Event-Triggered Observer-Based $\mathcal{H}_\infty$ Consensus Control and Fault Detection of Multiagent Systems Under Stochastic False Data Injection Attacks

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***Abstract*—This paper investigates the event-triggered observer-based security consensus and fault detection problem for nonlinear multi-agent systems (MASs) under external disturbances and stochastic false data injection attacks (FDIAs) over a directed communication network. The randomly occurring FDIAs are modeled by random variables that follow the Bernoulli distribution. An observer-based event-triggered control strategy using only local measurements and information from neighboring agents is developed, where the Zeno behavior of event-triggered mechanism (ETM) is excluded. Interestingly, the observer errors are first regarded as disturbance and then attenuated by $\mathcal{H}_\infty$ norm bounds, together with the external disturbances. Meanwhile, it is worth highlighting here that the same information used by the state observers is also adopted to construct residuals with adaptive thresholds, whose aim is to detect faults occurring in any agents. In addition, the accuracy of the observer and the performance of the fault detection mechanism are improved by introducing the disturbance compensation mechanism. Finally, simulation results are provided to illustrate the effectiveness and advantages of the proposed strategy.**

***Index Terms*—Observer-based anti-disturbance control, event-triggered mechanism (ETM), fault detection mechanism, multi-agent systems (MASs), false data injection attacks (FDIAs).**

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I. Introduction

Over the past few years, the cooperative control of multi-agent systems (MASs) has become one of the important research directions in the control field due to their speedy developments in many areas, such as intelligent transportation, multiple robot systems, unmanned marine vehicles, and many industrial facilities [1]–[4]. However, with the increase in the amount of information transmitted by agents and the open network environment, MASs are seriously threatened by cyber attacks, which destroy the integrity and authenticity of information [5]. As a result, the performance of MAS is deteriorating or even becoming unstable. So, it is very important to solve the network security problem of MASs.

In multi-agent networks, the main types of attacks include denial-of-service (DoS) attacks, false data injection attacks (FDIAs), replay attacks, zero dynamic attacks, and so on [6], [7]. In recent years, the security problem of MAS subject to DoS attacks has received extensive attention and achieved a lot of results due to the fact that such attacks are easy to implement and highly destructive [8], [9]. Compared with DoS attacks, FDIAs destroy the integrity and accuracy of the data by tampering with the transmitted information, and have stronger concealment [10]. However, research on the security of MAS under FDIA is still in the initial stage [11]–[17]. In [11], [12], a control strategy with an adaptive attack compensator is proposed to ensure the uniformly ultimately boundedness of MAS under FDIA. It should be pointed out that the attack model considered by these results continues to act on the agent after the attack, which provides convenience for the construction of attack compensation mechanism. However, in practice, since the attacker needs to save energy to launch the next attack, it is impossible for the attacker to launch the attack continuously. To better reflect the reality, the model of FDIA that obeys Bernoulli distribution is considered in [13]–[15], and a distributed impulsive controller is proposed to deal with the control signals attacked by FDIA to ensure that the MAS can achieve mean-square bounded synchronization [14], [15]. In these theoretical research, it is generally assumed that the state information of the system is measurable. However, in practical applications, the state information is unmeasurable, and only the output information can be obtained [18]–[20]. On the other hand, the continuous control protocol will inevitably generate redundant data and cause network congestion. To overcome this difficulty, the event-triggered strategy has received widespread attention [21]–[23]. However, there are relatively few research results that consider the use of event-triggered strategy to solve the security problems of MAS under FDIA [5], [16], [17]. Furthermore, it is noteworthy that most of these results are based on undirected communication topology and do not take into account the impact of disturbance. Due to that the symmetry property of the Laplacian matrix is no longer valid in directed network topology, and disturbance is also one of the main factors affecting the performance of MASs [23], [24], as a result, these results will not be applicable in the case of directed topology and disturbance. Hence, under the directed topology graph, how to design an observer-based event-triggered control strategy when the MAS is subjected to external disturbance and FDIA is the core issue to be solved in this paper.

Additionally, as is well known that due to the interconnection among MASs, serious performance degradation or even instability may occur regardless of whether there is a fault in a single agent or multiple agents, so the research on fault detection of MAS has attracted increasing attention [25]–[29]. For linear MAS, a mixed $\mathcal{H}_-/\mathcal{H}_\infty$ distributed simultaneous fault detection and consensus control strategy based on linear matrix inequality (LMI) method are proposed in [25]. In [26], a fixed-time observer is designed to solve the fault detection problem for nonlinear MAS. In [27], a bank of interval observers are proposed to construct the distributed fault detection and isolation strategy, while the upper and lower bounds of faults need to be known. However, it is remarkable that the results in [25]–[27] are studied in a healthy network environment only. Since faults are usually detected through the cooperation mechanism of MAS based on the communication network, it is essential to study the fault detection mechanism of MAS under cyber attacks. In [28], a co-design of fault detection and consensus of MASs under hidden DoS attack is investigated, and a mixed $\mathcal{H}_-/\mathcal{H}_\infty$ performance index is established to balance the fault detection performance and consensus performance. Nevertheless, the above results use a fixed threshold in the fault detection logic, in which case, false alarms may occur when there is a non-zero initial observation error, measurement noise, or disturbance. To overcome this shortcoming, a distributed fault detection and isolation strategy for networked multi-robot systems based on local observers is proposed in [29]. Meanwhile, the same information (local and received from neighboring agents) used by the local observer is also used to construct the residual and derive the adaptive threshold. However, when the disturbance is large, the adaptive threshold will also become large, which will lead to the invalid of the fault detection mechanism. Therefore, it is more appealing for us to develop a feasible fault detection mechanism for MAS under disturbance and FDIA.

As pointed out above, the main difficulties in the current research are as follows: a) In a directed communication network, how to design an observer-based event-triggered control strategy so that nonlinear MAS can achieve consensus and reduce the wastage of communication resources in the presence of disturbance and FDIAs, and b) How to design a feasible fault detection mechanism to reduce false alarms and non-alarms when the nonlinear MAS is subjected to disturbance and FDIA. To overcome these difficulties, this paper studies the security consensus problem and fault detection problem for nonlinear MAS under disturbance and FDIA. Moreover, the highlights of this paper are as follows:

1) Event-triggered state observer for system state estimation: Taking into account the effects of external disturbance and stochastic FDIA, a state observer is designed to remove the assumption that the states of the agent are measurable as in [13]–[15]. In addition, it is worth mentioning that the state observation errors are first regarded as disturbance, and then attenuated by $\mathcal{H}_\infty$ norm bounds, together with the external disturbances.
2) Novel event-triggered mechanism (ETM): Contrary to the centralized ETM developed by [30], [31], the proposed ETM updates the control input at its own trigger time. Moreover, compared with the distributed ETM in [16] where the trigger condition has a constant threshold, a state-dependent threshold is used in the proposed trigger condition. Furthermore, an exponential term ℓe^{-t} as in (26) is introduced to make easier the avoidance of Zeno behavior.
3) Novel fault detection mechanism based on an adaptive threshold: In order to reduce the computational burden, the same information used in the state observer is also adopted to construct the residual. Compared with the fault detection logic using a fixed threshold in [25]–[28], an adaptive threshold is adopted to reduce the false alarm and non-alarm rates.
4) Disturbance compensation mechanism for observer and fault detection mechanism: Different from the method of using disturbance observer in [32], the proposed mechanism can reduce the difficulties caused by the unmeasurable states and improve the accuracy of the state observer. Note that, in the case of consensus analysis of multi-agent systems, an adaptive mechanism is adopted to estimate the upper bounds of the external disturbances, while in the case of fault detection, these upper bounds are required to be known in advance as in [27], [33], [34]. In addition, it is worth mentioning that the problem of large adaptive threshold caused by large external disturbance and causing non-alarms as in [29] can be reduced by adding a disturbance compensation mechanism, and thus improving the performance of fault detection.

Notations: $\mathbb{R}^n$ and $\mathbb{R}^{N\times N}$ denote n-dimensional Euclidean space and $N\times N$ real matrices, respectively. Let I_N ($\mathbf{I}_N$) be the $N\times N$ dimension identity matrix and $\mathbf{0}_N$ be the $N\times N$ dimension zero matrix. 1_N is a N-dimensional column vector of all 1. For a matrix M, M^T denotes its transpose, and $\mathrm{He}(M) = M + M^T$. For a symmetric matrix M, $\lambda_{\max}(M)$

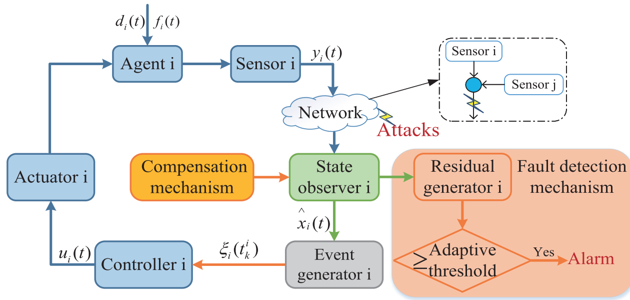


Fig. 1. Framework of the ith agent under false data injection attacks.

($\lambda_{\min}(M)$) is the maximum (minimum) eigenvalue of M; $M > 0$ means that M is a symmetric positive definite matrix. The symbols $\otimes$, $\|\cdot\|$, and $*$ respectively represent Kronecker product, Euclidean norm and the symmetric entries of a matrix. Define $diag\{\cdot\}$ as a diagonal matrix with the given diagonal entries and $col\{x_1,\ldots,x_N\} = [x_1^T,\ldots,x_N^T]^T$. The mathematical expectation of a stochastic variable is denoted as $\mathbb{E}\{\cdot\}$.

II. Problem Statement and Preliminaries

As shown in Fig. 1, the MAS configuration considered in this paper consists of sensors, state observers, ETM, controllers, actuators, and the fault detection mechanism, in which the communication network between agents may be subject to FDIAs [14], [15]. Besides, when a fault happens in an agent, the fault detection mechanism is to detect the fault promptly.

A. Graph Theory

The communication topology among the N followers of the MAS can be modeled by a directed graph $\mathcal{G} = (\mathcal{V},\mathcal{E},\mathcal{A})$, in which $\mathcal{V} = \{1,\ldots,N\}$, $\mathcal{E} \subseteq \mathcal{V}\times\mathcal{V}$ and $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ represent the node set, the edge set and the adjacency matrix, respectively. If $a_{ij} = 1$, it means that node i can receive information from node j through directed edge $\mathcal{E}_{ij}$ and node j is a neighbor of node i; otherwise, $a_{ij} = 0$. Let $\mathcal{N}_i = \{j \in \mathcal{V} : \mathcal{E}_{ij} \in \mathcal{E}\}$ denote the in-neighbor set of node i and assume that there is no self-loop in graph $\mathcal{G}$, i.e., $a_{ii} = 0$. Moreover, the matrix $\mathcal{D} = \mathrm{diag}\{d_1,\ldots,d_N\} \in \mathbb{R}^{N\times N}$ with $d_i = \sum_{j\in\mathcal{N}_i} a_{ij}$ is the degree matrix and the Laplacian matrix $\mathcal{L}$ is given as $\mathcal{L} = \mathcal{D} - \mathcal{A}$. For the graph $\bar{\mathcal{G}} = (\bar{\mathcal{V}},\bar{\mathcal{E}},\bar{\mathcal{A}})$ with a leader 0, define $\mathcal{A}_0 = \mathrm{diag}\{a_{10},\ldots,a_{N0}\}$ as the leader's adjacency matrix, where $a_{i0} = 1$ if the ith agent can receive leader's information, else $a_{i0} = 0$. Then, its Laplacian matrix $\bar{\mathcal{L}}$ can be expressed as $\bar{\mathcal{L}} = \begin{bmatrix} 0 & 0_{1\times N} \\ \mathcal{L}_2 & \mathcal{L}_1 \end{bmatrix}$, where $\mathcal{L}_1 = \mathcal{L} + \mathcal{A}_0$ and $\mathcal{L}_2 = -col\{a_{10},\ldots,a_{N0}\}$.

Assumption 1 ([9]): The graph $\bar{\mathcal{G}}$ has a directed spanning tree, and the leader is the root node.

B. System Model

Consider a multi-agent system with N followers, and the ith follower is characterized by

$$\begin{cases} \dot{x}_i(t) = Ax_i(t) + Bu_i(t) + Dd_i(t) + \phi(x_i(t)), \\ y_i(t) = Cx_i(t),\ i = 1,\ldots,N, \end{cases} \tag{1}$$

where $x_i(t) \in \mathbb{R}^n$, $u_i(t) \in \mathbb{R}^m$, and $y_i(t) \in \mathbb{R}^p$ represent the state, input, and output of the ith agent, respectively. $d_i(t)$ is the external disturbance which is bounded to $\bar{d}_i$, i.e., $\|d_i(t)\| \le \bar{d}_i$, and $\phi(x_i) = [\phi_1(x_i),\ldots,\phi_n(x_i)]^T$ is a nonlinear function vector. Besides, A, B, C, and D are constant matrices of commensurate dimensions. Assume that (A,B) is stabilizable and (A,C) is detectable.

The dynamics of the leader is described as

$$\begin{cases} \dot{x}_0(t) = Ax_0(t) + \phi(x_0(t)), \\ y_0(t) = Cx_0(t), \end{cases} \tag{2}$$

where $x_0(t) \in \mathbb{R}^n$ and $y_0(t) \in \mathbb{R}^p$ are the state and the output of the leader, and $\phi(x_0)$ denotes the nonlinear dynamics of the leader.

Assumption 2: For any $x_1(t), x_2(t) \in \mathbb{R}^n$, the nonlinear function $\phi(x_i)$ satisfies

$$\|\phi(x_2) - \phi(x_1)\| \le \|\Lambda(x_2 - x_1)\|, \tag{3}$$

where Λ is a symmetric positive definite matrix.

Assumption 3: The initial state of the agents $x(0) = \mathrm{col}\{x_1(0),\ldots,x_N(0)\}$ is unknown but bounded as $\|x(0)\| \le \Delta$, where Δ is a known positive constant.

Remark 1: Assumption 2 is called the Lipschitz condition and can be used to describe a class of nonlinear systems, such as complex circuits, robotic manipulators, and so on [35]. Assumption 3 is a common assumption in fault detection based on adaptive threshold method [33], [36] and, for example, it can indicate that the robots start from a known bounded region [29].

C. False Data Injection Attack Model

Since MASs are exposed to an open network environment, information transmission among agents is vulnerable to FDIAs, which leads to the destruction of the integrity and accuracy of the transmitted information. For example, for a large-scale smart grid system composed of N grid subregions, it is necessary to monitor the system in order to ensure its reliable operation, and state estimation is generally used for system monitoring by analyzing the measured values of meters and the power model to estimate the state of the power grid. However, FDIAs may destroy the measured value of the meter in transmission and change the result of state estimation, thus interfering with the normal operation of the smart grid [37]. To make the FDIA model consistent with the reality, let independent stochastic variables $\gamma_{ij}(t)$ indicate whether the attacker launches an attack on the transmission of sensor information between agents i and j, i.e.,

$$\gamma_{ij}(t) = \begin{cases} 1, & \text{if the information transmitted on} \\ & \text{the edge } \mathcal{E}_{ij} \text{ suffers from attacks,} \\ 0, & \text{otherwise,} \end{cases} \tag{4}$$

and the decision variable $\gamma_{ij}(t)$ obeys the Bernoulli distribution with the probabilities

$$\mathrm{Prob}\{\gamma_{ij}(t)=1\}=\chi_{ij},\quad \mathrm{Prob}\{\gamma_{ij}(t)=0\}=1-\chi_{ij},$$

where $\chi_{ij}\in[0,1)$ is a constant. Then, $\nu_i(t)\in\mathbb{R}^p$ is defined as the attack signal sent by the attacker. In practice, due to the fact that the attackers have limited energy and power, and can not launch continuous attacks, the following restriction holds:

$$\|\nu(t)\|\le\bar{\nu}, \tag{5}$$

where $\nu(t)=\mathrm{col}\{\nu_1(t),\ldots,\nu_N(t)\}$ and $\bar{\nu}$ is a known positive constant.

Remark 2: In some existing results (such as [10], [11]), FDIAs are modeled in a similar way as faults and disturbances, although they have different characteristics in essence. Under this model, the FDIAs considered in these papers act on the agent for a long time. But in engineering practice, the occurrence of attacks is often random. Therefore, it is more realistic to use sequences that obey Bernoulli distribution to model FDIAs. Futhermore, constraint (5) is a common limitation for FDIAs, which implies that the attacker has limited energy [14], [16].

To facilitate the control design, some lemmas and definitions are introduced first.

Lemma 1 ([38]): Under Assumption 1, there exists a positive definite matrix $\Theta=\mathrm{diag}\{\theta_1,\ldots,\theta_N\}$ such that the nonsingular M-matrix $\mathcal{L}_1$ satisfies

$$\tilde{\mathcal{L}}=\Theta\mathcal{L}_1+\mathcal{L}_1^T\Theta\ge\lambda_0 I_N>0, \tag{6}$$

where λ_0 is the minimum eigenvalue of $\tilde{\mathcal{L}}$, and $[\theta_1,\ldots,\theta_N]^T=(\mathcal{L}_1^T)^{-1}1_N$.

Lemma 2 ([39]): For any constant matrix $M_1\in\mathbb{R}^{q\times l}$, symmetric positive definite matrix $M_2\in\mathbb{R}^{l\times l}$ and scalar $\beta>0$, the following inequality holds:

$$2x^TM_1y\le\beta x^TM_1M_2M_1^Tx+\beta^{-1}y^TM_2^{-1}y, \tag{7}$$

where $x\in\mathbb{R}^q$ and $y\in\mathbb{R}^l$.

Definition 1 ([14]): For a given function $V(t)$, the infinitesimal operator $\Im$ is defined as

$$\Im V(t)=\lim_{\Delta t\to0^+}\frac{1}{\Delta t}\{\mathbb{E}\{V(t+\Delta t)|t\}-V(t)\}.$$

III. Main Results

In this section, an observer-based event-triggered control strategy is designed, which makes MAS (1) and (2) achieve a prescribed $\mathcal{H}_\infty$ consensus performance when subjected to external disturbance and FDIAs, and Zeno behavior is avoided. In addition, a fault detection mechanism based on adaptive threshold is proposed by using the designed state observer.

Define the state estimation error, output estimation error, consensus error, and the local neighborhood error as follows, respectively

$$e_{x_i}(t)=x_i(t)-\hat{x}_i(t), \tag{8a}$$
$$e_{y_i}(t)=y_i(t)-\hat{y}_i(t), \tag{8b}$$
$$\delta_i(t)=x_i(t)-x_0(t), \tag{8c}$$
$$\xi_i(t)=\sum_{j\in\mathcal{N}_i}a_{ij}(\hat{x}_j(t)-\hat{x}_i(t))+a_{i0}(x_0(t)-\hat{x}_i(t)), \tag{8d}$$

where $\hat{x}_i(t)$ and $\hat{y}_i(t)$ are the state and output of the observer, respectively, and will be designed later.

A. Design of Observer and Controller

The state observer for each follower is given by

$$\begin{cases}\dot{\hat{x}}_i(t)=A\hat{x}_i(t)+Bu_i(t)+Dg_i(t)+\phi(\hat{x}_i(t))+L\\ \qquad\times\left[\sum_{j\in\mathcal{N}_i}a_{ij}(e_{y_i}-e_{y_j}-\gamma_{ij}(t)\nu_i(t))+a_{i0}(e_{y_i}-e_{y_0})\right],\\ \hat{y}_i(t)=C\hat{x}_i(t),\ i\in\mathcal{V},\end{cases} \tag{9}$$

where L is the observer gain to be designed. The initial state is selected as $\hat{x}_i(0)=0$, and the adaptive disturbance compensation $g_i(t)$ is determined by

$$g_i(t)=\frac{W_1e_{y_i}(t)\hat{\bar{d}}_i^2(t)}{\|W_1e_{y_i}(t)\|\left\|\hat{\bar{d}}_i(t)\right\|+\vartheta_{i,1}}, \tag{10}$$

where $\hat{\bar{d}}_i(t)$ is the estimate of the disturbance bound $\bar{d}_i$ with

$$\dot{\hat{\bar{d}}}_i(t)=-\eta_i\vartheta_{i,1}\hat{\bar{d}}_i(t)+\eta_i\|W_1e_{y_i}(t)\|, \tag{11}$$

with $\eta_i>0$, $\vartheta_{i,1}>0$, and W_1 being a gain matrix to be designed. Note that the leader acts as a command generator, so it is assumed that the leader's state is measurable [19], i.e., $\hat{x}_0(t)=x_0(t)$, hence $e_{y_0}(t)=0$.

Remark 3: Since the output information of the neighboring agents of agent i can be attacked by FDIAs in the process of transmitting to agent i, attack signals are included in modeling the output information actually received by the observer (9), and this modeling method is widely used in the literature on stochastic FDIAs [14]–[16]. Besides, although the attack signals are unknown, they still have definite values in the actual situation, and as long as they meet the constraint (5), the designed observer (9) can achieve the desired state estimation objectives. Furthermore, it can be seen that the disturbance compensation mechanism $g_i(t)$ in (10) uses only the output information of agent i itself and the output signal of the observer. Compared with state information, output information is often more readily available, hence easier to apply in practice.

To reduce the network burden caused by the increase in agent information transmission, we aim to use an event-triggered strategy to reduce the frequent actions of the controller and save communication resources. Let $\{t_k^i\}$ denote the event-triggered time sequence of the ith agent. Based on sampled data and state observer (9), an event-triggered control strategy is designed as

$$u_i(t) = K\xi_i(t_k^i),\ t \in [t_k^i, t_{k+1}^i), \tag{12}$$

where K is the controller gain matrix, and the event-trigger rule of the sequence $\{t_k^i\}$ will be designed later.

Next, we prove the validity of the designed observer. By combining (1) and (9), one can get

$$\dot{e}_{x_i}(t) = Ae_{x_i}(t) + D(d_i(t) - g_i(t)) + \phi(x_i) - \phi(\hat{x}_i) - L \times \left[\sum_{j\in\mathcal{N}_i} a_{ij}(e_{y_i} - e_{y_j} - \gamma_{ij}(t)\nu_i(t)) + a_{i0}e_{y_i}\right]. \tag{13}$$

Denote $e_x(t) = \mathrm{col}\{e_{x_1}(t), \ldots, e_{x_N}(t)\}$, $\tilde{\gamma}_i(t) = \sum_{j\in\mathcal{N}_i}\gamma_{ij}(t)$, and $\tilde{\Upsilon} = \mathrm{diag}\{\tilde{\gamma}_1(t), \ldots, \tilde{\gamma}_N(t)\}$. Moreover, let

$$\Upsilon \triangleq \mathbb{E}[\tilde{\Upsilon}(t)] = \mathrm{diag}\left\{\sum_{j\in\mathcal{N}_i}\chi_{1j}, \ldots, \sum_{j\in\mathcal{N}_i}\chi_{Nj}\right\}. \tag{14}$$

Then, (13) can be written in the compact form as follows

$$\dot{e}_x(t) = (I_N \otimes A - \mathcal{L}_1 \otimes LC)e_x(t) + (I_N \otimes D)(d(t) - g(t)) + \phi(x) - \phi(\hat{x}) + (\tilde{\Upsilon} \otimes L)\nu(t), \tag{15}$$

where

$$d(t) = \mathrm{col}\{d_1(t), \ldots, d_N(t)\}, \quad g(t) = \mathrm{col}\{g_1(t), \ldots, g_N(t)\},$$
$$\phi(x) = \mathrm{col}\{\phi(x_1), \ldots, \phi(x_N)\}, \phi(\hat{x}) = \mathrm{col}\{\phi(\hat{x}_1), \ldots, \phi(\hat{x}_N)\},$$
$$\nu(t) = \mathrm{col}\{\nu_1(t), \ldots, \nu_N(t)\}.$$

Now, the observer design result is established in Theorem 1.

Theorem 1: Under Assumptions 1-2, for given positive scalars β, β_1, κ_1 and positive definite matrix R, the state observer (9) can ensure that state estimation error is uniformly ultimately bounded in the presence of external disturbance and FDIAs, if there exist symmetric positive definite matrix P_1 and matrix W_1 satisfying the following LMI conditions:

$$\begin{bmatrix} \mathrm{He}(P_1^{-1}A) + \frac{1}{\beta_1}\Lambda^T\Lambda + \beta\kappa_1 I_n - \frac{\kappa_1\lambda_0}{\lambda_{\max}(\Theta)}C^TR^{-1}C & P_1^{-1} \\ * & -\frac{1}{\beta_1}I_n \end{bmatrix} < 0, \tag{16}$$

$$\begin{bmatrix} -\varepsilon I & P_1^{-1}D - C^TW_1^T \\ * & -\varepsilon I \end{bmatrix} < 0, \tag{17}$$

where $\varepsilon > 0$ is a small constant, and λ_0 is as defined in Lemma 1. Furthermore, the observer gain is given by $L = \kappa_1 P_1 C^T R^{-1}$.

Proof: Let $e_{\bar{d}_i}(t) = \bar{d}_i - \hat{\bar{d}}_i(t)$ with $\bar{d}_i$ being the bound on $d_i(t)$. Then, by using (11), one has

$$\dot{e}_{\bar{d}_i}(t) = \eta_i\vartheta_{i,1}\hat{\bar{d}}_i(t) - \eta_i\|W_1e_{y_i}(t)\|. \tag{18}$$

Next, construct the Lyapunov function as

$$V_1(t) = e_x^T(t)(\Theta \otimes P_1^{-1})e_x(t) + \sum_{i=1}^N \frac{\theta_i}{\eta_i}e_{\bar{d}_i}^2(t). \tag{19}$$

where Θ is a diagonal matrix as in Lemma 1. Use Definition 1 to calculate the infinitesimal operator of $V_1(t)$ and take the mathematical expectation to get

$$\begin{aligned}\mathbb{E}\{\Im V_1(t)\} = \mathbb{E}\{&e_x^T[\Theta \otimes (P_1^{-1}A + A^TP_1^{-1}) - \Theta\mathcal{L}_1 \\ &\otimes P_1^{-1}LC - {\mathcal{L}_1}^T\Theta \otimes C^TL^TP_1^{-1}]e_x + 2e_x^T(\Theta \\ &\otimes P_1^{-1})[\phi(x) - \phi(\hat{x})] + 2e_x^T(\Theta \otimes P_1^{-1}D)d \\ &- 2e_x^T(\Theta \otimes P_1^{-1}D)g + 2e_x^T(\Theta\tilde{\Upsilon} \otimes P_1^{-1}L)\nu(t) \\ &+ 2\sum_{i=1}^N \frac{\theta_i}{\eta_i}e_{\bar{d}_i}(t)\dot{e}_{\bar{d}_i}(t)\}.\end{aligned} \tag{20}$$

Based on Lemma 2, Assumption 2 and (14), we have

$$\begin{aligned}&\mathbb{E}\{2e_x^T(\Theta\tilde{\Upsilon} \otimes P_1^{-1}L)\nu(t)\} \\ &= \mathbb{E}\{2\kappa_1e_x^T(\Theta \otimes I_n)(\tilde{\Upsilon} \otimes C^TR^{-1})\nu(t)\} \\ &= \mathbb{E}\{2\kappa_1e_x^T(\Theta \otimes I_n)(\Upsilon \otimes C^TR^{-1})\nu(t)\} \\ &\le \mathbb{E}\{\beta\kappa_1e_x^T(\Theta \otimes I_n)e_x + \beta^{-1}\kappa_1\nu^T(t)(\Upsilon \otimes R^{-1}C)(\Theta \otimes I_n) \\ &\quad \times (\Upsilon \otimes C^TR^{-1})\nu(t)\} \\ &\le \mathbb{E}\{\beta\kappa_1e_x^T(\Theta \otimes I_n)e_x\} \\ &\quad + \beta^{-1}\kappa_1\bar{\nu}^2\lambda_{\max}\big((\Upsilon \otimes R^{-1}C)(\Theta \otimes I_n)(\Upsilon \otimes C^TR^{-1})\big),\end{aligned} \tag{21}$$

and

$$\begin{aligned}&\mathbb{E}\{2e_x^T(\Theta \otimes P_1^{-1})[\phi(x) - \phi(\hat{x})]\} \\ &\le \mathbb{E}\{e_x^T[\Theta \otimes (\beta_1P_1^{-1}P_1^{-1} + \frac{1}{\beta_1}\Lambda^T\Lambda)]e_x\}.\end{aligned} \tag{22}$$

Additionally, in light of (17), (18) and the adaptive laws (10)-(11), it is clear that

$$
\begin{aligned}
&\mathbb{E}\Bigg\{2e_x^T(\Theta\otimes P_1^{-1}D)d-2e_x^T(\Theta\otimes P_1^{-1}D)g\\
&\quad+2\sum_{i=1}^N\frac{\theta_i}{\eta_i}e_{\bar{d}_i}(t)\dot{e}_{\bar{d}_i}(t)\Bigg\}\\
&\le\mathbb{E}\Bigg\{2\sum_{i=1}^N\left\|e_{x_i}^T(t)\theta_iP_1^{-1}D\right\|\bar{d}_i-2\sum_{i=1}^N e_{x_i}^T(t)\theta_iP_1^{-1}D\\
&\quad\times\frac{W_1e_{y_i}(t)\hat{\bar{d}}_i^2(t)}{\left\|W_1e_{y_i}(t)\right\|\left\|\hat{\bar{d}}_i(t)\right\|+\vartheta_{i,1}}-2\sum_{i=1}^N\theta_ie_{\bar{d}_i}(t)\left\|W_1e_{y_i}(t)\right\|\\
&\quad+2\sum_{i=1}^N\theta_i\vartheta_{i,1}e_{\bar{d}_i}(t)\hat{\bar{d}}_i(t)\Bigg\}\\
&\le\mathbb{E}\Bigg\{2\sum_{i=1}^N\theta_i\frac{\left\|W_1e_{y_i}(t)\right\|\left\|\hat{\bar{d}}_i(t)\right\|\vartheta_{i,1}}{\left\|W_1e_{y_i}(t)\right\|\left\|\hat{\bar{d}}_i(t)\right\|+\vartheta_{i,1}}+2\sum_{i=1}^N\theta_i\vartheta_{i,1}\\
&\quad\times\left[-\left(e_{\bar{d}_i}-\frac{1}{2}\bar{d}_i\right)^2+\frac{1}{4}\bar{d}_i^2\right]\Bigg\}\\
&\le 2\sum_{i=1}^N\theta_i\vartheta_{i,1}\left(1+\frac{1}{4}\bar{d}_i^2\right).
\end{aligned}\tag{23}
$$

Then, substituting (21)-(23) into (20), it yields that

$$\mathbb{E}\{\Im V_1(t)\}\le\mathbb{E}\{-e_x^T(\Theta\otimes\Xi_1)e_x\}+\Delta_1,$$

where $\Xi_1=-(P_1^{-1}A+A^TP_1^{-1}+\beta_1P_1^{-1}P_1^{-1}+\frac{1}{\beta_1}\Lambda^T\Lambda+\beta\kappa_1I_n-\frac{\kappa_1\lambda_0}{\lambda_{\max}(\Theta)}C^TR^{-1}C)$ and $\Delta_1=2\sum_{i=1}^N\theta_i\vartheta_{i,1}(1+\frac{1}{4}\bar{d}_i^2)+\beta^{-1}\kappa_1\bar{\nu}^2\lambda_{\max}((\Upsilon\otimes R^{-1}C)(\Theta\otimes I_n)(\Upsilon\otimes C^TR^{-1}))$. From (16), we can see that $\Xi_1>0$, thus, it is easy to obtain that

$$\mathbb{E}\{\Im V_1(t)\}\le-\lambda_{\min}(\Theta)\lambda_{\min}(\Xi_1)\mathbb{E}\{\|e_x\|^2\}+\Delta_1.$$

Applying the Lyapunov stability theory, it is obvious that state estimation error is uniformly ultimately bounded with bound $\mathbb{E}\{\|e_x(t)\|\}\le\sqrt{\frac{\Delta_1}{\lambda_{\min}(\Theta)\lambda_{\min}(\Xi_1)}}\triangleq\bar{\Delta}_1$, which completes the proof. ∎

Remark 4: Compared with [18], [38], [40], we estimate the upper bound $\bar{d}_i$ of disturbance $d_i(t)$ by the adaptive updating rule (11), and then use it to eliminate the effect of the disturbance. So the upper bound of disturbance does not need to be known in this part. Δ_1 depends on $\bar{d}_i$'s, so larger disturbance $d_i(t)$ leads to larger estimation error $\bar{\Delta}_1$. Δ_1 depends on FDIA as well. More frequent FDIA attacks (i.e., larger values in Υ in (14)) will lead to larger Δ_1, hence larger estimation error $\bar{\Delta}_1$. In the meanwhile, the bound of state estimation error $\bar{\Delta}_1$ can be made arbitrarily small by selecting appropriate design parameters. Furthermore, it should be pointed out that in order to facilitate the solution, the LMI (17) implements the equality constraint $P_1^{-1}D=C^TW_1^T$, and ε in (17) represents the similarity between $P_1^{-1}D$ and $C^TW_1^T$. The smaller ε is, the closer they will be. Hence, it is necessary to make ε small enough to reduce the error [18].

Remark 5: In addition to being an effective observer design for MASs under FDIAs, the observer design in (9) and the disturbance compensation approach in (10)-(11) are also a significant improvement over the well-known Extended State Observer (ESO) approach, which has been used extensively in the active disturbance rejection control literature [41]–[43]. Namely, the disturbances $d_i(t)$ are required to be bounded only, instead of the derivative of $d_i(t)$ being bounded as in ESO. The advantage can be seeing by noting that, while $\sin\omega t$ is bounded for all values of frequency ω, its derivative $\omega\cos\omega t$ will be unbounded if ω is not bounded.

B. $\mathcal{H}_\infty$ Consensus Performance Analysis

In this subsection, we study the design of the controller (12) to ensure that the MAS (1) and (2) achieve consensus with a desired $\mathcal{H}_\infty$ performance index in the presence of external disturbance and FDIAs, and propose a method to update the event-triggered time series $\{t_k^i\}$.

In order to achieve the control objective, define measurement error $e_i(t)=\xi_i(t_k^i)-\xi_i(t)$ with $\xi_i(t)$ as in (8d). Then, according to (8c) and substituting (12) into (1), we have

$$
\begin{aligned}
\dot{\delta}(t)=&(I_N\otimes A-\mathcal{L}_1\otimes BK)\delta(t)+(\mathcal{L}_1\otimes BK)e_x(t)\\
&+(I_N\otimes BK)e(t)+(I_N\otimes D)d(t)+\phi(x)-\tilde{\phi}(x_0),
\end{aligned}\tag{24}
$$

where

$$
\begin{aligned}
&\delta(t)=\mathrm{col}\{\delta_1(t),\ldots,\delta_N(t)\},e(t)=\mathrm{col}\{e_1(t),\ldots,e_N(t)\},\\
&\phi(x)-\tilde{\phi}(x_0)=\mathrm{col}\{\phi(x_1)-\phi(x_0),\ldots,\phi(x_N)-\phi(x_0)\}.
\end{aligned}
$$

Next, with the observer-based event-triggered control strategy (12), Theorem 2 can be obtained.

Theorem 2: Suppose that Assumptions 1-2 hold. Consider the MAS (1)-(2), under the controller (12) driven by the following event-triggered condition:

$$t_{k+1}^i=\inf\{t>t_k^i|z_i(t)>0\},\tag{25}$$

where

$$z_i(t)=\frac{\kappa_2}{\beta_4}\left\|B^TP_2e_i(t)\right\|^2-\hbar_i\left\|B^TP_2\xi_i(t)\right\|^2-\ell e^{-t},\tag{26}$$

with κ_2, β_4, $\hbar_i$, and ℓ being positive constants. If there exist symmetric positive definite matrix P_2 and positive scalars β_2, β_3, and σ such that

$$\begin{bmatrix}\Pi_{11}&P_2^{-1}\Lambda^T&P_2^{-1}\\ *&-\beta_3I_n&0\\ *&*&-I_n\end{bmatrix}<0,\tag{27}$$

$$\left[\beta_2\kappa_2+4\bar{\hbar}\left(\left(\tilde{\mathcal{N}}+\sqrt{N\tilde{\mathcal{N}}}\right)^2+1\right)\right]\lambda_{\max}(P_2BB^TP_2)<\sigma^2,\tag{28}$$

where $\Pi_{11} = \mathrm{He}(AP_2^{-1}) + \beta_3 I_n + \frac{1}{\sigma^2}DD^T - [\kappa_2\lambda_{\min}(\mathcal{L}_1 + \mathcal{L}_1^T) - \frac{\kappa_2}{\beta_2}\lambda_{\max}(\mathcal{L}_1\mathcal{L}_1^T) - \lambda\!\!\!\bar{}\,]BB^T$, $\bar{\hbar} = \max\{\hbar_i\}$, $\lambda\!\!\!\bar{}\, = \kappa_2\beta_4 + 4\bar{\hbar}((\tilde{\mathcal{N}} + \sqrt{N\tilde{\mathcal{N}}})^2 + 1)$, and $\tilde{\mathcal{N}} = \max_{i\in\mathcal{V}}\{|\mathcal{N}_i|\}$ represents the maximum number of neighbors of the agents and $|\mathcal{N}_i|$ denotes the number of agents in the set $\mathcal{N}_i$. Then, the MAS is asymptotically stable with $\mathcal{H}_\infty$ consensus performance,

$$\mathbb{E}\left\{\int_0^{t_f}\delta^T(t)\delta(t)dt\right\} \leq \mathbb{E}\left\{\sigma^2\int_0^{t_f}\varsigma^T(t)\varsigma(t)dt + V_2(0)\right\}, \tag{29}$$

where $\varsigma(t) = \mathrm{col}\{e_x(t), d(t)\}$, and $V_2(t)$ is defined in (31). In addition, Zeno behavior is excluded since the inter-event time is strictly positive, i.e.,

$$t_{k+1}^i - t_k^i = T_k^i \geq \frac{1}{\|A\|}\ln\left(\frac{\|A\|\sqrt{\frac{\beta_4}{\kappa_2}\ell e^{-t}}}{\|B^TP_2\|\alpha_k^i} + 1\right) > 0, \tag{30}$$

where α_k^i will be given in (42). Finally, the controller gain is given by $K = \kappa_2 B^T P_2$.

Proof: Consider the following Lyapunov function

$$V_2(t) = \delta^T(t)(I_N\otimes P_2)\delta(t) + N\ell e^{-t}. \tag{31}$$

where ℓ is a positive constant. Taking its infinitesimal operator and expectation along (24), one can deduce that

$$\begin{aligned}\mathbb{E}\{\Im V_2(t)\} = \mathbb{E}\{&\delta^T[I_N\otimes(P_2\,A + A^TP_2) - \mathcal{L}_1\otimes P_2BK \\ &- \mathcal{L}_1{}^T\otimes K^TB^TP_2]\delta + 2\delta^T(\mathcal{L}_1\otimes P_2BK)e_x \\ &+ 2\delta^T(I_N\otimes P_2BK)e + 2\delta^T(I_N\otimes P_2D)d \\ &+ 2\delta^T(I_N\otimes P_2)[\phi(x) - \tilde{\phi}(\hat{x})] - N\ell e^{-t}\}.\end{aligned} \tag{32}$$

Using $K = \kappa_2 B^TP_2$, it follows from Young's inequality that

$$\begin{aligned}&\mathbb{E}\{2\delta^T(\mathcal{L}_1\otimes\kappa_2P_2BB^TP_2)e_x\} \\ &\leq \mathbb{E}\left\{\frac{1}{\beta_2}\kappa_2\lambda_{\max}(\mathcal{L}_1\mathcal{L}_1^T)\delta^T(I_N\otimes P_2BB^TP_2)\delta + \beta_2\kappa_2 \right.\\ &\left.\quad\times\lambda_{\max}(P_2BB^TP_2)e_x^Te_x\right\},\end{aligned} \tag{33}$$

$$\begin{aligned}&\mathbb{E}\{2\delta^T(I_N\otimes P_2D)d\} \\ &\leq \mathbb{E}\left\{\frac{1}{\sigma^2}\delta^T(I_N\otimes P_2DD^TP_2)\delta + \sigma^2d^T(t)d(t)\right\},\end{aligned} \tag{34}$$

and

$$\begin{aligned}&\mathbb{E}\{2\delta^T(I_N\otimes\kappa_2P_2BB^TP_2)e\} \\ &\leq \mathbb{E}\left\{\frac{\kappa_2}{\beta_4}e^T(I_N\otimes P_2BB^TP_2)e + \beta_4\kappa_2\delta^T(I_N\otimes P_2BB^TP_2)\delta\right\}.\end{aligned} \tag{35}$$

By utilizing the event-triggered condition (25), for any $t \in [t_k^i, t_{k+1}^i)$, it is true that

$$\begin{aligned}&\frac{\kappa_2}{\beta_4}e^T(t)(I_N\otimes P_2BB^TP_2)e(t) \\ &\quad\leq \bar{\hbar}\sum_{i=1}^N\left\|B^TP_2\xi_i(t)\right\|^2 + N\ell e^{-t} \\ &\quad\leq 2\bar{\hbar}\left(\sum_{i=1}^N\left(\left\|B^TP_2q_i^a(t)\right\|^2 + \left\|B^TP_2q_i^b(t)\right\|^2\right)\right) + N\ell e^{-t},\end{aligned} \tag{36}$$

where $q_i^a(t) = \sum_{j\in\mathcal{N}_i}a_{ij}(\hat{x}_j(t) - \hat{x}_i(t))$ and $q_i^b(t) = x_0(t) - \hat{x}_i(t)$. Next, denote $q^a(t) = \mathrm{col}\{q_1^a(t),\ldots,q_N^a(t)\}$, $q^b(t) = \mathrm{col}\{q_1^b(t),\ldots,q_N^b(t)\}$. Then, we can conclude that the following properties are satisfied:

$$\begin{aligned}\left\|B^TP_2q_i^a(t)\right\| &\leq \sum_{j\in\mathcal{N}_i}\left(\left\|B^TP_2q_i^b(t)\right\| + \left\|B^TP_2q_j^b(t)\right\|\right) \\ &\leq \tilde{\mathcal{N}}\left\|B^TP_2q_i^b(t)\right\| \\ &\quad+ \sqrt{\tilde{\mathcal{N}}}\left\|(I_N\otimes B^TP_2)q^b(t)\right\|,\end{aligned} \tag{37}$$

$$\begin{aligned}\left\|(I_N\otimes B^TP_2)q^b(t)\right\|^2 &\leq 2\left\|(I_N\otimes B^TP_2)\delta(t)\right\|^2 \\ &\quad+ 2\left\|(I_N\otimes B^TP_2)e_x(t)\right\|^2.\end{aligned} \tag{38}$$

Thus, one has

$$\begin{aligned}\sum_{i=1}^N\left\|B^TP_2q_i^a(t)\right\|^2 &\leq \left(\tilde{\mathcal{N}} + \sqrt{N\tilde{\mathcal{N}}}\right)^2\left\|(I_N\otimes B^TP_2)q^b(t)\right\|^2 \\ &\leq 2\left(\tilde{\mathcal{N}} + \sqrt{N\tilde{\mathcal{N}}}\right)^2\Big[\left\|(I_N\otimes B^TP_2)\delta(t)\right\|^2 \\ &\quad+ \left\|(I_N\otimes B^TP_2)e_x(t)\right\|^2\Big].\end{aligned} \tag{39}$$

In order to establish the desired $\mathcal{H}_\infty$ consensus performance for the MAS, the following function is introduced:

$$\mathcal{J}(t) = \mathbb{E}\{\Im V_2(t) + \delta^T(t)\delta(t) - \sigma^2\varsigma^T(t)\varsigma(t)\}. \tag{40}$$

Then, combining with (32)-(40), it is easy to get

$$\begin{aligned}\mathcal{J}(t) \le \mathbb{E}\{&\delta^T[I_N \otimes (P_2A + A^TP_2 - \kappa_2\lambda_{\min}(\mathcal{L}_1 + \mathcal{L}_1^T)\\&\times P_2BB^TP_2 + \frac{\kappa_2}{\beta_2}\lambda_{\max}(\mathcal{L}_1\mathcal{L}_1^T)P_2BB^TP_2 + \beta_3P_2P_2\\&+ \frac{1}{\beta_3}\Lambda^T\Lambda + \frac{1}{\sigma^2}P_2DD^TP_2 + \bar{\lambda}P_2BB^TP_2)]\delta + [\beta_2\kappa_2\\&+ 4\bar{\hbar}((\tilde{\mathcal{N}} + \sqrt{N\tilde{\mathcal{N}}})^2 + 1)]\lambda_{max}(P_2BB^TP_2)e_x^Te_x\\&+ \sigma^2d^T(t)d(t) + \delta^T(t)\delta(t) - \sigma^2e_x^Te_x - \sigma^2d^T(t)d(t)\}.\end{aligned}$$

Using (27) and applying the Schur complement lemma, and premultiplying and postmultiplying P_2 on both sides of it, it can be inferred that (27) and (28) can guarantee $\mathcal{J}(t) < 0$, integration of which yields

$$\mathbb{E}\left\{V_2(t_f) - V_2(0) + \int_0^{t_f}[\delta^T(t)\delta(t) - \sigma^2\varsigma^T(t)\varsigma(t)]dt\right\} < 0. \tag{41}$$

Therefore, the MAS can achieve the desired $\mathcal{H}_\infty$ performance (29) with the proposed event-triggered control strategy (12).

In what follows, we will prove that Zeno behavior is excluded by proving that the minimum interval between any two event-intervals is positive. For $t \in [t_k^i, t_{k+1}^i)$, one has $\frac{d\|e_i(t)\|}{dt} = \frac{d}{dt}(e_i^Te_i)^{\frac{1}{2}} \le \|\dot{e}_i\|$. Similar to the proof process of Theorem 1, we know that the consensus error is also bounded. Assume that the bound is $\tilde{\Delta}$. So, we have

$$\begin{aligned}\|\dot{e}_i(t)\| &= \left\|-\dot{\xi}_i(t)\right\|\\&= \left\|\sum_{j\in\mathcal{N}_i}a_{ij}(\dot{\hat{x}}_j(t) - \dot{\hat{x}}_i(t)) + a_{i0}(\dot{x}_0(t) - \dot{\hat{x}}_i(t))\right\|\\&= \Bigg\|A\xi_i(t) + BK\left(\sum_{j\in\mathcal{N}_i}a_{ij}\big(\xi_j(t_k^i) - \xi_i(t_k^i)\big) - a_{i0}\xi_i(t_k^i)\right)\\&\quad + \sum_{j\in\mathcal{N}_i}a_{ij}\big(\phi(\hat{x}_j) - \phi(\hat{x}_i)\big) + a_{i0}(\phi(x_0) - \phi(\hat{x}_i)) + D\\&\quad \times\left(\sum_{j\in\mathcal{N}_i}a_{ij}(g_j(t) - g_i(t)) - a_{i0}g_i(t)\right) + L\\&\quad \times\Bigg[\sum_{j\in\mathcal{N}_i}a_{ij}\Bigg(\sum_{m\in\mathcal{N}_j}a_{jm}(e_{y_j} - e_{y_m} + \gamma_{jm}\nu_j) + a_{i0}e_{y_j}\\&\quad - \sum_{j\in\mathcal{N}_i}a_{ij}(e_{y_i} - e_{y_j} + \gamma_{ij}\nu_i) - a_{i0}e_{y_i}\Bigg)\\&\quad - a_{i0}\left(\sum_{j\in\mathcal{N}_i}a_{ij}(e_{y_i} - e_{y_j} + \gamma_{ij}\nu_i) + a_{i0}e_{y_i}\right)\Bigg]\Bigg\|\\&\le \|A\|\|e_i\| + \alpha_k^i,\end{aligned}$$

where

$$\begin{aligned}\alpha_k^i = \max_{t\in[t_k^i, t_{k+1}^i)}\Bigg\{&\Bigg\|A\xi_i(t_k^i) + BK\Bigg[\sum_{j\in\mathcal{N}_i}a_{ij}(\xi_j(t_k^i) - \xi_i(t_k^i))\\&- a_{i0}\xi_i(t_k^i)\Bigg] + D\Bigg[\sum_{j\in\mathcal{N}_i}a_{ij}(g_j(t) - g_i(t)) - a_{i0}g_i(t)\Bigg]\Bigg\|\\&+ (2\tilde{\mathcal{N}} + a_{i0})\|\Lambda\|(\bar{\Delta}_1 + \tilde{\Delta}) + \|LC\|(4\tilde{\mathcal{N}}^2 + 2a_{i0}\tilde{\mathcal{N}}\\&+ 2\tilde{\mathcal{N}} + a_{i0})\bar{\Delta}_1 + \|L\|(2\tilde{\mathcal{N}}^2 + a_{i0}\tilde{\mathcal{N}})\bar{\nu}\Bigg\} > 0,\end{aligned} \tag{42}$$

from which we have $\|e_i(t)\| \le \frac{\alpha_k^i}{\|A\|}(e^{\|A\|(t-t_k^i)} - 1)$. Thus, it is clear that

$$\left\|B^TP_2e_i(t)\right\| \le \frac{\left\|B^TP_2\right\|\alpha_k^i}{\|A\|}\left(e^{\|A\|(t-t_k^i)} - 1\right). \tag{43}$$

According to the event-triggered condition (25) and using (43), it follows that:

$$\sqrt{\frac{\beta_4}{\kappa_2}\ell e^{-t}} \le \frac{\left\|B^TP_2\right\|\alpha_k^i}{\|A\|}\left(e^{\|A\|(t_{k+1}^i - t_k^i)} - 1\right) \tag{44}$$

which implies that (30) holds. Hence, it is proved that Zeno behavior will not occur under the designed event-triggered condition. This ends the proof. ■

Remark 6: To simplify the solution of the matrix parameters in Theorem 2, we can first solve P_2 with (27) by adjusting the parameters β_2, β_3, β_4, σ, κ_2, and $\hbar_i$, then calculate $\lambda_{\max}(P_2BB^TP_2)$ and replace it into (28) to verify whether the condition is satisfied. If not satisfied, re-adjust these parameters, so as to avoid solving the two inequalities at the same time. In addition, compared with the undirected communication topology considered in [20], the control strategy proposed in this paper under the directed communication topology is more common because the undirected graph can be regarded as a special case of the directed graph if each agent can obtain the information of its neighbors. Moreover, the asymmetry of the Laplacian matrix and the consideration of external disturbances and stochastic FDIAs make this work more challenging than [20] in controller design and consensus analysis.

Remark 7: In the ETM (25), by analyzing different sets of parameters, we found that a smaller $\hbar_i$ and a larger $\frac{\kappa_2}{\beta_4}$ will trigger more points to ensure system performance. Therefore, we can adjust these parameters to achieve a balance between system performance and communication burden. Different from the centralized ETM [30], [31], the ETM (25) does not need global state information, thus reducing the unnecessary usage of communication resources. Moreover, compared with the constant threshold used in the distributed ETM by [16], a state-dependent threshold is used in the trigger function (26), which will further decrease the frequency of the control input updates and reduce the communication burden. In addition, compared with the ETM proposed in [30], by adding a timedependent term ℓe^{-t} to the trigger function (26), the Zeno behavior can be excluded easily.

Remark 8: It is mentioned in [44], [45] that as long as the variable is bounded and converges to zero or a bounded neighborhood of the origin after the system is stable, the variable can be regarded as a disturbance in the process of stability analysis. In Theorem 1, we can see that the state observation errors caused by the FDIAs are uniformly ultimately bounded, and the boundary can be arbitrarily small by adjusting the design parameters. So it is reasonable to regard the state observation error as disturbance in the process of consensus analysis.

C. Fault Detection Mechanism

When agents suffer from faults, it will lead to performance degradation or even instability of the MAS, so we propose a fault detection mechanism to detect faults promptly. Meanwhile, in order to avoid false alarms of the fault detection mechanism, an adaptive threshold is derived, which reduces the conservatism of the mechanism. In addition, as in [27], [33], [34], to develop the fault detection mechanism, it is assumed that $\bar{d}_i$ is known in advance. The fault agent model can be described as

$$\begin{cases} \dot{x}_i(t) = Ax_i(t) + Bu_i(t) + Dd_i(t) + Ff_i(t) + \phi(x_i) \\ y_i(t) = Cx_i(t),\ i = 1, \ldots, N \end{cases} \tag{45}$$

where $f_i(t)$ denote the fault signals. When the upper bound of disturbance $\bar{d}_i$ is known, the adaptive disturbance compensation $g_i(t)$ in (9) can be redesigned as

$$g_i(t) = \frac{W_2 e_{y_i}(t)\bar{d}_i^2}{\|W_2 e_{y_i}(t)\|\bar{d}_i + \vartheta_{i,2}} \tag{46}$$

where W_2 is the gain matrix whose design process is the same as W_1 in (10), and $\vartheta_{i,2}$ is the same positive constant as $\vartheta_{i,1}$ in (10). These two parameters need to be redesigned here, different from those in (10). Moreover, it follows from (9) and (45) that

$$\begin{aligned} \dot{e}_x(t) &= (I_N \otimes A - \mathcal{L}_1 \otimes LC)e_x(t) + (I_N \otimes D)(d(t) - g(t)) \\ &\quad + \phi(x) - \phi(\hat{x}) + (I_N \otimes F)f(t) + (\tilde{\Upsilon} \otimes L)\nu(t) \end{aligned} \tag{47}$$

where $f(t) = \mathrm{col}\{f_1(t), \ldots, f_N(t)\}$.

In order to detect the fault, define the residual $r_i(t) = \sum_{j \in \mathcal{N}_i} a_{ij}(e_{y_i} - e_{y_j}) + a_{i0}(e_{y_i} - e_{y_0})$, and one has

$$\mathbb{E}\{r(t)\} = \mathbb{E}\{(\mathcal{L}_1 \otimes C)e_x\}, \tag{48}$$

where $r(t) = \mathrm{col}\{r_1(t), \ldots, r_N(t)\}$. Then, we can write

$$r_i(t) = \Psi_i r(t). \tag{49}$$

where $\Psi_i = \{\mathbf{0}_p, \ldots, \underbrace{\mathbf{I}_p}_{i\text{th}}, \ldots, \mathbf{0}_p\} \in \mathbb{R}^{p \times Np}$. To this end, we can build an evaluation function based on residual signal $r_i(t)$ and detect the fault by comparing it with the adaptive threshold. Furthermore, it should be pointed out that defining $r_i(t)$ as residual can reduce the computational burden because the same term as $r_i(t)$ is also used in the observer (9).

The fault detection algorithm is stated in Theorem 3.

Theorem 3: The following fault detection logic can be used to detect the occurrence of faults in MAS (1) and (2):

$$\begin{cases} J_{r_i(t)} > J_{th_i}(t), f_i(t) \neq 0 \text{ or } f_j(t) \neq 0, j \in \mathcal{N}_i, \\ J_{r_i(t)} < J_{th_i}(t), \text{No fault}, \end{cases} \tag{50}$$

where $J_{r_i(t)} = \mathbb{E}\{\|r_i(t)\|^2\}$ is an evaluation function, $J_{th_i}(t)$ is the adaptive threshold that is designed as

$$\begin{aligned} J_{th_i}(t) &= \frac{\lambda_{\max}((\mathcal{L}_1^T \otimes C^T)\Psi_i^T \Psi_i(\mathcal{L}_1 \otimes C))}{\lambda_{\min}(\Theta)\lambda_{\min}(P_3^{-1})} \\ &\quad \times \Bigg\{ \lambda_{\max}(\Theta)\lambda_{\max}(P_3^{-1})\Delta^2 e^{-\frac{\lambda_{\min}(\Xi_2)}{\lambda_{\max}(P_3^{-1})}t} \\ &\quad + \frac{\kappa_3}{\beta_5}\varpi\bar{\nu}^2 \frac{\lambda_{\max}(P_3^{-1})}{\lambda_{\min}(\Xi_2)}\left(1 - e^{-\frac{\lambda_{\min}(\Xi_2)}{\lambda_{\max}(P_3^{-1})}t}\right)\Bigg\}, \end{aligned} \tag{51}$$

where P_3 is a symmetric positive definite matrix that ensures $\Xi_2 = -(P_3^{-1}A + A^T P_3^{-1} + \beta_6 P_3^{-1}P_3^{-1} + \frac{1}{\beta_6}\Lambda^T\Lambda + \beta_5\kappa_3 I_n - \frac{\kappa_3\lambda_0}{\lambda_{\max}(\Theta)}C^T R_1^{-1}C + \beta_7\lambda_{\max}(\Theta)P_3^{-1}P_3^{-1}) > 0$, $\varpi = \lambda_{\max}((\Upsilon \otimes R_1^{-1}C)(\Theta \otimes I_n)(\Upsilon \otimes C^T R_1^{-1}))$, Υ is as in (14), R_1 is a positive definite matrix, and β_5, β_6, β_7, κ_3 are positive constants. The observer gain L is redesigned as $L = \kappa_3 P_3 C^T R_1^{-1}$.

Proof: Select the Lyapunov function as

$$V_3(t) = e_x^T(t)(\Theta \otimes P_3^{-1})e_x(t). \tag{52}$$

By using a similar proof process as Theorem 1, we can get

$$\begin{aligned} &\mathbb{E}\{\Im V_3(t)\} \\ &= \mathbb{E}\Bigg\{ e_x^T \Bigg[\Theta \otimes \Bigg(P_3^{-1}A + A^T P_3^{-1} + \beta_6 P_3^{-1}P_3^{-1} + \frac{1}{\beta_6}\Lambda^T\Lambda \\ &\quad + \beta_5\kappa_3 I_n - \frac{\kappa_3\lambda_0}{\lambda_{\max}(\Theta)}C^T R_1^{-1}C + \beta_7\lambda_{\max}(\Theta)P_3^{-1}P_3^{-1}\Bigg)\Bigg] e_x \\ &\quad + \frac{\kappa_3}{\beta_5}\varpi\bar{\nu}^2 + \frac{1}{\beta_7}f^T(I_N \otimes F^T F)f \Bigg\} \\ &\leq -\frac{\lambda_{\min}(\Xi_2)}{\lambda_{\max}(P_3^{-1})}\mathbb{E}\{V_3(t)\} + \frac{\kappa_3}{\beta_5}\varpi\bar{\nu}^2 + \frac{1}{\beta_7}f^T(I_N \otimes F^T F)f. \end{aligned} \tag{53}$$

From (52), it is obvious that $\lambda_{\min}(\Theta)\lambda_{\min}(P_3^{-1})\mathbb{E}\{\|e_x\|^2\} \leq \mathbb{E}\{V_3(t)\} \leq \lambda_{\max}(\Theta)\lambda_{\max}(P_3^{-1})\mathbb{E}\{\|e_x\|^2\}$. Based on the above inequality and integrating (53) results in

$$\begin{aligned}&\lambda_{\min}(\Theta)\lambda_{\min}(P_3^{-1})\mathbb{E}\{\|e_x\|^2\}\\&\leq e^{-\frac{\lambda_{\min}(\Xi_2)}{\lambda_{\max}(P_3^{-1})}t}\lambda_{\max}(\Theta)\lambda_{\max}(P_3^{-1})\mathbb{E}\|e_x(0)\|^2\\&\quad+\frac{\kappa_3}{\beta_5}\varpi\bar{\nu}^2\int_0^t e^{-\frac{\lambda_{\min}(\Xi_2)}{\lambda_{\max}(P_3^{-1})}(t-\tau)}d\tau\\&\quad+\frac{1}{\beta_7}\int_0^t f^T(I_N\otimes F^TF)fe^{-\frac{\lambda_{\min}(\Xi_2)}{\lambda_{\max}(P_3^{-1})}(t-\tau)}d\tau. \qquad (54)\end{aligned}$$

Using (48) and (49), we have

$$\begin{aligned}\mathbb{E}\{r_i^T(t)r_i(t)\}&\leq\mathbb{E}\{e_x^T(\mathcal{L}_1^T\otimes C^T)\Psi_i^T\Psi_i(\mathcal{L}_1\otimes C)e_x\}\\&\leq\lambda_{\max}((\mathcal{L}_1^T\otimes C^T)\Psi_i^T\Psi_i(\mathcal{L}_1\otimes C))\mathbb{E}\{\|e_x\|^2\}. \qquad (55)\end{aligned}$$

Then, combining (54) and (55) and considering the initial condition $\hat{x}_i(0)=0$, it is obtained that

$$\begin{aligned}&\frac{\lambda_{\min}(\Theta)\lambda_{\min}(P_3^{-1})}{\lambda_{\max}((\mathcal{L}_1^T\otimes C^T)\Psi_i^T\Psi_i(\mathcal{L}_1\otimes C))}\mathbb{E}\{r_i^T(t)r_i(t)\}\\&\leq\lambda_{\max}(\Theta)\lambda_{\max}(P_3^{-1})\|x(0)\|^2e^{-\frac{\lambda_{\min}(\Xi_2)}{\lambda_{\max}(P_3^{-1})}t}\\&\quad+\frac{\kappa_3}{\beta_5}\varpi\bar{\nu}^2\frac{\lambda_{\max}(P_3^{-1})}{\lambda_{\min}(\Xi_2)}\left(1-e^{-\frac{\lambda_{\min}(\Xi_2)}{\lambda_{\max}(P_3^{-1})}t}\right)\\&\quad+\frac{1}{\beta_7}\int_0^t f^T(I_N\otimes F^TF)fe^{-\frac{\lambda_{\min}(\Xi_2)}{\lambda_{\max}(P_3^{-1})}(t-\tau)}d\tau. \qquad (56)\end{aligned}$$

Thus, if $J_{r_i(t)} > J_{th_i}(t)$, which means that the ith agent or one of its neighbors is faulty. This completes the proof. ∎

Remark 9: Similar to Theorem 1, the matrix parameters P_3 and W_2 can be solved by the following LMIs:

$$\begin{bmatrix}\Omega_{11} & P_3^{-1} & P_3^{-1}\\ * & -\frac{1}{\beta_6}I_n & 0\\ * & * & -\frac{1}{\beta_7\lambda_{\max}(\Theta)}I_n\end{bmatrix}<0, \qquad (57)$$

$$\begin{bmatrix}-\varepsilon I & P_3^{-1}D-C^TW_2^T\\ * & -\varepsilon I\end{bmatrix}<0, \qquad (58)$$

where $\Omega_{11}=P_3^{-1}A+A^TP_3^{-1}+\frac{1}{\beta_6}\Lambda^T\Lambda+\beta_5\kappa_3I_n-\frac{\kappa_3\lambda_0}{\lambda_{\max}(\Theta)}C^TR_1^{-1}C$, and β_5, β_6, β_7, κ_3, ε are positive constants. In addition, from the adaptive threshold (51), it can be seen that it will eventually tend to a constant related to the attack signal. The larger the attack signal, the greater the adaptive threshold, which may reduce the accuracy and promptness of the fault detection mechanism, or even non-alarms. However, due to the concealment and energy limitations of FDIAs, FDIAs are often smaller than fault signals, and we can make the adaptive threshold small enough to ensure the performance of the fault detection mechanism by adjusting parameters.

Remark 10: Note that, unlike the consensus analysis without requiring the upper bounds of the disturbances in Theorems 1 and 2, these upper bounds are required and are assumed to be known in advance in Theorem 3 to achieve fault detection. This is a common assumption in fault detection problems [27], [33], [34]. When the upper bound is known, disturbance compensation (46) can be used to eliminate the effect of disturbance on fault detection, so as to avoid the problem that the adaptive threshold contains an upper bound of disturbance [29], which degrades the performance of fault detection. In addition, the disturbance compensation (46) is also applicable to situations in consensus analysis where the disturbance upper bound is known.

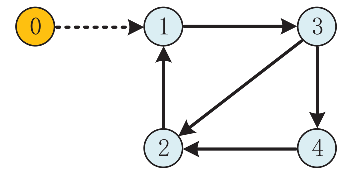


Fig. 2. Communication topology.

IV. Simulation Examples

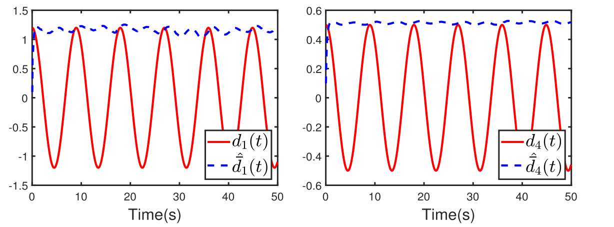
In this section, simulation results are given to verify the effectiveness of the main results. Consider the communication topology shown in Fig. 2, and the parameter matrices in (1) are as follows

$$A=\begin{bmatrix}-2.9 & 0.3 & 0.4 & 1.2\\-0.1 & -0.2 & 0.6 & 1.5\\1.2 & 2.1 & -2.8 & 3.4\\1 & -2 & -2.5 & -2.5\end{bmatrix}, B=\begin{bmatrix}0 & 0\\0 & 0\\-1 & 0.5\\-0.1 & 0.2\end{bmatrix},$$

$$C=\begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 1 & 1\end{bmatrix},\ D=[0.1\quad 0.1\quad -0.1\quad -0.1]^T.$$

The nonlinear function $\phi(x_i)$ is considered as $\phi(x_i)=[0,0,0,0.01\sin(x_{i1}(t))]^T$, and it follows from Assumption 2 that $\Lambda=\mathrm{diag}\{0.01,\ldots,0.01\}$. In addition, it is easy to obtain $\mathcal{L}_1$ from Fig. 2. Then, based on Lemma 1, we have $\Theta=\mathrm{diag}\{4,2.5,7,3.5\}$. Without loss of generality, we assume that the disturbances $d_1=1.2\cos(0.7t)$ and $d_4=0.5\cos(0.7t)$ acting on agent 1 and agent 4 all the time, respectively, and there are no disturbances to other agents. The probabilities of FDIAs occurring on each edge are chosen as $\chi_{12}=\chi_{23}=\chi_{43}=0.02$, $\chi_{24}=\chi_{31}=0.03$, and the attack characteristic is set to $\bar{\nu}=0.1$. Moreover, take the initial states of each agent as $x_0(0)=[-3,2.5,2,-2.8]^T$, $x_1(0)=[-0.3,0.4,-0.3,0.3]$, $x_2(0)=[0.4,-0.2,0.3,0.2]$, $x_3(0)=[0.4,0.1,0.3,-0.1]$, and $x_4(0)=[0.6,-0.3,0.3,-0.4]$.

First, we demonstrate the validity of the designed consensus control protocol (9), (10) and (12). Based on Theorem 1 and Theorem 2, select the parameters $\beta=0.08$, $\beta_1=0.1$, $\kappa_1=200$, $R=I_3$, $\beta_2=60$, $\beta_3=0.85$, $\varepsilon=0.00005$, $\sigma=0.387$ and the triggering parameters $\kappa_2=0.0034$, $\beta_4=1$, $\hbar_i=0.003$, $\ell=0.00001$. Then, the observer gain L and the disturbance bound estimation gain matrix W_1, the controller gain K and the weighting matrix P_2 are obtained as follows after solving (16)-(17) and (27)-(28), respectively



(a) $d_1(t)$ and $\hat{\tilde{d}}_1(t)$ (b) $d_4(t)$ and $\hat{\tilde{d}}_4(t)$

Fig. 3. The disturbance signals $d_i(t)$ and the adaptive parameters $\hat{\tilde{d}}_i(t)$ $(i = 1, 4)$.

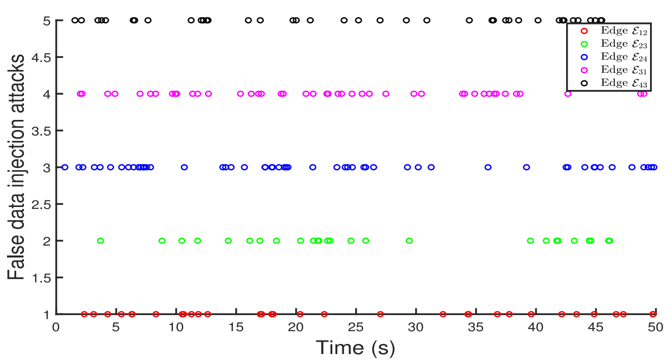


Fig. 4. The moment when the transmitted data on different edges is attacked by FDIAs.

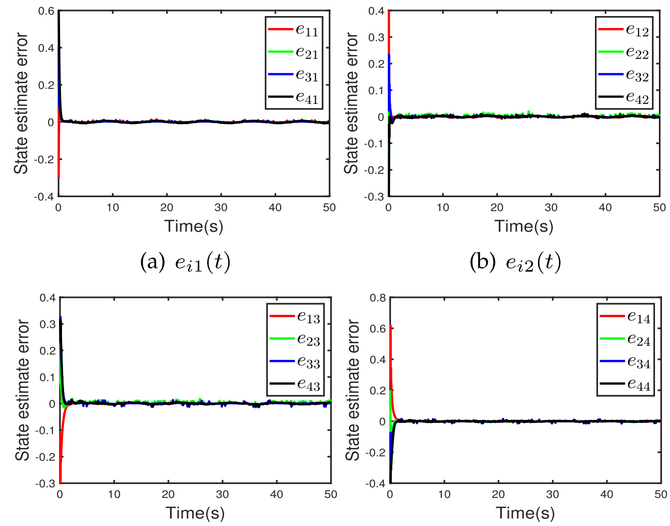
$$L = \begin{bmatrix} 5.6805 & 0.9824 & 2.3019 \\ 0.9824 & 14.1506 & -4.4867 \\ 0.5960 & 0.8026 & 8.3620 \\ 1.7059 & -5.2893 & 9.7204 \end{bmatrix},$$

$$W_1 = [4.5925 \quad 0.2255 \quad -2.7408],$$

$$K = \begin{bmatrix} 0.0004 & -0.0001 & -0.0016 & 0.0002 \\ -0.0001 & 0.0005 & 0.0007 & 0.0004 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 0.4952 & 0.1179 & -0.1308 & 0.2451 \\ 0.1179 & 1.5879 & -0.0550 & 0.8306 \\ -0.1308 & -0.0550 & 0.4903 & -0.1585 \\ 0.2451 & 0.8306 & -0.1585 & 1.0038 \end{bmatrix}.$$

Then, set $\eta_1 = 7$, $\vartheta_{1,1} = 0.017$, $\eta_4 = 0.7$, and $\vartheta_{4,1} = 0.05$. The disturbance signals $d_i(t)$ and the adaptive parameters $\hat{\tilde{d}}_i(t)$ $(i = 1, 4)$ are given in Fig. 3, it can be seen that $\hat{\tilde{d}}_i(t)$ can estimate the upper bound of disturbance. Fig. 4 depicts the moments when the information passed on different edges is attacked by FDIAs. Define $\tilde{J}(t) = \frac{1}{N}\sqrt{\sum_{i=1}^{N} \|x_i(t) - x_0(t)\|^2}$ as the consensus error of the system, then the observer errors and the consensus error $\tilde{J}(t)$ are shown in Fig. 5 and Fig. 6, respectively. It can be noticed that the designed observer and $\mathcal{H}_\infty$ controller perform very well even in the presence of FDIAs and external disturbance. Fig. 7 shows the event-triggered instants and release intervals of different agents, from which we can see that the proposed ETM (25) can save communication resources and excludes Zeno behavior. Fig. 8 displays the performance comparison of the observer (9) with or



(a) $e_{i1}(t)$ (b) $e_{i2}(t)$

(c) $e_{i3}(t)$ (d) $e_{i4}(t)$

Fig. 5. Observer errors of four agents under FDIAs and disturbances.

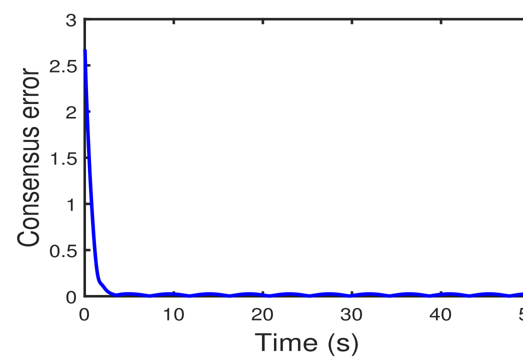


Fig. 6. Consensus error $\tilde{J}(t)$ under FDIAs and disturbances.

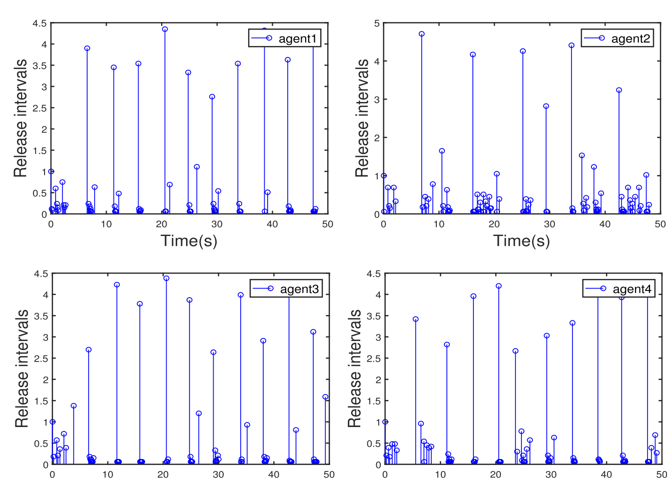


Fig. 7. Release instants and intervals of different agents.

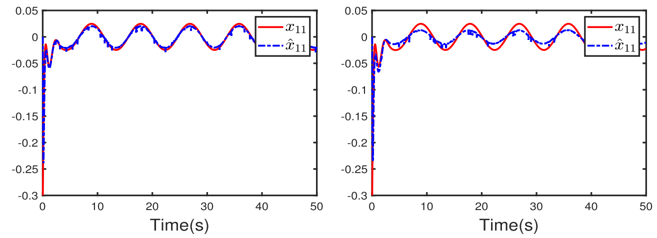


Fig. 8. The observer performance for the first element of the state $x_1(t)$ with (left) and without (right) disturbance compensation $g_i(t)$ in observer (9) under FDIAs and disturbances.

TABLE I
TRIGGER NUMBERS OF EACH AGENT UNDER DIFFERENT CONTROL SCHEMES

Control scheme	Agent 1	Agent 2	Agent 3	Agent 4
Event-triggered strategy (25)	76	100	87	87
Method in [16]	124	119	96	108
Method in [31]	122	122	122	122

without disturbance compensation $g_i(t)$ in (10), and it is obvious that when disturbance compensation is included, the observed values tend to the real values more accurately. Besides, the trigger numbers of each agent under different control schemes are summarized in TABLE I. Overall, the number of trigger points of the proposed method is less, which can better reduce the communication burden with the system performance not being adversely affected.

In the following, we illustrate the effectiveness of the proposed fault detection mechanism (50) and (51). The parameter matrix F in (45) is selected as $F = [3, -2, 0.3, -1.5]^T$. Besides, let $R_1 = I_3$, $\beta_5 = 0.1$, $\beta_6 = 0.2$, $\beta_7 = 0.004$, and $\kappa_3 = 120$. Then, by solving the LMIs (57)-(58), the parameter matrices P_3 and W_2 are obtained as follows

$$P_3 = \begin{bmatrix} 0.0645 & 0.0125 & 0.0035 & 0.0199 \\ 0.0125 & 0.1586 & 0.0053 & -0.0571 \\ 0.0035 & 0.0053 & 0.0949 & -0.0070 \\ 0.0199 & -0.0571 & -0.0070 & 0.1176 \end{bmatrix},$$

$$W_2 = [\,1.9785 \quad 0.0760 \quad -1.2211\,].$$

For the purpose of fault detection, it is assumed that faults $f_1(t)$ and $f_3(t)$ occur on agents 1 and 3, respectively, which are described as

$$f_1(t) = \begin{cases} 1.1 + 0.4\sin(3t), & 15\ s \le t \le 25\ s, \\ 0, & \text{otherwise}, \end{cases}$$
$$f_3(t) = \begin{cases} 0.9 + 0.2\cos(3t), & 30\ s \le t \le 40\ s, \\ 0, & \text{otherwise}. \end{cases}$$

The residual functions $J_{r_i(t)}$ and adaptive threshold $J_{th_i}(t)$ of different agents are shown in Fig. 9. It can be observed clearly that the fault was detected on agent 1 at 15.06 s, and no fault was detected on agent 2, so we can infer that the fault

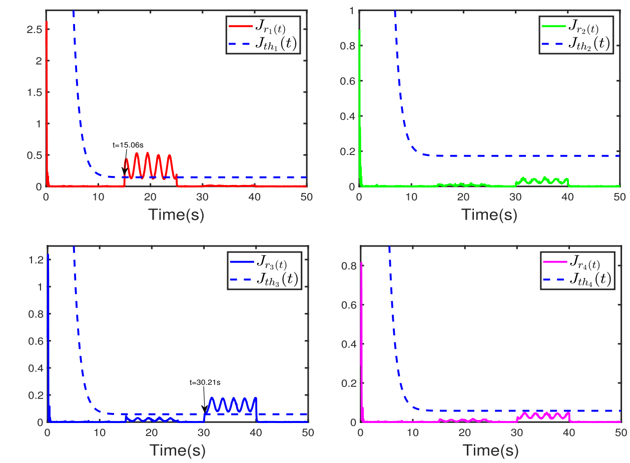


Fig. 9. The performance of the fault detection mechanism using the adaptive threshold design method proposed in Theorem 3.

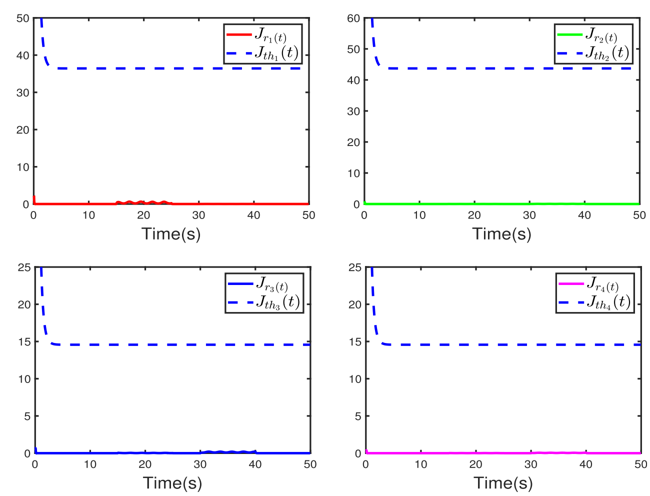


Fig. 10. The performance of the fault detection mechanism using the adaptive threshold design method proposed in [29].

occurs on agent 1. In addition, for $t > 25.02\ s$, the residual function is less than the adaptive threshold, which indicates that no fault occurs on agent 1. Similarly, we can conclude that there is a fault on agent 3 and was detected at 30.21 s, while no fault on agent 1 throughout the simulation. Fig. 10 shows the performance of the fault detection mechanism using the adaptive threshold design method of [29]. Obviously, due to the existence of external disturbance and FDIAs, the adaptive threshold is too large, which makes the fault detection mechanism invalid. Thus, the promptness and effectiveness of the proposed fault detection mechanism are verified.

V. CONCLUSION

In this paper, the security consensus problem for a class of nonlinear MASs with a directed communication topology under external disturbance, fault, and FDIA is addressed. A state observer is designed to estimate the state of the system, and an adaptive compensation mechanism is introduced to reduce the influence of disturbance on the accuracy of the observer. In order to reduce the communication burden, we proposed an event-triggered control strategy based on this observer to ensure that the MAS can achieve the prescribed $\mathcal{H}_\infty$ performance with Zeno behavior excluded. In addition, when a fault occurs on the agent, a fault detection mechanism is constructed based on the state observer to detect the fault promptly. Finally, a simulation example shows the effectiveness of the proposed methods. Note that only the communication graph with directed spanning tree and static ETM are considered in this paper. Therefore, how to extend these results to time-varying topology and adaptive ETM will be the future research topics. In addition, it is also an interesting research direction to extend these results to multiple attacks and attack detection problems.

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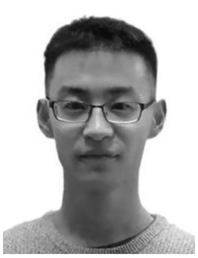

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