

# Dynamic vehicle redistribution and online price incentives in shared mobility systems

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## Abstract

This paper considers a combination of intelligent repositioning decisions and dynamic pricing for the improved operation of shared mobility systems. The approach is applied to London’s Barclays Cycle Hire scheme, which the authors have simulated based on historical data. Using model-based predictive control principles, dynamically varying rewards are computed and offered to customers carrying out journeys. The aim is to encourage them to park bicycles at nearby under-used stations, thereby reducing the expected cost of repositioning them using dedicated staff. In parallel, the routes that repositioning staff should take are periodically recomputed using a model-based heuristic. It is shown that it is possible to trade off reward payouts to customers against the cost of hiring staff to reposition bicycles, in order to minimize operating costs for a given desired service level.

## 1 Introduction

Public Bicycle Sharing (PBS) schemes offer the rental of bicycles as a means of public transportation in urban areas. They allow registered users to pick up a bicycle from one of many docking stations throughout the entire city, without any prior notice. The bicycle is returned to another station, after which the user’s intended destination is usually reached on foot. Short journeys are encouraged by charging users only a small fee for a short rental period (typically less than one hour), but then ramping up the cost significantly for longer use.

Such schemes have been introduced in major cities as an alternative to often slow and crowded mass transportation. Many have grown considerably in size in recent years [1, 2], and are becoming a major component of their cities’ public transportation systems. In 2008, for example, 120,000 daily journeys were being made using shared bicycles in Paris [3].

Most PBS schemes are still unable to recoup their full operational and investment costs solely from customer fees. According to [4], capital costs can be up to \$4,500 per bicycle, and annual operational costs up to \$1,700 per bicycle. Sometimes additional revenues from advertising can be used to mitigate this cost gap. However, in almost all cases additional funding from public sources is required [1, 5].

One of the major contributors to operational costs is the need to operate staffed trucks for manual relocation of bicycles, in order to balance the difference between supply and demand at various stations. If this effort were not made, the arrival and departure of customers would cause many stations to run full or empty, and the customer service rate would drop below acceptable levels [6, 7]. Since this redistribution of bicycles entails costs, a trade-off for the desired service level needs to be made.

The goal of this paper is to show how the system performance could be optimized by trading off two complementary methods. Firstly, we devise an algorithm to optimize the dynamic route-planning of multiple trucks for bicycle relocation. Secondly, on top of this manual repositioning, we propose a scheme that offers users price incentives based on the current and predicted state of the system, in order to encourage them to change the endpoint of their journeys. These incentives are set to shift bicycle drop-offs away from stations that are overfilled, and towards nearby stations that may have empty spaces. The price incentives are independent of the usual rental fees, which we assume to be a sunk cost for the user.

This paper’s main contributions can be summarized as follows:

1. A tailored routing algorithm that plans how trucks will be used to redistribute bikes amongst stations. Redistribution is performed in the dynamic setting, i.e. while the system is in operation. The heuristic chooses the actions of multiple trucks, with the aim of enabling as many extra journeys as possible to take place.
2. A dynamic incentives scheme where customers are encouraged to change their target station in exchange for a payment. Changes to journey length may be inconvenient, and we assume customers accept or reject such incentives based on the value of their time and the payment offered.

Both the truck routes and the price incentives are recomputed online at periodic time instances. For both components, a predictive model of the expected near-future evolution of the system state is used to optimize their actions over a finite, receding horizon. The optimization goal is to maximize the number of additional journeys enabled via repositioning, taking into account available resources and cost trade-offs. At each re-optimization step, up-to-date information on the current state of the system is used to plan all future operational decisions. This is shown schematically in Fig. 1.

We evaluate the trade-off between these two methods using a Monte Carlo model of the London Cycle Hire PBS scheme, constructed from detailed historical usage information.

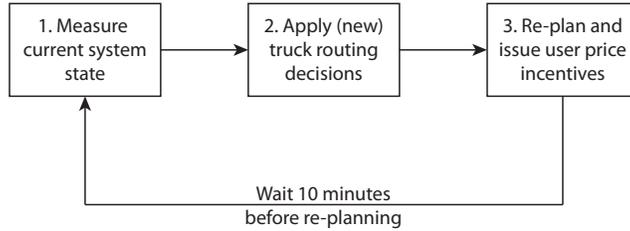


Figure 1: Schematic of the online optimization scheme presented in this paper. In step 2, truck routes are planned based on a model of how the bike movements will evolve from the current state, and if necessary new orders are issued. In step 3, the current system state and the future truck actions are taken into account, and new price incentives for users are computed, based on a trade-off between payouts and system performance.

Our results suggest that service level improvements may be attained using price incentives alone, and that increases in either the customer payouts or the number of repositioning trucks deliver diminishing returns to service levels. Unsurprisingly, higher service levels can be reached on weekends in comparison to weekdays.

The paper is organized as follows. In Section 2 we explain how a model of the London PBS scheme was derived using historical data. In Section 3 we develop a metric for the utility of repositioning actions based on the expected ability to serve additional future customers. The results are used in Section 4 to develop a heuristic for determining the routes of repositioning trucks. In Section 5 we develop a model-based controller for computing the price incentives offered to customers. The two approaches for repositioning are compared using a Monte Carlo simulation framework in Section 6. Section 7 draws conclusions on the performance of the scheme.

## 2 System model

### 2.1 Historical data

The PBS system model used in this paper is based on London’s Barclays Cycle Hire scheme. For modeling, we used three data sets made publicly available by the Transport for London authority:<sup>1</sup>

1. 1.42 million rides spanning a period of 97 days (examples in Table 1),
2. Size and location of 354 stations actively used during the recorded period (examples in Table 2),

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<sup>1</sup><http://www.tfl.gov.uk/businessandpartners/syndication/default.aspx>

Table 1: Ride information samples

bike-id	start (date, station-id)	end (date, station-id)
3340	{2010-07-30 06:00:00, 47}	{2010-07-30 06:22:00, 47}
3870	{2010-07-30 06:00:00, 234}	{2010-07-30 06:14:00, 203}
1627	{2010-07-30 06:01:00, 149}	{2010-07-30 06:29:00, 293}
1695	{2010-07-30 06:02:00, 152}	{2010-07-30 06:06:00, 324}

Table 2: Station information samples

id	name	position (lat, lon)	size
1	River St, Clerkenwell	{51.5291, -0.1099}	18
2	Phillimore Gardens, Kensington	{51.4996, -0.1975}	34
3	Christopher St, Liverpool St	{51.5212, -0.0846}	33
4	St. Chad’s Street, King’s Cross	{51.5300, -0.1200}	22

- An initial station fill level recorded during nighttime when all bicycles were docked. In total, we estimate that the system contained 3708 bikes at the time for which historical data is available.

Analysis of the historical journeys reveals regular daily flow patterns, with a substantial difference between weekdays and weekends. Journeys are allowed between 6am and midnight. As expected, many customers commute to the city center in the morning and ride back to the outer districts in the late afternoon. This pattern, absent on weekends, causes two spikes in daily rental activities that are illustrated in Fig. 2.

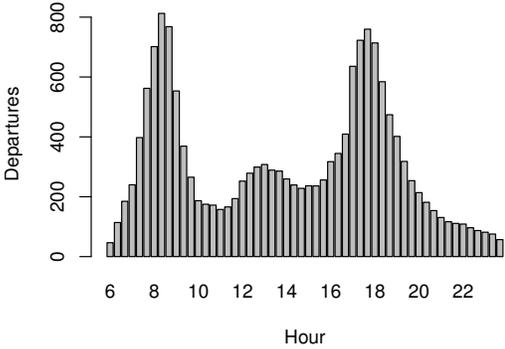


Figure 2a: Average departure rates on weekdays

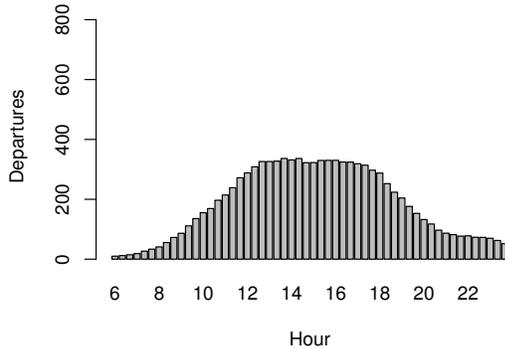


Figure 2b: Average departure rates on weekends

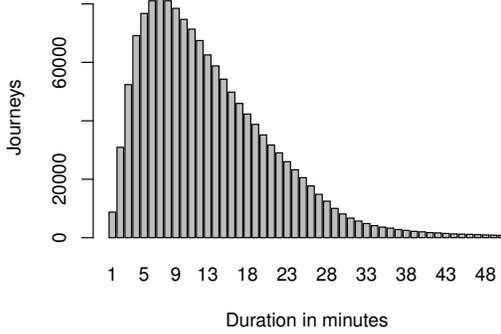


Figure 3a: Distribution of the journey duration in the historical data

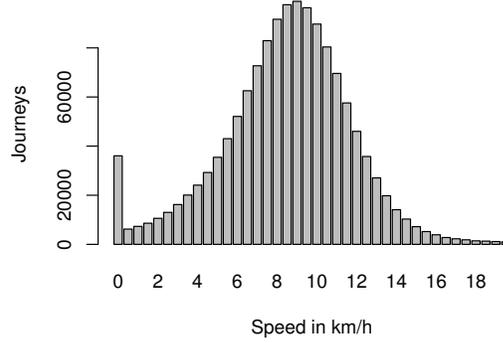


Figure 3b: Journey speed distribution (Euclidean station distances)

## 2.2 Model parameters

In our model, we define a set  $S$  containing all stations  $s \in S$ . Time  $t$  is assumed to be discrete and indexed on a one-minute level where  $T_{\text{hist}}$  denotes all time steps of the observed period. We distinguish between workdays and days on weekends by the binary variable  $w \in \{\textit{weekday}, \textit{weekend}\}$  and split every day into 72 slices  $k \in K$  of 20 minutes each. Time is mapped to day-type and timeslice using  $w(t)$  and  $k(t)$ , respectively. All customer departure and arrival events are counted in matrices of dimension  $|S| \times |S|$ . The sum of departing customers going from station  $i$  to  $j$  in a timeslice  $k$  and on a day  $w$  is  $D_{i,j}(k, w)$ ; similarly the sum of customers who arrive at station  $j$  coming from  $i$  is  $A_{i,j}(k, w)$ . The average number of such arrival events  $\Lambda_{i,j}(t)$  and departure events  $M_{i,j}(t)$  at time  $t$  in the historical data can thus be expressed as

$$M_{i,j}(t) = \frac{D_{i,j}(k(t), w(t))}{|\{t' \in T_{\text{hist}} : k(t') = k(t), w(t') = w(t)\}|}, \quad (1)$$

$$\Lambda_{i,j}(t) = \frac{A_{i,j}(k(t), w(t))}{|\{t' \in T_{\text{hist}} : k(t') = k(t), w(t') = w(t)\}|}. \quad (2)$$

Here, the denominator gives the duration of the recorded history for a given timeslice and day-type indicated by  $t$ .

Based on these average numbers of events happening per time step, customer departures are exponentially distribution with time-varying parameter  $M_{i,j}(t)$ . This parameter fit is based on the following implicit assumptions:

- 100% service rate for departures in the historical data. Potential customers who could not rent a bicycle due to an empty station are excluded, as they are not recorded in the historical data. This assumptions is justified to some degree by the considerable repositioning effort made by the operator of the London PBS scheme [8].

- Independence of customer arrival. Departure of customers vary with time and type of the day, but do not depend on other departures. As a caveat, this does not accurately model customer groups, for example tourists.
- Effects of season, weather, events, etc. are disregarded, but could easily be included in a more detailed model.

If a customer has departed at a station, the probability distribution of his destinations is given by their relative frequency in the historical data, as recorded in  $\Lambda_{i,j}(t)$ .

The total expected departure  $\mu_s(t)$  and arrival  $\lambda_s(t)$  at each station and the *net change of fill level*  $\eta_s(t)$  during time step  $t$  is therefore

$$\mu_s(t) = \sum_{\bar{s} \in S} M_{s,\bar{s}}(t), \quad \lambda_s(t) = \sum_{\bar{s} \in S} \Lambda_{s,\bar{s}}(t), \quad (3)$$

$$\eta_s(t) = \lambda_s(t) - \mu_s(t). \quad (4)$$

In order to simulate the system, the following assumptions about the behavior of customers are needed, in addition to their arrival rates:

- Customers who want to depart from a station that turns out to be empty leave without starting a journey. They do not wait for a bicycle to be returned, nor do they walk on to a neighboring station.
- The travel time between any two stations  $i$  and  $j$  is always equal to the average travel time extracted from the historical data. Figure 3 depicts the historical distribution of travel speeds and journey durations.
- Customers who arrive at their target station wanting to return their bicycle when the station is full ride on to one of the neighboring stations (chosen according to his perceived utility, as described in Section 2.3). If this station is also full, a customer will go on to the next station, but he does not return to any station already visited.

### 2.3 Customer decision model

In order to investigate the effects of offering price incentives, a model of how the customer reacts is required. We assume all customers place a value on the additional time they would spend travelling if they were to accept an incentive. This is equivalent to penalizing a longer travel distance. The additional distance a customer has to travel if he changes his target station consists of the additional distance he has to bike, plus an additional walking distance to his final destination. We assume this final destination lies at the center of mass of the Voronoi region around each station (Figure 4). The Voronoi region is the polytope that contains all points closer to a given station than to any other.

Let the center of mass of the Voronoi region around each station  $s_i$  be denoted by  $m_i$ , and let  $d_{\text{eucl}}$  be the Euclidean distance on the map. Assuming that the walking speed is half

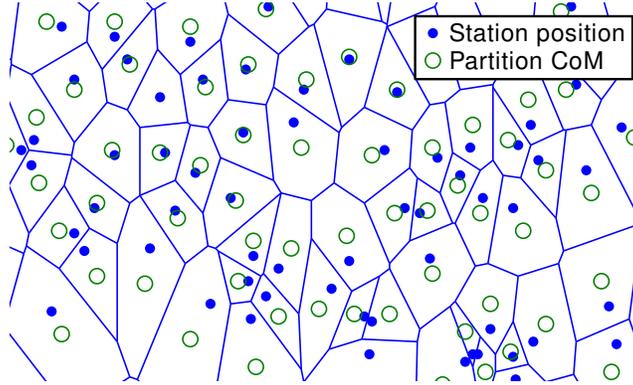


Figure 4: Voronoi partitioning with centers of mass (CoM) of the London bike sharing system

the cycling speed, the *effective distance*  $\tilde{d}_{i,j}$  between two stations  $s_i$  and  $s_j$  can be expressed as

$$\tilde{d}_{i,j} = d_{\text{eucl}}(s_i, s_j) + 2 \cdot d_{\text{eucl}}(s_j, m_j) - 2 \cdot d_{\text{eucl}}(s_i, m_i). \quad (5)$$

The incentives to go to a neighboring station are offered to customers upon their arrival. Each customer decides whether to take an incentive by maximizing his personal benefit based on the incentive payout and the customer's perceived cost of additionally traveled distance. This implies customer rationality and makes the choice independent from the original pricing of journeys. For each end station  $s \in S$ , the set of neighboring stations to which a price incentive could be offered is  $N_s$ , and  $p_{s,n}$  denotes the amount of money offered to from station  $s$  to neighbor  $n \in N_s$ . In addition, let  $\tilde{N}_s$  be the set of stations  $\tilde{s}$  which have  $s$  in their neighbor set. The following model of customer reactions is used.

1. The marginal cost of travel  $c$  for each arriving customer is drawn from a uniform distribution  $C \sim U[0, c_{\max}]$ , where we have used  $c_{\max} = \text{£}20/\text{km}$  in our simulations.<sup>2</sup>
2. The customer selects the best offer of maximum value as

$$n^* = \arg \max_{n \in N_s} p_{s,n} - d_{s,n} c. \quad (6)$$

3. If the original target station is full, i.e. the customer cannot return his bike there, he always chooses the best incentive to go to  $n^*$ . If there is space, he takes an incentive only if the perceived value of the best incentive is positive, i.e. if  $p_{s,n^*} - d_{s,n^*} c > 0$ .

The probability  $\pi(s, n, p_s)$  of an arriving customer taking an incentive to neighbor  $n \in N_s$  for a given payout vector  $p$  depends on the distribution of perceived travel costs  $c$ . First,

<sup>2</sup>Future work could incorporate more detailed models of how customers value their time (see [9, 10]). In addition, instead of having a single distribution, one could also differentiate between customer types (e.g. those commuting to work and those riding during leisure times) by introducing a time-varying component.

the offering to go to  $n$  must have the highest perceived value amongst all incentives offered to neighboring stations. Second, assuming the station  $s$  is not full, the perceived cost for traveling the additional distance must be lower than the relevant payout  $p_{s,n}$ .

$$\begin{aligned} \pi(s, n, p_s) = P & \left( p_{s,n} \geq c \cdot \tilde{d}_{s,n} \wedge p_{s,n} - (c \cdot \tilde{d}_{s,n}) \right. \\ & \left. \geq p_{s,n'} - (c \cdot \tilde{d}_{s,n'}), \forall n' \in N_s \right) \end{aligned} \quad (7)$$

### 3 Utility of changes in station fill level

In this paper, two methods are considered for influencing the distribution bicycles in the PBS: manual repositioning (Section 4) and price-led repositioning via incentives (Section 5). However, as a basis for both algorithms, it is necessary to estimate any change in the stations' fill levels will bring about. Since the system is stochastic, it is not straightforward to assess these benefits. In this section we introduce a function that estimates the utility of changes in fill levels for a given station.

Raviv and Kolka [11] have done related work in order to determine the best fill level of each station in a static repositioning setting. Their approach tracks the probability of all possible fill levels based on a discrete approximation of the underlying continuous birth-death process. However, if we were to adopt this method, the dimension of the resulting optimization problem would significantly increase the computational complexity of the proposed approaches in Sections 4 and 5. Therefore we propose a simpler approach.

We make the simplifying assumption that arrivals and departures are deterministic and given by the expected net change  $\eta_s(t)$ . Furthermore, we define the utility of changes to a station's fill level as the difference in the number of customers expected to be served successfully at that station within a long enough (but finite) time horizon. The benefit of any repositioning action (adding or taking away bikes at a single station) at the current time can then be evaluated based on this notion.

For a given station  $s$  and starting time  $t_0$ , we precompute the expected future fill level  $f_t^s$  over a time horizon with  $t = t_0, \dots, t_0 + T_{\text{util}}$ , where  $T_{\text{util}} = 24$  hours is the look-ahead period considered. The expected fill level is governed by the following dynamics:

$$f_{t+1}^s = \max(0, \min(f_t^s + \eta_s(t), f_{\text{max}}^s)), \quad (8)$$

where  $f_{\text{max}}^s$  denotes the maximum capacity of the station, and the max and min functions ensure that the station never becomes "more than full" or "less than empty". The quantity  $\eta_s(t)$  is the net arrival rate defined by (4).

Adding or taking away bikes from the station at the current time changes how many customers can be served later on in the time horizon, since the station will become empty

or full at different times in the future. For a current fill level  $f_{t_0}^s$  and a change in fill level  $\Delta f$  to be made at  $t_0$ , Algorithm 1 computes the utility  $u(s, t, f_t^s, \Delta f)$  by comparing the two cases of different initial fill level. The algorithm moves through the time horizon forward in time, and in each time step compares the amount of change  $\Delta$  and  $\tilde{\Delta}$  resulting from the system dynamics and the station size constraints. If the amount of change is lower in the original case, a station size constraint is being hit earlier than in the adapted case and vice versa. The difference between  $\Delta$  and  $\tilde{\Delta}$  is the difference in customers that could be served successfully in that time period. The procedure aborts if the end of the time horizon has been reached or if the fill level of the station becomes equal in both cases (e.g. both are full or empty).

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**Algorithm 1** Computing the utility of repositioning

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**Require:**  $s \in S, T_{\text{util}}$  ▷ Relevant station and horizon length,  
1:  $f_{\text{max}}^s$  ▷ Maximum capacity of station  $s$ ,  
2:  $\eta_s(t)$  ▷ Expected net arrival of customers,  
3:  $f_t^s \forall t \in \{t_0, \dots, t_0 + T_{\text{util}}\}$  ▷ Precomputed fill levels in the original case according to (8),  
4:  $\tilde{f}_{t_0}^s$  ▷ Starting fill level in the case with repositioning  
5: **procedure** REPOSITIONINGUTILITY( $t, \tilde{f}_t^s$ )  
6:   **if**  $\tilde{f}_t^s = f_t^s$  **or**  $t \geq T_{\text{util}}$  **then**  
7:     **return** 0  
8:   **end if**  
9:    $\tilde{f}_{t+1}^s \leftarrow \max(0, \min(\tilde{f}_t^s + \eta_s(t), f_{\text{max}}^s))$   
10:    $\Delta \leftarrow |f_t^s - \tilde{f}_{t+1}^s|$   
11:    $\tilde{\Delta} \leftarrow |\tilde{f}_t^s - \tilde{f}_{t+1}^s|$   
12:   **return**  $\tilde{\Delta} - \Delta + \text{REPOSITIONINGUTILITY}(t + 1, \tilde{f}_{t+1}^s)$   
13: **end procedure**

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Although the worst-case time complexity of Algorithm 1 is linear with respect to the horizon length, it would take too long to use online in the later optimization steps. Storing the results in a lookup table is intractable, especially if the expected future fill levels are treated as continuous values. However, fast computation can be achieved by constructing a simpler function, making use of a lookup table of dimension  $2 \times |S| \times T_{\text{util}}$ , from which this utility can be determined. Figure 5 shows an example of such a utility function for an empty station,  $u(s, t, f_t^s = 0, \Delta f)$ . For simplicity,  $\Delta f$  is also relaxed to be non-integral. The validity of this parameterisation is now proven.

**Theorem 1.** *For any station  $s \in S$  at time  $t$ , there exist two fill levels  $\underline{f}_t^s, \overline{f}_t^s \in [0, f_{\text{max}}^s]$*

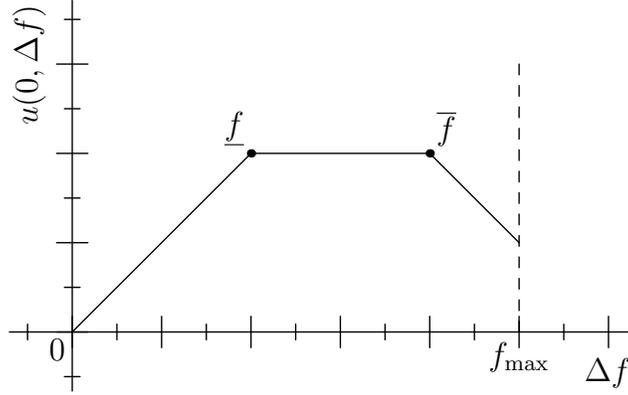


Figure 5: Example for an empty station ( $f = 0$ ), showing the utility “plateau” between the fill levels  $\underline{f}_t^s$  and  $\bar{f}_t^s$ . In this case maximum utility results from adding 4 to 8 bicycles at  $t_0$ , based on the expected net arrival rate over the time horizon.

independent from the initial fill level  $f_t^s$  such that for the utility of change in fill level  $\Delta f$

$$\frac{\partial u(s, t, f_t^s, \Delta f)}{\partial \Delta f} = \begin{cases} 1, & \text{if } f_t^s + \Delta f < \underline{f}_t^s \\ 0, & \text{if } \underline{f}_t^s \leq f_t^s + \Delta f \leq \bar{f}_t^s \\ -1, & \text{if } f_t^s + \Delta f > \bar{f}_t^s. \end{cases} \quad (9)$$

The utility of no change of the station’s fill level is understood to be zero,  $u(s, t, f_t^s, 0) := 0$ . The (possibly empty) interval  $[\underline{f}_t^s, \bar{f}_t^s]$  of utility is called the “plateau” of constant maximum utility.

*Proof.* Assume the station will become full within the time horizon  $T_{\text{util}}$  for initial fill level  $f_t^s + \Delta f$ . Adding  $\delta$  bicycles to the station implies that  $\delta$  additional expected customers who want to return their bicycles have to be rejected. Therefore, the utility  $u(s, t, f_t^s, \Delta f + \delta)$  decreases monotonically with slope -1 for any  $\delta \geq 0, f_t^s + \Delta f + \delta \leq f_{\text{max}}^s$ .

$$u(s, t, f_t^s, \Delta f + \delta) = u(s, t, f_t^s, \Delta f) - \delta$$

A similar case can be made for fill levels where the station is running empty. Only, the utility decreases when more bicycles are removed with a slope of  $u$  equal to 1. It follows naturally that for a given initial fill level there are thresholds  $\underline{f}_t^s, \bar{f}_t^s$ , with  $\underline{f}_t^s < \bar{f}_t^s$ , where the station first starts to run empty or full within the horizon. Within the interval  $[\underline{f}_t^s, \bar{f}_t^s]$ , the station’s capacity constraints are not hit and the utility function must therefore be constant, leading to a “plateau” of the type shown in Fig. 5.  $\square$

It can be shown that  $u(s, t, f_t^s, \Delta f)$  can be computed for each station using only three calls to Algorithm 1. Sections 4 and 5 will make use of this characterization of station fill

utilities in order to choose how trucks reposition bikes and price incentives can be offered to customers.

The system dynamics (8) are based on deterministic net arrival of customers. This results in a coarser model than for example the probabilistic approach of [11]. However, this simple parameterization is attractive in that it leads to tractable optimization problems, as will be seen in Sections 4 and 5.

## 4 A dynamic truck-routing algorithm

This section describes an algorithm for intelligent operation of a fleet of  $R$  trucks, which move bikes between stations as needed. Their objective is to increase the system utility (as defined in Section 3), and hence ultimately the system’s service level, as much as possible.

The problem of manual relocation of vehicles in shared vehicle systems is not new. It originated in pilot car-sharing projects, such as the French *Praxitèle* [12,13], *Intellishare* [14] and *Honda ICVS* [15]. However, the repositioning algorithms used in car-sharing projects do not translate directly to the public bicycle-sharing scheme considered in this paper. Firstly, each of these algorithms exhibits certain characteristics that are specific to its corresponding vehicle-sharing system, for example charging times of electric vehicles. Secondly, car-sharing systems tend to be much smaller in their network size than the PBS considered in our paper, and the proposed algorithms cannot easily be scaled to several hundred stations.

Intelligent repositioning in bicycle-sharing schemes has also received prior attention in the literature [1]. The proposed approaches can be separated into static and dynamic approaches.

In the static repositioning approach, an optimal route is computed in order to attain a predefined fill level for each station, prior to customers interacting with the system (e.g. during the night). For example, [16] present a solution for the routing of a single truck, and [17] consider the case of multiple trucks. The advantage of this approach is that there is ample time to compute a good truck routing solution, and this solution could serve as a reliable basis for computing the price incentives (Section 5). However, static repositioning has shown too little flexibility to react to unforeseen variations in the demand pattern, caused for example by unexpected weather conditions.

In the dynamic repositioning approach, the truck routing is planned in a receding horizon fashion while the system is in full operation. This allows the planning to react online to unexpected changes in the system’s state. As such it is a more suitable approach to our problem. Rair and Miller-Hooks [18] use a stochastic formulation for dynamic repositioning based on stationary distributions for customer arrivals and departures. However, their approach is not suitable to the case of our PBS, since these distributions vary considerably

according to the distinct daily flow patterns described in Section 1. Contardo et al. [19] present another dynamic repositioning approach with time-varying, yet deterministic future flow patterns. But the computational complexity of their approach is prohibitive for our system, because it is too expensive to simulate the system with multiple trucks over a long time horizon.

Whilst many PBS schemes redistribute bicycles during the night, nighttime operation is restricted in London in several key areas [20]. Therefore, this paper considers the dynamic repositioning case only. It is a variation of the routing problem with pickups and deliveries for one commodity, taken from [21]. It is illustrated in Figure 6. In order to determine a truck route, time is discretized into 5-minute intervals and the planning problem is considered on the time-expanded network (Section 4.1) During each interval, customer behavior is assumed to be time-varying, but deterministic. A receding planning horizon is considered, which is the maximum of the truck visiting 4 stations and  $T_{\text{truck}} = 40$  min. The period for re-optimizing of the truck routes (“implementation horizon”) is chosen as  $T_{\text{impl}} = 30$  min, based on a trade-off between computation time and performance quality. Note that as shown in Figure 6, the planning horizon is longer than the re-optimization cycle. This improves the performance of truck journeys beginning shortly before the next re-optimization as these journey are likely to end within the longer planning horizon where subsequent opportunity is still considered.

To solve the routing problem for a single truck (Section 4.2) over the finite planning horizon, we adopt a two-step approach. First, for each truck we construct a tree of “promising route candidates” using a greedy heuristic. This means that truck routes are extended by stations that promise high ratios of utility added per time to travel. Second, for each of the promising routes, the optimal number of bikes to be loaded or unloaded at each stop is optimized as the complete routes have become known. Then the route providing the highest utility improvement is selected. Finally, we extend this algorithm in a simple manner to multiple trucks (Section 4.3).

#### 4.1 Modeling repositioning truck routes on a time-expanded network

We now describe how the truck routes can be modeled as a time-expanded network on a graph  $G = (V, A)$  [15, 19], which is the basis of our truck-routing algorithm. The vertices  $V$  of the graph consist of tuples  $v = \{(s, t), s \in S, t \in T\} \in V$ .<sup>3</sup> The (expected future) fill level  $f$  for each vertex  $v$  (i.e. station  $s$  at time  $t$ ) is generated according to (8). The arcs  $a = (v_1, v_2) \in A$  of  $G$  correspond to possible journeys a truck is able to take.

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<sup>3</sup>The notation  $s(v)$  is used to access the component  $s$  of the tuple  $v = (s, t)$ .

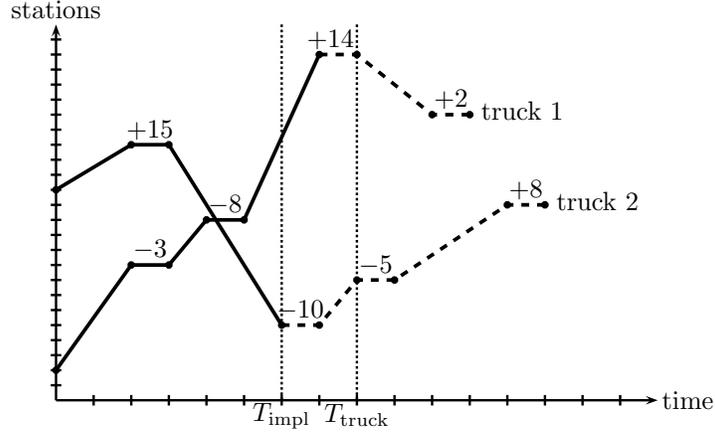


Figure 6: Illustration of the dynamic truck-routing algorithm for  $R = 2$  trucks. The time axis is discretized into 5 min-intervals. The planning horizon is the maximum of  $N_{\text{truck}} = 4$  stations visited by the truck and  $T_{\text{truck}} = 40$  min; the implementation horizon is  $T_{\text{impl}} = 30$  min. Each line represents the route of one truck, where journeys are indicated by a solid line if they start within the implementation horizon (i.e. they are definitely executed) or by a dashed line if they start with the planning horizon (i.e. they may be subject to change at future re-plannings). The trucks wait at each stop for 5 min in order to load or unload bikes. The number of bikes loaded and unloaded at each stop is indicated as well.

In our model, the time it takes for a truck to traverse an arc is discretized to multiples of 5 minutes. It is computed based on the Euclidean distance (in km)  $d_{\text{eucl}}(s_i, s_j)$  between two stations  $s_i, s_j \in S$ , assuming an average speed of 15 km/h for the truck in city traffic. Including an additional 5 min for bicycle handling after reaching the station, the resulting *effective journey time* (in time steps of 5 min) for a truck to go from station  $s_i$  to  $s_j$  is

$$\bar{d}(s_i, s_j) := \lceil d_{\text{eucl}}(s_i, s_j) / 1.25 \rceil + 1. \quad (10)$$

Note that the dividing factor of 1.25 results from converting distance (in km) into time steps (of 5 min). The repositioning trucks have limited operation hours during the day, set to 7 am – 10 pm. All trucks are constrained to start at a maintenance depot in the morning, and also to finish at this depot at the end of the working day. As a consequence, vertices from which no combination of arcs leads back to the depot on time are excluded from the graph. An example of a network of stations and a corresponding time-expanded network are shown in Figures 7 and 8.

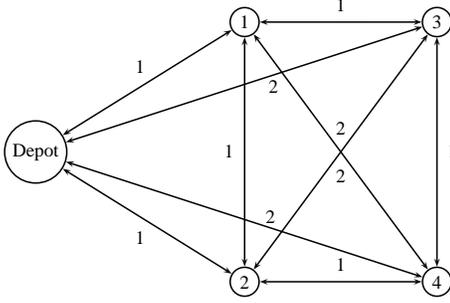


Figure 7: Example station graph, with arcs weighted according to distance.

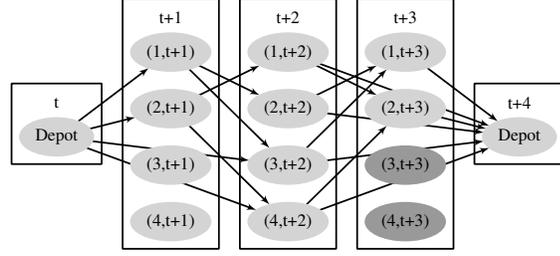


Figure 8: Example time-expanded network. Dark-grey vertices are marked “dead” due to the terminal condition.

## 4.2 Computing single-truck routes

In this Section, we present an algorithm for finding a repositioning route for a single truck in the dynamic case. The amount of time available for repositioning is assumed to be fixed. In contrast to the static repositioning case, the goal is not to attain a defined system state as little resources as possible, but rather to optimally invest the available resources, i.e. operational hours of the trucks. Thus, based on the utility of fill level changes defined in Section 3, the ratio of added utility per invested time of a truck  $r$  is maximized.

**Constructing a tree of promising candidate routes** Every truck  $r \in R$  can hold a maximum load  $l_{\max} = 20$  bikes and holds  $l_t^r \in \{0, \dots, l_{\max}\}$  bicycles at time  $t$ . The goal is to determine a truck’s *repositioning actions*  $\rho_i = (v_i, \Delta f_i, l_i^r) \in P : v_i \in V, l_i \in \{0, \dots, l_{\max}\}, i \in \{1, \dots, N_{\text{truck}}\}$ . The truck load  $l_i^r$  is defined as the number of bicycles after the repositioning action  $\Delta f_i$  performed at the station and time indicated by the vertex of the time-expanded network  $v(\rho_i)$ . In particular, for every pair of actions  $\rho_i, \rho_{i+1}$  there must exist an arc  $a \in A$  with

$$v_1(a) = v_i \wedge v_2(a) = v_{i+1}, \quad \forall i \in \{1, \dots, N_{\text{truck}} - 1\}. \quad (11)$$

Moreover, the following consistency constraints for station fill levels and truck loads must hold:

$$f_{t(v_i)}^{s(v_i)} = f_{t(v_i)-1}^{s(v_i)} + \Delta f_i \in [0, f_{\max}^{s(v_i)}], \forall i \in \{1, \dots, N_{\text{truck}}\} \quad (12a)$$

$$l_{i+1}^r = l_i^r - \Delta f_{i+1} \in \{0, \dots, l_{\max}\}, \forall i \in \{1, \dots, N_{\text{truck}} - 1\} \quad (12b)$$

Starting from an initial repositioning position  $\rho_1$ , the possible next steps can be represented by a tree graph  $\Phi$ , where each node  $\phi$  represents a specific repositioning action and each leaf node determines a unique truck route. The tree of all possible routes has a branching factor of  $|S - 1|$ , since a route could possibly lead to any of the other stations for

the next repositioning action. So there are  $|S - 1|^{N_{\text{truck}} - 1}$  possible combinations of stations for a route of length  $N_{\text{truck}}$  (where the initial position  $\rho_1$  is already known). For systems consisting of several hundred stations it is thus not viable to test every possible combination. The complexity of the problem is reduced by concentrating on a subset of possible routes corresponding to the most promising repositioning actions. It works by constructing a pruned version of the route tree. Starting from the trucks initial position (at the root), the tree is recursively extended at each of its leaf nodes until it has reached the desired height of  $N_{\text{truck}}$  (or the journey has a minimum duration of  $T_{\text{truck}}$ ):

1. The current leaf node is  $\phi = (v_\phi, \Delta f_\phi, l_\phi)$ . First, we compute the “value per unit distance” the truck might bring by going to any of the other stations. The set  $\tilde{V} := \{v \in V : \exists a \in A, v_1(a) = v_\phi, v_2(a) = v\}$  contains the vertices of the time-expanded network the truck would reach by going to any of the other stations. Since we know the current load of bicycles  $l_\phi^r$ , the best action to be performed at  $\tilde{v} \in \tilde{V}$  can be computed with a “greedy” approach as

$$\Delta f^*(\tilde{v}, \phi) = \begin{cases} \max \left( l_\phi^r - l_{\max}, \left[ \bar{f}_{t(\tilde{v})}^{s(\tilde{v})} - f(\tilde{v}) \right] \right), & \text{if } f(\tilde{v}) > \bar{f}(\tilde{v}) \\ \min \left( l_\phi^r, \left[ \underline{f}_{t(\tilde{v})}^{s(\tilde{v})} - f(\tilde{v}) \right] \right), & \text{if } f(\tilde{v}) < \underline{f}(\tilde{v}) \\ 0, & \text{else.} \end{cases} \quad (13)$$

We choose the  $K$  vertices with the best ratio  $\Delta f^*(\tilde{v}, \phi) / \bar{d}(v_\phi, \tilde{v})$  and add them as leaves of  $\phi$  in the form of route steps. The set of new leaf nodes is  $\Phi_\phi$ .

2. In addition, we add stations that could serve as an intermittent depot. Going there may not yield a direct utility. But the possibility to bring or take bicycles may be of use at other stations of the route. We choose

$$\tilde{v}_{store} = \arg \max_{\tilde{v} \in \tilde{V}: s(\tilde{v}) \neq s(v_\phi)} \frac{\min \left( l_{\text{depot}}, \bar{f}_{t(\tilde{v})}^{s(\tilde{v})} - f(\tilde{v}) \right)}{\bar{d}(v_\phi, \tilde{v})} \quad (14a)$$

$$\tilde{v}_{pick} = \arg \max_{\tilde{v} \in \tilde{V}: s(\tilde{v}) \neq s(v_\phi)} \frac{\min \left( l_{\text{depot}}, f(\tilde{v}) - \underline{f}_{t(\tilde{v})}^{s(\tilde{v})} \right)}{\bar{d}(v_\phi, \tilde{v})} \quad (14b)$$

and add them to the set of leafs  $\Phi_\phi$  with a repositioning action of zero. To prevent that  $\tilde{v}_{store}, \tilde{v}_{pick}$  are only set to very large stations, we cap the maximum intermittent depot size considered to some  $l_{\text{depot}} \leq l_{\max}$ . How stations that may serve as intermittent depots are incorporated into the actions at other steps of the route is explained in the following.

3. If the depth of the recursive procedure has not yet reached the final depth  $N_{\text{truck}}$ , it is repeated for every  $\phi' \in \Phi_\phi$ . Before entering the recursive procedure at  $\phi'$ , the

corresponding repositioning action  $\Delta f_{\phi'}$  is incorporated into the predicted future fill levels of the time-expanded network. These changes, of course, have to be unwound between the  $\phi' \in \Phi_{\phi}$ .

If even more aggressive tree pruning is necessary to comply with computational constraints, the similar Beam Search [22,23] can be applied. It leads to linear complexity in the route length, but at the expense of discarding more potentially optimal solutions. In Beam Search, a greedy approach is used to determine promising next steps as well. But only  $K$  leaf nodes are added in total to all nodes of the same height. Resorting to Beam Search was, however, not necessary for the route length horizon used in the sample setting of this paper.

**Refining truck loading actions** The repositioning actions  $\rho_i$ ,  $i = 1, \dots, N_{\text{truck}}$  of the promising routes candidates in  $\Phi$  stem from a greedy heuristic that could not know about stations visited later in the route. Knowing the complete routes, their respective action profile should be further refined. As a motivating example, it may be beneficial to pick up more bicycles than the utility function of a single station  $u(f, \Delta f)$  (see Section 3) originally indicated. That is, if taking more bicycles has zero utility locally (the fill level remains within the utility plateau), but a subsequent stations in the route can make use of the additional bicycles. The problem of choosing optimal actions can be formulated as a manageable quadratic program (QP).

$s(i), t(i)$	The station and time at the $i$ -th step in the route for $i \in \{1, \dots, N_{\text{truck}}\}$ .
$f_i$	The expected fill level of station $s(i)$ at time $t(i)$ .
$\Delta f_i$	The action performed at step $i$ . This is the optimization variable.
$\underline{f}(i), \overline{f}(i)$	Beginning and end of the utility plateau of station $s(i)$ , as described in Section 3.
$\Delta \underline{f}(i), \Delta \overline{f}(i)$	Difference between the new fill level $f_i + \Delta f_i$ and the plateau beginning/end $\overline{f}_{t(i)}^{s(i)}, \underline{f}_{t(i)}^{s(i)}$ . The difference is defined to grow positively going outwards from the respective side of the plateau.
$\Delta \underline{f}'(i), \Delta \overline{f}'(i)$	Auxiliary variables containing the absolute difference from the plateau beginning $\Delta \underline{f}'(i) =  \Delta \underline{f}(i) $ or end $\Delta \overline{f}'(i) =  \Delta \overline{f}(i) $ . They are correctly set by the solver minimizing costs within the bounds set in (16d) and (16e).
$l_0, l_{\max}$	Starting fill level of the repositioning truck and the maximum truck load capacity.
$q \gg 2l_{\max}^2 + 1$	Scaling factor for penalizing repositioning actions.
$\min \sum_{i=1}^{N_{\text{truck}}} \Delta \underline{f}(i) + \Delta \underline{f}'(i) + \Delta \overline{f}(i) + \Delta \overline{f}'(i) + \sum_{i=1}^{N_{\text{truck}}} \Delta f_i^2 / q \quad (15)$	

such that

$$0 \leq l_{i_0}^r - \sum_{i'=1}^i \Delta f_{i'} \leq l_{\max}, \quad \forall i \in \{1, \dots, N_{\text{truck}}\} \quad (16a)$$

$$\Delta \underline{f}(i) = \underline{f}(i) - f_i - \Delta f_i, \quad \forall i \in \{1, \dots, N_{\text{truck}}\} \quad (16b)$$

$$\Delta \bar{f}(i) = -\bar{f}(i) + f_i + \Delta f_i, \quad \forall i \in \{1, \dots, N_{\text{truck}}\} \quad (16c)$$

$$-\Delta \underline{f}'(i) \leq \Delta \underline{f}(i) \leq \Delta \underline{f}'(i), \quad \forall i \in \{1, \dots, N_{\text{truck}}\} \quad (16d)$$

$$-\Delta \bar{f}'(i) \leq \Delta \bar{f}(i) \leq \Delta \bar{f}'(i), \quad \forall i \in \{1, \dots, N_{\text{truck}}\} \quad (16e)$$

The linear part of the objective function (15) evaluates repositioning actions according to the utility definition of Section 3. The quadratic part of (15) minimizes the action of the truck operators (they take the fewest bicycles possible). It is scaled such that actions with a positive utility will still be performed; However, it prevent actions causing negative utility at one station and the equal positive utility at another step in the route. This also ensures that stations will not be pushed outwards from the plateau and actions remain feasible. Equation (16a) ensures that the fill level of trucks stays within the capacity constraints. If computation time allows, an integer constraint  $\Delta f_n \in \mathbb{Z}, \forall n$  can be added to reflect the discrete number of bikes. This renders the optimization problem into a mixed-integer quadratic program (MIQP). In our approach, we manually fit the solution to the truck load constraints based on the relaxed QP solution by clipping any non-integer parts. In test runs no or only very little differences from the MIQP were observed.

We now determine the best set of repositioning actions for all promising routes and choose the route that results in the best overall utility increase per unit time.

### 4.3 Routing multiple trucks

Co-optimized routes for several trucks are too difficult to compute online within the time constraints of the PBS system. Therefore, we resort to optimizing multiple truck routes sequentially where actions of prior trucks can be treated as known. These actions are manifest in the future fill levels stored in the graph of the time-expanded network.

The basic idea is to repeat adding route steps for each truck until it reaches a minimum number of  $N_{\text{truck}}$  steps, or a journey time of  $T_{\text{truck}}$ . This is performed sequentially, starting with the truck who has the minimum time-index for the last step in his route and has not yet reached the required route length. The reason for this sequential procedure is to prevent the collision of two truck routes, which can be detrimental to the performance of the algorithm as explained below. Assume, without loss of generality, that the trucks  $r \in R$  are ordered according to the current computation of their routes. If truck  $r$  chooses to go to a station  $s(v)$  to which a truck  $r' < r$  has already planned to go at a later time

$t(v') > t(v), s(v') = s(v)$ , then  $r'$  has made his choice based on false assumptions about the station's fill level. These collisions can be handled by

- Removing all routing steps that were added during the last route-search step from  $\Omega_{r'}$ .
- Removing all but the first routing steps that were added during the last route-search step from  $\Omega_r$ .

So collisions are prevented and since at least one new routing step is preserved per detected collision, so our algorithm will eventually reach  $T_{\text{truck}}$  for all trucks.

## 5 Dynamic price incentives for users

Customers themselves might contribute to the rebalancing of a PBS scheme if offered an appropriate payment. In this paper we consider how payments could be offered to customers to change the endpoint of their journey to a nearby station in a way that improves the overall service level. To this end, we take the model of how customers accept (or choose between) price offers, as described in Section 2.3, and then form an optimization problem trading off the expected payouts and the expected improvement in service level. The solution of this optimization problem is a set of price offers that are presented to any customer arriving at a given station. It seems reasonable to reduce the complexity of this optimization problem by limiting the number of prices offered (= decision variables) to 10 per station; i.e. for each station, only 10 prices are quoted for going to selected neighboring stations.

We assume that means of communicating the price incentives and for making payments are available. A payment infrastructure is already central to existing systems, like the *Oyster Card* for London's public transport network. New information capabilities could be added to the kiosk terminals used for rental, and/or the mobile applications many customers already use.

Using price incentives to induce a desired behavior in the users of shared mobility systems has been examined in multiple contexts in recent literature. The work of [14] examines user-based repositioning in a shared mobility system. However, the approach of splitting and merging rides can only be applied to cars and not to public bike hire schemes. Incentives for bike sharing schemes are investigated by [24], where users pick two stations at random and go to the more empty.

In this paper, we propose a novel scheme that is based on Model Predictive Control (MPC). A short summary of MPC is given in Section 5.1. In Section 5.2 we then show how the customer reaction model from Section 2 can be linearized in order to obtain a tractable MPC problem formulation. Finally, Section 5.3 explains the details of the corresponding optimization problem that has to be solved in a receding-horizon fashion in order to determine

the real-time price incentives.

## 5.1 Model Predictive Control

Here we provide a short introduction to Model Predictive Control (MPC [25]), giving the basic explanations required to describe the controller developed in Section 5.3. The fundamental idea is to employ a model of a given system in order to optimize the inputs given to the system over a finite control horizon. Only the first input is then applied to the system, and the scheme continues by measuring the new state of the system and solving another finite-horizon optimization problem (“MPC problem”).

The two main aspects of MPC comprise good control decisions for the system with respect to anticipated future events (by the optimization), and feedback in the case of unforeseen disturbances or model inaccuracies (by re-optimizing periodically for the control actions). An important strength of MPC is its ability to incorporate a model of the system dynamics and to handle constraints on the states and control inputs. For example, in the case of the PBS this is advantageous because it allows upper and lower bounds to be placed on the computed price incentives (control inputs) and the stations’ fill levels (system state).

Let the system state at time step  $t$  (i.e. a vector containing the fill levels of all stations) be denoted by  $x(t)$ . The future state evolves as some function of the current state and the control inputs  $u(t)$  (i.e. a vector of the price incentives offered to the users), so that the subsequent state is given by  $x(t + 1) = f_t(x(t), u(t))$ . Here  $f_t$  represents only a simplified model of the actual system dynamics, which is subject to model uncertainty and disturbances. Note that the function  $f_t$  depends on the predicted customer interaction and is therefore time-varying (recall that customer interaction shows time-varying patterns). The actual deviations from the predicted customer interaction is uncertain, and thus considered as a disturbance to our model. Moreover, given that the truck routes are known (from Section 4), the functions  $f_t$  also contain changes to the system state affected by manual repositioning.

Assume the current time to be  $t = 0$  without loss of generality. Now we wish to choose a *finite series* of inputs  $u(t)$  where  $t = 0, \dots, T - 1$  so that the system behaves optimally over the finite time horizon  $T$ , starting from the measured current state  $x(0)$ . This is done by solving an optimization problem, trading off the perceived cost of having suboptimal system states  $c_t^x(x(t))$ , and the cost of applying the control input  $c_t^u(u(t))$ :

$$\min_{u(0), \dots, u(T-1)} \sum_{t=1}^T c_t^x(x(t)) + \sum_{t=0}^{T-1} c_t^u(u(t)) \quad (17)$$

such that

$$x(t + 1) = f_t(x(t), u(t)), \quad t = 0, \dots, T - 1 \quad (18a)$$

$$u(t) \in \mathcal{U}_t, \quad t = 0, \dots, T-1 \quad (18b)$$

$$x(t) \in \mathcal{X}_t, \quad t = 1, \dots, T \quad (18c)$$

where the functions  $c_t^x$  and  $c_t^u$  are called *stage costs* for the state and input respectively, and sets  $\mathcal{X}_t$  and  $\mathcal{U}_t$  represent any constraints that may be present on the state and input.

Although solving problem (17) gives a series of inputs  $u(t), t = 0, \dots, T-1$ , only  $u(0)$  is applied to the system. In the next time step, a new measurement of the current state is made, and a new series of planned control inputs are determined by resolving the MPC problem in light of the new information. For this reason, MPC is also known as “Receding Horizon Optimal Control”.

To make problem (17) tractable, the system model must often be simplified. In particular, non-linear dynamics  $f_t(x(t), u(t))$  make the problem non-convex. For many systems, though, good control performance can still be achieved if the model is linearized.

## 5.2 Simplified model of customer behavior

We now derive an approximate model of customer behavior that can be used in the context of MPC. Customer behavior means their response to price incentives, as described in (7) and therefore enters into the system dynamics  $f_t(x(t), u(t))$ . However this response is nonlinear, which as described in the preceding section leads to a non-convex MPC problem (17).

We first explain the origin of this nonlinearity. Consider two neighbor stations  $n', n'' \in N_s$  with equal distance to  $s$ , for which the incentives offered from station  $s$  are equal,  $p_{s,n'} = p_{s,n''}$ . If there are customers equally willing to go to  $n'$  or  $n''$ , an infinitesimally small increase in  $p_{s,n'}$  would cause all those customers to choose  $n'$  if we assume they act totally rationally. The customer reaction to incentives is thus discontinuous in the prices, and the true behavior model (7) will not lead to a tractable optimization problem.

To formulate a tractable MPC problem we approximate  $\pi(s, n, p_s)$  in a linear fashion and choose a convenient set  $N_s$  of  $N$  nearby neighbors for each station, so that  $|N_s| = N$ . The linearization  $\bar{\pi}(s, n, p_s)$  is computed using Algorithm 2, which creates samples of customer reactions to random incentive offers (to all neighboring stations) and a performs a least-squares fit between observed behavior and the linear model. The linear dependency on offered incentives, where customers reject station  $s$  and go to neighbor  $n$  instead, is defined by vectors  $\tilde{\pi}_{s,n}$  of size  $N$ , for each  $s \in S, n \in N_s$ .

$$\bar{\pi}(s, n, p_s) = \tilde{\pi}_{s,n}^\top p_s \quad (19)$$

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**Algorithm 2** Fitting the linear customer behavior model

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**Require:**  $s \in S$ ,

- 1:  $N_s$ , ▷  $|N_s| = N$  nearest neighbours around station  $s$
  - 2:  $\text{TAKEN\_INCENTIVE}(s, N_s, p)$ , ▷ Neighbor chosen by the customer as described in Section 2.3
  - 3:  $p_{\max}$ , ▷ Maximum payout
  - 4:  $P \gg 0$ , ▷ Number of generated payout vectors (samples)
  - 5:  $C \gg 0$ , ▷ Number of customer behavior samples
  - 6:  $\Omega$  ▷ Set of samples. Each sample is a tuple of two vectors: The offered payouts  $p$  to the  $N$  neighbours and the percentage of customers taking a certain incentive  $\delta$ .
  - 7: **for**  $i = 1$  to  $P$  **do**
  - 8:    $p \leftarrow p_{\max} \cdot \text{RAND}(N) \in [0, p_{\max}]^N$  ▷ Payout vector
  - 9:    $e \leftarrow \{0\}^N$  ▷ Initialize behavior count
  - 10:   **for**  $c = 1$  to  $C$  **do**
  - 11:      $n' \leftarrow \text{TAKEN\_INCENTIVE}(s, N_s, p) \in \{\mathbb{N}^+, \emptyset\}$
  - 12:      $e_{n'} \leftarrow e_{n'} + 1$
  - 13:   **end for**
  - 14:    $\delta \leftarrow e/C$  ▷ Fraction taking a certain incentive
  - 15:    $\Omega(i) \leftarrow (p, \delta)$
  - 16: **end for**
  - 17: **for**  $n = 1$  to  $N$  **do**
  - 18:    $\tilde{\pi}_{s,n} \leftarrow \arg \min_{\pi} \sum_{i \in \{1, \dots, P\}} (\pi^\top \Omega(i)_p - \Omega(i)_{\delta,n})^2$
  - 19: **end for**
-

### 5.3 Computing dynamic price incentives

In this subsection we formulate an MPC problem, the solution of which gives the price incentives  $p_s(t)$  that should be offered to customers for  $t = 0, \dots, T-1$ , where  $p_s(0)$  are the prices to be issued immediately, and  $p_s(1), \dots, p_s(T-1)$  are prices planned for subsequent steps.

The number  $f_s(t)$  of bikes present at station  $s$  at time  $t$  evolves according to the original arrival rate  $\lambda_s(t)$  and net change  $\eta_s(t)$  described in Section 2, along with a modification  $\gamma(s, \lambda_s(t), p_s(t))$  due to customers taking price incentives and another,  $\Delta f_s(t)$  due to trucks adding or taking away bikes from the stations.  $\tilde{N}_s$  denotes the set of stations having  $s$  as one of their nearest neighbors.

$$\begin{aligned} \gamma(s, \lambda_s(t), p_s(t)) &= \sum_{\tilde{n} \in \tilde{N}_s} \pi(\tilde{n}, s, p_{\tilde{n}}(t)) \cdot \lambda_{\tilde{n}}(t) \\ &\quad - \sum_{n \in N_s} \pi(s, n, p_s(t)) \cdot \lambda_s(t) \end{aligned} \quad (20)$$

$$f_s(t+1) = f_s(t) + \eta_s(t) + \gamma(s, \lambda(t), p(t)) + \Delta f_s(t). \quad (21)$$

Note that  $\sum_{s \in S} \gamma(s, \lambda_s(t), p_s(t)) = 0$  since the total number of bikes in the system must be constant. Also, the controller assumes that customers who take an incentive go from their originally intended destination to the new one within the same time step. We do this to make the MPC problem easier to solve, and assume it does not distort predictions of customer actions too much.

We now specify the components of MPC problem (17). Using the linearized model of customer reactions to incentives from Section 5.2, and defining quadratic stage costs  $c_t^x$  and  $c_t^u$ , the MPC problem becomes a quadratic program. Under the assumption of a linear customer response to prices, the expected payouts are a quadratic function of the prices, and the input cost in the MPC problem represents a real monetary cost to the system operator. The state cost aims to penalize loss of customer service. The resulting MPC problem is a quadratic program (QP) and can be stated as follows:

$$\min_{p(t)} \sum_{t=1}^{T_{\text{price}}} \sum_s Q_s(t) \tilde{f}_s(t)^2 + \sum_{t=0}^{T_{\text{price}}-1} \sum_s R_s(t) p_s(t)^2 \quad (22)$$

such that

$$\begin{aligned} \tilde{f}_s(t) &= f_s(t) - \frac{1}{2} (\underline{f}_t^s + \overline{f}_t^s), \quad \forall s \in S, \forall t \\ f_s(t+1) &= f_s(t) + \eta_s(t) + \Delta f_s(t) \\ &\quad + \sum_{\tilde{n} \in \tilde{N}_s} (\tilde{\pi}_{\tilde{n},s}^\top p_{\tilde{n}}(t)) \lambda_{\tilde{n}}(t) \end{aligned} \quad (23a)$$

$$- \sum_{n \in N_s} (\tilde{\pi}_{s,n}^\top p_s(t)) \lambda_s(t), \quad \forall s \in S, \forall t \quad (23b)$$

$$\sum_{n \in N_s} \tilde{\pi}_{s,n}^\top p_s(t) \leq 1, \quad \forall s \in S, \forall t \quad (23c)$$

$$0 \leq p_{s,n}(t) \leq p_{\max}, \quad \forall s \in S, \forall t, \quad (23d)$$

$$n \in \{1, \dots, N\}$$

The cost weights  $Q_s(t)$  and  $R_s(t)$  in the cost function (22) are used to penalize deviation  $\tilde{f}(t)$  from the optimal state  $\frac{1}{2}(\underline{f}_t^s + \bar{f}_t^s)$ , and the cost caused by the incentives payout, respectively. A weighting factor  $\alpha$  is used to adjust the relative costs associated with having stations deviate from their optimal point of operation in the middle of the utility plateau (which we assume to lead to a lower service level), and cash payouts:

$$Q_s(t) = 1/(\bar{f}_t^s - \underline{f}_t^s), \quad (24)$$

$$R_s(t) = \alpha \sum_{n \in N_s} \tilde{\pi}_{s,n}^\top \lambda_s(t). \quad (25)$$

A lower value of  $\alpha$  leads to a lower relative penalty for paid incentives, likely leading to higher price incentives applied.

Equation (23a) transforms the number of bikes at each station to a quantity measured relative to the “best” fill level, the middle of the station’s utility plateau. The predicted system states within the horizon are defined by (23b). It includes the expected arrival and departure rates as well as the linearized model of customer behavior. Equation (23c) limits the payouts such that no more than 100% of arriving customers take an incentive to one of the neighbours, and (23d) ensures that payouts are at most  $p_{\max}$ .

## 6 Simulation

### 6.1 Simulation setting

Based on the assumptions and the system model developed in Section 2, a Monte-Carlo simulation is used to compare the two approaches for bike repositioning. It is important to note that although simplified models of the system are used to choose truck actions and prices, we use the *full* model derived from historical data as described in Section 2 to simulate the actual behavior of customers. We simulate first a sequence of weekdays, then a sequence of weekend days, bearing in mind that demand patterns differ significantly between the two. Every simulation run consists of a 24h burn-in period starting from the initial system configuration, in order to reduce the dependence of our results on this initial configuration. Then, three consecutive days are simulated, for which the statistics gathered

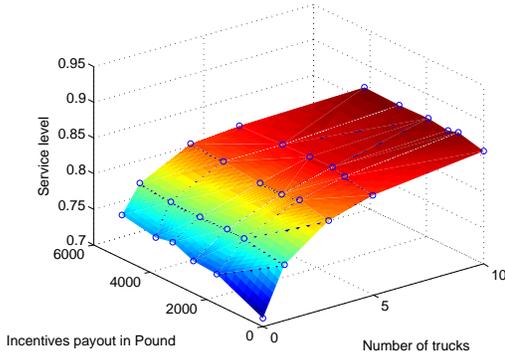


Figure 9a: Service level for weekdays, as a function of number of trucks and total payouts.

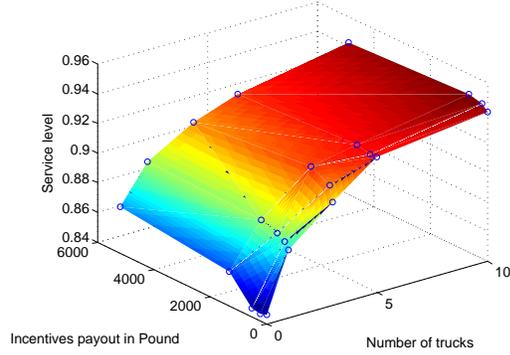


Figure 9b: Service level for weekend days, as a function of number of trucks and total payouts.

are presented below. In accordance to [20], redistribution with trucks is performed during 8am–10pm.

## 6.2 Simulation results

The resulting service level is computed as follows:

$$\text{Service level} = \frac{\text{Potential customers} - \text{No-service events}}{\text{Potential customers}} \quad (26)$$

When simulating three consecutive weekdays, about 49,800 potential customers are generated on average, and for three consecutive weekend days (e.g. a Bank Holiday weekend) about 29,900. The number of total no-service events is the sum of customers who could not rent a bike at an empty station and customers who wanted to return their bike at a full station.

We varied number of trucks used for repositioning and the level of price incentives given out (via the choice of state cost weight  $\alpha$  in the price controller). Figure 9 shows how the service level reported by the simulations varied as a result.

As expected, adding more trucks as well as paying out more in incentives has a positive effect on the service level. However, with increasing service level, adding trucks and incentives payouts becomes less efficient. Comparing the two simulations, it appears that the usage peaks caused by commuters were responsible for most of the service shortfalls observed. Most events where a customer could not be served were concentrated on only a few stations.

Figures 10 and 11 show the split of no-service events into “empty events” (where customers wanting to rent a bike arrive at an empty station) and “full events” (where customers

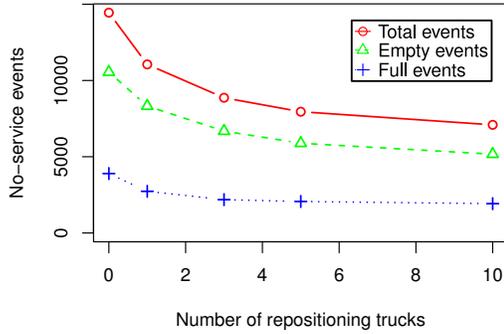


Figure 10a: No-service events for different numbers of repositioning trucks (no incentives) for three consecutive weekdays

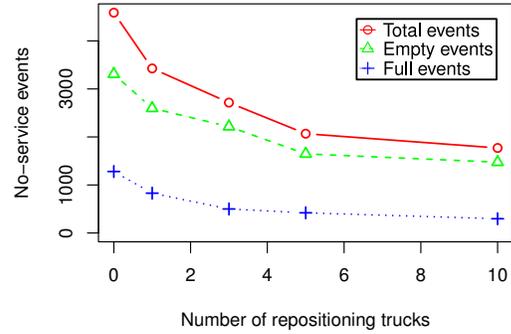


Figure 10b: No-service events for different numbers of repositioning trucks (no incentives) for three consecutive weekend days

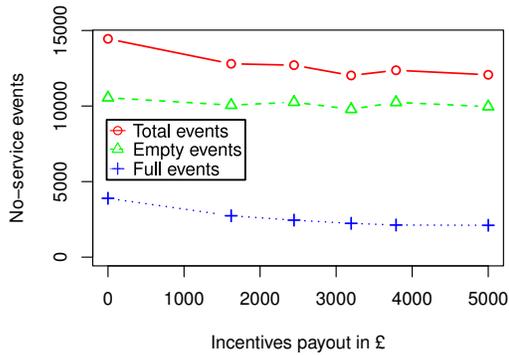


Figure 11a: No-service events during simulations for weekdays with 0 repositioning trucks. Over the course of 72h ca. 49,800 potential customers arrive.

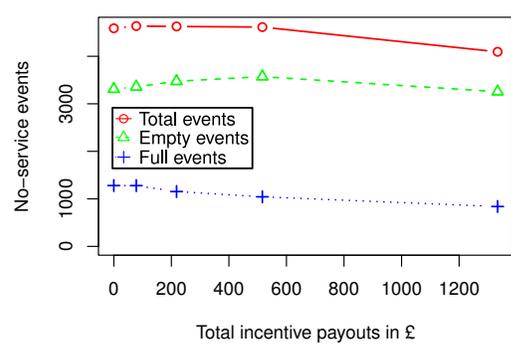


Figure 11b: No-service events during simulations of weekend days with 0 repositioning trucks. Over the course of 72h ca. 29,900 potential customers arrive.

wanting to return a bike arrive at a full station). Since the number of full events is considerably lower than the number of empty events, it seems plausible that adding more bikes could have a positive effect on the service rate.

## 7 Conclusions

This paper considered how a Public Bicycle Sharing scheme could be managed using a combination of intelligently routed repositioning trucks and redistribution incentives for customers. The truck routes and price incentives were computed using model-based receding horizon optimization principles, which took account of expected future customer behavior. As the number of trucks was increased, diminishing gains to service level were reported for added trucks and customer incentive payouts. Customer payments were shown to be a means of reducing service shortfalls, particularly when few repositioning trucks were in operation.

Our results suggest that price incentives are viable for repositioning bicycles in a PBS when the commuting rush hour is less prominent. For the London PBS, price incentives alone were shown to be enough to keep the service level above 87% on weekends without the use of staff. On weekdays however, when many customers use the PBS to commute to work, price incentives alone are not sufficient to lift the service level substantially.

The price control algorithm could be developed further in several ways. Firstly, a field trial could be used to improve the accuracy of the customer decision model upon which our controller is based. This would reveal the range of price elasticities customers exhibit, and also indicate to what extent customer responses to prices are irrational. Secondly, some simplifying assumptions (for example deterministic customer arrival, linearized customer reaction to incentives) could be replaced by more detailed models. However, it is unsure how much performance can be gained here, as the prediction horizon for optimization might have to be shortened considerably to account for the increase in computational complexity.

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