}^{n}} \| P_{S_{n}} \times P_{\tilde{Z}^{n}}\right) \\
& =\sum_{s_{n} \in \mathcal{S}_{n}} \frac{1}{\left|\mathcal{S}_{n}\right|} D\left(P_{\tilde{Z}^{n} \mid S_{n}}\left(\cdot \mid s_{n}\right) \| P_{\tilde{Z}^{n}}\right) \\
& =I\left(S_{n} ; \tilde{Z}^{n}\right),
\end{aligned}
$$

where $D(\cdot \| \cdot)$ is the divergence, and $I(\cdot ; \cdot)$ is the mutual information [19]. The coding system investigate in this paper is depicted in Fig. 1

In this paper, we are interested in the trade-off among the rate the dummy randomness, and the rates of the common, private, and confidential messages.

Definition 1: The rate quadruple $\left(R_{d}, R_{0}, R_{1}, R_{s}\right)$ is said to be achievable if there exists a sequence of Alice's deterministic encoder $f_{n}: \mathcal{K}_{n} \times \mathcal{L}_{n} \times \mathcal{S}_{n} \times \mathcal{A}_{n} \rightarrow \mathcal{X}_{1}^{n}$, Charlie's deterministic encoder $f_{n}^{\prime}: \mathcal{K}_{n} \rightarrow \mathcal{X}_{2}^{n}$, Bob's decoder $g_{n}: \mathcal{Y}^{n} \rightarrow \mathcal{K}_{n} \times \mathcal{L}_{n} \times$ $\mathcal{S}_{n}$, and Eve's decoder $\phi_{n}: \mathcal{Z}^{n} \rightarrow \mathcal{K}_{n}$ such that

$$
\begin{align*}
\lim _{n \rightarrow \infty} P_{e r r}\left(f_{n}, f_{n}^{\prime}, g_{n}\right) & =0  \tag{2}\\
\lim _{n \rightarrow \infty} P_{e r r}\left(f_{n}, f_{n}^{\prime}, \phi_{n}\right) & =0  \tag{3}\\
\lim _{n \rightarrow \infty} D\left(f_{n}, f_{n}^{\prime}\right) & =0  \tag{4}\\
\limsup _{n \rightarrow \infty} \frac{1}{n} \log \left|\mathcal{A}_{n}\right| & \leq R_{d}  \tag{5}\\
\liminf _{n \rightarrow \infty} \frac{1}{n} \log \left|\mathcal{K}_{n}\right| & \geq R_{0}  \tag{6}\\
\lim _{n \rightarrow \infty} \frac{1}{n} \log \left|\mathcal{L}_{n}\right| & =R_{1}  \tag{7}\\
\liminf _{n \rightarrow \infty} \frac{1}{n} \log \left|\mathcal{S}_{n}\right| & \geq R_{s} \tag{8}
\end{align*}
$$

Then the achievable region $\mathcal{R}$ is defined as the set of all achievable rate quadruples.

The following is our main result in this paper.
Theorem 2: Let $\mathcal{R}^{*}$ be a closed convex set consisting of those quadruples $\left(R_{d}, R_{0}, R_{1}, R_{s}\right)$ for which there exist auxiliary random variables $(U, V)$ such that

$$
\begin{aligned}
& \left(U, X_{2}\right) \leftrightarrow V \leftrightarrow X_{1} \\
& (U, V) \leftrightarrow\left(X_{1}, X_{2}\right) \leftrightarrow(Y, Z)
\end{aligned}
$$

and

$$
\begin{align*}
R_{0} \leq & \min \left[I\left(U, X_{2} ; Y\right), I\left(U, X_{2} ; Z\right)\right]  \tag{9}\\
R_{1}+R_{s} \leq & I\left(U, V ; Y \mid X_{2}\right),  \tag{10}\\
R_{0}+R_{1}+R_{s} \leq & I\left(V ; Y \mid U, X_{2}\right) \\
& +\min \left[I\left(U, X_{2} ; Y\right), I\left(U, X_{2} ; Z\right)\right],  \tag{11}\\
R_{s} \leq & I\left(V ; Y \mid U, X_{2}\right)-I\left(V ; Z \mid U, X_{2}\right),  \tag{12}\\
R_{1}+R_{d} \geq & I\left(X_{1} ; Z \mid U, X_{2}\right),  \tag{13}\\
R_{d} \geq & I\left(X_{1} ; Z \mid U, V, X_{2}\right) . \tag{14}
\end{align*}
$$

Then we have $\mathcal{R}=\mathcal{R}^{*}$. Moreover, it may be assumed that the ranges of $U$ and $V$ may be assumed to satisfy

$$
\begin{aligned}
|\mathcal{U}| & \leq\left|\mathcal{X}_{1}\right|\left|\mathcal{X}_{2}\right|+3 \\
|\mathcal{V}| & \leq\left|\mathcal{X}_{1}\right|^{2}\left|\mathcal{X}_{2}\right|^{2}+4\left|\mathcal{X}_{1}\right|\left|\mathcal{X}_{2}\right|+3
\end{aligned}
$$

Proof: See Section III
Remark 3: As we will find in the achievability proof of the main theorem, the private message can be used as dummy randomness to protect the confidential message from Eve. Thus, if we define the achievability rate region $\mathcal{R}$ by replacing Eq. (7) with

$$
\liminf _{n \rightarrow \infty} \frac{1}{n} \log \left|\mathcal{L}_{n}\right| \geq R_{1}
$$

region $\hat{\mathcal{R}}$ is broader than region $\mathcal{R}$. Indeed, $\hat{\mathcal{R}}$ is a closed convex set consisting of those quadruple $\left(R_{d}, R_{0}, R_{1}, R_{s}\right)$ for which there exist auxiliary random variables $(U, V)$ satisfying the same conditions as Theorem 2 except Eq. (13).

Remark 4: Eq. (14) means that here is a certain amount of dummy randomness that cannot be substituted by the private message. Note that the difference between the private message and the dummy randomness is whether Bob needs to decode it or not.

When there is no randomness constraint, region
$\mathcal{R}_{\infty}=\left\{\left(R_{0}, R_{1}, R_{s}\right): \exists R_{d} \geq 0\right.$ s.t. $\left.\left(R_{d}, R_{0}, R_{1}, R_{s}\right) \in \mathcal{R}\right\}$
coincide with the result obtained by Liang et. al. [8].
Corollary 5: ([8]) Region $\mathcal{R}_{\infty}$ is a closed convex set consisting of those triplet $\left(R_{0}, R_{1}, R_{s}\right)$ for which there exist auxiliary random variables $(U, V)$ such that

$$
\begin{aligned}
& \left(U, X_{2}\right) \leftrightarrow V \leftrightarrow X_{1} \\
& (U, V) \leftrightarrow\left(X_{1}, X_{2}\right) \leftrightarrow(Y, Z)
\end{aligned}
$$

and

$$
\begin{aligned}
R_{0} \leq & \min \left[I\left(U, X_{2} ; Y\right), I\left(U, X_{2} ; Z\right)\right] \\
R_{1}+R_{s} \leq & I\left(U, V ; Y \mid X_{2}\right) \\
R_{0}+R_{1}+R_{s} \leq & I\left(V ; Y \mid U, X_{2}\right) \\
& +\min \left[I\left(U, X_{2} ; Y\right), I\left(U, X_{2} ; Z\right)\right], \\
R_{s} \leq & I\left(V ; Y \mid U, X_{2}\right)-I\left(V ; Z \mid U, X_{2}\right) .
\end{aligned}
$$

## III. Proof of Main Results

## A. Proof of Direct Part of Theorem 2

The direct part of Theorem 2 follows from the following Corollary 7 and Lemma 8 .

We first show the following.
Lemma 6: Let $\mathcal{R}^{(i n)}$ be a closed convex set consisting of those quadruples $\left(R_{d}, R_{0}, R_{1}, R_{s}\right)$ for which there exist $r_{1} \geq$ 0 and auxiliary random variables $(U, V)$ such that

$$
\begin{aligned}
& \left(U, X_{2}\right) \leftrightarrow V \leftrightarrow X_{1} \\
& (U, V) \leftrightarrow\left(X_{1}, X_{2}\right) \leftrightarrow(Y, Z)
\end{aligned}
$$

and

$$
\begin{aligned}
R_{0}+r_{1} & \leq I\left(U, X_{2} ; Z\right) \\
R_{1}-r_{1}+R_{s} & \leq I\left(V ; Y \mid U, X_{2}\right) \\
R_{1}+R_{s} & \leq I\left(U, V ; Y \mid X_{2}\right) \\
R_{0}+R_{1}+R_{s} & \leq I\left(U, V, X_{2} ; Y\right) \\
R_{1}-r_{1} & \geq I\left(V ; Z \mid U, X_{2}\right) \\
R_{d} & \geq I\left(X_{1} ; Z \mid U, V, X_{2}\right)
\end{aligned}
$$

Then we have $\mathcal{R}^{(i n)} \subset \mathcal{R}$.
Proof: See Section III-B
We note the following observation. From the definition of the problem, if

$$
\left(R_{d}-r_{d}, R_{0}, R_{1}-r_{s}+r_{d}, R_{s}+r_{s}\right) \in \mathcal{R}
$$

for some $r_{d}, r_{s} \geq 0$, then we also have $\left(R_{d}, R_{0}, R_{1}, R_{s}\right) \in \mathcal{R}$. Thus, Lemma 6 implies the following corollary.

Corollary 7: Let $\tilde{\mathcal{R}}^{(i n)}$ be a closed convex set consisting of those quadruples $\left(R_{d}, R_{0}, R_{1}, R_{s}\right)$ for which there exist $r_{1}, r_{d}, r_{s} \geq 0$ and $(U, V)$ such that

$$
\begin{aligned}
& \left(U, X_{2}\right) \leftrightarrow V \leftrightarrow X_{1} \\
& (U, V) \leftrightarrow\left(X_{1}, X_{2}\right) \leftrightarrow(Y, Z)
\end{aligned}
$$

and

$$
\begin{aligned}
R_{0}+r_{1} & \leq I\left(U, X_{2} ; Z\right) \\
R_{1}-r_{1}+r_{d}+R_{s} & \leq I\left(V ; Y \mid U, X_{2}\right) \\
R_{1}+r_{d}+R_{s} & \leq I\left(U, V ; Y \mid X_{2}\right) \\
R_{0}+R_{1}+r_{d}+R_{s} & \leq I\left(U, V, X_{2} ; Y\right) \\
R_{1}-r_{1}-r_{s}+r_{d} & \geq I\left(V ; Z \mid U, X_{2}\right) \\
R_{d}-r_{d} & \geq I\left(X_{1} ; Z \mid U, V, X_{2}\right) .
\end{aligned}
$$

Then we have $\tilde{\mathcal{R}}^{(i n)} \subset \mathcal{R}$.
By using the Fourier-Motzkin elimination, we can also show the following.

Lemma 8: We have

$$
\mathcal{R}^{*} \subset \tilde{\mathcal{R}}^{(i n)}
$$

## Proof: See Appendix C

## B. Proof of Lemma 6

For a while, we consider the case with $n=1$ and omit the superscript and subscript to simplify the notation. We first split the private message as $\mathcal{L}=\mathcal{I} \times \mathcal{J}$. For each common message $k \in \mathcal{K}$, we randomly generate codeword $x_{2 k}$ according to distribution $P_{X_{2}}$. We denote such a code $\mathcal{C}_{0}$. For each $k$ and each $i \in \mathcal{I}$, we randomly generate codeword $u_{k i}$ according to distribution $P_{U \mid X_{2}}\left(\cdot \mid x_{2 k}\right)$. We denote such a code $\mathcal{C}_{1}$. For each $(k, i)$ and for each $(j, s) \in \mathcal{J} \times \mathcal{S}$, we randomly generate
codeword $v_{k i j s}$ according to distribution $P_{V \mid U X_{2}}\left(\cdot \mid u_{k i}, x_{2 k}\right)$. We denote such a code $\mathcal{C}_{2}$. For each $(k, i, j)$ and for each $a \in \mathcal{A}$, we randomly generate codeword $x_{1 k i j s a}$ according to distribution $P_{X_{1} \mid V}\left(\cdot \mid v_{k i j s}\right)$. We denote such a code $\mathcal{C}_{3}$.

Let

$$
\begin{aligned}
& \mathcal{T}_{0}=\left\{\left(u, x_{2}, z\right): \frac{P_{Z \mid U X_{2}}\left(z \mid u, x_{2}\right)}{P_{Z}(z)} \geq e^{\alpha_{0}}\right\}, \\
& \mathcal{T}_{1}=\left\{\left(u, v, x_{2}, y\right): \frac{P_{Y \mid U V X_{2}}\left(y \mid u, v, x_{2}\right)}{P_{Y \mid U X_{2}}\left(y \mid u, x_{2}\right)} \geq e^{\alpha_{1}}\right\}, \\
& \mathcal{T}_{2}=\left\{\left(u, v, x_{2}, y\right): \frac{P_{Y \mid U V X_{2}}\left(y \mid u, v, x_{2}\right)}{P_{Y \mid X_{2}}\left(y \mid x_{2}\right)} \geq e^{\alpha_{2}}\right\}, \\
& \mathcal{T}_{3}=\left\{\left(u, v, x_{2}, y\right): \frac{P_{Y \mid U V X_{2}}\left(y \mid u, v, x_{2}\right)}{P_{Y}(y)} \geq e^{\alpha_{3}}\right\},
\end{aligned}
$$

and let $\mathcal{T}=\mathcal{T}_{1} \cap \mathcal{T}_{2} \cap \mathcal{T}_{3}$. Eve decodes only $k$ by using the indirect decoding proposed in [20]. Eve's decoding region is defined by

$$
\begin{aligned}
& \mathcal{D}_{k}= \\
& \quad\left\{z: \exists i\left(u_{i k}, x_{2 i}, z\right) \in \mathcal{T}_{0}, \forall \hat{k} \neq k \forall \hat{i}\left(u_{\hat{k} \hat{i}}, x_{2 \hat{k}}, z\right) \notin \mathcal{T}_{0}\right\},
\end{aligned}
$$

i.e., $\phi(z)=k$ if $z \in \mathcal{D}_{k}$. Bob decodes $(k, i, j, s)$. Bob's decoding region is defined by

$$
\begin{aligned}
& \mathcal{D}_{k i j s}=\left\{y:\left(u_{k i}, v_{k i j s}, x_{2 k}, y\right) \in \mathcal{T}\right. \\
& \left.\quad \forall(\hat{k}, \hat{i}, \hat{j}, \hat{s}) \neq(k, i, j, s)\left(u_{\hat{k} \hat{i}}, v_{\hat{k} \hat{i} \hat{j} \hat{s}}, x_{2 \hat{k}}, y\right) \notin \mathcal{T}\right\}
\end{aligned}
$$

i.e., $g(y)=(k, i, j, s)$ if $y \in \mathcal{D}_{k i j s}$.

Then we have the following.
Lemma 9: We have

$$
\begin{align*}
& \mathbb{E}_{\mathcal{C}_{0} \mathcal{C}_{1} \mathcal{C}_{2} \mathcal{C}_{3}}\left[P_{\text {err }}(f, g)\right] \\
& \leq \quad P_{U V X_{2} Y}\left(\mathcal{T}_{1}^{c}\right)+P_{U V X_{2} Y}\left(\mathcal{T}_{2}^{c}\right)+P_{U V X_{2} Y}\left(\mathcal{T}_{3}^{c}\right) \\
& \quad+\left|\mathcal { J } \left\|\mathcal { S } \left|e^{-\alpha_{1}}+|\mathcal{I}\|\mathcal{J}\| \mathcal{S}| e^{-\alpha_{2}}+|\mathcal{K}\|\mathcal{I}\| \mathcal{J} \| \mathcal{S}| e^{-\alpha_{3}}\right.\right.\right. \tag{15}
\end{align*}
$$

$$
\begin{align*}
& \mathbb{E}_{\mathcal{C}_{0} \mathcal{C}_{1} \mathcal{C}_{2} \mathcal{C}_{3}}\left[P_{e r r}(f, \phi)\right] \\
& \quad \leq \quad P_{U X_{2} Z}\left(\mathcal{T}_{0}^{c}\right)+|\mathcal{K} \| \mathcal{I}| e^{-\alpha_{0}} \tag{16}
\end{align*}
$$

and

$$
\begin{align*}
& \mathbb{E}_{\mathcal{C}_{0} \mathcal{C}_{1} \mathcal{C}_{2} \mathcal{C}_{3}}[D(f)] \\
& \leq \frac{1}{\theta|\mathcal{A}|^{\theta}} e^{\psi\left(\theta \mid P_{Z \mid X_{1} X_{2}}, P_{X_{1} \mid V}, P_{U V X_{2}}\right)} \\
& \quad+\frac{1}{\theta^{\prime} \mid \mathcal{J} \theta^{\theta^{\prime}}} e^{\psi\left(\theta^{\prime} \mid P_{Z \mid U V X_{2}}, P_{V \mid U X_{2}}, P_{U X_{2}}\right)} \tag{17}
\end{align*}
$$

where

$$
\begin{aligned}
& \psi\left(\theta \mid P_{Z \mid X_{1} X_{2}}, P_{X_{1} \mid V}, P_{U V X_{2}}\right) \\
& =\log \sum_{u, v, x_{2}} P_{U V X_{2}}\left(u, v, x_{2}\right) \sum_{z} \\
& \quad\left(\sum_{x_{1}} P_{X_{1} \mid V}\left(x_{1} \mid v\right) P_{Z \mid X_{1} X_{2}}\left(z \mid x_{1}, x_{2}\right)^{1+\theta}\right) \\
& \quad P_{Z \mid U V X_{2}}\left(z \mid u, v, x_{2}\right)^{-\theta}
\end{aligned}
$$

and

$$
\begin{aligned}
& \psi\left(\theta^{\prime} \mid\right.\left.P_{Z \mid U V X_{2}}, P_{V \mid U X_{2}}, P_{U X_{2}}\right) \\
&= \log \sum_{u, x_{2}} P_{U X_{2}}\left(u, x_{2}\right) \sum_{z} \\
&\left(\sum_{v} P_{V \mid U X_{2}}\left(v \mid u, x_{2}\right) P_{Z \mid U V X_{2}}\left(z \mid u, v, x_{2}\right)^{1+\theta}\right) \\
& \quad P_{Z \mid U X_{2}}\left(z \mid u, x_{2}\right)^{-\theta}
\end{aligned}
$$

Proof: See Appendix B
We apply Lemma 9 for asymptotic case. For $\left(R_{d}, R_{0}, R_{1} R_{s}\right) \in \mathcal{R}^{(i n)}$ and arbitrary small $\delta>0$, we set $\left.\left|\mathcal{K}_{n}\right|=\left\lfloor e^{n\left(R_{0}-\delta\right)}\right\rfloor,\left|\mathcal{I}_{n}\right|=e^{n\left(r_{1}-\delta\right)}\right\rfloor$, $\left|\mathcal{J}_{n}\right|=\left\lfloor e^{n\left(R_{1}-r_{1}+2 \delta\right)}\right\rfloor, \quad\left|\mathcal{S}_{n}\right|=\left\lfloor e^{n\left(R_{s}-4 \delta\right)}\right\rfloor$, $\left|\mathcal{A}_{n}\right|=\left\lfloor e^{n\left(R_{d}+2 \delta\right)}\right\rfloor, \quad \alpha_{0}=I\left(U, X_{2} ; Z\right)-\delta$, $\alpha_{1}=I\left(V ; Y \mid U, X_{2}\right)-\delta, \alpha_{2}=I\left(U, V ; Y \mid X_{2}\right)-\delta$, $\alpha_{3}=I\left(U, V, X_{2} ; Y\right)-\delta$. Then,

$$
\begin{aligned}
\left|\mathcal{J}_{n}\right|\left|\mathcal{S}_{n}\right| e^{-\alpha_{1} n} & \leq e^{-n\left(I\left(V ; Y \mid U, X_{2}\right)-R_{1}+r_{1}-R_{s}+\delta\right)} \\
\left|\mathcal{I}_{n}\right|\left|\mathcal{J}_{n}\right|\left|\mathcal{S}_{n}\right| e^{-\alpha_{2} n} & \leq e^{-n\left(I\left(U, V ; Y \mid X_{2}\right)-R_{1}-R_{s}+2 \delta\right)} \\
\left|\mathcal{K}_{n} \| \mathcal{I}_{n}\right|\left|\mathcal{J}_{n}\right|\left|\mathcal{S}_{n}\right| e^{-\alpha_{3} n} & \leq e^{-n\left(I\left(U, V, X_{2} ; Y\right)-R_{0}-R_{1}-R_{s}+3 \delta\right)} \\
\left|\mathcal{K}_{n}\right|\left|\mathcal{I}_{n}\right| e^{-\alpha_{0} n} & \leq e^{-n\left(I\left(U, X_{2} ; Z\right)-R_{0}-r_{1}+\delta\right)}
\end{aligned}
$$

converge to 0 asymptotically. Furthermore, by the law of large numbers, $P_{U V X_{2} Y}^{n}\left(\mathcal{T}_{1, n}^{c}\right), \quad P_{U V X_{2} Y}^{n}\left(\mathcal{T}_{2, n}^{c}\right), \quad P_{U V X_{2} Y}^{n}\left(\mathcal{T}_{3, n}^{c}\right)$, and $P_{U X_{2} Z}^{n}\left(\mathcal{T}_{0, n}^{c}\right)$ also converge to 0 asymptotically.

Since

$$
\psi^{\prime}\left(0 \mid P_{Z \mid X_{1} X_{2}}, P_{X_{1} \mid V}, P_{U V X_{2}}\right)=I\left(X_{1} ; Z \mid U, V, X_{2}\right)
$$

there exists $\theta_{0}>0$ such that

$$
\begin{aligned}
& \frac{\psi\left(\theta_{0} \mid P_{Z \mid X_{1} X_{2}}, P_{X_{1} \mid V}, P_{U V X_{2}}\right)}{\theta_{0}} \\
& \quad \leq \quad I\left(X_{1} ; Z \mid U, V, X_{2}\right)+\delta \leq R_{d}+\delta
\end{aligned}
$$

which implies

$$
-\frac{\theta_{0}}{n} \log \left|\mathcal{A}_{n}\right|+\psi\left(\theta_{0} \mid P_{Z \mid X_{1} X_{2}}, P_{X_{1} \mid V}, P_{U V X_{2}}\right) \leq \delta
$$

Thus,

$$
\frac{1}{\theta_{0}\left|\mathcal{A}_{n}\right|^{\theta_{0}}} e^{n \psi\left(\theta_{0} \mid P_{Z \mid X_{1} X_{2}}, P_{X_{1} \mid V}, P_{U V X_{2}}\right)}
$$

exponentially converges to 0 . Similarly, since

$$
\psi^{\prime}\left(0 \mid P_{Z \mid U V X_{2}}, P_{V \mid U X_{2}}, P_{U X_{2}}\right)=I\left(V ; Z \mid U, X_{2}\right)
$$

there exists $\theta_{0}^{\prime}>0$ such that

$$
\begin{aligned}
& \frac{\psi\left(\theta_{0}^{\prime} \mid P_{Z \mid U V X_{2}}, P_{V \mid U X_{2}}, P_{U X_{2}}\right)}{\theta_{0}^{\prime}} \\
& \quad \leq \quad I\left(V ; Z \mid U, X_{2}\right)+\delta \leq R_{1}-r_{1}+\delta
\end{aligned}
$$

which implies

$$
-\frac{\theta_{0}^{\prime}}{n} \log \left|\mathcal{J}_{n}\right|+\psi\left(\theta_{0}^{\prime} \mid P_{Z \mid U V X_{2}}, P_{V \mid U X_{2}}, P_{U X_{2}}\right) \leq-\delta
$$

Thus,

$$
\frac{1}{\theta_{0}^{\prime}\left|\mathcal{J}_{n}\right|_{0}^{\theta_{0}^{\prime}}} e^{n \psi\left(\theta_{0}^{\prime} \mid P_{Z \mid U V X_{2}}, P_{V \mid U X_{2}}, P_{U X_{2}}\right)}
$$

exponentially converges to 0 asymptotically. This completes a proof of the lemma.

## C. Proof of Converse Part of Theorem 2

Suppose that $\left(R_{d}, R_{0}, R_{1}, R_{s}\right) \in \mathcal{R}$. Then, for arbitrary $\gamma>$ 0 , there exists $n$ such that

$$
\begin{aligned}
n\left(R_{0}-\gamma\right) & \leq \log \left|\mathcal{K}_{n}\right| \\
n\left(R_{1}+R_{s}-\gamma\right) & \leq \log \left|\mathcal{L}_{n}\right|\left|\mathcal{S}_{n}\right| \\
n\left(R_{0}+R_{1}+R_{s}-\gamma\right) & \leq \log \left|\mathcal{K}_{n}\right|\left|\mathcal{L}_{n}\right|\left|\mathcal{S}_{n}\right|, \\
n\left(R_{s}-\gamma\right) & \leq \log \left|\mathcal{S}_{n}\right| \\
n\left(R_{1}+R_{d}+\gamma\right) & \geq \log \left|\mathcal{L}_{n}\right|\left|\mathcal{A}_{n}\right| \\
n\left(R_{d}-\gamma\right) & \geq \log \left|\mathcal{A}_{n}\right|
\end{aligned}
$$

By combining these inequalities with the following Lemma 10 and Lemma 11, we have the converse part of the theorem. The statement about the range size of $U$ and $V$ can be proved in the same manner as [8]. It should be noted that Eqs. (97-(12) are derived in the same manner as [8] and the construction of the auxiliary random variable are also the same. Eqs. (13) and (14) are additionally proved in this paper by using the fact that Alice's encoder is deterministic given the dummy randomness.

Lemma 10: There exists $\varepsilon_{n} \rightarrow 0$ such that

$$
\begin{aligned}
& \log \left|\mathcal{K}_{n}\right| \\
& \quad \leq I\left(K_{n}, X_{2}^{n} ; Y^{n}\right)+n \varepsilon_{n} \\
& \log \left|\mathcal{K}_{n}\right| \\
& \quad \leq I\left(K_{n}, X_{2}^{n} ; Z^{n}\right)+n \varepsilon_{n} \\
& \log \left|\mathcal{L}_{n}\right|\left|\mathcal{S}_{n}\right| \\
& \quad \leq I\left(L_{n}, S_{n} ; Y^{n} \mid K_{n}, X_{2}^{n}\right)+n \varepsilon_{n} \\
& \log \left|\mathcal{K}_{n}\right|\left|\mathcal{L}_{n}\right|\left|\mathcal{S}_{n}\right| \\
& \quad \leq I\left(K_{n}, L_{n}, S_{n}, X_{2}^{n} ; Y^{n}\right)+n \varepsilon_{n} \\
& \log \left|\mathcal{K}_{n}\right|\left|\mathcal{L}_{n}\right|\left|\mathcal{S}_{n}\right| \\
& \quad \leq I\left(L_{n}, S_{n} ; Y^{n} \mid K_{n}, X_{2}^{n}\right)+I\left(K_{n}, X_{2}^{n} ; Z^{n}\right)+2 n \varepsilon_{n} \\
& \log \left|\mathcal{S}_{n}\right| \\
& \quad \leq I\left(L_{n}, S_{n} ; Y^{n} \mid K_{n}, X_{2}^{n}\right) \\
& \quad-I\left(L_{n}, S_{n} ; Z^{n} \mid K_{n}, X_{2}^{n}\right)+4 n \varepsilon_{n} \\
& \log \left|\mathcal{L}_{n}\right|\left|\mathcal{A}_{n}\right| \\
& \geq I\left(X_{1}^{n} ; Z^{n} \mid K_{n}, X_{2}^{n}\right)-2 n \varepsilon_{n} \\
& \log \left|\mathcal{A}_{n}\right| \\
& \quad \geq I\left(X_{1}^{n} ; Z^{n} \mid K_{n}, L_{n}, S_{n}, X_{2}^{n}\right)
\end{aligned}
$$

Proof: By using Fano's inequality, we have

$$
\begin{aligned}
\log \left|\mathcal{K}_{n}\right| & =H\left(K_{n}\right) \\
& =I\left(K_{n} ; Y^{n}\right)+H\left(K_{n} \mid Y^{n}\right) \\
& \leq I\left(K_{n}, X_{2}^{n} ; Y^{n}\right)+n \varepsilon_{n}
\end{aligned}
$$

By using Fano's inequality and by noting that $\left(K_{n}, X_{2}^{n}\right)$ and ( $L_{n}, S_{n}$ ) are independent, we have

$$
\begin{aligned}
\log \left|\mathcal{L}_{n}\right|\left|\mathcal{S}_{n}\right| & =H\left(L_{n}, S_{n}\right) \\
& =I\left(L_{n}, S_{n} ; Y^{n}\right)+H\left(L_{n}, S_{n} \mid Y^{n}\right) \\
& \leq I\left(L_{n}, S_{n} ; K_{n}, X_{2}^{n}, Y^{n}\right)+n \varepsilon_{n} \\
& =I\left(L_{n}, S_{n} ; Y^{n} \mid K_{n}, X_{2}^{n}\right)+n \varepsilon_{n}
\end{aligned}
$$

By using Fano's inequality, we also have

$$
\begin{aligned}
\log \left|\mathcal{K}_{n}\right|\left|\mathcal{L}_{n} \| \mathcal{S}_{n}\right| & =H\left(K_{n}, L_{n}, S_{n}\right) \\
& \leq I\left(K_{n}, L_{n}, S_{n}, X_{2}^{n} ; Y^{n}\right)+n \varepsilon_{n}
\end{aligned}
$$

and

$$
\begin{aligned}
& \log \left|\mathcal{K}_{n}\right|\left|\mathcal{L}_{n}\right|\left|\mathcal{S}_{n}\right| \\
& \quad=H\left(L_{n}, S_{n} \mid K_{n}\right)+H\left(K_{n}\right) \\
& \quad \leq I\left(L_{n}, S_{n} ; Y^{n} \mid K_{n}\right)+I\left(K_{n}, X_{2}^{n} ; Z^{n}\right)+2 n \varepsilon_{n} \\
& \quad=I\left(L_{n}, S_{n} ; Y^{n} \mid K_{n}, X_{2}^{n}\right)+I\left(K_{n}, X_{2}^{n} ; Z^{n}\right)+2 n \varepsilon_{n}
\end{aligned}
$$

where the last equality follows from the fact that $X_{2}^{n}$ is a determined from $K_{n}$. By using the security condition and Fano's inequality, we have

$$
\begin{align*}
& I\left(S_{n} ; Z^{n} \mid K_{n}\right) \\
& \quad=I\left(S_{n}, K_{n} ; Z^{n}\right)-I\left(K_{n} ; Z^{n}\right) \\
& \quad=I\left(S_{n} ; Z^{n}\right)+I\left(K_{n} ; Z^{n} \mid S_{n}\right)-I\left(K_{n} ; Z^{n}\right) \\
& \quad \leq I\left(S_{n} ; Z^{n}\right)+H\left(K_{n} \mid Z^{n}\right) \\
& \quad \leq 2 n \varepsilon_{n} \tag{18}
\end{align*}
$$

By using Fano's inequality and by using Eq. (18), we have

$$
\begin{aligned}
& \log \left|\mathcal{S}_{n}\right| \\
&= H\left(S_{n} \mid K_{n}\right) \\
& \leq I\left(S_{n} ; Y^{n} \mid K_{n}\right)+n \varepsilon_{n} \\
&= I\left(L_{n}, S_{n} ; Y^{n} \mid K_{n}\right)-I\left(L_{n} ; Y^{n} \mid S_{n}, K_{n}\right)+n \varepsilon_{n} \\
& \leq I\left(L_{n}, S_{n} ; Y^{n} \mid K_{n}\right)-H\left(L_{n} \mid S_{n}, K_{n}\right)+2 n \varepsilon_{n} \\
& \leq I\left(L_{n}, S_{n} ; Y^{n} \mid K_{n}\right)-I\left(S_{n} ; Z^{n} \mid K_{n}\right) \\
&-H\left(L_{n} \mid S_{n}, K_{n}\right)+4 n \varepsilon_{n} \\
& \leq I\left(L_{n}, S_{n} ; Y^{n} \mid K_{n}\right)-I\left(L_{n}, S_{n} ; Z^{n} \mid K_{n}\right)+4 n \varepsilon_{n} \\
&= I\left(L_{n}, S_{n} ; Y^{n} \mid K_{n}, X_{2}^{n}\right) \\
&-I\left(L_{n}, S_{n} ; Z^{n} \mid K_{n}, X_{2}^{n}\right)+4 n \varepsilon_{n} .
\end{aligned}
$$

By noting that $f_{n}$ is a deterministic function and by using Eq. 18), we have

$$
\begin{aligned}
& \log \left|\mathcal{L}_{n}\right|\left|\mathcal{A}_{n}\right| \\
& \quad \geq H\left(X_{1}^{n} \mid K_{n}, S_{n}\right) \\
& \quad \geq I\left(X_{1}^{n} ; Z^{n} \mid K_{n}, S_{n}\right) \\
& \quad=I\left(X_{1}^{n}, S_{n} ; Z^{n} \mid K_{n}\right)-I\left(S_{n} ; Z^{n} \mid K_{n}\right) \\
& \quad \geq I\left(X_{1}^{n} ; Z^{n} \mid K_{n}\right)-2 n \varepsilon_{n} \\
& \quad=I\left(X_{1}^{n} ; Z^{n} \mid K_{n}, X_{2}^{n}\right)-2 n \varepsilon_{n}
\end{aligned}
$$

Finally, by noting that $f_{n}$ is a deterministic function, we have

$$
\begin{aligned}
\log \left|\mathcal{A}_{n}\right| & \geq H\left(X_{1}^{n} \mid K_{n}, L_{n}, S_{n}\right) \\
& \geq I\left(X_{1}^{n} ; Z^{n} \mid K_{n}, L_{n}, S_{n}\right) \\
& =I\left(X_{1}^{n} ; Z^{n} \mid K_{n}, L_{n}, S_{n}, X_{2}^{n}\right)
\end{aligned}
$$

Lemma 11: For fixed $n$, let $T$ be the random variable that is uniformly distributed on $\{1, \ldots, n\}$ and is independent of the other random variables. Define the following random variables:

$$
\begin{aligned}
U_{t} & =\left(K_{n}, X_{2}^{n}, Y_{1}^{t-1}, Z_{t+1}^{n}\right) \\
V_{t} & =\left(L_{n}, S_{n}, U_{t}\right) \\
U & =\left(U_{T}, T\right) \\
V & =\left(V_{T}, T\right) \\
X_{1} & =X_{1 T} \\
X_{2} & =X_{2 T} \\
Y & =Y_{T} \\
Z & =Z_{T}
\end{aligned}
$$

Then, we have

$$
\begin{align*}
& I\left(K_{n}, X_{2}^{n} ; Y^{n}\right) \\
& \quad \leq n I\left(U, X_{2} ; Y\right)  \tag{19}\\
& I\left(K_{n}, X_{2}^{n} ; Z^{n}\right) \\
& \quad \leq n I\left(U, X_{2} ; Z\right)  \tag{20}\\
& I\left(L_{n}, S_{n} ; Y^{n} \mid K_{n}, X_{2}^{n}\right) \\
& \quad \leq n I\left(U, V ; Y \mid X_{2}\right)  \tag{21}\\
& I\left(K_{n}, L_{n}, S_{n}, X_{2}^{n} ; Y^{n}\right) \\
& \quad \leq n I\left(U, V, X_{2} ; Y\right)  \tag{22}\\
& I\left(L_{n}, S_{n} ; Y^{n} \mid K_{n}, X_{2}^{n}\right)+I\left(K_{n}, X_{2}^{n} ; Z^{n}\right) \\
& \quad \leq \quad n\left[I\left(V ; Y \mid U, X_{2}\right)+I\left(U, X_{2} ; Z\right)\right]  \tag{23}\\
& I\left(L_{n}, S_{n} ; Y^{n} \mid K_{n}, X_{2}^{n}\right)-I\left(L_{n}, S_{n} ; Z^{n} \mid K_{n}, X_{2}^{n}\right) \\
& \quad \leq n\left[I\left(V ; Y \mid U, X_{2}\right)-I\left(V ; Z \mid U, X_{2}\right)\right]  \tag{24}\\
& I\left(X_{1}^{n} ; Z^{n} \mid K_{n}, X_{2}^{n}\right) \\
& \quad \geq n I\left(X_{1} ; Z \mid U, X_{2}\right)  \tag{25}\\
& I\left(X_{1}^{n} ; Z^{n} \mid K_{n}, L_{n}, S_{n}, X_{2}^{n}\right) \\
& \quad \geq n I\left(X_{1} ; Z \mid U, V, X_{2}\right) \tag{26}
\end{align*}
$$

## Proof:

a) Proof of Eq. (19):

$$
\begin{aligned}
& I\left(K_{n}, X_{2}^{n} ; Y^{n}\right) \\
& \quad=\sum_{t=1}^{n} I\left(K_{n}, X_{2}^{n} ; Y_{t} \mid Y_{1}^{t-1}\right) \\
& \quad \leq \sum_{t=1}^{n} I\left(K_{n}, X_{2}^{n}, Y_{1}^{t-1}, Z_{t+1}^{n} ; Y_{t}\right) \\
& \quad=\sum_{t=1}^{n} I\left(U_{t} ; Y_{t}\right) \\
& \quad=n I\left(U_{T} ; Y_{T} \mid T\right) \\
& \quad=n I\left(U_{T}, T ; Y_{T}\right) \\
& \quad=n I(U ; Y)
\end{aligned}
$$

b) Proof of Eq. (20):

$$
\begin{aligned}
& I\left(K_{n}, X_{2}^{n} ; Z^{n}\right) \\
& \quad=\sum_{t=1}^{n} I\left(K_{n}, X_{2}^{n} ; Z_{t} \mid Z_{t+1}^{n}\right) \\
& \quad \leq \sum_{t=1}^{n} I\left(K_{n}, X_{2}^{n}, Y_{1}^{t-1}, Z_{t+1}^{n} ; Z_{t}\right) \\
& \quad=\sum_{t=1}^{n} I\left(U_{t} ; Z_{t}\right) \\
& \quad=n I\left(U_{T} ; Z_{T} \mid T\right) \\
& \quad=n I\left(U_{T}, T ; Z_{T}\right) \\
& \quad=n I(U ; Z) .
\end{aligned}
$$

c) Proof of Eq. (21):

$$
\begin{aligned}
I\left(L_{n},\right. & \left.S_{n} ; Y^{n} \mid K_{n}, X_{2}^{n}\right) \\
= & \sum_{t=1}^{n}\left[H\left(Y_{t} \mid K_{n}, X_{2}^{n}, Y_{1}^{t-1}\right)\right. \\
& \left.-H\left(Y_{t} \mid K_{n}, L_{n}, S_{n}, X_{2}^{n}, Y_{1}^{t-1}\right)\right] \\
\leq & \sum_{t=1}^{n}\left[H\left(Y_{t} \mid X_{2 t}\right)\right. \\
& \left.-H\left(Y_{t} \mid K_{n}, L_{n}, S_{n}, X_{2}^{n}, Y_{1}^{t-1}, Z_{t+1}^{n}\right)\right] \\
= & \sum_{t=1}^{n} I\left(K_{n}, L_{n}, S_{n}, X_{2}^{n} ; Y_{1}^{t-1}, Z_{t+1}^{n} ; Y_{t} \mid X_{2 t}\right) \\
= & \sum_{t=1}^{n} I\left(U_{t}, V_{t} ; Y_{t} \mid X_{2 t}\right) \\
= & n I\left(U_{T}, V_{T} ; Y_{T} \mid X_{2 T}, T\right) \\
= & n I\left(U, V ; Y \mid X_{2}\right)
\end{aligned}
$$

d) Proof of Eq. (22):

$$
\begin{aligned}
& I\left(K_{n}, L_{n}, S_{n}, X_{2}^{n} ; Y^{n}\right) \\
& \quad=\sum_{t=1}^{n} I\left(K_{n}, L_{n}, S_{n}, X_{2}^{n} ; Y_{t} \mid Y_{1}^{t-1}\right) \\
& \quad \leq \sum_{t=1}^{n} I\left(K_{n}, L_{n}, S_{n}, X_{2}^{n}, Y_{1}^{t-1}, Z_{t+1}^{n} ; Y_{t}\right) \\
& \quad=\sum_{t=1}^{n} I\left(U_{t}, V_{t}, X_{2 t} ; Y_{t}\right) \\
& \quad=n I\left(U_{T}, V_{T}, X_{2 T} ; Y_{T} \mid T\right) \\
& \quad=n I\left(U, V, X_{2} ; Y\right) .
\end{aligned}
$$

e) Proof of Eq. (23):

$$
\begin{aligned}
I\left(L_{n},\right. & \left.S_{n} ; Y^{n} \mid K_{n}, X_{2}^{n}\right)+I\left(K_{n}, X_{2}^{n} ; Z^{n}\right) \\
= & \sum_{t=1}^{n}\left[I\left(L_{n}, S_{n} ; Y_{t} \mid K_{n}, X_{2}^{n}, Y_{1}^{t-1}\right)\right. \\
& \left.+I\left(K_{n}, X_{2}^{n} ; Z_{t} \mid Z_{t+1}^{n}\right)\right] \\
\leq & \sum_{t=1}^{n}\left[I\left(L_{n}, S_{n}, Z_{t+1}^{n} ; Y_{t} \mid K_{n}, X_{2}^{n}, Y_{1}^{t-1}\right)\right. \\
& -I\left(Y_{1}^{t-1} ; Z_{t} \mid K_{n}, X_{2}^{n}, Z_{t+1}^{n}\right)
\end{aligned}
$$

$$
\begin{aligned}
&\left.+I\left(K_{n}, X_{2}^{n}, Y_{1}^{t-1} ; Z_{t} \mid Z_{t+1}^{n}\right)\right] \\
&= \sum_{t=1}^{n}\left[I\left(L_{n}, S_{n} ; Y_{t} \mid K_{n}, X_{2}^{n}, Y_{1}^{t-1}, Z_{t+1}^{n}\right)\right. \\
&+I\left(Z_{t+1}^{n} ; Y_{t} \mid K_{n}, X_{2}^{n}, Y_{1}^{t-1}\right) \\
&-I\left(Y_{1}^{t-1} ; Z_{t} \mid K_{n}, X_{2}^{n}, Z_{t+1}^{n}\right) \\
&\left.+I\left(K_{n}, X_{2}^{n}, Y_{1}^{t-1} ; Z_{t} \mid Z_{t+1}^{n}\right)\right] \\
& \stackrel{(a)}{=} \quad \sum_{t=1}^{n}\left[I\left(L_{n}, S_{n} ; Y_{t} \mid K_{n}, X_{2}^{n}, Y_{1}^{t-1}, Z_{t+1}^{n}\right)\right. \\
&\left.+I\left(K_{n}, X_{2}^{n}, Y_{1}^{t-1} ; Z_{t} \mid Z_{t+1}^{n}\right)\right] \\
& \leq \sum_{t=1}^{n}\left[I\left(L_{n}, S_{n} ; Y_{t} \mid K_{n}, X_{2}^{n}, Y_{1}^{t-1}, Z_{t+1}^{n}\right)\right. \\
&=\left.+I\left(K_{n}, X_{2}^{n}, Y_{1}^{t-1}, Z_{t+1}^{n} ; Z_{t}\right)\right] \\
&= n\left[I\left(V_{T} ; Y_{T} \mid U_{T}, X_{2 T}, T\right)+I\left(V ; Y \mid U, X_{2}\right)+I\left(U, X_{2} ; Z\right)\right],
\end{aligned}
$$

where we used Csiszár's sum identity [21] in (a).
f) Proof of Eq. (24):

$$
\begin{aligned}
I\left(L_{n},\right. & \left.S_{n} ; Y^{n} \mid K_{n}, X_{2}^{n}\right)-I\left(L_{n}, S_{n} ; Z^{n} \mid K_{n}, X_{2}^{n}\right) \\
= & \sum_{t=1}^{n}\left[I\left(L_{n}, S_{n} ; Y_{t} \mid K_{n}, X_{2}^{n}, Y_{1}^{t-1}\right)\right. \\
& \left.-I\left(L_{n}, S_{n} ; Z_{t} \mid K_{n}, X_{2}^{n}, Z_{t+1}^{n}\right)\right] \\
\stackrel{(a)}{=} & \sum_{t=1}^{n}\left[I\left(L_{n}, S_{n} ; Y_{t} \mid K_{n}, X_{2}^{n}, Y_{1}^{t-1}\right)\right. \\
& +I\left(Z_{t+1}^{n} ; Y_{t} \mid K_{n}, L_{n}, S_{n}, X_{2}^{n}, Y_{1}^{t-1}\right) \\
& -I\left(Y_{1}^{t-1} ; Z_{t} \mid K_{n}, L_{n}, S_{n}, X_{2}^{n}, Z_{t+1}^{n}\right) \\
& -I\left(L_{n}, S_{n} ; Z_{t} \mid K_{n}, X_{2}^{n}, Z_{t+1}^{n}\right) \\
= & \sum_{t=1}^{n}\left[I\left(L_{n}, S_{n}, Z_{t+1}^{n} ; Y_{t} \mid K_{n}, X_{2}^{n}, Y_{1}^{t-1}\right)\right. \\
& \left.-I\left(L_{n}, S_{n}, Y_{1}^{t-1} ; Z_{t} \mid K_{n}, X_{2}^{n}, Z_{t+1}^{n}\right)\right] \\
= & \sum_{t=1}^{n}\left[I\left(L_{n}, S_{n} ; Y_{t} \mid K_{n}, X_{2}^{n}, Y_{1}^{t-1}, Z_{t+1}^{n}\right)\right. \\
& +I\left(Z_{t+1}^{n} ; Y_{t} \mid K_{n}, X_{2}^{n}, Y_{1}^{t-1}\right) \\
& -I\left(Y_{1}^{t-1} ; Z_{t} \mid K_{n}, X_{2}^{n}, Z_{t+1}^{n}\right) \\
& \left.-I\left(L_{n}, S_{n} ; Z_{t} \mid K_{n}, X_{2}^{n}, Y_{1}^{t-1}, Z_{t+1}^{n}\right)\right] \\
(b) & \sum_{t=1}^{n}\left[I\left(L_{n}, S_{n} ; Y_{t} \mid K_{n}, X_{2}^{n}, Y_{1}^{t-1}, Z_{t+1}^{n}\right)\right. \\
= & \left.\quad-I\left(L_{n}, S_{n} ; Z_{t} \mid K_{n}, X_{2}^{n}, Y_{1}^{t-1}, Z_{t+1}^{n}\right)\right] \\
= & n\left[I\left(V_{T} ; Y_{T} \mid U_{T}, X_{2 T}, T\right)-I\left(V_{T} ; Z_{T} \mid U_{T}, X_{2 T}, T\right)\right] \\
= & \left.n\left(V ; Y \mid U, X_{2}\right)-I\left(V ; Z \mid U, X_{2}\right)\right]
\end{aligned}
$$

where (a) and (b) follow from Csiszár's sum identity [21].
g) Proof of Eq. (25):

$$
\begin{aligned}
& I\left(X_{1}^{n} ; Z^{n} \mid K_{n}, X_{2}^{n}\right) \\
& \quad=\quad \sum_{t=1}^{n}\left[H\left(Z_{t} \mid K_{n}, X_{2}^{n}, Z_{t+1}^{n}\right)\right. \\
& \left.\quad-H\left(Z_{t} \mid K_{n}, X_{1}^{n}, X_{2}^{n}, Z_{t+1}^{n}\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
\stackrel{(a)}{\geq} & \sum_{t=1}^{n}\left[H\left(Z_{t} \mid K_{n}, X_{2}^{n}, Y_{1}^{t-1}, Z_{t+1}^{n}\right)\right. \\
& \left.-H\left(Z_{t} \mid K_{n}, X_{1 t}, X_{2}^{n}, Y_{1}^{t-1}, Z_{t+1}^{n}\right)\right] \\
= & \sum_{t=1}^{n} I\left(X_{1 t} ; Z_{t} \mid K_{n}, X_{2}^{n}, Y_{1}^{t-1}, Z_{t+1}^{n}\right) \\
= & \sum_{t=1}^{n} I\left(X_{1 t} ; Z_{t} \mid U_{t}, X_{2 t}\right) \\
= & n I\left(X_{1 T} ; Z_{T} \mid U_{T}, X_{2 T}, T\right) \\
= & n I\left(X_{1} ; Z \mid U, X_{2}\right),
\end{aligned}
$$

where (a) follows from the fact that $\left(K_{n}, X_{11}^{t-1}, X_{1(t+1)}^{n}\right.$, $\left.X_{21}^{t-1}, X_{2(t+1)}^{n}, Y_{1}^{t-1}, Z_{t+1}\right),\left(X_{1 t}, X_{2 t}\right)$, and $Z_{t}$ form Markov chain.
h) Proof of Eq. (26):

$$
\begin{aligned}
I\left(X_{1}^{n} ;\right. & \left.Z^{n} \mid K_{n}, L_{n}, S_{n}, X_{2}^{n}\right) \\
= & \sum_{t=1}^{n}\left[H\left(Z_{t} \mid K_{n}, L_{n}, S_{n}, X_{2}^{n}, Z_{t+1}^{n}\right)\right. \\
& \left.-H\left(Z_{t} \mid K_{n}, L_{n}, S_{n}, X_{1}^{n}, X_{2}^{n}, Z_{t+1}^{n}\right)\right] \\
\stackrel{(a)}{\geq} & \sum_{t=1}^{n}\left[H\left(Z_{t} \mid K_{n}, L_{n}, S_{n}, X_{2}^{n}, Y_{1}^{t-1}, Z_{t+1}^{n}\right)\right. \\
& \left.-H\left(Z_{t} \mid K_{n}, L_{n}, S_{n}, X_{1 t}, X_{2}^{n}, Y_{1}^{t-1}, Z_{t+1}^{n}\right)\right] \\
= & \sum_{t=1}^{n} I\left(X_{1 t} ; Z_{t} \mid K_{n}, L_{n}, S_{n}, X_{2}^{n}, Y_{1}^{t-1}, Z_{t+1}^{n}\right) \\
= & \sum_{t=1}^{n} I\left(X_{1 t} ; Z_{t} \mid U_{t}, V_{t}, X_{2 t}\right) \\
= & n I\left(X_{1 T} ; Z_{T} \mid U_{T}, V_{T}, X_{2 T}, T\right) \\
= & n I\left(X_{1} ; Z \mid U, V, X_{2}\right),
\end{aligned}
$$

where (a) follows from the fact that $\left(K_{n}, L_{n}, S_{n}, X_{11}^{t-1}\right.$, $\left.X_{1(t+1)}^{n}, X_{21}^{t-1}, X_{2(t+1)}^{n}, Y_{1}^{t-1}, Z_{t+1}\right),\left(X_{1 t}, X_{2 t}\right)$, and $Z_{t}$ form Markov chain.

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## Appendix

## A. Channel Resolvability

Since we use a result of the channel resolvability problem [22] in the proof of our main result, we review the channel resolvability problem in this appendix. For simplicity of notation, we consider the so-called one-shot case, i.e., the block length is $n=1$. In the channel resolvability problem, for the input distribution $P_{X}$ of the channel $P_{Z \mid X}$, we want to simulate the response $P_{Z}$ of the channel, where

$$
P_{Z}(z)=\sum_{x} P_{X}(x) P_{Z \mid X}(z \mid x)
$$

The simulation is conducted by a deterministic map $\varphi: \mathcal{B} \rightarrow$ $\mathcal{X}$, and uniform random number $B$ on $\mathcal{B}$. Let

$$
P_{\tilde{Z}}(z)=\sum_{b \in \mathcal{B}} \frac{1}{|\mathcal{B}|} P_{Z \mid X}(z \mid \varphi(b))
$$

be the output distribution with map $\varphi$. The purpose of the resolvability problem is to construct a map such that $D\left(P_{\tilde{Z}} \| P_{Z}\right)$ is small.

In [16], the following random coding construction of a map was proposed. We split the alphabet as $\mathcal{B}=\mathcal{M}_{1} \times \mathcal{M}_{2}$. Let $P_{V X}$ be a distribution such that the marginal is $P_{X}$. We first randomly generate $\left|\mathcal{M}_{2}\right|$ codewords $v_{1}, \ldots, v_{\left|\mathcal{M}_{2}\right|}$ according to the distribution $P_{V}$. We denote the generated code by $\mathcal{C}_{2}$. Then, for each $1 \leq i \leq\left|\mathcal{M}_{2}\right|$, we randomly generate $\left|\mathcal{M}_{1}\right|$ codewords $x_{i 1}, \ldots, x_{i\left|\mathcal{M}_{1}\right|}$ according to the distribution $P_{X \mid V}\left(\cdot \mid v_{i}\right)$. We denote the generated code by $\mathcal{C}_{1}$. For this construction we have the following lemma.

Lemma 12: ([|6]) For $0<\theta, \theta^{\prime} \leq 1$, we have

$$
\begin{aligned}
\mathbb{E}_{\mathcal{C}_{1} \mathcal{C}_{2}} & {\left[D\left(P_{\tilde{Z}} \| P_{Z}\right)\right] } \\
\leq & \frac{1}{\theta\left|\mathcal{M}_{1}\right|^{\theta}} e^{\psi\left(\theta \mid P_{Z \mid X}, P_{X \mid V}, P_{V}\right)} \\
& +\frac{1}{\theta^{\prime}\left|\mathcal{M}_{2}\right|^{\theta^{\prime}}} e^{\psi\left(\theta^{\prime} \mid P_{Z \mid V}, P_{V}\right)}
\end{aligned}
$$

where

$$
\begin{aligned}
& \psi\left(\theta \mid P_{Z \mid X}, P_{X \mid V}, P_{V}\right) \\
&= \log \sum_{v} P_{V}(v) \sum_{z} \\
&\left(\sum_{x} P_{X \mid V}(x \mid v) P_{Z \mid X}(z \mid x)^{1+\theta}\right) P_{Z \mid V}(z \mid v)^{-\theta}
\end{aligned}
$$

and

$$
\begin{aligned}
& \psi\left(\theta^{\prime} \mid P_{Z \mid V}, P_{V}\right) \\
& \quad=\log \sum_{z}\left(\sum_{x} P_{X}(x) P_{Z \mid X}(z \mid x)^{1+\theta^{\prime}}\right) P_{Z}(x)^{-\theta^{\prime}}
\end{aligned}
$$

## B. Proof of Lemma 9

a) Proof of Eq. (15): We first not the following observation. By taking the average over randomly generated codes, we have

$$
\begin{aligned}
& \mathbb{E}_{\mathcal{C}_{0} \mathcal{C}_{1} \mathcal{C}_{2} \mathcal{C}_{3}}\left[P_{\text {err }}(f, g)\right] \\
& =\mathbb{E}_{\mathcal{C}_{0} \mathcal{C}_{1} \mathcal{C}_{2} \mathcal{C}_{3}}\left[\sum_{k, i, j, s, a} \frac{1}{|\mathcal{K}||\mathcal{I}||\mathcal{J}||\mathcal{S}||\mathcal{A}|}\right. \\
& \left.\quad P_{Y \mid X_{1} X_{2}}\left(\mathcal{D}_{k i j s a}^{c} \mid x_{1 k i j s a}, x_{2 k}\right)\right] \\
& =\mathbb{E}_{\mathcal{C}_{0} \mathcal{C}_{1} \mathcal{C}_{2}}\left[\sum_{k, i, j, s, a} \frac{1}{|\mathcal{K}||\mathcal{I}||\mathcal{J}||\mathcal{S}||\mathcal{A}|}\right. \\
& \mathbb{E}_{\mathcal{C}_{3}}\left[P_{Y \mid X_{1} X_{2}}\left(\mathcal{D}_{k i j s a}^{c} \mid x_{1 k i j s a}, x_{2 k}\right)\right]
\end{aligned}
$$

$$
\begin{align*}
=\mathbb{E}_{\mathcal{C}_{0} \mathcal{C}_{1} \mathcal{C}_{2}}[ & \sum_{k, i, j, s} \frac{1}{|\mathcal{K}||\mathcal{I}||\mathcal{J}||\mathcal{S}|} \\
& \left.P_{Y \mid V X_{2}}\left(\mathcal{D}_{k i j s a}^{c} \mid v_{k i j s}, x_{2 k}\right)\right] \tag{27}
\end{align*}
$$

Let $\mathcal{T}_{u v x_{2}}=\left\{y:\left(u, v, x_{2}, y\right) \in \mathcal{T}\right\}$. Then, we have

$$
\begin{aligned}
& \mathbb{E}_{\mathcal{C}_{0} \mathcal{C}_{1} \mathcal{C}_{2}}\left[\sum_{k, i, j, s} \frac{1}{|\mathcal{K}||\mathcal{I}||\mathcal{J}||\mathcal{S}|} P_{Y \mid V X_{2}}\left(\mathcal{D}_{k i j s a}^{c} \mid v_{k i j s}, x_{2 k}\right)\right] . \\
& \leq \mathbb{E}_{\mathcal{C}_{0} \mathcal{C}_{1} \mathcal{C}_{2}}\left[\sum_{k, i, j, s} \frac{1}{|\mathcal{K}||\mathcal{I}||\mathcal{J}||\mathcal{S}|}\right. \\
& \left\{P_{Y \mid V X_{2}}\left(\mathcal{T}_{u_{k i} v_{k i j s} x_{2 k}}^{c} \mid v_{k i j s}, x_{2 k}\right)\right. \\
& \left.\left.+\sum_{\substack{(\hat{k}, \hat{i}, \hat{j}, \hat{s}) \\
\neq(k, i, j, s)}} P_{Y \mid V X_{2}}\left(\mathcal{T}_{u_{\hat{k} \hat{i}} v_{\hat{k} \hat{\imath} \hat{j} \hat{s}} x_{2 \hat{k}}} \mid v_{k i j s}, x_{2 k}\right)\right\}\right] \\
& \leq \mathbb{E}_{\mathcal{C}_{0} \mathcal{C}_{1} \mathcal{C}_{3}}\left[\sum_{k, i, j, s} \frac{1}{|\mathcal{K}||\mathcal{I}||\mathcal{J} \| \mathcal{S}|}\right. \\
& \left\{P_{Y \mid V X_{2}}\left(\mathcal{T}_{u_{k i} v_{k i j s} x_{2 k}}^{c} \mid v_{k i j s}, x_{2 k}\right)\right. \\
& +\sum_{(\hat{j}, \hat{s}) \neq(j, s)} P_{Y \mid V X_{2}}\left(\mathcal{T}_{u_{k i} v_{k i \hat{\jmath} \hat{s}} x_{2 k}} \mid v_{k i j s}, x_{2 k}\right) \\
& +\sum_{\hat{i} \neq i} \sum_{(\hat{j}, \hat{s})} P_{Y \mid V X_{2}}\left(\mathcal{T}_{u_{k \hat{i}} v_{k \hat{i} \hat{j} \hat{s}} x_{2 k}} \mid v_{k i j s}, x_{2 k}\right) \\
& +\sum_{\hat{k} \neq k} \sum_{(\hat{i}, \hat{j}, \hat{s})} P_{Y \mid V X_{2}}\left(\mathcal{T}_{\left.\left.\left.u_{\hat{k} \hat{\imath}} v_{\hat{k} \hat{\imath} \hat{\jmath} \hat{s}} x_{2 \hat{k}} \mid v_{k i j s}, x_{2 k}\right)\right\}\right]}\right. \\
& \leq \sum_{k, i, j, s} \frac{1}{|\mathcal{K}\|\mathcal{I}\| \mathcal{J} \| \mathcal{S}|}\left\{P_{U V X_{2} Y}\left(\mathcal{T}^{c}\right)\right. \\
& +|\mathcal{J}||\mathcal{S}| \sum_{u, v, x_{2}} P_{U V X_{2}}\left(u, v, x_{2}\right) P_{Y \mid U X_{2}}\left(\mathcal{T}_{u v x_{2}} \mid u, x_{2}\right) \\
& +|\mathcal{I}||\mathcal{J}||\mathcal{S}| \sum_{u, v, x_{2}} P_{U V X_{2}}\left(u, v, x_{2}\right) P_{Y \mid X_{2}}\left(\mathcal{T}_{u v x_{2}} \mid x_{2}\right) \\
& +|\mathcal{K}||\mathcal{I}||\mathcal{J}||\mathcal{S}| \sum_{u, v, x_{2}} P_{U V X_{2}}\left(u, v, x_{2}\right) P_{Y}\left(\mathcal{T}_{u v x_{2}}\right) \\
& \leq P_{U V X_{2} Y}\left(\mathcal{T}^{c}\right)+|\mathcal{J} \| \mathcal{S}| e^{-\alpha_{1}} \\
& +\left|\mathcal { I } \left\|\mathcal{J}| | \mathcal{S}\left|e^{-\alpha_{2}}+|\mathcal{K}\|\mathcal{I}\| \mathcal{J}|\right| \mathcal{S} \mid e^{-\alpha_{3}},\right.\right.
\end{aligned}
$$

where we used

$$
\begin{aligned}
P_{Y \mid U X_{2}}\left(y \mid u, x_{2}\right) & \leq P_{Y \mid U V X_{2}}\left(y \mid u, v, x_{2}\right) e^{-\alpha_{1}} \\
P_{Y \mid X_{2}}\left(y \mid x_{2}\right) & \leq P_{Y \mid U V X_{2}}\left(y \mid u, v, x_{2}\right) e^{-\alpha_{2}} \\
P_{Y}(y) & \leq P_{Y \mid U V X_{2}}\left(y \mid u, v, x_{2}\right) e^{-\alpha_{3}}
\end{aligned}
$$

for $y \in \mathcal{T}_{u v x_{2}}$ in the last inequality.
b) Proof of Eq. (16): Let $\mathcal{T}_{0, u x_{2}}=\left\{z:\left(u, x_{2}, z\right) \in \mathcal{T}_{0}\right\}$. In a similar manner as Eq. (27), we have

$$
\begin{aligned}
& \mathbb{E}_{\mathcal{C}_{0} \mathcal{C}_{1} \mathcal{C}_{2} \mathcal{C}_{3}}\left[P_{\text {err }}(f, \phi)\right] \\
&= \mathbb{E}_{\mathcal{C}_{0} \mathcal{C}_{1}}\left[\sum_{k, i} \frac{1}{|\mathcal{K}||\mathcal{I}|} P_{Z \mid U X_{2}}\left(\mathcal{D}_{k}^{c} \mid u_{k i}, x_{2 k}\right)\right] \\
& \leq \mathbb{E}_{\mathcal{C}_{0} \mathcal{C}_{1}}\left[\sum _ { k , i } \frac { 1 } { | \mathcal { K } | | \mathcal { I } | } \left\{P_{Z \mid U X_{2}}\left(\mathcal{T}_{0, u_{k i} x_{2 k}}^{c} \mid u_{k i}, x_{2 k}\right)\right.\right. \\
&\left.\left.+\sum_{\hat{k} \neq k} \sum_{\hat{i}} P_{Z \mid U X_{2}}\left(\mathcal{T}_{0, u_{\hat{k} \hat{i}} x_{2 \hat{k}}} \mid u_{k i}, x_{2 k}\right)\right\}\right] \\
& \leq \sum_{k, i} \frac{1}{|\mathcal{K}||\mathcal{I}|}\left\{P_{U X_{2} Z}\left(\mathcal{T}_{0}^{c}\right)\right. \\
&\left.+|\mathcal{K}||\mathcal{I}| \sum_{u, x_{2}} P_{U X_{2}}\left(u, x_{2}\right) P_{Z}\left(\mathcal{T}_{0, u x_{2}}\right)\right\} \\
& \leq P_{U X_{2} Z}\left(\mathcal{T}_{0}^{c}\right)+|\mathcal{K}||\mathcal{I}| e^{-\alpha_{0}},
\end{aligned}
$$

where we used

$$
P_{Z}(z) \leq P_{Z \mid U X_{2}}\left(z \mid u, x_{2}\right) e^{-\alpha_{0}}
$$

for $z \in \mathcal{T}_{0, u x_{2}}$ in the last inequality.
c) Proof of Eq. (17): By using the monotonicity of the divergence, we have

$$
\begin{aligned}
D(f) & =D\left(P_{S \tilde{Z}} \| P_{S} \times P_{\tilde{Z}}\right) \\
& \leq D\left(P_{K I S \tilde{Z}} \| P_{S} \times P_{K I \tilde{Z}}\right) \\
& =\sum_{k, i} \frac{1}{|\mathcal{K}||\mathcal{I}|} D\left(P_{S \tilde{Z} \mid K I}(\cdot, \cdot \mid k, i) \| P_{S} \times P_{\tilde{Z} \mid K I}(\cdot \mid k, i)\right) \\
& =\sum_{k, i, s} \frac{1}{|\mathcal{K}||\mathcal{I}||\mathcal{S}|} D\left(P_{\tilde{Z} \mid K I S}(\cdot \mid k, i, s) \| P_{\tilde{Z} \mid K I}(\cdot \mid k, i)\right) .
\end{aligned}
$$

For each $(k, i)$, we use the relation

$$
\begin{aligned}
& \sum_{s} \frac{1}{|\mathcal{S}|} D\left(P_{\tilde{Z} \mid K I S}(\cdot \mid k, i, s) \| P_{\tilde{Z} \mid K I}(\cdot \mid k, i)\right) \\
&+D\left(P_{\tilde{Z} \mid K I}(\cdot \mid k, i) \| P_{Z \mid U X_{2}}\left(\cdot \mid u_{k i}, x_{2 k}\right)\right) \\
&= \sum_{s} \frac{1}{|\mathcal{S}|} D\left(P_{\tilde{Z} \mid K I S}(\cdot \mid k, i, s) \| P_{Z \mid U X_{2}}\left(\cdot \mid u_{k i}, x_{2 k}\right)\right) .
\end{aligned}
$$

By using Lemma 12 for input distributions $P_{V \mid U X_{2}}\left(\cdot \mid u_{k i}, x_{2 k}\right)$ and $P_{X_{1} \mid V}$ and channel $P_{Z \mid X_{1} X_{2}}$, we have

$$
\begin{aligned}
\mathbb{E}_{\mathcal{C}_{2} \mathcal{C}_{3}} & {[D(f)] } \\
\leq & \sum_{k, i} \frac{1}{|\mathcal{K}||\mathcal{I}|} \\
& {\left[\frac{1}{\theta|\mathcal{A}|^{\theta}} e^{\psi\left(\theta \mid P_{Z \mid X_{1} X_{2}}\left(\cdot \mid \cdot, x_{2 k}\right), P_{X_{1} \mid V}, P_{V \mid U X_{2}}\left(\cdot \mid u_{k i}, x_{2 k}\right)\right)}\right.} \\
& \left.+\frac{1}{\theta^{\prime}|\mathcal{J}|^{\theta^{\prime}}} e^{\psi\left(\theta^{\prime} \mid P_{Z \mid U V X_{2}}\left(\cdot \mid \cdot, u_{k i}, x_{2 k}\right), P_{V \mid U X_{2}}\left(\cdot \mid u_{k i}, x_{2 k}\right)\right)}\right] .
\end{aligned}
$$

By taking the average over $\mathcal{C}_{0}$ and $\mathcal{C}_{1}$, and by noting
$\mathbb{E}_{\mathcal{C}_{0} \mathcal{C}_{1}}\left[\sum_{k, i} \frac{1}{|\mathcal{K}||\mathcal{I}|} \mathbf{1}\left[u_{k i}=u, x_{2 k}=x_{2}\right]\right]=P_{U X_{2}}\left(u, x_{2}\right)$,
we have

$$
\begin{aligned}
& \mathbb{E}_{\mathcal{C}_{0} \mathcal{C}_{1} \mathcal{C}_{2} \mathcal{C}_{3}}[D(f)] \\
& \leq \sum_{u, x_{2}} P_{U X_{2}}\left(u, x_{2}\right) \\
& {\left[\frac{1}{\theta|\mathcal{A}|^{\theta}} e^{\psi\left(\theta \mid P_{Z \mid X_{1} X_{2}}\left(\cdot \mid \cdot, x_{2 k}\right), P_{X_{1} \mid V}, P_{V \mid U X_{2}}\left(\cdot \mid u_{k i}, x_{2 k}\right)\right)}\right.} \\
&\left.+\frac{1}{\theta^{\prime}|\mathcal{J}|^{\theta^{\prime}}} e^{\psi\left(\theta^{\prime} \mid P_{Z \mid U V X_{2}}\left(\cdot \mid \cdot, u_{k i}, x_{2 k}\right), P_{V \mid U X_{2}}\left(\cdot \mid u_{k i}, x_{2 k}\right)\right)}\right] \\
& \leq \frac{1}{\theta|\mathcal{A}|^{\theta}} e^{\psi\left(\theta \mid P_{Z \mid X_{1} X_{2}}, P_{X_{1} \mid V}, P_{U V X_{2}}\right)} \\
&+\frac{1}{\theta^{\prime}|\mathcal{J}|^{\theta^{\prime}}} e^{\psi\left(\theta^{\prime} \mid P_{Z \mid U V X_{2}}, P_{V \mid U X_{2}}, P_{U X_{2}}\right)} .
\end{aligned}
$$

## C. Proof of Lemma 8

By using the Fourier-Motzkin elimination, we can show that $\left(R_{d}, R_{0}, R_{1}, R_{s}\right) \in \tilde{\mathcal{R}}^{(i n)}$ if and only if

$$
\begin{align*}
R_{0} \leq & I\left(U, X_{2} ; Z\right) \\
R_{0}+R_{s} \leq & I\left(V ; Y \mid U, X_{2}\right)-I\left(V ; Z \mid U, X_{2}\right) \\
& +I\left(U, X_{2} ; Y\right)  \tag{28}\\
R_{1}+R_{s} \leq & I\left(U, V ; Y \mid X_{2}\right) \\
R_{0}+R_{1}+R_{s} \leq & I\left(V ; Y \mid U, X_{2}\right) \\
& +\min \left[I\left(U, X_{2} ; Y\right), I\left(U, X_{2} ; Z\right)\right] \\
R_{s} \leq & I\left(V ; Y \mid U, X_{2}\right)-I\left(V ; Z \mid U, X_{2}\right),(29  \tag{29}\\
R_{d}+R_{1} \geq & I\left(X_{1} ; Z \mid U, X_{2}\right) \\
R_{d} \geq & I\left(X_{1} ; Z \mid U, V, X_{2}\right)
\end{align*}
$$

are satisfied. By adding the inequality

$$
R_{0} \leq I\left(U, X_{2} ; Y\right)
$$

this inequality and Eq. (29) imply that Eq. (28) is redundant. Thus, we have $\mathcal{R}^{*} \subset \tilde{\mathcal{R}}^{(i n)}$.

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