Corrections

Corrections to "Link-State Routing With Hop-By-Hop Forwarding Can Achieve Optimal Traffic Engineering"

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WE PROVIDE a correction for the article [1]. As part of the development in [1], we solve the classic multiple commodity flow (MCF) problem (with $D(s,t)$ as the traffic demand from source s to destination t and $c_{u,v}$ as the capacity of link (u,v)) to find the optimal commodity flow $\{f_{u,v}^{t*}\}$ where $f_{u,v}^{t*}$ is an optimal flow (destined to t) on link (u,v) to minimize a convex objective function on the total flow on each link. $\widetilde{c}_{u,v} \triangleq \sum_t f_{u,v}^{t*}$ is called *necessary capacity* on each link (u,v). There could be multiple optimal solutions of $\{f_{u,v}^{t*}\}$ as long as $\sum_t f_{u,v}^{t*} = \widetilde{c}_{u,v}$. Among those optimal solutions, the one maximizing the so-called Network Entropy can be determined using one weight per link for a given traffic demand. The solution results in a routing protocol, PEFT (Penalizing Exponential Flow spliTting), that splits traffic over multiple paths with an exponential penalty on longer paths.

However, PEFT requires sending non-zero traffic on all outgoing links. The requirement in [1] does not hold if there exists no optimal commodity flow $\{f_{u,v}^{t*}\}$ where every $f_{u,v}^{t*} > 0$. Specifically, [1, Equation (12) in page 1721] does not hold in such cases, which may readily arise. For example, in an optimal commodity flow, one of $f_{u,v}^{t*}$ and $f_{v,u}^{t*}$ should be always 0 since the smaller one can be canceled from the other one.

In this correction, we analyze how close to optimality PEFT can get in such cases. A more compact formulation of the Network Entropy Maximization problem (**NEM-P**) (1) is introduced in [2], which is equivalent to the original NEM formulation based on path-enumeration [1]. The NEM-P is similar to the MCF problem though the capacity constraint in MCF, $\sum_{t\in\mathbb{V}} f_{u,v}^t \le c_{u,v}$ is replaced with $\sum_{t\in\mathbb{V}} f_{u,v}^t \le \widetilde{c}_{u,v}$. This ensures any feasible solution to NEM-P to be an optimal solution to MCF and provides the flexibility of putting one additional objective function $-\sum_{t\in\mathbb{V}}\left(\sum_{(u,v)\in\mathbb{E}} f_{u,v}^t \log\left(\frac{f_{u,v}^t}{\sum_w f_{u,w}^t}\right)\right)$ to create the NEM-P problem (1) below.

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NEM-P

$$\max \quad -\sum_{t\in\mathbb{V}}\left(\sum_{(u,v)\in\mathbb{E}} f_{u,v}^t \log\left(\frac{f_{u,v}^t}{\sum\limits_{w|(u,w)\in\mathbb{E}} f_{u,w}^t}\right)\right) \tag{1a}$$

$$\text{s.t.} \quad \sum_{v:(s,v)\in\mathbb{E}} f_{s,v}^t - \sum_{u:(u,s)\in\mathbb{E}} f_{u,s}^t = D(s,t), \forall s \neq t \tag{1b}$$

$$\sum_{t\in\mathbb{V}} f_{u,v}^t \le \widetilde{c}_{u,v}, \forall (u,v)\in\mathbb{E} \tag{1c}$$

$$\text{with domain for } f_{u,v}^t \text{ as } R_+. \tag{1d}$$

NEM-P is obviously a convex optimization problem though its global optimal may still be obtained at the boundary of of the constraint set (due to some $f_{u,v}^{t*} = 0$). However, for each $f_{u,v}^{t*} = 0$, we can manually inject a small amount of additional traffic from u to t and force its routing to be the concatenation of (u,v) and a sub-path from v to t, and relax necessary capacities to support this routing. Thus we construct a new MCF problem with all $f_{u,v}^{t*} > 0$, and then can apply NEM/PEFT as in [1], [2].

In the following Theorem 1, we show that PEFT can route any traffic demand arbitrarily close to an optimal flow solution. The above construction is the key to the proof.

Theorem 1: Assume there is an optimal solution to the MCF problem, then for the corresponding necessary capacities ($\{\widetilde{c}_{u,v}\}$) and any given small positive number, $\epsilon > 0$, PEFT can route the demands requiring capacities less than $\widetilde{c}_{u,v} + \epsilon$ on each link (u,v).

Proof: For the original network $G(\mathbb{V},\mathbb{E})$, without loss of generality, we assume there exists a path from v to t, otherwise, we can replace all the occurrences of variable $f_{u,v}^t$ with 0 in the objective and constraints without affecting the MCF problem. Starting from any optimal solution $\{f_{u,v}^{t*}\}$ for the MCF problem, and any given small positive number, ϵ, we initialize a counter $H(u,v,t) = 0$ for each $(u,v)\in\mathbb{E}$ and $t\in\mathbb{V}$. For every $f_{u,v}^{t*} = 0$, we find a minimum-hop path (or any loop-free path) from v to t (or $v = t$), then we increase $H(u',v',t)$ by one for all the links (u',v') along (u,v) and the path from v to t. After checking all $f_{u,v}^{t*} = 0$, we can compute $H(u,v) = \sum_{t\in\mathbb{V}} H(u,v,t)$, and let $H^* = \max_{(u,v)\in\mathbb{E}} H(u,v)$. Finally, for each $f_{u,v}^{t*} = 0$, we inject an additional demand ϵ/H^* from u to t, and set the initial routing path for the additional demand as link (u,v) concatenated with the above minimum-hop path from v to t. We define the relaxed necessary capacity as $\widehat{c}_{u,v} \triangleq \widetilde{c}_{u,v} + \epsilon H(u,v)/H^* \leq \widetilde{c}_{u,v} + \epsilon$, and increase traffic demand to $\widehat{D}(s,t) \triangleq D(s,t) + \left(\epsilon/H^* \sum_{(s,v)\in\mathbb{E}} |f^{t*}_{s,v} = 0|\right)$, where $|f^{t*}_{s,v} = 0|$ is 1 if $f^{t*}_{u,v} = 0$, and 0 otherwise. Similarly, we have the relaxed flow solution $\widehat{f}^t_{u,v} \triangleq f^{t*}_{u,v} + \epsilon H(u,v,t)/H^* > 0$.

To route the traffic under the above relaxed necessary capacity $\widehat{c}_{u,v}$ using PEFT, we can define a relaxed NEM problem (NEM-R) (2):

NEM-R

$$\max \quad -\sum_{t\in\mathbb{V}} \left(\sum_{(u,v)\in\mathbb{E}} f^t_{u,v} \log \left(\frac{f^t_{u,v}}{\sum\limits_{w|(u,w)\in\mathbb{E}} f^t_{u,w}} \right) \right) \tag{2a}$$

$$\text{s.t.} \quad \sum_{v:(s,v)\in\mathbb{E}} f^t_{s,v} - \sum_{u:(u,s)\in\mathbb{E}} f^t_{u,s} = \widehat{D}(s,t), \forall s \neq t \tag{2b}$$

$$\sum_{t\in\mathbb{V}} f^t_{u,v} \leq \widehat{c}_{u,v}, \forall (u,v) \in \mathbb{E} \tag{2c}$$

$$\text{with domain for } f^t_{u,v} \text{ as } R_+. \tag{2d}$$

NEM-R is a convex optimization problem with compact domain for $f^t_{u,v}$, thus it must have a global optimum. It has differentiable objective and constraint functions and there is a strictly feasible solution $\{\widehat{f}^t_{u,v} > 0\}$ (i.e., satisfying Slater's condition [3]), therefore KKT is the sufficient and necessary condition for optimality. Thus there exist a set of Lagrange multipliers $\{\mu_{s,t}\}$ for (2b), and $\{\lambda_{u,v} \geq 0\}$ for (2c), such that

$$-\log(f^t_{u,v}) + \log\left(\sum_w f^t_{u,w}\right) - (\lambda_{u,v} - \mu_{u,t} + \mu_{v,t}) = 0$$
$$\Rightarrow \quad \frac{f^t_{u,v}}{\sum\limits_w f^t_{u,w}} = e^{-(\lambda_{u,v} - \mu_{u,t} + \mu_{v,t})}.$$

If we use $\lambda_{u,v}$ as the weight $w_{u,v}$ for link (u,v), then a router can compute the distance d^t_u from any node u to node t, and the quantity $h^t_{u,v} \triangleq d^t_v + \lambda_{u,v} - d^t_u$. Therefore,

$$\begin{aligned}
1 &= \sum_{(u,v)\in\mathbb{E}} \frac{f^t_{u,v}}{\sum\limits_w f^t_{u,w}} \\
\Rightarrow \quad 1 &= \sum_{(u,v)\in\mathbb{E}} e^{-(\lambda_{u,v} - \mu_{u,t} + \mu_{v,t})} \\
\Rightarrow \quad e^{-\mu_{u,t}} &= \sum_{(u,v)\in\mathbb{E}} e^{-\lambda_{u,v} - \mu_{v,t}} \\
\Rightarrow \quad e^{-\mu_{u,t} + d^t_u} &= \sum_{(u,v)\in\mathbb{E}} e^{-\lambda_{u,v} - \mu_{v,t} - d^t_v + d^t_v + d^t_u} \\
&= \sum_{(u,v)\in\mathbb{E}} e^{-(\lambda_{u,v} + d^t_v - d^t_u)} e^{-\mu_{v,t} + d^t_v} \\
&= \sum_{(u,v)\in\mathbb{E}} \left(e^{-h^t_{u,v}} e^{-\mu_{v,t} + d^t_v} \right).
\end{aligned} \tag{3}$$

Note that, in [1, Equation (18)], a PEFT flow is represented by Υ^t_u, which is recursively defined as $\Upsilon^t_u = \sum_{(u,v)\in\mathbb{E}} \left(e^{-h^t_{u,v}} \Upsilon^t_v \right)$. Thus from solving the NEM-R problem, we could have $e^{-\mu_{u,t} + d^t_u} = \Upsilon^t_u$. Then Υ^t_u and $f^t_{u,v}$ can be computed in the same way as PEFT (refer to [1], [2] for more details).

In summary, by defining and solving the NEM-R problem, we show that PEFT can route no less traffic demands ($\widehat{D}(s,t) \geq D(s,t)$) by using an arbitrarily small amount of capacities (ϵ) than necessary capacities ($\widehat{c}_{u,v} \leq \widetilde{c}_{u,v} + \epsilon$). ∎

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