Investment and Pricing with Spectrum Uncertainty: A Cognitive Operator's Perspective

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Abstract—This paper studies the optimal investment and pricing decisions of a cognitive mobile virtual network operator (C-MVNO) under *spectrum supply uncertainty*. Compared with a traditional MVNO who often leases spectrum via long-term contracts, a C-MVNO can acquire spectrum dynamically in short-term by both *sensing* the empty "spectrum holes" of licensed bands and *dynamically leasing* from the spectrum owner. As a result, a C-MVNO can make flexible investment and pricing decisions to match the current demands of the secondary unlicensed users. Compared to dynamic spectrum leasing, spectrum sensing is typically cheaper, but the obtained useful spectrum amount is random due to primary licensed users' stochastic traffic. The C-MVNO needs to determine the optimal amounts of spectrum sensing and leasing by evaluating the trade off between cost and uncertainty. The C-MVNO also needs to determine the optimal price to sell the spectrum to the secondary unlicensed users, taking into account wireless heterogeneity of users such as different maximum transmission power levels and channel gains. We model and analyze the interactions between the C-MVNO and secondary unlicensed users as a Stackelberg game. We show several interesting properties of the network equilibrium, including threshold structures of the optimal investment and pricing decisions, the independence of the optimal price on users' wireless characteristics, and guaranteed fair and predictable QoS among users. We prove that these properties hold for general SNR regime and general continuous distributions of sensing uncertainty. We show that spectrum sensing can significantly improve the C-MVNO's expected profit and users' payoffs.

Index Terms—Cognitive radio, spectrum trading, spectrum sensing, dynamic spectrum leasing, spectrum pricing, Stackelberg game, Subgame Perfect equilibrium.

1 INTRODUCTION

WIRELESS spectrum is typically considered as a scarce resource, and is traditionally allocated through static licensing. Field measurements show that, however, most spectrum bands are often under-utilized even in densely populated urban areas ([2]). To achieve more efficient spectrum utilization, people have proposed various dynamic spectrum access approaches including hierarchical-access and dynamic exclusive use ([3]-[7]). Hierarchical-access allows a secondary (unlicensed) network operator or users to opportunistically access the spectrum without affecting the normal operation of the spectrum owner who serves the primary (licensed) users. Dynamic exclusive use allows a spectrum owner to dynamically transfer and trade the usage right of its licensed spectrum to a third party (e.g., a secondary network operator or a secondary end-user) in the spectrum market. This paper considers a secondary operator who obtains spectrum resource via both spectrum sensing as in the hierarchical-access approach and *dynamic spectrum leasing* as in the dynamic exclusive use approach.

Spectrum sensing obtains awareness of the spectrum usage and existence of primary users, by using geolocation and database, beacons, or cognitive radios (e.g., [8]–[11]). The primary users are oblivious to the presence of secondary cognitive network operators or users. The secondary network operator or users can sense and utilize the unused "spectrum holes" in the licensed spectrum without violating the usage rights of the primary users (e.g., [4], [7]). Since the secondary operator or users does not know the primary users' activities before sensing, the amount of useful spectrum obtained through sensing is uncertain (e.g. [12]–[15]).

With dynamic spectrum leasing, a spectrum owner allows secondary users to operate in their temporarily unused part of spectrum in exchange of economic return (e.g., [5], [7], [16]). The dynamic spectrum leasing can be short-term or even real-time (e.g., [17]–[19]), and can be at a similar time scale of the spectrum sensing operation.

In this paper, we study the operation of a cognitive radio network that consists a cognitive mobile virtual network operator (C-MVNO) and a group of secondary unlicensed users. The word "virtual" refers to the fact that the operator does not own the wireless spectrum bands or even the physical network infrastructure. The C-MVNO serves as the interface between the spectrum owner and the secondary end-users. The word "cognitive" refers to the fact that the operator can obtain spectrum resource through both spectrum sensing using the cognitive radio technology and dynamic spectrum leasing from the spectrum owner. The operator then

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resells the obtained spectrum (bandwidth) to secondary users to maximize its profit. The proposed model is a hybrid of the hierarchical-access and dynamic exclusive use models. It is applicable in various network scenarios, such as achieving efficient utilization of the TV spectrum in IEEE 802.22 standard [20]. This standard suggests that the secondary system should operate on a point-to-multipoint basis, i.e., the communications will happen between secondary base stations and secondary customer-premises equipment. The base stations can be operated by one or several C-MVNOs introduced in this paper.

Compared with a traditional MVNO who only leases spectrum through long-term contracts, a C-MVNO can dynamically adjust its sensing and leasing decisions to match the changes of users' demand at a short time scale. Moreover, sensing often offers a cheaper way to obtain spectrum compared with leasing. The cost of sensing mainly includes the sensing time and energy, and does not include explicit cost paid to the spectrum owner. With a mature spectrum sensing technology, sensing cost should be reasonable low (otherwise there is no point of using cognitive radio). Spectrum leasing, however, involves direct negotiation with the spectrum owner. When the spectrum owner determines the cost of leasing, it needs to calculate its opportunity cost, i.e., how much revenue the spectrum can provide if the spectrum owner provides services directly over it. It is reasonable to believe that the leasing cost is more expensive than the sensing cost in most cases¹. Although sensing is cheaper, the amount of spectrum obtained through sensing is often uncertain due to the stochastic nature of primary users' traffic. It is thus critical for a C-MVNO to find the right balance between cost and uncertainty.

Our key results and contributions are summarized as follows. For simplicity, we refer to the C-MVNO as "operator", secondary users as "users", and "dynamic leasing" as "leasing".

- A Stackelberg game model: We model and analyze the interactions between the operator and the users in the spectrum market as a Stackelberg game. As the leader, the operator makes the sensing, leasing, and pricing decisions sequentially. As the followers, users then purchase bandwidth from the operator to maximize their payoffs. By using backward induction, we prove the existence and uniqueness of the equilibrium, and show how various system parameters (i.e., sensing and leasing costs, users' transmission power and channel conditions) affect the equilibrium behavior.
- Threshold structures of the optimal investment and pricing decisions: At the equilibrium, the operator will sense the spectrum only if the sensing cost is cheaper than a threshold. Furthermore, it will lease some spectrum only if the resource obtained

1. The analysis of this paper also covers the case where sensing is more expensive than leasing, which is a trivial case to study.

through sensing is below a threshold. Finally, the operator will charge a constant price to the users if the total bandwidth obtained through sensing and leasing does not exceed a threshold. The thresholds are easy to compute and the corresponding decisions rules are easy to implement in practice.

- *Fair and predictable QoS*: The operator's optimal pricing decision is independent of the users' wireless characteristics. Each user receives a payoff that is proportional to its channel gain and transmission power, which leads to the same signal-to-noise (SNR) for all users.
- *Impact of spectrum sensing*: We show that the availability of sensing always increases the operator's profit in the *expected* sense. The actual realization of the profit at a particular time heavily depends on the spectrum sensing results. Users always get better payoffs when the operator performs spectrum sensing.

Section 2 introduces the network model and problem formulation. In Section 3, we analyze the game model through backward induction. We discuss various insights obtained from the equilibrium analysis and present some numerical results in Section 4. In Section 5, we show the impact of spectrum sensing on both the operator and users. We conclude in Section 6 and outline some future research directions.

1.1 Related Work

There is a growing interest in studying the investment and pricing decisions of cognitive network operators recently. Several auction mechanisms have been proposed to study the investment problems of cognitive network operators (e.g., [21], [22]). Other recent results studied the pricing decisions of the cognitive network operators who interact with a group of secondary users (e.g., [23]-[30]). [21] considered users' queueing delays and obtained most results through simulations. [23] presented a recent survey on the spectrum sharing games of network operators and cognitive radio networks. [24] studied the competition among multiple service providers without modeling users' wireless details. [25] considered a pricing competition game of two operators and adopted a simplified wireless model for the users. [26] derived users' demand functions based on the acceptance probability model for the users. [27] explored demand functions based on both quality-sensitive and price-sensitive buyer population models. [28] formulated the interaction between one primary user (monopolist) and multiple secondary users as a Stackelberg game. The primary user uses some secondary users as relays and leases its bandwidth to those relays to collect revenue. [29] studied a multiple-level spectrum market among primary, secondary, and tertiary services where global information is not available. [30] considered the short-term spectrum trading between multiple primary users and multiple secondary users. The spectrum buy-



Fig. 1. Operator's Investment in Spectrum Sensing and Leasing

ing behaviors of secondary users are modeled as an evolutionary game, while selling behaviors of primary users are modeled as a noncooperative game. [26]–[30] obtained most interesting results through simulations. There are only few papers (e.g., [19], [29], [31]) that jointly considered the spectrum investment and service pricing problem as this paper. None of the above work considered the impact of supply uncertainty due to spectrum sensing.

Our model of spectrum uncertainty is related to the random-yield model in supply chain management (e.g., [32]–[34]). The unique wireless aspects of the system model lead to new solutions and insights in our problem.

Our paper represents a first attempt of understanding how spectrum uncertainty impacts the economic decisions of an cognitive radio operator. To obtain sharp insights, we focus on a stylized model where a monopolist operator faces a group of secondary users. There are many more interesting research issues in this area. Some are further discussed in Section 6.

2 NETWORK MODEL

2.1 Background on Spectrum Sensing and Leasing

To illustrate the opportunity and trade-off of spectrum sensing and leasing, we consider a spectrum owner who divides its licensed spectrum into two types:

- *Service Band*: This band is reserved for serving the spectrum owner's primary users (PUs). Since the PUs' traffic is stochastic, there will be some unused spectrum which changes dynamically. The operator can sense and utilize the unused portions. There are no explicit communications between the spectrum owner and the operator.
- *Transference Band*: The spectrum owner temporarily does not use this band. The operator can lease the bandwidth through explicit communications with the spectrum owner. No sensing is allowed in this band.

Due to the short-term property of both sensing and leasing, the operator needs to make both the sensing and leasing decisions in each time slot.

The example in Fig. 1 demonstrates the dynamic opportunities for spectrum sensing, the uncertainty of sensing outcome, and the impact of sensing or leasing

TABLE 1 Key Notations

Symbol	Physical Meaning		
B_s	Sensing bandwidth		
B_l	Leasing bandwidth		
C_s	Unit sensing cost		
C_l	Unit leasing cost		
$\alpha \in [0, 1]$	Sensing realization factor		
$\mathcal{I} = \{1, \cdots, I\}$	Set of secondary users		
π	Unit price		
w_i	User <i>i</i> 's bandwidth allocation		
r_i	User <i>i</i> 's data rate		
P_i^{\max}	User <i>i</i> 's maximum transmission power		
h_i	User <i>i</i> 's channel gain		
n_0	Noise power density		
$g_i = P_i^{\max} h_i / n_0$	User <i>i</i> 's wireless characteristic		
$SNR_i = g_i/w_i$	User <i>i</i> 's SNR		
$G = \sum_{i \in \mathcal{I}} g_i$	Users' aggregate wireless characteristics		
R	Operator's profit		

decisions. The spectrum owner's entire band is divided into small 34 channels².

- Time slot 1: PUs use channels 1-4 and 11-15. The operator is unaware of this and senses channels 3-8. As a result, it obtains 4 unused channels (5 8). It leases additional 9 channels (20 28) from the transference band.
- Time slot 2: PUs change their behavior and use channels 1 6. The operator senses channels 5 14 and obtains 8 unused channels (7 14). It leases additional 5 channels (23 27) from the transference band.

In this paper, we will only study the operator's decisions within a single time slot. We choose the time slot length such that primary users' activities remain roughly unchanged within a single time slot. This means that it is enough for the operator to sense at the beginning of each time slot. For traffic types such as TV programs, data transfer, and even VoIP voice sessions, the length of the time slot can be reasonable long. For readers who are interested in the optimization of the time slot length to balance sensing and data transmission, see [14].

2.2 Notations and Assumptions

We consider a cognitive network with one operator and a set $\mathcal{I} = \{1, \ldots, I\}$ of users. The operator has the cognitive capability and can sense the unused spectrum. One way to realize this is to let the operator construct a sensor network that is dedicated to sensing the radio environment in space and time [35]. The operator will collect the sensing information from the sensor network and provide it to the unlicensed users, or providing "sensing as service". If the operator owns several base stations, then each base station is responsible for collecting sensing information in a certain geographical area. As mentioned in [35], there has been significant current

^{2.} Channel 16 is the guard band between the service and transference bands.



Fig. 2. A Stackelberg Game

research efforts in the context of an European project SENDORA [36], which aims at developing techniques based on sensor networks for supporting coexistence of licensed and unlicensed wireless users in a same area. The users are equipped with software defined radios and can tune to transmit in a wide range of frequencies as instructed by the operator, but do not necessarily have the cognitive sensing capacity. Since the secondary users do not worry about sensing, they can spend most of their time and energy on actual data transmissions. Such a network structure puts most of the implementation complexity at the operator side and reduces the user equipment complexity, and thus might be easier to implement in practice than a "full" cognitive network.

The key notations of this paper are listed in Table 1 with some explanations as follows.

- *Investment decisions* B_s and B_l: the operator's sensing and leasing bandwidths, respectively.
- Sensing realization factor α : when the operator senses a total bandwidth of B_s , only a proportion of $\alpha \in$ [0,1] is unused and can be used by the operator. α is a random variable and depends on the primary users' activities. With perfect sensing results, users can use bandwidth up to $B_s \alpha$ without generating interferences to the primary users.
- Cost parameters C_s and C_l : the operator's fixed sensing and leasing costs per unit bandwidth, respectively. Sensing cost C_s depends on the operator's sensing technologies. When the operator senses spectrum, it needs to spend time and energy on channel sampling and signal processing ([37]). Sensing over different channels often needs to be done sequentially due to the potentially large number of channels open to opportunistic spectrum access and the limited power/hardware capacity of cognitive radios ([38]). The larger sensing bandwidth and the more channels, the longer time and higher energy it requires ([39]). For simplicity, we assume that total sensing cost is linear in the sensing bandwidth B_s . Leasing cost C_l is determined through the negotiation between the operator and the spectrum owner and is assumed to be larger than C_s .
- *Pricing decision* π: the operator's choice of price per unit bandwidth to the users.

2.3 A Stackelberg Game

We consider a Stackelberg Game between the operator and the users as shown in Fig. 2. The operator is the Stackelberg leader: it first decides the sensing amount B_s in Stage I, then decides the leasing amount B_l in Stage II (based on the sensing result $B_s\alpha$), and then announces the price π to the users in Stage III (based on the total supply $B_s\alpha + B_l$). Finally, the users choose their bandwidth demands to maximize their individual payoffs in Stage IV.

We note that "sensing followed by leasing" is optimal for the operator to maximize profit. Since sensing is cheaper than leasing, the operator should lease only if sensing does not provide enough resource. If the operator determines sensing and leasing simultaneously, then it is likely to "over-lease" (compared with "sensing followed by leasing") to avoid having too little resource when α is small. "Leasing before sensing" can not improve the operator's profit either due to the same reason.

3 BACKWARD INDUCTION OF THE FOUR-STAGE GAME

The Stackelberg game falls into the class of dynamic game, and the common solution concept is the Subgame Perfect Equilibrium (SPE, or simply as *equilibrium* in this paper). A general technique for determining the SPE is the backward induction ([40]). We will start with Stage IV and analyze the users' behaviors given the operator's investment and pricing decisions. Then we will look at Stage III and analyze how the operator makes the pricing decision given investment decisions and the possible reactions of the users in Stage IV. Finally we proceed to derive the operator's optimal leasing decision in Stage II and then the optimal sensing decision in Stage I. The backward induction captures the sequential dependence of the decisions in four stages.

3.1 Spectrum Allocation in Stage IV

In Stage IV, end-users determine their bandwidth demands given the unit price π announced by the operator in stage III.

Each user can represent a transmitter-receiver node pair in an ad hoc network, or a node that transmits to the operator's base station in an uplink scenario. We assume that users access the spectrum provided by the operator through FDM (Frequency-division multiplexing) or OFDM (Orthogonal frequency-division multiplexing) to avoid mutual interferences. User *i*'s achievable rate (in nats) is³:

$$r_i(w_i) = w_i \ln\left(1 + \frac{P_i^{\max}h_i}{n_0 w_i}\right),\tag{1}$$

3. We assume that the operator only provides bandwidth without restricting the application types. This assumption has been commonly used in dynamic spectrum sharing literature, e.g., [16], [19], [24], [41].

where w_i is the allocated bandwidth from the operator, P_i^{\max} is user *i*'s maximum transmission power, n_0 is the noise power per unit bandwidth, h_i is user *i*'s channel gain (between user *i*'s own transmitter and receiver in an ad hoc network, or between user *i*'s transmitter to the operator's base station in an uplink scenario). To obtain rate in (1), user *i* spreads its maximum transmission power P_k^{\max} across the entire allocated bandwidth w_i . To simplify the notation, we let $g_i = P_i^{\max}h_i/n_0$, thus g_i/w_i is the user *i*'s signal-to-noise ratio (SNR). Here we focus on best-effort users who are interested in maximizing their data rates. Each user only knows its local information (i.e., P_i^{\max} , h_i , and n_0) and does not know anything about other users.

From a user's point of view, it does not matter whether the bandwidth has been obtained by the operator through spectrum sensing or dynamic leasing. Each unit of allocated bandwidth is perfectly reliable for the user.

To obtain closed-form solutions, we first focus on the high SNR regime where SNR \gg 1. This is motivated by the fact that users often have limited choices of modulation and coding schemes, and thus may not be able to decode a transmission if the SNR is below a threshold. In the high SNR regime, the rate in (1) can be approximated as

$$r_i(w_i) = w_i \ln\left(\frac{g_i}{w_i}\right). \tag{2}$$

Although the analytical solutions in Section 3 are derived based on (2), we emphasize that *all the major engineering insights remain true in the general SNR regime.* A formal proof is in Section 4.

A user *i*'s *payoff* is a function of the allocated bandwidth w_i and the price π ,

$$u_i(\pi, w_i) = w_i \ln\left(\frac{g_i}{w_i}\right) - \pi w_i, \tag{3}$$

i.e., the difference between the data rate and the linear payment (πw_i). Payoff $u_i(\pi, w_i)$ is concave in w_i , and the unique bandwidth *demand* that maximizes the payoff is

$$w_i^*(\pi) = \arg\max_{w_i \ge 0} u_i(\pi, w_i) = g_i e^{-(1+\pi)},$$
 (4)

which is always positive, linear in g_i , and decreasing in price π . Since g_i is linear in channel gain h_i and transmission power P_i^{\max} , then a user with a better channel condition or a larger transmission power has a larger demand.

Equation (4) shows that each user i achieves the same SNR:

$$\operatorname{SNR}_i = \frac{g_i}{w_i^*(\pi)} = e^{(1+\pi)}.$$

but a different payoff that is linear in g_i ,

$$u_i(\pi, w_i^*(\pi)) = g_i e^{-(1+\pi)}$$

We denote users' aggregate wireless characteristics as $G = \sum_{i \in \mathcal{I}} g_i$. The users' total demand is

$$\sum_{i \in \mathcal{I}} w_i^*(\pi) = G e^{-(1+\pi)}.$$
 (5)

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Fig. 3. Different intersection cases of $D(\pi)$ and $S(\pi)$

Next, we will consider how the operator makes the investment (sensing and leasing) and pricing decisions in Stages I-III based on the total demand in eq. $(5)^4$. In particular, we will show that the operator will always choose a price in Stage III such that the total demand (as a function of price) does not exceed the total supply.

3.2 Optimal Pricing Strategy in Stage III

In Stage III, the operator determines the optimal pricing considering users' total demand (5), given the bandwidth supply $B_s \alpha + B_l$ obtained in Stage II. The operator profit is

$$R(B_s, \alpha, B_l, \pi) = \min\left(\pi \sum_{i \in \mathcal{I}} w_i^*(\pi), \pi \left(B_l + B_s \alpha\right)\right) - \left(B_s C_s + B_l C_l\right), \quad (6)$$

which is the difference between the revenue and total cost. The min operation denotes the fact that the operator can only satisfy the demand up to its available supply. The objective of Stage III is to find the optimal price $\pi^* (B_s, \alpha, B_l)$ that maximizes the profit, that is,

$$R_{III}(B_s, \alpha, B_l) = \max_{\pi \ge 0} R(B_s, \alpha, B_l, \pi).$$
(7)

The subscript "III" denotes the best profit in Stage III.

Since the bandwidths B_s and B_l are given in this stage, the total cost $B_sC_s + B_lC_l$ is already fixed. The only optimization is to choose the optimal price π to maximize the revenue, i.e.,

$$\max_{\pi \ge 0} \min\left(\pi \sum_{i \in \mathcal{I}} w_i^*(\pi), \pi \left(B_l + B_s \alpha\right)\right).$$
(8)

The solution of problem (8) depends on the bandwidth investment in Stages I and II. Let us define $D(\pi) = \pi \sum_{i \in \mathcal{I}} w_i^*(\pi)$ and $S(\pi) = \pi (B_l + B_s \alpha)$. Figure 3 shows three possible relationships between these two terms, where $S_j(\pi)$ (for j = 1, 2, 3) represents each of the three possible choices of $S(\pi)$ depending on the bandwidth $B_l + B_s \alpha$:

• $S_1(\pi)$ (excessive supply): No intersection with $D(\pi)$;

^{4.} We assume that the operator knows the value of *G* through proper feedback mechanism from the users.

TABLE 2 Optimal Pricing Decision and Profit in Stage III

Total Bandwidth Obtained in Stages I and II	Optimal Price $\pi^*(B_s, \alpha, B_l)$	Optimal Profit $R_{III}(B_s, \alpha, B_l)$
Excessive Supply Regime: $B_l + B_s \alpha \ge Ge^{-2}$	$\pi^{ES} = 1$	$R_{III}^{ES}(B_s, \alpha, B_l) = Ge^{-2} - B_s C_s - B_l C_l$
Conservative Supply Regime: $B_l + B_s \alpha < Ge^{-2}$	$\pi^{CS} = \ln\left(\frac{G}{B_l + B_s \alpha}\right) - 1$	$R_{III}^{CS}(B_s, \alpha, B_l) = (B_l + B_s \alpha) \ln\left(\frac{G}{B_l + B_s \alpha}\right) - B_s(\alpha + C_s) - B_l(1 + C_l)$

- $S_2(\pi)$ (excessive supply): intersect once with $D(\pi)$ where $D(\pi)$ has a non-negative slope;
- S₃(π) (conservative supply): intersect once with D(π) where D(π) has a negative slope.

In the excessive supply regime, $\max_{\pi\geq 0} \min(S(\pi), D(\pi)) = \max_{\pi\geq 0} D(\pi)$, i.e., the max-min solution occurs at the maximum value of $D(\pi)$ with $\pi^* = 1$. In this regime, the total supply is larger than the total demand at the best price choice. In the *conservative supply* regime, the max-min solution occurs at the unique intersection point of $D(\pi)$ and $S(\pi)$. The above observations lead to the following result.

Theorem 1: The optimal pricing decision and the corresponding optimal profit at Stage III can be characterized by Table 2.

The proof of Theorem 1 is given in Appendix A. Note that in the excessive supply regime, some bandwidth is left unsold (i.e., $S(\pi^*) > D(\pi^*)$). This is because the acquired bandwidth is too large, and selling all the bandwidth will lead to a very low price that decreases the revenue (the product of price and sold bandwidth). The profit can be apparently improved if the operator acquires less bandwidth in Stages I and II. Later analysis in Stages II and I will show that the equilibrium of the game must lie in the conservative supply regime if the sensing cost is non-negligible.

3.3 Optimal Leasing Strategy in Stage II

In Stage II, the operator decides the optimal leasing amount B_l given the sensing result $B_s \alpha$:

$$R_{II}(B_s, \alpha) = \max_{B_l \ge 0} R_{III}(B_s, \alpha, B_l).$$
(9)

We decompose problem (9) into two subproblems based on the two supply regimes in Table 2,

 Choose *B_l* to reach the excessive supply regime in Stage III:

$$R_{II}^{ES}(B_s, \alpha) = \max_{B_l \ge \max\{Ge^{-2} - B_s \alpha, 0\}} R_{III}^{ES}(B_s, \alpha, B_l).$$
(10)

2) Choose *B*_l to reach the conservative supply regime in Stage III:

$$R_{II}^{CS}(B_s, \alpha) = \max_{0 \le B_l \le Ge^{-2} - B_s \alpha} R_{III}^{CS}(B_s, \alpha, B_l),$$
(11)

To solve subproblems (10) and (11), we need to consider the bandwidth obtained from sensing.

• *Excessive Supply* $(B_s \alpha > Ge^{-2})$: in this case, the feasible sets of both subproblems (10) and (11) are

empty. In fact, the bandwidth supply is already in the excessive supply regime as defined in Table II, and it is optimal not to lease in Stage II.

• Conservative Supply $(B_s \alpha \leq Ge^{-2})$: first, we can show that the unique optimal solution of subproblem (10) is $B_l^* = Ge^{-2} - B_s \alpha$. This means that the optimal objective value of subproblem (10) is no larger than that of subproblem (11), and thus it is enough to consider subproblem (11) in the conservative supply regime only.

Base on the above observations and some further analysis, we can show the following:

Theorem 2: In Stage II, the optimal leasing decision and the corresponding optimal profit are summarized in Table 3.

The proof of Theorem 2 is given in Appendix B. Table 3 contains three cases based on the value of $B_s\alpha$: (CS1), (CS2), and (ES3). The first two cases involve solving the subproblem (11) in the conservative supply regime, and the last one corresponds to the excessive supply regime. Although the decisions in cases (CS2) and (ES3) are the same (i.e., zero leasing amount), we still treat them separately since the profit expressions are different.

It is clear that we have an optimal *threshold* leasing policy here: the operator wants to achieve a total bandwidth equal to $Ge^{-(2+C_l)}$ whenever possible. When the bandwidth obtained through sensing is not enough, the operator will lease additional bandwidth to reach the threshold; otherwise the operator will not lease.

3.4 Optimal Sensing Strategy in Stage I

In Stage I, the operator will decide the optimal sensing amount to maximize its expected profit by taking the uncertainty of the sensing realization factor α into account. The operator needs to solve the following problem

$$R_I = \max_{B_s \ge 0} R_{II} \left(B_s \right)$$

where $R_{II}(B_s)$ is obtained by taking the expectation of α over the profit functions in Stage II (i.e., $R_{II}^{CS1}(B_s, \alpha)$, $R_{II}^{CS2}(B_s, \alpha)$, and $R_{II}^{ES3}(B_s, \alpha)$ in Table 3).

To obtain closed-form solutions, we assume that the sensing realization factor α follows a uniform distribution in [0, 1]. In Section 4.1, we prove that the major engineering insights also hold under any general distribution.

To avoid the trivial case where sensing is so cheap that it is optimal to sense a huge amount of bandwidth,

TABLE 3 Optimal Leasing Decision and Profit in Stage II

Given Sensing Result $B_s \alpha$ After Stage I	Optimal Leasing Amount B_l^*	Optimal Profit $R_{II}(B_s, \alpha)$
(CS1) $B_s \alpha \leq G e^{-(2+C_l)}$	$B_l^{CS1} = Ge^{-(2+C_l)} - B_s \alpha$	$R_{II}^{CS1}(B_s, \alpha) = Ge^{-(2+C_l)} + B_s(\alpha C_l - C_s)$
(CS2) $B_s \alpha \in (Ge^{-(2+C_l)}, Ge^{-2}]$	$B_l^{CS2} = 0$	$R_{II}^{CS2}(B_s, \alpha) = B_s \alpha \ln\left(\frac{G}{B_s \alpha}\right) - B_s(\alpha + C_s)$
(ES3) $B_s \alpha > Ge^{-2}$	$B_l^{ES3} = 0$	$R_{II}^{ES3}(B_s,\alpha) = Ge^{-2} - B_s C_s$



Fig. 4. Expected profit in Stage II under different sensing and leasing costs

we further assume that the sensing cost is non-negligible and is lower bounded by $C_s \ge (1 - e^{-2C_l})/4$.

To derive function $R_{II}(B_s)$, we will consider the following three intervals:

1) Case I: $B_s \in [0, Ge^{-(2+C_l)}]$. In this case, we always have $B_s \alpha \leq Ge^{-(2+C_l)}$ for any value $\alpha \in [0, 1]$, which corresponds to case (CS1) in Table 3. The expected profit is

R

which is a linear function of B_s . If $C_s > C_l/2$, $R_{II}^1(B_s)$ is linearly decreasing in B_s ; if $C_s < C_l/2$, $R_{II}^1(B_s)$ is linearly increasing in B_s .

R¹¹_{II}(B_s) is linearly increasing in B_s.
2) Case II: B_s ∈ (Ge^{-(2+C_l)}, Ge⁻²]. Depending on the value of α, B_sα can be in either case (CS1) or case (CS2) in Table 3. The expected profit is

$$R_{II}^{2}(B_{s}) = E_{\alpha \in \left[0, \frac{Ge^{-(2+C_{l})}}{B_{s}}\right]} \left[R_{II}^{CS1}(B_{s}, \alpha)\right] \\ + E_{\alpha \in \left[\frac{Ge^{-(2+C_{l})}}{B_{s}}, 1\right]} \left[R_{II}^{CS2}(B_{s}, \alpha)\right] \\ = \frac{B_{s}}{2} \ln\left(\frac{G}{B_{s}}\right) - \frac{B_{s}}{4} + \frac{B_{s}}{4} \left(\frac{Ge^{-(2+C_{l})}}{B_{s}}\right)^{2} - E_{s}$$

 $R_{II}^2(B_s)$ is a strictly concave function of B_s since its second-order derivative

$$\frac{\partial^2 R_{II}^2(B_s)}{\partial B_s^2} = \frac{1}{2B_s} \left[\left(\frac{Ge^{-(2+C_l)}}{B_s} \right)^2 - 1 \right] < 0$$

as $B_s > Ge^{-(2+C_l)}$ in this case.

Case III: B_s ∈ (Ge⁻²,∞). Depending on the value of α, B_sα can be any of the three cases in Table 3.

The expected profit is

$$\begin{split} R_{II}^{3}(B_{s}) = & E_{\alpha \in \left[0, \frac{Ge^{-(2+C_{l})}}{B_{s}}\right]} \left[R_{II}^{CS1}(B_{s}, \alpha)\right] \\ &+ E_{\alpha \in \left[\frac{Ge^{-(2+C_{l})}}{B_{s}}, \frac{Ge^{-2}}{B_{s}}\right]} \left[R_{II}^{CS2}(B_{s}, \alpha)\right] \\ &+ E_{\alpha \in \left[\frac{Ge^{-2}}{B_{s}}, 1\right]} \left[R_{II}^{ES3}(B_{s}, \alpha)\right] \\ &= \left(\frac{G}{e^{2}}\right)^{2} \frac{e^{-2C_{l}} - 1}{4B_{s}} - B_{s}C_{s} + \frac{G}{e^{2}}. \end{split}$$

Because its first-order derivative

$$\frac{\partial R_{II}^3(B_s)}{\partial B_s} = \left(\frac{Ge^{-2}}{B_s}\right)^2 \frac{1 - e^{-2C_l}}{4} - C_s < 0,$$

as $B_s > Ge^{-2}$ in this case, $R_{II}^3(B_s)$ is decreasing in B_s and achieves its maximum at $B_s = Ge^{-2}$.

To summarize, the operator needs to maximize

$$R_{II}(B_s) = \begin{cases} R_{II}^1(B_s), & \text{if } 0 \le B_s \le Ge^{-(2+C_l)}; \\ R_{II}^2(B_s), & \text{if } Ge^{-(2+C_l)} < B_s \le Ge^{-2}; \\ R_{II}^3(B_s), & \text{if } B_s > Ge^{-2}. \end{cases}$$
(12)

We can verify that Case II always achieves a higher optimal profit than Case III. This means that the optimal sensing will only lead to either case (CS1) or case (CS2) in Stage II, which corresponds to the conservative supply regime in Stage III. This confirms our previous intuition that equilibrium is always in the conservative supply regime under a non-negligible sensing cost, since some resource is wasted in the excessive supply regime (see discussions in Section 3.2).

Table 4 shows that the sensing decision is made in the following two cost regimes:

• High sensing cost regime $(C_s > C_l/2)$: it is optimal not to sense. Intuitively, the coefficient 1/2 is due to the uniform distribution assumption of α , i.e., on average obtaining one unit of available bandwidth through sensing costs $2C_s$.

 $B_sC_{\bullet}.Low$ sensing cost regime $(C_s \in \left[\frac{1-e^{-2C_l}}{4}, \frac{C_l}{2}\right])$: the optimal sensing amount B_s^{L*} is the unique solution to the following equation:

$$\frac{\partial R_{II}^2(B_s)}{\partial B_s} = \frac{1}{2} \ln \left(\frac{1}{B_s/G}\right) - \frac{3}{4} - C_s - \left(\frac{e^{-(2+C_l)}}{2B_s/G}\right)^2 = 0.$$
(13)

The uniqueness of the solution is due to the strict concavity of $R_{II}^2(B_s)$ over B_s . We can further show that B_s^{L*} lies in the interval of $[Ge^{-(2+C_l)}, Ge^{-2}]$

TABLE 4 Choice of Optimal Sensing Amount in Stage I

	Optimal Sensing Decision B_s^*	Expected Profit R_I
High Sensing Cost Regime: $C_s \ge C_l/2$	$B_{s}^{*} = 0$	$R_I^H = Ge^{-(2+C_l)}$
Low Sensing Cost Regime: $C_s \in \left[(1 - e^{-2C_l})/4, C_l/2\right]$	$B_s^* = B_s^{L*}$, solution to eq. (13)	R_I^L in eq. (14)

and is linear in *G*. Finally, the operator's optimal expected profit is

$$R_{I}^{L} = \frac{B_{s}^{L*}}{2} \ln\left(\frac{G}{B_{s}^{L*}}\right) - \frac{B_{s}^{L*}}{4} + \frac{1}{4B_{s}^{L*}} \left(\frac{G}{e^{2+C_{l}}}\right)^{2} - B_{s}^{L*}C_{s}$$
(14)

Based on these observations, we can show the following:

Theorem 3: In Stage I, the optimal sensing decision and the corresponding optimal profit are summarized in Table 4. The optimal sensing amount B_l^* is linear in G.

Figure 4 shows two possible cases for the function $R_{II}(B_s)$. The vertical dashed line represents $B_s = e^{-(2+C_l)}$. For illustration purpose, we assume G = 1, $C_l = 2$, and $C_s = \{0.8, 1.2\}$. When the sensing cost is large (i.e., $C_s = 1.2 > C_l/2$), $R_{II}(B_s)$ achieves its optimum at $B_s = 0$ and thus it is optimal not to sense. When the sensing cost is small (i.e., $C_s = 0.8 < C_l/2$), $R_{II}(B_s)$ achieves its optimum at $B_s > e^{-(2+C_l)}$ and it is optimal to sense a positive amount of spectrum.

4 EQUILIBRIUM SUMMARY AND NUMERICAL RESULTS

Based on the discussions in Section 3, we summarize the operator's equilibrium sensing/leasing/pricing decisions and the equilibrium resource allocations to the users in Table 5. Several interesting observations are as follows.

Observation 1: Both the optimal sensing amount B_s^* (either 0 or B_s^{L*}) and leasing amount B_l^* are linear in the users' aggregate wireless characteristics $G = \sum_{i \in \mathcal{I}} P_i^{\max} h_i / n_0$.

The linearity enables us to normalize optimal sensing and leasing decisions by users' aggregate wireless characteristics, and study the relationships between the normalized optimal decisions and other system parameters as in Figs. 5 and 6.

Figure 5 shows how the normalized optimal sensing decision B_s^*/G changes with the costs. For a given leasing cost C_l , the optimal sensing decision B_s^* decreases as the sensing cost C_s becomes more expensive, and drops to zero when $C_s \ge C_l/2$. For a given sensing cost C_s , the optimal sensing decision B_s^* increases as the leasing cost C_l becomes more expensive, in which case sensing becomes more attractive.

Figure 6 shows how the normalized optimal leasing decision B_l^*/G depends on the costs C_l and C_s as well as the sensing realization factor α in the low sensing cost regime (denoted by "L"). In all cases, a higher

value α means more bandwidth is obtained from sensing and there is a less need to lease. Figure 6 confirms the threshold structure of the optimal leasing decisions in Section 3.3, i.e., no leasing is needed whenever the bandwidth obtained from sensing reaches a threshold. Comparing different curves, we can see that the operator chooses to lease more as leasing becomes cheaper or sensing becomes more expensive. For high sensing cost regime, the optimal leasing amount only depends on C_l and is independent of C_s and α , and thus is not shown here.

Observation 2: The optimal pricing decision π^* in Stage III is independent of users' aggregate wireless characteristics *G*.

Observation 2 is closely related to Observation 1. Since the total bandwidth is linear in *G*, the "average" resource allocation per user is "constant" at the equilibrium. This implies that the price must be independent of the user population change, otherwise the resource allocation to each individual user will change with the price accordingly.

Observation 3: The optimal pricing decision π^* in Stage III is non-increasing in α in the low sensing cost regime.

First, in the low sensing cost regime where the sensing result is poor (i.e., α is small as the third column in Table 5), the operator will lease additional resource such that the total bandwidth reaches the threshold $Ge^{-(2+C_l)}$. In this case, the price is a constant and is independent of the value of α . Second, when the sensing result is good (i.e., α is large as in the last column in Table 5), the total bandwidth is large enough. In this case, as α increases, the amount of total bandwidth increases, and the optimal price decreases to maximize the profit.

Figure 7 shows how the optimal price changes with various costs and α in the low sensing cost regime. It is clear that price is first a constant and then starts to decrease when α is larger than a threshold. The threshold decreases in the optimal sensing decision of B_s^{L*} : a smaller sensing cost or a higher leasing cost will lead to a higher B_s^{L*} and thus a smaller threshold.

It is interesting to notice that the equilibrium price only changes in a time slot where the sensing realization factor α is large. This means that although operator has the freedom to change the price in every time slot, the actual variation of price is much less frequent. This leads to less overhead and makes it easier to implement in practice. Figure 8 illustrates this with different sensing costs and α realizations. The left two subfigures correspond to the realizations of α and the corresponding prices with $C_s = 0.48$ and $C_l = 1$. As the sensing

TABLE 5 The Operator's and Users' Equilibrium Behaviors

Sensing Cost Regimes	High Sensing Cost: $C_s \geq \frac{C_l}{2}$	Low Sensing Cost: 1-	$\frac{e^{-2C_l}}{4} \le C_s \le \frac{C_l}{2}$
Optimal Sensing Amount B_s^*	0	$B_s^{L*} \in [Ge^{-(2+C_l)}, Ge^{-2}]$	²], solution to eq. (13)
Sensing Realization Factor α	$0 \le \alpha \le 1$	$0 \le \alpha \le Ge^{-(2+C_l)}/B_s^{L*}$	$\alpha > Ge^{-(2+C_l)}/B_s^{L*}$
Optimal Leasing Amount B_l^*	$Ge^{-(2+C_l)}$	$Ge^{-(2+C_l)} - B_s^{L*}\alpha$	0
Optimal Pricing π^*	$1 + C_l$	$1 + C_l$	$\ln\left(\frac{G}{B_s^{L*\alpha}}\right) - 1$
Expected Profit R _I	$R_I^H = Ge^{-(2+C_l)}$	R_I^L in eq. (14)	R_I^L in eq. (14)
User i's SNR	$e^{(2+C_l)}$	$e^{(2+C_l)}$	$\frac{G}{B_s^{L*}\alpha}$
User <i>i</i> 's Payoff	$q_i e^{-(2+C_l)}$	$q_i e^{-(2+C_l)}$	$q_i(B_s^{L*}\alpha/G)$



Fig. 5. Optimal sensing amount B_s^* Fig. 6. Optimal leasing amount B_l^* as Fig. 7. Optimal price π^* as a function as a function of C_s , $and \alpha$.



Fig. 8. Optimal price π^* over time with different sensing costs and α realizations

cost C_s is quite high in this case, the operator does not rely heavily on sensing. As a result, the variability of α (in the upper subfigure) has very small impact on the equilibrium price (in the lower subfigure). In fact, the price only changes in 11 out 50 time slots, and the maximum amplitude variation is around 10%. The right two figures correspond to the case where $C_s = 0.35$ and $C_l = 1$. As sensing cost is cheaper in this case, the operator senses more and the impact of α on price is larger. The price changes in 30 out of 50 time slots, and the variation in amplitude can be as large as 30%.

Observation 4: The operator will sense the spectrum only if the sensing cost is lower than a threshold. Furthermore, it will lease additional spectrum only if the spectrum obtained through sensing is below a threshold.

Observation 5: Each user *i* obtains the same SNR independent of g_i and a payoff linear in g_i .

Observation 5 shows that users obtains fair and predictable resource allocation at the equilibrium. In fact, a user does not need to know anything about the total number and payoffs of other users in the system. It can simply predict its QoS if it knows the cost structure of the network (C_s and C_l)⁵. Such property is highly desirable in practice.

Finally, users achieve the same high SNR at the equilibrium. The SNR value is either $e^{(2+C_l)}$ or $G/(B_s^{L*}\alpha)$, both of which are larger than e^2 . This means that the approximation ratio $\ln(\text{SNR}_i)/\ln(1+\text{SNR}_i) > \ln(e^2)/\ln(1+e^2) \approx 94\%$. The ratio can even be close to one if the price π is high.

In Sections 3.1 and 3.4, we made the high SNR regime approximation and the uniform distribution assumption of α to obtain closed-form expressions. Next we show that relaxing both assumptions will not change any of the major insights.

5. The analysis of the game, however, does not require the users to know C_s or C_l .

4.1 Robustness of the Observations

Theorem 4: Observations 1-5 still hold under the general SNR regime (as in (1)) and any general distribution of α .

Proof: We represent a user i's payoff function in the general SNR regime,

$$u_i(\pi, w_i) = w_i \ln\left(1 + \frac{g_i}{w_i}\right) - \pi w_i.$$
(15)

The optimal demand $w_i^*(\pi)$ that maximizes (15) is $w_i^*(\pi) = g_i/Q(\pi)$, where $Q(\pi)$ is the unique positive solution to $F(\pi, Q) := \ln(1+Q) - \frac{Q}{1+Q} - \pi = 0$. We find the inverse function of $Q(\pi)$ to be $\pi(Q) = \ln(1+Q) - \frac{Q}{1+Q}$. By applying the implicit function theorem, we can obtain the first-order derivative of function $Q(\pi)$ over π as

$$Q'(\pi) = -\frac{\partial F(\pi, Q)/\partial \pi}{\partial F(\pi, Q)/\partial Q} = \frac{(1+Q(\pi))^2}{Q(\pi)},$$
 (16)

which is always positive. Hence, $Q(\pi)$ is increasing in π . User *i*'s optimal payoff is

$$u_i(\pi, w_i^*(\pi)) = \frac{g_i}{Q(\pi)} [\ln(1 + Q(\pi)) - \pi].$$
(17)

As a result, a user's optimal SNR equals $g_i/w_i^*(\pi) = Q(\pi)$ and is *user-independent*. The total demand from all users equals $G/Q(\pi)$, and the operator's investment and pricing problem is

$$R^{*} = \max_{B_{s} \ge 0} E_{\alpha \in [0,1]} [\max_{B_{l} \ge 0} \max_{\pi \ge 0} (\min\left(\pi \frac{G}{Q(\pi)}, \pi(B_{l} + B_{s}\alpha)\right) - B_{s}C_{s} - B_{l}C_{l})].$$
(18)

Define $\widetilde{R^*} = \frac{R^*}{G}$, $\widetilde{B}_l = \frac{B_l}{G}$, and $\widetilde{B}_s = \frac{B_s}{G}$. Then solving (18) is equivalent to solving

$$\widetilde{R^*} = \max_{\widetilde{B_s} \ge 0} E_{\alpha \in [0,1]} [\max_{\widetilde{B_l} \ge 0} \max_{\pi \ge 0} (\min\left(\frac{\pi}{Q(\pi)}, \pi(\widetilde{B_l} + \widetilde{B_s}\alpha)\right) - \widetilde{B_s}C_s - \widetilde{B_l}C_l)].$$
(19)

In Problem (19), it is clear that the operator's optimal decisions on leasing, sensing and pricing do not depend on users' aggregate wireless characteristics. This is true for any continuous distribution of α . And a user's optimal payoff in eq. (17) is linear in g_i since $Q(\pi)$ is independent of users' wireless characteristics. This shows that Observations 1, 2, and 5 hold for the general SNR regime and any general distribution of α . We can also show that Observations 3 and 4 hold in the general case, with a detailed proof in Appendix C.

5 THE IMPACT OF SPECTRUM SENSING UN-CERTAINTY

The key difference between our model and most existing literature (e.g., [19], [21], [22], [24], [26], [27]) is the possibility of obtaining resource through the cheaper but uncertain approach of spectrum sensing. Here we will elaborate the impact of sensing on the performances of operator and users by comparing with the *baseline case*

where sensing is not possible. Note that in the high sensing cost regime it is optimal not to sense, as a result, the performance of the operator and users will be the same as the baseline case. Hence we will focus on the low sensing cost regime in Table 5.

Observation 6: The operator's optimal *expected* profit always benefits from the availability of spectrum sensing in the low sensing cost regime.

Figure 9 illustrates the normalized optimal expected profit as a function of the sensing cost. We assume leasing cost $C_l = 2$, and thus the low sensing cost regime corresponds to the case where $C_s \in [0.2, 1]$ in the figure. It is clear that sensing achieves a better optimal expected profit in this regime. In fact, sensing leads to 250% increase in profit when $C_s = 0.2$. The benefit decreases as the sensing cost becomes higher. When sensing becomes too expensive, the operator will choose not to sense and thus achieve the same profit as in the baseline case.

Theorem 5: The operator's realized profit (i.e., the profit for a given α) is a strictly increasing function in α in the low sensing cost regime. Furthermore, there exists a threshold $\alpha_{th} \in (0, 1)$ such that the operator's realized profit is larger than the baseline approach if $\alpha > \alpha_{th}$.

Proof: As in Table 5, we have two cases in the low sensing cost regime:

• If $\alpha \leq Ge^{-(2+C_l)}/B_s^{L*}$, then substituting B_s^{L*} into $R_{II}^{CS1}(B_s, \alpha)$ in Table 3 leads to the realized profit

$$R_{II}^{CS1}(\alpha) = Ge^{-(2+C_l)} - B_s^{L*}C_s + B_s^{L*}\alpha C_l,$$

which is strictly and linearly increasing in α .

• If $\alpha \geq Ge^{-(2+C_l)}/B_s^{L*}$, then substituting B_s^{L*} into $R_{II}^{CS2}(B_s, \alpha)$ in Table 3 leads to the realized profit

$$R_{II}^{CS2}(\alpha) = B_s^{L*} \alpha \left(\ln \left(\frac{G}{B_s^{L*} \alpha} \right) - 1 \right) - B_s^{L*} C_s.$$

Because the first-order derivative

$$\frac{\partial R_{II}^{CS2}(\alpha)}{\partial \alpha} = B_s^{L*} \left(\ln \left(\frac{G}{B_s^{L*} \alpha} \right) - 2 \right) > 0,$$

as $B_s^{L*} \leq Ge^{-2}$, $R_{II}^{CS2}(\alpha)$ is strictly increasing in α . We can also verify that $R_{II}^{CS1}(\alpha) = R_{II}^{CS2}(\alpha)$ when $\alpha = Ge^{-(2+C_l)}/B_s^{L*}$. Therefore, the realized profit is a continuous and strictly increasing function of α .

Next we prove the existence of threshold α_{th} . First consider the extreme case $\alpha = 0$. Since the operator obtains no bandwidth through sensing but still incurs some cost, the profit in this case is lower than the baseline case. Furthermore, we can verify that $R_{II}^{CS2}(1) > R_{I}^{H}$ in Table 5, thus the realized profit at $\alpha = 1$ is always larger than the baseline case. Together with the continuity and strictly increasing nature of the realized profit function, we have proven the existence of threshold of α_{th} .

Figure 10 shows the realized profit as a function of α for different costs. The realized profit is increasing in α in both cases. The "crossing" feature of the two increasing curves is because the optimal sensing B_s^* is larger under a cheaper sensing cost ($C_s = 0.5$), which leads to larger



Fig. 9. Operator's normalized optimal Fig. 10. Operator's normalized opti- Fig. 11. User *i*'s normalized optimal *expected* profit as a function of C_s mal *realized* profit as a function of α . realized payoff as a function of α . and C_l .

realized profit loss (gain, respectively) when $\alpha \rightarrow 0$ ($\alpha \rightarrow 1$, respectively). This shows the tradeoff between improvement of expected profit and the large variability of the realized profit.

Theorem 6: Users always benefit from the availability of spectrum sensing in the low sensing cost regime.

Proof: In the baseline approach without sensing, the operator always charges the price $1 + C_l$. As shown in Table 5, the equilibrium price π^* with sensing is always no larger than $1 + C_l$ for any value of α . Since a user's payoff is strictly decreasing in price, the users always benefit from sensing.

Figure 11 shows how a user *i*'s normalized realized payoff u_i^*/g_i changes with α . The payoff linearly increases in α when α becomes larger than a threshold, in which case the equilibrium price becomes lower than $1+C_l$. A smaller sensing cost C_s leads to more aggressive sensing and thus more benefits to the users.

6 CONCLUSIONS AND FUTURE WORK

This paper represents some initial results towards understanding the new business models, opportunities, and challenges of the emerging cognitive virtual mobile network operators (C-MVNOs) under supply uncertainty. Here we focus on studying the trade-off between the cost and uncertainty of spectrum investment through sensing and leasing. We model the interactions between the operator and the users by a Stackelberg game, which captures the wireless heterogeneity of users in terms of maximum transmission power levels and channel gains.

We have discovered several interesting features of the game equilibrium. We show that the operator's optimal sensing, leasing, and pricing decisions follow nice threshold structures. The availability of sensing always increases the operator's expected profit, despite that the realized profit in each time slot will have some variations depending on the sensing result. Moreover, users always benefit in terms of payoffs when sensing is performed by the operator.

To keep the problem tractable, we have made several assumptions throughout this paper. Some assumptions can be (easily) generalized without affecting the main insights.

- *Imperfect sensing*: we can incorporate imperfect spectrum sensing (i.e., miss-detection and false-positive) into the model, which will change the uncertainty of the spectrum sensing. Given that our results work for any distribution of the sensing realization *α*, it is likely that such generalization does not change the major insights.
- *Learning*: we can also consider the interactions of multiple time slots, where the sensing realizations of previous time slots can be used to update the distributions of α in future time slots. Again, the per slot decision model introduced in this paper is still applicable with a time-dependent α distribution input.

Generalizations of some other assumptions, however, lead to more challenging new problems.

- *Incomplete information*: when the operator does not know the information of the users, the system needs to be modeled as a dynamic game with incomplete information. More elaborate economic models such as screening and signaling [42] become relevant.
- *Time scale separation*: it is possible that dynamic leasing is performed at a different (much larger) time scale compared with spectrum sensing. In that case, the operator has to make the leasing decision first, and then make several sequential sensing decisions. This leads to a dynamic decision model with more stages and tight couplings across sequential decisions.
- *Operator competition*: There may be multiple C-MVNOs providing services in the same geographic area. In that case, the operators need to attract the users through price competition. Also, if they sense and lease from the same spectrum owner, the operators may have overlapping or conflicting resource requests. Although we have obtained some preliminary results along this line in [43], more studies are definitely desirable.

Through the analytical and simulation study of an

idealized model in this paper, we have obtained various interesting engineering and economical insights into the operations of C-MVNOs. We hope that this paper can contribute to the further understanding of proper network architecture decisions and business models of future cognitive radio systems.

APPENDIX A PROOF OF THEOREM 1

Given the total bandwidth $B_l + B_s \alpha$, the objective of Stage III is to solve the optimization problem (8), i.e., $\max_{\pi \ge 0} \min(D(\pi), S(\pi))$. First, by examining the derivative of $D(\pi)$, i.e., $\partial D(\pi)/\partial \pi = (1 - \pi)Ge^{-(1+\pi)}$, we can see that the continuous function $D(\pi)$ is increasing in $\pi \in [0, 1]$ and decreasing in $\pi \in [1, +\infty]$, and $D(\pi)$ is maximized when $\pi = 1$. Since $S(\pi)$ always increases in π and $D(\pi)$ is concave over $\pi \in [0, 1]$, $S(\pi)$ intersects with $D(\pi)$ if and only if $\frac{\partial D(\pi)}{\partial \pi} > \frac{\partial S(\pi)}{\partial \pi}$ at $\pi = 0$, i.e., $B_l + B_s \alpha < Ge^{-1}$.

Next we divide our discussion into the intersection case and the non-intersection case:

- Given B_l+B_sα ≤ Ge⁻¹, S(π) intersects with D(π). By solving equation S(π) = D(π) the intersection point is π = ln (G/B_l+B_sα) − 1. There are two subcases:
 - when $B_l + B_s \alpha \leq Ge^{-2}$, $S(\pi)$ intersects with $D(\pi)$, and $\min(D(\pi), S(\pi))$ is maximized at the intersection point, i.e., $\pi^* = \ln\left(\frac{G}{B_l + B_s \alpha}\right) 1$. (See $S_3(\pi)$ in Fig. 3.)
 - when $B_l + B_s \alpha \ge Ge^{-2}$, $S(\pi)$ intersects with $D(\pi)$, and $\min(D(\pi), S(\pi))$ is maximized at the maximum value of $D(\pi)$, i.e., $\pi^* = 1$. (See $S_2(\pi)$ in Fig. 3.)
- 2) Given B_l+B_sα ≥ Ge⁻¹, S(π) doesn't intersect with D(π). Then min(D(π), S(π)) is maximized at the maximum value of D(π), i.e., π* = 1. (See S₁(π) in Fig. 3.)

APPENDIX B PROOF OF THEOREM 2

Given the sensing result $B_s \alpha$, the objective of Stage II is to solve the decomposed two subproblems (10) and (11), and select the best one with better optimal performance. Since $R_{III}^{ES}(B_s, \alpha, B_l)$ in subproblem (10) is linearly decreasing in B_l , its optimal solution always lies at the lower boundary of the feasible set (i.e., $B_l^* = \max\{Ge^{-2} - B_s\alpha, 0\}$). We compare the optimal profits of two subproblems (i.e., $R_{II}^{ES}(B_s, \alpha)$ and $R_{II}^{CS}(B_s, \alpha)$) for different sensing results:

- Given B_sα > Ge⁻², the obtained bandwidth after Stage I is already in excessive supply regime. Thus it is optimal not to lease for subproblem (10) (i.e., B_I^{ES3} = 0 of case (ES3) in Table 3).
- 2) Given $0 \le B_s \alpha \le Ge^{-2}$, the optimal leasing decision for subproblem (11) is $B_l^* = Ge^{-2} B_s \alpha$ and

we have $R_{III}^{ES}(B_s, \alpha, B_l) = R_{III}^{CS}(B_s, \alpha, B_l)$ when $B_l = Ge^{-2} - B_s \alpha$, thus the optimal objective value of (10) is always no larger than that of (11) and it is enough to consider the conservative supply regime only. Since

$$\frac{\partial^2 R^{CS}_{III}(B_s,\alpha,B_l)}{\partial B_l^2} = -\frac{1}{B_l + B_s \alpha} < 0,$$

 $R_{III}^{CS}(B_s, \alpha, B_l)$ is concave in $0 \le B_l \le Ge^{-2} - B_s \alpha$. Thus it is enough to examine the first-order condition

$$\frac{\partial R_{III}^{CS}(B_s, \alpha, B_l)}{\partial B_l} = \ln\left(\frac{G}{B_l + B_s \alpha}\right) - 2 - C_l = 0,$$

and the boundary condition $0 \le B_l \le Ge^{-2} - B_s \alpha$. This results in optimal leasing decision $B_l^* = \max(Ge^{-(2+C_l)} - B_s \alpha, 0)$ and leads to $B_l^{CS1} = Ge^{-(2+C_l)} - B_s \alpha$ and $B_l^{CS2} = 0$ of cases (CS1) and (CS2) in Table 3.

By substituting B_l^{CS1} and B_l^{CS2} into $R_{III}^{CS}(B_s, \alpha, B_l)$ in Table 2, we derive the corresponding optimal profits $R_{II}^{CS1}(B_s, \alpha)$ and $R_{II}^{CS2}(B_s, \alpha)$ in Table 3. $R_{II}^{ES3}(B_s, \alpha)$ can also be obtained by substituting B_l^{ES3} into $R_{III}^{ES}(B_s, \alpha, B_l)$.

APPENDIX C SUPPLEMENTARY PROOF OF THEOREM 4

In this section, we prove that Observations 3 and 4 hold for the genera case (i.e., the general SNR regime and a general distributions of α). We first show that Observation 4 holds for the general case.

C.1 Threshold structure of sensing

It is not difficult to show that if the sensing cost is much larger than the leasing cost, the operator has no incentive to sense but will directly lease. Thus the threshold structure on the sensing decision in Stage I still holds for the general case. We ignore the details due to space limitations.

C.2 Threshold structure of leasing

Next we show the threshold structure on leasing in Stage II also holds. Similar as in the proof of Theorem 1, we define $D(\pi) = \pi \frac{G}{Q(\pi)}$ and $S(\pi) = \pi (B_s \alpha + B_l)$.

• We first show that $D(\pi)$ is increasing when $\pi \in [0, 0.468]$ and decreasing when $\pi \in [0.468, +\infty)$. To see this, we take the first-order derivative of $D(\pi)$ over π ,

$$D'(\pi) = \frac{2Q(\pi)^2 + Q(\pi) - (1 + Q(\pi))^2 \ln(1 + Q(\pi))}{Q(\pi)^3}$$

which is positive when $Q(\pi) \in [0, 2.163)$ and negative when $Q(\pi) \in [2.163, +\infty)$. Since eq. (16) shows that $Q(\pi)$ is increasing in π and $\pi(Q) \mid_{Q=2.163} = 0.468$, as a result $D(\pi)$ is increasing in $\pi \in [0, 0.468]$



Fig. 12. Different intersection cases of $S(\pi)$ and $D(\pi)$ in the general SNR regime.



Fig. 13. The relation between the normalized total bandwidth B/G and the derivative of the revenue D'(B/G).

and decreasing in $\pi \in [0.468, +\infty)$. In other words, $D(\pi)$ is maximized at $\pi = 0.468$.

- Next we derive the operator's optimal pricing decision in Stage III. Figure 12 shows two possible intersection cases of S(π) and D(π). B_{th1} is defined as the total bandwidth obtained in Stages I and II (i.e., B_sα + B_l) such that S(π) intersects with D(π) at π = 0.468. Here is how the optimal pricing is determined:
 - If $B_s \alpha + B_l \ge B_{th1}$ (e.g., $S_1(\pi)$ in Fig. 12), the optimal price is $\pi^* = 0.468$. The total supply is no smaller (and often exceeds) the total demand.
 - If $B_s \alpha + B_l < B_{th1}$ (e.g., $S_2(\pi)$ in Fig. 12), the optimal price occurs at the unique intersection point of $S(\pi)$ and $D(\pi)$ (where $D(\pi)$ has a negative first-order derivative). The total supply equals total demand.
- Now we are ready to show the threshold structure of the leasing decision.
 - If the sensing result from Stage I satisfies $B_s \alpha \ge B_{th1}$, then the operator will not lease. This is because leasing will only increase the total cost without increasing the revenue, since the optimal price is fixed at $\pi^* = 0.468$ and thus revenue is also fixed at $D(\pi^*)$.
 - Let us focus on the case where the sensing result from Stage I satisfies $B_s \alpha < B_{th1}$. Let us define $B = B_s \alpha + B_l$, then we have $B = G/Q(\pi)$ and $\pi = \ln(1 + G/B) - G/(G + B)$. This enables us

to rewrite $D(\pi)$ as a function of total resource B only,

$$D(B) = B\left[\ln\left(1 + \frac{G}{B}\right) - \frac{G}{G+B}\right].$$

The first-order derivative of D(B) is

$$D'(B) = \ln\left(1 + \frac{1}{B/G}\right) - \frac{1}{1 + B/G} - \frac{1}{(1 + B/G)^2},$$
(20)

which denotes the increase of revenue D(B) due to unit increase in bandwidth B. Since obtaining each unit bandwidth has a cost of C_l in Stage II, the operator will only lease positive amount of bandwidth if and only if $D'(B_s \alpha) > C_l$. To facilitate the discussions, we will plot the function of D'(B/G) in Fig. 13, with the understanding that D'(B/G) = D'(B)G. The intersection point of B/G = 0.462 in Fig. 13 corresponds to the point of $\pi = 0.468$ in Fig. 12. The positive part of D'(B) on the left side of B/G = 0.462 in Fig. 13 corresponds to the part of $D(\pi)$ with a negative first-order derivative in Fig. 12. For any value C_l , Fig. 13 shows that there exists a unique threshold $B_{th2}(C_l)$ such that $D'(B_{th2}(C_l)/G) =$ C_lG_l , i.e., $D'(B_{th2}(C_l)) = C_l$. Then the optimal leasing amount will be $B_{th2}(C_l) - B_s \alpha$ if the bandwidth obtained from sensing $B_s \alpha$ is less than $B_{th2}(C_l)$, otherwise it will be zero.

C.3 Threshold structure of pricing and Observation 3

Based on the proofs above, we show that Observation 3 also holds for the general case as follows. Let us denote the optimal sensing decision as B_s^* , and consider two sensing realizations α_1 and α_2 in time slots 1 and 2, respectively. Without loss of generality, we assume that $\alpha_1 < \alpha_2$.

- If B^{*}_s α₂ ≥ B_{th1}, then the optimal price in time slot 2 is π^{*} = 0.468 (see Fig. 12). The optimal price in time slot 1 is always no smaller than 0.468.
- If B^{*}_sα₁ < B^{*}_sα₂ < B_{th1}, then we need to consider three subcases:
 - If $B_s^* \alpha_1 < B_s^* \alpha_2 \leq B_{th2}(C_l)$, then the operator will lease up to the threshold in both time slots, i.e., $B_l^* = B_{th2}(C_l) - B_s^* \alpha_1$ in time slot 1 and $B_l^* = B_{th2}(C_l) - B_s^* \alpha_2$ in time slot 2. Then optimal prices in both time slots are the same.
 - If $B_s^* \alpha_1 \leq B_{th2}(C_l) < B_s^* \alpha_2$, then the operator will lease $B_l^* = B_{th2}(C_l) B_s^* \alpha_1$ in time slot 1 and will not lease in time slot 2. Thus the total bandwidth in time slot 1 is smaller than that of time slot 2, and the optimal price in time slot 1 is larger.
 - If $B_{th2}(C_l) \leq B_s^* \alpha_1 < B_s^* \alpha_2$, then the operator in both time slots will not lease and total bandwidth in time slot 1 is smaller, and the optimal price in time slot 1 is larger.

To summarize, the optimal price π^* in Stage III is non-increasing in α . And the operator will charge a constant price ($\pi^* = 0.468$) to the users as long as the total bandwidth obtained through sensing and leasing does not exceed the threshold $B_{th2}(C_l)$.

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