Secure Mining of Association Rules in Horizontally Distributed Databases

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Abstract. We propose a protocol for secure mining of association rules in horizontally distributed databases. The current leading protocol is that of Kantarcioglu and Clifton [12]. Our protocol, like theirs, is based on the Fast Distributed Mining (FDM) algorithm of Cheung et al. [6], which is an unsecured distributed version of the Apriori algorithm. The main ingredients in our protocol are two novel secure multi-party algorithms — one that computes the union of private subsets that each of the interacting players hold, and another that tests the inclusion of an element held by one player in a subset held by another. Our protocol offers enhanced privacy with respect to the protocol in [12]. In addition, it is simpler and is significantly more efficient in terms of communication rounds, communication cost and computational cost.

Key words: Privacy Preserving Data Mining, Distributed Computation, Frequent Itemsets, Association Rules

1 Introduction

We study here the problem of secure mining of association rules in horizontally partitioned databases. In that setting, there are several sites (or players) that hold homogeneous databases, i.e., databases that share the same schema but hold information on different entities. The goal is to find all association rules with given minimal support and confidence levels that hold in the unified database, while minimizing the information disclosed about the private databases held by those players.

That goal defines a problem of secure multi-party computation. In such problems, there are M players that hold private inputs, x_1, \ldots, x_M , and they wish to securely compute $y = f(x_1, \ldots, x_M)$ for some public function f. If there existed a trusted third party, the players could surrender to him their inputs and he would perform the function evaluation and send to them the resulting output. In the absence of such a trusted third party, it is needed to devise a protocol that the players can run on their own in order to arrive at the required output y. Such a protocol is considered perfectly secure if no player can learn from his view of the protocol more than what he would have learnt in the idealized setting where the computation is carried out by a trusted third party. Yao [21] was the first to propose a generic solution for this problem in the case of two players. Other generic solutions, for the multi-party case, were later proposed in [2,4,10].

In our problem, the inputs are the partial databases, and the required output is the list of association rules with given support and confidence. As the above mentioned generic solutions rely upon a description of the function f as a Boolean circuit, they can be applied only to small inputs and functions which are realizable by simple circuits. In more complex settings, such as ours, other methods are required for carrying out this computation. In such cases, some relaxations of the notion of perfect security might be inevitable when looking for practical protocols, provided that the excess information is deemed benign (see examples of such protocols in e.g. [12,20,23]).

Kantarcioglu and Clifton studied that problem in [12] and devised a protocol for its solution. The main part of the protocol is a sub-protocol for the secure computation of the union of private subsets that are held by the different players. (Those subsets include candidate itemsets, as we explain below.) That is the most costly part of the protocol and its implementation relies upon cryptographic primitives such as commutative encryption, oblivious transfer, and hash functions. This is also the only part in the protocol in which the players may extract from their view of the protocol information on other databases, beyond what is implied by the final output and their own input. While such leakage of information renders the protocol not perfectly secure, the perimeter of the excess information is explicitly bounded in [12] and it is argued that such information leakage is innocuous, whence acceptable from practical point of view.

Herein we propose an alternative protocol for the secure computation of the union of private subsets. The proposed protocol improves upon that in [12] in terms of simplicity and efficiency as well as privacy. In particular, our protocol does not depend on commutative encryption and oblivious transfer (what simplifies it significantly and contributes towards reduced communication and computational costs). While our solution is still not perfectly secure, it leaks excess information only to a small number of coalitions (three), unlike the protocol of [12] that discloses information also to some single players. In addition, we claim that the excess information that our protocol may leak is less sensitive than the excess information leaked by the protocol of [12].

The protocol that we propose here computes a parameterized family of functions, which we call threshold functions, in which the two extreme cases correspond to the problems of computing the union and intersection of private subsets. Those are in fact general-purpose protocols that can be used in other contexts as well. Another problem of secure multi-party computation that we solve here as part of our discussion is the problem of determining whether an element held by one player is included in a subset held by another.

1.1 Preliminaries

Let D be a transaction database. As in [12], we view D as a binary matrix of N rows and L columns, where each row is a transaction over some set of items, $A = \{a_1, \ldots, a_L\}$, and each column represents one of the items in A. (In other words, the (i, j)th entry of D equals 1 if the *i*th transaction includs the item a_j , and 0 otherwise.) The database D is partitioned horizontally between M players, denoted P_1, \ldots, P_M . Player P_m holds the partial database D_m that contains $N_m = |D_m|$ of the transactions in $D, 1 \le m \le M$. The unified database is $D = D_1 \cup \cdots \cup D_M$, and $N = \sum_{m=1}^M N_m$.

An itemset X is a subset of A. Its global support, supp(X), is the number of transactions in D that contain it. Its local support, $supp_m(X)$, is the number of transactions in D_m that contain it. Clearly, $supp(X) = \sum_{m=1}^{M} supp_m(X)$. Let s be a real number between 0 and 1 that stands for a required threshold support. An itemset X is called s-frequent if $supp(X) \ge sN$. It is called locally s-frequent at D_m if $supp_m(X) \ge sN_m$.

For each $1 \leq k \leq L$, let F_s^k denote the set of all k-itemsets (namely, itemsets of size k) that are s-frequent, and $F_s^{k,m}$ be the set of all k-itemsets that are locally s-frequent at D_m , $1 \leq m \leq M$. Our main computational goal is to find, for a given threshold support $0 < s \leq 1$, the set of all s-frequent itemsets, $F_s := \bigcup_{k=1}^{L} F_s^k$. We may then continue to find all (s, c)-association rules, i.e., all association rules of support at least sN and confidence at least c. (Recall that if X and Y are two disjoint subsets of A, the support of the corresponding association rule $X \Rightarrow Y$ is $supp(X \cup Y)$ and its confidence is $supp(X \cup Y)/supp(X)$.)

1.2 The Fast Distributed Mining algorithm

The protocol of [12], as well as ours, are based on the Fast Distributed Mining (FDM) algorithm of Cheung et al. [6], which is an unsecured distributed version of the Apriori algorithm. Its main idea is that any *s*-frequent itemset must be also locally *s*-frequent in at least one of the sites. Hence, in order to find all globally *s*-frequent itemsets, each player reveals his locally *s*-frequent itemsets and then the players check each of them to see if they are *s*-frequent also globally. The stages of the FDM algorithm are as follows:

- (1) **Initialization:** It is assumed that the players have already jointly calculated F_s^{k-1} . The goal is to proceed and calculate F_s^k .
- (2) Candidate Sets Generation: Each P_m generates a set of candidate kitemsets $B_s^{k,m}$ out of $F_s^{k-1,m} \cap F_s^{k-1}$ — the (k-1)-itemsets that are both globally and locally frequent, using the Apriori algorithm.
- (3) **Local Pruning:** For each $X \in B_s^{k,m}$, P_m computes $supp_m(X)$ and retains only those itemsets that are locally *s*-frequent. We denote this collection of itemsets by $C_s^{k,m}$.
- (4) Unifying the candidate itemsets: Each player broadcasts his $C_s^{k,m}$ and then all players compute $C_s^k := \bigcup_{m=1}^M C_s^{k,m}$.
- (5) **Computing local supports.** All players compute the local supports of all itemsets in C_s^k .
- (6) **Broadcast Mining Results:** Each player broadcasts the local supports that he computed. From that, everyone can compute the global support of every itemset in C_s^k . Finally, F_s^k is the subset of C_s^k that consists of all globally *s*-frequent *k*-itemsets.

1.3 Overview and organization of the paper

The FDM protocol violates privacy in two stages: In Stage 4, where the players broadcast the itemsets that are locally frequent in their private databases, and in Stage 6, where they broadcast the sizes of the local supports of candidate itemsets. Kantarcioglu and Clifton [12] proposed secure implementations of those two stages. Our improvement is with regard to the secure implementation of Stage 4, which is the more costly stage of the protocol, and the one in which the protocol of [12] leaks excess information. In Section 2 we describe [12]'s secure implementation of Stage 4. We then describe our alternative implementation and we proceed to compare the two implementations in terms of privacy of efficiency. In the next two short Sections 3 and 4 we describe briefly, for the sake of completeness, [12]'s implementation of the two remaining stages of the distributed protocol: The identification of those candidate itemsets that are globally *s*-frequent, and then the derivation of all (s, c)-association rules. Section 5 includes a review of related work. We conclude the paper in Section 6.

Like in [12] we assume that the players are semi-honest; namely, they follow the protocol but try to extract as much information as possible from their own view. (See [11,18,23] for a discussion and justification of that assumption.) We too, like [12], assume that M > 2. (The case M = 2 is discussed in [12, Section 5]; the conclusion is that the problem of secure computation of frequent itemsets and association rules in the two-party case is unlikely to be of use.)

2 Secure computation of all locally frequent itemsets

Here we discuss the secure computation of the union $C_s^k = \bigcup_{m=1}^M C_s^{k,m}$. We describe the protocol of [12] (Section 2.1) and then our protocol (Sections 2.2–2.3). We analyze the privacy of the two protocols in Section 2.4, their communication cost in Section 2.5, and their computational cost in Section 2.6.

2.1 The protocol of [12] for the secure computation of all locally frequent itemsets

Protocol 1 (UNIFI-KC hereinafter) is the protocol that was suggested by Kantarcioglu and Clifton [12] for computing the unified list of all locally frequent itemsets, $C_s^k = \bigcup_{m=1}^M C_s^{k,m}$, without disclosing the sizes of the subsets $C_s^{k,m}$ nor their contents. It is based on two ideas: Hiding the sizes of the subsets $C_s^{k,m}$ by means of fake itemsets, and hiding their content by means of encryption. Let $Ap(F_s^{k-1})$ denote the set of k-itemsets which the Apriori algorithm generates when applied on F_s^{k-1} . Clearly, $C_s^{k,m} \subseteq Ap(F_s^{k-1})$, whence $|C_s^{k,m}| \leq |Ap(F_s^{k-1})|$, for all $1 \leq m \leq M$. Therefore, after each player computes $C_s^{k,m}$, he adds to it fake itemsets until its size becomes $|Ap(F_s^{k-1})|$ in order to hide the number of locally frequent itemsets that he has. In order to hide the actual itemsets, they use a commutative encryption algorithm. (A commutative encryption means that $E_{K_1} \circ E_{K_2} = E_{K_2} \circ E_{K_1}$ for any pair of keys K_1 and K_2 .) We proceed to describe **Protocol 1** (UNIFI-KC) Unifying lists of locally Frequent Itemsets [12]

Input: Each player P_m has an input set $\overline{C_s^{k,m} \subseteq Ap(F_s^{k-1})}, 1 \le m \le M$.

- **Output:** $C_s^k = \bigcup_{m=1}^M C_s^{k,m}$.
- 1: Phase 0: Getting started
- 2: The players decide on a commutative cipher and each player P_m , $1 \le m \le M$, selects a random secret encryption key K_m .
- 3: The players select a hash function h and compute h(x) for all $x \in Ap(F_s^{k-1})$.
- 4: If there exist $x_1 \neq x_2 \in Ap(F_s^{k-1})$ for which $h(x_1) = h(x_2)$, select a different h. 5: Build a lookup table $T = \{(x, h(x)) : x \in Ap(F_s^{k-1})\}.$
- 6: Phase 1: Encryption of all itemsets
- 7: for all Player P_m , $1 \le m \le M$, do
- 8: Set $X_m = \emptyset$.
- for all $x \in C_s^{k,m}$ do 9:
- Player P_m computes $E_{K_m}(h(x))$ and adds it to X_m . 10:
- 11: end for
- 12:Player P_m adds to X_m faked itemsets until its size becomes $|Ap(F_s^{k-1})|$.
- 13: end for
- 14: for i = 2 to M do
- 15: P_m sends a permutation of X_m to P_{m+1} .
- 16: P_m receives from P_{m-1} the permuted X_{m-1} .
- 17: P_m computes a new X_m as the encryption of the permuted X_{m-1} using K_m .
- 18: end for

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19: Phase 2: Merging itemsets
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- 20: Each odd player sends his encrypted sets to player P_1 .
- 21: Each even player sends his encrypted sets to player P_2 .
- 22: P_1 unifies all sets that were sent by the odd players and removes duplicates.
- 23: P_2 unifies all sets that were sent by the even players and removes duplicates.
- 24: P_2 sends his permuted list of itemsets to P_1 .
- 25: P_1 unifies his list of itemsets and the list received from P_2 and then removes duplicates from the unified list. Denote the final list by EC_s^k .
- 26: Phase 3: Decryption
- 27: for m = 1 to M 1 do
- P_m decrypts all itemsets in EC_s^k using K_m . 28:
- P_m sends the permuted (and K_m -decrypted) EC_s^k to P_{m+1} . 29:
- 30: end for
- 31: P_M decrypts all itemsets in EC_s^k using E_M ; denote the resulting set by C_s^k .
- 32: P_M uses the lookup table T to remove from C_s^k faked itemsets.
- 33: P_M broadcasts C_s^k .

the protocol. Since all protocols that we present here involve cyclic communication rounds, P_{M+1} always means P_1 , while P_0 means P_M .

In the preliminary Phase 0 (Steps 2-5) the players select the needed cryptographic primitives: They jointly select a commutative cipher, and each player selects a corresponding random private key. In addition, they select a hash function h to apply on all itemsets prior to encryption. It is essential that h will not experience collusions on $Ap(F_s^{k-1})$ in order to make it invertible on $Ap(F_s^{k-1})$. Hence, if such collusions occur (an event of a very small probability), a different

hash function must be selected. At the end, the players compute a lookup table with the hash values of all candidate itemsets in $Ap(F_s^{k-1})$, which will be used later on to find the preimage of a given hash value.

In Phase 1, all players compute a composite encryption of the hashed sets $C_s^{k,m}$, $1 \le m \le M$. First (Steps 7-13), each player P_m hashes all itemsets in $C_s^{k,m}$ and then encrypts them using the key K_m . (Hashing is needed in order to prevent leakage of algebraic relations between itemsets, see [12, Appendix].) Then, he adds to the resulting set faked itemsets until its size becomes $|Ap(F_s^{k-1})|$. We denote the resulting set by X_m . Then (Steps 14-18), the players start a loop of M-1 cycles, where in each cycle they perform the following operation: Player P_m send a permutation of X_m to the next player P_{m+1} ; Player P_m receives from P_{m-1} a permutation of the set X_{m-1} and then computes a new X_m as $X_m = E_{K_m}(X_{m-1})$. At the end of this loop, P_m holds an encryption of the hashed $C_s^{k,m+1}$ using all M keys. Due to the commutative property of the selected cipher, Player P_m holds the set $\{E_M(\cdots (E_2(E_1(h(x))))\cdots): x \in C_s^{k,m+1}\}$.

In Phase 2 (Steps 20-25), all odd players send the sets that they have to P_1 , who unifies them. Consequently, P_1 will hold all hashed and encrypted itemsets of the even players. Similarly, all even players send the sets that they have to P_2 , who also unifies them; hence, P_2 will hold all hashed and encrypted itemsets of the odd players. At this stage, both P_1 and P_2 remove duplicates, as those stand (with high probability) for itemsets that were frequent in more than one site. Then, player P_2 permutes his list and sends it to P_1 who unifies it with the list that he got. Therefore, at the completion of this stage P_1 holds the union set $C_s^k = \bigcup_{m=1}^M C_s^{k,m}$ hashed and then encrypted by all encryption keys, together with some fake itemsets that were used for the sake of hiding the sizes of the sets $C_s^{k,m}$; those fake itemsets are not needed anymore and will be removed after decryption in the next phase.

In phase 3 (Steps 27-33) a similar round of decryptions is initiated. At the end, the last player who performs the last decryption uses the lookup table T that was constructed in Step 5 in order to identify and remove the fake itemsets and then to recover C_s^k . Finally, he broadcasts C_s^k to all his peers.

2.2 A secure multiparty protocol for computing the OR

Protocol UNIFI-KC securely computes of the union of private subsets of some publicly known ground set $(Ap(F_s^{k-1}))$. Such a problem is equivalent to the problem of computing the OR of private vectors. Indeed, if we let n denote the size of the ground set, then the private subset that player P_m , $1 \le m \le M$, holds may be described by a binary vector $\mathbf{b}_m \in \mathbb{Z}_2^n$, and the union of the private subsets is described by the OR of those private vectors, $\mathbf{b} := \bigvee_{m=1}^M \mathbf{b}_m$. We present here a protocol for computing that function which is simpler than UNIFI-KC and employs less cryptographic primitives.

The protocol that we present (Protocol 2) computes a wider range of functions, which we call threshold functions. **Definition 1.** Let b_1, \ldots, b_M be M bits and $1 \le t \le M$ be an integer. Then

$$T_t(b_1, \dots, b_M) = \begin{cases} 1 & \text{if } \sum_{m=1}^M b_m \ge t \\ 0 & \text{if } \sum_{m=1}^M b_m < t \end{cases}$$
(1)

is called the t-threshold function. Given binary vectors $\mathbf{b}_m = (b_m(1), \ldots, b_m(n)) \in \mathbb{Z}_2^n$, we let $T_t(\mathbf{b}_1, \ldots, \mathbf{b}_M)$ denote the binary vector in which the *i*th component equals $T_t(b_1(i), \ldots, b_M(i)), 1 \leq i \leq n$.

The OR and AND functions are the 1- and M-threshold functions, respectively; i.e.,

$$\bigvee_{m=1}^{M} \mathbf{b}_m = T_1(\mathbf{b}_1, \dots, \mathbf{b}_M), \text{ and } \bigwedge_{m=1}^{M} \mathbf{b}_m = T_M(\mathbf{b}_1, \dots, \mathbf{b}_M).$$

Those special cases may be used, as we show in Section 2.3, to compute in a privacy-preserving manner unions and intersections of subsets.

The main idea behind Protocol 2 (THRESHOLD henceforth), which is based on the protocol suggested in [5] for secure computation of the sum, is to compute shares of the sum vector and then use those shares to securely verify the threshold conditions in each component. Since the sum vector may be seen as a vector over \mathbb{Z}_{M+1} , each player starts by creating random shares in \mathbb{Z}_{M+1}^n of his input vector (Step 1). In Step 2 all players send to all other players the corresponding shares in their input vector. Then (Step 3), player P_{ℓ} , $1 \leq \ell \leq M$, adds the shares that he got and arrives at his share, \mathbf{s}_{ℓ} , in the sum vector. Namely, if we let $\mathbf{a} :=$ $\sum_{m=1}^{M} \mathbf{b}_m$ denote the sum of the input vectors, then $\mathbf{a} = \sum_{\ell=1}^{M} \mathbf{s}_{\ell} \mod (M+1)$; furthermore, any M-1 vectors out of $\{\mathbf{s}_1, \ldots, \mathbf{s}_M\}$ do not reveal any information on the sum \mathbf{a} . In Steps 4-5, all players, apart from the last one, send their shares to P_1 who adds them up to get the share \mathbf{s} . Now, players P_1 and P_M hold additive shares of the sum vector $\mathbf{a}: P_1$ has \mathbf{s}, P_M has \mathbf{s}_M , and $\mathbf{a} = (\mathbf{s} + \mathbf{s}_M)$ mod (M + 1). It is now needed to check for each component $1 \leq i \leq n$ whether $(s(i) + s_M(i)) \mod (M + 1) < t$. Equivalently, we need to check whether

$$(s(i) + s_M(i)) \mod (M+1) \in \{j : 0 \le j \le t-1\}.$$
(2)

The inclusion in (2) is equivalent to

$$s(i) \in \Theta(i) := \{(j - s_M(i)) \mod (M+1) : 0 \le j \le t - 1\}.$$
 (3)

The value of s(i) is known only to P_1 while the set $\Theta(i)$ is known only to P_M . The problem of verifying the set inclusion in Eq. (3) can be seen as a simplified version of the *privacy-preserving keyword search*, which was solved by Freedman et. al. [15]. In the case of the the OR function, t = 1, which is the relevant case for us, the set $\Theta(i)$ is of size 1, and therefore it is the problem of oblivious string comparison, a problem that was solved in e.g. [9]. However, we claim that, since M > 2, there is no need to invoke neither of the secure protocols of [15] or

Protocol 2 (THRESHOLD) Secure computation of the *t*-threshold function

Input: Each player P_m has an input binary vector $\mathbf{b}_m \in \mathbb{Z}_2^n$, $1 \le m \le M$. **Output:** $b := T_t(b_1, ..., b_M).$

- 1: Each P_m selects M random share vectors $\mathbf{b}_{m,\ell} \in \mathbb{Z}_{M+1}^n$, $1 \leq \ell \leq M$, such that $\sum_{\ell=1}^{M} \mathbf{b}_{m,\ell} = \mathbf{b}_m \mod (M+1).$
- 2: Each P_m sends $\mathbf{b}_{m,\ell}$ to P_ℓ for all $1 \leq \ell \neq m \leq M$.
- 3: Each P_{ℓ} computes $\mathbf{s}_{\ell} = (s_{\ell}(1), \dots, s_{\ell}(n)) := \sum_{m=1}^{M} \mathbf{b}_{m,\ell} \mod (M+1).$
- 4: Players P_{ℓ} , $2 \leq \ell \leq M-1$, send \mathbf{s}_{ℓ} to P_1 . 5: P_1 computes $\mathbf{s} = (s(1), \dots, s(n)) := \sum_{\ell=1}^{M-1} \mathbf{s}_{\ell} \mod (M+1)$.
- 6: for i = 1, ..., n do
- If $(s(i) + s_M(i)) \mod (M+1) < t$ set b(i) = 0 otherwise set b(i) = 1. 7:

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8: end for
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9: Output $\mathbf{b} = (b(1), \dots, b(n)).$

[9]. Indeed, as M > 2, the existence of other semi-honest players can be used to verify the inclusion in Eq. (3) much more easily. This is done in Protocol 3 (SETINC) which we proceed to describe next.

Protocol SETINC starts with players P_1 and P_M agreeing on a keyed hash function $h_K(\cdot)$ (e.g., HMAC [3]), a corresponding secret key K, and a long random bit string r (Steps 1-2). Then (Steps 3-4), P_1 converts his sequence of elements $\mathbf{s} = (s(1), \ldots, s(n))$ into a sequence of corresponding "signatures" $\mathbf{s}' = (s'(1), \ldots, s'(n))$, where $s'(i) = h_K(r, i, s(i))$ and P_M does a similar conversions to the subsets that he holds. P_1 then sends \mathbf{s}' to P_2 and P_M sends to P_2 a random permutation of each of the subsets $\Theta'(i)$, $1 \leq i \leq n$. Finally, P_2 performs the relevant inclusion verifications on the signature values. If he finds out that for a given $1 \le i \le n$, $s'(i) \in \Theta'(i)$, he may infer, with high probability, that $s(i) \in \Theta(i)$, whence he sets b(i) = 0. If, on the other hand, $s'(i) \notin \Theta'(i)$, then, with certainty, $s(i) \notin \Theta(i)$, whence he sets b(i) = 1.

Two comments are in order:

- (1) If the index i had not been part of the input to the hash function (Steps 3-4), then two equal components in P_1 's input vector, say s(i) = s(j), would have been mapped to two equal signatures, s'(i) = s'(j). Hence, in that case player P_2 would have learnt that in P_1 's input vector the *i*th and *j*th components are equal. To prevent such leakage of information, we include the index i in the input to the hash function.
- (2) An event in which $s'(i) \in \Theta'(i)$ while $s(i) \notin \Theta(i)$ indicates a collusion; specifically, it implies that there exist $\theta' \in \Theta(i)$ and $\theta'' \in \Omega \setminus \Theta(i)$ for which $h_K(r, i, \theta') = h_K(r, i, \theta'')$. Hash functions are designed so that the probability of such collusions is negligible, whence the risk of a collusion can be ignored. However, it is possible for player P_M to check upfront the selected random values (K and r) in order to verify that for all $1 \le i \le n$, the sets $\Theta'(i) = \{h_K(r, i, \theta) : \theta \in \Theta(i)\}$ and $\Theta''(i) = \{h_K(r, i, \theta) : \theta \in \Omega \setminus \Theta(i)\}$ are disjoint.

Protocol 3 (SETINC) Set Inclusion computation

Input: P_1 has a vector $\mathbf{s} = (s(1), \ldots, s(n))$ and P_M has a vector $\boldsymbol{\Theta} = (\boldsymbol{\Theta}(1), \ldots, \boldsymbol{\Theta}(n))$, where for all $1 \leq i \leq n$, $s(i) \in \Omega$ and $\boldsymbol{\Theta}(i) \subseteq \Omega$ for some ground set Ω .

- **Output:** The vector $\mathbf{b} = (b(1), \dots, b(n))$ where b(i) = 0 if $s(i) \in \Theta(i)$ and b(i) = 1 otherwise, $1 \le i \le n$.
- 1: P_1 and P_M agree on a keyed-hash function $h_K(\cdot)$ and on a secret key K.
- 2: P_1 and P_M agree on a long random bit string r.
- 3: P_1 computes $\mathbf{s}' = (s'(1), \dots, s'(n))$, where $s'(i) = h_K(r, i, s(i)), 1 \le i \le n$.
- 4: P_M computes $\Theta' = (\Theta'(1), \ldots, \Theta'(n))$, where $\Theta'(i) = \{h_K(r, i, \theta) : \theta \in \Theta(i)\}, 1 \le i \le n.$
- 5: P_1 sends to P_2 the vector \mathbf{s}' .
- 6: P_M sends to P_2 the vector Θ' in which each $\Theta(i)$ is randomly permuted.
- 7: For all $1 \le i \le n$, P_2 sets b(i) = 0 if $s'(i) \in \Theta'(i)$ and b(i) = 1 otherwise.
- 8: P_2 broadcasts the vector $\mathbf{b} = (b(1), \dots, b(n))$.

We refer hereinafter to the combination of Protocols THRESHOLD and SET-INC as Protocol THRESHOLD-C; namely, it is Protocol THRESHOLD where the inequality verifications in Steps 6-8 are carried out by Protocol SETINC. Then our claims are as follows:

Theorem 1. Assume that the M > 2 players are semi-honest and that the keyed hash function in Protocol SETINC is preimage-resistant. Then:

- (a) Protocol THRESHOLD-C is correct (i.e., it computes the threshold function). (b) Let $C \subset \{P_1, P_2, \ldots, P_M\}$ be a coalition of players.
 - (i) If $P_2 \notin C$ and at least one of P_1 and P_M is not in C either, then Protocol THRESHOLD-C is perfectly private with respect to C.
 - (ii) If $P_2 \in C$ but $P_1, P_M \notin C$, the protocol is computationally private with respect to C.
 - (iii) Otherwise, C includes at least two of the players P_1, P_2, P_M ; such coalitions may learn the sum $\mathbf{a} = \sum_{m=1}^{M} \mathbf{b}_m$, but no further information beyond the sum.

Proof.

(a) Protocol THRESHOLD operates correctly if the inequality verifications in Step 7 are carried out correctly, since $(s(i) + s_M(i)) \mod (M + 1)$ equals the *i*th component a(i) in the sum vector $\mathbf{a} = \sum_{m=1}^{M} \mathbf{b}_m$. The inequality verification is correct if Protocol SETINC is correct. The latter protocol is indeed correct if the randomly selected K and r are such that for all $1 \leq i \leq n$, the sets $\Theta'(i) = \{h_K(r, i, \theta) : \theta \in \Theta(i)\}$ and $\Theta''(i) = \{h_K(r, i, \theta) : \theta \in \Omega \setminus \Theta(i)\}$ are disjoint. (As discussed earlier, such a verification can be carried out upfront, and most all selections of K and r are expected to pass that test.)

(b) Any single player P_{ℓ} , $1 \leq \ell \leq M$, learns in the course of the protocol his share \mathbf{s}_{ℓ} of the sum \mathbf{a} in a *M*-out-of-*M* secret sharing scheme for \mathbf{a} (see Step 3 in Protocol THRESHOLD). Two players learn more information: P_1 receives the shares $\mathbf{s}_2, \ldots, \mathbf{s}_{M-1}$ (Step 4 in Protocol THRESHOLD) and P_2 receives the signatures \mathbf{s}' and $\mathbf{\Theta}'$ during Protocol SETINC.

- 10 T. Tassa
 - (i) If $P_2, P_1 \notin C$ then the players in C have, at most, the shares $\mathbf{s}_3, \ldots, \mathbf{s}_M$. Since the secret sharing scheme is perfect, any number of shares which is less than M reveals no information on \mathbf{a} , since the missing shares were chosen at random. If $P_2, P_M \notin C$, then the worst scenario is that in which $P_1 \in C$; in that case, the coalition knows the shares $\mathbf{s}_1, \ldots, \mathbf{s}_{M-1}$. Once again, as the missing share \mathbf{s}_M was chosen at random by P_M , the shares $\mathbf{s}_1, \ldots, \mathbf{s}_{M-1}$ reveal no information on \mathbf{a} .
- (ii) If $P_2 \in C$ and $P_1, P_M \notin C$, then C has at most the M 2 additive shares $\mathbf{s}_2, \ldots, \mathbf{s}_{M-1}$. The additional knowledge that P_2 holds enables, in theory, the recovery of the missing shares \mathbf{s}_1 and \mathbf{s}_M and then the recovery of $\mathbf{a} = \sum_{\ell=1}^{M} \mathbf{s}_{\ell}$. Indeed, by scanning all possible keys K of the keyed hash function and all possible random strings r, P_2 may find a key K and a string r for which the signature values that he got from P_1 and P_M (namely, \mathbf{s}' and $\mathbf{\Theta}'$) are consistent with the signature scheme and the elements of $\Omega = \{0, 1, \ldots, M\}$. Hence, the protocol does not provide perfect privacy in the information-theoretic sense with respect to such coalitions. However, since such a computation is infeasible, and as the hash function is preimage-resistant, the protocol provides computational privacy with respect to such coalitions.
- (iii) If $P_1, P_M \in C$, then by adding \mathbf{s} (known to P_1) and \mathbf{s}_M (known to P_M), they will get the sum \mathbf{a} . No further information on the input vectors $\mathbf{b}_1, \ldots, \mathbf{b}_M$ may be deduced from the inputs of the players in such a coalition; specifically, every set of vectors $\mathbf{b}_1, \ldots, \mathbf{b}_M$ that is consistent with the sum \mathbf{a} is equally likely. Coalitions C that include either P_1, P_2 or P_2, P_M can also recover \mathbf{a} . Indeed, P_2 knows \mathbf{s}' and Θ' and P_1 or P_M knows h_K , K and r. Hence, a coalition of P_2 with either P_1 or P_M may recover from those values the preimages \mathbf{s} and Θ . Hence, such a coalition can recover \mathbf{s} and \mathbf{s}_M , and consequently \mathbf{a} . As argued before, the shares available for such coalitions do not reveal any further information about the input vectors $\mathbf{b}_1, \ldots, \mathbf{b}_M$.

The susceptibility of Protocol THRESHOLD-C to coalitions is not very significant because of two reasons:

- The entries of the sum vector **a** do not reveal information about specific input vectors. Namely, knowing that a(i) = p only indicates that p out of the M bits $b_m(i)$, $1 \le m \le M$, equal 1, but it reveals no information regarding which of the M bits are those.
- There are only three players that can collude in order to learn information beyond the intention of the protocol. Such a situation is far less severe than a situation in which any player may participate in a coalition, since if it is revealed that a collusion took place, there is a small set of suspects.

$\mathbf{2.3}$ An improved protocol for the secure computation of all locally frequent itemsets

As before, we denote by F_s^{k-1} the set of all globally frequent (k-1)-itemsets, and by $Ap(F_s^{k-1})$ the set of k-itemsets that the Apriori algorithm generates when applied on F_s^{k-1} . All players can compute that set and decide on an ordering of it. (Since all itemsets are subsets of $A = \{a_1, \ldots, a_L\}$, they may be viewed as binary vectors in $\{0,1\}^L$ and, as such, they may be ordered lexicographically.) Then, since the sets of locally frequent k-itemsets, $C_s^{k,m}$, $1 \le m \le M$, are subsets of Ap (F_s^{k-1}) , they may be encoded as binary vectors of length $n_k := |Ap(F_s^{k-1})|$. The binary vector that encodes the union $C_s^k := \bigcup_{m=1}^M C_s^{k,m}$ is the OR of the vectors that encode the sets $C_s^{k,m}$, $1 \le m \le M$. Hence, the players can compute the union by invoking Protocol THRESHOLD-C on their binary input vectors. This approach is summarized in Protocol 4 (UNIFI). (Replacing T_1 with T_M in Step 2 will result in computing the intersection of the private subsets.)

Protocol 4 (UNIFI) Unifying lists of locally Frequent Itemsets

Input: Each player P_m has an input subset $C_s^{k,m} \subseteq Ap(F_s^{k-1}), 1 \le m \le M$. **Output:** $C_s^k = \bigcup_{m=1}^M C_s^{k,m}$.

1: Each player P_m encodes his subset $C_s^{k,m}$ as a binary vector \mathbf{b}_m of length $n_k =$ $|Ap(F_s^{k-1})|$, in accord with the agreed ordering of $Ap(F_s^{k-1})$.

2: The players invoke Protocol THRESHOLD-C to compute $\mathbf{b} = T_1(\mathbf{b}_1, \dots, \mathbf{b}_M) =$ $\bigvee_{m=1}^{M} \mathbf{b}_{m}.$ 3: C_{s}^{k} is the subset of $Ap(F_{s}^{k-1})$ that is described by **b**.

2.4Privacy

We begin by analyzing the privacy offered by Protocol UNIFI-KC. That protocol does not respect perfect privacy since it reveals to the players information that is not implied by their own input and the final output. In Step 11 of Phase 1 of the protocol, each player augments the set X_m by fake itemsets. To avoid unnecessary hash and encryption computations, those fake itemsets are random strings in the ciphertext domain of the chosen commutative cipher. The probability of two players selecting random strings that will become equal at the end of Phase 1 is negligible; so is the probability of Player P_m to select a random string that equals $E_{K_m}(h(x))$ for a true itemset $x \in Ap(F_s^{k-1})$. Hence, every encrypted itemset that appears in two different lists indicates with high probability a true itemset that is locally s-frequent in both of the corresponding sites. Therefore, Protocol UNIFI-KC reveals the following excess information:

- (1) P_1 may deduce for any subset of the even players, the number of itemsets that are locally supported by all of them.
- (2) P_2 may deduce for any subset of the odd players, the number of itemsets that are locally supported by all of them.

11

- 12 T. Tassa
- (3) P_1 may deduce the number of itemsets that are supported by at least one odd player and at least one even player.
- (4) If P_1 and P_2 collude, they reveal for any subset of the players the number of itemsets that are locally supported by all of them.

As for the privacy offered by Protocol UNIFI, we consider two cases: If there are no collusions, then, by Theorem 1, Protocol UNIFI offers perfect privacy with respect to all players P_m , $m \neq 2$, and computational privacy with respect to P_2 . This is a privacy guarantee better than that offered by Protocol UNIFI-KC, since the latter protocol does reveal information to P_1 and P_2 even if they do not collude with any other player.

If there are collusions, both Protocols UNIFI-KC and UNIFI allow the colluding parties to learn forbidden information. In both cases, the number of "suspects" is small — in Protocol UNIFI-KC only P_1 and P_2 may benefit from a collusion while in Protocol UNIFI only P_1 , P_2 and P_M can extract additional information if two of them collude (see Theorem 1). In Protocol UNIFI-KC, the excess information which may be extracted by P_1 and P_2 is about the number of common frequent itemsets among any subset of the players. Namely, they may learn that, say, P_2 and P_3 have many itemsets that are frequent in both of their databases (but not which itemsets), while P_2 and P_4 have very few itemsets that are frequent in their corresponding databases. The excess information in Protocol UNIFI is different: If any two out of P_1 , P_2 and P_M collude, they can learn the sum of all private vectors. That sum reveals for each specific itemset in $Ap(F_s^{k-1})$ the number of sites in which it is frequent, but not which sites. Hence, while the colluding players in Protocol UNIFI-KC can distinguish between the different players and learn about the similarity or dissimilarity between them, Protocol UNIFI leaves the partial databases totally indistinguishable, as the excess information that it leaks is with regard to the itemsets only.

To summarize, given that Protocol UNIFI reveals no excess information when there are no collusions, and, in addition, when there are collusions, the excess information still leaves the partial databases indistinguishable, it offers enhanced privacy preservation in comparison to Protocol UNIFI-KC.

2.5 Communication cost

Here and in the next section we analyze the communication and computational costs of Protocols UNIFI-KC and UNIFI. In doing so, we let $n_k := |Ap(F_s^{k-1})|$ and $n := \sum_{k=2}^{L} n_k$; also, the *k*th iteration refers to the iteration in which F_s^k is computed from F_s^{k-1} ($2 \le k \le L$).

We start with Protocol UNIFI-KC. Let t denote the number of bits required to represent an itemset. Clearly, t must be at least $\log_2 n_k$ for all $2 \le k \le L$. However, as Protocol UNIFI-KC hashes the itemsets and then encrypts them, tshould be at least the recommended ciphertext length in commutative ciphers. RSA [17], Pohlig-Hellman [16] and ElGamal [7] ciphers are examples of commutative encryption schemes. As the recommended length of the modulus in all of them is at least 1024 bits, we take t = 1024.

During Phase 1 of Protocol UNIFI-KC, there are M-1 rounds of communication, where in each one of them each of the M players sends to the next player a message; the length of that message in the kth iteration is tn_k . Hence, the communication cost of this phase in the kth iteration is $(M-1)Mtn_k$. During Phase 2 of the protocol all odd players send their encrypted itemsets to P_1 and all even players send their encrypted itemsets to P_2 . Then P_2 unifies the itemsets he got and sends them to P_1 . Hence, this phase takes 2 more rounds and its communication cost in the kth iteration is bounded by $1.5Mtn_k$. In the last phase a similar round of decryptions is initiated. The unified list of all encrypted true and fake itemsets may contain in the kth iteration up to Mn_k itemsets. Hence, that phase involves M - 1 rounds with communication cost of no more than $(M-1)Mtn_k$. To summarize: Protocol UNIFI-KC entails 2M communication rounds (in each of the iterations) and the communication cost in the kth iteration is $\Theta(M^2 t n_k)$. (In fact, since the majority of the itemsets are expected to be fake itemsets, the communication cost in the decryption phase is close to $(M-1)Mtn_k$ and then the overall communication cost is roughly $2M^2tn_k$.)

We now turn to analyze the communication cost of Protocol UNIFI. It consists of 3 communication rounds: One for Step 2 of Protocol THRESHOLD that it invokes, one for Step 4 of that protocol, and one for the threshold verifications in Steps 6-8 (in which Protocol SETINC is invoked). Hence, it improves upon Protocol UNIFI-KC that entails 2M communication rounds.

In the kth iteration, the length of the vectors in Protocol THRESHOLD is n_k ; each entry in those vectors represents a number between 0 and M - 1, whence it may be encoded by $\log_2 M$ bits. Therefore:

- The communication cost of Step 2 in Protocol THRESHOLD is $M(M 1)(\log_2 M)n_k$ bits.
- The communication cost of Step 4 in Protocol THRESHOLD is $(M-2)(\log_2 M)n_k$ bits.
- Steps 6-8 in Protocol THRESHOLD are carried out by invoking Protocol SET-INC. The communication cost of Steps 5-6 in the latter protocol is $2|h|n_k$, where |h| is the size in bits of the hash function's output. (Recall that when Protocol SETINC is called in the framework of Protocol THRESHOLD-C, the size of each of the sets $\Theta(i)$ is 1.)

Hence, the overall communication cost of Protocol THRESHOLD-C in the kth iteration is $((M^2 - 2)(\log_2 M) + 2|h|)n_k$ bits.

As discussed earlier, a plausible setting of t would be t = 1024. A typical value of |h| is 160. With those parameters, the improvement factor in terms of communication cost, as offered by Protocol UNIFI with respect to Protocol UNIFI-KC, is approximately

$$\frac{2M^2 t n_k}{(M^2 \log_2 M + 2|h|) n_k} = \left(\frac{\log_2 M}{2t} + \frac{|h|}{M^2 t}\right)^{-1} = \left(\frac{\log_2 M}{2048} + \frac{0.15625}{M^2}\right)^{-1} \,.$$

For M = 4 we get an improvement factor of roughly 93, while for M = 8 we get a factor of about 256.

2.6 Computational cost

In Protocol UNIFI-KC each of the players needs to perform hash evaluations as well as encryptions and decryptions. As the cost of hash evaluations is significantly smaller than the cost of commutative encryption, we focus on the cost of the latter operations. In Steps 9-11 of the protocol, player P_m performs $|C_s^{k,m}| \leq n_k = |Ap(F_s^{k-1})|$ encryptions (in the kth iteration). Then, in Steps 14-18, each player performs M-1 encryptions of sets that include n_k items. Hence, in Phase 1 in the kth iteration, each player performs between $(M-1)n_k$ and Mn_k encryptions. In Phase 3, each player decrypts the set of items EC_s^k . EC_s^k is the union of the encrypted sets from all M players, where each of those sets has n_k items — true and fake ones. Clearly, the size of EC_s^k is at least n_k . On the other hand, since most of the items in the M sets are expected to be fake ones, and the probability of collusions between fake items is negligible, it is expected that the size of EC_s^k would be close to Mn_k . So, in all its iterations $(2 \leq k \leq L)$, Protocol UNIFI-KC requires each player to perform an overall number of close to 2Mn (but no less than Mn) encryptions or decryptions, where, as before $n = \sum_{k=2}^{L} n_k$. Since commutative encryption is typically based on modular exponentiation, the overall computational cost of the protocol is $\Theta(Mt^3n)$ bit operations per player.

In Protocol THRESHOLD, which Protocol UNIFI calls, each player needs to generate (M-1)n (pseudo)random $(\log_2 M)$ -bit numbers (Step 1). Then, each player performs (M-1)n additions of such numbers in Step 1 as well as in Step 3. Player P_1 has to perform also (M-2)n additions in Step 5. Therefore, the computational cost for each player is $\Theta(Mn\log_2 M)$ bit operations. In addition, Players 1 and M need to perform n hash evaluations.

Estimating the practical gain in computation time Here, we estimate the practical gain in computation time, as offered by Protocol UNIFI in comparison to Protocol UNIFI-KC. We adopt the same estimation methodology that was used in [12, Section 6.2]. We measured the times to perform the basic operations used by the two protocols on an Intel(R) Core(TM)2 Quad CPU 2.67 GHz personal computer with 8 GB of RAM:

- Modular addition took (on average) 0.762 μ s (microseconds);
- random byte generation took 0.0126 $\mu {\rm s};$
- equality verification between two 160-bit values took 0.13 μ s;
- modular exponentiation with modulus of t = 1024 bits took 12251 μ s; and
- computing HMAC on an input of 512 bits took on average 15.7 μ s.

As in [12], we estimate the added computational cost due to the secure computations in the two protocols when implemented in the experimental setting that was used in [6]. In that experimental setting, the number of sites was M = 3, the number of items was L = 1000 and the unified database contained N = 500,000transactions. Using a support threshold of s = 0.01, [6] reports that $n = \sum_{k=2}^{L} n_k$ was just over 100,000. In Protocol UNIFI-KC each player performs roughly 2Mn encryptions and decryptions. Hence, the corresponding encryption time per player is $2\cdot 3\cdot 100,000\cdot$ $12251\ \mu$ s in this setting, i.e., approximately 7350 seconds. In comparison, Protocol UNIFI requires each player to generate $(M-1)n\log_2 M$ pseudo random bits $(50,000\ \text{bytes}$ in this setting, which mean $630\ \mu$ s), and perform (2M-2)n additions (400,000 modular additions in this case, which mean $0.305\ \text{seconds}$). P_1 has to perform (M-2)n additional additions $(0.076\ \text{seconds})$; P_1 and P_M need to perform $100,000\ \text{HMAC}$ computations $(1.57\ \text{seconds})$; and $P_2\ \text{performs}\ 100,000$ verifications of equality between 160-bit values $(0.013\ \text{seconds})$. The overall computational overhead for the least busy player (P_2) is $0.318\ \text{seconds}$ and for the busiest player (P_1) is $1.951\ \text{seconds}$. Hence, compared to $7350\ \text{seconds}$ for each player in Protocol UNIFI-KC, the improvement in computational time overhead is overwhelming.

3 Identifying the globally *s*-frequent itemsets

Protocol UNIFI-KC yields the set C_s^k that consists of all itemsets that are locally *s*-frequent in at least one site. Those are the *k*-itemsets that have potential to be also globally *s*-frequent. In order to reveal which of those itemsets is globally *s*-frequent there is a need to securely compute the support of each of those itemsets. That computation must not reveal the local support in any of the sites. Let *x* be one of the candidate itemsets in C_s^k . Then *x* is globally *s*-frequent if and only if

$$\Delta(x) := supp(x) - sN = \sum_{m=1}^{M} (supp_m(x) - sN_m) \ge 0.$$
 (4)

Kantarcioglu and Clifton considered two possible settings. If the required output includes all globally s-frequent itemsets, as well as the sizes of their supports, then the values of $\Delta(x)$ can be revealed for all $x \in C_s^k$. In such a case, those values may be computed using a secure summation protocol (e.g. [5]). The more interesting setting, however, is the one where the support sizes are not part of the required output. We proceed to discuss it.

Let q be an integer greater than 2N. Then, since $|\Delta(x)| \leq N$, the itemset $x \in C_s^k$ is s-frequent if and only if $\Delta(x) \mod q < q/2$. The idea is to verify that inequality by starting an implementation of the secure summation protocol of [5] on the private inputs $\Delta_m(x) := supp_m(x) - sN_m$, modulo q. In that protocol, all players jointly compute random additive shares of the required sum $\Delta(x)$ and then, by sending all shares to, say, P_1 , he may add them and reveal the sum. If, however, P_M withholds his share of the sum, then P_1 will have one random share, $s_1(x)$, of $\Delta(x)$, and P_M will have a corresponding share, $s_M(x)$; namely, $s_1(x) + s_M(x) = \Delta(x) \mod q$. It is then proposed that the two players execute the generic secure circuit evaluation of [21] in order to verify whether

$$(s_1(x) + s_M(x)) \mod q \le q/2.$$
 (5)

Those circuit evaluations may be parallelized for all $x \in C_s^k$.

We observe that inequality (5) holds if and only if

$$s_1(x) \in \Theta(x) := \{(j - s_M(x)) \mod q : 0 \le j \le \lfloor (q - 1)/2 \rfloor\}.$$
 (6)

As $s_1(x)$ is known only to P_1 while $\Theta(x)$ is known only to P_M , the verification of the set inclusion in (6) can theoretically be carried out by means of Protocol SETINC. However, the ground set Ω in this case is \mathbb{Z}_q , which is typically a large set. (Recall that when Protocol SETINC is invoked from UNIFI, the ground set Ω is \mathbb{Z}_{M+1} , which is typically a small set.) Hence, Protocol SETINC is not useful in this case, and, consequently, Yao's generic protocol remains, for the moment, the simplest way to securely verify inequality (5). Yao's protocol is designed for the two-party case. In our setting, as M > 2, there exist additional semi-honest players. (This is the assumption on which Protocol SETINC relies.) An interesting question which arises in this context is whether the existence of such additional semi-honest players may be used to verify inequalities like (5), even when the modulus is large, without resorting to costly protocols such as oblivious transfer.

4 Identifying all (s, c)-association rules

Once all s-frequent itemsets are found (F_s) , we may proceed to look for all (s, c)-association rules (rules with support at least sN and confidence at least c). For $X, Y \in F_s$, the corresponding association rule $X \Rightarrow Y$ has confidence at least c if and only if $supp(X \cup Y)/supp(X) \ge c$, or, equivalently,

$$C_{X,Y} := \sum_{m=1}^{M} \left(supp_m(X \cup Y) - c \cdot supp_m(X) \right) \ge 0.$$
(7)

Since $|C_{X,Y}| \leq N$, then by taking q = 2N + 1, the players can verify inequality (7), in parallel, for all candidate association rules, as described in Section 3.

5 Related work

Previous work in privacy preserving data mining has considered two related settings. One, in which the data owner and the data miner are two different entities, and another, in which the data is distributed among several parties who aim to jointly perform data mining on the unified corpus of data that they hold.

In the first setting, the goal is to protect the data records from the data miner. Hence, the data owner aims at anonymizing the data prior to its release. The main approach in this context is to apply data perturbation [1,8]. The idea is that the perturbed data can be used to infer general trends in the data, without revealing original record information.

In the second setting, the goal is to perform data mining while protecting the data records of each of the data owners from the other data owners. This is a problem of secure multi-party computation. The usual approach here is cryptographic rather than probabilistic. Lindell and Pinkas [14] showed how to securely

build an ID3 decision tree when the training set is distributed horizontally. Lin et al. [13] discussed secure clustering using the EM algorithm over horizontally distributed data. The problem of distributed association rule mining was studied in [20,22] in the vertical setting, where each party holds a different set of attributes, and in [12] in the horizontal setting. Also the work of [18] considered this problem in the horizontal setting, but they considered large-scale systems in which, on top of the parties that hold the data records (resources) there are also managers which are computers that assist the resources to decrypt messages; another assumption made in [18] that distinguishes it from [12] and the present study is that no collusions occur between the different network nodes resources or managers.

6 Conclusion

We proposed a protocol for secure mining of association rules in horizontally distributed databases that improves significantly upon the current leading protocol [12] in terms of privacy and efficiency. One of the main ingredients in our proposed protocol is a novel secure multi-party protocol for computing the union (or intersection) of private subsets that each of the interacting players hold. Another ingredient is a protocol that tests the inclusion of an element held by one player in a subset held by another. The latter protocol exploits the fact that the underlying problem is of interest only when the number of players is greater than two.

One research problem that this study suggests was described in Section 3; namely, to devise an efficient protocol for set inclusion verification that uses the existence of a semi-honest third party. Such a protocol might enable to further improve upon the communication and computational costs of the second and third stages of the protocol of [12], as described in Sections 3 and 4. Another research problem that this study suggests is the extension of those techniques to the problem of mining generalized association rules [19].

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- 18 T. Tassa
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