# Mining Maximal Dynamic Spatial Co-Location Patterns

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Abstract—A spatial co-location pattern represents a subset of spatial features with instances that are prevalently located together in a geographic space. Although many algorithms for mining spatial co-location patterns have been proposed, the following selected problems remain: 1) these methods miss certain meaningful patterns (e.g., {Ganoderma\_lucidum<sub>new</sub>, {*water\_hyacinth<sub>new</sub>*(increase), maple\_tree<sub>dead</sub>} and algae<sub>dead</sub>(decrease)}) and obtain the wrong conclusion if the instances of two or more features increase/decrease (i.e., new/dead) in the same/approximate proportion, which has no effect on the prevalent patterns; 2) because the number of prevalent spatial co-location patterns is quite large, the efficiency of existing methods is low in mining prevalent spatial co-location patterns. Therefore, we first propose the concept of the dynamic spatial co-location pattern that can reflect the dynamic relationships among spatial features. Second, we mine a small number of prevalent maximal dynamic spatial colocation patterns that can derive all prevalent dynamic spatial co-location patterns, which can improve the efficiency of obtaining all prevalent dynamic spatial co-location patterns. Third, we propose an algorithm for mining prevalent maximal dynamic spatial co-location patterns and two pruning strategies. Finally, the effectiveness and efficiency of the proposed method and the pruning strategies are verified by extensive experiments over real/synthetic datasets.

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*Index Terms*—Association rule mining, spatial co-location pattern, dynamic pattern, maximal pattern.

## I. INTRODUCTION

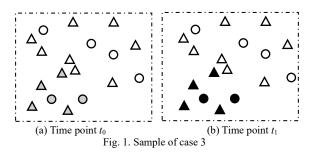
PATIAL co-location pattern mining is an important component of association rule mining [1,2] in machine learning [3,4,5,6]. A spatial co-location pattern represents a subset of spatial features whose instances are prevalently located together in a geographic space. Mining of the spatial co-location pattern is significant. For example, if a city planner cannot find the prevalent pattern {school, supermarket, restaurant} near the "school", this indicates that we need to build a new "supermarket" or "restaurant" around the "school". Other application domains include public health [7], public transportation [8,9], environmental management [10], social media services [11,12], location services [13,14], and multimedia [15,16,17,18,19], among others.

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Although many methods for mining spatial co-location patterns exist, they cannot find the dynamic relationships among spatial features. On the one hand, the existing methods miss certain meaningful patterns. Case 1: "Ganoderma lucidum" grows on the "maple tree", which was dead. However, existing methods mine patterns from the set of coexisting plants such that the meaningful pattern {*Ganoderma\_lucidum<sub>new</sub>*, *maple\_tree<sub>dead</sub>*} was missed. Case 2: For mutually inhibitory features such as "water hyacinth" and "algae", the instances of "algae" decrease with the increase in the instances of "water hyacinth" in the same zone. However, because the participation index is always unchanged for existing methods, these methods obtain the prevalent pattern {water hyacinth, algae} regardless of the increase/decrease in instances of "water hyacinth"/"algae". On the other hand, existing methods obtain the wrong conclusion that the instances of two or more features increase/decrease (i.e., new/dead) in the same/approximate proportion, which has no effect on prevalent patterns. Case 3: One application of the prevalent spatial co-location pattern judges whether the environment was polluted or not by comparing prevalent patterns at different time points. As shown in Fig. 1, the instances of two features were dead (black shadow) by an equivalent (or approximate) percentage because of environment disruption, but the existing methods determine that the environment has not been polluted because they obtain the same prevalent patterns with the same participation index at two-time points (i.e.,  $t_0$  and  $t_1$ ). In conclusion, finding the dynamic relationships among spatial features (i.e., dynamic spatial co-location patterns) is a promising topic.



In similar existing methods, the negative/sequential/strongsymbiotic patterns with the above cases appear to be similar, but essential differences exist between them. Both the dynamic pattern and the negative pattern [34,35] can mine mutual exclusion relationship, but the features in the negative pattern cannot coexist. Both the dynamic pattern and the sequential pattern [36,37] are all about time, but the latter is to look for prevalent subsequences from sequential databases. In contrast, the dynamic spatial co-location pattern represents a dynamic relationship between features, which exists in a symbiont circle. The strong-symbiotic pattern [38] belongs to a portion of the dynamic spatial colocation pattern, so it can only mine a portion of the dynamic spatial co-location patterns (i.e., { $A_{new}$ ,  $B_{new}$ }) and misses other dynamic spatial co-location patterns (i.e., { $A_{new}$ ,  $B_{dead}$ }

## and $\{A_{dead}, B_{dead}\}$ ).

Moreover, because the amount of spatial data is always quite large, the number of prevalent dynamic spatial colocation pattern is also large, and thus the efficiency of the existing methods is low in mining the prevalent spatial colocation patterns. It is necessary to find certain representative patterns that can derive all prevalent patterns and whose number is small. The prevalent maximal pattern is a compact representation of the prevalent pattern, and the number of prevalent maximal patterns is far less than the number of all prevalent patterns. Therefore, mining the prevalent maximal spatial co-location patterns that can derive all prevalent spatial co-location patterns is more efficient than mining all prevalent spatial co-location patterns using the existing methods. Although selected methods [39,40,41,42,43,44] can mine the prevalent maximal spatial co-location patterns, they still require large numbers of calculations and connections for table instances as well as general methods of mining prevalent spatial co-location patterns.

In summary, the existing methods cannot find the dynamic relationships among spatial features (i.e., dynamic spatial co-location pattern), and the efficiency of mining prevalent dynamic spatial co-location patterns by the existing methods is rather low. Therefore, in this paper, we propose a method for mining the prevalent maximal dynamic spatial colocation pattern and make the following contributions:

- 1. The existing methods cannot find the dynamic relationships among spatial features, and thus we propose the concept of the dynamic spatial co-location pattern (Dc for short) that can reflect the dynamic relationships among spatial features and can solve the problems in Case 1/Case 2/Case 3.
- 2. The prevalent maximal patterns can be used to derive all prevalent patterns, and the number of prevalent maximal patterns is far less than the number of all prevalent patterns. Therefore, we mine the prevalent maximal dynamic spatial co-location patterns rather than all prevalent dynamic spatial co-location patterns, which is more efficient than mining all prevalent dynamic spatial co-location patterns using the existing methods.
- 3. Because a large number of calculations and connections are necessary for table instances in the existing methods for mining maximal patterns, these methods have low efficiency. To improve the efficiency of mining maximal patterns, we propose an algorithm for mining the prevalent maximal dynamic spatial colocation patterns, in which the calculation and connection for table instances are turned into the calculation and connection of dynamic features whose number is far less than that of the instances. Moreover, we propose two pruning strategies to further improve the efficiency.
- 4. We verified the effectiveness of our algorithm (i.e., we can find the dynamic relationships among spatial features), the representativeness of the prevalent maximal *Dc*, the efficiency of our algorithm (i.e., comparison with the join-based method), and the efficiency of two pruning strategies over real/synthetic datasets.

## II. RELATED WORK

Although many methods for mining spatial co-location pattern have been proposed, no method exists that can mine

the dynamic spatial co-location patterns. S. Shekhar et al. [20,21] defined the spatial co-location pattern for the first time and proposed the join-based algorithm. Subsequently, certain methods focused on many other interesting research directions, such as high utility patterns [22,23,24], redundancy reduction [25], improved efficiency [26], causal rules [27], competitive pairs [28], fuzzy objects [29], uncertain data [30,31,32,33], etc. However, the existing methods miss certain meaningful patterns (e.g., {*Ganoderma* lucidum<sub>new</sub>, *maple tree*<sub>*dead*</sub>} and {*water hyacinth<sub>new</sub>*(increase), *algae<sub>dead</sub>*(decrease)}) and obtain the wrong conclusion that the instances of two or more features increase/decrease (i.e., new/dead) in the same/approximate proportion, which has no effect on the prevalent patterns.

The dynamic spatial co-location pattern in this paper might appear to a negative pattern [34,35], sequential pattern [36,37] or strong symbiotic pattern [38], but their essences are different. The features in the negative pattern [34,35] cannot coexist, whereas the features in the dynamic spatial co-location pattern must coexist. Sequential patterns [36,37] represent prevalent repeated paths between items, which exist in the form of a sequence, whereas the dynamic spatial co-location pattern represents a dynamic relationship between features which exist in a symbiont circle. In a strong symbiosis pattern [38], at least one feature benefits from the pattern, so it belongs to a portion of the dynamic spatial colocation pattern, and the method of mining strong symbiosis patterns can only mine a small portion of dynamic spatial colocation patterns (i.e.,  $\{A_{new}, B_{new}\}$ ), and other dynamic patterns (i.e.,  $\{A_{new}, B_{dead}\}$  and  $\{A_{dead}, B_{dead}\}$ ) cannot be mined. In conclusion, the methods for mining a negative pattern, sequential pattern or strong symbiotic pattern cannot mine the dynamic spatial co-location pattern in this paper.

TABLE I
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	CATEGORY OF METHO	DS
Categories	Innovations	Literatures
Traditional	Origin	[20,21]
co-location	high utility patterns	[22,23,24]
pattern	redundancy reduction	[25]
-	improved efficiency	[26]
	causal rules	[27]
	competitive pairs	[28]
	fuzzy objects	[29]
	uncertain data	[30,31,32,33]
Similar	Negative pattern	[34,35]
methods	Sequential pattern	[36,37]
	Strong-symbiotic	[38]
Related methods	maximal pattern	[39,40,41,42,43,44]

Although certain methods can mine the prevalent maximal spatial co-location pattern, they still require a large number of calculations and connections for table instances as well as general methods of mining prevalent spatial co-location patterns. Wang et al. [39] proposed an order-clique-based approach for mining maximal co-location pattern, and based on this approach, Yao et al. [40,41] proposed an ordered-instance-clique approach. Dai et al. [42] used an index structure similar to four binary trees to mine the maximal spatial co-location patterns. Bao et al. [43] mined the top-k longer size maximal co-location patterns. Wang et al. [44] mined the maximal sub-prevalent co-location patterns, which introduced star participation instances to measure the prevalence of co-location patterns, i.e., spatially correlated

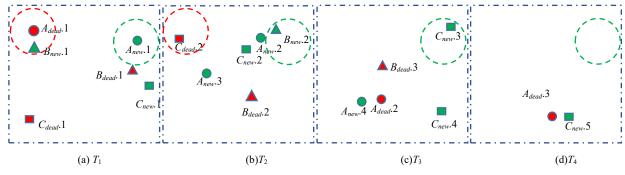


Fig. 2. Sample distribution datasets of new/dead instance

instances that cannot form cliques were also properly considered. However, the above methods still require a large number of calculations and connections for table instances. In contrast, we propose an algorithm for mining prevalent maximal dynamic spatial co-location patterns, which is based on a degree-based approach for the maximum clique/maximal co-location patterns [45,46] and turns the calculation and connection for table instances into calculation and connection of dynamic features, the number of which is far less than that of instances.

## **III. BASIC DEFINITIONS**

**Definition 1. (Dynamic Feature/Instance)** A dynamic feature represents the new/dead object in a certain area, denoted as  $Df_{[i]}$ , which is a new or dead object (e.g.,  $A_{new}$ ,  $A_{dead}$  are two dynamic features in Fig. 2). A dynamic instance is an instance of a dynamic feature at a specific location, denoted as  $Df_{[i]}$ . j (e.g.,  $A_{new}$ .1 and  $A_{new}$ .2 are two instances of the dynamic feature  $A_{new}$  in Fig. 2).

**Definition 2. (Dynamic Distance Threshold,**  $D_d$ ) If the distance between two dynamic instances is less than  $D_d$  as designated by experts, it is considered that the two dynamic instances have a relationship, and otherwise, they have no relationship.

**Definition 3. (Time Span and Time Span Constraint)** *Time span* is the time difference between two adjacent dynamic datasets and represents the time interval in which certain instances have changed, which is designated by experts. The *time span constraint* of a dynamic feature is the length of time in which the feature influences the surrounding dynamic features, denoted as *span*  $(Df_{[i]})$ . Furthermore, if the *time span constraint* of  $Df_{[i]}$  is equal to k time spans, it can be represented as *span*  $(Df_{[i]}) = k$  (*time spans*).

The effect of a new object  $Df_{[i]}$  on the surrounding dynamic features is the life cycle of  $Df_{[i]}$ . For example, we assume that the *time span* is 3 years and that the life cycle of  $A_{new}$  is 75 years, and thus *span* ( $A_{new}$ ) =25(*time spans*).

The effect of a dead object  $Df_{[i]}$  on the surrounding dynamic features is a one-*time span* because the dead object has little effect on the surrounding dynamic features, and the one-*time span* can sufficiently cover the time interval in which a dead object influences the surrounding instances. Thus *span*  $(A_{dead})=1$  (*time span*).

**Definition 4.** (Dynamic Spatial Neighborhood Relationship,  $D_R$ ) For two dynamic instances, if the distance between them is less than  $D_d$ , which is designated by experts, and the time difference between them is less than the maximum of the *time span constraints* of all dynamic features, it is considered that the two dynamic instances satisfy the dynamic spatial neighborhood relationship  $D_R$ .

$$D_R(A_{new}.1, B_{dead}.2) \Leftrightarrow$$
  
distance  $(A_{new}.1, B_{dead}.2) \leq D_d$ 

$$\Delta T(A_{new}, 1, B_{dead}, 2) < max (span (A_{new}), span (B_{dead}))$$

**Definition 5. (Dynamic Spatial Co-Location Pattern**, Dc) A dynamic spatial co-location pattern contains multiple new/dead features and can reflect the dynamic relationships among dynamic features, denoted as Dc. For example,  $Dc_{[i]} = \{A_{new}, B_{dead}\}$  is a size-2 dynamic spatial co-location pattern. **Definition 6. (Dynamic Row Instance and Dynamic Table Instance)** For a dynamic spatial co-location pattern Dc and a set of dynamic instances DI, if a one-to-one match exists between each dynamic instance in DI and each dynamic feature in Dc and any two dynamic instances in DIsatisfy the dynamic spatial neighborhood relationship, we say that DI is a dynamic row-instance of Dc, denoted as dynamic row-instance (Dc). The dynamic table-instance of Dc consists of all distinct dynamic row-instances of Dc, denoted as dynamic table-instance (Dc).

**Example 1.** For  $Dc_{[i]} = \{A_{new}, B_{new}\}$ , if both  $(A_{new}.1, B_{new}.2)$ and  $(A_{new}.2, B_{new}.2)$  satisfy the dynamic spatial neighborhood relationship, they are the dynamic row-instance of  $Dc_{[i]}$ , and  $\{\{A_{new}.1, B_{new}.2\}, \{A_{new}.2, B_{new}.2\}\}$  is the dynamic tableinstance of  $Dc_{[i]}$ , denoted as dynamic table-instance $(Dc_{[i]})$  $=\{\{A_{new}.1, B_{new}.2\}, \{A_{new}.2, B_{new}.2\}\}$ .

**Definition 7. (Dynamic Participation Ratio (DPR)/ Index** (**DPI**)) The dynamic participation ratio DPR (Dc,  $Df_{[i]}$ ) of dynamic feature  $Df_{[i]}$  in a size-k dynamic spatial co-location pattern  $Dc = \{Df_{[1]}, Df_{[2]} \dots Df_{[k]}\}$  is defined as follows:

$$DPR(Dc, Df_{[i]}) = \frac{\pi_{Df_{[i]}}(dynamic\_table\_instance(Dc))}{dynamic\_table\_instance(\{Df_{[i]}\})}$$

where  $\pi$  is the relational projection operation with a duplication elimination. *DPI(Dc)* of *Dc* is defined as shown:

$$DPI(Dc) = min_{i=1}^{\kappa} \{DPR(Dc, Df_{[i]})\},\$$

If DPI(Dc) is greater than a given minimum prevalence threshold  $min\_prev$  that is designated by experts and is used to judge whether the pattern occurs prevalently or not, we say that Dc is a prevalent dynamic spatial co-location pattern. **Example 2.** For  $Dc_{[i]} = \{A_{new}, B_{new}\}$ , dynamic tableinstance $(Dc_{[i]}) = \{\{A_{new}, 1, B_{new}, 2\}, \{A_{new}, 2, B_{new}, 2\}\}$ . In  $Dc_{[i]}$ , the number of dynamic instances of  $A_{new}$  and  $B_{new}$  are 2 and 1, respectively. In contrast, from Fig. 2, the total number of dynamic instances of  $A_{new}$  and  $B_{new}$  are 4 and 2, respectively. Therefore,  $DPR(Dc_{[i]}, A_{new}) = 2/4 = 0.5$ ,  $DPR(Dc_{[i]}, B_{new}) = 1/2$ = 0.5 and thus  $DPI(Dc_{[i]}) = min\{DPR(Dc_{[i]}, A_{new}), DPR(Dc_{[i]}, B_{new})\} = 0.5$ . If  $min\_prev=0.3$ ,  $Dc_{[i]} = \{A_{new}, B_{new}\}$  is a size-2 prevalent dynamic spatial co-location pattern.

**Definition 8. (Prevalent Maximal Dynamic Spatial Co-Location Pattern)** Given a prevalent dynamic spatial colocation pattern  $Dc = \{Df_{[I]}, \dots, Df_{[v]}\}$ , for any  $Df_{[i]} \in Df$  and  $Df_{[i]} \notin Dc$ , if any  $Dc \cup Df_{[i]}$  is not a prevalent dynamic spatial co-location pattern, then Dc is a prevalent maximal dynamic spatial co-location pattern.

**Definition 9. (Dynamic Spatial Feature Clique,** Dfc) Given a dynamic spatial feature set  $Dfc = \{Df_{[i]}, \dots, Df_{[v]}\}$ , if any size-2 pattern  $Dc_{[i]} = \{Df_{[i]}, Df_{[k]}\}$  ( $Df_{[i]}, Df_{[k]} \in Dfc$  and  $j \neq k$ ) is prevalent, then Dfc is a dynamic spatial feature clique. **Definition 10. (Maximal Dynamic Spatial Feature Clique)** Given a dynamic spatial feature set  $Dfc = \{Df_{[i]}, \dots, Df_{[v]}\}$ , for any  $Df_{[i]} \in Df$  and  $Df_{[i]} \notin Dfc$ , if  $Dfc \cup Df_{[i]}$  is not a dynamic spatial feature clique, then Dfc is a maximal dynamic spatial feature clique.

## IV. MINING PREVALENT MAXIMAL DC (ALGORITHM MDC)

We propose an algorithm for mining the prevalent maximal dynamic spatial co-location pattern (i.e., Algorithm MDC), which is divided into three sub algorithms (i.e., *Algorithm 1, Algorithm 2, Algorithm 3*). For convenience of description, the dynamic spatial co-location pattern is abbreviated as *Dc* in this paper.

Because a large number of calculations and connections exist for table instances in the existing methods, these methods have low efficiency. To improve the efficiency of mining patterns, after obtaining the size-2 prevalent dynamic spatial co-location patterns, we convert them to a dynamic feature graph DG by *Algorithm 1* such that the calculation and connection for the table instances are turned into the calculation and connection of the dynamic features. Subsequently, we obtain the set of maximal dynamic feature clique Dfc from the dynamic feature graph DG by *Algorithm* 2. Finally, each maximal dynamic feature clique Dfc as a candidate maximal Dc is verified by *Algorithm 3*, and thus we can obtain prevalent maximal Dc.

## A. First Sub-Algorithm

First, we generate the distribution dataset of dynamic instances (i.e., new/dead instances) from the distribution dataset of spatial instances at different time points (*step 1* and *Example 3*). Second, with  $D_d$ , *time span* and *time span constraint* (i.e., life cycle) of each dynamic feature, we can confirm whether any two dynamic instances have a dynamic spatial neighborhood relationship or not and obtain a set of all dynamic neighborhood relationships (*step 2* and *Example 4*). Second, we obtain the dynamic table-instance of all size-2 *Dc* by arranging all dynamic neighborhood relationships, and thus we obtain the size-2 prevalent *Dc* by definition 8 and *min\_prev* (*step 3/step 4* and *Example 5*). Finally, we transform all size-2 prevalent *Dc* to dynamic feature graph *DG* (*step 5* and *Example 6*).

Algorithm 1: Generating Dynamic Feature Graph DG

**Example 3.** Given the distribution dataset of spatial instances at different time points (i.e.,  $t_1, t_2, \ldots, t_n$ ), because this paper studies the dynamic relationship among features, we obtain *n*-1 dynamic datasets that contain only new/dead instances by comparing two datasets at  $t_i$  and  $t_{i+1}$ . For instance, we can obtain a dynamic dataset in Fig. 2(a) by comparing the two datasets at  $t_1$  and  $t_2$ . The new/dead (green/red in Fig. 2) categorizations of the same object are denoted by two dynamic features (i.e.,  $A_{new}/A_{dead}$ ), respectively. One instance of feature A exists in  $t_1$  and disappears at  $t_2$ , and it is used as a dead instance in  $T_1$ , denoted by Adead.1. Similarly, if one instance of feature A appears at  $t_2$  for the first time, it is used as a new instance in  $T_1$ , denoted by  $A_{new.}$  1. As shown in Fig. 2, the distribution datasets of new/dead instances are obtained by comparing the datasets at 5 time points.

**Example 4.** Suppose  $D_d = k$  and  $span(A_{new}) = 3(time spans)$ (the time difference between  $T_1$  and  $T_2$  is one *time span*). To obtain the neighborhood instances of  $A_{new}$ .1 in Fig. 2(a), we should confirm whether Anew.1 and all other dynamic instances in Fig. 2(a)(b)(c)(d) (according to definition 3 and  $span(A_{new})=3(time spans))$  satisfy the dynamic spatial neighborhood relationship or not by definition 4. Subsequently, we determine that  $B_{new.}2$  and  $C_{new.}3$  are the neighborhood instances of  $A_{new.}$  1. Similarly, to obtain the neighborhood instances of Adead.1 in Fig. 2(a), we should confirm whether  $A_{dead}$ . 1 and all other dynamic instances in Fig. 2(a)(b) (according to definition 3, and thus span  $(A_{dead})=1$  (time span)) satisfy the dynamic spatial neighborhood relationship or not by definition 4. Finally, we find that  $B_{new}$ .1 and  $C_{dead}$ .2 are the neighborhood instances of Adead. 1.

**Example 5.** By arranging all neighborhood dynamic instance pairs of  $A_{new}$  and  $B_{new}$  (i.e.,  $A_{new}$ .1 and  $A_{new}$ .2 are the neighborhood dynamic instances of  $B_{new}$ .2), we obtain the dynamic table-instances { $\{A_{new}.1, B_{new}.2\}, \{A_{new}.2, B_{new}.2\}$ } of  $Dc_{[i]} = \{A_{new}, B_{new}\}$ . Suppose  $min\_prev=0.3$  according to definition 7, because  $DPI(Dc_{[i]}) = 0.5>0.3$ , and thus  $Dc_{[i]} = \{A_{new}, B_{new}\}$  is a size-2 prevalent Dc.

**Example 6.** Suppose certain size-2 prevalent dynamic spatial co-location patterns exist such as  $\{A_{new}, B_{new}\}$ ,  $\{A_{new}, C_{new}\}$ ,  $\{A_{dead}, B_{new}\}$ ,  $\{A_{dead}, B_{dead}\}$ ,  $\{A_{dead}, C_{dead}\}$  and  $\{B_{new}, C_{dead}\}$ . In Fig. 3, each dynamic feature in all size-2 prevalent Dc and each size-2 prevalent Dc are treated as a vertex and an edge, respectively. For instance, because  $\{A_{new}, B_{new}\}$  is a size-2 prevalent Dc, we should connect  $A_{new}$  and  $B_{new}$ . In contrast,  $A_{new}$  does not connect to  $C_{dead}$  because  $\{A_{new}, C_{dead}\}$  is not a prevalent Dc. Finally, we obtain a dynamic feature graph DG, as shown in Fig. 3.

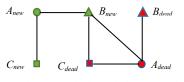


Fig. 3. Sample dynamic feature graph

# B. Second Sub-Algorithm

The maximal Dfc (dynamic feature clique) is treated as the candidate prevalent maximal Dc and can be obtained from DG by *Algorithm2*, which is proposed based on the degreebased approach for the maximum clique/maximal colocation patterns [45,46].

**Input:** (1)  $Df = \{Df_{[1]}, Df_{[2]}, ..., Df_{[n]}\}$ :a set of dynamic spatial features; (2)  $St = \{St_{[1]}, St_{[2]}, ..., St_{[n]}\}$ :the distribution dataset of spatial instances at different time points; (3)  $Lc = \{Lc_{[1]}, Lc_{[2]}, ..., Lc_{[n]}\}$ :a set of life cycle of all dynamic features; (4) $D_d$ :a dynamic distance threshold; (5)min\_prev.a minimum DPI threshold.

Output: DG: dynamic feature graph.

**Variables:**  $S_T$ : the distribution dataset of dynamic instances at different time points.

<sup>1:</sup> *S*<sub>T</sub>=Gen\_dynamic\_instance\_distribution(*St*)

<sup>2:</sup>  $\delta_{DR}$  = Gen\_dynamic\_neighborhood (*Df*, *S<sub>T</sub>*, *D<sub>d</sub>*, *Lc*, *time\_span*)

<sup>3:</sup>  $\delta_{dynamic table_instance} = \text{Gen_dynamic_table_instance} (Df, \delta_{DR})$ 

<sup>4:</sup>  $\delta_{dynamic\_size2}$ =Gen\_size2\_prevalent\_ $Dc(Df, \delta_{dynamic\_table\_instance}, min\_prev)$ 5: DG = Gen dynamic feature graph ( $\delta_{dynamic\_size\_2}$ )

The core idea of *Algorithm2* is described as follows. First, the vertex with maximum degree in DG is selected as  $V_{max}$ , and other vertices are divided into two categories (adjacent and non-adjacent vertices). Second,  $V_{max}$  is treated as a node in one candidate Dfc, and all of the adjacent vertices of  $V_{max}$  form a subgraph (i.e.,  $sub\_DG_{[1]}$ ). This process is applied recursively for  $sub\_DG_{[1]}$ . Third, the non-adjacent vertices of  $V_{max}$  are treated successively as  $V_{max}'$ , which is the same as  $V_{max}$ . For *i*-th  $V_{max}'$ , the adjacent vertices of  $V_{max}'$  form a subgraph (i.e.,  $sub\_DG_{[2i]}$ ), and  $sub\_DG_{[2i]}$  is subjected to a recursive process similar to that of  $sub\_DG_{[1]}$ .

In the recursive process, starting from the vertex with maximum degree ( $V_{max}$ ) can accelerate the speed of finding all maximal Dfc. Because each vertex in the maximal Dfc must connect to all other vertices in the maximal Dfc,  $V_{max}$  is always located in a maximal Dfc that contains many vertices. On the one hand, starting from  $V_{max}$  can quickly reduce the edges in  $sub_DG_{[1]}$ , and thus it accelerates the process of finding the maximal Dfc in  $sub_DG_{[1]}$ . On the other hand, starting from  $V_{max}$  always first finds the maximal Dfc that contains many vertices such that  $sub_DG_{[2]}$  is far less than original DG, and therefore, it can accelerate the process of finding the maximal Dfc in  $sub_DG_{[2]}$ .

Algorithm 2: Generating Maximal Dynamic Feature Clique
Input: DG: dynamic feature graph.
<b>Output:</b> $\delta_{maximal\_dynamic\_clique}$ : the set of maximal Dfc.
<i>Variables:</i> (1) <i>Df</i> : a set of dynamic spatial features;(2) <i>can Dfc</i> : a set of
candidate vertices in dynamic feature cliques; (3) $V_{max}$ : a vertex with max
degree; (4) link_Df: a set of adjacent vertices of $V_{max}$ ; (5) not_link_Df: a
set of non-adjacent vertices of $V_{max}$ ; (6) $V_{max}$ : vertex in not_link_Df is
regarded as $V_{max}$ ' successively, which is the same as $V_{max}$ ; (7) second_Df:
a set of adjacent vertices of $V_{max}$ '.
Clique ( <i>Df</i> , <i>can_Dfc</i> , <i>DG</i> )
1: $V_{max} = \text{Get}_{max} \text{degree}_{Df}(Df, DG)$
2: <i>link_Df</i> , <i>not_link_Df</i> =Get_link_or_unlink_ <i>V<sub>max</sub></i> ( <i>Df</i> , <i>DG</i> )
3: If (Exist_side ( <i>link_Df</i> ))
Clique ( <i>link_Df</i> , <i>can_Dfc</i> + $V_{max}$ , <i>DG</i> )
Else
$\delta_{maximal\_dynamic\_clique} + (can\_Dfc + V_{max} + link\_Df)$
4: While (Exist dynamic feature (not link Df, DG))
$V_{max}' = \overline{G}et$ and $\overline{d}elete$ first $\overline{D}f(not \ link \ Df, DG)$
second $Df = Get link V_{max'}$ (not link $Df$ , link $Df$ , DG)
If (Exist side(second Df, DG))
Clique (second $Df$ , can $Dfc + V_{max}', DG$ )
Else
$\delta_{maximal \ dynamic \ clique} + (can_Dfc + V_{max}' + second_Df)$

**Example 7.** For *DG* in Fig. 3,  $V_{max}$  [1] =  $B_{new}$ , and other vertices can be divided into adjacent/non-adjacent vertices. For the adjacent portion of  $B_{new}$ ,  $B_{new}$  is treated as one node in the candidate maximal Dfc (i.e.,  $can Dfc=\{B_{new}\}$ ), and the adjacent vertices form a subgraph (i.e., sub  $DG_{[1]} = \{A_{new}, A_{new}\}$ Adead, Cdead}). Algorithm2 executes a recursion process for sub  $DG_{[1]}$ ,  $V_{max[11]}=A_{dead}$ , and for the adjacent portion of  $A_{dead}$ ,  $A_{dead}$  is treated as one node in candidate maximal Dfc(i.e., *can\_Dfc*={ $B_{new}$ ,  $A_{dead}$ }, and the adjacent vertices form a subgraph (i.e., sub  $DG_{[11]} = \{C_{dead}\}$ ). Because no edge exists in sub  $DG_{[11]}$ , Algorithm2 adds the remaining nodes to the candidate maximal Dfc and obtains the first maximal Dfc (i.e.,  $Dfc_{[1]} = \{B_{new}, A_{dead}, C_{dead}\}$ . For the non-adjacent portion of  $A_{dead}$ , the candidate maximal Dfc is  $\{B_{new}\}$ , and no edge exists in  $sub_DG_{[12]} = \{A_{new}\}$ . Similarly, we obtain the second maximal Dfc (i.e.,  $Dfc_{[2]} = \{B_{new}, A_{new}\}$ ). For the non-adjacent vertices (i.e.,  $C_{new}$  and  $B_{dead}$ ) of  $B_{new}$ , the candidate maximal *Dfc* is  $\{\}, C_{new}$  and  $B_{dead}$  are successively treated as  $V_{max'}$ , and their adjacent vertices form subgraphs sub  $DG_{[21]} = \{A_{new}\}$ and sub  $DG_{[22]} = \{A_{dead}\}$ , respectively. Thus, we can obtain

 $Dfc_{[3]} = \{C_{new}, A_{new}\}$  and  $Dfc_{[4]} = \{B_{dead}, A_{dead}\}$ . Finally, we can obtain all maximal Dfc (i.e.,  $\{B_{new}, A_{dead}, C_{dead}\}, \{B_{new}, A_{new}\}, \{C_{new}, A_{new}\}$  and  $\{B_{dead}, A_{dead}\}$ ).

## C. Third Sub-Algorithm

Each maximal Dfc is treated as a candidate maximal Dc (i.e.,  $Dc_{[i]}$ ); therefore, we need to verify whether it is prevalent or not. First, we obtain the dynamic table instance of the candidate maximal  $Dc_{[i]}$ , which can be obtained by the dynamic table-instance of the size-2 prevalent Dc (*Example* 8). Second, we calculate the DPI of the candidate maximal  $Dc_{[i]}$  and compare it with *min\_prev*. Third, if the  $Dc_{[i]}$  is prevalent, we add it to the set of prevalent maximal Dc; otherwise, it is decomposed into a size-k-1 pattern ( $Dc_{[i]}$  is a size-k pattern) (*Example 9*), and each size-k-1 pattern is treated as a candidate maximal Dc and passes through a verification process (except the one that already exists in the set of prevalent maximal Dc).

 $\delta_{clique\_size2\_prevalent}$ =Get\_dynamic\_table\_instance( $Df_{[0]}, Df_{[t]}, \delta_{size2\_prevalent}$ ) 4: common\_instance = Get\_common\_instance\_ $Df_{[0]}$  ( $\delta_{clique\_size2\_prevalent}$ ) 5:  $\delta_{cci}$ =Get\_clique\_dynamic\_instance(common\_instance,  $\delta_{clique\_size2\_prevalent}$ )

6: for (i = 1; i < n - 1; i + +)for (j = i + 1; j < n; j + +)

 $\delta_{cci} = \text{Verifying}(\delta_{cci}, \delta_{clique\_dynamic\_size\ 2}, Df_{[i]}, Df_{[j]})$ 7: If (prevalently( $\delta_{cci}$ ))

- $\delta_{prevalent\_dynamic\_maximal\_Dc} = \delta_{prevalent\_dynamic\_maximal\_Dc} + clique$ 8: Else if (size (clique) > 3)  $\delta_{size\_k-1} = \text{Split} (clique)$
- $\delta_{size\_k-1} = \text{Non\_prevalent}(\delta_{size\_k-1}, \delta_{prevalent\_dynamic\_maximal\_Dc}, \delta_{maximal\_Dfc})$  $\delta_{maximal\_Dfc} = \delta_{maximal\_Dfc} + \delta_{size\_k-1}$

Example 8. Given the maximal Dfc, which is also a candidate maximal Dc (i.e.,  $Dc_{[i]} = \{A_{dead}, B_{new}, C_{dead}\}$ ), we first obtain the common instances from the dynamic table instances of selected size-2 prevalent subpatterns (i.e.,  $Dc_{[i1]} = \{A_{dead}, B_{new}\}$  and  $Dc_{[i2]} = \{A_{dead}, C_{dead}\}$ ). For example, from the dynamic table-instance( $Dc_{[i1]}$ )={{ $A_{dead}$ , 1,  $B_{new}$ , 1},  $\{A_{dead}, 1, B_{new}, 2\}, \{A_{dead}, 2, B_{new}, 1\}\}$  and the dynamic tableinstance $(Dc_{[i2]}) = \{ \{A_{dead}, 1, C_{dead}, 2\} \}$ , we can obtain the common instances (i.e., A<sub>dead</sub>.1) of Dc<sub>[i1]</sub> and Dc<sub>[i2]</sub>. Second, with the common dynamic instances, we select the dynamic row-instances of the size-2 prevalent subpatterns to construct the candidate dynamic row-instances of  $Dc_{[i]}$ . For example, with  $A_{dead}$ .1, we select the dynamic row-instances set  $\{\{A_{dead}, 1, B_{new}, 1\}, \{A_{dead}, 1, B_{new}, 2\}\}$  and  $\{\{A_{dead}, 1, B_{new}, 2\}\}$  $C_{dead}$ , 2}} and subsequently construct the dynamic rowinstances of  $Dc_{[i]}$  (i.e., { $A_{dead}$ .1,  $B_{new}$ .1,  $C_{dead}$ .2} and { $A_{dead}$ .1,  $B_{new}$ .2,  $C_{dead}$ .2}. Third, we verify the dynamic row-instances by the other size-2 prevalent subpattern (i.e.,  $Dc_{[i3]} = \{B_{new}, \}$  $C_{dead}$ . For example, because only the dynamic row-instance  $\{B_{new}, 1, C_{dead}, 2\}$  exists in the dynamic table-instance of  $Dc_{[i3]}$ while another dynamic row-instance  $\{B_{new}, 2, C_{dead}, 2\}$  does not exist, only { $A_{dead}$ .1,  $B_{new}$ .1,  $C_{dead}$ .2} is a real dynamic

**Algorithm 3:** Verifying Prevalent Maximal Spatial Co-Location Pattern **Input:** (1) $\delta_{maximal_Dfc}$ : the set of maximal Dfc which is regarded as candidate prevalent maximal Dc;(2)min\_prev: minimum prevalent threshold;(3) $\delta_{size_2-prevalent}$ : size-2 prevalent Dc.

**Output:**  $\delta_{prevalent\_dynamic\_maximal\_Dc}$ : the set of prevalent Dc.

**Variables:** (1) clique: one maximal  $Dfc;(2)Df_{[n]}$ : a set of dynamic features in clique; (3) $\delta_{clique}$  size 2 prevalent: a set of dynamic table instances of size-2 prevalent Dc which contains  $Df_{[0]};(4)$  common\_instance: common instances of  $Df_{[0]}$  in size-2 prevalent Dc which contains  $Df_{[0]};(5)\delta_{cci}$ : validated dynamic table instance of  $Dfc;(6)\delta_{size_k-1}$ : a set of size-k-1 subpatterns of Dfc.

While (not\_empty( $\delta_{maximal_Dfc}$ ))

<sup>1:</sup> *clique* = Get\_one\_clique( $\delta_{maximal_Dfc}$ )

<sup>2:</sup> *Df*<sub>[n]</sub> = Get\_dynamic\_feature(*clique*)

<sup>3:</sup> for (i = 1; i < n; i + +)

row-instance of  $Dc_{[i]}$ , and thus we obtain the dynamic tableinstance( $Dc_{[i]}$ )={ $\{A_{dead}.1, B_{new}.1, C_{dead}.2\}$ }.

**Example 9.** Supposing that a size-4 candidate Dc (i.e.,  $Dc_{[i]}=\{A_{dead}, B_{new}, C_{dead}, D_{new}\}$ ) is not prevalent, we divide the Dc into size-3 candidate Dc (i.e.,  $\{A_{dead}, B_{new}, C_{dead}\}$ ,  $\{A_{dead}, B_{new}, D_{new}\}$ ,  $\{A_{dead}, C_{dead}, D_{new}\}$  and  $\{B_{new}, C_{dead}, D_{new}\}$ . If candidate pattern  $\{A_{dead}, B_{new}, C_{dead}\}$  already exists in the set of prevalent maximal Dc, then it is deleted from the set of subpatterns. We add the size-3 candidate patterns  $\{A_{dead}, B_{new}, C_{dead}, D_{new}\}$  to the set of candidate Dc.

## V. PRUNING STRATEGIES

**Theorem 1.** For a dynamic feature  $Df_{[j]}$  in the candidate prevalent maximal  $Dc_{[i]}$ , if  $DPR(Dc_{[i]}, Df_{[j]}) \le min\_prev$ , then  $Dc_{[i]}$  is non-prevalent.

*Proof.* From definition 7,  $DPI(Dc_{[i]})$  is the minimum value among dynamic participation ratios of all dynamic features in  $Dc_{[i]}$ . Therefore, if  $DPR(Dc_{[i]}, Df_{[j]}) < min_prev$ , then  $DPI(Dc_{[i]}) \leq DPR(Dc_{[i]}, Df_{[j]}) \leq min_prev$ , namely,  $Dc_{[i]}$  is non-prevalent.

**Pruning Strategy 1.** During verification of a candidate prevalent maximal *Dc*, if the dynamic participation ratio of any dynamic feature is smaller than *min\_prev*, then we can stop verification and confirm that the candidate maximal *Dc* is nonprevalent.

**Example 10.** During verification of the size-4 candidate prevalent maximal Dc (i.e.,  $Dc_{[i]}=\{A_{dead}, B_{new}, C_{dead}, D_{new}\}$ ), we can calculate the dynamic participation ratio of any dynamic feature in the process of obtaining the common instances of  $A_{dead}$  and verifying the dynamic row-instance of  $Dc_{[i]}$ . If  $DPR(Dc_{[i]}, A_{dead}) < min_prev$ , then we can stop verification and confirm that  $Dc_{[i]}$  is non-prevalent.

**Theorem 2.** Given a candidate prevalent maximal  $Dc_{[i]}$  and its superpattern  $Dc_{[i]}'$  (i.e.,  $Dc_{[i]} \subseteq Dc_{[i]}'$  and  $Dc_{[i]} \neq Dc_{[i]}'$ ), if  $Dc_{[i]}$  is non-prevalent, then  $Dc_{[i]}'$  is also non-prevalent.

*Proof.* Pattern  $Dc_{[i]}$  is non-prevalent, and we know that a dynamic feature  $Df_{[i]} \in Dc_{[i]}$  exists and  $DPR(Dc_{[i]}, Df_{[j]}) < min_prev$ . Therefore, for its superpattern  $Dc_{[i]}'$  (i.e.,  $Dc_{[i]} \subseteq Dc_{[i]}'$  and  $Dc_{[i]} \neq Dc_{[i]}'$ ), the existence of inheritance leads to  $DPR(Dc_{[i]}', Df_{[i]}) \le DPR(Dc_{[i]}, Df_{[i]}) \le min_prev$ , and thus  $Dc_{[i]}'$  is non-prevalent.

**Pruning Strategy 2.** If multiple candidate prevalent maximal dynamic spatial co-location patterns have a common subpattern, we can first verify their common subpattern, and if the common subpattern is non-prevalent, all candidate prevalent maximal spatial co-location patterns are non-prevalent.

**Example 11.** Given a candidate prevalent maximal dynamic spatial co-location patterns  $Dc_{[i]} = \{A_{dead}, B_{new}, C_{dead}, D_{new}\}$  and  $Dc_{[j]} = \{A_{dead}, B_{new}, C_{dead}, E_{dead}\}$ , we can first verify the common subpattern  $Dc_{[i]} = \{A_{dead}, B_{new}, C_{dead}\}$ , and if  $Dc_{[i]}$  is non-prevalent, then both  $Dc_{[i]}$  and  $Dc_{[j]}$  are non-prevalent. If  $Dc_{[ij]}$  is prevalent, then we verify the other portions of  $Dc_{[i]}$  and  $Dc_{[j]}$ .

#### VI. COMPLEXITY ANALYSIS

To analyze the complexity, the upper limits of certain parameters will need to be determined. For the original datasets (i.e., n time points), the number of instances at each time point is no more than I. For the dynamic datasets (i.e., n-1 time points), the number of dynamic instances in each dynamic dataset is I', and there are 2\*F dynamic features

(new/dead). For any dynamic feature, its dynamic instances and *time span constraint* are no more than i and n, respectively.

## A. Time Complexity

In Algorithm1, from step 1 to step 5, the corresponding complexities are  $O(I^*I^*(n-1)), O(I'^{2*}(n-1)^2), O(I'^{2*}(n-1)^{2/2}),$  $O(F^*F)$  and  $O(F^*F)$ , and thus the time complexity of Algorithm1 is  $O(I^{2*n})+O(I^{2*n^2})$ , where F is much less than I, so selected portions have been omitted. In Algorithm2, for any dynamic feature  $Df_{[i]}$ , the number of dynamic feature cliques that are related to  $Df_{[i]}$  is no more than F, and there are 2\*F dynamic features. Therefore, the number of dynamic feature cliques is no more than  $F^{*2}F$ . Moreover, from the literature [45], the time complexity of obtaining a maximum clique is  $O(1.442^{F})$ , and thus the time complexity of Algorithm2 is  $O(1.442^{F*}F^2)$ . In Algorithm3, the number of dynamic feature cliques is  $F^{*2}F$  (from Algorithm2), any dynamic feature clique is decomposed once (on average), and therefore, its number is no more than  $F^{*}2^{*}F^{*}F$  after decomposition. Moreover, for one dynamic feature clique, the time complexity of verification is  $O(F^2)$ , so the time complexity of Algorithm3 is  $O(F^5)$ . Therefore, the time complexity of Algorithm MDC is  $O(I^{2}*n) + O(I^{2}*n^{2}) +$  $O(1.442^{F}*F^{2}) + O(F^{5}).$ 

## **B.** Space Complexity

The space complexity values of the storing instances, dynamic features, dynamic instances, adjacent instance set, dynamic table-instances, dynamic feature graph, dynamic feature cliques, common code and decomposed maximal cliques are  $O(I^*n)$ ,  $O(2^*F)$ ,  $O(I'^*(n-1))$ ,  $O(I'^2*(n-1)^2/2)$ ,  $O(I'^2*(n-1)^2/2)$ ,  $O(F^*F)$ ,  $O(F^*2^*F)$ , O(i) and O(F), respectively. Moreover, during searching of the dynamic feature clique, the space complexity is  $O(F^2)$ . Therefore, the space complexity of Algorithm MDC is  $O(I^*n) + O(I'^2*n^2) + O(F^*2^*F)$ , where I', I and F are much less than I,  $I'^*n$  and I, respectively, and thus certain portions have been omitted.

#### VII. EXPERIMENTAL EVALUATION

Various experiments over both real and synthetic datasets were conducted to verify the effectiveness of Algorithm MDC (i.e., we can find the dynamic relationships among spatial features), the representativeness of the prevalent maximal Dc, the efficiency of Algorithm MDC (i.e., comparison with the join-based algorithm), and the efficiency of the two pruning strategies.

## A. Experiments on Real Dataset

In this section, we verify the effectiveness of the Algorithm MDC, namely, whether the Algorithm MDC can mine the dynamic relationships among spatial features (i.e., Dc) from the real datasets.

The real dataset is sourced from the Wuhua district of Kunming, Yunnan province, China, in the most recent 30 years. Specifically, Df= {"School", "Park", "Hospital", "Hotel", "Supermarket", "KTV", "Bank"}, where "Bank" includes bank business halls and ATMs, and "Hospital" includes clinics, pharmacies, etc. The *life cycle* of all dynamic feature is {30,30,15,9,6,6,6}, the number of new/dead instances is approximately 1500, and the *time span* is 3 years. If  $D_d$  is 1 km and *min\_prev* is 0.4, we can obtain all prevalent Dc as shown in Table II, which can be derived from the prevalent maximal Dc.

## TABLE II

PREVALENT DYNAMIC SPATIAL CO-LOCATION PATTERNS

	$A_{new}, B_{new}$	$A_{dead}, B_{dead}$	$A_{new}, B_{dead}$
size-2	{school <sub>new</sub> , supermarket <sub>new</sub> }	$\{school_{dead}, bank_{dead}\}$	{hotel <sub>dead</sub> , supermarket <sub>new</sub> }
	{school <sub>new</sub> , bank <sub>new</sub> }	{hospital <sub>dead</sub> , supermarket <sub>dead</sub> }	{supermarket <sub>new</sub> , KTV <sub>dead</sub> }
	$\{school_{new}, KTV_{new}\}$	{ $hotel_{dead}, bank_{dead}$ }	{ $hospital_{new}, KTV_{dead}$ }
	$\{KTV_{new}, bank_{new}\}$	{ $hotel_{dead}, KTV_{dead}$ }	$\{school_{new}, KTV_{dead}\}$
	{hospital <sub>new</sub> , bank <sub>new</sub> }	$\{KTV_{dead}, bank_{dead}\}$	
	{hospital <sub>new</sub> , supermarket <sub>new</sub> }		
	{ $hotel_{new}, bank_{new}$ }		
	{ $hotel_{new}, KTV_{new}$ }		
	$\{part_{new}, hotel_{new}\}$		
size-3	{ $hotel_{new}, KTV_{new}, bank_{new}$ }	$\{hotel_{dead}, KTV_{dead}, bank_{dead}\}$	{hotel <sub>dead</sub> , supermarket <sub>new</sub> , $KTV_{dead}$ }

Traditional methods can obtain the prevalent pattern {"School", "Park", "Hospital", "Hotel", "Supermarket", "KTV", "Bank"} at each time point (i.e.,  $t_0, t_1, t_2, \ldots, t_n$ ) when  $D_d=1$  km and min\_prev=0.4, namely, all features are always coexistent at each time point, and the result is not meaningful. In contrast, from the experimental results of Algorithm MDC in Table II, we can obtain the following meaningful information:

 The instances of "Bank" increase (or decrease) with the increase (or decrease) of the instances of "School", "Hotel", "Hospital" and "KTV", which means that Algorithm MDC can find the dynamic relationships among spatial features such as {water\_hyacinthnew(increase), algaedead (decrease)}.

2) Life service (e.g., "*Hospital*" and "*Supermarket*") has a mutual exclusion relationship with entertainment (e.g., "*KTV*"), namely, the instances of "*Hospital*" and "*Supermarket*" increase (or decrease) with the decrease (or increase) of the instances of "*KTV*", which represents the adjustment of an urban regional structure. This result means that Algorithm MDC can find the dynamic relationships among spatial features such as {*Ganoderma\_lucidum<sub>new</sub>*, *maple\_treedead*}.

3) The instances of "Hotel", "KTV" and "Bank" always appear/disappear simultaneously, which indicates that they have strong symbiotic relationships, and they reflect the economic prosperity/recession in this region because they denote the level of regional economic development. This result means that Algorithm MDC can effectively avoid the wrong conclusion that the instances of two or more features increase/decrease (i.e., new/dead) in the same/approximate proportion, which has no effect on prevalent patterns.

In conclusion, Algorithm MDC can obtain certain meaningful patterns such as {*water\_hyacinth<sub>new</sub>*(increase), *algae<sub>dead</sub>*(decrease)} and {*Ganoderma\_lucidum<sub>new</sub>*, *maple\_tree<sub>dead</sub>*}, and avoid the wrong conclusion that the instances of two or more features increase/decrease (i.e., new/dead) in the same/approximate proportion, which has no effect on prevalent patterns."

## B. Experiments on Synthetic Datasets

In this section, we examine the representation of the prevalent maximal Dc for all prevalent Dc, the efficiency of Algorithm MDC, and the performance of the pruning strategies.

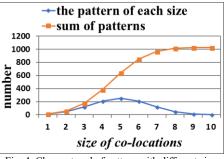
We randomly generate synthetic datasets, where the *time span* is 3, there are 11 time points, and the distribution area of spatial instances is 1000\*1000. By default, the number of dynamic instances and dynamic features are 10000 and 10,

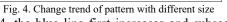
respectively; the *life cycle* of all dynamic features is  $\{9,3,30,15,27,24,30,3,24,18\}$ ; and *min\_prev* and  $D_d$  are 0.1 and 35, respectively.

1) Change Trend of Maximal Dc and Dc

We analyze the change trend of the number of size-k ( $k \in [1,10]$ ) patterns (i.e., the pattern of each size) and the sum of the number of patterns from size-1 to size-k (i.e., sum of patterns), as shown in Fig. 4.

Furthermore, the change trend of the number of maximal Dc and that of Dc are approximate to the blue line (the pattern of each size) and red line (i.e., sum of patterns), respectively. Suppose both of the size-k patterns and its lowsize patterns are prevalent, and the size-k+1 patterns are nonprevalent. On the one hand, the subsets of the size-k patterns are prevalent and its superset are non-prevalent at this time, and by the definition of the maximal Dc, the size-k patterns are maximal prevalent patterns, which leads to the observation that the change trend of the number of prevalent maximal Dc is approximate to that of the size-k patterns (i.e., the pattern of each size (blue line)). On the other hand, all prevalent patterns include patterns from size-1 to size-k, which leads to the observation that the change trend of the number of prevalent Dc is approximate to the red line (i.e., sum of patterns). Finally, although the analysis of change trend starts from the supposed premise, its real change trend is actually approximate to the lines shown in Fig. 4.





In Fig. 4, the blue line first increases and subsequently decreases, which represents the change trend of the number of prevalent maximal Dc mined by our method. The red line continues to increase, which represents the change trend of the number of prevalent Dc mined by traditional methods. First, with the increase of size-k, the gap between the number of prevalent maximal Dc and that of prevalent Dc becomes larger, which means that mining the prevalent maximal Dc is more efficient than mining the prevalent Dc. Moreover, with the increase of size-k, the number of prevalent Dc significantly increases, which leads to the observation that the execution time of the traditional method is unacceptable,

and thus we only display the complete trend (i.e., the prevalent maximal Dc first increases and subsequently decreases) in Fig. 5(a), which is treated as a sample example and partial trend (i.e., the prevalent maximal Dc increases) in other comparison experiments.

## 2) Representativeness of Prevalent Maximal Dc

We compare the number of prevalent maximal Dc with the number of prevalent Dc over the change in *number of dynamic instances*,  $D_d$ , *min\_prev*, and *number of dynamic features*, as shown in Table III, Table IV, Table V and Table VI (Comparisons in Fig. 5 is more intuitive). With the increase in *number of dynamic instances*, the numbers of prevalent maximal Dc and prevalent Dc increase, which lead to the increase of size-k of the prevalent maximal Dc and prevalent maximal Dc first increases and subsequently decreases and that of the prevalent Dc continues to increase, as shown in Fig. 5(a) (or Table III) and Fig. 4.

TABLE III

REPRESENTATIVENES	S OF VA	ARIED N	UMBER	OF DY	NAMIC	INSTAN	CES
dynamic instances	5k	6k	7k	8k	9k	10k	111
prevalent maximal Dc	99	131	154	191	162	142	99
prevalent Dc	193	270	414	580	670	720	870

TABLE IV						
REPRESENTATIVENESS OF VARIED $Dd$						
Dd	15	20	25	30	35	
prevalent maximal Dc	44	73	115	153	170	
prevalent Dc	44	101	214	376	583	

	TAB	LE V			
REPRESENTAT	IVENES	SS OF V	ARIED	min pre	ev
min prev	0.25	0.20	0.15	0.10	0.05
prevalent maximal Dc	93	104	116	126	139
prevalent Dc	306	376	460	583	767

TABLE VI

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RESENTATIVENESS C	F VARIE	D NUMB	ER OF DY	NAMIC F	EATURE
amic features	9	10	11	12	13
alent maximal Dc	67	152	316	658	1096
valent Dc	416	752	1070	2059	3349
arent De	410	152	1070	2057	5.

Because the execution time of the traditional method is unacceptable, we only display a partial trend (i.e., the prevalent maximal Dc increases) in other comparison experiments (more detailed information is given in the previous section). From Fig. 5(b)(c)(d) (or Table IV, Table V and Table VI), on the one hand, the number of prevalent maximal Dc is far less than the number of all prevalent Dc, which means that mining the prevalent maximal Dc by our method is more efficient than mining the prevalent Dc by traditional methods. On the other hand, with the change of  $D_d$ , min\_prev and number of dynamic features, the gap between the number of prevalent maximal Dc and that of prevalent Dc becomes larger, and the difference in efficiency between mining the prevalent maximal Dc by our method and mining the prevalent Dc by the traditional method is increasingly obvious.

# 3) Efficiency of Algorithm MDC

We compare the running times for mining the prevalent Dc by Algorithm MDC and the join-based algorithm [15,16] over the change in *number of dynamic instances*,  $D_d$ , *min\_prev* and *number of dynamic features*, as shown in Fig. 6. Because the number of prevalent Dc is small on the original parameters, the advantage of Algorithm MDC is not obvious compared with traditional method. Moreover, with the change in these parameters, the number of prevalent Dc increases, and the difference in efficiency between mining the prevalent maximal Dc by our method and mining the prevalent Dc by the traditional method is becomes more obvious.

## 4) Performance of Pruning Strategies

We compare the efficiency before and after pruning via pruning strategies 1 and 2, which can effectively accelerate the process of mining the prevalent maximal Dc by Algorithm MDC, as shown in Fig. 7.

#### VIII. CONCLUSION

Because the existing methods cannot mine the dynamic relationships among spatial features, and the number of prevalent patterns is too large, this paper proposes 1) the definition of a dynamic spatial co-location pattern, 2) the idea of mining the prevalent maximal patterns instead of all prevalent patterns (the former can be used to derive the latter), and 3) an algorithm for mining the maximal dynamic spatial co-location patterns (i.e., Algorithm MDC) based on the maximal dynamic feature clique.

Mining the dynamic spatial co-location pattern can remedy the defects of the existing methods. The existing methods miss certain meaningful patterns, such as {Ganoderma lucidum<sub>new</sub>, *maple tree<sub>dead</sub>*} and {*water\_hyacinth<sub>new</sub>*(increase), *algae*<sub>dead</sub>(decrease)}, and obtain the wrong conclusion that the instances of two or more features increase/decrease (i.e., new/dead) in the same/approximate proportion, which has no effect on prevalent patterns. Therefore, we propose the dynamic spatial co-location pattern Dc, which can reflect the dynamic relationships among spatial features similar to the above three types of dynamic changes.

Compared with mining the prevalent Dc, mining the prevalent maximal Dc that can derive all prevalent Dc is more efficient. The number of prevalent patterns is large, which makes the efficiency of the existing methods low. Therefore, we introduce the prevalent maximal pattern into the process of mining the prevalent Dc because the prevalent maximal patterns are compact representations of all

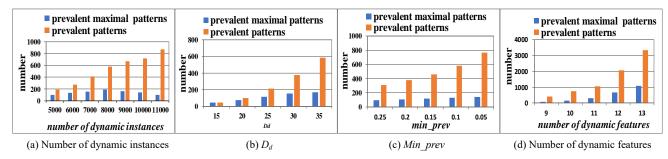


Fig. 5. Representativeness of prevalent maximal Dc

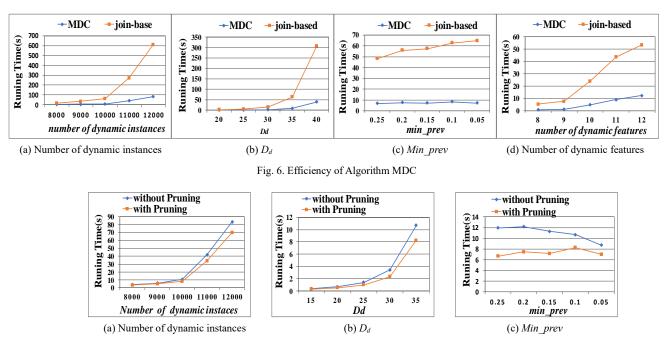


Fig. 7. Performance of pruning strategies

prevalent patterns and can be used to derive all prevalent patterns. The gap between the number of prevalent maximal Dc and that of the prevalent Dc is large, and thus the difference in efficiency between mining the prevalent maximal Dc by our method and mining the prevalent Dc by the traditional method is obvious.

We propose an algorithm (i.e., Algorithm MDC) for mining the prevalent maximal Dc to avoid the many connections and computations in the existing methods. We convert the size-2 prevalent patterns into a dynamic feature graph (DG) by Algorithm1 such that the calculation and connection for the table instances are turned into the calculation and connection of the dynamic features. We obtain the set of maximal dynamic feature clique Dfc from the dynamic feature graph DG by Algorithm2. Finally, the maximal dynamic feature cliques as candidate maximal dynamic spatial co-location patterns are verified by Algorithm3, and we can obtain the prevalent maximal Dc. Moreover, we propose two pruning strategies to improve the efficiency of Algorithm MDC.

The experimental results from a real dataset and a synthetic dataset show that our algorithm can effectively mine the prevalent maximal Dc, that the number of prevalent maximal Dc is much less than the number of all prevalent Dc and that the performance of Algorithm MDC is better than the join-based [15,16].

The biggest limitation of our method is that parameters are designated by domain experts, such as *Dd*, *min\_prev*, *time\_span*, which is the common problem of mining spatial co-location pattern and is also the future research directions: 1) parameters might be learned from the dataset that can reduce the subjectively of the parameter designated by experts as much as possible; 2) methods for how to set a more reasonable *time\_span* can be considered to maximize the value/meaning of the dynamic spatial co-location patterns; 3) more efficient approaches to mining the prevalent maximal dynamic spatial co-location patterns can be designed. Our study also opens the door to exploring the dynamic relationships among spatial features.

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