Comments and Corrections

Correction to "Boolean Compressed Sensing and Noisy Group Testing"

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***Abstract*— In this paper, a correction of Lemma III.1 in [1] is presented.**

***Index Terms*— Compressed sensing, group testing, channel coding.**

I. Achievable Bound

Lemma III.1 stated in the probability of error analysis in [1] has an error, however, in this note we present a corrected version of this lemma. This version leads to the same main achievability result (Theorem III.1), as well as to a less stringent bound. All order-wise conditions for the sufficient number of tests T in the paper hold as is with the new lemma.

The corrected lemma is as follows.

Lemma 1:

$$P(E_i) \leq 2^{-T\left(E_o(\rho)-\rho\frac{\log\binom{N-K}{i}}{T}-\frac{\log\binom{K}{i}}{T}\right)}$$

Note that the difference from the original Lemma is that ρ does not multiply the last term in the exponent. The lemma above is established by modifying inequality (c) on p. 1887 that upper bounds $\Pr[E_i|\omega_0 = 1, \mathbf{X}_{S_1}, Y^T]$ such that $\binom{N-K}{i}$ is replaced with $\binom{N-K}{i}^{\rho}$, which follows from inequality (b). Following the rest of the proof in Section III.E establishes the lemma.

As a result, the sufficiency bound in Theorem III.1 can now be replaced with a less stringent bound,

$$T > (1+\epsilon)\cdot \max_{i:(\mathcal{S}^1,\mathcal{S}^2)\in\Xi_S^{\{i\}}} \frac{\log\binom{N-K}{i}}{I(X_{\mathcal{S}^1}; X_{\mathcal{S}^2}, Y)}$$

where only the $\binom{K}{i}$ term is removed in the numerator. In turn the original sufficiency bound also holds.

To see that the sufficiency bound above holds, observe that it is sufficient to show that

$$Tf(\rho) = TE_o(\rho) - \rho\log\binom{N-K}{i} - \log\binom{K}{i} \to \infty,$$

however for this to hold, it is sufficient that $TE_o(\rho) - \rho\log\binom{N-K}{i} \to \infty$, as $\log\binom{K}{i}$ is a constant for the fixed K regime. The condition above can in turn be proven exactly following the proof of Theorem III.1 where $\binom{N-K}{i}\binom{K}{i}$ is replaced with $\binom{N-K}{i}$.

It can then be observed that all achievability results for fixed K hold without any modifications, including theorems V.1, V.3, V.4, VI.1 and VI.4.

II. $K = o(N)$ Scaling Regime

The achievability bounds for the $K = o(N)$ regime as presented in theorems V.2, VI.2 and VI.5 also hold without any modifications. These can be proven using the revised lemma presented above, along with the lower bounds on $I(X_{\mathcal{S}^1}; X_{\mathcal{S}^2}, Y)$ and the second derivatives $|E_o''(\psi)|$ that are derived in Appendix C.

Using Theorem V.2 as an example, we see that a sufficient condition for recovery is

$$T\left(E_o(\rho) - \rho\frac{\log\binom{N-K}{i}}{T} - \frac{\log\binom{K}{i}}{T} - \frac{\log K}{T}\right) \to \infty,$$

where $E_o(\rho) \geq \rho\frac{i}{K} - \frac{\rho^2}{2}\frac{i}{K}\log^2\frac{K}{i}$ for the noiseless model. A simple analysis then shows that the above condition holds for $\rho = \frac{1}{\log^2(K/i)}$ and the choice of T in the theorem statement.

III. Acknowledgment

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References

[1] G. K. Atia and V. Saligrama, "Boolean compressed sensing and noisy group testing," *IEEE Trans. Inf. Theory*, vol. 58, no. 3, pp. 1880–1901, Mar. 2012.

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