A Polynomial Time Solution to Minimum Forwarding Set Problem in Wireless Networks under Unit Disk Coverage Model

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***Abstract*— Network-wide broadcast (simply broadcast) is a frequently used operation in wireless ad hoc networks. One promising practical approach for energy efficient broadcast is to use localized algorithms to minimize the number of nodes involved in the propagation of the broadcast messages. In this context, the minimum forwarding set problem (MFSP) (also known as multi-point relay (MPR) problem) has received a considerable attention in the research community. Even though the general form of the problem is shown to be NP-complete, the complexity of the problem has not been known under the practical application context of ad hoc networks. In this paper, we present a polynomial time algorithm to solve the MFSP problem for wireless network under unit disk coverage model. We prove the existence of some geometrical properties for the problem and then propose a polynomial time algorithm to build an optimal solution based on these properties. To the best of our knowledge, our algorithm is the first polynomial time solution to the MFSP problem under the unit disk coverage model. We believe that the work presented in this paper will have an impact on the design and development of new algorithms for several wireless network applications including energy efficient multicast, broadcast, and topology control protocols for wireless ad hoc networks and sensor networks.**

Keywords: Multi-point relays, minimum forwarding set problem, network wide broadcast, unit disk graphs.

I. Introduction

Wireless ad hoc networks (WANETs) are used to provide communication services in dynamic environments including active battlefield, search and rescue, and emergency relief. Energy and wireless bandwidth are two scarce WANET resources that need to be used efficiently. Energy is limited as the nodes typically operate on battery power. Wireless bandwidth is limited as the nodes share the same transmission medium which is open to collision and contention. Network-wide broadcast (simply broadcast) is a frequently used operation in WANETs. In addition to data dissemination, many protocols utilize broadcast to communicate control messages [1], [2], [3], [4]. As an example, popular WANET routing protocols, including OLSR, AODV, and DSR, use broadcast to discover and maintain routes between the nodes in a WANET. A naive implementation of the broadcast operation where each node involves in propagation of a broadcast message (i.e., network wide flooding) may cause a high level of energy and bandwidth consumption in WANETs [5].

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A. Problem Definition

Energy efficient broadcast problem received a significant attention from the research community and a large number of studies have been published in the area [6]. One promising approach that was proposed for energy efficient broadcast is the *neighbor designation* approach [7] where the goal is to prevent unnecessary transmission of broadcast packets for energy efficiency. Each node collects 2-hop neighborhood information and then identifies a subset of its 1-hop neighbors as forwarding nodes for relaying a broadcast message toward its 2-hop neighbors. The efficiency of neighbor designation approach depends on finding a minimum size forwarding node set among the 1-hop neighbors. This problem is referred to as *Minimum Forwarding Set Problem (MFSP)* [7], [8] and is formally defined as follows:

Definition 1 (Minimum Forwarding Set Problem (MFSP)): Consider a graph $G = (V, E)$ where V is the set of nodes and E is the set of links in the network. Given a node $v \in V$, let $N(v)$ and $N_2(v)$ represent the set of 1-hop and 2-hop neighbors of v, respectively. $N(v)$ and $N_2(v)$ are strict sets such that $v \notin N(v)$ and $N(v) \cap N_2(v) = \varnothing$. MFSP asks for a minimum-size subset S of $N(v)$ such that every node in $N_2(v)$ is within the coverage of at least one node in S. More formally, MFSP asks for a minimum cardinality set S such that $S \subseteq N(v)$ and $(\forall x \in N_2(v),\ \exists y \in S \mid x \in N(y))$.

A solution to the MFSP problem at a node v is $S \subseteq N(v)$ where S is a minimum cardinality set called *forwarding set*. Note that in an optimal solution, the assignment of a node $b \in N_2(v)$ to a node $s \in S$ requires that $b \in N(s)$. In other words, in the context of the wireless broadcast operation, b should be within the coverage range of s. Also note that, in certain cases, multiple different optimal solutions may exist.

B. Existing Solutions

The MFSP problem is shown to be NP-complete [7] with a reduction from the Set Cover problem. The heuristic proposed in [7] is an application of the well-known Chvatal's greedy algorithm for the Set Cover problem [9] and gives an approximation ratio of $(1 + ln(|S_i|_{max}))$ where $|S_i|_{max}$ is the size of the largest subset of $N_2(v)$ that is covered by a node $i \in N(v)$. Busson et al. [10] presented a stochastic analysis to argue that the heuristic in [7] performs near optimal for most practical scenarios.

Calinescu et al. [8] studied the problem under the assumption that nodes are distributed in 2-dimensional plane and they have a a unit disk coverage [11] for their transmissions. They proposed a 6-approximation algorithm that runs in $O(n\log n)$ time and a 3-approximation algorithm that runs in $O(n\log^2 n)$ time. In addition, they presented an exact $O(n\log^2 n)$ time algorithm for a special case of the MFSP problem when all 2-hop neighbors are in the same quadrant of a 2-dimensional coordinate space with respect to the broadcasting node.

Finally, Wu et al. [12] considered an extended version of the MFSP problem where a broadcasting node v collects 3-hop neighbor information to find a small number of 1-hop and/or 2-hop neighbors to cover the set of 2-hop neighbors. They proposed a heuristic that gives a constant local approximation ratio to identify an extended forwarding node set. We believe that the extended version of the problem introduces a new dimension to the original problem setup with some potential performance improvements depending on the availability of 3-hop neighborhood information. In this paper, we consider the original MFSP problem as defined above and leave the extended version of the problem for future work.

C. Our Contributions

In this paper, we present the first polynomial time algorithm to solve the MFSP problem under unit disk coverage model for wireless transmission. First, we introduce two properties named as *Two-Set Property* and *Non-Interleaving Property*. We then present an algorithm that uses a dynamic programming approach to build an optimal solution and prove its correctness. The algorithm has $O(n^3+n^2m)$ time complexity where $m=|N(v)|$ and $n=|N_2(v)|$ for a broadcasting node v. The current version of our algorithm works under the unit disk coverage model and therefore may have limited utility for real world wireless networks. However, the algorithm can be quite instrumental in evaluating the performance of more practical heuristics within simulation studies.

The rest of the paper is organized as follows. The next section is on the related work. Section III presents the problem setup and establishes some facts about intersecting unit disks. Section IV introduces the two geometric properties that we utilize in our algorithm. Section V presents our polynomial time exact algorithm and Section VI concludes the paper.

II. Related Work

The MFSP problem emerged within the context of network wide broadcast in WANETs. In this section, we present a brief summary of the related problems and refer our readers to [13], [14], [15] for more information on the existing literature on energy efficient broadcast operation in WANETs. The general case of the MFSP problem is an instance of the well-known NP-complete Set Cover problem [7]. Set Cover problem has been extensively studied in the literature and early approximation algorithms have been proposed for both unweighted version by Johnson [16] and by Lovasz [17], and for weighted version by Chvatal [9]. These algorithms give an approximation ratio of $1+ln(\Delta)$ where Δ is the cardinality of the maximum cardinality subset $(\max_{i\leq n}|S_i|)$. In [18], Hochbaum presents an algorithm for the weighted version with an approximation ratio of α where α represents the maximum number of subsets covering an element. The running time of this algorithm is $O(n^3)$. In [19], Bar-Yehuda and Even present an algorithm with a similar approximation ratio but an improved running time of $O(n^2)$. We refer readers to [20] for other approximation algorithms on the Set Cover problem.

The MFSP problem becomes a geometrical problem when we use unit disks to model the coverage area of wireless transmissions. Unit disk graphs (UDGs) are neither perfect nor planar graphs [11]. Thus, efficient algorithms proposed for planar and perfect graphs cannot be applied to UDGs. MFSP problem under the unit disk coverage assumption resembles to the well-known Minimum Dominating Set (MDS) problem. MDS problem for UDGs has been studied extensively. The problem is shown to be NP-complete for UDGs [11]. In [21], Marathe et al. present a linear time approximation algorithm with a constant-factor performance guarantee of 5. In [22], a polynomial-time approximation scheme (PTAS) with $((k+1)/k)^2$ guarantee is given for a constant k in $n^{O(k^3)}$. Minimum Connected Dominating Set (MCDS) problem is a different version of the problem in which the dominating set should be connected. In [23], Cheng et al. presented a PTAS for MCDS problem. In [24], Ambuhl et al. presented constant-factor approximation algorithms for the weighted versions of MDS and MCDS problems. These approximations do not apply to MFSP problem as the dominating nodes in MFSP should be chosen from only 1-hop neighbors.

Another related problem to MFSP problem is *covering with disks* which aims at finding a minimally sized set of unit disks to cover given points on the plane (disks can be placed arbitrarily). This problem is examined in [25] and a $O(l^2(l*\sqrt{2})^2.(2n)^{2(l\sqrt{2})^2+1})$ time approximation algorithm is given with a performance guarantee of $(1+1/l)^2$. The difference between this problem and our problem is in the selection of the disks. This problem selects arbitrary disks to cover given points, but in our problem we are bound to select disks from the set of on-hop neighboring nodes.

Another related problem to MFSP problem is the well known Disk Cover (DC) problem that tries to find a minimal size set of disks (from a given set of disks) to cover a given set of points on a plane [25]. In [26], authors present an algorithm with an approximation ratio of $O(1)$ and running time of $O(c^2 n\log n\log(n/c))$ where c represents the size of the optimal solution. MSFP problem is a special instance of the DC problem where disks are selected from a given set of 1-hop nodes.

Another related work in the context of wireless broadcast is Localized Broadcast Incremental Power Protocol (LBIP) [27]. In LBIP, nodes are assumed to have variable transmission power and the goal is to cover 2-hop neighbors with minimum energy. LBIP involves selection of forwarding nodes as well as determining transmission power levels for such nodes to achieve minimum energy usage. In our current work, we assume fixed transmission power (i.e., unit disk coverage) and our goal is to choose a minimum number of 1-hop neighbors to cover all 2-hop neighbors.

Finally, the most related work to our study in this paper is the previous work by Calinescu et al. [8]. In their work, they propose approximation algorithms to solve the MFSP problem (see Section I-B). In this paper, we use a similar setup and develop the first polynomial time algorithm for minimum forwarding node selection.

III. Preliminaries

A. The Practical Setup of the Problem

Most studies use a unit disk or a sphere to represent the shape of the effective coverage area of wireless transmissions [28]. This assumption, though may not always hold in practice, helps in gaining more insight to the problem within the practical context of wireless transmissions. In this paper we consider a similar setup and assume a unit disk coverage model for wireless transmissions. In addition, as most local knowledge based broadcast approaches [6], our approach requires the availability of 2-hop neighborhood information. The required information includes (1) the identities of the 1-hop and 2-hop neighbors and (2) a radial ordering (which we define in Section III-B) of the 2-hop neighbors with respect to the broadcasting node. The availability of the position information for the nodes is sufficient to compute the radial ordering of the 2-hop neighbors. One simple way of acquiring the position information is to use a GPS unit at each node. Another possibility is to use the distance and angle information between the neighboring nodes. The distance information can be calculated by using the transmission and reception power level within an energy consumption model [29] that is representative for the environment. The angle information between neighboring nodes can be measured by using multiple ultrasound receivers or directional antennas. Recently, Calinescu [30] proposed methods to calculate 2-hop neighborhood information (identities and positions) for the cases where GPS or distance and angle information is available with a message complexity of $O(n)$ where n is the total number of the nodes in the network.

B. Definitions

Consider an instance of MFSP problem at a node v. Let $N(v)$ and $N_2(v)$ represent the 1-hop and 2-hop neighbors of v, respectively. To avoid the introduction of excessive notation, let v also represent the location of the node v in a 2-dimensional space. Similarly, let each set $N(v)$ and $N_2(v)$ represent the set of points that 1-hop and 2-hop neighbors of v are located in 2-dimensional space. The coverage area of node v is a unit disk (i.e., a disk with a origin at point v and radius $r_1 = 1$) represented by D_v. Let $\bar{D}_v$ represent the area of the annulus with an origin at point v and radii $r_1 = 1$ and $r_2 = 2$, i.e., $\bar{D}_v = A(v, r_2) \setminus D_v$ where $A(v, r_2)$ is a disk with origin at point v and radius $r_2 = 2$ (see Figure 1-a). By this definition, we have $N(v) \subseteq D_v$ and $N_2(v) \subseteq \bar{D}_v$. Based on this setup, we introduce several definitions below.

Definition 2 (Radial Order): Radial order is the ordering of a set of points in $\bar{D}_v$ (or the nodes at those points) by using the angle that they make with the origin (point) v. Radial order is a cyclic order. If two or more points make the same angle with v, then their distance to v can be used to put them into a total order.

Consider the example scenario in Figure 1-(a) where $N_2(v) = \{a, b, c, d, e\}$. Starting from the exact south position, the nodes in $N_2(v)$ form a radial order as $(e < d < c < b < a)$. The geometrical properties introduced below and the algorithm presented in Section V-A use the radial ordering of the nodes in $N_2(v)$ in finding an optimal solution. As we discussed in Section III-A, a node v can compute the radial ordering of the nodes in $N_2(v)$ from the collected geographical location information from its neighbors. Therefore, from now on we assume that the radial ordering of the nodes in $N_2(v)$ is known by v.

Definition 3 (Radially Continuous Neighbor (RCN) Interval): One or more points in the area $\bar{D}_v$ that form a continuous interval in the radial order with respect to (w.r.t.) v are said to form a radially continuous neighbor (RCN) interval.

As an example, in Figure 1-(a), $(a > b > c)$ and $(e > a > b)$ form RCN intervals w.r.t. v but $(a > b > d)$ does not as $c \in N_2(v)$ separates this interval into two non-consecutive intervals.

Definition 4 (Radially Continuous Coverage Area (RCCA)): Consider a set $S \subseteq N(v)$. For a node $s \in S$, RCCA of s is a continuous subarea in $\bar{D}_v$, RCCA(s)$\subseteq \bar{D}_v$, such that s is the only node in S that can cover all the points in RCCA(s). A node $s \in S$ may have zero or more RCCAs.

Definition 5 (Connectivity Matrix): Consider an instance of MFPS problem at a node v. Let $N(v) = \{b_1, b_2, \ldots, b_m\}$ and $N_2(v) = \{a_1, a_2, \ldots, a_n\}$ be the 1-hop and 2-hop neighbors of v respectively. A connectivity matrix R is an $m \times n$ matrix that shows the connectivity relation between the nodes in $N(v)$ and $N_2(v)$. For a given $b_i \in N(v)$ and $a_j \in N_2(v)$, $R_{i,j} = 1$ if $a_j \in N(b_i)$ and $R_{i,j} = 0$ otherwise.

Definition 6 (Coverage Matrix): Let $m = |N(v)|$ and $n = |N_2(v)|$ for a node v. Using two hop neighborhood information, v generates a coverage matrix as a $m \times n$ matrix C. Each row in C corresponds to a 1-hop neighbor of v and each column corresponds to a 2-hop neighbor of v. An entry $C_{ef} = (a_p, \bar{p})$ represents the longest RCN interval in $N_2(v)$ that is covered by $e \in N(v)$ and that includes f in it. If $f \notin N(e)$, then $C_{ef} = \varnothing$.

Please see Figure 2 for an example of connectivity and coverage matrices for a node v.

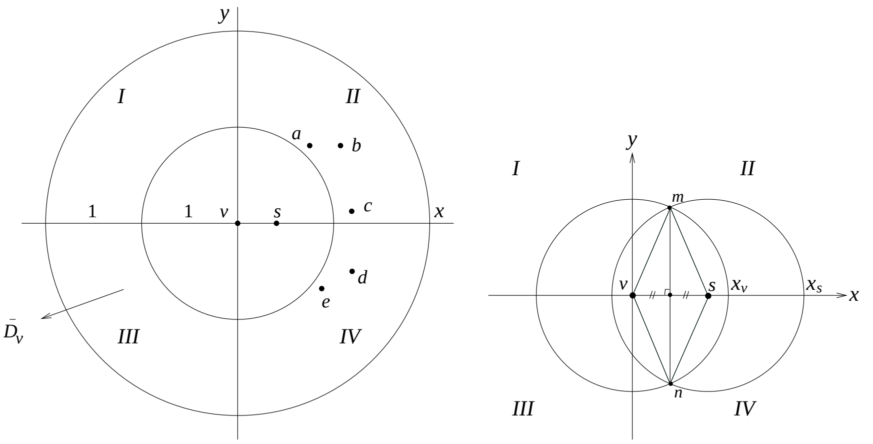
Definition 7 (Maximum Coverage Interval (MCI)): An MCI of a node $s \in N(v)$ is an RCN interval $\{a_i, \ldots, a_j\} \in N_2(v)$ that is completely covered by s such that s cannot cover neither of a_{i-1} and a_{j+1}. Note that s can have multiple MCIs in $N_2(v)$.

Definition 8 (Essential Coverage): Consider a node $s \in S \subseteq N(v)$ that covers a point $a_i \in \bar{D}_v$. s is said to be essential to cover a_i if no other node $t \in S$ covers a_i. If s is essential for a node (at a point) a_i in an MCI that it covers, than s is essential for this MCI. Similarly, a node $s \in S \subseteq N(v)$ is said to be essential to cover an RCCA(s).

Note that the essentiality of s in $\bar{D}_v$ is w.r.t. $S \subseteq N(v)$.

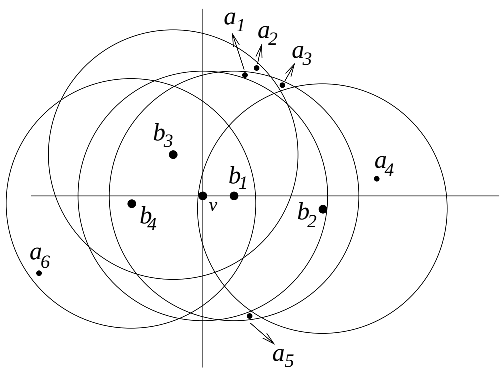
C. Intersection Characteristics of Two Unit Disks

Consider a unit disk centered at a point v. Let D_v represent the unit disk and the area covered by it and C_v represent



(a) (b)

Fig. 1. Some geometric relations of two intersecting unit disks.



Connectivity Matrix

	a_1	a_2	a_3	a_4	a_5	a_6
b_1	1	0	1	0	1	0
b_2	0	0	0	1	0	0
b_3	1	1	0	0	0	0
b_4	0	0	0	0	0	1

Coverage Matrix

	a_1	a_2	a_3	a_4	a_5	a_6
b_1	$(a_1,1)$	$\emptyset$	$(a_3,1)$	$\emptyset$	$(a_5,1)$	$\emptyset$
b_2	$\emptyset$	$\emptyset$	$\emptyset$	$(a_4,1)$	$\emptyset$	$\emptyset$
b_3	$(a_1,2)$	$(a_1,2)$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
b_4	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$(a_6,1)$

Fig. 2. An MFPS instance at v and its connectivity and coverage matrices.

the circle enclosing D_v. Assume that v is at the origin of a two-dimensional space which is divided into four sub-spaces, named as I, II, III, and IV, by the x and y coordinate axes as shown in Figure 1-(a). Consider a second point s that is on the x-axis to the east of v. Similar to the case for v, let D_s and C_s represent the unit disk and the enclosing circle for s.

When $|vs| = 2$, C_v and C_s are tangent to each other and D_v and D_s intersect at a single point. When $|vs| < 2$, C_v and C_s intersect twice and $D_v \cap D_s \neq \emptyset$ as shown in Figure 1-(b). Let m and n represent the intersection points of C_v and C_s. Note that, since s is on x-axis to the east of v, m and n have to be to the east of y-axis. This intersection forms two equal angles as $\widehat{mvn}$ and $\widehat{msn}$. Let α represent these two equal angles. When $|vs| \leq 1$, α is in $[\frac{2\pi}{3}, \pi]$ and when $1 < |vs| \leq 2$, α is in $[0, \frac{2\pi}{3})$. Also note that when $|vs| \leq 1$, the length of the arc $arc(mx_vn)$ (the segment of C_v corresponding to α; here x_v is the intersection of C_v with x axis) is in $[\frac{2\pi}{3}, \pi]$. Similarly, the length of the arc $arc(mx_sn)$ (the segment of C_s corresponding to $2\pi - \alpha$; here x_s is the intersection of C_s with x axis) is in $[\pi, \frac{4\pi}{3}]$. Finally, the line segment connecting m and n vertically is referred to as $Chord_{vs}$ and it divides the line segment between v and s into two equal parts.

D. Intersection Characteristics of Three Unit Disks

In this section, we consider the intersection characteristics of three unit disks in a special setup that is relevant to the MFSP problem. Consider an instance of MFSP problem at a node v and consider $\{s, t\} \in S \subseteq N(v)$. Similar to the above discussion, we assume that v defines a two-dimensional coordinate space and s lies to the exact east of v in this coordinate space. Note that s and t are neighbors of v, and $|vs| \leq 1$, $|vt| \leq 1$, and $|st| \leq 2$. For $|st| < 2$, C_s and C_t intersect twice and $D_s \cap D_t \neq \emptyset$. and for $|st| = 2$, C_s and C_t are tangent to each other, D_s and D_t intersect at a single point. Observe that since s lies to the east of v, the coverage area D_s beyond D_v (i.e., $D_{s/v} = D_s \setminus D_v$) lies in region $(II \cup IV)$. Similarly, let $D_{t/v}$ represent the coverage area D_t beyond D_v, i.e., $D_{t/v} = D_t \setminus D_v$. In addition, let $C_{s/v}$ and $C_{t/v}$ represent the segments of C_s and C_t outside the coverage area D_v, respectively.

From MFSP problem's point of view, we study the nature of the coverage area in $D_{s/v}$ that s is essential for (i.e., number and nature of RCCA(s) w.r.t. S) in the presence of intersections that $C_{s/v}$ may have with $C_{t/v}$. Note that $C_{s/v}$ and $C_{t/v}$ can have zero, one, or two intersections with each other.

Lemma 1: Let $S = \{s, t\} \subseteq N(v)$ in an instance of MFSP problem at v. If $C_{s/v}$ has no intersection with $C_{t/v}$, then

$D_{s/v} \cap D_{t/v} = \varnothing$.

`Proof of Lemma 1`: Let the intersection points of C_v and C_s be m and n. Let x_v^+ and x_s^+ be the points that C_v and C_s intersect x-axis on east of v, respectively (see Figure 3-(a)). Given that $C_{s/v}$ and $C_{t/v}$ have zero intersection, C_t and C_s intersect twice in D_v. Since D_t and D_v are both unit disks, C_t cannot be enclosed in D_v. Therefore, C_t should intersect C_v at two points, say $\bar{m}$ and $\bar{n}$. While tracing C_t clockwise direction, assume $\bar{m}$ is the point C_t enters into D_v and $\bar{n}$ the point that C_t exits D_v. Observe that $\bar{m}$ and $\bar{n}$ cannot be to east of $Chord_{vs}$ (i.e., the line crossing m and n) as otherwise C_t intersects $C_{s/v}$. Assume now that $D_{s/v} \cap D_{t/v} \neq \varnothing$. Since C_t cannot intersect $C_{s/v}$, this is only possible if C_t intersects $arc(mx_s^+n)$ twice where m and n are the intersection points of C_v and C_s. But, since C_v and C_t already intersected at $\bar{m}$ and $\bar{n}$ on west of $Chord_{vs}$, they cannot intersect on $arc(mx_s^+n)$. Hence $D_{s/v} \cap D_{t/v} \neq \varnothing$ is not possible. □

Corollary 1: Let $S = \{s,t\} \subseteq N(v)$ in an instance of MFSP problem at v. If $C_{s/v}$ has no intersection with $C_{t/v}$, then $D_{s/v}$ is an RCCA of s w.r.t. S.

Lemma 2: Let $S = \{s,t\} \subseteq N(v)$ in an instance of MFSP problem at v. If $C_{s/v}$ has one intersection with $C_{t/v}$, then s is essential to cover one single RCCA in $D_{s/v}$ w.r.t. S.

`Proof of Lemma 2`: From the previous section, the intersection of two disks D_s and D_t results in three coverage areas as (1) $D_s \setminus D_t$, (2) $D_t \setminus D_s$, and (3) $D_s \cap D_t$. We consider the parts of these coverage areas in $\bar{D}_v$ namely $D_{s/v/t} = (D_s \setminus D_t) \cap \bar{D}_v$, $D_{t/v/s} = (D_t \setminus D_s) \cap \bar{D}_v$, and $D_{st/v} = (D_s \cap D_t \cap \bar{D}_v)$, respectively (see Figure 3-(b)). Let p be the intersection point of $C_{s/v}$ and $C_{t/v}$. Consider a line l that originates at v and crosses p as in Figure 3-(b). The line l divides $(D_{s/v} \cup D_{t/v}) \cap \bar{D}_v$ into two areas such that the radial order of the points at both sides of l are disjoint from each other. In this case, s is essential to cover one RCCA that includes $D_{s/v/t}$ and part of $D_{st/v}$ below line l and t is essential to cover one RCCA that includes $D_{t/v/s}$ and part of $D_{st/v}$ above line l as in Figure 3-(b). □

Lemma 3: Let $S = \{s,t\} \subseteq N(v)$ in an instance of MFSP problem at v. If $C_{s/v}$ has two intersections with $C_{t/v}$, then s is essential to cover one or two RCCAs in $D_{s/v}$ w.r.t. S.

`Proof of Lemma 3`: If $C_{t/v}$ and $C_{s/v}$ intersect twice, C_t cannot intersect C_s in $D(v)$. Let m_1 and n_1 be the intersection points of $C_{t/v}$ and $C_{s/v}$ and $\bar{m}$ and $\bar{n}$ be the intersection points of C_v and C_t. We have two cases: (1) both $\bar{m}$ and $\bar{n}$ are on $arc(mx_v^+n)$ or (2) both $\bar{m}$ and $\bar{n}$ are on $arc(mx_v^-n)$. Assume the contrary that $\bar{m}$ (or $\bar{n}$) is on $arc(mx_v^+n)$ and $\bar{n}$ (or $\bar{m}$) is on $arc(mx_v^-n)$. This requires that $arc(\bar{m}\bar{n})$ intersects C_s in D_v which contradicts that C_t and C_s intersect twice in $\bar{D}_v$. We now examine the two cases.

`Case 1` ($\bar{m}$ `and` $\bar{n}$ `are on` $arc(mx_v^+n)$)`:` The arc $arc(\bar{m}x_t^-\bar{n})$ is in $D_v \cap D_s$. Assume the contrary that $arc(\bar{m}x_t^-\bar{n})$ is in $D_{v/s}$. Consider a walk on $arc(\bar{m}x_t^-\bar{n})$ starting at point x_t^- in clockwise direction. Given that C_t and C_v intersects at $\bar{m}$ and $\bar{n}$ on $arc(mx_v^+n)$, we cannot intersect $arc(mx_v^-n)$ while walking on C_t clockwise. Similarly, since C_t and C_s intersect at m_1 and n_1 on $arc(mx_s^+n)$, we cannot intersect $arc(mx_s^-n)$ while walking on C_t clockwise. This then makes it impossible to complete the walk as the $arc(\bar{m}x_t^-\bar{n})$ is assumed to be in $D_{v/s}$. As a result, $arc(\bar{m}x_t^-\bar{n}) \notin D_{v/s}$. Consider a walk on $arc(\bar{m}x_t^-\bar{n})$ starting at point x_t^- in clockwise direction. Note that we cannot intersect $arc(mx_s^-n)$ but intersect $arc(mx_v^+n)$ at a point $\bar{m}$. In our walk, before we intersect $\bar{n}$, we have to intersect m_1 and n_1 as C_t has to intersect C_s twice on $arc(mx_s^+n)$. Let the first intersection be m_1 and the second one be n_1. Finally, we intersect $\bar{n}$ to get back into $D_v \cap D_s$ and reach back to x_t^-. An example of the resulting coverage scenario for D_v, D_s and D_t is given in Figure 3-(c).

Consider the intersection points m_1 and n_1 and let l_1 and l_2 be two lines that originate at v and cross m_1 and n_1 respectively. Observe that l_1 and l_2 divide $D_{s/v} \cup D_{t/v}$ into three RCCAs such that part of $D_{s/v} \cup D_{t/v}$ between l_2 and l_1 is an RCCA that t is essential to cover and the parts of $D_{s/v} \cup D_{t/v}$ from n to l_2 and from l_1 to m are two RCCAs that s is essential to cover w.r.t. S.

`Case 2` ($\bar{m}$ `and` $\bar{n}$ `are on` $arc(mx_v^-n)$)`:` Given that s is on positive x-axis, t is in $(II \cup IV)$ as otherwise $C_{t/v}$ and $C_{s/v}$ cannot intersect twice. This requires that $x_t^- \in D_v$. Note that $x_t^- \notin (D_v \cap D_s)$ as otherwise C_t intersects either $arc(mx_s^-n)$ or $arc(mx_v^+n)$ contradicting our assumptions that C_t intersects C_s on $arc(mx_s^+n)$ or that C_t intersects C_v on $arc(mx_v^-n)$, respectively. A walk on C_t starting at x_t^- in clockwise direction first intersects C_v at a point $\bar{m}$ as it exits D_v. It needs to then intersect C_s at m_1 and n_1 before it enters into D_v at a point $\bar{n}$. Assume the contrary that it enters D_v at $\bar{n}$ without intersecting C_s outside D_v. This requires that C_t intersect C_s in D_v which is a contradiction. Thus, the sequence of intersections of C_t on this walk is $\bar{m}$, m_1, n_1, and $\bar{n}$. An example of the resulting coverage scenario for D_v, D_s and D_t is given in Figure 3-(d).

Consider the intersection points m_1 and n_1 and let l_1 and l_2 be two lines that originate at v and cross m_1 and n_1 respectively. Observe that l_1 and l_2 divide $D_{s/v} \cup D_{t/v}$ into three RCCAs such that part of $D_{s/v} \cup D_{t/v}$ between l_2 and l_1 is an RCCA that s is essential to cover and the parts of $D_{s/v} \cup D_{t/v}$ from n to l_2 and from l_1 to m are two RCCAs that t is essential to cover w.r.t. S. □

Lemma 4: Let $S = \{s,t\} \subseteq N(v)$ in an instance of MFSP problem at v. If $C_{s/v}$ has two intersections with $C_{t/v}$, then $t \in D_s$.

`Proof of Lemma 4`: Recall that s is on positive x-axis and the two intersection points m and n for C_v and C_s are on east of y-axis. This implies that $|arc(mx_s^+n)|$ is in $(\pi, \frac{4}{3}\pi)$. Given that $C_{t/v}$ intersects $C_{s/v}$ twice at points m_1 and n_1, $arc(m_1x_s^+n_1)$ is a segment of $arc(mx_s^+n)$ and $|arc(m_1x_s^+n_1)| \leq |arc(mx_s^+n)|$. From Section III-C, when C_s and C_t intersect twice, we have two cases: (1) $1 < |ts| < 2$ resulting that $|arc(m_1x_s^+n_1)|$ in $(\frac{4}{3}\pi, 2\pi)$ and (2) $|ts| < 1$ resulting that $|arc(m_1x_s^+n_1)|$ in $(\frac{2}{3}\pi, \frac{4}{3}\pi)$. The case $1 < |ts| < 2$ is not possible as it requires $|arc(m_1x_s^+n_1)|$ to be in $(\frac{4}{3}\pi, 2\pi)$ whereas we have $|arc(m_1x_s^+n_1)| \leq |arc(mx_s^+n)| < \frac{4}{3}\pi$. As a result, $|ts| < 1$ and $t \in D_s$. □

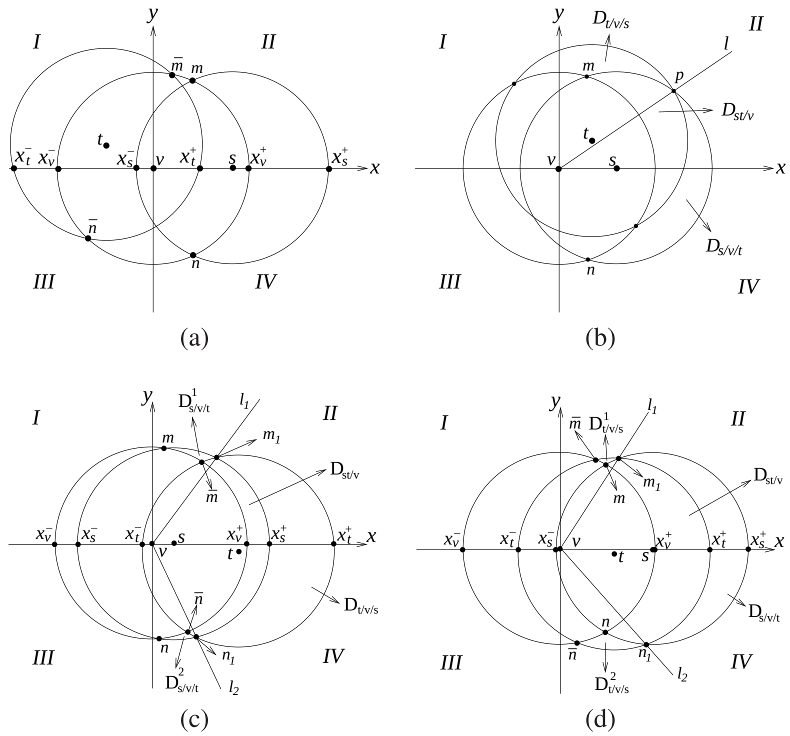


Fig. 3. Some geometric relations of three intersecting unit disks.

IV. Geometrical Properties

A. Two-Set Property

Theorem 1 (Two-Set Property): Given an instance of the MFSP problem (i.e., a node v and its 1-hop and 2-hop neighbor sets $N(v)$ and $N_2(v)$), the coverage relation presented in Figure 4-(a) is not possible.

`Proof of Theorem 1`: The scenario in Figure 4-(a) is related to coverage relation between b_1, b_2 and b_3 all in $N(v)$. In the figure $\{a_1, a_2, \ldots, a_6\} \in N_2(v)$ are a subset of radially ordered 2-hop neighbors of v. Observe that, in this setup, b_1 covers three disjoint MCIs, MCI_1 including a_1, MCI_3 including a_3, and MCI_5 including a_5. Let m and n be the intersection points between C_{b_1} and C_v, m_1 and n_1 be the ones between C_{b_1} and C_{b_2}, and m_2 and n_2 be the ones between C_{b_1} and C_{b_3}. Figure 4-(b) shows an example scenario corresponding to the coverage relation presented in Figure 4-(a). In the figure, the lines between b_is and a_is indicate the coverage of b_i on a_i.

Now, from Lemma 4, $\{b_2, b_3\} \in N(b_1)$. From the discussion in Section III-C, $|arc(mn)| < \frac{4}{3}\pi$. Similarly, the coverage relation in Figure 4-(a) requires that $|arc(m_1n_1)| > \frac{2}{3}\pi$ and $|arc(m_2n_2)| > \frac{2}{3}\pi$ contradicting $|arc(mn)| < \frac{4}{3}\pi$. As a result, the coverage relation shown in Figure 4-(a) is not possible. □

Assumption 1: Assume that the coverage relation presented in Figure 4-(c) is not possible.

The reason for this assumption is that our algorithm considers the interval of 2-hop nodes in $N_2(v)$ (i.e., (a_i, n)) as a *non-circular* interval, i.e., it ignores that a_i follows a_{i+n-1} in circular order. Note that, considering circularity of the nodes in $N_2(v)$, Figure 4-(a) implies that b_1 cannot be essential for more than two MCIs in an optimal solution. Similarly, assuming non-circularity, Figure 4-(c) also implies the same property for b_1.

B. Non-Interleaving Property

Definition 9 (Interleaving Coverage): Consider two nodes $\{b_1, b_2\} \in N(v)$ in an instance of MFSP problem at v. Assume b_1 covers $\{a_1, a_3\} \in N_2(v)$ but does not cover $\{a_2, a_4\} \in N_2(v)$. Similarly, assume b_2 covers $\{a_2, a_4\} \in N_2(v)$ but does not cover $\{a_1, a_3\} \in N_2(v)$. Finally, assume that the radial order between the nodes in $N_2(v)$ is as $(a_1 > a_2 > a_3 > a_4)$. The coverage of this form between the nodes b_1 and b_2 is called an interleaving coverage.

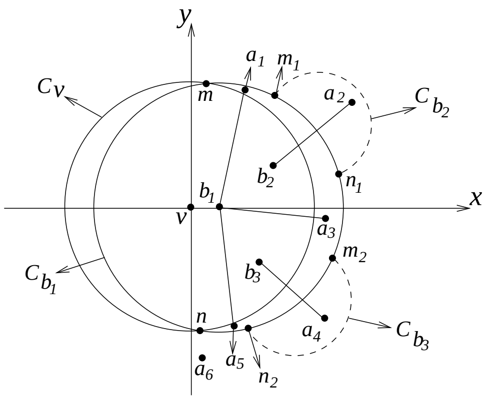
Theorem 2 (Non-Interleaving Property): In an instance of the MFSP problem (i.e., a node v and its 1-hop and 2-hop neighbor sets $N(v)$ and $N_2(v)$), no two nodes $\{b_1, b_2\} \in N(v)$ can have interleaving coverage, i.e., the connectivity matrix in Figure 4-(d) is not feasible.

`Proof of Theorem 2`: Note that interleaving is considered between *any* two nodes $\{b_1, b_2\} \in N(v)$. If $C_{b_1/v}$ and $C_{b_2/v}$ intersect zero times, b_1 and b_2 both have two disjoint coverage areas. If $C_{b_1/v}$ and $C_{b_2/v}$ intersect once, based on Lemma 2, b_1 (and b_2) covers a single RCCA. When this RCCA includes some node $a_i \in N_2(v)$, then b_1 (and b_2) has one single MCI (including such node a_i). Finally, when they intersect twice, based on Lemma 3, one of the nodes, say b_1, can cover two MCIs and the other node b_2 can cover one MCI. Note that, as shown in Figure 3-(c), the two intersections radially separate the coverage areas $D^1_{b_1/v/b_2}$, $D_{b_2/v/b_1}$, and $D^2_{b_1/v/b_2}$ and no points $q \in D_{b_2/v}$ can exist at west of the two areas $D^1_{b_1/v/b_2}$ and $D^2_{b_1/v/b_2}$. □

Connectivity Matrix

	a_1	a_2	a_3	a_4	a_5	a_6
b_1	1	0	1	0	1	0
b_2	0	1	0	*	*	*
b_3	*	*	0	1	0	*

(a)



(b)

Connectivity Matrix

	a_1	a_2	a_3	a_4	a_5
b_1	1	0	1	0	1
b_2	0	0	0	1	0
b_3	0	1	0	0	0

(c)

Connectivity Matrix

	a_1	a_2	a_3	a_4
b_1	1	0	1	0
b_2	0	1	0	1

(d)

Fig. 4. Unfeasible connectivity scenarios.

V. Solution to MFSP Problem

A. Construction

In this section, we present a dynamic programming algorithm to solve the MFSP problem under unit disk coverage model. The algorithm consists of two parts $ALG1$ and $ALG2$ as shown in Figure 5. In an instance of MFSP problem at a node v, all 2-hop neighbors of v form a circular interval of length $n = |N_2(v)|$. $ALG1$ works with non-circular intervals. $ALG1$ assumes that Assumption 1 and Theorem 2 hold for a given (a_i, n). Hence, the solution of $ALG1$ for an interval not satisfying Assumption 1 may not be optimal.

Let $S_{min}(a_i, j)$ be a list of 1-hop neighbors of v that cover the interval (a_i, j). $ALG1$ uses $N_{min}(a_i, n) = |S_{min}(a_i, n)|$ for ease-of-presentation. Below are the steps of the algorithm in finding a solution for an interval (a_i, j) where $\{i, j\} \in [1, n]$:

1) **Step 1:** The best possible solution for (a_i, j) is that the entire interval is covered by a single node $e \in N(v)$. This can be checked by searching the column $f = a_i$ of the coverage matrix. If there exists an MCI $C_{ef} = (a_p, \bar{p})$ that completely includes the interval (a_i, j), then the corresponding one hop neighbor e can be assigned to cover the interval (a_i, j) in the solution. Since this is an assignment with minimum size, i.e., $N_{min}(a_i, j) = 1$, there is no need to check for the other cases below.
2) **Step 2:** In this step, we split the interval (a_i, j) to two consecutive sub-intervals as (a_i, k) and $(a_{i+k},\ j - k)$. We can combine the optimal solutions of these intervals and this will be a solution to (a_i, j). We consider each possible case for splitting the interval (a_i, j) into two intervals. There are $j - 1$ possible cases. Since we are interested in minimum cardinality solution, we take the minimum one in cardinality.
3) **Step 3:** In this step, we pick a special 1-hop node s which covers a_i and a_{i+j-1} (end nodes of (a_i, j)) and find the MCIs $(a_p, \bar{p})$ and $(a_q, \bar{q})$ that s covers such that $a_i \in (a_p, \bar{p})$ and $a_{i+j-1} \in (a_q, \bar{q})$. A solution in this case can be given by $\{s\} \cup S_{min}(a_r, \bar{r})$ where $(a_r, \bar{r}) = \{a_{p+\bar{p}}, \ldots, a_{q-1}\}$. We find such solutions for all possible s and save it as the optimal solution if it is better than the current solution.

Starting from $j = 1$, $ALG1$ applies the above procedure for all intervals of lengths up to $j = n$. For an interval (a_i, j), it considers possible solution scenarios by applying the above procedure. Among those solutions, it chooses the one that gives the minimum size solution as $S_{min}(a_i, j)$. At the end, the algorithm returns a solution as $S_{min}(a_i, n)$. The running time of the algorithm is $O(n^3 + n^2m)$ where $O(n^3)$ comes from Step 2 and $O(n^2m)$ comes from Steps 1 and 3. $ALG2$ calls $ALG1$ n times for (a_i, n) with $i = [1, n]$ resulting in the overall complexity $O(n^4 + n^3m)$. The overall run time complexity can be reduced to $O(n^3 + n^2m)$ by modifying $ALG1$ to compute $S_{min}(a_i, n)$ for all (a_i, n) in one call by changing the line 8 to `"FOR i:=1 to n"` and by having $ALG2$ to choose the minimum size $S_{min}(a_i, n)$ returned by $ALG1$. The space complexity of the algorithm is $\Theta(n^2k)$ where k represents an upper bound for the number of forwarding nodes in an optimal solution ($k < min(n, m)$). This bound can be reduced to $\Theta(n^2)$ by saving special indices instead of forwarding node sets for each interval. For each interval, after calculating the optimal solution, we save an index to represent how the optimal solution is found. If the optimal solution is found in Step 1, the 1-hop node covering the whole interval is saved. For Step 2, the optimal split point is saved. For Step 3, 1-hop node and the identity of the middle interval is saved. In this way, the entry for each interval is in constant size and space complexity is $\Theta(n^2)$. In this method interval entries should be traversed back to find the elements of forwarding set. Recall that 2-hop neighborhood information is an input to the algorithm and Calinescu [30] proposes methods to calculate 2-hop neighborhood information with a message complexity of $O(n)$. After the execution of the algorithm, the node v includes the identities of the selected forwarding nodes into the broadcast message. Hence, this step does not incur any additional message overhead.

B. Proof of Correctness: Part 1

Theorem 3: Given an (a_i, j) where Assumption 1 and Theorem 2 hold, $ALG1$ finds an optimal solution to (a_i, j) provided that optimal solutions to all continuous subintervals of (a_i, j) are known.

ALG1 {**Input:** $index$, $N(v) = \{b_1, \dots, b_m\}$, $N_2(v) = \{a_1, \dots, a_n\}$, C={$m \times n$ coverage matrix}}
{
    /* initialization */
    FOR $i := 1$ to n
        $N_{min}(a_i, 1) := 1$, $L_{min}(a_i, 1) := (b)$, $S_{min}(a_i, 1) := L_{min}(a_i, 1)$ where $b \in N(v)$ and $a_i \in N(b)$

    /* main body of the algorithm */
    FOR $j := 2$ to n /* for each round */
        FOR $i := index$ to $(index + n - j)$ /* for each interval (a_i, j)*/
        $N_{min}(a_i, j) := \infty$, $L_{min}(a_i, j) := \emptyset$

        /* Step 1: */
        FOR $e := 1$ to m
            IF C_{ei} includes (a_i, j) as a subinterval **THEN**
                $N_{min}(a_i, j) := 1$, $L_{min}(a_i, j) := (b_e)$

        IF $(N_{min}(a_i, j) \neq 1)$ **THEN**
            /* Step 2: Check for alternative solutions */
            FOR $k := 1$ to $j - 1$ /* for all possible splits of the interval (a_i, j) */
                IF $(N_{min}(a_i, k) + N_{min}(a_{i+k}, j - k) < N_{min}(a_i, j))$ **THEN**
                    $N_{min}(a_i, j) := N_{min}(a_i, k) + N_{min}(a_{i+k}, j - k)$
                    $L_{min}(a_i, j) := (S_{min}(a_i, k)) + (S_{min}(a_{i+k}, j - k))$ /* + is list append */

            /* Step 3: Check if a $b_e \in N(v)$ covers both ends of interval (a_i, j) */
            FOR $e := 1$ to m
                Consider coverage matrix entries $C_{ei} = (a_p, \bar{p})$ and $C_{e(i+j-1)} = (a_q, \bar{q})$
                IF $(C_{ei} \neq \varnothing)$ **AND** $(C_{e(i+j-1)} \neq \varnothing)$ **THEN**
                    Let $(a_r, \bar{r}) = (a_{p+\bar{p}}, a_{p+\bar{p}+1}, \dots, a_{q-1})$ be the subinterval of (a_i, j) that b_e does not cover
                    IF $(N_{min}(a_r, \bar{r}) + 1 < N_{min}(a_i, j))$ **THEN**
                        $N_{min}(a_i, j) := N_{min}(a_r, \bar{r}) + 1$
                        $L_{min}(a_i, j) := (S_{min}(a_r, \bar{r})) + (b_e)$ /* + is list append */

        $S_{min}(a_i, j) = L_{min}(a_i, j)$
    /* End of FOR */
    Return $S_{min}(a_{index}, n)$
}

ALG2 {**Input:**$N(v) = \{b_1, \dots, b_m\}$, $N_2(v) = \{a_1, \dots, a_n\}$, C={$m \times n$ coverage matrix}}
{
    $MFS(v)$=**ALG1**{1, $N(v)$, $N_2(v)$, C}
    FOR $i := 2$ to n
        Sol=**ALG1**{i, $N(v)$, $N_2(v)$, C}
        IF $|Sol| < |MFS(v)|$ **THEN**
            $MFS(v) = Sol$
    Return $MFS(v)$
}

Fig. 5. Outline of the algorithms $ALG1$ and $ALG2$.

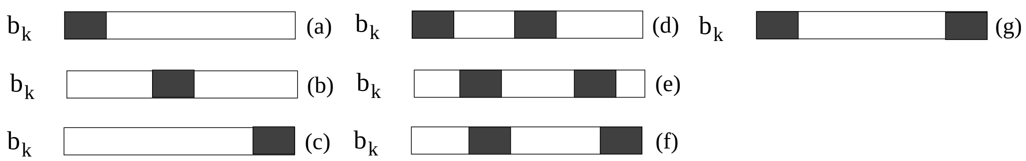


Fig. 6. Possible MCIs b_k is essential for in S.

`Proof of Theorem 3`: Let S be an optimal solution for (a_i, j). Let $b_k \in S$ be a node that covers the largest MCI starting from a_i, i.e., b_k covers $(a_i, \bar{x})$ and for any $b_l \neq b_k \in S$ covering an MCI $(a_i, \tilde{x})$, $\tilde{x} \leq \bar{x}$. Note that we know only the existence of such a b_k and do not need to know its identity. We analyze the relations between the coverage characteristics of b_k and all other nodes in S in all possible coverage cases. Note that b_k is essential for at least one and at most two MCIs in (a_i, j). It is essential for at least one MCI because it is in S. It is essential for at most two MCIs by Assumption 1. Figure 6 presents possible configurations of MCIs where b_k is essential for in S. In the following, we first show that the cases in Figure 6-(e),(f) are not possible and then present how our algorithm handles the other cases.

Claim 1: The MCIs that b_k is essential for cannot be as in Figure 6-(e),(f).

`Proof of Claim 1`: Assume that this is not the case and b_k is essential for two MCIs $(a_z, \bar{z})$ and $(a_d, \bar{d})$ as in Figure 7-(a) or -(b). Note that these cases correspond to the cases in Figure 6-(e),(f). Let $b_l \in S$ be a node covering a_y and $b_m \in S$ be a node covering a_c in the figure. Note that since b_k does not cover either a_y or a_c, $b_l \neq b_m$ due to non-interleaving property. Note also that since b_k covers the largest MCI $(a_i, \bar{x})$, b_l cannot cover this MCI completely. Finally, since b_k is assumed to be essential for $(a_z, \bar{z})$ and $(a_d, \bar{d})$, neither b_l nor b_m can cover these MCIs completely. This then causes a violation of Assumption 1 by b_k where the coverage relation between b_k, b_l, and b_m is similar to the one in Figure 4-(c). □

Case 1: Assume b_k is essential for only one MCI on one of the ends of the interval, e.g., $(a_i, \bar{x})$ as in Figure 7-(a) (or $(a_d, \bar{d})$ as in Figure 7-(b)). We explain the behavior of the algorithm using the first case and the same argument applies for the second case. Since b_k is not essential for any other nodes in $(a_y, j - \bar{x})$, $S \setminus \{b_k\}$ covers $(a_y, j - \bar{x})$. Consider an optimal solution $S_{opt}(a_y, j - \bar{x})$ for the interval $(a_y, j - \bar{x})$. Note that $|S_{opt}(a_y, j - \bar{x})|$ cannot be larger than $|S \setminus \{b_k\}|$ as the former is an optimal solution and the latter is a solution covering $(a_y, j - \bar{x})$, i.e., $|S_{opt}(a_y, j - \bar{x})| \leq |S \setminus \{b_k\}| = |S| - 1$. Similarly, an optimal solution $S_{opt}(a_i, \bar{x})$ for the interval $(a_i, \bar{x})$ can not be larger than $|\{b_k\}|$ as the former is an optimal solution and the latter is a solution covering $(a_i, \bar{x})$. This makes $S_{opt}(a_i, \bar{x}) \cup S_{opt}(a_y, j - \bar{x})$ an optimal solution for (a_i, j) as we assumed that S is an optimal solution for (a_i, j).

This case covers the essential coverage scenarios of b_k in S as presented in Figure 6-(a),-(c). If the optimal solution S is of this nature, since we know the optimal solutions for all intervals of length up to $j - 1$, the presented algorithm finds this solution in Step 2 by looking at each split points for the interval (a_i, j).

Case 2: Assume that b_k is essential for two MCIs $(a_i, \bar{x})$ and $(a_d, \bar{d})$ at both ends of (a_i, j) as shown in Figure 7-(b). In this case, $S \setminus \{b_k\}$ covers $(a_y, j - (\bar{x} + \bar{d}))$ since b_k is not essential for that part. Consider an optimal solution $S_{opt}(a_y, j - (\bar{x} + \bar{d}))$ for the interval $(a_y, j - (\bar{x} + \bar{d}))$. Note that $|S_{opt}(a_y, j - (\bar{x} + \bar{d}))|$ cannot be larger than $|S \setminus \{b_k\}|$ as the former is an optimal solution and the latter is a solution $(a_y, j - (\bar{x} + \bar{d}))$, i.e., $|S_{opt}(a_y, j - (\bar{x} + \bar{d}))| \leq |S \setminus \{b_k\}| = |S| - 1$. This then requires that $\{b_k\} \cup S_{opt}(a_y, j - (\bar{x} + \bar{d}))$ is an optimal solution for (a_i, j) as we started with an assumption that S is an optimal solution for (a_i, j), i.e., $|S_{opt}(a_y, j - (\bar{x} + \bar{d}))| + |\{b_k\}| \leq |S|$.

This case covers the essential coverage scenarios of b_k in S as presented in Figure 6-(g). If the optimal solution S is of this nature, since we know the optimal solutions for all intervals of length up to $j - 1$, the presented algorithm finds this solution in Step 3.

Case 3: Assume that b_k is essential for two MCIs $(a_i, \bar{x})$ and $(a_z, \bar{z})$ as shown in Figure 7-(a) or -(b). In this scenario, we divide the nodes in S into two subgroups, S_1 and S_2 in a way that $S_1 \cup S_2 = S$ and $S_1 \cap S_2 = \varnothing$. Our goal is to create S_1 and S_2 such that S_1 covers $(a_i, \bar{x} + \bar{y} + \bar{z})$ and S_2 covers $(a_c, j - (\bar{x} + \bar{y} + \bar{z}))$. When we assign a node b_x to S_1 (or S_2), if we can guarantee that there exist another node b_y in S_2 (or S_1) which covers all the nodes that b_x covers in $(a_c, j - (\bar{x} + \bar{y} + \bar{z}))$ (or $(a_i, \bar{x} + \bar{y} + \bar{z})$), then this partition will give two sets S_1 and S_2 with the desired properties. Note that b_k is the only node covering $(a_i, \bar{x})$ in S as in Figure 7-(a) or -(b). Then, there is a node, say b_m, in S which covers a_c.

Claim 2: b_k should cover all nodes that b_m covers in $(a_i, \bar{x} + \bar{y} + \bar{z})$.

`Proof of Claim 2`: b_k covers a node in $(a_i, \bar{x})$ and a node in $(a_z, \bar{z})$ that are not covered by b_m. b_m covers a_c that is not covered by b_k. If b_k does not cover a node that b_m covers in $(a_y, \bar{y})$, this leads to an interleaving coverage between b_k and b_m contradicting Theorem 2. □

Claim 3: b_m should cover all the nodes that b_k covers in $(a_c, j - (\bar{x} + \bar{y} + \bar{z}))$.

`Proof of Claim 3`: Recall that b_k does not cover a_y. Let b_l be a node covering a_y. Note that $b_l \neq b_m$ as otherwise b_k and b_l have an interleaving coverage. Now, b_k covers nodes in $(a_i, \bar{x})$ and in $(a_z, \bar{z})$ which b_l does not cover. Note also that b_m covers a_c which is not covered by b_k. If b_m does not cover a node $a_d \in (a_c, j - (\bar{x} + \bar{y} + \bar{z}))$ $(a_d \neq a_c)$ that b_k covers, then this causes a violation of Assumption 1 where the coverage relation between b_k, b_l, and b_m is similar to the one in Figure 4-(c). Thus, b_m covers all such $a_d \in (a_c, j - (\bar{x} + \bar{y} + \bar{z}))$. □

Based on Claims 2 and 3, we can put b_k into S_1 and b_m into S_2. For any other node b_o:

1) If b_o covers a node in $(a_c, j - (\bar{x} + \bar{y} + \bar{z}))$ which is not covered by b_k, then $b_o \in S_2$ and b_k covers all nodes that b_o covers in $(a_i, \bar{x} + \bar{y} + \bar{z})$ due to Theorem 2.
2) If b_o covers a node in $(a_y, \bar{y})$ which is not covered by b_k, then $b_o \in S_1$ and b_k covers all nodes that b_o covers in $(a_c, j - (\bar{x} + \bar{y} + \bar{z}))$ due to Theorem 2. From Claim 3, b_m covers all nodes that b_k covers in $(a_c, j - (\bar{x} + \bar{y} + \bar{z}))$. Therefore, b_m covers all nodes that b_o covers in $(a_c, j - (\bar{x} + \bar{y} + \bar{z}))$.

Note that b_o should be in one of the above two cases as otherwise $b_o \notin S$. Based on the above construction, S_1 covers $(a_i, \bar{x} + \bar{y} + \bar{z})$ and S_2 covers $(a_c, j - (\bar{x} + \bar{y} + \bar{z}))$. Consider optimal solutions $S_{opt}(a_i, \bar{x} + \bar{y} + \bar{z})$ and $S_{opt}(a_c, j - (\bar{x} + \bar{y} + \bar{z}))$. We have $|S_{opt}(a_i, \bar{x} + \bar{y} + \bar{z})| \leq |S_1|$ and $|S_{opt}(a_c, j - (\bar{x} + \bar{y} + \bar{z}))| \leq |S_2|$. Since $S = S_1 \cup S_2$ is an optimal solution, by Step 2 of $ALG1$, we have $|L_{min}(a_i, j)| \leq |S_{opt}(a_i, \bar{x} + \bar{y} + \bar{z})| + |S_{opt}(a_c, j - (\bar{x} + \bar{y} + \bar{z}))| \leq |S_1| + |S_2| = |S|$.

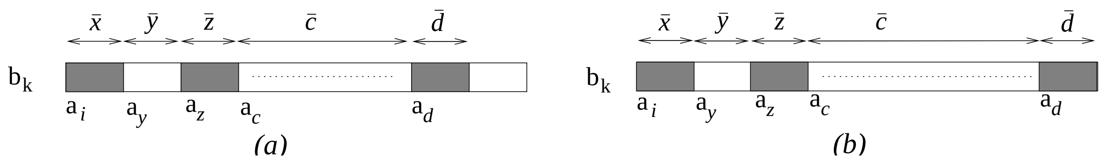


Fig. 7. Coverage characteristics of b_k in S.

This case covers the essential coverage scenarios of b_k in S as presented in Figure 6-(d). If the optimal solution S is of this nature, since we know the optimal solutions for all intervals of length up to $j-1$, the presented algorithm finds this solution in Step 2.

Case 4: We now examine the case where b_k is essential for one interval (e.g., $(a_z, \bar{z})$ in Figure 7) which corresponds to the case in Figure 6-(b). Similar to the above discussion, we again divide S into S_1 and S_2 and try to put nodes b_x into one of these two sets such that $S_1 \cup S_2 = S$ and $S_1 \cap S_2 = \varnothing$. Let $b_m \in S$ be a node covering a_c and $b_l \in S$ be a node covering a_y as in Figure 7. Note that b_l cannot cover $(a_i, \bar{x})$ as otherwise b_k would not be covering the largest interval including a_i. Note also that $b_l \neq b_m$ as otherwise b_k and b_l would have an interleaving coverage.

Claim 4: We claim that b_k or b_l should cover all nodes that b_m covers in $(a_i, \bar{x}+\bar{y}+\bar{z})$.

`Proof of Claim 4`: Note that if b_m does not cover $(a_i, \bar{x})$, then the proof is the same as the proof of Claim 2 above. If b_m covers $(a_i, \bar{x})$, it can cover some node in $(a_y, \bar{y})$ and some node in $(a_c, \bar{c})$ which are not covered by b_k. If this is the case, b_l should cover all the nodes b_m covers in $(a_y, \bar{y})$ as otherwise b_m violates Assumption 1. That is, b_m covers $(a_i, \bar{x})$ and some nodes in $a_{\tilde{y}} \neq a_y \in (a_y, \bar{y})$ which b_l does not cover; and b_l covers a_y which b_m does not cover. Based on this, b_l separates the coverage of b_m into two sets. In addition, b_m covers $a_{\tilde{y}} \neq a_y \in (a_y, \bar{y})$ and a_c which are not covered by b_k and b_k covers $(a_z, \bar{z})$ which b_m does not cover. The coverage scenario among the three nodes is similar to the case in Figure 4-(c) with b_m, b_l, and b_k in the position of b_1, b_2, and b_3 in the figure, respectively. □

Claim 5: b_m covers all nodes b_k covers in $(a_c, j-(\bar{x}+\bar{y}+\bar{z}))$.

`Proof of Claim 5`: b_k covers a node in $(a_i, \bar{x})$ and a node in $(a_z, \bar{z})$ which are not covered by b_l. b_l covers a_y which is not covered by b_k. b_m covers a_c which is not covered by b_k. If b_m does not cover a node that b_k covers in $(a_c, j - (\bar{x}+\bar{y}+\bar{z}))$, this will be a violation of Assumption 1 with a coverage scenario among b_k, b_l, and b_m similar to the case in Figure 4-(c) with b_k, b_l, and b_m in the position of b_1, b_2, and b_3 in the figure, respectively. □

Claim 6: b_k covers all nodes that b_l covers in $(a_c, j-(\bar{x}+\bar{y}+\bar{z}))$.

`Proof of Claim 6`: b_k covers nodes in $(a_i, \bar{x})$ and in $(a_z, \bar{z})$ which are not covered by b_l. b_l covers a_y that b_k does not cover. If b_k does not cover a node b_l covers in $(a_c, j-(\bar{x}+\bar{y}+\bar{z}))$, this will lead to an interleaving coverage between b_k and b_l contradicting Theorem 2. □

Combining Claims 5 and 6, b_m covers all nodes that b_l covers in $(a_c, j-(\bar{x}+\bar{y}+\bar{z}))$. Based on these results, we can put b_k and b_l into S_1 and b_m into S_2. For any other node b_o:

1) If b_o does not cover a node in $(a_y, \bar{y})$ which is not covered by b_k, then $b_o \in S_2$.
2) If b_o does not cover a node in $(a_c, j-(\bar{x}+\bar{y}+\bar{z}))$ which is not covered by b_k, then $b_o \in S_1$. b_m covers all nodes b_o covers in $(a_c, j-(\bar{x}+\bar{y}+\bar{z}))$.
3) If b_o covers nodes from both $(a_y, \bar{y})$ and $(a_c, j-(\bar{x}+\bar{y}+\bar{z}))$ which are not covered by b_k, then $b_o \in S_2$. In this case, b_l covers all nodes b_o covers in $(a_y, \bar{y})$. Now, to have a non-interleaving coverage between b_o and b_k, b_o should cover $(a_i, \bar{x})$ as b_k covers a node in $(a_z, \bar{z})$ that b_o does not cover. Next, if b_l does not cover all nodes b_o covers in $(a_y, \bar{y})$, b_o will violate Assumption 1. This case is similar to the case of b_m in the proof of Claim 4 and is therefore omitted.

Note that b_o should be in one of the above two cases as otherwise $b_o \notin S$. Similar to the discussion above, for optimal solutions $S_{opt}(a_i, \bar{x}+\bar{y}+\bar{z})$ and $S_{opt}(a_c, j-(\bar{x}+\bar{y}+\bar{z}))$, we have $|S_{opt}(a_i, \bar{x}+\bar{y}+\bar{z})| \leq |S_1|$ and $|S_{opt}(a_c, j-(\bar{x}+\bar{y}+\bar{z}))| \leq |S_2|$. Since $S = S_1 \cup S_2$ is an optimal solution, by Step 2 of $ALG1$, we have $|L_{min}(a_i, j)| \leq |S_{opt}(a_i, \bar{x}+\bar{y}+\bar{z})| + |S_{opt}(a_c, j-(\bar{x}+\bar{y}+\bar{z}))| \leq |S_1| + |S_2| = |S|$.

This case covers the essential coverage scenarios of b_k in S as presented in Figure 6-(b). If the optimal solution S is of this nature, since we know the optimal solutions for all intervals of length up to $j-1$, the presented algorithm finds this solution in Step 2. □

As a result, depending on the nature of the optimal solution for (a_i, j), one of the above mentioned four cases correspond to the optimal solution $S_{opt}(a_i, j)$. This concludes the proof of Theorem 3.

Lemma 5: $ALG1$ finds an optimal solution for (a_i, j) if Assumption 1 holds for (a_i, j) and its all continuous subintervals whose solutions contribute to calculate the solution for (a_i, j)[1].

`Proof of Lemma 5`: The proof is by induction on j. For $j = 1$, the optimal solution is found in Step 1 of $ALG1$. For the hypothesis case, we assume that we have the optimal solution for any (a_k, j') for $j' < j$ and $k \in [i, i+j'-1]$ where (a_k, j') contributes to the solution for (a_i, j). By Theorem 3 and the induction hypothesis, since Assumption 1 holds for (a_i, j) and we have the solutions for all contributing subintervals of (a_i, j), $ALG1$ finds an optimal solution for (a_i, j). □

[1]A solution of a subinterval contributes to the solution of (a_i, j) if it is used as part of the selected solution for (a_i, j). As an example, in Figure 8, the solution for $(a_1, 4)$ and $(a_5, 1)$ contribute to the solution of $(a_1, 5)$.

C. Proof of Correctness: Part 2

The example in Figure 8 shows that Assumption 1 may not hold for all (a_i, n). In the example, consider the interval $(a_2, 5)$. In this interval, s is essential to cover three MCIs namely $(a_2, 1)$, $(a_4, 1)$, and $(a_1, 1)$. The problem in this case is that the interval $(a_2, 5)$ starts at the middle of an MCI that s is essential for. Assumption 1 does not hold in this case and the solution returned by $ALG1$ is not optimal. On the other hand, please note that for another interval, $(a_1, 5)$, Assumption 1 is satisfied.

Theorem 4: Given an instance of the MFSP problem (i.e., a node v and its 1-hop and 2-hop neighbor sets $N(v)$ and $N_2(v)$), there exists at least one interval (a_i, n) where $i \in [1, n]$ such that $ALG1$ finds an optimal solution for (a_i, n).

`Proof of Theorem 4`: Let S be an optimal solution for an instance of MFSP problem. Let $b_k \in S$ be essential for an MCI $(a_i, \bar{x})$. In this case, we represent the nodes in $N_2(v)$ as an interval (a_i, n). By the definition of an MCI, since b_k is essential for an MCI including a_i, b_k cannot cover a_{i+n-1}, i.e., the node prior to a_i in circular order. From Theorem 1, $b_k \in S$ may be essential for another MCI $(a_z, \bar{z})$ in (a_i, n).

Case 1: (b_k is essential for $(a_i, \bar{x})$ only). This case is similar to **Case 1** in the previous section. That is, (a_i, n) is divided into two intervals $(a_i, \bar{x})$ and $(a_y, n - \bar{x})$ where $a_y = a_{i+\bar{x}}$. Following the same arguments as in **Case 1** in the previous section, we have $S_{opt}(a_i, n) = S_{opt}(a_i, \bar{x}) \cup S_{opt}(a_y, n - \bar{x})$. $ALG1$ finds an optimal solution for $(a_i, \bar{x})$ in Step 1 as b_k covers $(a_i, \bar{x})$. $ALG1$ finds an optimal solution for $(a_y, n-\bar{x})$ if Assumption 1 holds for $(a_y, n - \bar{x})$ and its all continuous subintervals whose solution contribute to the solution of $(a_y, n-\bar{x})$ by Lemma 5. Assume Assumption 1 does not hold for $(a_y, n - \bar{x})$ or any of its continuous subintervals whose solution contribute to the solution of $(a_y, n-\bar{x})$. Let us call the (sub)interval that violates Assumption 1 $(a_c, \bar{c})$. There should be a node b_m that violates Assumption 1 for $(a_c, \bar{c})$ similar to b_1 in Figure 4-(c). In this case, b_m should cover all the nodes in $(a_i, n) \setminus (a_c, \bar{c})$ as otherwise b_m would violate Theorem 1 in (a_i, n). If $b_m \notin S_{opt}(a_c, \bar{c})$, then it can be discarded. If $b_m \in S_{opt}(a_c, \bar{c})$, then $S_{opt}(a_i, n) = S_{opt}(a_y, n - \bar{x})$ which contradicts that $S_{opt}(a_i, n) = S_{opt}(a_i, \bar{x}) \cup S_{opt}(a_y, n-\bar{x})$. As a result, no b_m can violate Assumption 1 for any $(a_c, \bar{c})$. Since $ALG1$ finds optimal solutions $S_{opt}(a_i, \bar{x})$ and $S_{opt}(a_y, n-\bar{x})$, and $S_{opt}(a_i, n) = S_{opt}(a_i, \bar{x}) \cup S_{opt}(a_y, n - \bar{x})$ holds, $ALG1$ finds optimal solution for (a_i, n) in Step 2.

Case 2: (b_k is essential for $(a_i, \bar{x})$ and $(a_z, \bar{z})$.) This case is similar to **Case 3** in the previous section. That is, (a_i, n) is divided into two intervals $(a_i, \bar{x}+\bar{y}+\bar{z})$ and $(a_c, n-(\bar{x}+\bar{y}+\bar{z}))$ where $a_c = a_{i+\bar{x}+\bar{y}+\bar{z}}$. Since Theorem 1 holds for (a_i, n) and b_k does not cover a_{i+n-1}, we can use the arguments in **Case 3** in the previous section for this case again. From those arguments, we have $S_{opt}(a_i, n) = S_{opt}(a_i, \bar{x} + \bar{y} + \bar{z}) \cup S_{opt}(a_c, n - (\bar{x} + \bar{y} + \bar{z}))$. $ALG1$ finds an optimal solution for $(a_i, \bar{x} + \bar{y} + \bar{z})$ if Assumption 1 holds for $(a_i, \bar{x} + \bar{y} + \bar{z})$ and its all continuous subintervals whose solution contribute to the solution of $(a_i, \bar{x}+\bar{y}+\bar{z})$ by Lemma 5. Assume Assumption 1 does not hold for $(a_i, \bar{x} + \bar{y} + \bar{z})$ or any of its continuous subintervals whose solution contribute to its solution. Let us call this (sub)interval $(a_p, \bar{p})$. There should be a node b_m that violates Assumption 1 for $(a_p, \bar{p})$ similar to b_1 in Figure 4-(c). In this case, b_m should cover all the nodes in $(a_i, n) \setminus (a_p, \bar{p})$ as otherwise b_m would violate Theorem 1 in (a_i, n). If $b_m \notin S_{opt}(a_p, \bar{p})$, then it can be discarded. If $b_m \in S_{opt}(a_p, \bar{p})$, then $S_{opt}(a_i, n) = S_{opt}(a_i, \bar{x} + \bar{y} + \bar{z})$ which contradicts that $S_{opt}(a_i, n) = S_{opt}(a_i, \bar{x}+\bar{y}+\bar{z}) \cup S_{opt}(a_c, n-(\bar{x}+\bar{y}+\bar{z}))$. As a result, no such b_m can violate Assumption 1 in any $(a_p, \bar{p})$. $ALG1$ finds an optimal solution for $(a_c, n - (\bar{x} + \bar{y} + \bar{z}))$ with a similar argument. Since $ALG1$ finds optimal solutions $S_{opt}(a_i, \bar{x} + \bar{y} + \bar{z})$ and $S_{opt}(a_y, n - (\bar{x} + \bar{y} + \bar{z}))$, and $S_{opt}(a_i, n) = S_{opt}(a_i, \bar{x} + \bar{y} + \bar{z}) \cup S_{opt}(a_y, n - (\bar{x} + \bar{y} + \bar{z}))$ holds, $ALG1$ finds optimal solution for (a_i, n) in Step 2. □

Finally, since $ALG1$ finds the optimal solution for at least one (a_i, n), $ALG2$ returns this solution by choosing the minimum size solution returned by $ALG1$.

VI. Conclusions

In this paper, we have studied the minimum forwarding set problem (MFSP) in the context of wireless ad hoc networks. Leveraging the practical characteristics of the application environment, we have proposed a polynomial time algorithm to build an optimal solution to the MFSP problem under the unit disk coverage model for wireless transmission. We expect the work presented in this paper to have an impact on the design and development of new algorithms for several wireless network applications including energy efficient multicast and broadcast protocols; energy efficient topology control protocols; and energy efficient virtual backbone construction protocols for wireless ad hoc networks and sensor networks.

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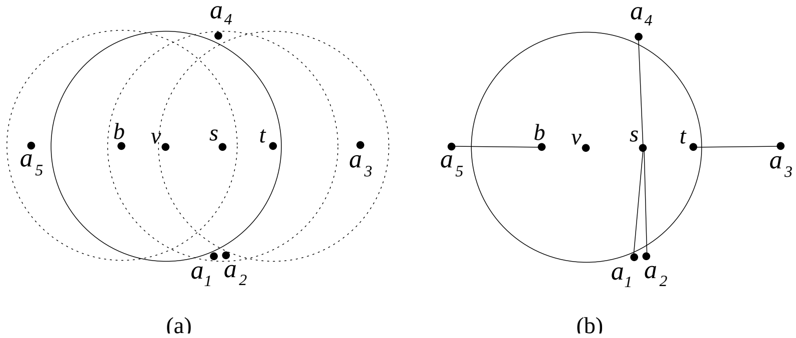

Fig. 8. An example on finalization.

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