# Toward an Automated Auction Framework for Wireless Federated Learning Services Market

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Abstract—In traditional machine learning, the central server first collects the data owners' private data together and then trains the model. However, people's concerns about data privacy protection are dramatically increasing. The emerging paradigm of federated learning efficiently builds machine learning models while allowing the private data to be kept at local devices. The success of federated learning requires sufficient data owners to jointly utilize their data, computing and communication resources for model training. In this paper, we propose an auction based market model for incentivizing data owners to participate in federated learning. We design two auction mechanisms for the federated learning platform to maximize the social welfare of the federated learning services market. Specifically, we first design an approximate strategy-proof mechanism which guarantees the truthfulness, individual rationality, and computational efficiency. To improve the social welfare, we develop an automated strategyproof mechanism based on deep reinforcement learning and graph neural networks. The communication traffic congestion and the unique characteristics of federated learning are particularly considered in the proposed model. Extensive experimental results demonstrate that our proposed auction mechanisms can efficiently maximize the social welfare and provide effective insights and strategies for the platform to organize the federated training.

*Index Terms*—federated learning, incentive mechanism, graph neural network, auction, automated mechanism design, wireless communication

### I. INTRODUCTION

C urrently, there are nearly 7 billion connected Internet-of-Things (IoT) devices<sup>1</sup> and 3 billion smartphones around the world. The devices continuously generate a large amount of fresh data. The traditional data analytics and machine learning requires all the data to be collected to a centralized data center/server, and then used for analysis or produce effective machine learning models. This is the actual practice now conducted by giant AI companies, including Amazon, Facebook, Google, etc.. However, this approach may raise concerns regarding the data security and privacy. Although various privacy preservation methods have been proposed, such as differential privacy [1] and secure multi-party computation (MPC) [2], a large proportion of people are still not willing to expose their private data which can be inspected by the server. This discourages the development of advanced AI technologies as well as new industrial applications. Motivated by the increasing privacy concern among data owners, Google introduced the concept of the *federated learning (FL)* [3]. The FL is a collaborative learning scheme that distributes the training process to individual users which then collaboratively train the shared model while keeping the data on their devices, thus alleviating the privacy issues.

A typical FL system is composed of two entities, including the *FL platform* and the *data owners*. Each data owner, e.g., mobile phone user, has a set of private data stored at its local device. The *local data* are used to train a local machine learning model where the initial model and hyper-parameters are preset by the FL platform. Once the local training is completed, each data owner just sends the trained model to the FL platform. Then, all received *local models* are aggregated by the FL platform to build a *global model*. The training process iterates until achieving the target performance or reaching the predefined number of iterations. Federated learning has three distinctive characteristics [4], [5]:

- A massive number of distributed FL participants are independent and uncontrollable, which is different from the traditional distributed training at a centralized data center.
- 2) The communication among devices, especially through the wireless channel, can be asymmetric, slow and unstable. The assumption of a perfect communication environment with a high information transmission rate and negligible packet loss is not realistic. For example, the Internet upload speed is typically much slower than download speed. Some participants may consequently drop out due to disconnection to the Internet, especially using the mobile phone through congested wireless communication channels [6].
- 3) The local data is not independent and identically distributed (Non-IID), which significantly affects the learning performance [7], [8]. Since data owners' local data cannot be accessed and fused by the FL platform and may follow different distributions, assuming all local datasets are IID is impractical.

As implied by the first characteristic above, an important prerequisite for a successful FL task is the participation of a large base of data owners that contribute sufficient training data. Therefore, establishing an FL services market is necessary for the sustainable development of the FL

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<sup>&</sup>lt;sup>1</sup>https://iot-analytics.com/state-of-the-iot-update-q1-q2-2018-number-ofiot-devices-now-7b/

community. We propose an auction based market model to facilitate commercializing federated learning services among different entities. Specifically, the FL platform first initiates and announces an FL task. When receiving the information of the FL tasks, each data owner determines the service value by evaluating its local data quality and the computing and communication capabilities. Then, data owners report their types including bids representing the services value and their resources information to the FL platform. According to the received types, the platform selects a set of FL workers from data owners and decides the service payments. Finally, the FL platform coordinates the selected FL workers to conduct model training.

In this paper, we mainly investigate the federated learning in the wireless communication scenario and design applicable auction mechanisms to realize the trading between the FL platform and the data owners. From the system perspective, we aim to maximize their total utility, i.e., social welfare. For an efficient and stable business ecosystem of the FL services market, there are several critical issues about FL task allocation and pricing. First, which data owner can participate in the federated training as an FL worker? Due to the unique features listed above, the FL platform should consider data owners' reported data size and non-IID degree of data. Also, the limited wireless spectrum resource need to be reasonably allocated since the large population of participated data owners may exacerbate the communication congestion. Second, how to set reasonable payments for data owners such that they can be incentivized to undertake the FL tasks? Auction is an efficient method for pricing and task allocation [9]. The payment amount should satisfy individual rationality, which means there is no loss to data owners from trading. We should also consider how to make data owners truthfully expose their private types. The truthfulness property can stabilize the market, prevent possible manipulation and may significantly reduce the communication overhead and improve the learning efficiency. The major contributions of this paper can be summarized as follows:

- Based on real-world datasets and experiments, we define and verify a data quality function that reflects the impacts of local data volume and distribution on the federated training performance. The earth mover's distance (EMD) [7] is used as the metric to measure the non-IID degree of the data. Moreover, we consider the wireless channel sharing conflicts among data owners.
- We propose an auction framework for the wireless federated learning services market. From the perspective of the FL platform, we formulate the social welfare maximization problem which is a combinatorial NP-hard problem.
- We first design a reverse multi-dimensional auction (RMA) mechanism as an approximate algorithm to maximize the social welfare. To further improve the social welfare and the efficiency, we novelly develop an automated deep reinforcement learning based auction (DRLA) mechanism which is integrated with the graph neural network (GNN). According to the data owners'

requested wireless channels, we construct a conflict graph for the usage of GNN. Both mechanisms, i.e., RMA and DRLA, are theoretically proved to be strategyproof, i.e. truthful and individually rational.

• Demonstrated by our simulation results, the proposed auction mechanisms can help the FL platform make practical trading strategies to efficiently coordinate data owners to invest their data and computing resources in the federated learning while optimizing the social welfare of the FL services market. Particularly, the automated DRLA mechanism shows significant improvement in social welfare compared with the RMA mechanism.

To the best of our knowledge, this is the first work that studies the auction based wireless FL services market and applies the GNN and deep reinforcement learning (DRL) in the design of a truthful auction mechanism to solve a combinatorial NP-hard problem.

The rest of this paper is organized as follows. Section II reviews related work. The system model of the FL services market and the social welfare maximization problem are introduced in Section III. Section IV proposes the designed reverse multi-dimensional auction mechanism. In Section V, the automated auction mechanism based on GNN and DRL is presented in detail. Section VI presents and analyzes simulation results based on real-world and synthetic datasets. Finally, Section VII concludes this paper.

### II. RELATED WORK

Due to the resource constraints and the heterogeneity of devices, some papers have discussed the optimal allocation of the resources and tasks to improve the efficiency of federated training. The relevant issues mainly include client selection, computation offloading and incentive mechanism. The authors in [10] designed a protocol called FedCS. The FedCS protocol has a resource request phase to gather information such as computing power and wireless channel states from a subset of randomly selected clients, i.e., FL workers. To tradeoff the accuracy and efficiency, the FL platform optimally selects a set of clients that are able to punctually finish the local training. Compared with the protocol that ignores the client selection, the FedCS can achieve higher performance. Besides improving the training efficiency, the authors in [11], [12] discussed the fairness issue that if a protocol selects the clients by the computing power, the final trained model would more cater to the data distribution of clients with high computational capability. Based on the original federated averaging (FedAvg) algorithm [13], a q-FedAvg training algorithm was proposed in [12] to give the client with low performance a higher weight in optimizing the objective function. For computation offloading, the authors in [14] combined the DRL and the FL to optimally allocate the mobile edge computing (MEC) resources. The client can use the DRL to intelligently decide whether to perform the training locally or offload it to the edge server. The simulation results showed that the DRL based approaches can achieve similar average utilities in FL and centralized learning. With respect to the incentive mechanism design, the authors in [15] proposed a Stackelberg game model

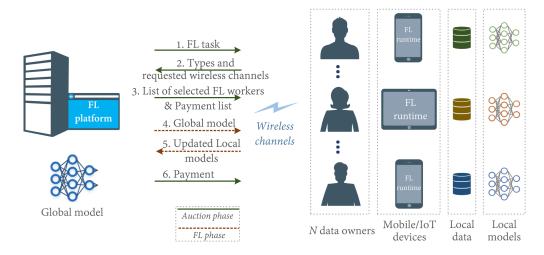


Fig. 1. Federated learning services market.

to investigate the interactions between the server and the mobile devices in a cooperative relay communication network. The mobile devices determine the price per unit of data for individual profit maximization, while the server chooses the size of training data to optimize its own profit. The simulation results demonstrate that the interaction can finally reach an equilibrium, and the cooperative communication scheme can reduce the congestion and improve the energy efficiency. In a similar setting of [15], the authors in [16] proposed a contract theory method to incentivize the mobile devices to take part in the FL and contribute high-quality data. The mobile users can only choose the contract matching their own types to maximize the utility. However, the above incentive mechanisms did not consider the non-IID data or the wireless channel constraints which are taken into account in this paper.

Different from the Stackelberg game and contract theory, the auction mechanism allows the data owner to actively report its type and has been applied in various application scenarios [17]. Thus, the FL platform can sufficiently understand their status and requests to optimize the target performance metric, such as the social welfare of the market or the platform's revenue. To design a new auction mechanism for higher performance or other properties that manually designed auction mechanism cannot realize, the automated mechanism design [18], [19]Â assisted by machine learning techniques is gaining popularity. In [20], the authors used the multi-layer neural network to model an auction with the guarantee of individual rationality (IR) and incentive compatibility<sup>2</sup> (IC). The proposed deep learning based framework successfully recovered all known analytical solutions to classical multiitem auction settings, and discovered new mechanisms for settings where the optimal analytical solution is unknown. In [21], the authors proposed a strategyproof mechanism to deploy the mobile base station based on the deep learning technique, which significantly improved the social welfare of the wireless powered spatial crowdsourcing system. The study of using DRL to solve combinatorial problems over

the graph was initialized in [22]. The authors first calculated the graph embedding and then trained a deep Q network to optimize several classical NP-hard problems in a greedy style. Since the wireless channel conflicts among the data owners are represented by a conflict graph in this paper, we propose an automated auction mechanism based on DRL and GNN to optimize the social welfare of FL services market while meeting the requirement of IC and IR.

### III. System Model: Federated Learning Services Market

### A. Preliminary Knowledge of Federated Learning

As illustrated in Fig. 1, we focus on a representative monopoly FL services market structure which consists of one *FL platform* and a community of N data owners  $\mathcal{N} =$  $\{1, \ldots, N\}$ . The *platform* performs publishing the FL task and selecting data owners as FL workers. Each data owner i maintains a set of private local data  $\mathcal{D}_i$  and has a local FL runtime to train a *local model*  $\mathbf{w}_i$ . We use  $\mathcal{W} \subseteq \mathcal{N}$  to denote the set of W FL workers selected from data owners. Different from the traditional centralized training that collects all local data  $\mathcal{D}_{\mathcal{W}} = \bigcup_{i \in \mathcal{W}} \mathcal{D}_i$ , the FL platform only collects and aggregates the updated local models  $\cup_{i \in \mathcal{W}} \mathbf{w}_i$  from workers to generate a global model  $\mathbf{w}_{q}$ . We assume that the data owners are honest to use their real private data to do training and submit the true local models to the platform. The FL training process generally contains the following 3 steps, where Steps 2 and 3 form an iterative loop between the platform and the workers.

- Step 1 (task initialization): The platform determines the training task, i.e., the target application, and the corresponding data requirements. Meanwhile, it specifies the hyper-parameters of the machine learning model and the training process. Then, the platform transmits the task information and the initial global model  $\mathbf{w}_g^0$  to all workers.
- Step 2 (local model training and update): Based on the global model  $\mathbf{w}_g^k$ , where k denotes the current global epoch index, each worker respectively uses the local data

<sup>&</sup>lt;sup>2</sup>In this paper, truthfulness and incentive compatibility are used interchangeably.

and device to update the local model parameters  $\mathbf{w}_i^k$ . The worker *i*'s goal in epoch *k* is to make parameters  $\mathbf{w}_i^k$  that minimize the predefined loss function  $L(\mathbf{w}_i^k)$ , i.e.,

$$\mathbf{w}_i^{k^*} = \arg\min_{\mathbf{w}_i^k} L(\mathbf{w}_i^k).$$
(1)

• Step 3 (global model aggregation and update): The platform receives and aggregates the local models from workers, and then sends the updated global model parameters  $\mathbf{w}_{g}^{k+1}$  back. The platform aims to minimize the global loss function  $L(\mathbf{w}_{a}^{k})$ , i.e.,

$$L(\mathbf{w}_g^k) = \frac{1}{W} \sum_{i \in \mathcal{W}} L(\mathbf{w}_i^k).$$

Steps 2-3 repeat until the global loss converges. Note that the federated training process can be adopted for various machine learning approaches based on the gradient descent method such as Support Vector Machines (SVM), convolutional neural network, and linear regression. The worker *i*'s local training dataset  $\mathcal{D}_i$  usually contains a set of  $n_i$  feature vectors  $\mathbf{x}_i = \{x_1, \ldots, x_{n_i}\}$  and a set of corresponding labels  $\mathbf{y} = \{y_1, \ldots, y_{n_i}\}$ . Let  $\hat{y}_j = f(x_j; \mathbf{w})$  denote the predicted result from the model  $\mathbf{w}$  using data vector  $x_j$ . We focus on the neural network model in which a common loss function is the mean square error (MSE) defined as

$$l(\mathbf{w}_{i}^{k}) = \frac{1}{n_{i}} \sum_{j=1}^{n_{i}} (y_{j} - f(x_{j}; \mathbf{w}_{i}^{k}))^{2}.$$
 (2)

Global model aggregation is the core part of the FL scheme. In this paper, we apply the classical federated averaging algorithm (FedAvg) [3] in Algorithm 1. According to (1), the worker *i* trains the local model on minibatches sampled from the original local dataset (lines 4-8). At the *k*th iteration, the platform minimizes the global loss using the averaging aggregation which is formally defined as

$$\mathbf{w}_{g}^{k} = \frac{1}{\sum_{i \in \mathcal{W}} n_{i}} \sum_{i \in \mathcal{W}} n_{i} \mathbf{w}_{i}^{k}.$$
(3)

As the hyper-parameters of Algorithm 1,  $\delta_B$  is the local

Algorithm 1 Federated averaging algorithm (FedAvg) [3]

**Input:** Local minibatch size  $\delta_B$ , number of local epochs  $\delta_l$ , number of global epochs  $\delta_g$ , and learning rate  $\eta$ .

- **Output:** Global model  $\mathbf{w}_g$ .
- 1: [Worker i]
- 2: LocalTraining $(i, \mathbf{w}_i)$ :
- 3: Split the local dataset  $\mathcal{D}_i$  to minibatches and include them into the set  $\mathcal{B}_i$ .
- 4: for each local epoch from 1 to  $\delta_l$  do (stochastic gradient descent (SGD))
- 5: **for** each minibatch in  $\mathcal{B}_i$  **do**
- 6:  $\mathbf{w} \leftarrow \mathbf{w} \eta l'(\mathbf{w})$  (*l'* is the gradient of *l* on the minibatch.) 7: **end for**
- 8: end for

9:

- 10: [Platform]
- 11: Initialize  $\mathbf{w}_{g}^{0}$

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12: for each global epoch k from 1 to \delta_g do
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13: Randomly choose a subset of \delta_s workers from W
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14: for each worker i in the sampled subset parallely do
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15: \mathbf{w}_i^{k+1} \leftarrow \text{LocalTraining}(i, \mathbf{w}_g^k)
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 $\begin{array}{ll} & 17: \quad \mathbf{w}_g^k = \frac{1}{\sum_{i \in \mathcal{W}} n_i} \sum_{i \in \mathcal{W}} n_i \mathbf{w}_i^k & \quad (\text{Averaging aggregation}) \\ & 18: \text{ end for} \end{array}$ 

minibatch size,  $\delta_l$  is the number of local epochs and  $\delta_g$  is

the number of global epochs and  $\eta$  is the learning rate.

### B. Local Data Evaluation

The evaluation of local data is the first step for both the data owners and the platform in the valuation of FL service. The data owner needs to calculate the cost of collecting the local data. The local data cost not only comes from the deployment of sensing devices, e.g., IoT gadgets and smart phones, but also from the data pre-processing that requires costly human intervention for data annotation and cleaning, e.g., redundancy elimination and anomaly detection. Hence, the data cost  $c_i^d$ has a unit cost  $\gamma_i > 0$  of local data. The local data cost  $c_i^d$ can be written as

$$c_i^{\rm d} = d_i \gamma_i \tag{4}$$

where  $d_i > 0$  is the data owner *i*'s local data size.

The platform cares about the data quality and needs a metric to quantify data owners' potential contributions to the task completion. Due to the unique features of local data in FL, we focus on two critical attributes of local data: one is the data size and the other one is the data distribution. According to [23] and the experimental validation in [24], data size plays an essential role in improving the data quality where more data generally means better prediction performance. With respect to the data distribution, the conventional centralized learning, e.g., data center learning, usually assumes that the training data are independently and identically distributed (IID). However, the local data are user-specific and usually non-IID in the FL scenario. The characteristic of non-IID dominantly affects the performance, e.g., prediction accuracy, of the trained FL model [3]. Indicated in [7], the accuracy reduction is mainly due to the weights divergence which can be quantized by the earth mover's distance (EMD) metric. A large EMD value means that the weights divergence is high which adversely affects the global model quality. We consider an L class classification problem defined over a compact space  $\mathcal{X}$  and a label space  $\mathcal{Y}$ . The data owner *i*'s data samples  $\mathcal{D}_i = \{\mathbf{x}_i, \mathbf{y}_i\}$  distribute over  $\mathcal{X} \times \mathcal{Y}$  following the distribution  $\mathbb{P}_i$ . Let  $\sigma_i$  denote the EMD of  $\mathcal{D}_i$ . Specifically, given the actual distribution  $\mathbb{P}_a$  for the whole population, the EMD  $\sigma_i$ is calculated by [7]

$$\sigma_i = \sum_{j \in \mathcal{Y}} \left\| \mathbb{P}_i(y=j) - \mathbb{P}_a(y=j) \right\|.$$
(5)

The actual distribution  $\mathbb{P}_a$  is actually used as a reference distribution. It can be the public knowledge or announced by the platform which has sufficient historical data to estimate  $\mathbb{P}_a$ .

Let  $\boldsymbol{\sigma} = \{\sigma_1, \ldots, \sigma_N\}$  denote the set of all data owner's EMD value. With the data size and the EMD metric, the FL platform can measure its data utility. The real-world experimental results in Section VI indicate that the relationship between the *model quality q*, e.g., prediction accuracy, and the selected workers' total data size *D* and average EMD  $\Delta$  can be well represented by the following function:

$$q(\mathcal{W}) = q(D(\mathcal{W}), \Delta(\mathcal{W}))$$
$$= \alpha(\Delta) - \kappa_1 e^{-\kappa_2(\kappa_3 D)^{\alpha(\Delta)}}$$
(6)

where D and  $\Delta$  are functions of the set of workers  $\mathcal{W}$ , i.e., the total data size  $D(\mathcal{W}) = \sum_{i \in \mathcal{W}} d_i$  and the average EMD metric  $\Delta(\mathcal{W}) = \frac{\sum_{i \in \mathcal{W}} \sigma_i}{|\mathcal{W}|}$  with  $\Delta(\emptyset) = 0$ , and  $\alpha(\Delta) = \kappa_4 \exp(-(\frac{\Delta + \kappa_5}{\kappa_6})^2) < 1$ .  $\kappa_1, \ldots, \kappa_6 > 0$  are positive curve fitting parameters. The curve fitting approach for determining the function of machine learning quality is typical in the literature and a similar function has been adopted in other works, such as [25]. In the experiment presented in Section VI-A, the data utility function (6) fits well when  $\sigma$ falls in the  $[0, \sigma_{\max}]$ . To guarantee good service quality,  $\sigma_{\max}$ can be set as the maximum EMD that the platform can accept. The first term  $\alpha(\Delta)$  reflects that the increasing average EMD metric causes the degradation of the model performance. The exponential term  $-\kappa_1 e^{-\kappa_2(\kappa_3 D)^{\alpha(\Delta)}}$  captures the diminishing marginal returns when the total data size increases. Hereby, we define the platform's data utility  $\varphi$  as a linear function of q as

$$\varphi(\mathcal{W}) = \varphi(D(\mathcal{W}), \Delta(\mathcal{W}))$$
  
=  $\kappa_7 q(\mathcal{W})$   
=  $\kappa_7 \left( \alpha(\Delta) - \kappa_1 e^{-\kappa_2(\kappa_3 D)^{\alpha(\Delta)}} \right)$  (7)

where  $\kappa_7$  represents the profit per unit performance.

### C. Auction based FL Services Market

To recruit enough qualified workers for successful federated training, the FL platform<sup>3</sup> conducts an auction. Figure 1 depicts the auction supported the FL process. For simplicity, we assume that the data owners' computing and storage capabilities, i.e., the CPU frequency and memory, can meet the FL platform's minimum requirement of the training speed and the local model size. Since the communication delay seriously degrades the efficiency of FL [6], the platform requires the FL worker<sup>4</sup> to immediately transmit back the updated local model at the transmission rate R bits/s when the local training is completed. With the fixed model size, this is actually equivalent to requiring the workers to finish the model transmission in a fixed time.

As described in Step 1 in Section III-A, the platform first initializes the global neural network model with size M and hyper-parameters, such as  $\delta_l, \delta_g$  and R. Then, the platform announces the auction rule and advertises the FL task to the data owners. Then, the data owners report their type profile  $\mathbf{T} = {\mathbf{t}_1, \ldots, \mathbf{t}_N}$  and the requested wireless channel profile  $\mathcal{C} = {\mathcal{C}_1, \ldots, \mathcal{C}_N}$ . The data owner *i*'s type  $\mathbf{t}_i$  contains the bid  $b_i$  which reveals its private service cost/valuation  $c_i$ , the size  $d_i$  and EMD value  $\sigma_i$  of its possessed local data, i.e.,  $\mathbf{t}_i = {b_i, d_i, \sigma_i}$ .  $\mathcal{C}_i$  is the set of data owner *i*'s requested wireless channels to communicate with the FL platform.

Since this paper focuses on the resource allocation of the FL system, we assume that there is no adverse attack in the FL training. The data owners cannot provide services with higher data quality than their truly owned. They will not report higher data size or lower EMD metric to the platform. Otherwise,

this would be seen as the model update poisoning attack [6]. Based on the received types, the platform has to select workers and notifies all data owners the service allocation, i.e., the set of FL workers W, and the corresponding payments  $\mathbf{p} = \{p_1, \ldots, p_N\}$  to each data owner. The workers are considered to be single-minded at the channel allocation. That is, the data owner *i* only accepts the set of its requested channels if it wins the auction. The payment for a data owner failing the auction is set to be zero, i.e.,  $p_i = 0$  if  $i \notin W$ . Once the auction results are released, an FL session starts and the selected workers train the local model using their own local data. Meanwhile, the platform keeps aggregating the local models and updating the global model. Finally, the platform pays the workers when the FL session is completed.

### D. Service Cost in the FL Market

Besides the local data cost defined in (4), the data owner also needs to calculate the costs of computation and communication to estimate its service cost if it becomes the worker. According to our previous experimental results about the energy consumption of the FL training [26, Figure 2], the data owner *i*'s computational cost  $c_i^{\rm p}$  is defined as a linear function of the data size  $d_i$ , which is written as

$$c_i^{\rm p} = d_i \delta_{\rm l} \delta_{\rm g} M \alpha_i \tag{8}$$

where  $\alpha_i$  is the data owner *i*'s unit computational cost. Since the structures of the global model and the local model are the same when applying FedAvg, we use *M* to denote the model size. With respect to the communication cost, we ignore the communication overhead and assume the channel is slowfading and stable. Since this paper focuses on the design of incentive mechanism, we consider a frequency-division multiple-access (FDMA) communication scheme. This is also for simplicity and minimum communication interference. Nonetheless, other more sophisticated wireless communication configurations can be adopted with slight modification in the cost function.

According to Shannon's formula [27], the data owner i's communication power cost is

$$P_i^{\rm m} = \frac{(2^{\frac{m}{BC_i}} - 1)BC_i}{h_i},\tag{9}$$

where B is the channel bandwidth,  $C_i = |C_i|$  is the number of data owner *i*'s requested channels,  $BC_i$  is the total bandwidth,  $h_i = \frac{\tilde{h}_i^2}{\psi_0}$  is the normalized channel power gain,  $\tilde{h}_i$  is the channel gain between the data owner *i* and the FL platform (as a base station), and  $\psi_0$  is the one-sided noise power spectral density. The total cost for communication is

$$c_i^{\rm m} = P_i^{\rm m} \frac{M}{R} \delta_{\rm g} \beta_i \tag{10}$$

$$=\frac{(2^{\frac{R}{BC_i}}-1)BC_iM\delta_g\beta_i}{h_iR}\tag{11}$$

where  $\frac{M}{R}\delta_{g}$  is the total time for model transmission,  $\beta_{i}$  is the data owner *i*'s unit energy cost for communication. The channel conditions of different subcarriers for each data owner can be perfectly estimated. That is,  $h_{i}$  is known by both the

<sup>&</sup>lt;sup>3</sup>We use "FL platform" and "platform" interchangeably.

<sup>&</sup>lt;sup>4</sup>Note that the FL worker refers to as the data owner that has been selected by the platform to perform the FL training.

data owner *i* and the platform. Adding all costs in (4), (8) and (10) together, the data owner *i*'s total service  $\cot c_i$  is

$$c_{i} = c_{i}^{d} + c_{i}^{p} + c_{i}^{m}$$
$$= d_{i}\gamma_{i} + d_{i}\delta_{l}\delta_{g}M\alpha_{i} + \frac{\left(2^{\frac{R}{BC_{i}}} - 1\right)BC_{i}M}{h_{i}R}\delta_{g}\beta_{i}.$$
 (12)

Since our proposed auction mechanisms are truthful (to be proved later), the reported bid  $b_i$  is equal to the true service cost  $c_i$ , i.e.,  $b_i = c_i$ .

Similarly, the FL platform has the computational cost  $\hat{c}^{\rm p}$  for model averaging and the communication cost  $\hat{c}^{\rm m}$  for global model transmission defined as follows:

$$\hat{c}^{\mathrm{p}}(\mathcal{W}) = \delta_{\mathrm{g}} M(W-1)\hat{\alpha},\tag{13}$$

$$\hat{c}^{\mathrm{m}}(\mathcal{W}) = \sum_{i \in \mathcal{W}} \frac{(2^{\frac{R}{BC_i}} - 1)BC_i M}{h_i R} \delta_{\mathrm{g}} \hat{\beta}, \qquad (14)$$

where  $\hat{\alpha}$  and  $\hat{\beta}$  are respectively the unit costs for computation and communication. Hence, we have the platform's total cost as follows

$$\hat{c}(\mathcal{W}) = \hat{c}^{\mathrm{p}} + \hat{c}^{\mathrm{m}} \tag{15}$$

$$= \delta_{\mathrm{g}} M(W-1)\hat{\alpha} + \sum_{i \in \mathcal{W}} \frac{(2^{\frac{R}{BC_i}} - 1)BC_i M}{h_i R} \delta_{\mathrm{g}} \hat{\beta}.$$
(16)

### E. Social Welfare Optimization and Desired Economic Properties

With the data utility and the service cost introduced in Sections III-B and III-D, we can obtain the utility functions of all entities. The FL platform's utility is the data utility minus the total cost and the total payments to workers, which is written as

$$\hat{u} = \varphi(D, \Delta) - \hat{c} - \sum_{i \in \mathcal{W}} p_i.$$
(17)

The data owner *i*'s utility is the difference between its payment  $p_i$  and service cost  $c_i$ , which is expressed as

$$u_i = p_i - c_i. \tag{18}$$

In Section IV, we design the auction mechanism to maximize the *social welfare* which can be regarded as the FL system efficiency [28] and is defined as the sum of the platform's utility and the data owners' utilities. Formally, the social welfare maximization problem is

$$\max_{\mathcal{W}\subseteq\mathcal{N}} S(\mathcal{W}) = \hat{u} + \sum_{i\in\mathcal{W}} u_i \tag{19}$$

$$=\varphi(D(\mathcal{W}),\Delta(\mathcal{W})) - \hat{c}(\mathcal{W}) - \sum_{i\in\mathcal{W}} c_i \quad (20)$$

s.t. 
$$C_i \cap C_j = \emptyset, \forall i, j \in \mathcal{W}, i \neq j.$$
 (21)

As we consider the FDD communication scheme, the constraint in (21) requires that the sets of workers allocated channels have no conflict with each other. For an efficient and stable FL market, the following economic properties should be guaranteed.

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• *Truthfulness (Incentive compatibility, IC).* The data owner *i* has no incentive to report a fake type for a higher utility. Formally, with other data owner's types fixed, the condition for the truthfulness is

$$u_i(t'_i) \le u_i(t_i), \forall t'_i \ne t_i,$$

where  $t_i = (b_i, d_i, \sigma_i)$  is data owner *i*'s true type and  $t'_i = (b'_i, d'_i, \sigma'_i)$  is a false type.

- Individual rationality (IR). No data owner will suffer a deficit from its FL service provision, i.e., u<sub>i</sub>(t<sub>i</sub>) ≥ 0,∀i ∈ N.
- *Computational efficiency (CE).* The auction algorithm can be completed in polynomial time.

### IV. REVERSE MULTI-DIMENSIONAL AUCTION MECHANISM FOR FEDERATED TRAINING

In this section, we first design a truthful auction mechanism, called Reverse Multi-dimensional auction (RMA) mechanism, to maximize the social welfare defined in (19). As presented in Algorithm 2, the RMA generally follows a randomized and greedy way to choose the FL workers and decides the payments. It consists of three consecutive phases: dividing (lines 2-9), worker selection (lines 12-20) and service payment determination (lines 21-41).

The RMA first divides the workers into G groups, i.e.,  $\{\Theta_1, \ldots, \Theta_j, \ldots, \Theta_G\}$ , according to the EMD metric. Each group consecutively covers an EMD interval  $\epsilon = \frac{\sigma_{\max}}{G}$ . That is, the data owner i whose EMD value  $\sigma_i$  falls in  $[(j-1)\frac{\sigma_{\max}}{G}, j\frac{\sigma_{\max}}{G})$  will be put in the group  $\Theta_j$ . Meanwhile, we define a *virtual EMD value* for the data owner i in group j by the corresponding interval midpoint, i.e.,  $\tilde{\sigma}^j = \frac{(2j-1)\sigma_{\max}}{2G}$ . For group j, the virtual social welfare  $\tilde{S}^j(\mathcal{W})$  is calculated by using the virtual EMD value as follows:

$$\widetilde{S}^{j}(\mathcal{W}) = \widetilde{\varphi}^{j}(\mathcal{W}) - \hat{c}(\mathcal{W}) - \sum_{i \in \mathcal{W}} b_{i}$$
(22)

$$=\varphi(D(\mathcal{W}),\widetilde{\Delta}^{j}(\mathcal{W})) - \hat{c}(\mathcal{W}) - \sum_{i\in\mathcal{W}} b_{i} \qquad (23)$$

where  $\widetilde{\varphi}^{j}(\mathcal{W}) = \varphi(D(\mathcal{W}), \widetilde{\Delta}^{j}(\mathcal{W}))$  and  $\widetilde{\Delta}^{j}(\mathcal{W}) = \frac{\sum_{i \in \mathcal{W}} \widetilde{\sigma}^{j}}{|\mathcal{W}|} = \widetilde{\sigma}^{j} = \frac{(2j-1)\sigma_{\max}}{2G}$ . Let  $\mathcal{L}(\mathcal{W})$  denote the set of workers that have channel conflicts with the worker set  $\mathcal{W}$ . We introduce the *marginal virtual social welfare density*  $V_{i}^{j}(\mathcal{W})$  for the worker *i* in group *j* defined as

$$V_{i}^{j}(\mathcal{W}) = \frac{\widetilde{S}^{j}(\mathcal{W} \cup \{i\}) - \widetilde{S}^{j}(\mathcal{W})}{|\mathcal{L}(\{i\})|}$$

$$= \frac{1}{|\mathcal{L}(\{i\})|} \left( \kappa_{1}\kappa_{7} e^{-\kappa_{2}(\kappa_{3}\sum_{k \in \mathcal{W}} d_{k})^{\alpha(\tilde{\Delta}^{j})}} - \kappa_{1}\kappa_{7} e^{-\kappa_{2}(\kappa_{3}\sum_{k \in \mathcal{W} \cup \{i\}} d_{k})^{\alpha(\tilde{\Delta}^{j})}} - \hat{c}(\{i\}) - b_{i} \right).$$

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For the sake of brevity, we simply call it *marginal density*. We use  $W_o$  to denote the set of already selected workers from other groups. In each group j, the RMA first excludes the workers that are conflicted with  $W_o$ , i.e.,  $\tilde{\Theta}_j = \Theta_j \setminus \mathcal{L}(W_o)$ . Then, the RMA finds and sorts the data owners which have no channel conflict with each other in  $\tilde{\Theta}_j$  by non-increasing order of the marginal density:

$$V_1^j(\mathcal{U}_0 \cup \mathcal{W}_o) \ge V_2^j(\mathcal{U}_1 \cup \mathcal{W}_o) \ge \cdots$$

 $\geq V_k^j(\mathcal{U}_{k-1} \cup \mathcal{W}_o) \geq \cdots \geq V_{K'}^j(\mathcal{U}_{K'-1} \cup \mathcal{W}_o) \quad (26)$ where  $\mathcal{U}_{k-1}$  is the set of first k-1 sorted data owners and  $\mathcal{U}_0 = \emptyset$ . There are totally K' data owners in the sorting and the kth data owner has the largest marginal density  $V_k^j(\mathcal{U}_{k-1} \cup \mathcal{W}_o)$ 

#### Algorithm 2 Reverse Multi-dimensional auction (RMA)

**Input:** G,  $\overline{E}$  and  $\mathbf{t} = \{t_1, \ldots, t_i, \ldots, t_N\}$  with  $t_i = \{b_i, d_i, e_i, C_i\}$ . Output: The set of FL workers W and the service payment p. 1: begin  $\epsilon \leftarrow \frac{\sigma_{\max}}{G}, \mathcal{U} \leftarrow \emptyset, \mathcal{W}_o \leftarrow \emptyset, \mathcal{G} \leftarrow \emptyset$ 2: 3: for j = 1 to G do  $\Theta_j \leftarrow \emptyset, \mathcal{W}_j \leftarrow \emptyset, \mathcal{G} \leftarrow \mathcal{G} \cup \{j\}$ 4: 5: end for 6: for each  $i \in \mathcal{N}$  do  $\begin{array}{l} p_i \leftarrow 0, j \leftarrow \lceil e_i/\epsilon \rceil \\ \bar{\gamma}_i \leftarrow \frac{(2j-1)\epsilon}{2} \end{array}$ 7: 8: 9. end for while  $\mathcal{G} \neq \emptyset$  do 10: Uniformly select j from  $\mathcal{G}$ 11:  $\mathcal{G} \leftarrow \mathcal{G} \setminus \{j\}, \, \tilde{\Theta}_j \leftarrow \Theta_j \setminus \mathcal{L}(\mathcal{W}_o), \, \mathcal{U} \leftarrow \emptyset$ 12: 13: while  $\Theta_i \neq \emptyset$  do  $k^* \leftarrow \arg\max_{k \in \tilde{\Theta}_i} V_k^j(\mathcal{U} \cup \mathcal{W}_o)$ 14: if  $V_{k^*}^j(\mathcal{U}\cup\mathcal{W}_o)<0$  then 15: 16: break end if 17:  $\mathcal{U} \leftarrow \mathcal{U} \cup \{k^*\}, \, \tilde{\Theta}_j \leftarrow \tilde{\Theta}_j \setminus (\mathcal{L}(\{k^*\} \cup \{k^*\}))$ 18:  $\mathcal{W}_i \leftarrow \mathcal{U}$ 19: 20: end while 21: for each  $i \in \mathcal{W}_j$  do  $\Theta_{j_{-i}}^{-i} \leftarrow \Theta_{j_{-i}} \setminus (\{i\} \cup \mathcal{L}(\mathcal{W}_{o})), \ \tilde{\Theta}_{j}^{-i} \leftarrow \Theta_{j}^{-i}, \ \mathcal{T} \leftarrow \emptyset$ 22: if  $\tilde{\Theta}_i^{-i} = \emptyset$  then 23:  $p_i \leftarrow \arg_{b_i} V_i^j(\mathcal{T} \cup \mathcal{W}_o) = 0$  end if 24: 25: while  $\tilde{\Theta}_{i}^{-i} \neq \emptyset$  do 26.  $i_{k^*} \leftarrow \arg \max_{i_k \in \tilde{\Theta}_j^{-i}} V_{i_k}^j(\mathcal{T} \cup \mathcal{W}_o)$ 27: if  $V_{i_{k^*}}^j(\mathcal{T}\cup\mathcal{W}_o)<0$  then 28:  $p_i \leftarrow \max\{p_i, \arg_{b_i} V_i^j(\mathcal{T} \cup \mathcal{W}_o) = 0\}$ 29: 30: break else if  $i \in \mathcal{L}(\{i_{k^*}\})$  then 31:  $p_i \leftarrow \arg_{b_i} V_i^j(\mathcal{T} \cup \mathcal{W}_o) = V_{i_{1,*}}^j(\mathcal{T} \cup \mathcal{W}_o)$ 32: break 33. 34: end if  $\begin{array}{l} & \underset{p_i \leftarrow \max\{p_i, \arg_{b_i} V_i^j(\mathcal{T} \cup \mathcal{W}_o) = V_{i_{k^*}}^j(\mathcal{T} \cup \mathcal{W}_o)\} \\ & \mathcal{T} \leftarrow \mathcal{T} \cup \{i_{k^*}\}, \tilde{\Theta}_j^{-i} \leftarrow \tilde{\Theta}_j^{-i} \setminus (\{i_{k^*}\} \cup \mathcal{L}(\{i_{k^*}\})) \end{array} \end{array}$ 35: 36: if  $\tilde{\Theta}_i^{-i} = \emptyset$  then 37:  $p_i \leftarrow \max\{p_i, \arg_{b_i} V_i^j(\mathcal{T} \cup \mathcal{W}_o) = 0\}$ end if 38: 39: 40: end while 41: end for  $\mathcal{W}_o \leftarrow \mathcal{W}_o \cup \mathcal{W}_i$ 42: 43: end while 44:  $\mathcal{W} \leftarrow \mathcal{W}_o$ 45: end

in  $\hat{\Theta}_j \setminus \mathcal{U}_{k-1}$  while having no channel conflict with data owners in  $\mathcal{U}_{k-1}$ . From the sorting, the RMA aims to find the set  $\mathcal{U}_{K_s}$ containing  $K_s$  data owners as workers, such that  $V_{K_s}^j(\mathcal{U}_{K_s-1} \cup \mathcal{W}_o) > 0$  and  $V_{K_s+1}^j(\mathcal{U}_{K_s} \cup \mathcal{W}_o)$  (lines 12-19).

Once the set of workers in group j has been determined, the RMA re-executes the worker selection on the set of data owners in group j (except the data owner i), i.e.,  $\tilde{\Theta}_j^{-i} = \tilde{\Theta}_j \setminus \{i\}$ , to calculate the payment  $p_i$  for worker i (lines 22-34). Similarly, the RMA sort data owners in  $\tilde{\Theta}_j^{-i} = \tilde{\Theta}_j \setminus \{i\}$ as follows:

$$V_{i_{1}}^{j}(\mathcal{T}_{0}\cup\mathcal{W}_{o}) \geq V_{i_{2}}^{j}(\mathcal{T}_{1}\cup\mathcal{W}_{o}) \geq \cdots$$
$$\geq V_{i_{k}}^{j}(\mathcal{T}_{k-1}\cup\mathcal{W}_{o}) \geq \cdots \geq V_{i_{K''}}^{j}(\mathcal{T}_{K''-1}\cup\mathcal{W}_{o}) \quad (27)$$

where  $\mathcal{T}_{k-1}$  is the set of the first k-1 data owners in the sorting and  $\mathcal{T}_0 = \emptyset$ . From the sorting, we select the first  $K_p$  data owners as the workers where the  $K_p$ th data owner  $i_{K_p}$  is (1) the first one that has a non-negative marginal density

and channel conflicts with worker i, i.e.,  $i \in \mathcal{L}(\{i_{K_p}\})$  and  $V_{i_{K_p}}^j(\mathcal{T}_{i_{K_p-1}} \cup \mathcal{W}_o) \geq 0$ , or (2) the last one that satisfies  $i \notin \mathcal{L}(\{i_k\})$  and  $V_{i_{K_p}}^j(\mathcal{T}_{i_{K_p-1}} \cup \mathcal{W}_o) \geq 0$ . If the data owner  $i_{K_p}$  is chosen by the condition (1), the payment  $p_i$  is set to be the bid value such that the worker i and the data owner  $i_{K_p}$  have equal marginal density on  $\mathcal{T}_{i_{K_p-1}} \cup \mathcal{W}_o$ , i.e.,  $p_i \leftarrow \arg_{b_i} V_i^j(\mathcal{T}_{i_{K_p-1}} \cup \mathcal{W}_o) = V_{i_{K_p}}^j(\mathcal{T}_{i_{K_p-1}} \cup \mathcal{W}_o)$  (lines 31-33). If data owner  $i_{K_p}$  is chosen by condition (2),  $p_i$  is set to be the maximum value such that  $V_i^j(\mathcal{T}_{i_{K_1}} \cup \mathcal{W}_o) \geq V_{i_k}^j(\mathcal{T}_{i_{k-1}} \cup \mathcal{W}_o)$ ,  $\exists k \in \{1, \ldots, K_p\}$  or  $V_i^j(\mathcal{T}_{K_p} \cup \mathcal{W}_o) \geq 0$  (lines 28-30 and 35-39).

The dividing phase decomposes the original auction mechanism  $\mathbf{M}_o$  into a set of G sub-auctions. We use  $\mathbf{M}_{i \in \{1, \dots, G\}}$ to denote the sub-auction mechanism for group j. Since the data owners in each group have the same EMD value and the reported channel information is true, only the bid and the data size  $(b_i, d_i)$  in the type  $t_i$  can be manipulated. Thus, each sub-auction can be reduced to a deterministic reverse multi-unit auction where each data owner *i* bids  $b_i$  to sell  $d_i$ data units. Reflected in the data utility function in (6), the  $d_i$ data units here essentially represent the data owner i's service quality. Here, again, the data owners are *single-minded*, which means they can only sell the reported amount of data units. The deterministic auction mechanism here means the same input types will deterministically generate the same unique output. As the randomization is applied over a collection of deterministic mechanisms (line 11), the original auction mechanism  $\mathbf{M}_{0}$  is a randomized auction mechanism [29]. Our design rationale of each sub-auction is formally presented in Theorem 1 which adopts the characterizations for the truthful forward multi-unit auction presented in [30, Section 9.5.4].

**Theorem 1.** In the reverse multi-unit and single-minded setting, an auction mechanism is truthful if it satisfies the following two properties:

- Monotonicity: If a bidder *i* wins with type (b<sub>i</sub>, d<sub>i</sub>), then it will also win with any type which offers at most as much price for at least as many items. That is, bidder *i* will still win if the other bidders do not change their types and bidder *i* changes its type to some (b'<sub>i</sub>, d'<sub>i</sub>) with b<sub>i</sub> ≥ b'<sub>i</sub> and d<sub>i</sub> ≤ d'<sub>i</sub>.
- 2) Critical payment: The payment of a winning type  $(b_i, d_i)$  by bidder *i* is the largest value needed in order to sell  $d_i$  items, i.e., the supremum of  $b'_i$  such that  $(b'_i, d_i)$  is still a winning type, when the other bidders do not change their types.

We next show the desired properties of the RMA, including the truthfulness (Proposition 1), the individual rationality (Proposition 2) and the computational efficiency (Proposition 3).

**Proposition 1.** The RMA mechanism is universally truthful (incentive compatible).

**Proof:** We first investigate the truthfulness of the subauction  $M_j$ . Since the RMA guarantees that data owners in the same group have the same virtual EMD value and the group selection is random (line 11), data owners have no incentive to report false EMD value. Therefore, we just need to discuss the truthfulness of the reported data size and the bid. According to Theorem 1, it suffices to prove that the worker selection of  $M_j$  is monotone, and the payment  $p_i$  is the critical value for the data owner *i* to win the auction. Given a fixed EMD value  $\Delta$ , we construct a function o(z) as

$$o(z) = \kappa_1 \kappa_7 \mathrm{e}^{-\kappa_2 (\kappa_3 z)^{\alpha(\Delta)}} \tag{28}$$

where  $x \in \mathbb{R}^+$  and  $\alpha(\Delta) \in (0, 1)$ ,  $\kappa_1, \kappa_2, \kappa_3, \kappa_7 \in (0, +\infty)$ are parameters. The first derivative and the second derivative of o(z) are receptively

$$\frac{\mathrm{d}o(z)}{\mathrm{d}z} = -\kappa_1 \kappa_2 \kappa_3 \kappa_7 (\kappa_3 z)^{\alpha(\Delta)-1} \alpha(\Delta) \mathrm{e}^{-\kappa_2 (\kappa_3 z)^{\alpha(\Delta)}}, \quad (29)$$
$$\frac{\mathrm{d}^2 o(z)}{\mathrm{d}z^2} = \kappa_1 \kappa_2 \kappa_7 \alpha(\Delta) \mathrm{e}^{-\kappa_2 (\kappa_3 z)^{\alpha(\Delta)}} \\ (\kappa_3 z)^{\alpha(\Delta)} (\kappa_2 \alpha(\Delta) (\kappa_3 z)^{\alpha(\Delta)} - \alpha(\Delta) + 1).$$

Since  $1 > \alpha(\Delta) > 0$  and  $\kappa_1, \kappa_2 > 0$ , we can find that  $\frac{do(z)}{dz} < 0$  and  $\frac{d^2o(z)}{dz^2} > 0$  which means o(z) is a convex and monotonically decreasing function. Note that expanding  $\mathcal{W}$  is equivalent to increasing the total data size  $D(\mathcal{W}) = \mathcal{W} = \sum_{k \in \mathcal{W}} d_k$ . Substituting  $z = \sum_{k \in \mathcal{W}} d_k$ and  $z = \sum_{k \in \mathcal{W} \cup \{i\}} d_k$  into o(z), we can find  $V_i^j(\mathcal{W}) = \frac{o(\sum_{k \in \mathcal{W}} d_k) - o(\sum_{k \in \mathcal{W} \cup \{i\}} d_k) - \hat{c}(\{i\}) - b_i}{|\mathcal{L}(\{i\})|}$  which is monotonically decreasing with  $\mathcal{W}$  since  $\sum_{k \in \mathcal{W} \cup \{i\}} d_k > \sum_{k \in \mathcal{W}} d_k$  and the monotonically decreasing  $V_i^j(\mathcal{W}) = \sum_{k \in \mathcal{W}} d_k$  and the monotonically decreasing  $V_i^j(\mathcal{W}) = \sum_{k \in \mathcal{W}} d_k$  and the monotonically decreasing  $V_i^j(\mathcal{W}) = \sum_{k \in \mathcal{W}} d_k$  and the monotonically decreasing  $V_i^j(\mathcal{W}) = \sum_{k \in \mathcal{W}} d_k$  and the monotonically decreasing  $V_i^j(\mathcal{W}) = \sum_{k \in \mathcal{W}} d_k$  and  $V_i^j(\mathcal{W}) = \sum$ the monotonicity and convexity of o(z). It is also clear that the marginal density  $V_i^{\mathcal{I}}(\mathcal{W})$  defined in (24) is monotonically decreasing with the bid  $b_i$  while monotonically increasing with  $d_i$ . As the data owner i takes the ith place in the sorting (26), if it changes the type from  $t_i$  to  $t'_i$  by lowering its bid from  $b_i$  to  $b_i^ (b_i > b_i^-)$  or raising the reported data size from  $d_i$  to  $d_i^+$  ( $d_i^+ > d_i$ ), it will have a larger marginal density  $V_{i'}^{\mathcal{I}}(\mathcal{U}_{i-1}) > V_{i}^{\mathcal{I}}(\mathcal{U}_{i-1})$ . Since  $V_{i}^{\mathcal{I}}(\mathcal{W})$  is a decreasing function of  $\mathcal{W}$ , the data owner *i*'s marginal density can only increase when it is at a higher rank in the sorting (26), i.e.,  $V_{i'}^{j}(\mathcal{U}_{i-k}) > V_{i'}^{j}(\mathcal{U}_{i-1}), \forall k \in \{2, 3, ..., i\}.$  Thus, we have proved the monotonicity condition required by Theorem 1.

We next prove that  $p_i$  calculated by Algorithm 2 is the critical payment, which means that with  $d_i$  fixed, bidding a higher price  $b_i^+ > p_i$  causes the worker i to fail the auction. As mentioned above, the final payment  $p_i$  depends on the data owner  $i_{K_p}$  in the sorting (27). If the  $K_p$ th worker has channel conflict with the worker i, summiting a higher bid  $b_i^+$  makes worker i be ranked after data owner  $i_{K_p}$ , i.e.,  $V_i^j(\mathcal{T}_{K_p-1}\cup\mathcal{W}_o) < V_{i_{K_p}}^j(\mathcal{T}_{K_p-1}\cup\mathcal{W}_o),$  and then worker iwould be removed from the candidate pool in the subsequent selection. If the data owner  $i_{K_p}$  has no channel conflict with the data owner i, a higher bid  $b_i^+ > p_i$  still causes  $V_i^j(\mathcal{T}_{k-1} \cup \mathcal{W}_o) < V_{i_k}^j(\mathcal{T}_{k-1} \cup \mathcal{W}_o), \forall k \in \{1, 2, \dots, K_p\}$  and  $V_i^j(\mathcal{T}_{K_p} \cup \mathcal{W}_o) < 0$ , which apparently cannot lead the data owner i to win the auction. Thus, the truthfulness of the subauction  $M_i$  is proved. Since each sub-auction  $M_i$  is truthful and the original auction mechanism  $\mathbf{M}_{o}$  is a randomization over the collection of the sub-auctions, we can finally prove that the RMA mechanism is universally truthful [31, Definition 9.38].

Proposition 2. The RMA mechanism is individually rational.

**Proof:** Let  $i_i$  denote the worker *i*'s replacement in the payment determination process, i.e., the *i*th data owner in the sorting (27). As the data owner  $i_i$  must be after the *i*th place in the sorting (27) or even not in the sorting if worker *i* wins the auction, we have  $V_i^j(\mathcal{T}_{i-1} \cup \mathcal{W}_o) > V_{i_i}^j(\mathcal{T}_{i-1} \cup \mathcal{W}_o)$ . As shown in the Algorithm 2, the payment  $p_i$  for worker *i* is the maximum winning bid  $b'_i$ , which means the corresponding marginal density  $V_{i'}^j(\mathcal{T}_{i-1} \cup \mathcal{W}_o)$  satisfies  $V_i^j(\mathcal{T}_{i-1} \cup \mathcal{W}_o) > V_{i_i}^j(\mathcal{T}_{i-1} \cup \mathcal{W}_o) > V_{i_i}^j(\mathcal{T}_{i-1} \cup \mathcal{W}_o) > V_{i_i}^j(\mathcal{T}_{i-1} \cup \mathcal{W}_o)$ . Since  $V_i^j(\mathcal{W})$  is monotonically decreasing with the bid  $b_i$  (see the proof for Proposition 1), we have  $p_i = b'_i \geq b_i = c_i$ , which means the worker *i*'s utility  $u_i$  defined in (18) is non-negative, i.e.,  $u_i(t_i) \geq 0$ . Therefore, we can guarantee the individual rationality of each sub-auction  $M_j$  and the original RMA mechanism  $M_o$ .

### **Proposition 3.** The RMA mechanism is computationally efficient.

**Proof:** For each sub-auction  $M_j$  (lines 12-41) in Algorithm 2, finding the workers in group  $\Theta_j$  with the maximum marginal density has the time complexity of  $O(|\Theta_j|)$  (line 14). Since the number of workers is at most  $|\Theta_j|$ , the worker selection process (the while-loop lines 13-20) has the time complexity of  $O(|\Theta_j|^2)$ . In the payment determination process (lines 21-41), each for-loop executes similar steps as the while-loop in lines 13-20 and the payment determination process generally has the time complexity of  $O(|\Theta_j|^3)$ . Dominated by the for-loop (lines 21-41), the time complexity of a sub-auction (Algorithm 2) is  $O(|\Theta_j|^3)$ . Since  $\sum_{j \in \{1,...,G\}} |\Theta_j| = N$  and  $\frac{N^3}{G^2} \leq \sum_{j \in \{1,...,G\}} |\Theta_j|^3 \leq N^3$ , the running time of the original RMA  $M_o$  is bounded by polynomial time  $O(N^3)$ .

### V. DEEP REINFORCEMENT LEARNING BASED AUCTION MECHANISM (DRLA)

Although the RMA mechanism can guarantee the IC, IR and CE, its achieved social welfare is still restricted. The reasons are that the randomization may degrade the social welfare performance and the channel conflicts among workers is not well represented and exploited. Resolving these issues is very challenging. In this section, we attempt to utilize the powerful artificial intelligence (AI) to establish an automated mechanism for improving the social welfare while ensuring the IC and IR. Specifically, we first use the graph neural network (GNN) [32] to exploit the conflict relationships and generate effective embeddings. Based on the embeddings, we propose a deep reinforcement learning (DRL) framework to design truthful auction mechanisms in order to improve the social welfare.

## A. Feature engineering with embeddings of wireless spectrum conflict graph

Although the bid  $b_i$ , data size  $d_i$  and EMD  $\sigma_i$  in data owner's type  $t_i$  and the channel information  $h_i$  are already continuous variables, the information of requested wireless channels  $C_i$  is a discrete variable which restricts directly applying the DRL approach. Therefore, we construct a spectrum

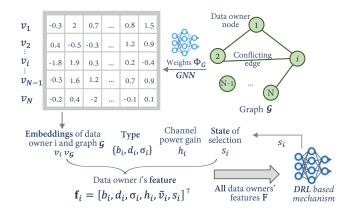


Fig. 2. Feature engineering based on GNN.

conflict graph  $\mathcal{G}$  [33] to represent the channel conflicting relationship among the data owners. We here denote the data owner *i*'s *original feature* by  $\mathbf{f}_i^{\text{o}}$ , i.e.,  $\mathbf{f}_i^{\text{o}} = [b_i, d_i, \sigma_i, h_i, C_i]^{\top}$ .  $\top$  is the transpose operator and the square brackets [·] is the operators of incorporating the inside elements to a vector.

As illustrated in Fig. 2, each node in the graph  $\mathcal{G}$  is a data owner and each undirected edge represents the conflicting relationship between two connected data owners. Due to the differences in some aspects, such as hardware or wireless channel occupancy, each data owner may have different demands for wireless channels. Taking an example with 3 data owners, the data owners 1, 2 and 3 respectively request channels  $C_1 = \{1, 4, 6\}, C_2 = \{2, 5, 6\}$  and  $C_3 = \{3, 7\}$ . Since the data owners 1 and 2 are single-minded and both of them request the channel 6, they are conflicting in wireless channels and there should be an edge between data owners 1 and 2. The data owner 3 has no channel conflicting with any other worker's requested channels, so there is no edge connected to data owner 3.

To map the discrete channel information to continuous embeddings, we specifically apply a multi-layer Graph Convolutional Network (GCN) [22] in which the l+1th layer output  $\mathbf{H}^{(l+1)}$  is calculated by

$$\mathbf{H}^{(l+1)} = \operatorname{ReLU}(\mathbf{B}^{-\frac{1}{2}}\hat{\mathbf{A}}\mathbf{B}^{-\frac{1}{2}}\mathbf{H}^{(l)}\Phi_{G}^{(l)}), \qquad (31)$$

where  $\mathbf{H}^0 = \mathbf{1}^{N \times \varpi_{\mathcal{G}}}$  is an all-ones matrix and  $\hat{\mathbf{A}} = \mathbf{A} + \mathbf{I}$  denotes the adjacency matrix  $\mathbf{A} \in \mathbb{N}^{N \times N}$  with self-connections.  $\mathbf{I} \in \mathbb{N}^{N imes N}$  is an identity matrix and  $\mathbf{B} = \sum_{j=0} \hat{\mathbf{A}}_{ij}$  is the diagonal degree matrix of  $\hat{\mathbf{A}}_{ij}$ .  $\boldsymbol{\Phi}_G$  is the trainable parameter set of the GCN where  $\Phi_G^{(l)} \in \mathbb{R}^{\varpi_G \times \varpi_G}$  is the trainable weight matrix of the *l*th layer. We use the rectified linear units  $\operatorname{ReLU}(\cdot) = \max(0, \cdot)$  [34] as an activation function. Then, the embedding  $v_i \in \mathbb{R}^{\varpi_{\mathcal{G}} \times 1}$  of each data owner (node) *i* generated by the GCN can be obtained from the output of the last layer  $\mathbf{H}^{(\hat{l}+1)} = [v_1, v_2, \dots, v_N]^{\top}$ , where  $\hat{l}$  is the total number of layers. Based on the node embeddings, the embedding of the graph  $\mathcal{G}$  is encoded by  $v_{\mathcal{G}} = \sum_i v_i \in \mathbb{R}^{\varpi_{\mathcal{G}} \times 1}$ . We concatenate the node embedding and the graph embedding as the data owner *i*'s final embedding  $\tilde{v}_i = [v_i, v_{\mathcal{G}}]^{\top} \in \mathbb{R}^{2\varpi_{\mathcal{G}} \times 1}$ . Apart from the data owner's own type information, the state  $s_i$  which is output by the DRL algorithm and indicates whether the worker i wins the DRL based auction should be included in the input feature. If the data owner i is selected to be included in the set of workers, we set  $s_i = 1$ ; otherwise,  $s_i = 0$ . Finally, the data owner *i* is represented by its feature  $\mathbf{f}_i = [b_i, d_i, \sigma_i, C_i, h_i, \tilde{v}_i, s_i]^\top \in \mathbb{R}^{(2\varpi_{\mathcal{G}}+5)\times 1}$  which incorporates its type, embeddings, state and channel information. In addition, we use  $\mathbf{F} = [\mathbf{f}_1, \dots, \mathbf{f}_N]^\top \in \mathbb{R}^{N \times (2\varpi_{\mathcal{G}}+5)}$  and  $\mathbf{F}^{\text{o}} = [\mathbf{f}_1^{\text{o}}, \dots, \mathbf{f}_N^{\text{o}}]$ to respectively represent all data owners' features and original features.

### B. Automated mechanism under deep Q-learning framework

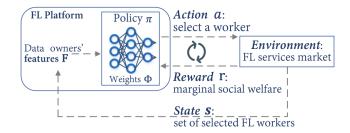


Fig. 3. DRL based framework auction mechanism.

Generally, we adopt the deep Q-learning [22], [35] framework to design an auction mechanism that possesses the properties of IC and IR and solves the NP-hard social welfare maximization problem. Similar to the RMA, the DRL based auction mechanism applies the greedy scheme which finds the workers step by step. At the step m (starting from 1), it selects a worker that has no channel conflict with the candidate set  $\mathcal{V}^m$  and maximizes an *evaluation function* Q on  $\mathcal{V}^m$ . After  $\hat{m}$ steps reaching the *termination condition*, the worker selection process ends and the final worker set is  $\hat{\mathcal{V}} = \mathcal{V}^{\hat{m}}$ . At step 1, the initial candidate set is  $\mathcal{V}^1 = \emptyset$ .

The core assumption for the DRLA mechanism is that the data owners' original features  $\mathbf{F}^{\circ}$  follow a distribution  $\mathbb{D}$ . When the service provider, i.e., the auctioneer, trains the DRLA network, it can obtain the data owners' original features from a historical real-world dataset or a priori known distribution [22].

As illustrated in Fig. 3, the proposed DRL framework is composed of the state, action, reward, policy and environment parts defined as follows:

- States: the state s<sup>m</sup> = {s<sub>1</sub><sup>m</sup>,...,s<sub>i</sub><sup>m</sup>,...,s<sub>N</sub><sup>m</sup>} consists of the aforementioned workers' states at step m, indicating whether they have joined the candidate set V<sup>m</sup>, i.e., s<sub>i</sub><sup>m</sup> = 1 if i ∈ V<sup>m</sup> and 0 otherwise. We use function V to express this transformation, s<sup>m</sup> = V(V<sup>m</sup>).
- Actions: the action a<sup>m</sup> is a data owner, i.e., a node in the graph G, which is picked by the FL platform at step m and not in the candidate set V<sup>m</sup>.
- State transition: the state transition from the current state  $s^m$  to the next state  $s^{m+1}$  is determined by the *m*th action which means setting  $s_{a^m} = 1$  and putting the data owner (node)  $a^m$  into the set  $\mathcal{V}^m$ , i.e.,  $\mathcal{V}^{m+1} = \mathcal{V}^m \cup \{a^m\}$ .
- *Rewards*: the reward function  $r^m(s^m, a^m)$  at state  $s^m$  is the increased social welfare contributed by the action  $a^m$ , which is defined as

1

$$S^m(\mathbf{s}^m, a^m) = S(\mathcal{V}^m \cup \{a^m\}) - S(\mathcal{V}^m)$$
(32)

where  $S(\cdot)$  is the social welfare function in (19). Then, the cumulative reward  $R = \sum_{m=1}^{\hat{m}} r^m(\mathbf{s}^m, a^m)$  is equal to our optimization target, i.e., the final achieved social welfare  $S(\mathcal{V}^{\hat{m}})$ .

• *Policy*: different from the traditional Q-learning [36], which uses a Q-table, the adopted DRL trains a deep neural network (DNN)  $Q(\mathbf{s}^m, a^m | \Phi_Q)$  to evaluate the quality of action  $a^m$  under state  $\mathbf{s}^m$ . That is, the input of DNN is any state and action pair and the output is the corresponding quality value.  $\Phi_Q$  denotes the trainable parameter set of the DNN. Thus,  $\Phi = [\Phi_G, \Phi_Q]$  represents all trainable parameters of the proposed *DRLA networks*. Based on the evaluation function Q, we use the greedy policy  $\pi(a^m | \mathbf{s}^m) = \arg \max_{a^m \in \mathcal{N} \setminus (\mathcal{V}^m \cup \mathcal{L}(\mathcal{V}^m))} Q(\mathbf{s}^m, a^m)$  to choose the action  $a^m$  under state  $\mathbf{s}^m$ . In the specific algorithm, we apply  $\varsigma$ -greedy policy. That is, the DRL based auction at step m randomly chooses a worker from  $\mathcal{V}^m$  with probability  $\varsigma$ , or implement the policy  $\pi(a^m | \mathbf{s}^m)$ with probability  $1 - \varsigma$ .

With the Q function in the classical deep Q-learning [22], [37] and the prepared data owners' features **F**, the DNN based evaluation function  $Q(\mathbf{s}^m, a^m | \mathbf{\Phi}_Q)$  is designed as

$$Q(\mathbf{s}^{m}, a^{m} | \mathbf{\Phi}) = Q(\mathbf{s}^{m}, a^{m} | \mathbf{\Phi}_{G}, \mathbf{\Phi}_{Q})$$
(33)  
= ReLU( $[\tilde{\mathbf{v}}, \mathbf{s}, \mathbf{C}, \mathbf{h}]^{\top} \Phi_{Q}^{1} ) \Phi_{Q}^{2}$   
-  $\mathbf{b}^{\top} e^{\Phi_{Q}^{3}} + g(\mathbf{d}, \boldsymbol{\sigma}) e^{\Phi_{Q}^{4}} )$ (34)

where  $\tilde{\mathbf{v}} = [\tilde{v}_1, \dots, \tilde{v}_N]^\top$ ,  $\mathbf{s} = [s_1, \dots, s_N]^\top$ ,  $\mathbf{C} = [C_1, \dots, C_N]^\top$ ,  $\mathbf{h} = [h_1, \dots, h_N]^\top$ ,  $\mathbf{b} = [b_1, \dots, b_N]^\top$ ,  $\mathbf{d} = [d_1, \dots, d_N]^\top$  and  $\boldsymbol{\sigma} = [\sigma_1, \dots, \sigma_N]^\top$ . Besides the parameters  $\Phi_Q^1 \in \mathbb{R}^{(2\varpi_{\mathcal{G}}+3)\times(2\varpi_{\mathcal{G}}+3)}, \Phi_Q^2, \Phi_Q^3 \in \mathbb{R}^{(2\varpi_{\mathcal{G}}+3)\times 1}$  and  $\Phi_Q^4 \in \mathbb{R}$ , the evaluation function integrates a monotonic neural network function  $g(\mathbf{d}, \boldsymbol{\sigma}) \in \mathbb{R}$  described by

$$g(\mathbf{d}, \boldsymbol{\sigma}) = \max_{j \in \{1, \dots, J\}} \min_{k \in \{1, \dots, K\}} \{ \text{ReLU}(\text{ReLU}( [\mathbf{d}, -\boldsymbol{\sigma}]^\top e^{\Phi_Q^{5_{jk1}}} + \Phi_Q^{6_{jk1}}) e^{\Phi_Q^{5_{jk2}}} + \Phi_Q^{6_{jk2}}) \}$$
(35)

where J and K are positive integral hyper-parameters that adjust the approximate accuracy and the complexity of the monotonic network g,  $\Phi_Q^{5_{jk1}} \in \mathbb{R}^{2 \times K}$ ,  $\Phi_Q^{6_{jk1}} \in \mathbb{R}^{N \times K}$ are the parameters in the first layer, and  $\Phi_Q^{5_{jk2}} \in \mathbb{R}^{K \times 1}$ ,  $\Phi_{Q}^{6_{jk2}} \in \mathbb{R}^{N imes 1}$  are the parameters in the second hidden layer. The exponential operations in (34) and (35), i.e.,  $e^{\Phi_Q^3}$ ,  $e^{\Phi_Q^4}$ ,  $e^{\Phi_Q^{4jk1}}$  and  $e^{\Phi_Q^{4jk2}}$ , guarantee that the coefficients of input features  $-\mathbf{b}$ ,  $[\mathbf{d}, -\boldsymbol{\sigma}]$  and g are positive. According to the characterizations of the monotonic network in [38], [39],  $g(\mathbf{d}, \boldsymbol{\sigma})$  and  $Q(\mathbf{s}^m, a^m | \boldsymbol{\Phi})$  are monotonically decreasing with the data owner's EMD value and monotonically increasing with the data size.  $Q(\mathbf{s}^m, a^m | \boldsymbol{\Phi})$  is clearly monotonically decreasing with bid. The DNN Q can be seen as a scoring function [40], [41] that calculates the data owner i's score  $Q_i$ . This is convenient for the FL platform to sort and select the data owners. Actually,  $q(\mathbf{d}, \boldsymbol{\sigma})$  maps the data owner *i*'s data size  $d_i$  and EMD  $\sigma_i$  to a new metric value  $g_i$ . Analogous to the RMA, the setting in the DRLA mechanism is transformed to that each data owner wants to sell  $g_i$  units of data at price

 $b_i$ .  $g_i$  can be regarded as the data owner i's normalized data size. Thus, the truthfulness and individual rationality of the DRLA mechanism are also guaranteed by Theorem 1 where we accordingly replace  $d_i$  with  $g_i$ . With the FL platform's greedy policy, the trained deep Q-learning model with fixed parameters always chooses the maximum  $Q_i$  at each step and thus possesses the monotonicity in worker selection. The payment  $p_i$  is set to the worker *i*'s critical bid, i.e., the maximum bid  $b_i$  that keeps  $Q_i$  as a winning score. Similar to the RMA (lines 21-41), the payment determination is applying the deep Q-learning model on the set of data owners except the worker *i* to find the critical data owner  $\overline{i}$  which is the last data owner selected or the first one having channel conflicts with worker i. Due to the monotonicity of the bid in Q function, the FL platform can efficiently calculate the bid  $\bar{b}_i$  such that worker's new score  $\bar{Q}_i$  is equal to the critical data owner  $\bar{i}$ 's score  $Q_{\overline{i}}$ .

#### Algorithm 3 DRLA training algorithm **Input:** Distribution $\mathbb{D}$ or real dataset Output: The set of FL workers $\mathcal{W}$ . 1: Initialize parameters $\Phi$ , $\Phi^-$ , experience replay memory $\mathcal{M}$ 2: begin 3: for episode k = 1 to $L_1$ do Draw $\mathbf{F}^{\mathrm{o}}$ from distribution $\mathbb D$ or real dataset 4: 5: Initialize the candidate set to be empty $\mathcal{V}^m = \emptyset$ 6: for step m = 1 to $L_2$ do $\widehat{\mathcal{V}}^{\hat{m}} \leftarrow \mathcal{V}^m \cup \mathcal{L}(\mathcal{V}^m)$ 7: $\int$ randomly choose $a^m \in \mathcal{N} \setminus \widehat{\mathcal{V}}^m, w.p.\varsigma$ 8: $\operatorname{arg\,max}_{a^m \in \mathcal{N} \setminus \widehat{\mathcal{V}}^m} Q(\mathbf{s}^m, a^m), \quad w.p. \ 1 - \varsigma$ $\mathcal{V}^m \leftarrow \mathcal{V}^m \cup \{a^m\}$ 9: $\mathbf{s}^{m+1} \leftarrow V(\mathcal{V}^{\tilde{m}})$ 10: 11: Execute action $a^m$ to obtain reward $r^m$ if $r^m < 0$ or $\mathcal{N} \setminus \widehat{\mathcal{V}}^m = \emptyset$ then 12: 13: break 14: end if 15: if $m \geq \mu_{\rm u}$ then Store $\{s^{m-\mu_u}, a^{m-\mu_u}, r^{m-\mu_u}, s^m, \mathbf{F}\}$ in $\mathcal{M}$ 16: Sample minibatch $\mathcal B$ from memory $\mathcal M$ 17: 18: Update $\Phi$ by SGD over (36) 19: end if 20: end for 21: if $k \mod \mu_r = 0$ then 22: $\Phi^- \leftarrow \Phi$ 23: end if 24: end for 25: end

The training process of our proposed DRL based auction mechanism is presented in Algorithm 3. At the beginning of each episode, the platform first samples a set of worker's original features  $\mathbf{F}^{o}$  from a known distribution  $\mathbb{D}$  or a realworld dataset. Training the DNN, i.e., the evaluation function, can help the FL platform to establish the optimal policy that finds the best action at the current state. The standard Q-learning updates the Q function parameters based on the immediate reward  $r^m$  at the *m*th step of an episode. However, the standard update method is myopic since our objective is to optimize the total social welfare, i.e., the accumulated reward R. Thus, our deep Q-learning training shifts to use the additive reward from the past  $\mu_{u}$  steps, i.e.,  $R_{\mu_{u}}^{m} = \sum_{m-\mu_{u}}^{m} r^{m}$ . To improve the training stability, an experience replay memory  $\mathcal{M}$  is created for storing the experiences, e.g.,  $\{s^{m-\mu_u}, a^{m-\mu_u}, R^m_{\mu_u}, s^m, \mathbf{F}\}$ . Moreover, a single DNN may also lead to the overestimation [42] since the FL platform's action is selected and evaluated by the same Q function. To address this issue, we apply the double deep Q learning (DDQL) [42]. Specifically, we have two DRLA networks, including the original evaluation DRLA network with parameters  $\Phi$  and an additional target DRLA network with parameters  $\Phi^-$ . The parameters  $\Phi$  of the evaluation DNN can be updated by using the gradient descent at each step after  $\mu_u$  steps in an episode to minimize the following square loss function:

$$\operatorname{loss} = (\hat{Q} - Q(\mathbf{s}^{m-\mu_{\mathrm{u}}}, a^{m-\mu_{\mathrm{u}}} | \boldsymbol{\Phi}))^{2}.$$
(36)

The target value  $\hat{Q}$  is defined as

$$\hat{Q} = R^m_{\mu_u} + \lambda^{\mu_u} (Q(\mathbf{s}^m, \arg\max_{a^m \in \mathcal{N} \setminus \widehat{\mathcal{V}}^m} Q(\mathbf{s}^m, a^m | \mathbf{\Phi}) | \mathbf{\Phi}^-)),$$
(37)

where  $\mu_{\rm u} \leq m \leq \hat{m}$  and  $\lambda$  is a discount factor. The target DNN resets its parameters  $\Phi^- = \Phi$  at every  $\mu_{\rm r}$  episodes. The termination condition for the training in an episode is that the immediate reward  $r^m$  becomes negative, i.e.,  $r^m < 0$ , or there is no worker to select, i.e.,  $\mathcal{N} \setminus \hat{\mathcal{V}}^m = \emptyset$ . For higher robustness of convergence, we use stochastic gradient descent (SGD) to train the evaluation DNN over a minibatch  $\mathcal{B}$  of  $\mu_{\rm B}$  experiences randomly drawn from memory  $\mathcal{M}$ . The proposed DRLA mechanism actually adopts the classical DRL framework proposed in [35] which has the stable convergence in training large neural networks using the reinforcement learning signal and the SGD method. In Section VI, we present the experimental result on the convergence in training the DRLA network.

### VI. EXPERIMENTAL RESULTS AND DISCUSSIONS

In this section, we first conduct a federated learning experiment based on real-world data to verify the proposed data utility function. From the simulation results, we then examine the performance of the proposed RMA and DRLA mechanisms.

### A. Verification for Data Utility Function

To verify the data utility function defined in (6), we use the convolutional neural network (CNN) model on the classical MNIST dataset<sup>5</sup> to develop a federated handwritten digit recognition service. For simplicity in our experiments, we first consider 2 workers following the FedAvg algorithm (Algorithm 1) and cooperating to train the CNN model with two convolutional and two fully-connected layers. The federated learning rate is  $\eta = 0.01$  and the number of global epochs and local epochs are fixed at  $\delta_g = 10$  and  $\delta_l = 5$ . The MNIST dataset contains 60,000 training samples and 10,000 testing samples for 10 digit labels from 0 to 9. It is reasonable to assume that each label essentially has an equal occurrence probability. So we set the actual distribution for the whole population  $\mathbb{P}$ , i.e., the benchmark for measuring the EMD value, as  $\mathbb{P}(y = j) = 0.1, \forall j \in \{0, \dots, 9\}$ . The worst EMD  $\sigma_{\rm max}$  that the FL platform can accept is set to be 1.2, i.e.,  $\sigma_{\rm max} = 1.2$ . We vary the normalized total data size D



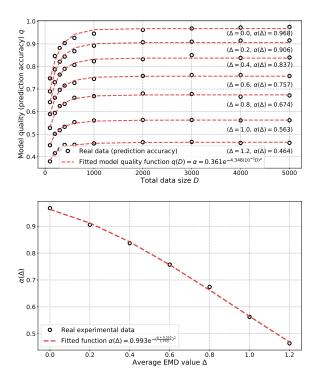


Fig. 4. Estimation of the data utility function in (6).

and the average EMD value  $\Delta$  by changing each worker *i*' local data size and number of labels. Each presented result is averaged over 100 instances. The data utility here is the prediction accuracy. Fig. 4 demonstrates that the data utility function in (6) well fits the real experiment results. Based on the experiment, we set  $\kappa_1 = 0.361$ ,  $\kappa = 4.348$ ,  $\kappa_3 = 10^{-3}$ ,  $\kappa_4 = 0.993$ ,  $\kappa_5 = 0.31$ ,  $\kappa_6 = 1.743$ ,  $\kappa_7 = 100$ ,  $\delta_g = 10$ ,  $\delta_1 = 5$  and M = 0.5 in the following simulations.

### B. Performance of RMA and DRLA mechanisms

We conduct simulations to evaluate the performance of our proposed strategyproof auction mechanisms, including the manually designed RMA and the automated DRLA. Unless otherwise stated, the simulation parameters are configured as follows. There are N = 50 data owners joining in the auction for participation in the federated learning activity. We assume that the noise power spectral density level  $\psi_0$  is -130 dBm/Hz, and the dynamic range of the channel power gain  $h^2$  is from -90 dB to -100 dB. Hereby, we uniformly generate data owner *i*'s normalized channel power gain  $h_i$  from  $[10^6, 10^7]$ , data size  $d_i$  from [0, 10], EMD value  $\sigma_i$  from [0, 1.2], both unit data collection cost  $\gamma_i$  and unit data computational cost  $\alpha_i$  from  $[10^{-5}, 10^{-4}]$ , and unit data transmission cost from  $[10^{-2}, 10^{-1}]$ . Here, the worker's maximum data size  $d_{\text{max}} =$ 10 and maximum EMD value  $\sigma_{max} = 1.2$ . The platform's unit costs for computing and transmission are set as  $\hat{\alpha} = 5 \times 10^{-2}$ and  $\hat{\beta} = 5 \times 10^{-5}$ . With respect to the wireless channels, we fix the average number of channels per worker  $\overline{C}$  at 2 and then the set of total available channels is  $C = \{1, \dots, 100\}$ . Each data owner' requested channel set  $C_i$  are uniformly sampled from C and the corresponding number of channels  $C_i$  also follows uniform distribution in [2,6]. We prepare 5,000 samples for the DRLA model training, 100 samples for validation and 1,000 samples for testing and evaluating the performance of both the RMA and DRLA mechanisms. For RMA mechanism, we set the number of groups as G = 10. We implement the DRLA mechanism integrated with a 2layer GCN and a monotonic network where K = 8, J = 8and  $\varpi_{\mathcal{G}} = 64$ . We use the ADAM optimizer [43] with a learning rate of 0.001 and minibatch of 128 and linearly anneal the exploration probability  $\varsigma$  from 0.9 to 0.05 when training the DRLA model. As illustrated in Figure 5, we plot our proposed DRLA mechanism's convergence curves with respect to the held-out validation performance for different number of data owners. The performance of DRLA mechanism, i.e., the achieved social welfare or the accumulated reward, can quickly converge to a stable high value after training with a few hundreds of minibatches. All the following experimental results are the mean values based on the 1,000 testing samples.

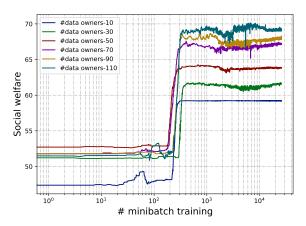


Fig. 5. DRLA convergence measured by the held-out validation performance.

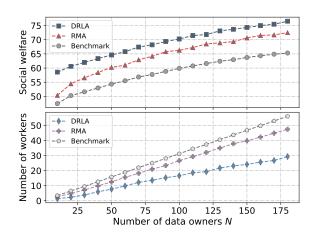


Fig. 6. Impact of number of data owners N.

In addition to the social welfare metric, we are interested in the number of selected workers W which reflects the fairness and the satisfaction rate of data owners. Figure 6 demonstrates the impact of the number of data owners N on the social welfare S and the number of workers W. We observe that the social welfare and number of workers in the RMA and DRLA mechanisms both increase with growing data owners at a diminishing rate. The reason is two folds: First, the

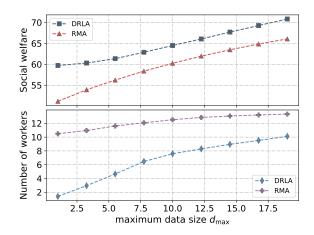


Fig. 7. Impact of maximum data size  $d_{\text{max}}$ .

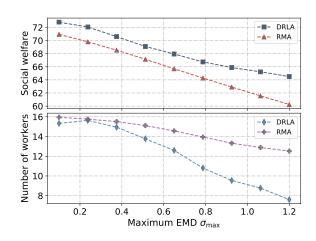


Fig. 8. Impact of maximum EMD  $\sigma_{max}$ .

greedy algorithms only choose the worker that can improve the social welfare. Second, a larger base of interested workers will bring more competition in the auction and that more workers would also reduce the remaining workers' marginal social welfare density. Although the DRLA can achieve higher social welfare than that of RMA mechanism, the DRLA is less fair since it is more capable of sequentially finding out the data owner with larger marginal social welfare. As mentioned in Section I, there is few research work discussing the auction mechanism dedicated for wireless federated learning. Since the key challenging issue in this paper is about the channel conflict, we attempt to apply a well-known strategyproof spectrum auction mechanism proposed in [44] as the benchmark. The benchmark auction mechanism decides the allocation only based on the bidders' bid prices while avoiding the channel conflicts among the bidders. As shown in the Figure 6, the social welfare achieved by the benchmark auction mechanism is lower than the proposed auction mechanisms, although it provides better fairness. However, this paper focuses on the social welfare optimization and thus the benchmark mechanism is not suitable to be directly applied in our wireless federated learning scenario.

As illustrated in Figs. 7 and 8, we investigate the impact of the worker's maximum data size  $d_{\text{max}}$  and EMD value  $\sigma_{\text{max}}$  on the social welfare. Note that  $d_{\text{max}}$  and  $\sigma_{\text{max}}$  are

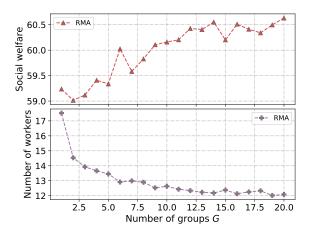


Fig. 9. Impact of the number of groups G on the RMA mechanism performance.

adjustable parameters and preset by the FL platform before the auction. It is clear that the social welfare increases when the FL platform raises its requirement of data quality by larger data size and lower EMD value. Certainly, the precondition is that there are enough data owners that satisfy the requirement. It is interesting to note that when  $\sigma_{\max}$  is large, the DRLA can drop more data owners with low data quality (high  $\sigma_i$ ) to keep better social welfare than that of the RMA. In Fig. 9, we vary the number of groups G to show its impact on the performance of the RMA mechanism. With the G growing, the achieved social welfare is increasing while less workers are selected. More groups means the virtual EMD difference among data owners is widening, so the RMA mechanism can recognize more data owners with low original EMD value. When such data owner with high data quality is found, there is less need to choose other data owners with poor data quality.

### VII. CONCLUSION

In this paper, we have proposed an auction based market model for trading federated learning services in the wireless environment. We have designed a reverse multi-dimensional auction (RMA) mechanism for maximizing the social welfare of the federated learning services market. The RMA mechanism not only considers workers' bid prices for providing training services but also takes each worker's own multiple attributes, including the data size, EMD, and wireless channel demand, into account. To well evaluate each workers' value, we have introduced a data quality function verified by real world experiments to characterize the relationship between the accuracy performance and the size and average EMD of all local data. To further improve the social welfare, we have proposed a deep reinforcement learning based auction (DRLA) mechanism which uses the graph neural network to effectively extract useful features from worker's reported types and automatically determines the service allocation and payment. Both the proposed RMA mechanism and the DRLA mechanism possess the economic properties of truthfulness and individual rationality.

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