Economic Analysis of Rollover and Shared Data Plans

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Abstract—In today's growing data market, wireless service providers (WSPs) compete severely to attract users by announcing innovative data plans. Two of the most popular innovative data plans are rollover and shared data plans, where the former plan allows a user to keep his unused data quota to next month and the latter plan allows users in a family to share unused data. As a pioneer to provide such data plans, a WSP faces immediate revenue loss from existing users who pay less overage charges due to less data over-usage, but his market share increases gradually by attracting new users and those under the other WSPs. In some countries, WSPs have asymmetric timing for providing such innovative data plans, while some other markets' WSPs have symmetric timing or no planning. This raises the question of why and when the competitive WSPs should offer the new data plans. This paper provides game theoretic modelling and analysis of the WSPs' timing of offering innovative data plans, by considering new user arrival and dynamic user churn between WSPs. Our equilibrium analysis shows that the WSP with small market share prefers to announce the innovative data plan first to attract more users, while the WSP with large market share prefers to announce later to avoid the immediate revenue loss. In a market with many new users, WSPs with similar market shares will offer the data plans simultaneously, but these WSPs facing few new users may not offer any new plan. Perhaps surprisingly, WSPs' profits can decrease with new user number and they may not benefit from the option of innovative data plans. Finally, unlike rollover data plan, we show that the timing of shared data plan further depends on the composition of users.

Index Terms—Networks economics, data plan upgrade timing, user dynamics, Nash equilibrium

1 INTRODUCTION

THE development of mobile technology has been driving the rapid growth of mobile devices (e.g., smartphones and iPad) and the users' data consumption. To attract more users and increase market shares, competitive wireless service providers (WSPs) have announced new alluring data plans to save users' costs. Rollover data plan is one of the most popular innovative data plans, which allows the users to keep the current month's unused data quota to next month without any additional charge. Take the USA market as an example. AT&T has freely upgraded its users' data plans to include rollover, where a user's left data from last month can be used after the current month's data quota is used up. Another popular innovative plan is shared data plan, which allows two or more users (devices) to share a common pool of data quota in each month. For example, China Mobile allows two users in the same city to combine their data plans without additional charge. Both rollover and shared data plans are attractive to (heavy) users by reducing their expected data costs in different ways: the former seeks inner-user data sharing by pooling an individual's monthly data quota over time, while the latter seeks inter-user data sharing by pooling two or more users' data quota in each month.

To a WSP's point of view, both rollover and shared data plans reduce its overage charges from heavy users, but can increase its market share by attracting new users

and those from its WSP competitors. In reality, we observe that the WSPs' timing of announcing the innovative data plans is very different in different countries. For example, in the saturated data market of the USA, T-Mobile with small market share is a pioneer to announce its rollover data plan in 2014. Though its immediate revenue from existing users reduced, it successfully increased its market share by more than 1 million users in merely a quarter [1]. The major WSPs, AT&T and Verizon in the same market unveiled their rollover data plans sequentially in 2015 and 2016, respectively. Different from the USA market, China's market is fast growing with many new users annually, and we observe that China Mobile and China Unicom offered the rollover data plan at the same time [2]. However, in some saturated markets such as Singapore, the major WSPs SingTel and Starhub have similar market shares and neither provides the rollover data plan. Similar situations happened to the shared data plan case. For example, in some countries (e.g., the UK), the WSPs with diverse market shares sequentially provided the shared data plans [3], while in some countries (e.g., China), WSPs offered the shared data plans almost at the same time [4], [5]. This motivates us to ask the key question in this paper: Why and when should the competitive WSPs announce the innovative data plans?

There are some recent works discussing shared and rollover data plans from a user's or a single WSP's perspective ([6], [7], [8], [9]). [6] models and numerically analyzes a user's choice between individual and shared data plan. In [7], the authors study a monopoly WSP's decision on the adoption of shared data plan, by examining the cost for such a new service. [8] analyzes the benefits of rollover data for a monopoly WSP and its users. And the work in [9] further designs the price of the rollover data plan to

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maximize the WSP's revenue. However, all these works only consider the monopoly WSP, and the competition between the WSPs (common in almost all data markets) has not been investigated yet. There are some prior works on generic network's competition or user dynamics (e.g., [10], [11], [12]). The work in [10] studies two interconnected service providers' timing of upgrading architecture, where one provider's upgrade also benefits the other due to the network effect. This "free-rider" effect gives the other operator a temptation to postpone its upgrade or even not upgrade. The work in [11] studies the cellular operators' timing of 4G network upgrade and shows that operators select different upgrade times to avoid severe competition. However, these works focus on the upgrade competition between symmetric operators, where operators are assumed to have identical market share. [12] studies the duopoly competition in spectrum leasing market but only looked at a static model in one-shot. [13] analyzes the user dynamics under innovative mobile social services rather than innovative data plans. In our paper, we look at a different problem without free riding effect and will show that the disparity of market shares will significantly affect the WSPs' timing of offering innovative data plans under users dynamics.

In this paper, we analyze the timing of competitive WSPs to announce the rollover or shared data plan, and answer why the WSPs in some countries prefer to offer simultaneously yet some other countries have asymmetric timing or no planning for innovative data plans. Our key novelty and main contributions are summarized as follows.

- *Economics of innovative data plans in competitive markets:* To our best knowledge, this is the first paper to provide game theoretic modelling and analysis of innovative data plans in competitive data markets. We practically model the dynamic user arrival and user churn between WSPs, and analytically characterize the pros and cons of innovative data plans for each WSP. We formulate the WSPs' interactions as a non-cooperative game, where each WSP seeks the trade-off between the increase of market share and the loss of overage charges.
- Equilibrium timing for innovative data plans: By analyzing the non-cooperative timing game, our equilibrium analysis shows that the WSP with small market share would like to offer the rollover or shared data plan first to attract significantly more users, while the WSP with large market share prefers to announce later to avoid the immediate revenue (overage charge) loss. In a fast-growing market with many new user arrivals, we show that WSPs with similar market shares will offer the data plans simultaneously. However, given few new users, these WSPs may not offer any new plan, as the benefit from attracting new users cannot compensate for the immediate revenue loss. Perhaps surprisingly, we show that WSPs may not benefit from the option of innovative data plans, and their profits can decrease with new user number.
- Comparison between rollover and shared data plans: In rollover data plan, a WSP's profit change is not affected by heavy users, while the timing of the shared

The rest of this paper is organized as follows. The system model and problem formulation is given in Section 2. In Section 3, WSPs' optimal decisions under rollover data plan is analyzed and the equilibrium rollover time is presented. In Section 4, we study the WSPs' equilibrium timing under shared data plan. Section 5 concludes this paper.

heavy) and (heavy, light) families.

2 SYSTEM MODEL AND PROBLEM FORMULATION

We consider a typical wireless data service market including competitive WSPs. For ease of exposition, we consider the two WSPs case as in [10], [11]. Later in Section 3, we also discuss the case with arbitrary number of symmetric WSPs. In reality, the data prices are similar for the WSPs with similar service quality. For example, in the USA, the subscription fee for 2 GB data is 20 USD per month for both Verizon and T-mobile. Thus, we assume that the WSPs offer the two-part tariff data plan (P, B, p) with the same data price to their users, where a user is given a monthly data cap *B* at a fixed lump-sum fee *P*, and should pay for extra data beyond *B* at a costly unit price *p*. Without considering any future new user, there are 2N users currently and WSP $i, i \in \{1, 2\}$ covers $2N_i$ users with current market share $\eta_i = N_i/N$.

In the following, we will first introduce the user model in the traditional data plan before the introduction of any innovative data plan. Then we will characterize the WSPs' pros and cons for upgrading traditional plans to rollover and shared data plans, and formulate the game theoretic models for the WSPs' timing competition. The key parameters which will be used throughout the paper are summarized in Table 1.

TABLE 1: Key Notations and Their Physical Meaning

Symbol	Description	
В	subscribed data quota	
P	subscription fee	
p	unit price for extra data	
α	percentage of heavy users	
D_l, D_h	maximum monthly data usage of light users,	
	heavy users	
d_l, d_h	minimum monthly data usage of light users,	
	heavy users	
u_l, u_h	monthly data usage of light users, heavy users	
η_i	initial market share for WSP <i>i</i>	
η_0	initial proportion of new users to existing users	
λ_0, λ	new user arrival rate and churn rate	
S	discount rate over time	

In both the existing and new user pools, we classify the data users into two types according to their monthly data usage behaviors: $\alpha \in (0, 1)$ percentage of heavy users who may exceed the data quota and $1 - \alpha$ percentage of light users who will not exceed the data quota. For example, according to [14], more than 25% of AT&T customers (heavy users) paid an overage charge within six months. A light

user's monthly data usage u_l is a random variable on his possible usage range $d_l \leq u_l \leq D_l$, where d_l and D_l are the minimum and maximum monthly data usage, respectively. As a light users seldom exceeds the data cap B (i.e., $D_l \leq B$), his expected monthly cost is $\mathbb{E}C_l = P$. Similarly, a heavy user's monthly data usage u_h is a random variable with probability density function $f_h(u_h)$ on his possible usage range $d_h \leq u_h \leq D_h$. In the existing twopart tariff, given $D_h > B$, a heavy user may overuse and his monthly cost by consuming u_h is

$$C_h = P + p(u_h - B)^+,$$
 (1)

where $(x)^{+} = \max(x, 0)$.

By taking the expectation of (1) with respect to u_h , the heavy user's expected cost is

$$\mathbb{E}C_h = P + p \int_B^{D_h} (u_h - B) f_h(u_h) du_h.$$
 (2)

Take the uniform distribution as in [15] as an example, e.g., $f_h(u_h) = \frac{1}{D_h - d_h}$ for a heavy user, the heavy user's expected cost is

$$\mathbb{E}C_h = P + p \frac{(D_h - B)^2}{2D_h}.$$
(3)

Note that the data usage distribution will only affect the users' expected cost reduction under innovative data plan and will not affect the WSPs' equilibrium timing analysis as shown in Sections 3 and 4.

Besides the existing 2N users, there are potentially $2N_0$ new users and they can add proportion $\eta_0 = N_0/N$ to the market, where η_0 may be even larger than 1. The number of new users is different across countries, e.g., large in China but small in USA. These new users will gradually join the market in the long run. Many of them are heavy users who are sensitive to the overage charge of the existing plan, but they will consider joining the innovative data plan (if any) for cost-saving. Besides the new user arrival, the existing users of a WSP will also consider switching to the other WSP's innovative data plan. Due to the switching cost c_s for changing the subscribed WSP (e.g., contract termination fee), the existing users will switch from one WSP to the other (first providing innovative data plan) gradually rather than all at once [11]. As new user arrival or existing user churn between WSPs takes time, we denote the new user arrival rate as λ_0 and existing user churn rate as λ , respectively, both of which depend on the user's cost reduction/saving Δ under the new innovative data plan. Note that we will estimate Δ according to the corresponding innovative data plan as specified later. If the cost reduction is large, the users will switch to the innovative data plan with large rates λ, λ_0 . Moreover, for the users who already subscribed to the WSP's innovative data plan to enjoy the cost reduction Δ , they will not switch to the other WSP even when the latter party also offers the innovative data plan later due to the switching cost and lock-in effect. This lock-in effect actually motivates the WSPs to compete to first offer innovative data plans.

Mathematically, we use differential equations to model the impact of switching cost c_s and its resultant lock-in effect. Suppose WSP *i* upgrades earlier than WSP *j*, i.e., $T_i < T_j$, WSP *j*'s left user share/proportion $\eta_j(t)$ is dynamically shrinking and follows

$$\frac{d\eta_j(t)}{dt} = -\lambda(c_s, \Delta)\eta_j(t), t \in [T_i, T_j],$$
(4)

where $\eta_j(t)$ starts with η_j initially and $\lambda(c_s, \Delta)$ is the user churn rate. In general, $\lambda(c_s, \Delta)$ increases with cost reduction Δ under the new data plan, and decreases with c_s which keeps users from churning. Note that the user churn rate is also affected by the difference between the two WSPs' wireless service qualities (e.g., signal coverage and the delay of data delivery), and the users will churn to the WSP with better service quality at a larger churn rate. Our analysis methods in Sections 3 and 4 still apply for the asymmetric churn rates case, though the analysis is more complicated with more cases. After T_j , WSP j's users also enjoy the cost reduction and will no longer churn to WSP i to avoid switching cost. By solving (4), we have

$$\eta_j(t) = e^{-\lambda(c_s,\Delta)(t-T_i)}\eta_j, t \in [T_i, T_j].$$
(5)

Similar to [11], we can validate the value of λ by using real data.

Similarly, given the initial proportion of new users η_0 , the dynamic of the left user proportion $\eta_0(t)$ of new users is

$$\frac{d\eta_0(t)}{dt} = -\lambda_0(\Delta)\eta_0(t), t \in [T_i, \infty), \tag{6}$$

where $\lambda_0(\Delta)$ increases with the cost reduction Δ and does not depend on c_s , since new users are not locked in any WSP initially. By solving (6), we have

$$\eta_0(t) = e^{-\lambda_0(\Delta)(t-T_i)}\eta_0, t \in [T_i, \infty).$$
(7)

Then, for WSP *i* who upgrades first at time T_i , its user share $\eta_i(t)$ at time *t* increases from its initial market share η_i and follows

$$\eta_i(t) = \eta_i + (1 - e^{-\lambda(c_s, \Delta)(t - T_i)})\eta_j + (1 - e^{-\lambda_0(\Delta)(t - T_i)})\eta_0,$$

$$t \in [T_i, T_j]$$
(8)

In the following, we will formulate the WSPs' timing problems for providing rollover and shared data plans, respectively.

2.1 Problem Formulation for Rollover Data Plan

To attract users from the new user pool and the other WSP, a WSP can provide rollover data plan at a proper time. Note that light users under-use the data quota and upgrading to rollover plan or not does not change their costs and subscription. The rollover upgrade from the existing plan only affects the heavy users' subscription (from the new user pool and the other WSP). Thus, we only need to consider the heavy users who can reduce their overage charges by using the rollover. We consider the typical rollover data plan (e.g., provided by AT&T), where the WSP allows its users to keep unused data quota $(B - u_h(t - 1))^+$ only from last month t-1 to next month t, and the rollover data is consumed only after the current month's data quota is used up. At time t, the heavy user's data quota now increases from B to $B + (B - u_h(t-1))^+$ and his monthly cost by consuming random $u_h(t)$ in this month is

$$C_h^r(t) = P + p \Big(u_h(t) - B - (B - u_h(t-1))^+ \Big)^+.$$
 (9)

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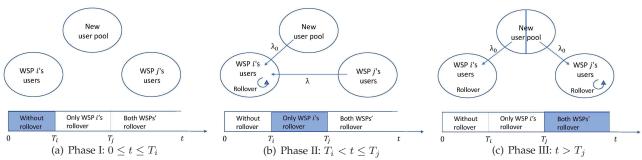


Fig. 2: Illustration of dynamic (heavy) user subscription when WSP *i* chooses rollover upgrade earlier than WSP *j* ($T_i < T_j$): Phase I without rollover data, Phase II with WSP *i*'s rollover service, Phase III with both WSPs' rollover services.

By taking expectation of (9) with respect to both $u_h(t)$ and $u_h(t-1)$, the heavy user's expected cost under the rollover data plan is

$$\mathbb{E}C_h^r = P + p \int_{\xi}^{D_h} \left(u_h(t) - \xi \right) f_h(u_h(t)) du_h(t).$$
(10)

where $\xi = B + \int_{a}^{B} (B - u_h(t-1)) f_h(u_h(t-1)) du_h(t-1)$.

Take the uniform distribution as an example, we derive the heavy user's expected cost under the rollover data plan as follows.

Lemma 2.1. For the uniform distribution, the heavy user's expected cost under the rollover data plan is

$$\mathbb{E}C_{h}^{r} = \begin{cases} P + p\frac{2(D_{h} - B)^{3}}{3D_{h}^{2}}, \text{ if } B < D_{h} \le 2B; \\ P + p(\frac{2(D_{h} - B)^{3}}{3D_{h}^{2}} - \frac{(D_{h} - 2B)^{3}}{6D_{h}^{2}}), \text{ if } D_{h} > 2B, \end{cases}$$
(11)

which is smaller than traditional cost $\mathbb{E}C_h$ in (3).

As shown in Fig. 1, the cost reduction $\mathbb{E}C_h - \mathbb{E}C_h^r$ after adding rollover data first increases with the maximum data usage D_h as the heavy user is more likely to rollover data and save cost. The cost reduction then decreases as there is less left data to rollover.

Due to the cost saving, heavy users will gradually churn to the WSP with rollover data upgrade. Fig. 2 illustrates how existing heavy users switch from WSP *i* to WSP *j* and how new (heavy) users choose WSPs over the time horizon *t*. Time 0 means the earliest possible time for data plan upgrade. Here we suppose WSP *i* upgrades earlier than WSP *j* (i.e., $T_i \leq T_j$) in Fig. 2.¹ As explained earlier, we do not consider existing and new light users, as they are not affected by the rollover data plan and do not matter the WSPs' upgrade timing. There are three phases for dynamic user subscription:

- Phase I (0 ≤ t ≤ T_i) as in Fig. 2(a). No WSP offers the rollover data plan. Thus no existing heavy user switches between WSPs and no new heavy user joins any network.
- Phase II (*T_i* < *t* ≤ *T_j*) as in Fig. 2(b). WSP *i* offers the rollover data plan at time *T_i* but WSP *j* has not. The existing heavy users of WSP *i* benefit from the upgrade to rollover data plan immediately from time *T_i*, and the heavy users of WSP *j* and the new user

pool gradually join WSP *i* at rates λ and λ_0 , respectively. The number of heavy users switched from WSP *j* to *i* from time T_i to *t* is $2\alpha N_j(1 - e^{-\lambda(t-T_i)})$, where there are originally $2\alpha N_j$ users in WSP *j*. The number of new heavy users joining WSP *i* is $2\alpha N_0(1 - e^{-\lambda_0(t-T_i)})$.

• Phase III $(t > T_j)$ as in Fig. 2(c). Both WSPs offer rollover data plans, and no existing users will switch to the other WSP. The new heavy users are equally likely to subscribe to both WSPs (for cost saving) since T_j . There are only $2\alpha N_0 e^{-\lambda_0 (T_j - T_i)}$ left in the new heavy user pool at time T_j , and the numbers of new heavy users subscribed to each WSP from time T_j to t are the same, i.e., $\alpha N_0 e^{-\lambda_0 (T_j - T_i)} (1 - e^{-\lambda_0 (t - T_j)})$.

Based on the dynamic subscription of users over the three phases, we are ready to characterize the profits of WSPs. In reality, each WSP values its future profit less importantly due to depreciation. We denote the discount rate over time as S > 0, and model the discount factor till time t as e^{-St} as in [16]. Without offering rollover data plan (i.e., $T_i = T_j = \infty$), the long-term profit of WSP i, i = 1, 2, from its existing heavy users is

$$R_i = \int_0^\infty 2\alpha N_i \mathbb{E} C_h e^{-St} dt = 2\alpha N_i \mathbb{E} C_h \frac{1}{S}, \qquad (12)$$

where $\mathbb{E}C_h$ is given in (3).

Given WSP *j*'s rollover time $T_j \ge 0$, WSP *i* can choose to rollover earlier or later than WSP *j*. As shown in Fig. 2, WSP *i*'s long-term profits through the three time phases for $T_i \le T_j$ and $T_i > T_j$ are given in (13) and (14), respectively. Based on these results, we describe the non-cooperative timing game for rollover data plan as follows.

- Players: WSPs 1 and 2.
- Strategy spaces: WSP *i* ∈ {1,2} can choose rollover time *T_i* from the feasible set *T_i* = [0,∞].
- Payoff functions: WSP *i* ∈ {1, 2} wants to maximize its profit defined in (13) and (14).

2.2 Problem Formulation for Shared Data Plan

To attract more subscribers, a WSP can also choose to provide shared data plan at a proper time. We assume the average family size for choosing a shared data plan is two, and our analysis can also be extended to the case of three or more members in an average family. By combining

$$R_{i}^{r,\leq} = \int_{0}^{T_{i}} 2\alpha N_{i} \mathbb{E}C_{h} e^{-St} dt + \int_{T_{i}}^{T_{j}} \left(2\alpha N_{j} (1 - e^{-\lambda(t-T_{i})}) \mathbb{E}C_{h}^{r} + 2\alpha N_{i} \mathbb{E}C_{h}^{r} + 2\alpha N_{0} (1 - e^{-\lambda_{0}(t-T_{i})}) \mathbb{E}C_{h}^{r} \right) e^{-St} dt + \int_{T_{j}}^{\infty} \left(2\alpha N_{j} (1 - e^{-\lambda(T_{j}-T_{i})}) \mathbb{E}C_{h}^{r} + 2\alpha N_{i} \mathbb{E}C_{h}^{r} + \alpha N_{0} e^{-\lambda_{0}(T_{j}-T_{i})} (1 - e^{-\lambda_{0}(t-T_{j})}) \mathbb{E}C_{h}^{r} + 2\alpha N_{0} (1 - e^{-\lambda_{0}(t-T_{j})}) \mathbb{E}C_{h}^{r} + 2\alpha N_{0} (1 - e^{-\lambda_{0}(t-T_{j})}) \mathbb{E}C_{h}^{r} + 2\alpha N_{0} (1 - e^{-\lambda_{0}(t-T_{j})}) \mathbb{E}C_{h}^{r} \right) e^{-St} dt.$$

$$R_{i}^{r,>} = \int_{0}^{T_{j}} 2\alpha N_{i} \mathbb{E}C_{h} e^{-St} dt + \int_{T_{j}}^{T_{i}} 2\alpha N_{i} e^{-\lambda(t-T_{j})} \mathbb{E}C_{h} e^{-St} dt + \int_{T_{i}}^{\infty} \left(2\alpha N_{i} e^{-\lambda(T_{i}-T_{j})} + \alpha N_{0} e^{-\lambda_{0}(T_{i}-T_{j})} (1 - e^{-\lambda_{0}(t-T_{i})}) \right) \mathbb{E}C_{h}^{r} e^{-St} dt.$$

$$(13)$$

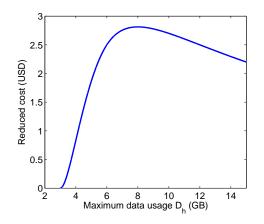


Fig. 1: Heavy user's expected cost reduction $\mathbb{E}C_h - \mathbb{E}C_h^r$ vs his maximum data usage D_h when B = 3 GB, p = 10 USD/GB.

two individual users' data cap B and lump sum fee P, the shared data plan is (2P, 2B, p) for the whole family. Suppose two users (no matter heavy or light users) form a family randomly, and each user is a heavy user with probability α . Based on different combinations of heavy and light users, the proportions of (heavy, heavy), (heavy, light) and (light, light) families are α^2 , $2\alpha(1-\alpha)$ and $(1-\alpha)^2$, respectively. Compared to the rollover data plan where a WSP's profit change is only affected by heavy users, the timing of the shared data plan here depends on the composition of light and heavy users in a family. A light user in a (heavy, light) family can also change to the WSP with shared data plan to reduce the cost of the heavy user in the same family. We note that in each of the (light, light) families, the two users do not benefit from the shared data plan. This type of families does not affect the WSPs' profit change after the shared data plan, and we only need to consider the (heavy, heavy) and (heavy, light) families when deciding the timing of offering shared data plan. Similarly, we do not consider bachelordom users without family. For each type of families, the proportion of both family users choosing the same WSP *i* of market share η_i is η_i^2 , and we call such families as pure families. Similarly, the proportion of each user choosing a different WSP is $2\eta_i\eta_i$, and we call such families as mixed families.

Under the shared data plan, the family's monthly cost is

$$C_s = 2P + p(u - 2B)^+,$$
 (15)

where random variable u is the total family data usage: $u = u_h + u_l$ for a (heavy, light) family and $u = u_h + u_h$ for a (heavy, heavy) family.

By taking the expectation of (15) with respect to u, the expected costs of (heavy, light) and (heavy, heavy) families

are

$$\mathbb{E}C_{h,l} = 2P + p \int_{2B}^{D_l + D_h} (u - 2B) f(u) du, \qquad (16)$$

and

$$\mathbb{E}C_{h,h} = 2P + p \int_{2B}^{D_h + D_h} (u - 2B) f(u) du.$$
(17)

Take the uniform distribution of users' data usage as an example, the distribution of u for a (heavy, light) family is

$$f(u) = \begin{cases} \frac{u}{D_l D_h}, & \text{if } 0 \le u \le D_l; \\ \frac{1}{D_h}, & \text{if } D_l < u \le D_h; \\ \frac{D_l + D_h - u}{D_l D_h}, & \text{if } D_h < u \le D_l + D_h. \end{cases}$$
(18)

For a (heavy, heavy) family, its distribution of u is the similar as (18) by letting $D_l = D_h$. Then, we obtain the expected costs of (heavy, heavy) and (heavy, light) families as follows.

Lemma 2.2. For the uniform distribution of users' data usage, the (heavy, heavy) family's expected cost is

$$\mathbb{E}C_{h,h} = \begin{cases} 2P + \frac{4p}{3D_h^2}(D_h - B)^3, & \text{if } D_h \le 2B;\\ 2P + \frac{p}{D_h^2}(D_h^3 - 2BD_h^2 + \frac{4}{3}B^3), & \text{if } D_h > 2B. \end{cases}$$
(19)

and the (heavy, light) family's expected cost is

$$\mathbb{E}C_{h,l} = \begin{cases} 2P + \frac{p}{6D_h D_l} (D_h + D_l - 2B)^3, \text{ if } D_h \le 2B;\\ 2P + \frac{p}{D_h} (\frac{D_h}{2} - 2BD_h + 2B^2 + \frac{D_l^2}{6} + \frac{D_h D_l}{2} - BD_l), \text{ if } D_h > 2B. \end{cases}$$
(20)

Due to the overage cost saving, (heavy, heavy) and (heavy, light) families will gradually churn to the WSP with shared data upgrade. Fig. 3 illustrates how existing (heavy, heavy) and (heavy, light) families switch from one WSP to another and how new (heavy, heavy) and (heavy, light) families choose WSPs in the time horizon *t*. Here we suppose WSP *i* upgrades earlier than WSP *j* (i.e., $T_i \leq T_j$) in Fig. 3 and can simply skip Phase II if $T_i = T_j$. As explained earlier, here we do not count existing and new (light, light) families, as they are not affected by the shared data plan and do not matter the timing decisions.

- Phase I (0 ≤ t ≤ T_i) as in Fig. 3(a). No WSP offers the shared data plan. Thus, no existing user switches between WSPs and no new user joins any network.
- Phase II (*T_i* < *t* ≤ *T_j*) as in Fig. 3(b). WSP *i* offers the shared data plan at time *T_i* but WSP *j* has not. If both users in a family have subscribed to WSP *i*, they can upgrade to the shared data plan immediately at time *T_i*. Otherwise, the existing families (with at least one family member in WSP *j*) and the new families will join WSP *i* gradually at churn rates λ and λ₀,

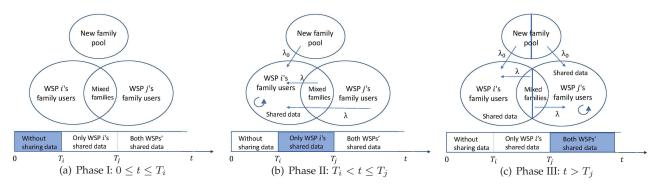


Fig. 3: Illustration of (heavy, heavy) and (heavy, light) families subscription under WSPs' shared data plans when WSP *i* upgrades earlier than WSP *j* ($T_i < T_j$): Phase I without shared data, Phase II with WSP *i*'s shared data service, Phase III with both WSPs' shared data services.

$$R_{i}^{s,\leq} = \left(\mathcal{E}N(\frac{1}{S}-\frac{1}{\lambda+S})+\eta_{i}^{2}N\frac{1}{\lambda+S}\mathcal{E}-\eta_{i}N\mathcal{D}\frac{1}{S}+\eta_{i}\eta_{j}N\mathcal{D}\frac{1}{\lambda+S}+N_{0}\mathcal{E}(\frac{1}{S}-\frac{1}{\lambda_{0}+S})\right)e^{-ST_{i}}-(1-\eta_{i})N\mathcal{E}(\frac{1}{S}-\frac{1}{\lambda+S})e^{-(\lambda+S)T_{j}+\lambda T_{i}}\\ -\frac{1}{2}N_{0}\mathcal{E}(\frac{1}{S}-\frac{1}{\lambda_{0}+S})e^{-(\lambda_{0}+S)T_{j}+\lambda_{0}T_{i}}+\eta_{i}N\mathcal{D}\frac{1}{S},$$

$$(22)$$

$$R_{i}^{s,>} = \eta_{i}N\mathcal{D}\frac{1}{S}-\eta_{i}N\mathcal{D}(\frac{1}{S}-\frac{1}{\lambda+S})e^{-ST_{j}}+\eta_{i}N(\mathcal{E}\frac{1}{S}-\eta_{j}\mathcal{E}\frac{1}{\lambda+S}-\eta_{i}\mathcal{D}\frac{1}{\lambda+S})e^{-(\lambda+S)T_{i}+\lambda T_{j}}+\frac{1}{2}N_{0}\mathcal{E}(\frac{1}{S}-\frac{1}{\lambda_{0}+S})e^{-(\lambda_{0}+S)T_{i}+\lambda_{0}T_{j}}.$$

$$(23)$$

respectively. Thus, the subscribed families to WSP i from T_i to t are: (i) $\eta_i^2 \alpha^2 N$ pure (heavy, heavy) families from WSP i; (ii) $\eta_j^2 \alpha^2 N(1 - e^{-\lambda(t-T_i)})$ pure (heavy, heavy) families from WSP j, where there are originally $\eta_j^2 \alpha^2 N$ pure (heavy, heavy) families in WSP j; (iii) $2\eta_i\eta_j\alpha^2 N(1 - e^{-\lambda(t-T_i)})$ mixed (heavy, heavy) families; (iv) $2\eta_i^2\alpha(1 - \alpha)N$ pure (heavy, light) families from WSP i; (v) $2\eta_j^2\alpha(1 - \alpha)N(1 - e^{-\lambda(t-T_i)})$ pure (heavy, light) families from WSP i; (vi) $4\eta_i\eta_j\alpha(1 - \alpha)N(1 - e^{-\lambda(t-T_i)})$ mixed (heavy, light) families; (vii) $\alpha^2 N_0(1 - e^{-\lambda_0(t-T_i)})$ new (heavy, heavy) families; (vii) $2\alpha(1 - \alpha)N_0(1 - e^{-\lambda_0(t-T_i)})$ new (heavy, heavy) families; (vii) $2\alpha(1 - \alpha)N_0(1 - e^{-\lambda_0(t-T_i)})$ new (heavy, light) families.

• Phase III $(t > T_j)$ as in Fig. 3(c). Both WSPs offer the shared data plan, and no pure families will switch to the other WSP. The mixed families and the new families are equally likely to subscribe to the two WSPs. Thus, a number $\eta_i \eta_j \alpha^2 N e^{-\lambda(T_j - T_i)} (1 - e^{-\lambda(t - T_j)})$ of mixed (heavy, heavy) families and a number $2\eta_i \eta_j \alpha (1 - \alpha) N e^{-\lambda(T_j - T_i)} (1 - e^{-\lambda(t - T_j)})$ of mixed (heavy, light) families will subscribe to each WSP from time T_j to t. And the number of new (heavy, heavy) families and (heavy, light) families subscribed to each WSP from time T_j to t are $\frac{1}{2}\alpha^2 N_0 e^{-\lambda_0(T_j - T_i)} (1 - e^{-\lambda_0(t - T_j)})$ and $\alpha (1 - \alpha) N_0 e^{-\lambda_0(T_j - T_i)} (1 - e^{-\lambda_0(t - T_j)})$, respectively.

Based on the dynamic subscription of users over the three phases, we can characterize the profits of WSPs as follows. Without offering shared data plan (i.e., $T_i = T_j = \infty$), the long-term profit of WSP i, i = 1, 2, from its existing (heavy, heavy) and (heavy, light) families is

$$R_i^s = \int_0^\infty (2\alpha\eta_i N \mathbb{E}C_h + 2\eta_i N\alpha(1-\alpha) \mathbb{E}C_l) e^{-St} dt$$

= $(2\alpha\eta_i N \mathbb{E}C_h + 2\eta_i N\alpha(1-\alpha) \mathbb{E}C_l) \frac{1}{S}.$ (21)

WSP *i*'s long-term profits through the three time phases for $T_i \leq T_j$ and $T_i > T_j$ cases are given in (22) and (23), respectively, where

$$\mathcal{D} = 2\alpha \mathbb{E}C_h + 2\alpha (1-\alpha) \mathbb{E}C_l, \qquad (24)$$

$$\mathcal{E} = \alpha^2 \mathbb{E} C_{h,h} + 2\alpha (1-\alpha) \mathbb{E} C_{h,l}.$$
 (25)

Similar to the timing game of rollover data plan, we use a noncooperative game to model the interactions between the two WSPs, where each of them seeks to maximize its profit defined in (22) and (23) by choosing the optimal timing of offering the shared data plan. In the following, we will analyze the timing equilibrium for the rollover data plan in Section 3 and shared data plan in Section 4.

3 TIMING EQUILIBRIUM FOR ROLLOVER DATA PLAN

By announcing the rollover data plan, a WSP loses its immediate revenue from its existing users as the overage charge reduces (from $\mathbb{E}C_h$ in (3) to $\mathbb{E}C_h^r$ in (11)), but it can gradually increase profit by attracting new users and those from the other WSP. Note that as existing users are mainly adults and potential new users could be teenagers who take time to mature, we expect $\lambda_0 < \lambda$ in practice. In this section, we first analyze WSP *i*'s best rollover timing given any WSP *j*'s rollover time T_j , where $i, j \in \{1, 2\}$ and $i \neq j$. Then, we derive the equilibrium rollover timing of both WSPs.

By examining the first-order conditions of WSP's profit before and after its competitor's rollover, we derive the conditions that WSP i will rollover first.

Lemma 3.1. Given WSP j's rollover time T_j , WSP i will upgrade to rollover data plan earlier than WSP j (i.e., $T_i \leq T_j$), if and only if one of the following conditions holds:

$$\left(2\alpha (N_i \mathbb{E}C_h - N\mathbb{E}C_h^r + N_j \frac{S}{\lambda + S} \mathbb{E}C_h^r) - 2\alpha N_0 \frac{\lambda_0}{\lambda_0 + S} \mathbb{E}C_h^r \right) e^{-ST_i} - 2\alpha N_j \left(\frac{\lambda}{S} - \frac{\lambda}{\lambda + S}\right) \mathbb{E}C_h^r e^{-ST_j} e^{-\lambda(T_j - T_i)} - \alpha N_0 \left(\frac{\lambda_0}{S} - \frac{\lambda_0}{\lambda_0 + S}\right) \mathbb{E}C_h^r e^{-ST_j} e^{-\lambda_0(T_j - T_i)} = 0.$$

$$(27)$$

(I) Reduction of overage charge after rollover is mild, i.e., $\kappa_i \leq 1$, where

$$\kappa_i = \frac{2\eta_i (\mathbb{E}C_h - \mathbb{E}C_h^r \frac{\lambda + S}{S})}{\eta_0 \mathbb{E}C_h^r \frac{\lambda_0}{S}}.$$
 (26)

(II) $\kappa_i > 1$ yet earlier upgrade's gain in market share still exceeds the later upgrade's benefit of overage charges (i.e., $R_i^{r,\leq}(T_i)$ in (13) is larger than $R_i^{r,>}(T_j + \frac{\log \kappa_i}{\lambda - \lambda_0})$ in (14)). Here, WSP *i*'s best upgrade time after T_j is $T_j + \frac{1}{\lambda - \lambda_0}$ $\frac{\log \kappa_i}{\lambda - \lambda_0}$, and the best time before T_j is $\hat{T}_i =$ $\arg \max(R_i^{r,\leq}(\tilde{T}_i), R_i^{r,\leq}(0), R_i^{r,\leq}(T_i)))$, where \tilde{T}_i is the solution to (27).

Proof: If $\kappa_i \leq 1$, we have $\frac{\partial R_i^{r,>}(T_i,T_j)}{\partial T_i} \leq 0$. Thus, WSP *i* will not offer rollover later than WSP *j*.

If $\kappa_i > 1$, WSP *i* will choose its upgrade time by comparing its profits before and after WSP *j*'s rollover. Note that $\frac{\partial^2 R_i^{r,>}(T_i,T_j)}{\partial T_i^2} < 0$. By letting $\frac{\partial R_i^{r,>}(T_i,T_j)}{\partial T_i} = 0$, we have WSP *i*'s best upgrade time after T_j as $T_j + \frac{\log \kappa_i}{\lambda - \lambda_0}$. Then, we derive the optimal time before T_j . Denote T_i as the solution to $\frac{\partial R_i^{r,\leq}(T_i,T_j)}{\tilde{\sigma}^{T_i}} = 0$, which can be written as (27). Note that T_i should be less than T_j and larger than 0. Thus, by comparing the profits of extreme and boundary points, we have the best time before T_j as $\hat{T}_i = \arg \max(R_i^{r,\leq}(\tilde{T}_i), R_i^{r,\leq}(0), R_i^{r,\leq}(T_i)).$ Therefore, WSP i will upgrade to rollover data plan earlier than WSP j if $R_i^{r,\leq}(\hat{T}_i) \ge R_i^{r,>}(T_j + \frac{\log \kappa_i}{\lambda - \lambda_0}).$ According to Lemma 3.1, WSP *i*'s best response can be

derived immediately in the following proposition.

Proposition 3.1. Given WSP j's rollover time T_j , WSP i's best rollover time $T_i^*(T_i)$ is

If $T_j = 0$, then

$$T_i^*(T_j) = \begin{cases} 0, & \text{if } \kappa_i \le 1;\\ \frac{\log \kappa_i}{\lambda - \lambda_0}, & \text{if } \kappa_i > 1. \end{cases}$$
(28)

If
$$T_j > 0$$
, then

$$T_i^*(T_j) = \begin{cases} \hat{T}_i, & \text{if (I) or (II) holds;} \\ T_j + \frac{\log \kappa_i}{\lambda - \lambda_0}, & \text{Otherwise.} \end{cases}$$
(29)

According to Proposition 3.1, WSP i will postpone its rollover time if the heavy user's overage cost reduction (from $\mathbb{E}C_h$ in (3) to $\mathbb{E}C_h^r$ in (11)) is large and the new user population is small. In reality, the heavy user's cost reduction after rollover is found not huge [17], and we consider an upper bound for the cost reduction, i.e., $\mathbb{E}C_h < \mathbb{E}C_h^r \frac{2\lambda+S}{S}$.²

Theorem 3.1. The WSPs' equilibrium timing of rollover data plans is summarized as follows (see Fig. 4):

- If $\mathbb{E}C_h \leq \mathbb{E}C_h^r \frac{\lambda+S}{S}$, the overage charge reduction after rollover is mild and the two WSPs will immediately upgrade their data plans (i.e., $T_i^{ne} = T_i^{ne} = 0$), as shown in Fig. 4(a).
- If $\mathbb{E}C_h > \mathbb{E}C_h^{r\lambda+\dot{S}}$, the WSPs' equilibrium rollover timing depends on the new user proportion η_0 and the WSPs' market shares:
 - 1) Large η_0 regime $(\eta_0 \ge \frac{2S}{\lambda_0}(\frac{\mathbb{E}C_h}{\mathbb{E}C_h} \frac{\lambda+S}{S}))$: both WSPs will offer rollover data plans immediately to attract many new (heavy) users (i.e., $T_i^{ne} = T_j^{ne} = 0$), as shown in Fig. 4(a).
 - Medium η_0 regime $(\bar{\eta}_0 < \eta_0 < \frac{2S}{\lambda_0}(\frac{\mathbb{E}C_h}{\mathbb{E}C_h} \frac{\lambda+S}{S})$ 2) with $\bar{\eta}_0$ as the solution to (30)): the median size of new users and the other WSP's many users allure the WSP with a small market share (less than η_r in (31)³) to upgrade immediately, and the other WSP purposely waits and upgrades at time $\frac{\log \kappa_i}{\lambda - \lambda_0} > 0, i \in \{1, 2\}.$ If the two WSPs have similar market shares (between η_r and $1 - \eta_r$), they will upgrade immediately to keep their existing users and attract new users (see Fig. 4(b)).
 - Small η_0 regime $(0 \le \eta_0 \le \overline{\eta}_0)$: the small size 3) of new users cannot allure the two WSPs with similar market shares (between $\bar{\eta}_r$ and 1 – $\bar{\eta}_r$ with $\bar{\eta}_r$ as the solution of (32)) to upgrade to rollover data plan (i.e., $T_i^{ne} = T_j^{ne} = \infty$). Only the WSP of small market share (less than $\bar{\eta}_r$) still chooses to upgrade to attract the other WSP's large market share, and the other will still upgrade after $\frac{\log \kappa_i}{\lambda - \lambda_0}$, $i \in \{1, 2\}$ to keep its market share to some extent (see Fig. 4(c)).

The proof of Theorem 3.1 is given in Appendix A of the supplemental material. Then, we can conclude that

- Both WSPs will offer rollover data plans immediately if the heavy user's cost reduction (i.e., $\mathbb{E}C_h - \mathbb{E}C_h^r$) is small or the new user population is large.
- The WSP with small market share prefers to rollover first as it can attract many users at least from the other WSP. While the WSP with large market share prefers late rollover to avoid the immediate revenue loss, and it still chooses rollover upgrade to keep its market share.
- Given few new users and large cost reduction from heavy users, the WSPs with similar market shares will not rollover as the profit received from the new users cannot compensate for the revenue loss due to users' cost reduction.

These results match well with our industry observations about rollover data plans. For example, in China, the quar-

3. $\tilde{\eta}_r$ in (31) is the solution of $R_i^{r,\leq}(T_i = 0, T_j = \frac{\log \kappa_j}{\lambda - \lambda_0}) =$ $R_i^{r,>}(T_j = 0, T_i = \frac{\log \kappa_i}{\lambda - \lambda_0})$ as a function of η_i .

^{2.} For the case when $\mathbb{E}C_h \geq \mathbb{E}C_h^r \frac{2\lambda+S}{S}$, we can show that the equilibrium analysis is similar, and in the equilibrium results that only the medium η_0 regime includes more tedious cases to quantify equilibrium timing.

$$\alpha N \mathbb{E}C_{h} - 2\alpha N \mathbb{E}C_{h}^{r} + \alpha N \frac{S}{\lambda + S} \mathbb{E}C_{h}^{r} - 2\alpha \eta_{0} N \frac{\lambda_{0}}{\lambda_{0} + S} \mathbb{E}C_{h}^{r} - \alpha N (\frac{\lambda}{S} - \frac{\lambda}{\lambda + S}) \mathbb{E}C_{h}^{r} \left(\frac{\eta_{0} \mathbb{E}C_{h}^{r} \frac{\lambda_{S}}{S}}{\mathbb{E}C_{h} - \mathbb{E}C_{h}^{r} \frac{\lambda + S}{S}}\right)^{\frac{\lambda + S}{\lambda - \lambda_{0}}}$$

$$+ \alpha \eta_{0} N (\frac{\lambda_{0}}{\lambda_{0} + S} - \frac{\lambda_{0}}{S}) \mathbb{E}C_{h}^{r} \left(\frac{\eta_{0} \mathbb{E}C_{h}^{r} \frac{\lambda_{0}}{S}}{\mathbb{E}C_{h} - \mathbb{E}C_{h}^{r} \frac{\lambda + S}{S}}\right)^{\frac{\lambda_{0} + S}{\lambda - \lambda_{0}}} = 0,$$

$$\underline{\eta_{r}} = \max \left(1 - \tilde{\eta_{r}}, \min \left(\max(1 - \bar{\eta_{r}}, \frac{N_{0} \mathbb{E}C_{h}^{r} \frac{\lambda_{0}}{S}}{2N(\mathbb{E}C_{h} - \mathbb{E}C_{h}^{r} \frac{\lambda + S}{S})}\right), \min(\bar{\eta_{r}}, 1 - \frac{N_{0} \mathbb{E}C_{h}^{r} \frac{\lambda_{0}}{S}}{2N(\mathbb{E}C_{h} - \mathbb{E}C_{h}^{r} \frac{\lambda + S}{S})}) \right) \right).$$

$$(31)$$

$$2\alpha (N_{i} \mathbb{E}C_{h} - N \mathbb{E}C_{h}^{r} + N_{j} \frac{S}{\lambda + S} \mathbb{E}C_{h}^{r} - N_{0} \frac{\lambda_{0}}{\lambda_{0} + S} \mathbb{E}C_{h}^{r}) - 2\alpha N_{j} (\frac{\lambda}{S} - \frac{\lambda}{\lambda + S}) \mathbb{E}C_{h}^{r} (\frac{1}{\kappa_{j}})^{\frac{\lambda + S}{\lambda - \lambda_{0}}} - \alpha N_{0} (\frac{\lambda_{0}}{S} - \frac{\lambda_{0}}{\lambda_{0} + S}) \mathbb{E}C_{h}^{r} (\frac{1}{\kappa_{j}})^{\frac{\lambda_{0} + S}{\lambda - \lambda_{0}}} = 0.$$

$$(32)$$

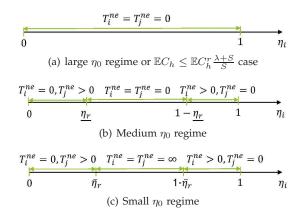


Fig. 4: Equilibrium rollover time vs new user proportion η_0 and WSP *i*'s market share.

terly new customers growth is larger than 10%, which falls into the large new user regime in Fig. 4(a). Thus the two major WSPs, China Mobile and China Unicom, announced the rollover data plan at the same time [18]. In the USA, the quarterly new customers growth is around 2% and the WSPs with diverse market shares chose rollover sequentially, i.e., T-mobile with the small market share announced rollover first in 2014, which is followed by AT&T in 2015 [19]. In a saturated market like Singapore, the quarterly new customers growth is trivial, which can be viewed as the small new user regime. So far, no WSP in this market has announced rollover data since the involved WSPs' market shares are similar.

3.1 Multiple WSPs' Equilibrium Timing

TABLE 2: Equilibrium Timing for three WSPs with market shares $\eta_1 = 0.1, \eta_2 = 0.3, \eta_3 = 0.6$. The expected cost $\mathbb{E}C_h^r$ under rollover data plan is fixed as 3, and we change the new user population η_0 and the initial cost $\mathbb{E}C_h$ to alter $\Delta = \mathbb{E}C_h - \mathbb{E}C_h^r$.

$\mathbb{E}C_h$	η_0	Equilibrium Timing
(3, 5.6]	0.3	$T_1^{ne} = T_2^{ne} = T_3^{ne} = 0$
(5.6, 8.5]	0.3	$0 = T_1^{ne} = T_2^{ne} < T_3^{ne}$
(8.5, 10]	0.3	$0 = T_1^{ne} < T_2^{ne} < T_3^{ne}$
(10, 18]	0.3	$0 = T_1^{ne} < T_2^{ne} = T_3^{ne}$
$(18,\infty]$	0.3	$T_1^{ne} = T_2^{ne} = T_3^{ne} = \infty$
5.5	0.1	$0 = T_1^{ne} = T_2^{ne} < T_3^{ne}$

For multiple asymmetric WSPs, we can numerically show the equilibrium rollover timing in Table 2, where the three WSPs have market shares $\eta_1 = 0.1$, $\eta_2 = 0.3$, $\eta_3 = 0.6$ initially. As shown in Table 2, when the overage charge cost reduction after rollover is mild, i.e., from initial cost

 $\mathbb{E}C_h \in (3, 5.6]$ to $\mathbb{E}C_h^r = 3$ under the rollover data plan, the three WSPs will immediately upgrade their data plans, i.e., $T_1^{ne} = T_2^{ne} = T_3^{ne} = 0$ to equally attract the sizable new users with large churn rate $\eta_0 = 0.3$. As the cost reduction increases, i.e., $\mathbb{E}C_h \in (5.6, 8.5]$, the WSPs with smaller market share will upgrade immediately to attract the new users and other WSP's large market share, i.e., $T_1^{ne} = T_2^{ne} = 0$, and the WSP with the largest market share purposely waits and upgrades at time $T_3^{ne} > 0$ to avoid the immediate revenue loss from its existing users. If the cost reduction further increases, i.e., $\mathbb{E}C_h \in (8.5, 10]$, only the WSP with the smallest market share will upgrade immediately and the other two WSPs will wait and upgrade successively according to their current market share, i.e., $0 < T_2^{ne} < T_3^{ne}$. For larger cost reduction regime $\mathbb{E}C_h \in (10, 18]$, only the WSP with the smallest market share will upgrade immediately, and the second largest WSP will further postpone its upgrade timing and upgrade together with the largest WSP 3, i.e., $T_2^{ne} = T_3^{ne} > 0$. For very large cost reduction regime $\mathbb{E}C_h \in (18,\infty]$, the WSPs will not upgrade to rollover data plan, i.e., $T_1^{ne} = T_2^{ne} = T_3^{ne} = \infty$, as the increased profit from the switched users and new users cannot compensate for the profit loss from existing users. Moreover, we show that for the mild cost reduction, i.e., $\mathbb{E}C_h = 5.5$, as the proportion of new users η_0 decreases from 0.3 to 0.1, the equilibrium rollover timing will change from $T_1^{ne} = T_2^{ne} = \overline{T_3^{ne}} = 0$ to $0 = \overline{T_1^{ne}} = \overline{T_2^{ne}} < \overline{T_3^{ne}}$. That is to say, the WSP 3 with the largest market share will postpone its upgrade time as the small new user population cannot justify the reduction of immediate overage charges due to rollover. The numerical results shown in Table 2 for multiple WSPs coincide with the key results of two-WSP case in Theorem 3.1 and Fig. 4. Moreover, for multiple symmetric WSPs, we have the following corollary, which is consistent with Theorem 3.1.

Corollary 3.1. For multiple symmetric WSPs, they will offer rollover immediately at the same time if $\mathbb{E}C_h - \mathbb{E}C_h^r \frac{\lambda+S}{S} < \eta_0 \mathbb{E}C_h^r \frac{\lambda_0}{S}$ for large η_0 or mild cost reduction.

Proof: Suppose there are $\mathcal{M} \geq 2$ symmetric WSPs. Note that a WSP's existing users are only affected by the WSP who offers rollover first as the existing users start to churn to the earlier rollover WSP gradually. For any WSP $i \in \mathcal{M}$, denote T_0 as the WSPs who offers rollover first. Then, WSP

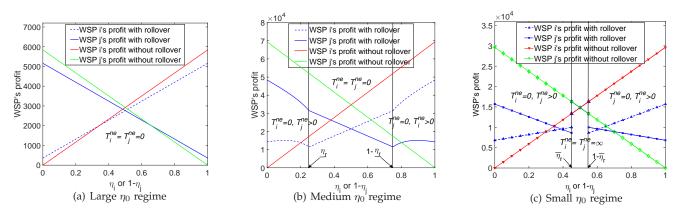


Fig. 5: WSPs' equilibrium profits at equilibrium rollover time versus WSP i's market share (η_i or $1 - \eta_j$) and new user proportion η_0 .

 $i \in \mathcal{M}$ will not offer rollover latest if

$$\frac{\partial R_i^{r,>}}{\partial T_i} = 2\alpha \frac{N}{\mathcal{M}} (\mathbb{E}C_h - \mathbb{E}C_h^r \frac{\lambda + S}{S}) e^{\lambda T_0 - (\lambda + S)T_i} - 2\alpha \frac{N_0}{\mathcal{M}} \mathbb{E}C_h^r \frac{\lambda_0}{S} e^{\lambda_0 T_0 - (\lambda_0 + S)T_i} < 0,$$
(33)

i.e., $\frac{N}{\mathcal{M}}(\mathbb{E}C_h - \mathbb{E}C_h^r \frac{\lambda + S}{S}) < \frac{N_0}{\mathcal{M}} \mathbb{E}C_h^r \frac{\lambda_0}{S}$. As a result, all WSPs prefer rollover earlier, which results in the equilibrium point that all WSPs offer rollover at the beginning.

Remark 3.1. Note that for multiple symmetric WSPs, they may have asymmetric upgrade timing. For example, for non-trivial overage charge cost reduction and not large new user population, e.g., $\mathbb{E}C_h = 6$, $\mathbb{E}C_h^r = 3$, $\eta_0 = 0.3$, two out of the three WSPs with market shares $\eta_1 = \eta_2 =$ η_3 will upgrade immediately to attract the new users and switched users, i.e., $0 = T_1^{ne} = T_2^{ne} < T_3^{ne}$. Given the equilibrium rollover timing of the two WSPs, if the third WSP still chooses to upgrade at the beginning, it can only attract the new users and no switched users from the other WSPs will join it. Therefore, it is better for the third WSP to wait and upgrade at time $T_3^{ne} > 0$ as the revenue received from the new users cannot compensate for the revenue loss from existing users.

3.2 WSPs' Profits at Equilibrium Timing

Now we are ready to discuss the duopoly WSPs' profits at the equilibrium. By substituting the equilibrium rollover time in Theorem 3.1 to (13) and (14), we can derive the equilibrium profits of the two WSPs. At the equilibrium rollover timing, the profit of WSP i, i = 1, 2 is given as follows:

• The WSP *i*'s profit for rollover at the same time (i.e., $T_i^{ne} = T_i^{ne} = 0$) is

$$\bar{R}_i^r = 2\alpha N_i \mathbb{E}C_h^r \frac{1}{S} + \alpha N_0 (\frac{1}{S} - \frac{1}{S + \lambda_0}) \mathbb{E}C_h^r.$$
 (34)

• The WSP *i*'s profit for first rollover (i.e., $T_i^{ne} = 0$, $T_j^{ne} = \frac{\log \kappa_j}{\lambda - \lambda_0}$) is

$$\bar{R}_{i}^{r,\leq} = 2\alpha N \mathbb{E}C_{h}^{r} \frac{1}{S} - 2\alpha N_{j} \mathbb{E}C_{h}^{r} \frac{1}{S+\lambda} - 2\alpha N_{j} \mathbb{E}C_{h}^{r} (\frac{1}{S} - \frac{1}{\lambda+S})(\frac{1}{\kappa_{j}})^{\frac{\lambda+S}{\lambda-\lambda_{0}}} + 2\alpha N_{0} (\frac{1}{S} - \frac{1}{S+\lambda_{0}}) \mathbb{E}C_{h}^{r} - \alpha N_{0} (\frac{1}{S} - \frac{1}{S+\lambda_{0}})(\frac{1}{\kappa_{j}})^{\frac{\lambda_{0}+S}{\lambda-\lambda_{0}}} \mathbb{E}C_{h}^{r}.$$

$$(35)$$

• The WSP *i*'s profit for late rollover (i.e., $T_i^{ne} = \frac{\log \kappa_i}{\lambda - \lambda_0}$, $T_i^{ne} = 0$) is

$$\bar{R}_{i}^{r,>} = 2\alpha N_{i} \frac{1}{\lambda+S} \mathbb{E}C_{h} \left(1 - \left(\frac{1}{\kappa_{i}}\right)^{\frac{\lambda+S}{\lambda-\lambda_{0}}}\right) \\ + 2\alpha N_{i} \frac{1}{S} \mathbb{E}C_{h}^{r} \left(\frac{1}{\kappa_{i}}\right)^{\frac{\lambda+S}{\lambda-\lambda_{0}}} \\ + \alpha N_{0} \left(\frac{1}{S} - \frac{1}{S+\lambda_{0}}\right) \left(\frac{1}{\kappa_{i}}\right)^{\frac{\lambda_{0}+S}{\lambda-\lambda_{0}}} \mathbb{E}C_{h}^{r}.$$
(36)

If the new user population is extremely large, it is manifest that the WSPs always gain profit by upgrading to rollover data. Next, we look at the non-trivial case that the number of new users is smaller than that of the existing users, i.e., $\eta_0 < 1$. By comparing WSP *i*'s profits before and after rollover at the equilibrium, we have the following proposition.

Proposition 3.2. At the equilibrium, there exists a unique threshold $\eta_i^{th} \in [0, 1]$ such that WSP *i* gains profit by data rollover if $\eta_i \leq \eta_i^{th}$, and loses profit if $\eta_i > \eta_i^{th}$.

The Proof of Proposition 3.2 is given in Appendix B of the supplemental material.

If a WSP has a small market share, it can still increase its profit by first rollover and attracting many users from the other WSP. While for the WSP with already a large market share, the small new user population cannot justify the reduction of overage charges due to rollover. In the following, we present numerical results in Figs. 5 and 6 to examine the WSPs's equilibrium profits.

Observation 1: A WSP's equilibrium profit can decrease with its market share.

As shown in Fig.5(a) and 5(b), if both WSPs choose rollover immediately, it is manifest that each WSP's profit

$$\frac{1}{2}\mathcal{D}\left(\frac{\lambda}{\lambda+S}+\frac{1}{2}\frac{S}{\lambda+S}\right)-\mathcal{E}\left(\frac{\lambda}{\lambda+S}+\frac{1}{4}\frac{S}{\lambda+S}\right)-\mathcal{E}\eta_{0}\frac{\lambda_{0}}{\lambda_{0}+S}-\frac{1}{2}\mathcal{E}\left(\frac{\lambda}{S}-\frac{\lambda}{\lambda+S}\right)\left(\frac{\mathcal{E}\eta_{0}\frac{\lambda_{0}}{S}}{\frac{1}{2}(\mathcal{D}-\mathcal{E})-\mathcal{E}\frac{\lambda}{S}}\right)^{\frac{\lambda+S}{\lambda-\lambda_{0}}} -\frac{1}{2}\mathcal{E}\eta_{0}\frac{\lambda_{0}}{S(\lambda_{0}+S)}\left(\frac{\mathcal{E}\eta_{0}\frac{\lambda_{0}}{S}}{\frac{1}{2}(\mathcal{D}-\mathcal{E})-\mathcal{E}\frac{\lambda}{S}}\right)^{\frac{\lambda_{0}+S}{\lambda-\lambda_{0}}} = 0.$$
(37)

$$\underline{\eta_s} = \max\left(1 - \tilde{\eta}_s, \min\left(\max(1 - \hat{\eta}_s, \frac{\mathcal{E}\frac{\lambda}{S} + \sqrt{(\mathcal{E}\frac{\lambda}{S})^2 + 2(\mathcal{D} - \mathcal{E})\mathcal{E}\eta_0\frac{\lambda_0}{S}}}{2(\mathcal{D} - \mathcal{E})}\right), \min(\hat{\eta}_s, 1 - \frac{\mathcal{E}\frac{\lambda}{S} + \sqrt{(\mathcal{E}\frac{\lambda}{S})^2 + 2(\mathcal{D} - \mathcal{E})\mathcal{E}\eta_0\frac{\lambda_0}{S}}}{2(\mathcal{D} - \mathcal{E})})\right)\right).$$
(38)

$$\mathcal{D}\eta_i(\frac{\lambda}{\lambda+S}+\eta_i\frac{S}{\lambda+S}) - \mathcal{E}(\frac{\lambda}{\lambda+S}+\eta_i^2\frac{S}{\lambda+S}) - \mathcal{E}\eta_0\frac{\lambda_0}{\lambda_0+S} - \eta_j\mathcal{E}(\frac{\lambda}{S}-\frac{\lambda}{\lambda+S})(\frac{1}{\kappa_j^s})^{\frac{\lambda+S}{\lambda-\lambda_0}} - \frac{1}{2}\mathcal{E}\eta_0\frac{\lambda_0^2}{S(\lambda_0+S)}(\frac{1}{\kappa_j^s})^{\frac{\lambda_0+S}{\lambda-\lambda_0}} = 0.$$
(39)

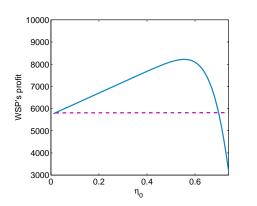


Fig. 6: WSP's equilibrium profit (the blue solid curve) under first rollover versus $\eta_0.$

increases with its market share. However, in Fig. 5(b), we observe that if WSP *i* provides rollover data plan first, its profit for first rollover may decrease with its market share η_i . This is because as η_i increases, WSP *j*'s market share η_j decreases and it brings forward its rollover time, which results in the number of new users and the existing users switched from WSP *j* becomes smaller. Similarly, in Fig. 5(c), the WSP *i*'s profit suddenly reduces as η_i increases across the threshold point $1 - \bar{\eta}_r$ (i.e., from medium to large market share). The reason is that if η_i is slightly larger than $1 - \bar{\eta}_r$, WSP *i* prefers rollover late and the revenue received from the few new users cannot compensate for the revenue loss due to its many existing users' cost reduction.

Observation 2: The WSP may not benefit from the new user population.

As shown in Fig. 6, the WSP's profit for earlier rollover first increases with η_0 , as it gains more revenue from more new users. However, as η_0 further increases, the WSP's profit for earlier rollover decreases. This is because the other WSP's rollover time is brought forward (with intensified competition), which reduces the number of new users and switched existing users subscribed to the WSP of earlier rollover.

4 TIMING EQUILIBRIUM FOR SHARED DATA PLAN

When deciding the timing for shared data plan, a WSP also faces the tradeoff between its overage charge loss from its existing users and the profit gain of market share (from new users and those from the other WSP). Recall that in the earlier upgrade profit in (22), we have introduced \mathcal{D} and \mathcal{E} in (24) and (25) to tell the families' expected costs before and

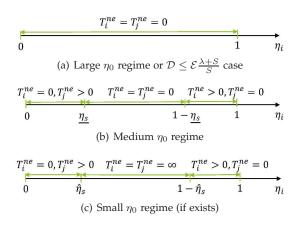


Fig. 7: Equilibrium timing vs new user population η_0 and WSP *i*'s market share.

after using the shared data plan, respectively. We note that $\mathcal{D} > \mathcal{E}$ as the (heavy, heavy) and (heavy, light) families' costs are reduced. One can imagine that if the cost reduction after offering the shared data plan is huge, the WSPs prefer not to offer due to large revenue loss. In the following analysis, we consider an upper bound for such cost reduction, i.e., $\mathcal{D} < \mathcal{E} \frac{4\lambda + S}{S}$.⁴ Similar to the timing analysis of the rollover data plan, we first analyze the WSPs' best responses and then derive the WSPs' equilibrium timing of shared data plan as follows.

Theorem **4.1.** The WSPs' equilibrium upgrade timing of shared data plan is given as follows (see Fig. 7):

- If D ≤ ε^{λ+S}/_S, the overage charge reduction after offering shared data plan is mild and the two WSPs will immediately upgrade their data plans (i.e., T^{ne}_i = T^{ne}_i = 0), as shown in Fig. 7(a).
- If D > ε^{λ+S}/_S, the WSPs' equilibrium share timing depends on the new user proportion η₀ and the WSPs' market shares:
 - 1) Large η_0 regime $(\eta_0 \ge \frac{2(\mathcal{D}S \mathcal{E}(\lambda + S))}{\mathcal{E}\lambda_0})$: both WSPs will offer shared data plan immediately to attract many new (heavy, heavy) and (heavy, light) families (i.e., $T_i^{ne} = T_j^{ne} = 0$), as shown in Fig. 7(a).
 - 2) Medium η_0 regime $(\min((\bar{\eta}_0^s)^+, (\frac{\mathcal{D}S \mathcal{E}(2\lambda + S)}{2\mathcal{E}\lambda_0})^+) < \eta_0 <$

4. For the case when $\mathcal{D} \geq \mathcal{E} \frac{4\lambda+S}{S}$, we can show that the equilibrium analysis is similar, and in the equilibrium results that only the medium η_0 regime includes more tedious cases to quantify equilibrium timing.

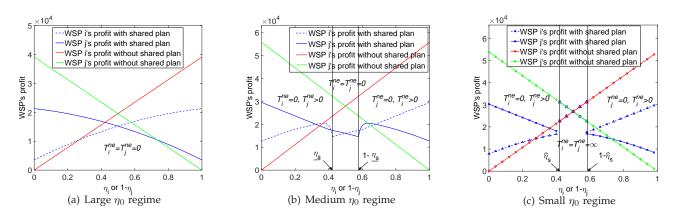


Fig. 8: WSPs' equilibrium profits at equilibrium share time versus WSP *i*'s market share (η_i or $1 - \eta_j$) and new user proportion η_0 .

 $\frac{2(\mathcal{D}S-\mathcal{E}(\lambda+S))}{\mathcal{E}\lambda_0} \text{ with } \bar{\eta}_0^s \text{ as the solution to} (37)): the median size of new users and the other WSP's many users allure the WSP with a small market share (less than <math>\underline{\eta}_s$ in $(38)^5$) to upgrade immediately, and the other WSP purposely waits for $\frac{\log \kappa_i^s}{\lambda-\lambda_0}, i \in \{1,2\} > 0$, where

$$\kappa_i^s = \frac{(\mathcal{D} - \mathcal{E})\eta_i^2 - \mathcal{E}\frac{\lambda}{S}\eta_i}{\frac{1}{2}\mathcal{E}\eta_0\frac{\lambda_0}{S}}.$$

If the two WSPs have similar market shares (between $\underline{\eta}_s$ and $1 - \underline{\eta}_s$), they will upgrade immediately to keep their existing users and attract new users (see Fig. 7(b)).

3) Small regime (0 \leq η_0 η_0 $\min((\bar{\eta}_0^s)^+, (\frac{\mathcal{D}S - \mathcal{E}(2\lambda + S)}{2\mathcal{E}\lambda_0})^+)):$ the small size of new users cannot allure the two WSPs with similar market shares (between $\hat{\eta}_s$ and $1 - \hat{\eta}_s$ with $\hat{\eta}_s$ as the solution to (39)) to upgrade to shared data plan (i.e., $T_i^{ne} = T_i^{ne} = \infty$). Only the WSP of small market share (less than $\hat{\eta}_s$) still chooses to upgrade to attract the other WSP's large market share, and the other will still upgrade after $\frac{\log \kappa_i^s}{\lambda - \lambda_0}$, $i \in \{1, 2\}$ to keep its market share to some extent (see Fig. 7(c)).

The proof of Theorem 4.1 is given in Appendix C of the supplemental material. Unlike Theorem 3.1 for the rollover data plan, here the small η_0 regime in Fig. 7(c) does not exist if $\frac{DS - \mathcal{E}(2\lambda + S)}{2\mathcal{E}\lambda_0} \leq 0$. Then, we only have large and medium η_0 regimes in Fig. 7(a) and 7(b), where at least one WSP will upgrade to shared data plan even for few new user population. Recall when $\eta_0 = 0$ in the rollover data case, the WSPs can only benefit from the heavy users, and the WSPs with similar market share will not offer rollover as shown in Fig. 4(c). However, this is not true for the shared data case as a WSP benefits from attracting (heavy,heavy) and (heavy,light) families from the other WSP. Here, the relative proportion between (heavy, heavy) and (heavy, light)

5. $\tilde{\eta}_s$ in (38) is the solution of $R_i^{s,\leq}(T_i = 0, T_j = \frac{\log \kappa_j^s}{\lambda - \lambda_0}) = R_i^{s,>}(T_j = 0, T_i = \frac{\log \kappa_i^s}{\lambda - \lambda_0})$ as a function of η_i for any given η_0 , with $R_i^{s,\leq}$ and $R_i^{s,>}$ given in (22) and (23) respectively.

families increases with the proportion of heavy users α . In the following proposition, we discuss this different result clearly given no new user arrival.

- **Proposition 4.1.** Even if $\eta_0 = 0$, both WSPs with similar market shares (between $\underline{\eta}_s$ and $1 \underline{\eta}_s$ in Fig. 7(b)) will still offer the shared data plan, once one of the following three conditions is true:
 - Large (heavy, heavy) family proportion with mild cost reduction: If (heavy, light) family's cost reduction is significant (i.e., $\frac{\mathbb{E}C_h + \mathbb{E}C_l}{\mathbb{E}C_{h,l}} > \frac{2\lambda + S}{S}$) but (heavy, heavy) family's cost reduction is mild (i.e., $\frac{2\mathbb{E}C_h}{\mathbb{E}C_{h,h}} \leq \frac{2\lambda + S}{S}$), then the WSPs will offer shared data plan when facing large proportion of (heavy, heavy) families (i.e., $\alpha \geq \frac{2(\mathbb{E}C_h + \mathbb{E}C_l \frac{2\lambda + S}{S} \mathbb{E}C_{h,l})}{2\mathbb{E}C_l + (\mathbb{E}C_{h,h} 2\mathbb{E}C_{h,l})}$.
 - Large (heavy, light) family proportion with mild cost reduction: If (heavy, light) family's cost reduction is mild (i.e., $\frac{\mathbb{E}C_h + \mathbb{E}C_l}{\mathbb{E}C_{h,l}} \leq \frac{2\lambda + S}{S}$) but (heavy, heavy) family's cost reduction is significant (i.e., $\frac{2\mathbb{E}C_h}{\mathbb{E}C_{h,h}} > \frac{2\lambda + S}{S}$), then the WSPs will offer shared data plan when facing small proportion of (heavy, heavy) families (i.e., $\alpha \leq \frac{2(\mathbb{E}C_h + \mathbb{E}C_l \frac{2\lambda + S}{S} \mathbb{E}C_{h,l})}{2\mathbb{E}C_l + (\mathbb{E}C_{h,h} 2\mathbb{E}C_{h,l})^2 \frac{2\lambda + S}{S}}$.
 - Mild cost reduction for both (heavy, heavy) and (heavy, light) families: If both (heavy, heavy) and (heavy, light) families' cost reductions are mild (i.e., $\frac{\mathbb{E}C_h + \mathbb{E}C_l}{\mathbb{E}C_{h,l}} \leq \frac{2\lambda + S}{S}$ and $\frac{2\mathbb{E}C_h}{\mathbb{E}C_{h,l}} \leq \frac{2\lambda + S}{S}$), then the WSPs will offer shared data plan for any $\alpha \in (0, 1)$.

The Proof of Proposition 4.1 is given in Appendix D of the supplemental material.

4.1 WSPs' Profits at Equilibrium Timing

By substituting the equilibrium timing in Theorem 4.1 to (22) and (23), we can derive the equilibrium profits of the two WSPs. At the equilibrium share time, the profit of WSP i, i = 1, 2 is given as follows:

 The WSP *i*'s profit for offering shared data plan at the same time, i.e., T_i^{ne} = T_j^{ne} = 0, is

$$\bar{R}_i^s = \eta_i N \mathcal{E} \frac{1}{S} + \eta_i \eta_j N (\mathcal{D} - \mathcal{E}) \frac{1}{\lambda + S} + \frac{1}{2} N_0 \mathcal{E} (\frac{1}{S} - \frac{1}{\lambda_0 + S}).$$
(40)

 The WSP *i*'s profit for offering shared data plan first, i.e., T^{ne}_i = 0 is

$$\begin{split} \bar{R}_i^{s,\leq} = & \mathcal{E}N(\frac{1}{S} - \frac{1}{\lambda+S}) + \eta_i^2 N \frac{1}{\lambda+S} \mathcal{E} \\ & + \eta_i \eta_j N \mathcal{D} \frac{1}{\lambda+S} + N_0 \mathcal{E}(\frac{1}{S} - \frac{1}{\lambda_0+S}) \\ & - (1-\eta_i) N \mathcal{E}(\frac{1}{S} - \frac{1}{\lambda+S}) (\frac{1}{\kappa_j^s})^{\frac{\lambda+S}{\lambda-\lambda_0}} \\ & - \frac{1}{2} N_0 \mathcal{E}(\frac{1}{S} - \frac{1}{\lambda_0+S}) (\frac{1}{\kappa_j^s})^{\frac{\lambda_0+S}{\lambda-\lambda_0}}. \end{split}$$
(41)

The WSP *i*'s profit for offering shared data plan late,
 i.e., T^{ne}_i = log κⁱ_i/(λ-λ₀), is

$$\begin{split} \bar{R}_{i}^{s,>} = & \eta_{i} N \mathcal{D} \frac{1}{\lambda + S} \left(1 - \left(\frac{1}{\kappa_{i}^{s}}\right)^{\frac{\lambda + S}{\lambda - \lambda_{0}}} \right) \\ & + \eta_{i} N \mathcal{E} \frac{1}{S} \left(\frac{1}{\kappa_{i}^{s}}\right)^{\frac{\lambda + S}{\lambda - \lambda_{0}}} \\ & + \eta_{i} \eta_{j} N (\mathcal{D} - \mathcal{E}) \frac{1}{\lambda + S} \left(\frac{1}{\kappa_{i}^{s}}\right)^{\frac{\lambda + S}{\lambda - \lambda_{0}}} \\ & + \frac{1}{2} N_{0} \mathcal{E} \left(\frac{1}{S} - \frac{1}{\lambda_{0} + S}\right) \left(\frac{1}{\kappa_{i}^{s}}\right)^{\frac{\lambda - S}{\lambda - \lambda_{0}}}. \end{split}$$
(42)

Fig. 8 numerically shows how the WSPs' equilibrium profits change with their market shares in the three η_0 regimes. As shown in Fig. 8(a) and 8(b), if both WSPs choose shared data immediately, each WSP's profit increases with its market share. However, as shown in Fig. 8(b), a WSP's profit for earlier share may decrease with its market share. This is because WSP *j* brings forward its upgrade time due to the reduced market share η_j , and thus less new users and WSP *j*'s users subscribe to WSP *i*. Similarly, in Fig. 8(c), the WSP *i*'s profit suddenly reduces as η_i increases across the threshold point $1 - \hat{\eta}_s$ (i.e., from medium to large market share). The reason is that WSP *i* with large market share faces large revenue loss of overage charge, which cannot be made up by the revenue gain from the few new users.

5 CONCLUSION

This paper analytically studies the competitive WSPs' timing of offering innovative data plans. For the WSP with small market share, it is better to upgrade first to attract more users (especially from the other WSP). While for the WSP with large market share, it is better to upgrade late to avoid immediate loss in overage charge. For the WSPs with similar market shares, they should upgrade immediately for large new user population and may not upgrade for small new user population. The launching of different innovative data plans also depends on their strategies. According to the composition of users in a family, the WSPs may upgrade to shared data plan simultaneously even when no new user arrives, which is different from the rollover data plan.

In the future, we plan to extend our results in two possible directions. First, we could consider a variety of separate rollover and shared data plans (e.g., for 2G/3G/4G) to fit heterogeneous users. Though the analysis is more involved by modeling mixed user churn flows from any existing plan to any innovative plan under the same or different WSPs, we still expect the WSP of small market share chooses innovative data plan first. Second, we can extend users'

static data usage model and consider that the innovative data plans may motivate users to increase their usage. In this case, the WSPs' overage charges reduce less severely and they have more incentives to announce innovative plans earlier.

APPENDIX A PROOF OF THEOREM 3.1

According to Proposition 3.1, we can see that the possible equilibrium rollover time for the WSPs are $(T_i^{ne} = T_j^{ne} = 0)$, $(T_i^{ne} = T_j^{ne} > 0)$, $(T_i^{ne} = 0, T_j^{ne} = \frac{\log \kappa_j}{\lambda - \lambda_0})$, $(T_j^{ne} = 0, T_i^{ne} = \frac{\log \kappa_j}{\lambda - \lambda_0})$, $(T_i^{ne} = T_j^{ne} = \infty)$. In the following, we will discuss the conditions for each equilibrium.

 $(T_i^{ne} = T_j^{ne} = 0)$ is the equilibrium if and only if

$$\frac{\partial R_i^{r,>}(T_i,0)}{\partial T_i} = 2\alpha N_i (\mathbb{E}C_h - \mathbb{E}C_h^r \frac{\lambda + S}{S}) e^{\lambda T_j - (\lambda + S)T_i} - \alpha N_0 \mathbb{E}C_h^r \frac{\lambda_0}{S} e^{\lambda_0 T_j - (\lambda_0 + S)T_i} \le 0,$$
(43)

and

$$\frac{\partial R_j^{r,>}(0,T_j)}{\partial T_j} = 2\alpha N_j (\mathbb{E}C_h - \mathbb{E}C_h^r \frac{\lambda+S}{S}) e^{\lambda T_i - (\lambda+S)T_j} - \alpha N_0 \mathbb{E}C_h^r \frac{\lambda_0}{S} e^{\lambda_0 T_i - (\lambda_0+S)T_j} \le 0,$$
(44)

i.e., $\kappa_i \leq 1$ and $\kappa_j \leq 1$.

It is manifest that when $\mathbb{E}C_h \leq \mathbb{E}C_h^r \frac{\lambda+S}{S}$, which means the heavy user's expected cost after rollover is close to that before rollover, both WSPs will rollover at the beginning.

Then, we show that if $T_i^{ne} = T_j^{ne}$ is the equilibrium, we have $T_i^{ne} = T_j^{ne} = 0$.

If $T_i^{ne} = T_j^{ne}$ is the equilibrium, if and only if $\kappa_i \leq 1$, $\kappa_j \leq 1$ and

$$\frac{\partial R_i^{\prime, -}}{\partial T_i}|_{T_i = T_j} \ge 0, \tag{45}$$

and

$$\frac{\partial R_j^{\prime, \geq}}{\partial T_j}|_{T_j=T_i} \ge 0, \tag{46}$$

i.e.,

$$2\alpha (N_{i}\mathbb{E}C_{h} - N\mathbb{E}C_{h}^{r} + N_{j}\frac{S}{\lambda + S}\mathbb{E}C_{h}^{r}) - 2\alpha N_{0}\frac{\lambda_{0}}{\lambda_{0} + S}\mathbb{E}C_{h}^{r}$$
$$- 2\alpha N_{j}(\frac{\lambda}{S} - \frac{\lambda}{\lambda + S})\mathbb{E}C_{h}^{r} + \alpha N_{0}(\frac{\lambda_{0}}{\lambda_{0} + S} - \frac{\lambda_{0}}{S})\mathbb{E}C_{h}^{r} \ge 0,$$
(47)

and

$$2\alpha(N_{j}\mathbb{E}C_{h} - N\mathbb{E}C_{h}^{r} + N_{i}\frac{S}{\lambda + S}\mathbb{E}C_{h}^{r}) - 2\alpha N_{0}\frac{\lambda_{0}}{\lambda_{0} + S}\mathbb{E}C_{h}^{r}$$
$$- 2\alpha N_{i}(\frac{\lambda}{S} - \frac{\lambda}{\lambda + S})\mathbb{E}C_{h}^{r} + \alpha N_{0}(\frac{\lambda_{0}}{\lambda_{0} + S} - \frac{\lambda_{0}}{S})\mathbb{E}C_{h}^{r} \ge 0,$$
(48)

According to $\kappa_i \leq 1$ and $\kappa_j \leq 1$, the proportion of new users should satisfy $\frac{S}{\lambda_0}(\frac{\mathbb{E}C_h}{\mathbb{E}C_h^r} - \frac{\lambda+S}{S}) \leq \eta_0 < \frac{2S}{\lambda_0}(\frac{\mathbb{E}C_h}{\mathbb{E}C_h^r} - \frac{\lambda+S}{S})$. Under this circumstance, by solving (47) and (48), we have $\eta_i > 0.5$ and $\eta_j = 1 - \eta_i > 0.5$, which can't be satisfied simultaneously. Therefore, $T_i^{ne} = T_j^{ne} = 0$.

The necessary condition for $(T_i^{ne} = 0, T_j^{ne} = \frac{\log \kappa_j}{\lambda - \lambda_0})$ is $\kappa_j > 1$ and $\frac{\partial R_i^{r,\leq}}{\partial T_i} |_{T_j = \frac{\log \kappa_j}{\lambda - \lambda_0}} < 0$, i.e., $g(\eta_i) < 0$, where

$$g(\eta_i) = 2\alpha (N_i \mathbb{E}C_h - N \mathbb{E}C_h^r + N_j \frac{S}{\lambda + S} \mathbb{E}C_h^r) - 2\alpha N_j (\frac{\lambda}{S} - \frac{\lambda}{\lambda + S}) \mathbb{E}C_h^r (\frac{1}{\kappa_j})^{\frac{\lambda + S}{\lambda - \lambda_0}} - \alpha N_0 (\frac{\lambda_0}{S} - \frac{\lambda_0}{\lambda_0 + S}) \mathbb{E}C_h^r (\frac{1}{\kappa_j})^{\frac{\lambda_0 + S}{\lambda - \lambda_0}} - 2\alpha N_0 \frac{\lambda_0}{\lambda_0 + S} \mathbb{E}C_h^r.$$

$$(49)$$

And the necessary condition for $(T_j^{ne} = 0, T_i^{ne} = \frac{\log \kappa_i}{\lambda - \lambda_0})$ is $\kappa_i > 1$ and $\frac{\partial R_j^{r,e}}{\partial T_j} \Big|_{T_i = \frac{\log \kappa_i}{\lambda - \lambda_0}} < 0$, i.e.,

$$2\alpha (N_{j}\mathbb{E}C_{h} - N\mathbb{E}C_{h}^{r} + N_{i}\frac{S}{\lambda + S}\mathbb{E}C_{h}^{r}) - 2\alpha N_{0}\frac{\lambda_{0}}{\lambda_{0} + S}\mathbb{E}C_{h}^{r}$$
$$- 2\alpha N_{i}(\frac{\lambda}{S} - \frac{\lambda}{\lambda + S})\mathbb{E}C_{h}^{r}(\frac{1}{\kappa_{i}})^{\frac{\lambda + S}{\lambda - \lambda_{0}}}$$
$$+ \alpha N_{0}(\frac{\lambda_{0}}{\lambda_{0} + S} - \frac{\lambda_{0}}{S})\mathbb{E}C_{h}^{r}(\frac{1}{\kappa_{i}})^{\frac{\lambda_{0} + S}{\lambda - \lambda_{0}}} < 0.$$
(50)

Otherwise, both WSPs will not rollover, i.e., $(T_i^{ne} = T_i^{ne} = \infty)$.

$$\begin{split} I_{j}^{ic} &= \infty \end{pmatrix}. \\ & \text{When } \mathbb{E}C_{h} > \mathbb{E}C_{h}^{r}\frac{\lambda+S}{S}, \text{ according to } \kappa_{i} \leq 1 \text{ and } \kappa_{j} \leq 1, \\ I_{i}^{ne} &= T_{j}^{ne} = 0 \text{ is the equilibrium if and only if } 1 - \frac{N_{0}\mathbb{E}C_{h}^{r}\frac{\lambda_{0}}{S}}{2N(\mathbb{E}C_{h}-\mathbb{E}C_{h}^{r}\frac{\lambda+S}{S})} \leq \eta_{i} \leq \frac{N_{0}\mathbb{E}C_{h}^{r}\frac{\lambda_{0}}{S}}{2N(\mathbb{E}C_{h}-\mathbb{E}C_{h}^{r}\frac{\lambda+S}{S})}. \\ & \Pi_{0} \geq \frac{2S}{\lambda_{0}} \left(\frac{\mathbb{E}C_{h}}{\mathbb{E}C_{h}^{r}} - \frac{\lambda+S}{S}\right), \text{ we have } \frac{N_{0}\mathbb{E}C_{h}^{r}\frac{\lambda+S}{S}}{2N(\mathbb{E}C_{h}-\mathbb{E}C_{h}^{r}\frac{\lambda+S}{S})} \geq 1, \text{ which means the condition always holds and } T_{i}^{ne} = T_{j}^{ne} = 0 \text{ is the equilibrium.} \end{split}$$

Since $g(\eta_i = 0) < 0$ and $g(\eta_i)$ decreases with η_0 , the solution of $g(\eta_i = 0) = 0$, denoted as $\bar{\eta}_r$, increases with η_0 . Therefore, if $g(\eta_i = 0.5) < 0$ for a given $\hat{\eta}_0$, then, for any $\eta_0 > \hat{\eta}_0$, $g(\eta_i = 0.5) < 0$, which means $\bar{\eta}_r > 0.5$. When $\eta_0 = \frac{S}{\lambda_0} \left(\frac{\mathbb{E}C_h}{\mathbb{E}C_h^r} - \frac{\lambda + S}{S}\right)$ and $\eta_i = 0.5$, we have $\kappa_j = 1$. Since $\mathbb{E}C_h > \mathbb{E}C_h^r \frac{\lambda + S}{S}$, it is easy to check that $g(\eta_i = 0.5) < 0$. Therefore, when $\frac{S}{\lambda_0} \left(\frac{\mathbb{E}C_h}{\mathbb{E}C_h^r} - \frac{\lambda + S}{S}\right) < \eta_0 < \frac{2S}{\lambda_0} \left(\frac{\mathbb{E}C_h}{\mathbb{E}C_h^r} - \frac{\lambda + S}{S}\right)$, i.e., $0.5 < \frac{N_0 \mathbb{E}C_h^r \frac{\lambda_0}{S}}{2N(\mathbb{E}C_h - \mathbb{E}C_h^r \frac{\lambda + S}{S})} < 1$, $\bar{\eta}_r > 0.5 > 1 - \frac{N_0 \mathbb{E}C_h^r \frac{\lambda_0}{S}}{2N(\mathbb{E}C_h - \mathbb{E}C_h^r \frac{\lambda + S}{S})}$. Then, when $0 \leq \eta_i < 1 - \frac{N_0 \mathbb{E}C_h^r \frac{\lambda_0}{S}}{2N(\mathbb{E}C_h - \mathbb{E}C_h^r \frac{\lambda + S}{S})}$, $g(\eta_i) \leq 0$ is always satisfied and thus $T_i^{ne} = 0, T_j^{ne} = \frac{\log \kappa_j}{\lambda - \lambda_0}$ is the equilibrium. Similarly, for the case when $\frac{N_0 \mathbb{E}C_h^r \frac{\lambda_0}{S}}{2N(\mathbb{E}C_h - \mathbb{E}C_h^r \frac{\lambda + S}{S})} < 1$, $1 - \frac{N_0 \mathbb{E}C_h^r \frac{\lambda_0}{S}}{2N(\mathbb{E}C_h - \mathbb{E}C_h^r \frac{\lambda + S}{S})} < 1$, $1 - \frac{N_0 \mathbb{E}C_h^r \frac{\lambda_0}{S}}{2N(\mathbb{E}C_h - \mathbb{E}C_h^r \frac{\lambda + S}{S})} < \frac{N_0 \mathbb{E}C_h^r \frac{\lambda_0}{S}}{2N(\mathbb{E}C_h - \mathbb{E}C_h^r \frac{\lambda + S}{S})}$. Thus, when $1 - \frac{N_0 \mathbb{E}C_h^r \frac{\lambda_0}{S}}{2N(\mathbb{E}C_h - \mathbb{E}C_h^r \frac{\lambda + S}{S})} \leq \eta_i \leq \frac{N_0 \mathbb{E}C_h^r \frac{\lambda_0}{S}}{2N(\mathbb{E}C_h - \mathbb{E}C_h^r \frac{\lambda + S}{S})}$. Thus, when $1 - \frac{N_0 \mathbb{E}C_h^r \frac{\lambda_0}{S}}{2N(\mathbb{E}C_h - \mathbb{E}C_h^r \frac{\lambda + S}{S})} \leq \eta_i \leq \frac{N_0 \mathbb{E}C_h^r \frac{\lambda + S}{S}}{N(\mathbb{E}C_h - \mathbb{E}C_h^r \frac{\lambda + S}{S})}$. Thus, upon the matrix solution as $\bar{\eta}_i$.

Define $\phi(\bar{\eta_0}) := g(\eta_i = 0.5)$ and denote its solution as $\bar{\eta}_0$. Since $\mathbb{E}C_h > \mathbb{E}C_h^r \frac{\lambda + S}{S}$, it is easy to check that $\phi(\eta_0 = 0) > 0$, and thus $\bar{\eta}_0 > 0$. Then, when $\eta_0 \le \bar{\eta}_0$, we have $\bar{\eta}_r \le 0.5$ and when $\eta_0 > \bar{\eta}_0$, we have $\bar{\eta}_r > 0.5$. We can check that $\phi(\eta_0 = \frac{S}{\lambda_0}(\frac{\mathbb{E}C_h}{\mathbb{E}C_h^r} - \frac{\lambda + S}{S})) < 0$ when $\mathbb{E}C_h > \mathbb{E}C_h^r \frac{\lambda + S}{S}$, which results in $\bar{\eta}_0 < \frac{S}{\lambda_0}(\frac{\mathbb{E}C_h}{\mathbb{E}C_h^r} - \frac{\lambda + S}{S})$. When $\bar{\eta}_0 < \eta_0 \le \frac{S}{\lambda_0}(\frac{\mathbb{E}C_h}{\mathbb{E}C_h^r} - \frac{\lambda + S}{S})$.

 $\frac{\lambda+S}{S}), \quad \frac{N_0 \mathbb{E}C_h^r \frac{\lambda_0}{S}}{2N(\mathbb{E}C_h - \mathbb{E}C_h^r \frac{\lambda+S}{S})} \leq 1 - \frac{N_0 \mathbb{E}C_h^r \frac{\lambda_0}{S}}{2N(\mathbb{E}C_h - \mathbb{E}C_h^r \frac{\lambda+S}{S})}. \text{ Thus,}$ when $\max(1 - \bar{\eta}_r, \frac{N_0 \mathbb{E}C_h^r \frac{\lambda_0}{S}}{2N(\mathbb{E}C_h - \mathbb{E}C_h^r \frac{\lambda+S}{S})}) < \eta_i < \min(\bar{\eta}_r, 1 - N_0)$ $\frac{N_0 \mathbb{E}C_h^r \frac{\lambda_0}{2}}{2N(\mathbb{E}C_h - \mathbb{E}C_h^r \frac{\lambda+S}{2})}), \text{ all the conditions } \kappa_i > 1, \ \kappa_j > 1,$ $\frac{\partial R_j^{r,\leq}}{\partial T_j}|_{T_i=\frac{\log \kappa_i}{\lambda=\lambda_0}} < 0 \text{ and } \frac{\partial R_i^{r,\leq}}{\partial T_i}|_{T_j=\frac{\log \kappa_j}{\lambda=\lambda_0}} < 0 \text{ are satisfied}$ and the WSPs choose to rollover early or late by comparing their profits. For any $\eta_0 \in (\bar{\eta}_0, \frac{S}{\lambda_0}(\frac{\mathbb{E}C_h}{\mathbb{E}C_r} - \frac{\lambda+S}{S})),$ denote $\tilde{\eta}_r$ as the solution of $\bar{R}_i^{r,\leq}(\eta_i) - \bar{R}_i^{r,\tilde{S}}(\eta_i) = 0$. Then, when WSP *i*'s market share is large $(\eta_i > \tilde{\eta}_r)$, it chooses to rollover late $(\bar{R}_i^{r,\leq}(\eta_i) < \bar{R}_i^{r,>}(\eta_i))$; when WSP *i*'s market share is small ($\eta_i < \tilde{\eta}_r$), it chooses to rollover early $(\bar{R}_i^{r,\leq}(\eta_i) > \bar{R}_i^{r,>}(\eta_i))$. Therefore, when $\eta_i < \tilde{\eta}_r$ and $\eta_j > \tilde{\eta}_r$, i.e., $\eta_i < \min(\tilde{\eta}_r, 1 - \tilde{\eta}_r)$, $T_i^{ne} = 0$, $T_j^{ne} = \frac{\log \kappa_j}{\lambda - \lambda_0}$ is the equilibrium and when $\eta_j < \tilde{\eta}_r$ and $\eta_i > \tilde{\eta}_r$, i.e., $\eta_i > \max(\tilde{\eta}_r, 1 - \tilde{\eta}_r), T_i^{ne} = 0, T_j^{ne} = \frac{\log \kappa_j}{\lambda - \lambda_0}$ is the equilibrium. If $\tilde{\eta}_r \geq 0.5$, the conditions $\eta_i < \tilde{\eta}_r, \eta_j < \tilde{\eta}_r$ are satisfied when $1 - \tilde{\eta}_r < \eta_i < \tilde{\eta}_r$, then, both WSPs choose to rollover early ($T_i^{ne} = T_i^{ne} = 0$); If $\tilde{\eta}_r < 0.5$, the conditions $\eta_i > \tilde{\eta}_r, \eta_j > \tilde{\eta}_r$ are satisfied when $\tilde{\eta}_r < \eta_i < 1 - \tilde{\eta}_r$, then, both WSPs choose to rollover late $(T_i^{ne} = T_i^{ne} = \infty)$. According to WSP *i*'s profits for first and late rollover, i.e.,

$$\bar{R}_{i}^{r,\leq} = 2\alpha N \mathbb{E}C_{h}^{r} \frac{1}{S} - 2\alpha N_{j} \mathbb{E}C_{h}^{r} \frac{1}{S+\lambda} - 2\alpha N_{j} \mathbb{E}C_{h}^{r} (\frac{1}{S} - \frac{1}{\lambda+S})(\frac{1}{\kappa_{j}})^{\frac{\lambda+S}{\lambda-\lambda_{0}}} + 2\alpha N_{0} (\frac{1}{S} - \frac{1}{S+\lambda_{0}}) \mathbb{E}C_{h}^{r} - \alpha N_{0} (\frac{1}{S} - \frac{1}{S+\lambda_{0}})(\frac{1}{\kappa_{j}})^{\frac{\lambda_{0}+S}{\lambda-\lambda_{0}}} \mathbb{E}C_{h}^{r},$$
(51)

and

$$\bar{R}_{i}^{r,>} = 2\alpha N_{i} \frac{1}{\lambda + S} \mathbb{E}C_{h} \left(1 - \left(\frac{1}{\kappa_{i}}\right)^{\frac{\lambda + S}{\lambda - \lambda_{0}}}\right) + 2\alpha N_{i} \frac{1}{S} \mathbb{E}C_{h}^{r} \left(\frac{1}{\kappa_{i}}\right)^{\frac{\lambda + S}{\lambda - \lambda_{0}}} + \alpha N_{0} \left(\frac{1}{S} - \frac{1}{S + \lambda_{0}}\right) \left(\frac{1}{\kappa_{i}}\right)^{\frac{\lambda_{0} + S}{\lambda - \lambda_{0}}} \mathbb{E}C_{h}^{r},$$
(52)

when $\eta_0 \in (\bar{\eta}_0, \frac{S}{\lambda_0}(\frac{\mathbb{E}C_h}{\mathbb{E}C_h^r} - \frac{\lambda+S}{S})), \bar{R}_i^{r,\leq}(\eta_i) - \bar{R}_i^{r,>}(\eta_i)$ decreases with η_i . Define $\psi(\eta_0) := \bar{R}_i^{r,\leq}(\eta_i = 0.5) - \bar{R}_i^{r,>}(\eta_i = 0.5)$. When $\mathbb{E}C_h < \mathbb{E}C_h^r \frac{2\lambda+S}{S}$, we can check that $\psi(\eta_0) \ge 0$ always holds for $\eta_0 \in (0, \frac{S}{\lambda_0}(\frac{\mathbb{E}C_h}{\mathbb{E}C_h^r} - \frac{\lambda+S}{S}))$, which means $\tilde{\eta}_r \ge 0.5$ for $\eta_0 \in (\bar{\eta}_0, \frac{S}{\lambda_0}(\frac{\mathbb{E}C_h}{\mathbb{E}C_h^r} - \frac{\lambda+S}{S}))$.

Note that $T_i^{ne} = 0$, $T_j^{ne} = \frac{\log \kappa_j}{\lambda - \lambda_0}$ is the equilibrium if $\eta_i < \min(\bar{\eta}_r, 1 - \frac{N_0 \mathbb{E}C_h^r \lambda_0}{2N(\mathbb{E}C_h - \mathbb{E}C_h^r \lambda_S^+)})$ and $T_j^{ne} = 0$, $T_i^{ne} = \frac{\log \kappa_i}{\lambda - \lambda_0}$ is the equilibrium if $\eta_i > \max(1 - \bar{\eta}_r, \frac{N_0 \mathbb{E}C_h^r \lambda_S}{2N(\mathbb{E}C_h - \mathbb{E}C_h^r \lambda_S^+)})$. For $\eta_0 \le \bar{\eta}_0$, as $\bar{\eta}_r \le 0.5$ and $\frac{N_0 \mathbb{E}C_h^r \lambda_S}{2N(\mathbb{E}C_h - \mathbb{E}C_h^r \lambda_S^+)} < 0.5$, then, we have $T_i^{ne} = 0$, $T_j^{ne} = \frac{\log \kappa_j}{\lambda - \lambda_0}$ when $0 \le \eta_i \le \bar{\eta}_r$ and $T_j^{ne} = 0$, $T_i^{ne} = \frac{\log \kappa_j}{\lambda - \lambda_0}$ when $1 - \bar{\eta}_r \le \eta_i \le 1$. When $\bar{\eta}_r < \eta_i < 1 - \bar{\eta}_r$, all the equilibrium conditions can't be satisfied and thus $T_i^{ne} = T_j^{ne} = \infty$.

APPENDIX B PROOF OF PROPOSITION 3.2

When $\mathbb{E}C_h \leq \mathbb{E}C_h^r \frac{\lambda+S}{S}$, the WSPs rollover at the same time. If $\bar{R}_i^r(\eta_i = 1) \ge R_i(\bar{\eta_i} = 1)$, which means the cost reduction $\mathbb{E}C_h - \mathbb{E}C_h^r$ is very small and the number of new users N_0 is large, the WSPs always gain profit by rollover data. Otherwise, there exists an unique threshold η_i^{th} such that when $\eta_i \leq \eta_i^{th}$, WSP *i* gains profit and when $\eta_i > \eta_i^{th}$, WSP *i* losses profit by rollover data.

In the following, we discuss the situation when $\mathbb{E}C_h >$ $\mathbb{E}C_h^r \frac{\lambda+S}{S}$. When $\eta_i = 0$, the WSP's profit before rollover is 0, which is smaller than its profit after rollover.

As R_i^r increases with η_0 , we have

$$\bar{R}_{i}^{r}(\eta_{i}=1) \leq 2\alpha N \mathbb{E}C_{h}^{r} \frac{1}{S} + \alpha N(\frac{1}{S} - \frac{1}{S+\lambda_{0}})\mathbb{E}C_{h}^{r}$$

$$< 2\alpha N \mathbb{E}C_{h}^{r} \frac{\lambda + S}{S} \frac{1}{S} \leq 2\alpha N \mathbb{E}C_{h} \frac{1}{S}$$

$$= R_{i}(\eta_{i}=1).$$
(53)

Thus, when $\eta_i = 1$, the WSP's profit for rollover at the same time is smaller than its profit before rollover.

By noting that the WSP's profit linearly increases with η_i , there is only one intersection of $R_i^r(\eta_i)$ and $R_i(\eta_i)$ for large η_0 .

For any η_i , by comparing \bar{R}_i^r and $\bar{R}_i^{r,\leq}$, it is obvious that $\bar{R}_i^r < \bar{R}_i^{r,\leq}.$

Note that

$$\bar{R}_{i}^{r,>}(\eta_{i}=1) < 2\alpha N \frac{1}{\lambda+S} \mathbb{E}C_{h} + \alpha N(\frac{1}{S} - \frac{1}{S+\lambda_{0}}) \mathbb{E}C_{h}^{r}$$
$$< 2\alpha N \mathbb{E}C_{h} \frac{1}{S} = R_{i}(\eta_{i}=1).$$
(54)

For medium η_0 , if the WSP *i* chooses to rollover late, then $\eta_i > 0.5$. Therefore, we have $\bar{R}_i^{r,>}(\eta_i) < R_i$ due to $\eta_0 < 1$ and $\mathbb{E}C_h > \mathbb{E}C_h^r \frac{\lambda + S}{S}$.

Note that $R_i(\eta_i)$ increases faster than $\bar{R}_i^r(\eta_i)$. Therefore, there is an unique intersection of the WSP's profits before and after rollover for medium η_0 .

For small η_0 , when $\bar{\eta}_r < \eta_i < 1 - \bar{\eta}_r$, the WSPs choose not to rollover data. Thus, the unique intersection of the WSP's profits before and after rollover is the η_i^{th} such that $R_i(\eta_i) = \bar{R}_i^{r,\leq}(\eta_i).$

APPENDIX C **PROOF OF THEOREM 4.1**

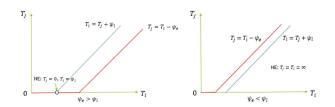


Fig. 9: Two WSPs' best responses to each other and the equilibrium rollover time.

Note that if a WSP offers shared data plan first and the other offers later, the equilibrium will converge to $(T_i^{ne} = 0, T_i^{ne} > 0)$ or $(T_i^{ne} = T_i^{ne} = \infty)$ as shown in Fig. 9. Therefore, the possible equilibrium share time for the WSPs are $(T_i^{ne} = T_j^{ne} = 0)$, $(T_i^{ne} = 0, T_j^{ne} = \frac{\log \kappa_j^s}{\lambda - \lambda_0})$, $(T_j^{ne} = 0, T_i^{ne} = \frac{\log \kappa_i^s}{\lambda - \lambda_0})$, and $(T_i^{ne} = T_j^{ne} = \infty)$, where $T_i^{ne} = \frac{\log \kappa_i^s}{\lambda - \lambda_0} \text{ is obtained by letting } \frac{\partial R_i^{s,>}}{\partial T_i}|_{T_j=0} = 0.$ $(T_i^{ne} = T_j^{ne} = 0) \text{ is the equilibrium if and only if } \partial P_i^{s,>(T_i=0)}$

 $\frac{\partial R_i^{s,>}(T_i,0)}{\partial T_i} \leq 0 \text{ and } \frac{\partial R_j^{s,>}(T_j,0)}{\partial T_j} \leq 0, \text{ i.e., both } \eta_i, \eta_j \text{ satisfies}$

r

$$\eta_i, \eta_j \le \frac{\mathcal{E}\frac{\lambda}{S} + \sqrt{(\mathcal{E}\frac{\lambda}{S})^2 + 2(\mathcal{D} - \mathcal{E})\mathcal{E}\eta_0 \frac{\lambda_0}{S}}}{2(\mathcal{D} - \mathcal{E})}.$$
 (55)

Note the if $\mathcal{D} \leq \mathcal{E} \frac{\lambda+S}{S}$ or $\eta_0 \geq \frac{2(\mathcal{D}S - \mathcal{E}(\lambda+S))}{\mathcal{E}\lambda_0}$, (55) is always satisfied due to $\frac{\mathcal{E}\frac{\lambda}{S} + \sqrt{(\mathcal{E}\frac{\lambda}{S})^2 + 2(\mathcal{D} - \mathcal{E})\mathcal{E}\eta_0 \frac{\lambda_0}{S}}}{2(\mathcal{D} - \mathcal{E})}$

In the following, we discuss the situation when $\mathcal{D} > \mathcal{E}\frac{\lambda+S}{S}$ and $\eta_0 < \frac{2(\mathcal{D}S-\mathcal{E}(\lambda+S))}{\mathcal{E}\lambda_0}$. $(T_i^{ne} = 0, T_j^{ne} = \frac{\log \kappa_j^s}{\lambda - \lambda_0})$ is the equilibrium if $\kappa_j^s > 1$ and

 $\frac{\partial R_{i}^{s,\leq}}{\partial T_{i}}\big|_{T_{j}=\frac{\log\kappa_{j}^{s}}{\lambda-\lambda_{0}}}<0\text{, i.e.,}$

$$v(\eta_i) = \mathcal{D}\eta_i \left(\frac{\lambda}{\lambda+S} + \eta_i \frac{S}{\lambda+S}\right) - \mathcal{E}\left(\frac{\lambda}{\lambda+S} + \eta_i^2 \frac{S}{\lambda+S}\right) - \mathcal{E}\eta_0 \frac{\lambda_0}{\lambda_0+S} - \eta_j \mathcal{E}\left(\frac{\lambda}{S} - \frac{\lambda}{\lambda+S}\right) \left(\frac{1}{\kappa_j^s}\right)^{\frac{\lambda+S}{\lambda-\lambda_0}} - \frac{1}{2} \mathcal{E}\eta_0 \frac{\lambda_0^2}{S(\lambda_0+S)} \left(\frac{1}{\kappa_j^s}\right)^{\frac{\lambda_0+S}{\lambda-\lambda_0}} < 0.$$
(56)

Denote $\hat{\eta}_s$ as the solution to $v(\eta_i) = 0$ and $\bar{\eta}_0^s$ as the solution to $\chi(\eta_0) := v(\eta_i = 0.5) = 0$. Note that $v(\eta_i = 0) < 0$ 0 and $v(\eta_i)$ decreases with η_0 , thus $\hat{\eta}_s$ increases with η_0 . Then, when $\eta_0 \leq \bar{\eta}_0^s$, $\hat{\eta}_s \leq 0.5$ and when $\eta_0 > \bar{\eta}_0^s$, $\hat{\eta}_s > 0.5$. Therefore, if $0 \leq \eta_0 \leq \min((\bar{\eta}_0^s)^+, (\frac{\mathcal{D}S - \mathcal{E}(2\lambda + S)}{2\mathcal{E}\lambda_0})^+)$, then, when $0 \leq \eta_i \leq \hat{\eta}_s$, $T_i^{ne} = 0$, $T_j^{ne} = \frac{\log \kappa_j^s}{\lambda - \lambda_0}$; When $\hat{\eta}_s < \eta_i < 1 - \hat{\eta}_s$, $T_i^{ne} = T_j^{ne} = \infty$; When $1 - \hat{\eta}_s \leq \eta_i \leq 1$,
$$\begin{split} T_{j}^{ne} &= 0, T_{i}^{ne} = \frac{\log \kappa_{i}^{s}}{\lambda - \lambda_{0}}.\\ \text{Then, we consider}\\ \min((\bar{\eta}_{0}^{s})^{+}, (\frac{\mathcal{D}S - \mathcal{E}(2\lambda + S)}{2\mathcal{E}\lambda_{0}})^{+}) \end{split}$$
the <If $\min(\hat{\eta}_s, 1 - \frac{\mathcal{E}_{S}^{\lambda} + \sqrt{(\mathcal{E}_{S}^{\lambda})^2 + 2(\mathcal{D} - \mathcal{E})\mathcal{E}\eta_0 \frac{\lambda_0}{S}}}{2(\mathcal{D} - \mathcal{E})}$ $\max(1 2(\mathcal{D} - \mathcal{E})$ $\hat{\eta}_s, \frac{\mathcal{E}\frac{\lambda}{S} + \sqrt{(\mathcal{E}\frac{\lambda}{S})^2 + 2(\mathcal{D} - \mathcal{E})\mathcal{E}\eta_0 \frac{\lambda_0}{S}}}{2(\mathcal{D} - \mathcal{E})}), \text{ according to the above}$ $2(\mathcal{D} - \mathcal{E})$ analysis, we can conclude that when $0 \le \eta_i < \min(\hat{\eta}_s, 1 - \eta_s)$ $\frac{\mathcal{E}\frac{\lambda}{S} + \sqrt{(\mathcal{E}\frac{\lambda}{S})^2 + 2(\mathcal{D} - \mathcal{E})\mathcal{E}\eta_0 \frac{\lambda_0}{S}}}{2(\mathcal{D} - \mathcal{E})}), \ T_i^{ne} = 0, T_j^{ne} = \frac{\log \kappa_j^s}{\lambda - \lambda_0}; \text{ when }$ $\underline{\mathcal{E}_{\overline{S}}^{\lambda}} + \sqrt{(\mathcal{E}_{\overline{S}}^{\lambda})^2 + 2(\overline{\mathcal{D}} - \overline{\mathcal{E}})\mathcal{E}\eta_0 \frac{\lambda_0}{S}}$ $\min(\hat{\eta}_s, 1)$ $\leq \eta_i \leq \max(1 - \eta_i)$ $+\sqrt{(\mathcal{E}\frac{\lambda}{S})^2+2(\mathcal{D}-\mathcal{E})\mathcal{E}\eta_0\frac{\lambda_0}{S}}$ $T_i^{ne} = 0$; when $\frac{\mathcal{E}\frac{\lambda}{S} + \sqrt{(\mathcal{E}\frac{\lambda}{S})^2 + 2(\mathcal{D} - \mathcal{E})\mathcal{E}\eta_0}}{2(\mathcal{D} - \mathcal{E})}$ $\max(1 \frac{\mathcal{E}\frac{\lambda}{S} + \sqrt{(\mathcal{E}\frac{\lambda}{S})^2 + 2(\mathcal{D} - \mathcal{E})\mathcal{E}\eta_0 \frac{\lambda_0}{S}}}{2(\mathcal{D} - \mathcal{E})} \Big) >$ $\frac{\log \kappa_i^s}{\lambda - \lambda_o}$. If min $(\hat{\eta}_s, 1 - \hat{\eta}_s)$ $\frac{\mathcal{E}\frac{\lambda}{S} + \sqrt{(\mathcal{E}\frac{\lambda}{S})^2 + 2(\mathcal{D} - \mathcal{E})\mathcal{E}\eta_0 \frac{\lambda_0}{S}}}{2(\mathcal{D} - \mathcal{E})}$ $\max(1 - \hat{\eta}_s,$), all the conditions

$$\eta_i, \eta_j > \frac{\mathcal{E}\frac{\Lambda}{S} + \sqrt{(\mathcal{E}\frac{\Lambda}{S})^2 + 2(\mathcal{D} - \mathcal{E})\mathcal{E}\eta_0 \frac{\Lambda_0}{S}}}{2(\mathcal{D} - \mathcal{E})}, \quad (57)$$

 $\frac{\partial R_j^{s,\leq}}{\partial T_j}\Big|_{T_i=\frac{\log \kappa_i^s}{\lambda-\lambda_0}} < 0 \text{ and } \frac{\partial R_i^{s,\leq}}{\partial T_i}\Big|_{T_j=\frac{\log \kappa_j^s}{\lambda-\lambda_0}} < 0 \text{ are satisfied}$ and the WSPs choose to offer shared data plan early or late by comparing their profits. Denote $\tilde{\eta}_s$ as the solution

of $\bar{R}_i^{s,\leq}(\eta_i) - \bar{R}_i^{s,>}(\eta_i) = 0$. WSP *i* choose shared data plan earlier if $\eta_i \leq \tilde{\eta}_s$ and later if $\eta_i > \tilde{\eta}_s$. Since $\mathcal{D} \geq \mathcal{E}\frac{4\lambda+S}{S}$, we have $\tilde{\eta}_s \geq 0.5$. Thus, when $\eta_i < 1 - \tilde{\eta}_s$, WSP *i* prefers to offer shared data plan first while WSP j prefers later, and when $\eta_i > \tilde{\eta}_s$, WSP *j* prefers earlier while WSP *i* prefers later. When $1 - \tilde{\eta}_s \leq \eta_i \leq \tilde{\eta}_s$, both WSPs prefer to offer shared data plan earlier and thus $T_i^{ne} = T_j^{ne} = 0$. Therefore, for the case $\min((\bar{\eta}_0^s)^+, (\frac{\mathcal{D}S - \mathcal{E}(2\lambda + S)}{2\mathcal{E}\lambda_0})^+) < \eta_0 < \frac{2(\mathcal{D}S - \mathcal{E}(\lambda + S))}{\mathcal{E}\lambda_0}$, we can conclude that when $0 \le \eta_i < \eta_s$, $T_i^{ne} = 0$, $T_j^{ne} = \frac{\log \kappa_j^s}{\lambda - \lambda_0}$; when $\underline{\eta_s} \le \eta_i \le 1 - \underline{\eta_s}$, $T_i^{\overline{ne}} = T_j^{ne} = 0$; when $1 - \underline{\eta_s} < \eta_i \le 1$, $T_j^{ne} = 0$, $T_i^{ne} = \frac{\log \kappa_i^s}{\lambda - \lambda_0}$.

APPENDIX D PROOF OF PROPOSITION 4.1

We only need to check the conditions when $\mathcal{D}S \leq \mathcal{E}(2\lambda + S)$, i.e.,

$$2S(\mathbb{E}C_h + \mathbb{E}C_l) - 2(2\lambda + S)\mathbb{E}C_{h,l}$$

$$\leq \alpha (2S\mathbb{E}C_l + (2\lambda + S)\mathbb{E}C_{h,h} - 2(2\lambda + S)\mathbb{E}C_{h,l}).$$
(58)

If
$$\frac{\mathbb{E}C_h + \mathbb{E}C_l}{\mathbb{E}C_{h,l}} > \frac{2\lambda + S}{S}$$
 and $\frac{2\mathbb{E}C_h}{\mathbb{E}C_{h,h}} \leq \frac{2\lambda + S}{S}$, we have
 $2S\mathbb{E}C_l + (2\lambda + S)\mathbb{E}C_{h,h} - 2(2\lambda + S)\mathbb{E}C_{h,l}$

$$\geq 2S(\mathbb{E}C_h + \mathbb{E}C_l) - 2(2\lambda + S)\mathbb{E}C_{h,l} > 0.$$

Thus, if $\alpha \geq \frac{2(\mathbb{E}C_h + \mathbb{E}C_l - \frac{2\lambda + S}{S} \mathbb{E}C_{h,l})}{2\mathbb{E}C_l + (\mathbb{E}C_{h,h} - 2\mathbb{E}C_{h,l}) \frac{2\lambda + S}{S}}$, (58) is satisfied. If $\frac{\mathbb{E}C_h + \mathbb{E}C_l}{\mathbb{E}C_{h,l}} \leq \frac{2\lambda + S}{S}$ and $\frac{2\mathbb{E}C_h}{\mathbb{E}C_{h,h}} > \frac{2\lambda + S}{S}$, we have $2S\mathbb{E}C_{l} + (2\lambda + S)\mathbb{E}C_{h,h} - 2(2\lambda + S)\mathbb{E}C_{h,l}$

$$< 2S(\mathbb{E}C_h + \mathbb{E}C_l) - 2(2\lambda + S)\mathbb{E}C_{h,l} \le 0.$$

Thus, if $\alpha \leq \frac{2(\mathbb{E}C_h + \mathbb{E}C_l - \frac{2\lambda + S}{S} \mathbb{E}C_{h,l})}{2\mathbb{E}C_l + (\mathbb{E}C_{h,h} - 2\mathbb{E}C_{h,l})\frac{2\lambda + S}{S}}$, (58) is satisfied. If $\frac{\mathbb{E}C_h + \mathbb{E}C_l}{\mathbb{E}C_{h,l}} \leq \frac{2\lambda + S}{S}$ and $\frac{2\mathbb{E}C_h}{\mathbb{E}C_{h,h}} \leq \frac{2\lambda + S}{S}$, we have $2S(\mathbb{E}C_h + \mathbb{E}C_l) - 2(2\lambda + S)\mathbb{E}C_{h,l} \leq 0$ and

$$2S\mathbb{E}C_l + (2\lambda + S)\mathbb{E}C_{h,h} - 2(2\lambda + S)\mathbb{E}C_{h,h}$$

$$\geq 2S(\mathbb{E}C_h + \mathbb{E}C_l) - 2(2\lambda + S)\mathbb{E}C_{h,l}.$$

Since $\alpha \in (0, 1)$, (58) always holds. If $\frac{\mathbb{E}C_h + \mathbb{E}C_l}{\mathbb{E}C_{h,l}} > \frac{2\lambda + S}{S}$ and $\frac{2\mathbb{E}C_h}{\mathbb{E}C_{h,h}} > \frac{2\lambda + S}{S}$, we have we have $2S(\mathbb{E}C_h + \mathbb{E}C_l) - 2(2\lambda + S)\mathbb{E}C_{h,l} > 0$ and

$$2S\mathbb{E}C_l + (2\lambda + S)\mathbb{E}C_{h,h} - 2(2\lambda + S)\mathbb{E}C_{h,l}$$

$$< 2S(\mathbb{E}C_h + \mathbb{E}C_l) - 2(2\lambda + S)\mathbb{E}C_{h,l}.$$

By noting that $\alpha \in (0, 1)$, (58) never holds.

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