On Time Headway Selection in Platoons Under the MPF Topology in the Presence of Communication Delays

Elham Abolfazli, Bart Besselink, *Member, IEEE*, and Themistoklis Charalambous, *Senior Member, IEEE*

***Abstract*— For platoons under the multiple-predecessor following (MPF) topology, communication delays can compromise both the internal stability and string stability. The most straightforward solution to guarantee stability is by increasing the time headway. However, time headway plays a significant role in road capacity and increasing its value is in contrast with the idea of platooning. In this study, internal stability and string stability of platoons suffering from communication delays are investigated and a lower bound for the time headway is proposed. Using this bound, platoons do not need to massively increase the time headway in order to compensate for the effects of communications delays. Finally, we evaluate the proposed lower bound on the time headway and the simulation results demonstrate its effectiveness.**

***Index Terms*— Platooning, time headway, multiple-predecessor following topology, communication delays.**

I. Introduction

PLATOONING has been shown to increase road-network capacity, decrease fuel consumption and greenhouse gas emissions, and improve safety; see [2]–[5] and references therein. For maintaining a stable vehicle platoon, it is necessary to guarantee both *internal* stability and *string* stability. Internal stability refers to the individual stability defined for each vehicle and describes the ability to converge to given desired trajectories, [6]. Regarding string stability, although many definitions have been proposed in the literature, such as [7]–[9], all entail the unique fact that disturbances must not amplify along the platoon.

It is known that the spacing policy, which defines the distance between two consecutive vehicles, can influence the ability of a platoon to attenuate the effects of disturbances and reach string stability [10]. The main three classes for spacing policy are: constant spacing policy (CSP), nonlinear distance (NLD) policy and constant time headway spacing policy (CTHP). In CSP, the desired inter-vehicle distance is constant and independent of vehicle velocity [11]. In NLD, the desired distance between two consecutive vehicles can be a nonlinear function of states of the vehicles [12], [13]. For example, in [14], a quadratic spacing policy is proposed and in [15], [16] an improved quadratic spacing policy is introduced, which can work without the restrictive assumption of having zero initial spacing errors. For CTHP, which is the class under consideration in this work, a linear function of speed, with the proportional gain, named *Time headway* (h), dictates the desired inter-vehicle distance [17].

A smaller time headway can increase the road throughput, but it could be dangerous for the platoon, since it could lead to internal and/or string instability and, hence, collisions. On the other hand, an increase in the time headway value leads to a larger inter-vehicle distance, which guarantees safety, but as a consequence, the capacity of the roads is decreased and the fuel consumption is increased; see for example, [18]. Therefore, finding the minimum employable time headway, which can guarantee string stability, is of paramount importance.

The communication topology of early-stage platoons consisted mainly of vehicles connected to their nearest predecessor and information was only obtained from radars. Such a topology is known as predecessor following (PF). However, in case information is sought from multiple predecessors, it is difficult to achieve so using only onboard measurements. Recent technological advancement facilitates a vehicle to receive information and be connected with many vehicles in the platoon by using vehicle-to-infrastructure (V2I) and vehicle-to-vehicle (V2V) communications, such as dedicated short-range communication (DSRC) and vehicular ad-hoc networks (VANETs) [19]. Therefore, various topologies are emerging, including predecessor leader following (PLF), two-predecessor following (TPF), two-predecessor-leader following (TPLF) and multiple-predecessor following (MPF). The influence of different information flow topologies on the internal stability has been studied in [20]. Regarding string stability, in [21], a vehicle platoon with CSP and in the presence of time lags is considered, wherein each vehicle, with a linear model, is connected to ‘r’ vehicles ahead. It is shown that the platoon is not string stable. Hence, the

Manuscript received 20 June 2020; revised 4 December 2020 and 26 February 2021; accepted 17 April 2021. Date of publication 22 June 2021; date of current version 8 July 2022. This work was supported in part by the Academy of Finland under Grant 320043 and in part by the European Commission through the H2020 Project Finest Twins under Agreement 856602. The work of Themistoklis Charalambous was supported by the Academy of Finland under Grant 317726. This article was presented in part at the IFAC World Congress in 2020. The Associate Editor for this article was G. Guo. *(Corresponding author: Elham Abolfazli.)*

Elham Abolfazli and Themistoklis Charalambous are with the Department of Electrical Engineering and Automation, School of Electrical Engineering, Aalto University, 02150 Espoo, Finland (e-mail: elham.abolfazli@aalto.fi; themistoklis.charalambous@aalto.fi).

Bart Besselink is with the Bernoulli Institute for Mathematics, Computer Science and Artificial Intelligence, University of Groningen, 9712 CP Groningen, The Netherlands (e-mail: b.besselink@rug.nl).

Digital Object Identifier 10.1109/TITS.2021.3087484

focus turns towards platoons under the MPF topology and with CTHP.

The advantages of multiple connected vehicles with CTHP have been widely studied in the literature. Studies revealed that the cooperation of multiple connected vehicles decreases the oscillations and collisions and improves the string stability of the platoon. Also, it is shown that connecting multiple vehicles results in a higher throughput [22]. In [23], a platoon with CTHP, linear model and time lag is considered, in which each vehicle can obtain information from multiple predecessor vehicles and the desired inter-vehicle distance is defined to be a function of the host vehicle's velocity. Then, a lower bound for the time headway is proposed for three scenarios; when the position, velocity and acceleration information of 'r' immediate preceding vehicles is used, when only the position and velocity of the 'r' immediately preceding vehicles is used and also when the information from the immediate and the r^{th} predecessors are available. It has been shown that the minimum time headway is dependent on the number of connected vehicles and hence, it demonstrates that using V2V communications and increasing the number of connected vehicles, a higher road capacity can be achieved. In [24], a new definition of desired inter-vehicle distance in a platoon with CTHP and time lag under the MPF topology is proposed which avoids inconsistencies in the inter-vehicle distance by extending this definition from the PF topology. Under this definition, a lower bound that guarantees internal stability and string stability is provided, which implies that an increasing number of connected cars can lead to a smaller time headway. The proposed controller and minimum time headway are obtained using a linear third-order model, but it is evaluated by simulations on both linear and nonlinear vehicle models.

Despite the advantages of V2V and V2I communications, wireless communications inevitably introduce time delays in vehicle platoons and can be a big challenge for control design, since both the internal and string stability are compromised in the presence of communication delays. The effect of delays on platoons has been extensively studied in the literature. For example, for CSP, a platoon with PLF topology and linear vehicle model is considered in [25] and it is shown that the system is string unstable when each vehicle sees different delays either in receipt of the lead vehicle information or the preceding vehicle information. Then, a controller is proposed which makes the platoon string stable, in the case when all vehicles see the same delay in both lead vehicle information and in preceding vehicle information. Also, in [26], a platoon with CSP under PLF topology is considered and it is shown that a tight formation and string stability is achievable if the information of the lead vehicle is received instantaneously. But, in the case of multiple broadcasting, if the leader states transmit with a delay, the platoon will be string unstable. For the nonlinear vehicle model, where parameters like the road slope, wind speed, rolling and grade resistances are considered, a controller for a platoon under the PLF topology is proposed in [27], that can guarantee disturbance string stability while having sensor and communication delays. In [28], an H-infinity control method is developed for heterogeneous platoons with uncertainty in the vehicle dynamics and uniform communication delay. The proposed controller guarantees string stability, but it has been shown that the communication delays have negative effects on vehicular platoon performances. In [29], by using the Markovian jumping system theory, a sampled-data control method is proposed for platoons subject to communication delays, switching topologies and external disturbances. In [30], a distributed nonlinear delay-dependent control method is proposed for platoons in the presence of heterogeneous time-varying delays. Also, an upper bound for the time delay is provided. A vehicle platoon in the presence of multiple time-varying communication delays is considered in [31] and a distributed adaptive collaborative control method is proposed. In [32], the NLD policy is used for a scenario of having a mixture of connected cruise control (CCC) vehicles and non-CCC vehicles and the effects of information delays on the longitudinal dynamics of connected vehicles have been investigated. Also, it is demonstrated that on condition of having the communication delays below a threshold, increasing the number of communication links between vehicles can improve the stability of the system.

Dealing with time delays has been studied for the platoons with CTHP as well. In [33], a Cooperative Adaptive Cruise Control (CACC) system is considered and for the third-order vehicle model, a controller is proposed which uses the position and velocity information of the predecessor that is coming from radar and also delayed acceleration information of the predecessor coming from the wireless communication network. The effects of sampling, zero-order hold (ZOH) and constant time delay on string stability are investigated and the maximum allowable time delays for different sampling times and time headway are characterized. In [34], for one vehicle look-ahead CACC system with the third-order linear model, identical local controllers are proposed which guarantee strict $\mathcal{L}_2$ string stability, in the presence of different types of delays, actuation, communication and sensor delay. In [35] and [36], platooning control has been solved by treating it as the problem of achieving consensus in a network with time delays. A second-order linear model is considered in [35] and the controller is designed as a coupling protocol which uses information coming from onboard sensors as well as the communication network. The internal stability is proven in the presence of time-varying delays and string stability is analyzed for the PLF topology with constant delays. Also, in [36], a third-order linear model is considered for a vehicular network affected by time-varying delays and a control algorithm, based on data coming from onboard sensors and the wireless communication network, is proposed which guarantees asymptotic and exponential stability of the system. Moreover, stability is analyzed in the case of switching topologies, for example when during a time period, one of the vehicles loses its connection with the leader. In addition to internal stability, it is shown that in the case of fixed delays, the system will be string stable. Due to the nonrational representation of time delays, they are approximated with Maclaurin series expansions in [37] and with Pade approximation in [38] and then the string stability is analyzed for the rational transfer function. In [39], the minimum acceptable value for the time headway, in the presence of parasitic delays and lags in a platoon, wherein each vehicle has only access to the position of its predecessor, is proposed. Although, by selecting that small time headway, which meets the requirements for string stability, a higher road throughput and fuel efficiency will be achieved, the effects of connections between more than two vehicles on the time headway are not investigated.

None of the aforementioned works considered the case when there exist time delays in a platoon with the MPF topology and apparently, the optimal time headway in the presence of both time delays and V2V communications is not analyzed. That is, on the one hand, in [39], considering time delays, the minimum time headway is proposed, but with the scenario of one vehicle look-ahead and on the other, in [24], considering platoons with MPF topology, the minimum time headway is found, but time delays are neglected. This paper focuses on those platoons, which have more connections among their vehicles, and proposes a minimum acceptable time headway that guarantees string stability in the presence of time delays. This proposed lower bound for the time headway is consistent with the results in [24] when the communication delays become zero. Also, because of having a direct acceleration error feedback, the proposed lower bound in this paper is smaller than the results in [39] when vehicles are only connected to their predecessors. The contributions of this paper are the following:

- The internal stability of the heterogeneous platoon under the MPF topology, where each vehicle is allowed to be connected to a different number of predecessors, is analyzed by means of Nyquist stability theory. In order to have a complete internal stability analysis for all possible values of time delays, we apply the Nyquist criterion to both the delay-free and delayed communications cases. The conditions that are found for the delay-free case, are consistent with the internal stability conditions proposed in the literature. Then, for the delayed communications case, a sufficient condition that guarantees the asymptotically stability of the platoon is proposed.
- For overcoming the problem of string instability in a platoon, where every vehicle is connected to *multiple* preceding vehicles via communication links suffering time delays, a lower bound on time headway is proposed for the case in which the communication delays are homogeneous. Hence, the disturbances acting on the lead vehicle do not propagate along the platoon and the knowledge of this lower bound allows the platoon to utilize its full potential, implying that the road capacity is maximized within the possibilities given by the platoon control system. Also, the proposed lower bound shows that by increasing the number of predecessors, a smaller lower bound on the time headway is achieved, which is in accordance to the results in [24].

The remainder of this paper is structured as follows. Section II presents some notation and mathematical preliminaries. Also, it gives the vehicle model and then a control law for a platoon with homogeneous time delays is proposed and the objectives of the paper are formulated. Section III analyzes the internal stability and string stability of the system and proposes a lower bound for the time headway. Section IV shows the simulation results verifying the suggested time headway. Finally, in Section V, we draw conclusions and discuss future directions.

II. Preliminaries and Problem Statement

A. Notation and Mathematical Preliminaries

Notation: Vectors and matrices are denoted by lowercase and uppercase letters, respectively. Integer and natural numbers sets are denoted by $\mathbb{Z}$ and $\mathbb{N}$, respectively. $\mathbb{Z}_0 \triangleq \{0, 1, 2, \ldots\}$, $\mathbb{Z}_k^n \triangleq \{k, k+1, \ldots, n\}$, and $\mathbb{N}^n \triangleq \{1, 2, \ldots, n\}$. Real and nonnegative real numbers sets are denoted by $\mathbb{R}$ and $\mathbb{R}_+$, respectively. $m \times n$ real matrices are denoted by $\mathbb{R}^{m\times n}$. For any matrix $A \in \mathbb{R}^{m\times n}$, $(m, n) \in \mathbb{N}\times\mathbb{N}$, we denote its transpose by A^T and its entries by a_{ij}, $i \in \mathbb{N}^m$, $j \in \mathbb{N}^n$ (i.e., $A = [a_{ij}]$). The $n \times n$ identity matrix is represented by I_n.

We model the information flow between platooning vehicles using a directed graph. A graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$, is a set of nodes $\mathcal{V}$ connected by edges $\mathcal{E}$, where $\mathcal{V} = \{v_1, v_2, ..., v_N\}$ represents all the following vehicles and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ represents the connections between each pair of following vehicles. The Laplacian matrix associated with $\mathcal{G}$ is defined as $\mathcal{L} = [l_{ij}]$, $i, j \in \mathbb{N}^N$, with

$$l_{ij} = \begin{cases} -a_{ij}, & i \neq j, \\ \sum_{k=1}^{N} a_{ik}, & i = j, \end{cases} \tag{1}$$

where $a_{ij} = 1$ if $(v_j, v_i) \in \mathcal{E}$ and $a_{ij} = 0$, otherwise. The connection $a_{ij} = 1$ means vehicle i can receive information from vehicle j. Also, we assume a uni-directional communication structure, i.e., vehicles are able to receive information only from their predecessors, and hence $a_{ij} = 0$ if $j > i$. Moreover, the connections between the vehicles and the leader can be modeled by

$$\mathcal{P} = \mathrm{diag}\{p_{11}, p_{22}, \ldots, p_{NN}\}, \tag{2}$$

where $p_{ii} = 1$ when vehicle i is connected to the leader and $p_{ii} = 0$, otherwise. Then, a new information topology matrix can be defined as

$$\mathcal{L}_p := \mathcal{L} + \mathcal{P}. \tag{3}$$

It is easy to see that $\mathcal{L}_p$ is a lower triangular matrix.

The following lemma is useful for later developments in the paper.

Lemma 1 [40]: Suppose A, B, C and D are matrices of dimension $n \times n$, $n \times m$, $m \times n$ and $m \times m$, respectively. Then, if A is invertible, for the block matrix we have

$$\det \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \det(A)\det(D - CA^{-1}B). \tag{4}$$

B. Vehicle Model

Consider a platoon of N vehicles with the following longitudinal model, as in, e.g., [8]

$$\begin{cases} \dot{p}_i(t) = v_i(t), \\ \dot{v}_i(t) = a_i(t), \\ \tau_i \dot{a}_i(t) + a_i(t) = u_i(t), \end{cases} \tag{5}$$

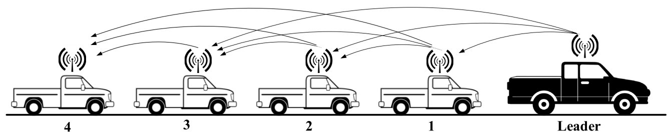


Fig. 1. A platoon under MPF topology.

where $p_i(t)$, $v_i(t)$, $a_i(t)$ and $u_i(t)$ are the position, velocity, acceleration and control input of the ith vehicle, respectively and $\tau_i > 0$ is the time lag in the powertrain.

It is assumed that vehicle i can use information from multiple predecessor vehicles, as shown in Fig. 1, where vehicle i, $i \in \mathbb{Z}_3^N$, is connected to three predecessor vehicles. For this topology, the desired distance between vehicle i and the l-th vehicle ahead of it is considered as [24]

$$d_{i,i-l}(t) = \sum_{k=i-l+1}^{i} (h_k v_k(t) + d_k), \tag{6}$$

where $h_k \geq 0$ is the time headway of vehicle k and $d_k > 0$ is the desired standstill gap between vehicle k and $k-1$.

C. Control Structure

The following linear feedback controller is used in [24] for vehicle i when there are no time-delays:

$$u_i(t) = -\sum_{l=1}^{r_i} \Bigg(k_{pi} \Big(p_i - p_{i-l} + \sum_{k=i-l+1}^{i} (h_k v_k + d_k) \Big) + k_{vi}(v_i - v_{i-l}) + k_{ai}(a_i - a_{i-l}) \Bigg). \tag{7}$$

Here, $r_i \leq i$ is the number of the vehicles directly ahead of vehicle i that send their information to it. In a heterogeneous platoon under the MPF topology, each vehicle is allowed to be connected to a different number of predecessors. The control parameters (k_{pi}, k_{vi}, k_{ai}) are tunable gains for feeding back distance, velocity and acceleration errors between vehicle i and the l-th vehicle ahead of i.

D. Problem Formulation

It is assumed that the controller of vehicle i has access to the difference between its own states and all the predecessors through wireless communication, which suffers from a homogeneous time-delay Δ. Then, based on the controller (7) proposed in [24], the following control law is proposed:

$$\begin{aligned} u_i'(t) = -\sum_{l=1}^{r_i} \Bigg(& k_{pi} \Big(p_i(t-\Delta) - p_{i-l}(t-\Delta) \\ & + \sum_{k=i-l+1}^{i} (h_k v_k(t-\Delta) + d_k) \Big) \\ & + k_{vi} \big(v_i(t-\Delta) - v_{i-l}(t-\Delta) \big) \\ & + k_{ai} \big(a_i(t-\Delta) - a_{i-l}(t-\Delta) \big) \Bigg), \end{aligned} \tag{8}$$

where $u_i'(t) = u_i(t-\Delta)$ is the actual control input acting on vehicle i. The main goal is to coordinate the motion of vehicles so that internal stability and the string stability are guaranteed. The former means the closed loop system is stable and accordingly, vehicles in the platoon can track the desired inter-vehicle distance and keep the desired velocity. The latter means that the disturbances do not propagate along the platoon. Mathematically, for internal stability we require the following conditions for $1 \leq l \leq r$,

$$\begin{cases} \lim_{t\to\infty} \Big(p_i(t) - p_{i-l}(t) + \sum_{k=i-l+1}^{i} (h_k v_k(t) + d_k) \Big) = 0, \\ \lim_{t\to\infty} \Big(v_i(t) - v_{i-l}(t) \Big) = 0, \\ \lim_{t\to\infty} \Big(a_i(t) - a_{i-l}(t) \Big) = 0. \end{cases}$$

For string stability under the MPF topology, we consider the following definition used in [24],

$$\|e_i(t)\|_2^2 \leq \frac{1}{r} \sum_{l=1}^{r} \|e_{i-l}(t)\|_2^2,$$

where

$$e_i(t) = p_i(t) - p_{i-1}(t) + h v_i(t) + d_i.$$

In the next section, we will analyze these two stability objectives.

III. Stability Analysis

In this section, we first find the closed loop dynamics, by following the work in [24]. Then we analyze the internal stability and the string stability.

We define the following errors

$$\begin{cases} \bar{p}_i(t) = p_i(t) - p_0(t) + \sum_{k=1}^{i} (h_k v_k(t) + d_k), \\ \bar{v}_i(t) = v_i(t) - v_0(t), \\ \bar{a}_i(t) = a_i(t) - a_0(t). \end{cases} \tag{9}$$

It is assumed that the lead vehicle moves at a constant speed, i.e., $u_0(t) = 0$ and $a_0(t) = 0$. This assumption on the lead vehicle has been proved to be necessary in many scenarios [41]. From (5), the dynamics of the error variables can be written as

$$\begin{cases} \dot{\bar{p}}_i(t) = \bar{v}_i(t) + \sum_{k=1}^{i} h_k \bar{a}_k(t), \\ \dot{\bar{v}}_i(t) = \bar{a}_i(t), \\ \dot{\bar{a}}_i(t) = -\dfrac{1}{\tau_i}\bar{a}_i(t) + \dfrac{1}{\tau_i} u_i(t). \end{cases} \tag{10}$$

Using (9) and after some algebraic manipulations, the control law (8) becomes

$$\begin{aligned} u_i'(t) = -\sum_{l=1}^{r_i} \Bigg(& k_{pi} \Big(\bar{p}_i(t-\Delta) - \bar{p}_{i-l}(t-\Delta) \Big) \\ & + k_{vi} \Big(\bar{v}_i(t-\Delta) - \bar{v}_{i-l}(t-\Delta) \Big) \\ & + k_{ai} \Big(\bar{a}_i(t-\Delta) - \bar{a}_{i-l}(t-\Delta) \Big) \Bigg). \\ = -k_{pi} \Bigg(& r_i \bar{p}_i(t-\Delta) - \sum_{l=1}^{r_i} \bar{p}_{i-l}(t-\Delta) \Bigg) \end{aligned}$$

$$- k_{vi}\left(r_i\bar{v}_i(t-\Delta) - \sum_{l=1}^{r_i}\bar{v}_{i-l}(t-\Delta)\right)$$
$$- k_{ai}\left(r_i\bar{a}_i(t-\Delta) - \sum_{l=1}^{r_i}\bar{a}_{i-l}(t-\Delta)\right). \quad (11)$$

By substituting (11) into (10) and defining augmented errors $\bar{p} = [\bar{p}_1, \bar{p}_2, \ldots, \bar{p}_N]^\top$, $\bar{v} = [\bar{v}_1, \bar{v}_2, \ldots, \bar{v}_N]^\top$, $\bar{a} = [\bar{a}_1, \bar{a}_2, \ldots, \bar{a}_N]^\top$, the dynamics model of the closed loop network can be recast as

$$\dot{\xi}(t) = A\xi(t) + A_\Delta \xi(t-\Delta),$$
$$\xi(t) = \Phi(t), \quad t \in [-\Delta, 0], \quad (12)$$

where $\xi = [\bar{p}^\top, \bar{v}^\top, \bar{a}^\top]^\top$ is the lumped state vector, $\Phi(\cdot) \in \mathcal{C}([-\Delta, 0], \mathbb{R}^\nu)$ represents the initial state of the system and A and $A_\Delta \in \mathbb{R}^{\nu\times\nu}$, $\nu = 3N$, are given as

$$A = \begin{bmatrix} 0 & I_N & H \\ 0 & 0 & I_N \\ 0 & 0 & -\mathrm{T} \end{bmatrix}, \quad (13)$$

$$A_\Delta = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -\mathrm{T}K_p\mathcal{L}_p & -\mathrm{T}K_v\mathcal{L}_p & -\mathrm{T}K_a\mathcal{L}_p \end{bmatrix}. \quad (14)$$

In (13) and (14),

$$K_m = \mathrm{diag}\{k_{m1}, \ldots k_{mN}\}, \quad m \in \{p, v, a\}, \quad (15)$$

collects the control parameters, whereas T captures the vehicle time constants as

$$\mathrm{T} = \mathrm{diag}\{1/\tau_1, \ldots 1/\tau_N\}. \quad (16)$$

Finally, the desired headway parameters are represented in H as

$$H = \begin{bmatrix} h_1 & 0 & \ldots & 0 \\ h_1 & h_2 & \ldots & 0 \\ \vdots & \vdots & \ddots & 0 \\ h_1 & h_2 & \ldots & h_N \end{bmatrix}. \quad (17)$$

A. Internal Stability Analysis

In this section, we will find a sufficient condition under which the vehicle platoon (12) is asymptotically stable.

Theorem 1: By selecting the control gains (k_{pi}, k_{vi}, k_{ai}) *such that the following conditions hold*

$$k_{pi} > 0, \quad (18a)$$
$$k_{ai} > 0, \quad (18b)$$
$$k_{ai} - \tau_i(k_{vi} + k_{pi}h_i) + \tau_i{}^2 k_{pi} \neq 0, \quad (18c)$$
$$k_{vi} + k_{pi}h_i \geq k_{pi}\tau, \quad (18d)$$

the closed loop system (12) *is asymptotically stable for any time delay* Δ *that satisfies the following inequality*

$$\Delta r_i(k_{vi} + k_{pi}h_i) < 1, \quad i \in \mathbb{N}. \quad (19)$$

Outline of the Proof: Due to having time delays, we analyze the internal stability using the Nyquist stability criterion. As a first step, because of the structure of the MPF topology, the characteristic equation (12) can be reduced to the level of individual vehicles and then we apply the Nqyuist criterion to the open loop transfer function. The proof is provided for two cases, perfect communications and delayed communications. See Appendix A for details.

Note that the condition is sufficient and not necessary. As a result, there might be delays for which (19) is not satisfied, but the system is internally stable.

In the case of $\Delta = 0$, condition (19) obviously holds. Also, the region defined by (18a), (18b) and (18d) is consistent with the internal stability conditions proposed in [24], in which $\Delta = 0$.

Remark 1: While the conditions seem to be counter-intuitive, the following statements can be inferred:

- *When a time delay* Δ *is given, it follows from* (19) *that stability cannot be guaranteed for large* h_i. *A possible reason is that the error in the speed difference is amplified.*
- *The higher the number of hops* r *that the information is transferred, the smaller the delay* Δ *that the network can tolerate.*
- *By substituting* (18d) *in* (19), *we obtain* $k_{pi}\tau\Delta r_i < 1$. *Therefore, in the case of increasing* r, *platoon stability can only be guaranteed for smaller values of* $\tau\Delta$.

B. Homogeneous String Stability Analysis

For string stability analysis, we assume that the platoon is homogeneous, which means that $\tau_i = \tau > 0$, $r_i = r$, $h_i = h$, $k_{pi} = k_p$, $k_{vi} = k_v$ and $k_{ai} = k_a$, $\forall i \in \mathbb{Z}_0^n$. Then from (5) and (8) we have

$$\tau\dddot{p}_i(t) + \ddot{p}_i(t)$$
$$= -\sum_{l=1}^{r}\Bigg(k_p\Big(p_i(t-\Delta) - p_{i-l}(t-\Delta)$$
$$+ \sum_{k=i-l+1}^{i}(hv_k(t-\Delta) + d_k)\Big) + k_v\Big(v_i(t-\Delta)$$
$$- v_{i-l}(t-\Delta)\Big) + k_a\Big(a_i(t-\Delta) - a_{i-l}(t-\Delta)\Big)\Bigg), \quad (20)$$

and

$$\tau\dddot{p}_{i-1}(t) + \ddot{p}_{i-1}(t)$$
$$= -\sum_{l=1}^{r}\Bigg(k_p\Big(p_{i-1}(t-\Delta)$$
$$- p_{i-1-l}(t-\Delta) + \sum_{k=i-l}^{i-1}(hv_k(t-\Delta) + d_k)\Big)$$
$$+ k_v\Big(v_{i-1}(t-\Delta) - v_{i-1-l}(t-\Delta)\Big)$$
$$+ k_a\Big(a_{i-1}(t-\Delta) - a_{i-1-l}(t-\Delta)\Big)\Bigg). \quad (21)$$

The time derivative of (20) is

$$\tau\dddot{v}_i(t) + \ddot{v}_i(t)$$
$$= -\sum_{l=1}^{r}\Bigg(k_p\Big(v_i(t-\Delta) - v_{i-l}(t-\Delta)$$

$$+\sum_{k=i-l+1}^{i} ha_k(t-\Delta)\Big) + k_v\Big(a_i(t-\Delta) - a_{i-l}(t-\Delta)\Big)$$
$$+k_a\Big(\dot{a}_i(t-\Delta) - \dot{a}_{i-l}(t-\Delta)\Big)\Big). \tag{22}$$

Then, by using the definition of spacing error, i.e., $e_i(t) = p_i(t) - p_{i-1}(t) + hv_i(t) + d_i$, calculating (20) − (21) + h × (22) and doing some algebraic manipulations, we obtain

$$\tau\dddot{e}_i(t) + \ddot{e}_i(t) + rk_a\ddot{e}_i(t-\Delta) + r\left(k_v + k_p h\right)\dot{e}_i(t-\Delta)$$
$$+rk_p e_i(t-\Delta) = \sum_{l=1}^{r}\Big(k_a\ddot{e}_{i-l}(t-\Delta)$$
$$+\left(k_v - k_p h(r-l)\right)\dot{e}_{i-l}(t-\Delta) + k_p e_{i-l}(t-\Delta)\Big). \tag{23}$$

Then, by taking the Laplace transform of (23), and after algebraic manipulations, we obtain

$$E_i(s) = \sum_{l=1}^{r} H_l(s)E_{i-l}(s), \tag{24}$$

where $E_i(s)$ is the Laplace transformation of $e_i(t)$ and

$$H_l(s) = \frac{k_a s^2 e^{-\Delta s} + \left(k_v - k_p h(r-l)\right) s e^{-\Delta s} + k_p e^{-\Delta s}}{\tau s^3 + s^2 + rk_a s^2 e^{-\Delta s} + r\left(k_v + k_p h\right) s e^{-\Delta s} + rk_p e^{-\Delta s}}. \tag{25}$$

It can be shown that for homogeneous platoons, $e_i/e_{i-l} = v_i/v_{i-l}$. In a platoon under the MPF topology, the spacing error of the vehicles are affected by their multiple predecessors. Therefore, in order to have string stability, in addition to $H_1(s)$, all the string stability functions $H_l(s)$ for $l \le r$ must be examined. Since (25) is identical for all vehicles, string stability of the platoon can be guaranteed if, [24]

$$\|H_l(j\omega)\|_\infty \le \frac{1}{r}, \quad \forall 1 \le l \le r, \tag{26}$$

where $H_l(j\omega)$ can be derived from (25) by substituting $s = j\omega$.

Now we are ready to present the second theorem on the string stability conditions.

Theorem 2: Consider system (5) *with input* (8) *that is internally stable. Then, the string stability specification* (26) *holds if all the following conditions are satisfied*:

$$k_v + k_p(h-\tau) \ge 0, \tag{27a}$$

$$2\tau\Delta - \Delta h - \tau h \le 0, \tag{27b}$$

$$k_a - \tau(k_v + k_p h) \le 0, \tag{27c}$$

$$\tau - 2rk_a\Delta \ge 0, \tag{27d}$$

$$1 + 2r\Big(k_a - \tau(k_v + k_p h)\Big) + 2r\Delta\Big(k_p(\tau - h) - k_v\Big) \ge 0, \tag{27e}$$

$$r^2k_p^2h^2(1-(r-l)^2) + 2r^2k_pk_vh(1+r-l)$$
$$-2rk_p \ge 0, \quad 1 \le l \le r \tag{27f}$$

$$k_a > 0, \tag{27g}$$

$$k_p > 0. \tag{27h}$$

For the region defined by (27)*, there exists a set of feedback gains k_p, k_v and k_a, such that string stability specification* (26) *holds if*

$$h \ge h_{\min} = \frac{2(\tau+\Delta)}{2rk_a + 1}. \tag{28}$$

Outline of the Proof: First, we will find the conditions in (27), which can guarantee the string stability. Then based on the region defined in (27), a lower bound for the time headway is proposed. See Appendix B for details.

Note that conditions (27a) and (27c)-(27g) are sufficient but not necessary for string stability, while conditions (27b) and (27h) do not affect string stability and only help to derive the lower bound for the time headway in (28).

Contrary to the controller used in [39], we have a direct acceleration error feedback. As a result, if $k_a \neq 0$, the lower bound presented by Theorem 2 is smaller than the lower bound in [39], even in the same scenario where $r = 1$.

Remark 2: Theorem 2 proves the conjecture made in [24]. As it is shown, the larger the delay Δ, the larger the minimum time headway h. This comes into contrast with the internal stability condition obtained in Theorem 1. Nevertheless, one can tune the triplet of control gains (k_p, k_v, k_a), such that both conditions can be satisfied.

Remark 3: Considering a set of parameters $\{k_a = 0.4, r = 3, \tau = 0.5s, \Delta = 0.2\ s, h = 0.45\ s\}$ that satisfies (27b) *and* (27d)*, the feasible region for (k_p, k_v), that satisfies conditions in* (27)*, is shown shaded in Fig. 2a. Also, in case of perfect communication, i.e., $\Delta = 0$, the feasible region for (k_p, k_v) that is proposed in [24] is shown in Fig. 2b. It can be seen that, to guarantee string stability, the acceptable region for the control parameters in the presence of time delays has become smaller. Indeed, in the case of $\Delta = 0$, condition* (27e) *is equivalent to condition [24]-(34c) and also, condition* (27a) *and* (27c) *are not needed; see Appendix B. Therefore, the proposed region defined in* (27) *is consistent with the necessary and sufficient region defined in [24] when the communication delays become zero.*

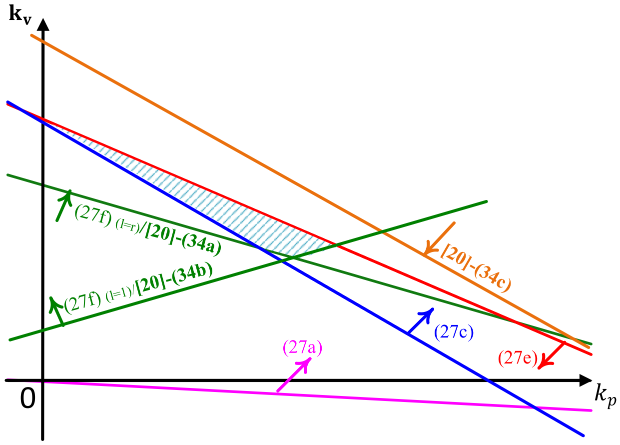
The relationship among r, Δ and $h_{\min}$ in Theorem 2 is shown in Fig. 3. In this figure, the values of τ and k_a are considered the same as in Remark 6 and the Numerical Results section.

IV. Numerical Results

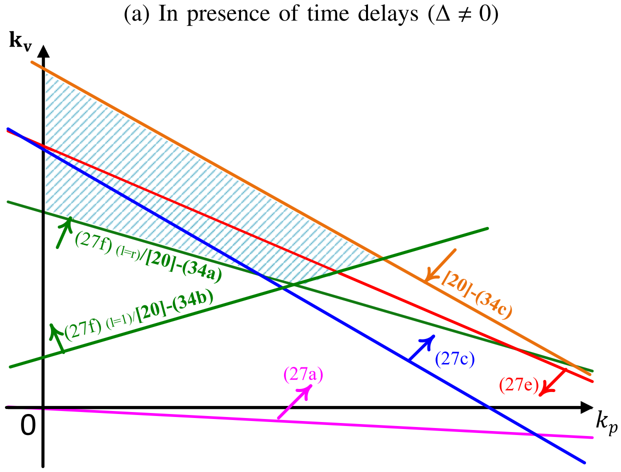
In this section, numerical simulations are presented to illustrate the results in our theorems, first for a small-scale platoon with 5 vehicles and next for a larger-scale platoon with 20 vehicles.

A. Small-Scale Platoons

The model used for each car is the nominal linear model (5), and the controller used in (8). We considered two values for the number of predecessors: $r = 1$ and $r = 3$. Vehicle i starts at the point $-id$ and moves to reach the desired distance as



(a) In presence of time delays ($\Delta \neq 0$)



(b) Perfect communication ($\Delta = 0$), obtained in [24]

Fig. 2. The feasible region of (k_p, k_v) that guarantees string stability.

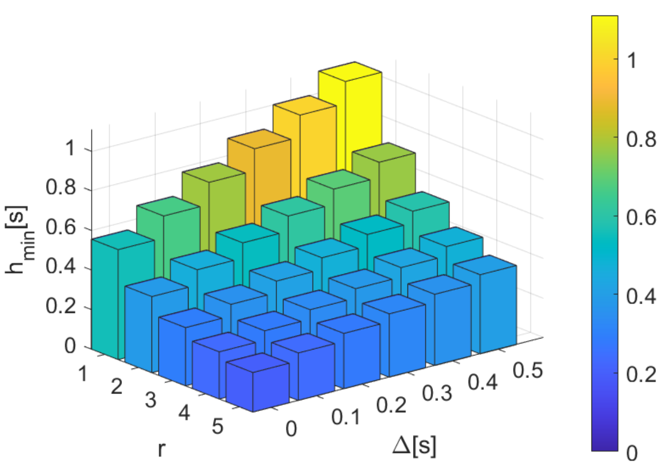


Fig. 3. Minimum acceptable time headway ($h_{\min}$) for different time delays (Δ) and different number of inter-vehicle connections (r).

well as the desired velocity, which is 20 m/s, same as the leader's velocity.

1) External Disturbances During the Steady State: In the first scenario, after 60 seconds, when the platoon has reached the steady state, a sinusoidal perturbation $u_0(t) = A_0 \sin(\omega_0\, t)$ acts on the leader for the duration of one cycle ($\frac{2\pi}{\omega_0}s$). The numerical values for system parameters are given in Table I.

TABLE I

MODEL PARAMETERS

N	d	τ	Δ	v_0	A_0	ω_0
5	5 m	0.5 s	0.2 s	20 m/s	10 m/s^2	1 rad/s

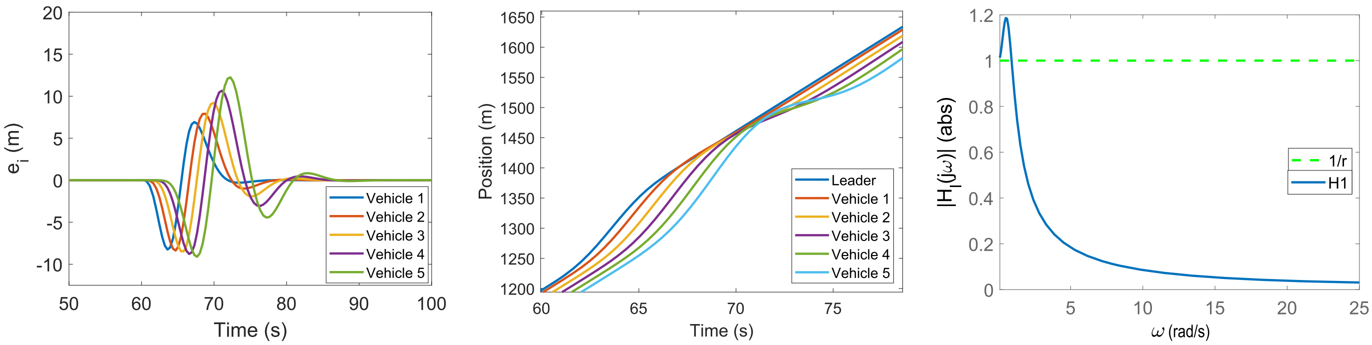
After choosing the acceleration control coefficient as $k_a = 0.4$, in case of $r = 1$, the minimum value for the time headway will be $h_{\min} = 0.78s$ and in case of $r = 3$, we will obtain $h_{\min} = 0.41s$. Other control parameters, (k_p, k_v) are selected in a way that conditions (27) are satisfied. Then, to verify the proposed criterion for the time headway, we simulate the platoon in two cases, i.e., when $h < h_{\min}$ and also when $h > h_{\min}$.

In Fig. 4, where each vehicle is connected only to its predecessor and $h_{\min} = 0.78\ s$, by choosing $h < h_{\min}$, it can be seen that the perturbations in the spacing error amplify as in Fig. 4(a) and the string stability transfer function surpasses $1/r$ in Fig. 4(c) and hence, the system is not string stable. Also, the disturbance has caused a collision in the platoon. But, as can be observed in Fig. 5, by choosing $h > h_{\min}$, the platoon remains string stable.

Next, we simulate a vehicle platoon with the MPF topology and $r = 3$. Fig. 6 demonstrates the system response after adding the disturbance in the case of $h < h_{\min}$ for this type of platoon. Using Theorem 2, the minimum time headway is $h_{\min} = 0.41s$. In Fig. 6(a), after the disturbance, large overshoots for the spacing error between vehicles can be seen, which is a sign of string instability. Fig. 6(b) is a more accurate representation of the platoon, that shows the position of each vehicle. We observe that after the disturbance, there would be a collision between the leader and vehicle 1 and also, the space between other vehicles decreases in an unsafe way. Also, the magnitude-frequency diagram of $H_l(j\omega)$ is shown in Fig. 6(c), where it is shown that the magnitudes surpass $1/r$ and thus, (26) does not hold. According to the simulation results, by considering $h < h_{\min}$, the platoon will be string unstable.

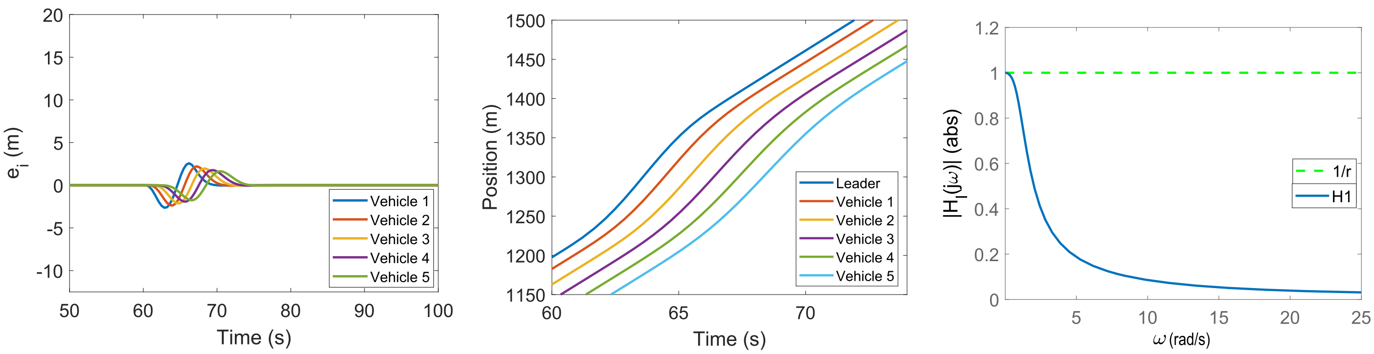
Fig. 7 depicts the platoon response for $h > h_{\min}$. It can be seen from Fig. 7(a) that the height of the peaks in the spacing errors after adding the disturbance is lower than the previous case. Besides, Fig. 7(b) shows that although the space between the leader and vehicle 1 decreases, there are no collisions between them. Finally, the magnitude-frequency diagram of $H_l(j\omega)$ when $h > h_{\min}$ is shown in Fig. 7(c). As can be seen in the figure, $|H_l(j\omega)|$ does not surpass the maximum acceptable value for string stability, i.e., $1/r$, which means (26) holds, and hence the platoon is string stable.

2) External Disturbances During the Transient State: In the second scenario, vehicles start from rest but at time $t = 10\ s$, before reaching the desired velocity, the same kind of disturbance that we had in the first scenario, acts on the leader. Fig. 8 shows the system response for this scenario, while having $r = 3$ and $h > h_{\min}$. Having both the external disturbances and the initial movements happen at the same



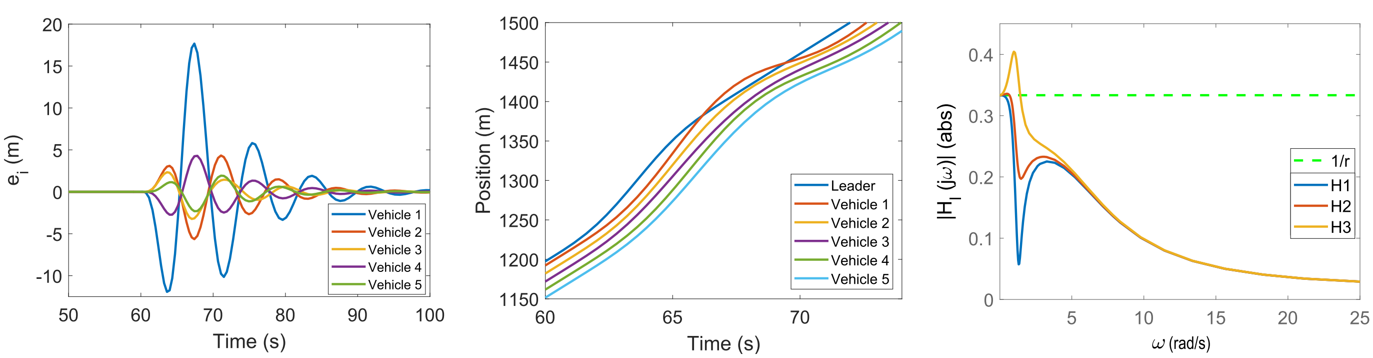
(a) Spacing Error (b) Position of vehicles (c) String stability transfer functions

Fig. 4. Platoon response, in the presence of sinusoidal disturbance acting on the leader at $t = 60\ s$, with $h < h_{\min}$ and $r = 1$.



(a) Spacing Error (b) Position of vehicles (c) String stability transfer functions

Fig. 5. Platoon response, in the presence of sinusoidal disturbance acting on the leader at $t = 60\ s$, with $h > h_{\min}$ and $r = 1$.



(a) Spacing Error (b) Position of vehicles (c) String stability transfer functions

Fig. 6. Platoon response, in the presence of sinusoidal disturbance acting on the leader at $t = 60\ s$, with $h < h_{\min}$ and $r = 3$.

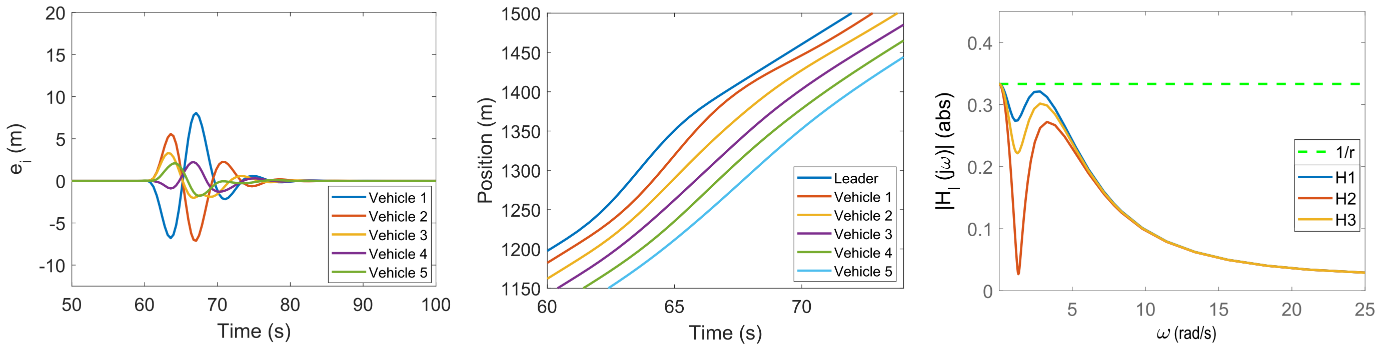
time is like having a superimposed disturbance with a higher magnitude. String stability, as the proof also manifests, does not depend on the magnitude of the disturbance and it can be seen in the figure that the platoon is string stable.

B. Large-Scale Platoons

A platoon with 20 vehicles is considered, where each vehicle can be connected to 10 vehicles ahead, i.e., $r = 10$. In this case, by considering the same k_a and by using Theorem 2, $h_{\min} = 0.156s$, which indicates the importance of multiple connection. Then after selecting $h = 0.16s$, we consider a similar scenario as in [24], which there in no disturbance but the speed profile of the leader is changing as

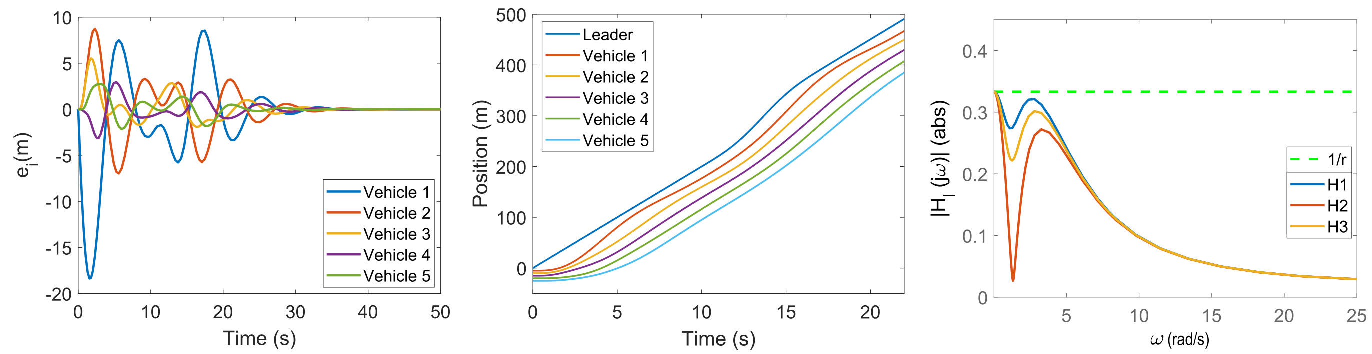
$$v_0(m/s) = \begin{cases} 20, & 0 \le t(s) < 60 \\ 20 - (t - 10), & 60 \le t(s) < 70 \\ 10, & 70 \le t(s) < 80 \\ 10 + (t - 40), & 80 \le t(s) < 90 \\ 20, & t(s) \le 90 \end{cases} \tag{29}$$

Fig. 9 shows that although the vehicles are grouped closely together, changes in the speed of the leader cannot cause a collision and the platoon still remains string stable.



(a) Spacing Error (b) Position of vehicles (c) String stability transfer functions

Fig. 7. Platoon response, in the presence of sinusoidal disturbance acting on the leader at $t = 60\ s$, with $h > h_{\min}$ and $r = 3$.



(a) Spacing Error (b) Position of vehicles (c) String stability transfer functions

Fig. 8. Platoon response, starting from rest, in the presence of sinusoidal disturbance acting on the leader at $t = 10\ s$, with $h > h_{\min}$ and $r = 3$.

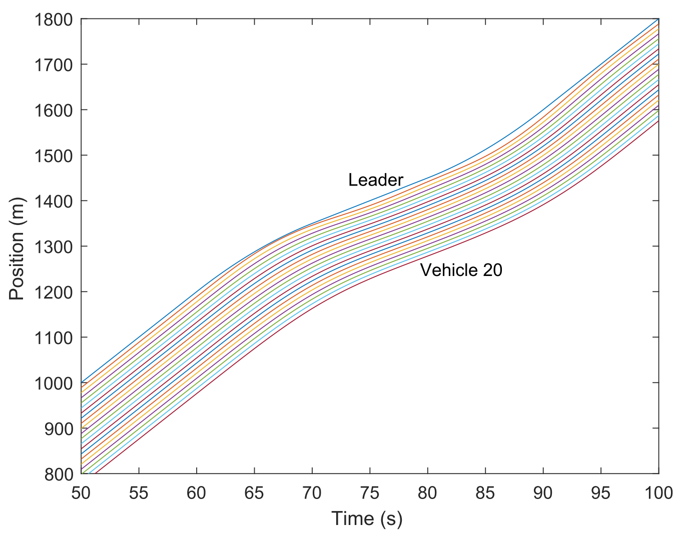


Fig. 9. Large scale platoon response, with $h > h_{\min}$ and $r = 10$.

V. Conclusion and Future Directions

In this paper, we have investigated the internal stability and string stability conditions for platoons under the MPF topology, which are affected by homogeneous constant communication delays. For this setup, first, by means of Nyquist stability theory, we provide a sufficient condition that guarantees internal stability. Next, based on string stability, we have formulated a lower bound for the time headway. The proposed lower bound demonstrates that by increasing the number of predecessor vehicles, a smaller lower bound on the time headway is achieved, which is in accordance to the results in [24]. The simulation results, which considered small- and large-scale platoons, illustrate the validity of our theoretical results and corroborate the importance of this lower bound.

This study reveals several open problems for the considered setup. As part of our future directions, we consider the following:

- Foremost, we plan to find conditions that guarantee string stability in platoons with heterogeneous constant and time-varying delays. This scenario serves as a general case in which different types of sensors are combined. For example, if radars are used for the predecessor vehicle and communication for the remaining $(r-1)$ vehicles then the lower bound on the time headway can be possibly reduced.
- As part of future work, we will investigate the effect of packet drops on the (internal and string) stability of platoons.
- Additionally, part of future work includes the analysis of platoons under the MPF topology with bidirectional connectivity. We plan to investigate the effect of receiving information from vehicles in the back on time headway.

APPENDIX

A. Proof of Theorem 1

By taking the Laplace transform of (12), we have

$$\Xi(s) = (sI_{3N} - A - A_\Delta e^{-\Delta s})^{-1}\xi(0). \tag{30}$$

By finding $\det(sI_{3N} - A - A_\Delta e^{-\Delta s})$, the characteristic equation of system (12) will be obtained as (31), shown at the bottom of the page. Then, by using Lemma 1, (31) can be written in the form of (32), as shown at the bottom of the page. For the derivation of the second determinant in the last equation of (32), Lemma 1 can be used again and hence we obtain (33), as shown at the bottom of the page. For the derivation of the final line in (33), we have considered the facts that matrices T, K_p, K_v and K_a are diagonal and matrices $\mathcal{L}_p$ and H are lower triangular. After decoupling (33) to N subsystems, we define

$$\begin{aligned} p_i'(s) = s^3 + \frac{1}{\tau_i}s^2 + s^2\frac{1}{\tau_i}(k_{ai}r_i e^{-\Delta s}) \\ + s\frac{1}{\tau_i}r_i\left(k_{vi} + k_{pi}h_i\right)e^{-\Delta s} + \frac{1}{\tau_i}r_i k_{pi}e^{-\Delta s}. \end{aligned} \tag{34}$$

As the roots of (30) are given by those of (34) (for $i \in \mathbb{N}^N$), we will proceed by stability analysis on the basis of (34).

In [24], the internal stability when the time delay is zero is analyzed using Routh–Hurwitz stability criterion. Although, this criterion has many advantages, it cannot analyze the systems with time delays. In this paper, we will use the Nyquist stability criterion for determining the stability of our system. This criterion can be applied to systems defined by non-rational functions, such as systems with delays. Also, it is less computational and more geometric and can work directly with experimental frequency response, even without having the transfer function.

The Nyquist stability criterion can be expressed as $Z = N + P$, where Z is the number of roots of the characteristic equation in right-hand side (RHS) of s-plane, N is the number of times that the Nyquist plot encircles $-1 + j0$ clockwise and P is the number of poles of the open loop transfer function in RHS of s-plane [42].

In order to have a complete and unified internal stability analysis for all possible values of time delays, we apply the Nyquist criterion to both the delay-free and delayed communications cases.

1) Internal Stability in Case of Perfect Communication: Neglecting the time delay, i.e., $\Delta = 0$, (34) can be written as

$$p_i'(s) = s^3 + \frac{1 + k_{ai}r_i}{\tau_i}s^2 + \frac{r_i\left(k_{vi} + k_{pi}h_i\right)}{\tau_i}s + \frac{r_i k_{pi}}{\tau_i} = 0. \tag{35}$$

Assuming $k_{pi} \neq 0$, it is easy to see that $s = 0$ cannot be a solution to (35). Thus, (35) can be written as

$$1 + \frac{1 + k_{ai}r_i}{\tau_i}\frac{1}{s} + \frac{r_i\left(k_{vi} + k_{pi}h_i\right)}{\tau_i}\frac{1}{s^2} + \frac{r_i k_{pi}}{\tau_i}\frac{1}{s^3} = 0, \tag{36}$$

The characteristic equation (36) can be rewritten as

$$1 + L(s) = 0, \tag{37}$$

where

$$L(s) = \frac{(a+b)s^2 + cs + d}{s^3}, \tag{38}$$

with

$$a = \frac{1}{\tau_i}, \quad b = \frac{k_{ai}r_i}{\tau_i}, \quad c = \frac{r_i\left(k_{vi} + k_{pi}h_i\right)}{\tau_i}, \quad d = \frac{r_i k_{pi}}{\tau_i}. \tag{39}$$

Then, instead of checking when the roots of (35) lie in the left half plane, to guarantee stability of system (12) with $\Delta = 0$, we apply the Nyquist criterion to $L(s)$, as the open loop transfer function. By replacing s by $j\omega$, we have

$$L(j\omega) = \frac{d - (a+b)\omega^2 + jc\omega}{-j\omega^3}. \tag{40}$$

$$\det(sI_{3N} - A - A_\Delta e^{-\Delta s}) = \begin{vmatrix} sI_N & -I_N & -H \\ 0 & sI_N & -I_N \\ \mathrm{T}K_p\mathcal{L}_p e^{-\Delta s} & \mathrm{T}K_v\mathcal{L}_p e^{-\Delta s} & sI_N + \mathrm{T} + \mathrm{T}K_a\mathcal{L}_p e^{-\Delta s} \end{vmatrix} \tag{31}$$

$$\begin{aligned} \det(sI_{3N} - A - A_\Delta e^{-\Delta s}) &= \det(sI_N)\det\left(\begin{bmatrix} sI_N & -I_N \\ \mathrm{T}K_v\mathcal{L}_p e^{-\Delta s} & sI_N + \mathrm{T} + \mathrm{T}K_a\mathcal{L}_p e^{-\Delta s} \end{bmatrix} - \begin{bmatrix} 0 \\ \mathrm{T}K_p\mathcal{L}_p e^{-\Delta s} \end{bmatrix}(sI_N)^{-1}\begin{bmatrix} -I_N & -H \end{bmatrix}\right) \\ &= \det(sI_N)\det\left(\begin{bmatrix} sI_N & -I_N \\ \mathrm{T}K_v\mathcal{L}_p e^{-\Delta s} + \frac{1}{s}\mathrm{T}K_p\mathcal{L}_p e^{-\Delta s} & sI_N + \mathrm{T} + \mathrm{T}K_a\mathcal{L}_p e^{-\Delta s} + \frac{1}{s}\mathrm{T}K_p\mathcal{L}_p H e^{-\Delta s} \end{bmatrix}\right) \end{aligned} \tag{32}$$

$$\begin{aligned} \det(sI_{3N} - A - A_\Delta e^{-\Delta s}) &= \det(sI_N)\det(sI_N)\det\Big(sI_N + \mathrm{T} + \mathrm{T}K_a\mathcal{L}_p e^{-\Delta s} + \frac{1}{s}\mathrm{T}K_p\mathcal{L}_p H e^{-\Delta s} \\ &\qquad + (\mathrm{T}K_v\mathcal{L}_p e^{-\Delta s} + \frac{1}{s}\mathrm{T}K_p\mathcal{L}_p e^{-\Delta s})(sI_N)^{-1}I_N\Big) \\ &= \det(s^3 I_N + s^2(\mathrm{T} + \mathrm{T}K_a\mathcal{L}_p e^{-\Delta s}) + s(\mathrm{T}K_v\mathcal{L}_p + \mathrm{T}K_p\mathcal{L}_p H)e^{-\Delta s} + \mathrm{T}K_p\mathcal{L}_p e^{-\Delta s}) \\ &= \prod_{i=1}^{N}\left(s^3 + s^2\frac{1}{\tau_i}\left(1 + k_{ai}r_i e^{-\Delta s}\right) + s\frac{1}{\tau_i}r_i\left(k_{vi} + k_{pi}h_i\right)e^{-\Delta s} + \frac{1}{\tau_i}r_i k_{pi}e^{-\Delta s}\right) \end{aligned} \tag{33}$$

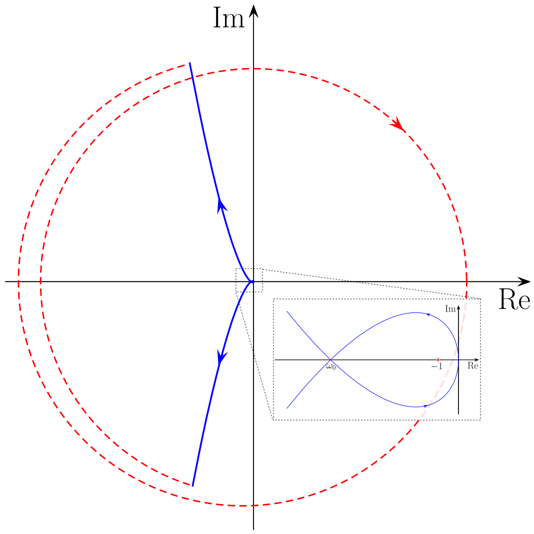


Fig. 10. Nyquist plot for some given parameters in case of perfect communication with no delays.

Since (38) has poles at the origin, in order to use Nyquist stability criterion, the phase angle of (40) should be investigated as $\omega \to 0^+$ and $\omega \to 0^-$. We have

$$\angle L(j\omega) = 90^\circ + \tan^{-1}\left(\frac{c\omega}{d-(a+b)\omega^2}\right). \quad (41)$$

For $\omega \to 0^+$, if $(c,d) > 0$, we will have

$$\tan^{-1}\left(\frac{c\omega}{d-(a+b)\omega^2}\right) > 0. \quad (42)$$

Therefore, $\angle L(j\omega) > 90^\circ$ as $\omega \to 0^+$ and in the same way, $\angle L(j\omega) < -90^\circ$ as $\omega \to 0^-$. As a result, for $\omega \to 0$ the Nyquist plot encircles the point $-1+j0$ twice in the clockwise direction. Fig. 10 demonstrates that to ensure the stability of system (12) with $\Delta = 0$, the Nyquist plot of $L(j\omega)$ should also encircle the point $-1+j0$ twice in the counterclockwise direction, which means it should cross the real axis at the left side of the point $-1+j0$ when $\omega \not\to 0$. For having this feature, we should find when the imaginary part of $L(j\omega)$ is zero. At this point, from (40) we have

$${\omega_0}^2 = d/(a+b), \quad (43)$$

where $\omega = \omega_0$ is the frequency when $L(j\omega)$ crosses the real line. At this point, the real part will be

$$\mathrm{Re}\{L(j\omega_0)\} = -c/{\omega_0}^2 = -c(a+b)/d. \quad (44)$$

Thus, for stability, we need to have $c(a+b)/d > 1$, and as a result, we need $(a+b) > 0$. By substituting from (39) into (44), we will find that the same condition for internal stability holds as the one given in [24] via the Routh-Hurwitz stability criterion.

2) Internal Stability in Presence of Time Delays: If the roots of (34) lie in the complex left-half plane, then the system is stable. It can be easily deduced that $s = 0$ and $s = -1/\tau_i$ cannot be the solution to (34), if

$$k_{pi} \neq 0, \quad (45a)$$

$$k_{ai} - \tau_i(k_{vi} + k_{pi}h_i) + {\tau_i}^2 k_{pi} \neq 0. \quad (45b)$$

Thus, after dividing (34) by $s^3 + \frac{1}{\tau_i}s^2$, the characteristic equation can be rewritten as

$$1 + \frac{be^{-\Delta s}}{s+a} + \frac{ce^{-\Delta s}}{s(s+a)} + \frac{de^{-\Delta s}}{s^2(s+a)} = 0, \quad (46)$$

where the notation (39) is used. The characteristic equation (46) can be written as

$$1 + G(s) = 0, \quad (47)$$

where the open loop transfer function, denoted by $G(s)$ is

$$G(s) = \frac{be^{-\Delta s}}{s+a} + \frac{ce^{-\Delta s}}{s(s+a)} + \frac{de^{-\Delta s}}{s^2(s+a)}, \quad (48)$$

which can be compactly written as

$$G(s) = \frac{bs^2 + cs + d}{s^2(s+a)}e^{-\Delta s}. \quad (49)$$

Now, for stability analysis of (34), we apply the Nyquist criterion on $G(s)$. After replacing s by $j\omega$ in (49), we obtain

$$G(j\omega) = \frac{d - b\omega^2 + jc\omega}{-\omega^2(j\omega + a)}e^{-j\omega\Delta}. \quad (50)$$

From (50), the phase of $G(j\omega)$ will be

$$\angle G(j\omega) = -180^\circ - \tan^{-1}(\frac{\omega}{a}) + \tan^{-1}(\frac{c\omega}{d-b\omega^2}) - \omega\Delta. \quad (51)$$

If $\angle G(j\omega) > -180^\circ$ as $\omega \to 0^+$, then it can be shown that the Nyquist plot of $G(j\omega)$ does not encircle the point $-1+j0$ as $\omega \to 0$. Hence, we investigate what happens as $\omega \to 0^+$. From (51), we know that $\angle G(j0^+) > -180^\circ$ if

$$\tan^{-1}(\frac{c\omega}{d-b\omega^2}) - \tan^{-1}(\frac{\omega}{a}) - \omega\Delta > 0. \quad (52)$$

Then, if

$$\tan^{-1}(\frac{c\omega}{d-b\omega^2}) - \tan^{-1}(\frac{\omega}{a}) > 0, \quad (53)$$

there exist a time delay Δ such that (52) holds. If

$$\frac{c\omega}{d-b\omega^2} - \frac{\omega}{a} = \frac{\omega(ac-d+b\omega^2)}{(d-b\omega^2)} > 0, \quad (54)$$

then, inequality (53) holds. By assuming

$$k_{pi} > 0, \quad (55a)$$

$$k_{vi} + k_{pi}(h_i - \tau) \geq 0, \quad (55b)$$

we will have $d > 0$ and $ac-d > 0$ and then, when $\omega \to 0^+$, inequality (54) holds. Therefore, there will be no encirclement around the point $-1+j0$ for the Nyquist plot when $\omega \to 0$. In Fig. 11, a Nyquist plot is provided for some given parameters (same as the numerical example), depicting that as long as (45) and (55) hold, there is no encirclement of the point $-1+j0$ as $\omega \to 0$.

Then, we should investigate if there is any encirclement as $\omega \neq 0$. After algebraic manipulation, (50) becomes

$$G(j\omega) = -\frac{\left(c\omega - a\omega(b - \frac{d}{\omega^2})\right) + j\left(ac + \omega^2(b - \frac{d}{\omega^2})\right)}{\omega(a^2+\omega^2)}e^{-j\Delta\omega}.$$

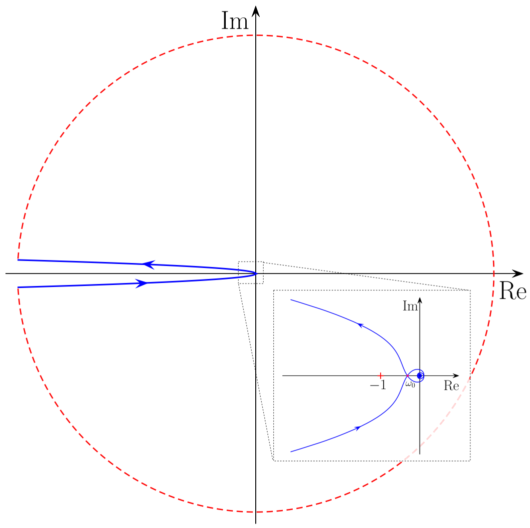


Fig. 11. Nyquist plot for some given parameters in case of communication with delays.

By substituting $e^{-j\Delta\omega} = \cos(\Delta\omega) - j\sin(\Delta\omega)$ and then defining

$$Y(\omega) \triangleq c\omega - a\omega(b - \frac{d}{\omega^2}),\ \ Z(\omega) \triangleq ac + \omega^2(b - \frac{d}{\omega^2}), \quad (56)$$

the imaginary part of $G(j\omega)$ is as follows

$$\text{Im}\{G(j\omega)\} = -\frac{Z(\omega)\cos(\Delta\omega) - Y(\omega)\sin(\Delta\omega)}{\omega(a^2+\omega^2)}. \quad (57)$$

Suppose that $G(j\omega)$ crosses the real line when $\omega = \omega_0$. From (57), we have

$$\tan(\Delta\omega_0) = \frac{Z(\omega_0)}{Y(\omega_0)}. \quad (58)$$

Now, we just need to find the conditions for which $\text{Re}\{G(j\omega_0)\} > -1$. For the sake of simplicity, we write Y and Z for $Y(\omega_0)$ and $Z(\omega_0)$, respectively. We have

$$\begin{aligned}\text{Re}\{G(j\omega_0)\} &= -\frac{Y\cos(\Delta\omega_0) + Z\sin(\Delta\omega_0)}{\omega_0(a^2+\omega_0{}^2)} \\ &= -\frac{Y\frac{\cos(\Delta\omega_0)}{\sin(\Delta\omega_0)}\sin(\Delta\omega_0) + Z\sin(\Delta\omega_0)}{\omega_0(a^2+\omega_0{}^2)} \\ &= -\frac{(Y^2+Z^2)\sin(\Delta\omega_0)}{(a^2+\omega_0{}^2)Z\omega_0}. \end{aligned} \quad (59)$$

Considering $k_{ai} > 0$, we will have $b > 0$ and by using (55), we will have $Z > 0$. Then, using the fact that $\sin(\Delta\omega) \leq \Delta\omega$ for $\omega \geq 0$, we can continue analyzing $\text{Re}\{G(j\omega_0)\}$ as follows

$$\text{Re}\{G(j\omega_0)\} \geq -\frac{\Delta(Y^2+Z^2)}{(a^2+\omega_0{}^2)Z}. \quad (60)$$

Defining

$$m \triangleq \frac{\Delta(Y^2+Z^2)}{(a^2+\omega_0{}^2)Z}, \quad (61)$$

a sufficient condition for stability is having $m < 1$. By substituting from (56) into (61) and simplifying, we obtain

$$m = \Delta\left(\frac{\omega_0^2(b - \frac{d}{\omega_0{}^2})^2 + c^2}{ac + \omega_0{}^2(b - \frac{d}{\omega_0{}^2})}\right). \quad (62)$$

Let $X \triangleq b - \frac{d}{\omega_0{}^2}$. Then fom (62) we have that

$$\frac{\Delta(c^2 + \omega_0^2 X^2)}{ac + \omega_0^2 X} < 1. \quad (63)$$

Since $ac - d > 0$, we can easily deduce that $ac + \omega_0^2\ X > 0$, and inequality (63) can be written as

$$\Delta(c^2 + \omega_0{}^2X^2) < ac + \omega_0{}^2X. \quad (64)$$

By multiplying both sides of (64) by Δ, and rearranging we obtain

$$(\Delta X)^2 - (\Delta X) + \frac{(\Delta c)^2 - a(\Delta c)}{\omega_0{}^2} < 0. \quad (65)$$

Since (65) is a quadratic inequality, for having $(\Delta X) \in \mathbb{R}$, we need a positive discriminant, i.e.,

$$1 - 4\frac{(\Delta c)^2 - a(\Delta c)}{\omega_0{}^2} > 0, \quad (66)$$

which can be rewritten as

$$(\Delta c)^2 - a(\Delta c) - \frac{\omega_0{}^2}{4} < 0. \quad (67)$$

Obviously, if (Δc) lies between the roots of (67), then (67) holds, i.e.,

$$\frac{a - \sqrt{a^2+\omega_0{}^2}}{2} < \Delta c < \frac{a + \sqrt{a^2+\omega_0{}^2}}{2}. \quad (68)$$

The left term of (68) is negative, Δ is positive and we can easily deduce from (55) that $c > 0$. Then we just need to have

$$\Delta c < \frac{a + \sqrt{a^2+\omega_0{}^2}}{2}. \quad (69)$$

Since we do not know ω_0 (which changes for different Δ), we provide a conservative bound, that is, since

$$\frac{a + \sqrt{a^2+\omega_0{}^2}}{2} > a,$$

then if $\Delta c < a$, then inequality (69) also holds. Therefore, if

$$\Delta\frac{c}{a} < 1, \quad (70)$$

then, the Nyquist plot of $G(j\omega)$ does not encircle the point $-1 + j0$ when ω is not close to 0. Therefore, there is no encirclement for all frequencies and the system is stable. By substituting from (39) into (70), condition (19) is obtained.

B. Proof of Theorem 2

The proof of Theorem 2 follows *mutatis mutandis* that of [39].

Inequality (26) is equivalent to

$$\max_{1\leq l\leq r} \|H_l(j\omega)\|_\infty^2 = \max_{1\leq l\leq r} \sup_{\omega\geq 0} |H_l(j\omega)|^2 \leq \frac{1}{r^2}. \quad (71)$$

Define

$$|H_l(j\omega)|^2 \triangleq \frac{N_l}{D_l}, \quad (72)$$

where

$$N_l = (k_p - k_a\omega^2)^2 + \big(k_v - k_p h(r-l)\big)^2\omega^2, \quad (73)$$

and

$$\begin{aligned} D_l = &\big(-\omega^2 - rk_a\omega^2\cos(\Delta\omega) \\ &+ r(k_v+k_ph)\omega\sin(\Delta\omega) + rk_p\cos(\Delta\omega)\big)^2 \\ &+ \big(-\tau\omega^3 + rk_a\omega^2\sin(\Delta\omega) \\ &+ r(k_v+k_ph)\omega\cos(\Delta\omega) - rk_p\sin(\Delta\omega)\big)^2. \end{aligned} \tag{74}$$

After some simplifications, we obtain that the inequality in (71) holds for $l \in \{1, r\}$ if

$$D_l - r^2N_l = M_6\omega^6 + M_5\omega^5 + M_4\omega^4 + M_3\omega^3 + M_2\omega^2 \geq 0, \tag{75}$$

where

$$M_6 = \tau^2, \tag{76a}$$
$$M_5 = -2\tau rk_a\sin(\Delta\omega), \tag{76b}$$
$$M_4 = 1 + 2r\Big(k_a - \tau(k_v+k_ph)\Big)\cos(\Delta\omega), \tag{76c}$$
$$M_3 = 2r\Big(k_p(\tau-h) - k_v\Big)\sin(\Delta\omega), \tag{76d}$$
$$\begin{aligned} M_2 = &\, r^2k_p^2h^2(1-(r-l)^2) + 2r^2k_pk_vh(1+r-l) \\ &- 2rk_p\cos(\Delta\omega). \end{aligned} \tag{76e}$$

Considering the fact that $\sin(\Delta\omega) \leq \Delta\omega$ for $\omega \geq 0$ and $\cos(\Delta\omega) \leq 1$, if $k_a > 0$, by considering

$$k_p(\tau-h) - k_v \leq 0, \tag{77}$$

and

$$k_a - \tau(k_v+k_ph) \leq 0, \tag{78}$$

then we need to have

$$D_l - r^2N_l \geq \omega^2(\overline{M}_4\omega^4 + \overline{M}_2\omega^2 + \overline{M}_0), \tag{79}$$

where

$$\begin{aligned} \overline{M}_4 &= \tau^2 - 2\tau rk_a\Delta, \\ \overline{M}_2 &= 1 + 2r\Big(k_a - \tau(k_v+k_ph)\Big) + 2r\Delta\Big(k_p(\tau-h) - k_v\Big), \\ \overline{M}_0 &= r^2k_p^2h^2(1-(r-l)^2) + 2r^2k_pk_vh(1+r-l) - 2rk_p. \end{aligned}$$

Having $\overline{M}_4, \overline{M}_2, \overline{M}_0 \geq 0$ results in string stability. Hence, there are three cases as follows

$$\overline{M}_4 \geq 0 \iff k_a \leq \frac{\tau}{2r\Delta} \tag{80a}$$
$$\overline{M}_2 \geq 0 \iff k_v \leq \frac{1 + 2rk_a + 2rk_p(\tau\Delta - \Delta h - \tau h)}{2r(\tau+\Delta)} \tag{80b}$$
$$\overline{M}_0 \geq 0 \iff k_v \geq \frac{1}{rh} - \frac{k_ph}{2}, \quad \text{if} \quad l = r. \tag{80c}$$

By combining (80b) and (80c), we obtain

$$\frac{1}{rh} - \frac{k_ph}{2} \leq k_v \leq \frac{1 + 2rk_a + 2rk_p(\tau\Delta - \Delta h - \tau h)}{2r(\tau+\Delta)}. \tag{81}$$

The above inequality implies that

$$\frac{1+2rk_a + 2rk_p(\tau\Delta - \Delta h - \tau h)}{2r(\tau+\Delta)} - \left(\frac{1}{rh} - \frac{k_ph}{2}\right) \geq 0. \tag{82}$$

After some simplifications, we obtain

$$\frac{h(2rk_a+1) - 2(\tau+\Delta) + rhk_p(2\tau\Delta - \Delta h - \tau h)}{2rh(\tau+\Delta)} \geq 0. \tag{83}$$

Then, by assuming $2\tau\Delta - \Delta h - \tau h \leq 0$ and considering $k_p > 0$, we can deduce that

$$h \geq h_{\min} = \frac{2(\tau+\Delta)}{2rk_a+1}. \tag{84}$$

ACKNOWLEDGMENT

In this article, the authors provide more detailed proofs, including a new proof for internal stability for the case in which the communication has no delays. Additionally, they extend the simulations to include the one-vehicle look-ahead scenario and large-scale platoons.

REFERENCES

[1] E. Abolfazli, B. Besselink, and T. Charalambous, "Reducing time headway in platoons under the MPF topology in the presence of communication delays," in *Proc. IFAC World Congr.*, 2020, pp. 15267–15274.
[2] A. A. Alam, A. Gattami, and K. H. Johansson, "An experimental study on the fuel reduction potential of heavy duty vehicle platooning," in *Proc. 13th Int. IEEE Conf. Intell. Transp. Syst.*, Sep. 2010, pp. 306–311.
[3] A. Alam, "Fuel-efficient heavy-duty vehicle platooning," Ph.D. dissertation, School Elect. Eng. (EES), Roy. Inst. Technol. (KTH), Faridabad, Haryana, 2014.
[4] A. Alam, B. Besselink, V. Turri, J. Mårtensson, and K. H. Johansson, "Heavy-duty vehicle platooning for sustainable freight transportation: A cooperative method to enhance safety and efficiency," *IEEE Control Syst. Mag.*, vol. 35, no. 6, pp. 34–56, Dec. 2015.
[5] S. van de Hoef, "Coordination of heavy-duty vehicle platooning," Ph.D. dissertation, School Elect. Eng. Comput. Sci. (EECS), Roy. Inst. Technol. (KTH), Faridabad, Haryana, 2018.
[6] S. Feng, Y. Zhang, S. E. Li, Z. Cao, H. X. Liu, and L. Li, "String stability for vehicular platoon control: Definitions and analysis methods," *Annu. Rev. Control*, vol. 47, pp. 81–97, Jan. 2019.
[7] D. Swaroop and J. K. Hedrick, "String stability of interconnected systems," *IEEE Trans. Autom. Control*, vol. 41, no. 3, pp. 349–357, Mar. 1996.
[8] J. Ploeg, N. van de Wouw, and H. Nijmeijer, "Lp string stability of cascaded systems: Application to vehicle platooning," *IEEE Trans. Control Syst. Technol.*, vol. 22, no. 2, pp. 786–793, Mar. 2014.
[9] B. Besselink and K. H. Johansson, "String stability and a delay-based spacing policy for vehicle platoons subject to disturbances," *IEEE Trans. Autom. Control*, vol. 62, no. 9, pp. 4376–4391, Sep. 2017.
[10] P. Wijnbergen and B. Besselink, "Existence of decentralized controllers for vehicle platoons: On the role of spacing policies and available measurements," *Syst. Control Lett.*, vol. 145, Nov. 2020, Art. no. 104796.
[11] S. Sheikholeslam and C. A. Desoer, "Longitudinal control of a platoon of vehicles with no communication of lead vehicle information: A system level study," *IEEE Trans. Veh. Technol.*, vol. 42, no. 4, pp. 546–554, Nov. 1993.
[12] G. Orosz, "Connected cruise control: Modelling, delay effects, and nonlinear behaviour," *Vehicle Syst. Dyn.*, vol. 54, no. 8, pp. 1147–1176, Aug. 2016.
[13] K. Santhanakrishnan and R. Rajamani, "On spacing policies for highway vehicle automation," *IEEE Trans. Intell. Transp. Syst.*, vol. 4, no. 4, pp. 198–204, Dec. 2003.
[14] H. E. Sungu, M. Inoue, and J.-I. Imura, "Nonlinear spacing policy based vehicle platoon control for local string stability and global traffic flow stability," in *Proc. Eur. Control Conf. (ECC)*, Jul. 2015, pp. 3396–3401.
[15] G. Guo, P. Li, and L.-Y. Hao, "Adaptive fault-tolerant control of platoons with guaranteed traffic flow stability," *IEEE Trans. Veh. Technol.*, vol. 69, no. 7, pp. 6916–6927, Jul. 2020.
[16] G. Guo, P. Li, and L.-Y. Hao, "A new quadratic spacing policy and adaptive fault-tolerant platooning with actuator saturation," *IEEE Trans. Intell. Transp. Syst.*, early access, Sep. 23, 2020, doi: 10.1109/TITS.2020.3023453.
[17] D. Swaroop and K. R. Rajagopal, "A review of constant time headway policy for automatic vehicle following," in *Proc. IEEE Intell. Transp. Syst. (ITSC)*, Aug. 2001, pp. 65–69.
[18] C. Nowakowski, D. Thompson, S. E. Shladover, A. Kailas, and X.-Y. Lu, "Operational concepts for truck maneuvers with cooperative adaptive cruise control," *Transp. Res. Rec., J. Transp. Res. Board*, vol. 2559, no. 1, pp. 57–64, Jan. 2016.

[19] S. E. Li, Y. Zheng, K. Li, and J. Wang, "An overview of vehicular platoon control under the four-component framework," in *Proc. IEEE Intell. Vehicles Symp. (IV)*, Jun. 2015, pp. 286–291.
[20] Y. Zheng, S. Eben Li, J. Wang, D. Cao, and K. Li, "Stability and scalability of homogeneous vehicular platoon: Study on the influence of information flow topologies," *IEEE Trans. Intell. Transp. Syst.*, vol. 17, no. 1, pp. 14–26, Jan. 2016.
[21] S. Konduri, S. Darbha, and P. R. Pagilla, "Vehicle platooning with multiple vehicle look-ahead information," *IFAC-PapersOnLine*, vol. 50, no. 1, pp. 5768–5773, 2017.
[22] A. Talebpour and H. S. Mahmassani, "Influence of connected and autonomous vehicles on traffic flow stability and throughput," *Transp. Res. C, Emerg. Technol.*, vol. 71, pp. 143–163, Oct. 2016.
[23] S. Darbha, S. Konduri, and P. R. Pagilla, "Benefits of V2V communication for autonomous and connected vehicles," *IEEE Trans. Intell. Transp. Syst.*, vol. 20, no. 5, pp. 1954–1963, May 2019.
[24] Y. Bian, Y. Zheng, W. Ren, S. E. Li, J. Wang, and K. Li, "Reducing time headway for platooning of connected vehicles via V2V communication," *Transp. Res. C, Emerg. Technol.*, vol. 102, pp. 87–105, May 2019.
[25] X. Liu, A. Goldsmith, S. S. Mahal, and J. K. Hedrick, "Effects of communication delay on string stability in vehicle platoons," in *Proc. IEEE Intell. Transp. Syst. (ITSC)*, Aug. 2001, pp. 625–630.
[26] A. A. Peters, R. H. Middleton, and O. Mason, "Leader tracking in homogeneous vehicle platoons with broadcast delays," *Automatica*, vol. 50, no. 1, pp. 64–74, Jan. 2014.
[27] L. Xu, W. Zhuang, G. Yin, and C. Bian, "Stable longitudinal control of heterogeneous vehicular platoon with disturbances and information delays," *IEEE Access*, vol. 6, pp. 69794–69806, 2018.
[28] F. Gao, S. E. Li, Y. Zheng, and D. Kum, "Robust control of heterogeneous vehicular platoon with uncertain dynamics and communication delay," *IET Intell. Transp. Syst.*, vol. 10, no. 7, pp. 503–513, Sep. 2016.
[29] S. Wen and G. Guo, "Sampled-data control for connected vehicles with Markovian switching topologies and communication delay," *IEEE Trans. Intell. Transp. Syst.*, vol. 21, no. 7, pp. 2930–2942, Jul. 2020.
[30] Y. Li, C. Tang, S. Peeta, and Y. Wang, "Nonlinear consensus-based connected vehicle platoon control incorporating car-following interactions and heterogeneous time delays," *IEEE Trans. Intell. Transp. Syst.*, vol. 20, no. 6, pp. 2209–2219, Jun. 2019.
[31] A. Petrillo, A. Salvi, S. Santini, and A. S. Valente, "Adaptive multi-agents synchronization for collaborative driving of autonomous vehicles with multiple communication delays," *Transp. Res. C, Emerg. Technol.*, vol. 86, pp. 372–392, Jan. 2018.
[32] L. Zhang and G. Orosz, "Motif-based design for connected vehicle systems in presence of heterogeneous connectivity structures and time delays," *IEEE Trans. Intell. Transp. Syst.*, vol. 17, no. 6, pp. 1638–1651, Jun. 2016.
[33] S. Öncü, J. Ploeg, N. van de Wouw, and H. Nijmeijer, "Cooperative adaptive cruise control: Network-aware analysis of string stability," *IEEE Trans. Intell. Transp. Syst.*, vol. 15, no. 4, pp. 1527–1537, Aug. 2014.
[34] D. Dileep, M. Fusco, J. Verhaegh, L. Hetel, J. Richard, and W. Michiels, "Achieving an $\mathcal{L}_2$ string stable one vehicle look-ahead platoon with heterogeneity in time-delays," in *Proc. 18th Eur. Control Conf. (ECC)*, Jun. 2019, pp. 1220–1226.
[35] M. di Bernardo, A. Salvi, and S. Santini, "Distributed consensus strategy for platooning of vehicles in the presence of time-varying heterogeneous communication delays," *IEEE Trans. Intell. Transp. Syst.*, vol. 16, no. 1, pp. 102–112, Feb. 2015.
[36] A. Salvi, S. Santini, and A. S. Valente, "Design, analysis and performance evaluation of a third order distributed protocol for platooning in the presence of time-varying delays and switching topologies," *Transp. Res. C, Emerg. Technol.*, vol. 80, pp. 360–383, Jul. 2017.
[37] F. Morbidi, P. Colaneri, and T. Stanger, "Decentralized optimal control of a car platoon with guaranteed string stability," in *Proc. Eur. Control Conf. (ECC)*, Jul. 2013, pp. 3494–3499.
[38] H. Xing, J. Ploeg, and H. Nijmeijer, "Padé approximation of delays in cooperative ACC based on string stability requirements," *IEEE Trans. Intell. Vehicles*, vol. 1, no. 3, pp. 277–286, Sep. 2016.
[39] L. Xiao and F. Gao, "Practical string stability of platoon of adaptive cruise control vehicles," *IEEE Trans. Intell. Transp. Syst.*, vol. 12, no. 4, pp. 1184–1194, Dec. 2011.
[40] C. D. Meyer, *Matrix Analysis and Applied Linear Algebra*, vol. 71. Philadelphia, PA, USA: SIAM, 2000.
[41] Y. Zheng, Y. Bian, S. Li, and S. E. Li, "Cooperative control of heterogeneous connected vehicles with directed acyclic interactions," *IEEE Intell. Transp. Syst. Mag.*, vol. 13, no. 2, pp. 127–141, May 2021.
[42] G. F. Franklin, J. D. Powell, A. Emami-Naeini, and J. D. Powell, *Feedback Control of Dynamic Systems*, vol. 3. Reading, MA, USA: Addison-Wesley, 1994.

Elham Abolfazli received the B.Sc. degree in electrical engineering from the Iran University of Science and Technology, Tehran, Iran, in 2012, and the M.Sc. degree in electrical and control engineering from the University of Tehran, Iran, in 2015. She is currently pursuing the Ph.D. degree with the Department of Electrical Engineering and Automation, School of Electrical Engineering, Aalto University, Finland. Her current research interests include networked control systems and connected vehicles.

Bart Besselink (Member, IEEE) received the M.Sc. degree *(cum laude)* in mechanical engineering and the Ph.D. degree from the Eindhoven University of Technology, Eindhoven, The Netherlands, in 2008 and 2012, respectively. He was a short-term Visiting Researcher with the Tokyo Institute of Technology, Tokyo, Japan, in 2012. From 2012 to 2016, he was a Post-Doctoral Researcher with the ACCESS Linnaeus Center and the Department of Automatic Control, KTH Royal Institute of Technology, Stockholm, Sweden. Since 2016, he has been an Assistant Professor with the Bernoulli Institute for Mathematics, Computer Science and Artificial Intelligence, University of Groningen, Groningen, The Netherlands. His main research interests include systems theory and model reduction for nonlinear dynamical systems and large-scale interconnected systems, with applications in intelligent transportation systems.

Themistoklis Charalambous (Senior Member, IEEE) received the B.A. and M.Eng. degrees in electrical and information sciences from the Trinity College, Cambridge University, in 2004 and 2005, respectively, and the Ph.D. degree from the Control Laboratory, Engineering Department, Cambridge University, in 2010. From September 2009 to September 2010, he was with the Human Robotics Group, Imperial College London, as a Research Associate for an academic year. From September 2010 to December 2011, he worked as a Visiting Lecturer with the Department of Electrical and Computer Engineering, University of Cyprus. He worked as a Post-Doctoral Researcher with the Department of Automatic Control, School of Electrical Engineering, KTH Royal Institute of Technology, from January 2012 to January 2015, and the Department of Electrical Engineering, Chalmers University of Technology, from April 2015 to December 2016. In January 2017, he joined the Department of Electrical Engineering and Automation, School of Electrical Engineering, Aalto University, as a tenure-track Assistant Professor. He has been a nominated Research Fellow of the Academy of Finland since September 2018 and a tenured Associate Professor since July 2020. His primary research targets the design and analysis of (wireless) networked control systems that are stable, scalable, and energy efficient.