

# Low Complexity and Provably Efficient Algorithm for Joint Inter and Intrasession Network Coding in Wireless Networks

Abdallah Khreishah, *Member, IEEE*, Issa Khalil, *Member, IEEE*, and Jie Wu, *Fellow, IEEE*

**Abstract**—The performance of wireless networks can be enhanced by performing network coding on the intermediate relay nodes. To enhance the throughput of large wireless networks, we can decompose them into a superposition of simple relay networks called *two-hop relay networks*. Previously, the capacity region of two-hop relay networks with multiple unicast sessions and limited feedback was characterized where packet erasure channels are used. A near-optimal coding scheme that exploits the broadcast nature and the diversity of the wireless links was proposed. However, the complexity of the scheme is exponential in terms of the number of sessions, as it requires the knowledge of the packets that are received by any subset of the receivers. In this paper, we provide a polynomial time coding scheme and characterize its performance using linear equations. The coding scheme uses random network coding to carefully mix intra and intersession network coding and makes a linear, not exponential, number of decisions. For two-hop relay networks with two sessions, we provide an optimal coding scheme that does not require the knowledge of the channel conditions. We also provide a linear programming formulation that uses our two-hop relay network results as a building block in large lossy multihop networks.

**Index Terms**—Network coding, lossy wireless networks, two-hop relay networks, capacity, fairness

## 1 INTRODUCTION

ONE of the major performance metrics is to maximize the throughput of the users while achieving fairness among them. The term *capacity*, or *capacity region*, of wireless networks can be used in this context [1], as it refers to the set of possible rates that can be achieved by the users simultaneously.

Different works have targeted the characterization of the capacity and the design of different algorithms that can achieve the capacity or a portion of it. These approaches can be classified into three main categories, namely: Cross-layer design, *intrasession network coding* (IANC), and *intersession network coding* (IRNC). The objective of cross-layer design is to jointly optimize the operations at different layers of the OSI reference model through the use of the queue length information at different nodes [2], [3]. IANC exploits link diversity, where links are lossy to maximize the capacity of wireless networks [4], [5], [6], [7], [8]. IRNC [9], [10], [11], [12], [13], [14] mixes packets of different flows to maximize the capacity by exploiting the broadcast nature of wireless links.

Using IANC, intermediate nodes perform coding on packets of the same flow. In IANC, the source node divides

the message it wants to send into batches, each having  $K$  packets of the form  $P_1, \dots, P_K$ . The source node keeps sending coded packets of the form  $\sum_{i=1}^K \alpha_i P_i$ , where  $\alpha_i, \forall i$  is a random coefficient chosen over a finite field of a large enough size, typically  $2^8 \cdot 2^{16}$ . Upon receiving a coded packet, the intermediate relay node checks to see if the coded packet is linearly independent to what it has received before. If so, it keeps the coded packet, otherwise, it drops the packet. When the destination receives any  $K$  linearly independent packets, this means that it can decode all of the packets of the batch. Therefore, it sends feedback to the source and tells it to stop sending from the current batch and to move to the next batch.

IRNC, on the other hand, exploits the broadcast nature of wireless links. Take Fig. 1 as an example; we have two flows, one of them between nodes  $A$  and  $E$  and the other one between nodes  $B$  and  $D$ . If the broadcast nature of wireless links is not exploited, we need four transmissions to send one packet in each flow. The relay node  $C$  can exploit the broadcast nature of its output links and reduce the number of transmissions to three using network coding by XORing the two packets, as shown in the figure.

To operate the network closer to the capacity, the three approaches have to be used jointly. One can think of cross-layer optimization as an orthogonal approach to the other approaches. Therefore, using IANC jointly and optimally with IRNC provides insight into how to achieve the objective of approaching the capacity of wireless networks. The work in [15] shows that achieving the capacity which requires the joint design of IANC and IRNC is NP hard, and linear coding is not sufficient for the problem [16]. Nonetheless, one can limit network coding to be in a single hop [9], where the coding node is a neighbor of the decoding node. Understanding network coding in this simple setting is important, because it provides distributed algorithms with large gains as we will see in the

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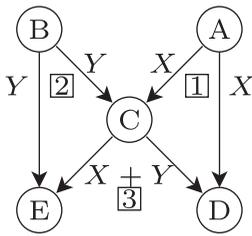


Fig. 1. A two-flow network.

simulations. We call this network setting a *two-hop relay network*.

IANC fails to resolve the bottleneck when different flows are using an intermediate node. On the other hand, most IRNC protocols do not consider lossy links. COPE [9] turns off IRNC when the links have loss rates above 20 percent. Rayanchu et al. [17] studied single-hop IRNC in lossy wireless networks. They considered only XOR operations, did not optimize overhearing, and treated every packet separately, not as a member of a flow. They showed that the problem is  $\#P$ -complete and provided several heuristics.

The work in [18] considered the joint design of IANC and IRNC in wireless networks. However, the benefit was marginal, and no theoretical analysis or guarantees were provided. In our previous work [19], and in [20], the joint IANC and IRNC in lossy two-hop relay networks is considered. The work's optimized overhearing, not limited to XOR, considered flows instead of packets and assumed limited feedback. The capacity region for the problem is characterized using linear equations when the number of sessions is less than three. For more than three sessions, a near-optimal coding scheme is provided, and its performance is characterized using linear equations. The complexity of the near-optimal scheme is exponential. Even though a near-optimal scheme is found, different problems are still open and need investigating.

In this work, we tackle some of these problems, and we have the following contributions: 1) We develop a polynomial time coding scheme for the two-hop relay network problem that makes a linear number of decisions and uses random network coding. We characterize the performance of the polynomial time scheme using linear constraints in terms of the links' delivery ratios. 2) For a two-hop relay network with two sessions, we provide a coding scheme that does not require the explicit knowledge of the delivery ratios of the links, and we show that the scheme achieves the capacity. 3) Since achieving the full capacity region of multihop wireless networks is an open problem, we formulate an achievable rate region for general lossy multihop networks when using any achievable scheme for two-hop relay networks as a building block. The formulation is also in terms of linear equations. We evaluate the effectiveness of our schemes in lossy wireless networks by simulating the different linear equations.

The rest of the paper is organized as follows: We provide examples to represent the benefits of the joint design of IANC and IRNC in Section 2. In Section 3, we describe our settings. We then present the polynomial time algorithm in Section 4. The channel loss oblivious scheme is presented in Section 5. We provide an extension to the multihop

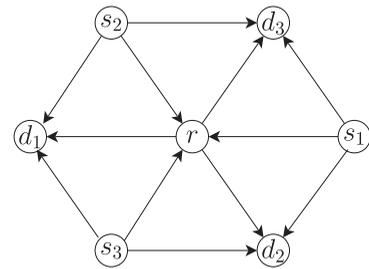


Fig. 2. Two-hop relay network with three flows.

networks case in Section 6 and present our simulation results in Section 7. We conclude the paper in Section 8.

## 2 JOINT IANC AND IRNC EXAMPLE

To illustrate the benefits of mixing IANC and IRNC, we use Figs. 1, 2, and 3 in the supplement document, which can be found on the Computer Society Digital Library at <http://doi.ieeecomputersociety.org/10.1109/TPDS.2012.215>. In all of these figures, source  $s_1$  ( $s_2$ ) wants to send packets to the destination node  $d_1$  ( $d_2$ ), respectively. The delivery rate of all of the links is 0.5. We assume that nodes  $s_1$  and  $s_2$  are scheduled for four time slots each. After that, node  $r$  is given enough time to deliver the packets that it has received from  $s_1$  ( $s_2$ ) to  $d_1$  ( $d_2$ ), respectively.

Fig. 1 represents the IANC solution. In Figs. 1a and 1b, each source node generates four randomly coded packets [21] and sends them in four time slots. Two linearly independent packets from each session are received by the relay node,  $r$ , after the four transmissions by each of the source nodes. Note that performing coding on packets of the same session is limited; thus, the packets overheard by the destination nodes in the first stage are useless. After that, the relay node takes another eight time slots to ensure that the destination nodes can decode their respective packets. We can perform the same operations on the packets  $x_3, x_4, y_3, y_4$ , which means that we need 32 time slots to deliver eight packets. Hence, the achievable total throughput using IANC is  $\frac{8}{32}$ .

Fig. 2 represents the IRNC solution. In this case, no coding is allowed among packets of the same session. Therefore, after four transmissions by  $s_1$ , two of the packets are received by the relay node  $r$  and two are overheard by  $d_2$ . Packet  $x_3$  is received by both  $d_2$  and  $r$ , packet  $x_4$  is neither received by any of  $r$  nor  $d_2$ , while each of  $x_1$  and  $x_2$  are received by a different node. This is due to the independence of the channels, which is illustrated in Fig. 2a. A similar situation occurs in the next four time slots for  $s_2$  packets, as illustrated in Fig. 2b. Since we only allow packets from different sessions to be coded together, the relay node can only code  $x_3$  and  $y_4$ . Therefore, it needs six time slots to deliver two packets to  $d_1$  and another two to  $d_2$ . This means that the achievable throughput would be  $\frac{8}{28}$ .

Fig. 3 represents how to jointly mix IANC and IRNC. The first two steps are similar to the IANC case. In the last stage, represented by Fig. 3c, node  $r$  mixes packets from different sessions together. Now, node  $d_1$  can generate the  $y$  part of the packets it received to decode  $x_1$  and  $x_2$ . Also, node  $d_2$  can generate the  $x$  part of the packets it received to decode  $y_1$  and

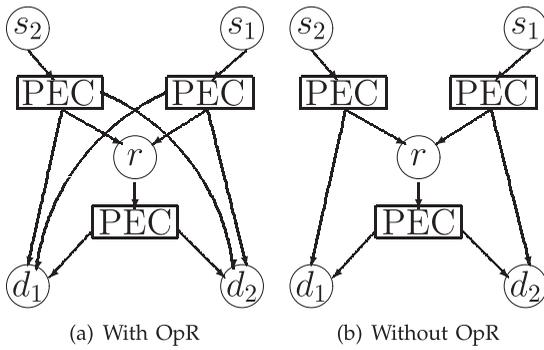


Fig. 3. Illustration of two-session relay networks.

$y_2$ . Therefore, we can achieve a rate of  $\frac{8}{24}$ . This simple example represents the benefits of combining IANC and IRNC.

Note that when IANC and IRNC are used jointly, the relay node is scheduled for the same amount of time as  $s_1$  and  $s_2$ , unlike the separate IANC and IRNC cases. This means that using IANC and IRNC jointly simplifies the design of the MAC. Also, here, the size of the batch is only two packets. When we use larger batches, the benefits of using IANC and IRNC jointly become larger, which we will show in the simulations.

Fig. 2 in this document represents a three sessions network. In this figure, using the same reasoning as above, and assuming that the delivery rates of all of the links is 0.5, IRNC alone achieves a total throughput of  $\frac{12}{44}$  and IANC alone achieves  $\frac{12}{48}$ . When we use IANC and IRNC jointly, the achievable throughput will be  $\frac{12}{32}$ . This example shows that as we increase the number of sessions, the gain of joint IANC and IRNC increases. We will further verify that using the simulations.

### 3 THE SETTING

In an  $N$ -session two-hop relay network, we have  $N$  sessions, each with a source and destination pair, where source  $s_i$  would like to send packets to destination  $d_i$ ,  $\forall i \in \{1, \dots, N\}$  with the help of the relay node  $r$ . Fig. 3a represents a two-session, two-hop relay network. PEC in the figure stands for *packet erasure channel*, such that the sent packet by the source of the PEC can be received by any subset of the receivers of the PEC. Each of  $s_i$  and  $r$  can use the corresponding PEC  $n$  times, respectively. Each of  $s_i$  would like to send  $n \times R_i$  packets, and we are interested in the largest achievable rate vector  $(R_1, \dots, R_N)$  that guarantees decodability of the packets sent by  $s_i$  at  $d_i$ ,  $\forall i$ , with close-to-1 probability for sufficiently large  $n$  and finite field size. In this paper, we use  $p_{uv}$  to represent the delivery ratio of link  $(u, v)$ , and we use  $X_i$  to represent the set of symbols representing the set of packets sent by node  $s_i$ .

To model the “reception report” suggested by practical implementations, we enforce the following sequential, round-based feedback schedule: Each of  $s_i$ ,  $\forall i \in \{1, \dots, N\}$  transmits  $n$  symbols, respectively. After the transmission of  $N \times n$  symbols,  $N$  reception reports are sent from  $d_1, \dots, d_N$ , back to the relay  $r$  so that  $r$  knows which packets have successfully arrived at which destinations. After the reception reports, no further feedback is allowed, and the relay  $r$  has to make its own decision of how to use the available  $n$  PEC usages to guarantee decodability at the

TABLE 1  
Summary of the Symbols Used for the Two-Hop Relay Network

Symbol	Definition
$N$	Number of sessions
$i, j, k, l, m$	Index for a session
$i^c$	Not session $i$
$n$	Number of time slots node $r$ is scheduled
$R_i$	Rate of session $i$
$X_i$	Symbols representing the set of packets for session $i$
$s_i$	Source of session $i$
$d_i$	Destination of session $i$
$r$	Relay node
$t(u)$	Fraction of the time node $u$ is scheduled
$p_{uv}$	Delivery rate between nodes $u$ and $v$
$X_i^J$	The packets sent by $s_i$ and overheard by $d_j$ , $\forall j \in J$ .
$R_i^J$	The rate of the packets represented by $X_i^J$ .

destinations. In our setting, we also assume that the success probability parameters of all PECs and all coding operations are known to all nodes. The only unknown parts are the values of the packets sent by  $s_i$ ,  $\forall i$ .

For the purpose of illustration, a simplified network setting is also depicted in Fig. 3b, in which the packets sent by  $s_i$  will not be overheard by the two-hop-away destination  $d_i$ . For future reference, we say that Fig. 3a admits OpR, as the packets can be overheard by the two-hop destinations, while Fig. 3b does not admit OpR. Without the loss of generality, we assume that  $p_{rd_i} \geq p_{rd_{i+1}}$ ,  $\forall i$ , which can be achieved by relabelling the sessions.

For convenience, we use  $nR_i^{I \setminus J}$  to denote the number of packets that have been received by the destination nodes of the sessions in set  $J$  and not received by the destination nodes of the sessions in set  $I$  after node  $s_i$  sends  $n$  packets. We also use  $X_i^{I \setminus J}$  to refer to the set of symbols representing these packets. For example,  $nR_2^{1 \setminus 3}$  is the number of packets not received by  $d_1$  and received by  $d_3$  when node  $s_2$  sends  $n$  packets. Also,  $X_2^{1 \setminus 3}$  is used to denote the set of the symbols that represent these packets. We also use  $X_i$  to refer to the symbols that represent the packets sent by node  $s_i$ .

The destination node  $d_i$  computes the null space of the packets it has received, randomly chooses a vector from the null space, and sends it back to the relay node. Upon receiving the vector, the relay node multiplies this vector with each of the packets it has received. If the multiplication result is zero, this means that the packet has been received by the destination node  $d_i$  with high probability; otherwise, it means that the packet has not been received by the relay node. The work in [6] makes the false-positive probability very small by using hash tables. This shows that our scheme does not require a perfect channel for the feedback messages. This also shows that our scheme requires a very low number of feedback messages.

Table 1 summarizes the symbols used for the two-hop relay network results used in Sections 4 and 5. To avoid the distraction of the proof details, we present the proofs in the Appendix, available in the online supplemental material.

### 4 LOW-COMPLEXITY ALGORITHM

In this section, we provide a low-complexity coding scheme to be implemented at the relay node. The scheme is described in Algorithm 1:

**Algorithm 1.** Low Complexity Algorithm

- 1:  $W_0 \leftarrow 0$ ;
- 2: **for**  $i \leftarrow 1$  **To**  $N$  **do**
- 3: **if**  $i = N$  **then**
- 4:    $W_i = n$
- 5: **else**
- 6:    $W_i \leftarrow n[\sum_{j:j<i} \frac{1}{Pr_{d_j}} (R_j^{(i+1)^c} + \sum_{k:i>k>j} R_j^{k^c k+1\dots i+1})]$
- 7: **end if**
- 8: Perform random linear network coding RLNC [21] on the packets that belong to the following sets  $\{X_1^{2\dots i}, \dots, X_{i-1}^i, X_i\}$ .
- 9: Send the coded packets in  $(W_i - W_{i-1})$  time slots.
- 10: **end for**

After the sources send their packets, the relay node receives one feedback packet from every destination so that it acquires the knowledge of the overheard packets. Based on this knowledge, the relay node finds the packets that belong to each set of the following:  $\{X_1^{2\dots i}, \dots, X_{i-1}^i, X_i\}$ ,  $\forall i \in \{1, \dots, N\}$ , which we call the  $i$ th collection of sets. For every  $i$ , the relay node performs random linear network coding (RLNC) [21] on the packets of the  $i$ th collection of sets and send the coded packets in  $n \times (W_i - W_{i-1})$  time slots. We call these coded packets the  $i$ th batch  $B_i$ . Also,  $W_i$ ,  $\forall i$  are auxiliary variables that we will use in the proofs. Note that we mix both IANC and IRNC here by coding packets of the same session and others among different sessions. The following theorem characterizes the performance of Algorithm 1. Note that in all of the results we have, we have the following constraint:  $R_i \leq p_{s_i r}$ . We do not explicitly write this constraint in the results for the sake of brevity.

**Theorem 1.** *The following set of rates can be achieved by the coding scheme in Algorithm 1:*

$$R_i + \sum_{j:j>i} R_j^{i^c} + \sum_{j:j<i} \frac{Pr_{d_i}}{Pr_{d_j}} \left( R_j^{i^c} + \sum_{k:i>k>j} R_j^{k^c k+1\dots i} \right) \leq Pr_{d_i}, \forall i. \quad (1)$$

**Examples.** When  $N = 2$ , the following set of equations characterizes the performance of Algorithm 1:

$$R_1 + R_2^{1^c} \leq Pr_{d_1}, \quad R_2 + R_1^{2^c} \frac{Pr_{d_2}}{Pr_{d_1}} \leq Pr_{d_2}.$$

When  $N = 3$ , the bound becomes

$$R_1 + R_2^{1^c} + R_3^{1^c} \leq Pr_{d_1}, \quad R_2 + R_1^{2^c} \frac{Pr_{d_2}}{Pr_{d_1}} + R_3^{2^c} \leq Pr_{d_2},$$

$$R_3 + (R_1^{3^c} + R_1^{2^c 3}) \frac{Pr_{d_3}}{Pr_{d_1}} + R_2^{3^c} \frac{Pr_{d_3}}{Pr_{d_2}} \leq Pr_{d_3}.$$

For  $N = 4$ , we have the following bound:

$$R_1 + R_2^{1^c} + R_3^{1^c} + R_4^{1^c} \leq Pr_{d_1},$$

$$R_2 + R_1^{2^c} \frac{Pr_{d_2}}{Pr_{d_1}} + R_3^{2^c} + R_4^{2^c} \leq Pr_{d_2},$$

$$R_3 + (R_1^{3^c} + R_1^{2^c 3}) \frac{Pr_{d_3}}{Pr_{d_1}} + R_2^{3^c} \frac{Pr_{d_3}}{Pr_{d_2}} + R_4^{3^c} \leq Pr_{d_3},$$

$$R_4 + (R_1^{4^c} + R_1^{3^c 4} + R_1^{2^c 3 4}) \frac{Pr_{d_4}}{Pr_{d_1}} + (R_2^{4^c} + R_2^{3^c 4}) \frac{Pr_{d_4}}{Pr_{d_2}} + R_3^{4^c} \frac{Pr_{d_4}}{Pr_{d_3}} \leq Pr_{d_4}.$$

Note that Algorithm 1 has a running time of  $O(N)$ .

The following corollary can be obtained from the examples above and from [19].

**Corollary 1.** *When there are two sessions, i.e.,  $N = 2$ , Algorithm 1 achieves the capacity region of the network.*

If we assume that the erasure patterns through the links are independent, we can characterize the performance of Algorithm 1 using a linear program where the only variables are  $R_i, \forall i$ , according to the following corollary:

**Corollary 2.** *The achievable rate of Algorithm 1 can be represented by the following linear program that only requires the knowledge of the delivery rates of the links:*

$$R_i + \sum_{j:j>i} [R_j - p_{s_j d_i}]^+ + \sum_{j:j<i} \frac{Pr_{d_i}}{Pr_{d_j}} \left[ R_j - \prod_{k:j<k\leq i} p_{s_j d_k} \right]^+ \leq Pr_{d_i}.$$

Here,  $[\cdot]^+$  is a projection on  $[0, \infty]$ .

**Proof.** To maximize overhearing, the term  $R_j^{i^c}$  in (1) can be rewritten as  $[R_j - p_{s_j d_i}]^+$ . The term

$$\left( R_j^{i^c} + \sum_{k:i>k>j} R_j^{k^c k+1\dots i} \right)$$

in (1) can be rewritten as  $R_j - R_j^{j+1\dots i}$ . Assuming that the channels are i.i.d., we have:

$$R_j - R_j^{j+1\dots i} = \left[ R_j - \prod_{k:j<k\leq i} p_{s_j d_k} \right]^+,$$

which completes the proof.  $\square$

When flexible scheduling is used, i.e., node  $u$  is scheduled for  $t(u)$  fraction of the time, Algorithm 1 can be modified to Algorithm 2.

**Algorithm 2.** Flexible Scheduling Algorithm

- 1:  $W_0 \leftarrow 0$
- 2: **for**  $i \leftarrow 1$  **To**  $N$  **do**
- 3: **if**  $i = N$  **then**
- 4:    $W_i = n$
- 5: **else**
- 6:    $W_i \leftarrow n[\sum_{j:j<i} \frac{1}{Pr_{d_j}} (R_j^{(i+1)^c} + \sum_{k:i>k>j} R_j^{k^c k+1\dots i+1})]$
- 7: **end if**
- 8: Perform RLNC on the packets that belong to the following sets  $\{X_1^{2\dots i}, \dots, X_{i-1}^i, X_i\}$ .
- 9: Send the coded packets in  $t(r) \times (W_i - W_{i-1})$  time slots.
- 10: **end for**

**Corollary 3.** *When flexible scheduling is used, i.e., node  $u$  is scheduled for  $t(u)$  fraction of the time, the achievable rate of Algorithm 2 is:*

$$R_i + \sum_{j:j>i} [R_j - t(s_j)p_{s_j d_i}]^+ + \sum_{j:j<i} \frac{Pr_{d_i}}{Pr_{d_j}} \left[ R_j - t(s_j) \prod_{k:j<k\leq i} p_{s_j d_k} \right]^+ \leq t(r)Pr_{d_i}.$$

If the two-hop relay network admits OpR, the relay network can perform coding on only the vectors in the complementary space of the vectors directly received by the intended receiver, which gives us the following corollary:

**Corollary 4.** *If the two-hop relay network admits OpR, the achievable rate of Algorithm 2 is:*

$$R_i + \sum_{j:j>i} [R_j - t(s_j)(p_{s_j d_j} + (1 - p_{s_j d_j})p_{s_j d_i})]^+ \\ + \sum_{j:j<i} \frac{p_{rd_i}}{p_{rd_j}} \left[ R_j - t(s_j) \left( p_{s_j d_j} + (1 - p_{s_j d_j}) \prod_{j<k<i} p_{s_j d_k} \right) \right]^+ \\ \leq t(s_1)p_{s_1 d_i} + t(r)p_{rd_i}.$$

## 5 A SCHEME WITH NO KNOWLEDGE OF LINK DELIVERY RATE

The joint IRNC and IANC schemes developed so far assume that the channel delivery rates of the links are known. However, this assumption is far from reality because it is not easy to estimate the channel conditions. Also, estimating channel conditions incurs a large overhead in addition to being inaccurate, because the channel conditions are varying and not stable.

For two session two-hop relay networks, we develop a coding scheme that achieves the capacity and does not need to know the delivery rates of the channels. Algorithm 3 represents the operations that the relay node has to perform to implement the coding scheme that does not require the knowledge of the channel conditions.

The challenge is as follows: The scheme that knows the delivery rates labels the sessions such that  $p_{rd_i} \geq p_{rd_{i+1}}$ . Therefore, in the scheme that does not know the delivery rates, node  $r$  randomly marks one of the sources as  $s'_1$  and the other one as  $s'_2$ . In Algorithm 3, we use  $X_i^j$  ( $X_i^{j'}$ ) to represent the set of packets sent by node  $s'_i$  and overheard (not overheard) by node  $d'_j$  ( $d'_j$ ). The scheme divides the batch of packets sent by  $s'_i$  into two different subbatches and uses simple feedback about the decodability of the subbatches to decide whether  $s'_1 = s_1$  or  $s'_1 = s_2$  and based on that what to do next.

**Algorithm 3.** Channel Loss Oblivious Scheme

- 1:  $r$  sends RLNC of  $X_1^2$  and  $X_2^1$  until either  $d'_1$  is able to recover  $X_1^2$  or  $d'_2$  is able to recover  $X_2^1$ .
- 2: **if** ( $d'_2$  is able to recover  $X_2^1$  and the removed symbols by  $d'_1$  can recover  $X_2^1$ ) or ( $d'_1$  is able to recover  $X_1^2$  and the removed symbols by  $d'_2$  cannot recover  $X_1^2$ ) **then**
- 3:  $r$  performs RLNC on  $X_2^{1c}$  and  $X_1^2$  and send them until  $d'_2$  can recover all of its packets.
- 4:  $r$  sends  $X_1^{2c}$  in the remaining time slots.
- 5: **else**
- 6:  $r$  performs RLNC on  $X_1^{2c}$  and  $X_2^1$  and send them until  $d'_1$  can recover all of its packets.
- 7:  $r$  send  $X_2^{1c}$  in the remaining time slots.
- 8: **end if**

The following theorem establishes the optimality of the coding scheme in Algorithm 3.

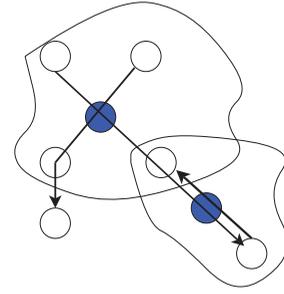


Fig. 4. An example of a multihop network to represent the superposition principle. The arrows represents flows. Note that any intersection of two or more flows results in a two-hop relay network, where we can perform joint IANC and IRNC.

**Theorem 2.** *The coding scheme in Algorithm 3 achieves the optimal solution for two sessions.*

## 6 EXTENSIONS TO THE MULTIHOP CASE

In this section, we build upon the two-hop relay network results to study the throughput and fairness benefits of using IANC and IRNC jointly in multihop wireless networks. We provide a linear programming formulation of an achievable rate region that uses the two-hop relay networks' results as building blocks. The linear program represents the superposition principle that is explained in Fig. 4. For each node, the linear program finds all possible two-hop relay networks in the neighborhood of the node. These nodes are represented by the filled circle nodes in the figure. In the figure, there are three flows represented by the arrows. The relay node is identified when it acts as an intermediate node for two or more flows. Based on that, we have two two-hop relay networks in the figure. We derive the linear program through three steps. In the first step, we formulate the problem with IANC only; then, we change the formulation to include fixed paths. In the third step, we add the joint IANC and IRNC.

The two-hop topology is formed when two or more flows use a node as an intermediate node. Therefore, as we increase the number of flows in the network, the number of existing two-hop network topologies increases. Many studies [9], [10], [11], [12], [13] have shown that the wireless networks contain many two-hop relay networks. These papers have used the superposition principle that is used in this paper. However, these papers used only IRNC as a coding scheme in the two-hop relay networks and assume lossless links.

For example, the work in [9] has shown that for their testbed, using the two-hop relay networks superposition approach, the achievable throughput can be enhanced by 3-4 $\times$ . Also, about 60 percent of the sent packets encounter at least one two-hop relay network. In their testbed, 50 percent of the two-hop relay networks are for two sessions, 25 percent are for three sessions, 20 percent are for four sessions, and the rest are for more than four sessions.

### 6.1 Settings

We consider a general multihop wireless network and represented it by a graph  $G = (V, E)$ , where  $V$  is the set of vertices representing the nodes, and  $E$  is the set of edges

TABLE 2  
Summary of the Symbols Used in the Multihop Case

Symbol	Definition
$u, v$	Intermediate nodes
$\mathbb{P}(i)$	Path used for session $i$
$V_1(u, i)$	Next hop node of $u$ on $\mathbb{P}(i)$
$V_2(u, i)$	Next hop node of $V_1(u, i)$ on $\mathbb{P}(i)$
$U_1(u, i)$	Previous hop node of $u$ on $\mathbb{P}(i)$
$U_2(u, i)$	Previous hop node of $U_1(u, i)$ on $\mathbb{P}(i)$
$y_{uv}(i)$	Rate of IANC packets for session $i$ through link $(u, v)$
$y'_{uv}(i)$	Same as $y_{uv}(i)$ , but for joint IANC and IRNC packets
$t(u, i)$	Fraction of the time node $u$ is scheduled to send IANC packets of session $i$
$t'(u)$	Fraction of the time node $u$ is scheduled to send joint IANC and IRNC packets
$P_u^J$	The probability that packet sent by node $u$ is overheard by any node in $J$
$c_k(u)$	The $k$ -th session that uses node $u$ .
$\gamma_{u,v}(i)$	By definition equals to $\frac{y_{uv}(i)}{P_{uv}}$ .

representing the links between the nodes. Transmission by a node can be overheard by multiple nodes, which we model by a hyperarc  $(u, J)$ , where  $u$  is the transmitter, and  $J$  is a subset of the set of direct receivers. There are  $N$  sessions in the network. For every session  $i$ , the source node  $s_i$  wants to send packets at rate  $R_i$  to the session's destination node  $d_i$ , possibly over multiple intermediate nodes. We use  $\mathbb{P}(i)$  to refer to the path used for session  $i$ . For every node  $u$  on path  $\mathbb{P}(i)$ ,  $V_1(u, i)$  ( $U_1(u, i)$ ) represents the next-hop (previous-hop) node, respectively, on that path, and  $V_2(u, i)$  ( $U_2(u, i)$ ) represents the next-hop (previous-hop), node of  $V_1(u, i)$  ( $U_1(u, i)$ ) on  $\mathbb{P}(i)$ , respectively.

We assume that every sent packet is either a packet formed by performing IANC for the packets of one session, or joint IRNC and IANC for different sessions' packets. This includes the case of sending noncoded packets as a special case. We use  $y_{uv}(i)$  to represent the rate of linearly independent IANC packets for session  $i$  that are sent by node  $u$  and can be decoded by node  $v$ , only if node  $v$  is  $d_i$ , or can be forwarded by  $v$ . Symbol  $y'_{uv}(i)$  represents the same as  $y_{uv}(i)$  but for packets with joint IANC and IRNC. The fraction of time that node  $u$  is scheduled for sending session  $i$  IANC coded packets is represented by  $t(u, i)$ .  $t'(u)$  represents the fraction of time that node  $u$  is scheduled to send joint IANC and IRNC packets. Symbol  $t(u)$  represents the fraction of time that node  $u$  is scheduled. To avoid the use of complex notations, we use, in this section,  $y_{uv}, y'_{uv}, \gamma_{uv}, V_1(u), V_2(u), U_1(u), U_2(u)$  to represent  $y_{uv}(i), y'_{uv}(i), \gamma_{uv}(i), V_1(u, i), V_2(u, i), U_1(u, i), U_2(u, i)$ , respectively. Table 2 represents the symbols used for the multihop case.

## 6.2 Formulation with Only IANC

Using IANC only, assuming that we do not have specified paths, the linear constraints that specify the capacity region are as follows:

$$\sum_{v:u \neq v} y_{vu} - \sum_{v:u \neq v} y_{uv} \leq \begin{cases} -R_i & u = s_i \\ 0 & \text{Else,} \end{cases} \quad \forall i, \forall u \in E \setminus d_i \quad (2)$$

$$\sum_{v \in J} y_{uv} \leq t(u, i) P_u^J, \quad \forall (u, J), \forall i, \quad (3)$$

where  $p_u^J$  is the probability that any node in  $J$  receives the packet. The constraints in (2) represent balance equations, such that the total received linearly independent packets and the total generated packets at a node should be, at most, equal to the total amount of sent linearly independent ones. Constraint (3) states that for any set of nodes that can receive the sent packets by a specific node, the total number of linearly independent packets per unit time that these nodes can forward is equal to the probability that any one of these nodes received the packet, which is  $p_u^J$ . If the paths are not specified, the solution of (2)-(3) will result in a back-pressure algorithm, which has bad delays, poor performance, and might not converge to the optimal solution, as noted in [22]. Therefore, in the following, we study the case of specified paths. The formulation becomes:

$$yU_{1(u)u} + yU_{2(u)u} - yuV_{1(u)} - yuV_{2(u)} \leq \begin{cases} -R_i & u = s_i \\ 0 & \text{Else,} \end{cases} \quad (4)$$

$$\forall i, \forall u \in E \setminus d_i,$$

$$y_{uv} \leq t(u, i) p_{uv}, \quad \forall u, v \in \mathbb{P}(i) \quad (5)$$

$$\begin{aligned} & yuV_{1(u)} + yuV_{2(u)} \\ & \leq t(u, i) (p_{uV_{1(u)}} + p_{uV_{2(u)}} - p_{uV_{1(u)}} p_{uV_{2(u)}}), \\ & \forall u \in \mathbb{P}(i). \end{aligned} \quad (6)$$

The above formulation can be obtained from (2) to (3) by noting that the only hyperarc for node  $u$  through the path for session  $i$  is the one with the receivers being  $V_1(u)$  and  $V_2(u)$ . This modeling agrees with practical implementations of IANC, as in [4], [6], which state that the overhearing of a node transmission over a path happens only for one- and two-hop away nodes.

## 6.3 Formulation with Joint IANC and IRNC

As it is hard to jointly use IANC and IRNC in multihop networks, we provide a restriction that allows us to use the two-hop relay networks results as a building block in large multihop networks. The restriction is that joint IANC and IRNC opportunities are limited to be in the form of two-hop relay networks; the coding node is the relay node, and the set of decoding nodes is a subset of the next-hop nodes of the relay node. Therefore, the general lossy multihop network can be decomposed into a superposition of IANC traffic alone and joint IANC and IRNC traffic in two-hop relay networks.

The assumption we have has the following implications on the capacity region: 1) In Fig. 5, if there are two sessions, one of them goes through the path  $v_1 w_2 v_2 r v_3$ , the other through  $u_1 u_2 r u_3$ , and joint IANC and IRNC can happen at node  $r$ . Due to our restriction, we assume that there is no side information from  $w_2$  to  $u_3$ , nor from  $u_1$  to  $v_3$ . If such side information exists, we ignore it. 2) If, in the same figure, there is another session that goes through  $w_1 w_2 w_3$ , and node  $w_2$  is acting as a relay node for performing joint IANC and IRNC between this session and the session that goes through  $v_1 w_2 v_2 r v_3$ , and the joint IANC and IRNC packets are overheard by  $r$ , then node  $r$  will consider these packets as useless and drop them.

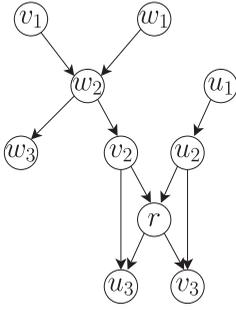


Fig. 5. Sample network.

Under the assumption that the channels are independent, for  $N$ -sessions two-hop relay networks, let:

$$\begin{aligned} \mathbf{t} &\triangleq [t(r), t(s_1), \dots, t(s_N)], \mathbf{p}_s \triangleq [p_{s_1r}, \dots, p_{s_Nr}], \\ \mathbf{p}_d &\triangleq [p_{rd_1}, \dots, p_{rd_N}], \mathbf{p}_{sd} \triangleq [\mathbf{p}_{s_1d}, \dots, \mathbf{p}_{s_Nd}], \end{aligned}$$

and  $\mathbf{p}_{s_1d} \triangleq [p_{s_1d_1}, \dots, p_{s_1d_{l-1}}, p_{s_1d_{l+1}}, \dots, p_{s_1d_N}]$ . In the case when  $d_l$  cannot overhear  $s_l$ , we use:

$$\begin{aligned} \text{Cap}A(\mathbf{t}, \mathbf{p}_{sd}, \mathbf{p}_s, \mathbf{p}_d) &= \{(R_1, \dots, R_N) : \text{The rates} \\ R_1, \dots, R_N &\text{ satisfy the linear programming constraints} \\ &\text{for the capacity region } A\}. \end{aligned}$$

$A$  is any achievable rate region for the two-hop relay network that uses IANC and IRNC jointly. It could be the optimal capacity region in [20], or an approximation of it, as in this paper. For example, when  $N = 2$ , the rates that satisfy:

$$\begin{aligned} R_1 &\leq \min(t(s_1)p_{s_1r}, t(r)p_{rd_1} - (R_2 - t(s_2)p_{s_2d_1})^+) \\ R_2 &\leq \min\left(t(s_2)p_{s_2r}, t(r)p_{rd_2} - (R_1 - t(s_1)p_{s_1d_2})^+ \frac{p_{rd_2}}{p_{rd_1}}\right), \end{aligned}$$

belong to  $\text{Cap}A$ , where  $A$  is the optimal capacity region.

When  $d_l$  can overhear  $s_l$ , and if  $d_l$  forwards  $\gamma_l p_{s_l d_l}$  linearly independent symbols of the overheard packets (or decodes them if it is the last destination of the packets) joint IRNC and IANC should happen for the symbols in the complementary spaces of the forwarded or decoded symbols. Letting  $\gamma \triangleq [\gamma_1, \dots, \gamma_N]$  and  $\mathbf{p}'_{sd} \triangleq [p_{s_1d_1}, \dots, p_{s_Nd_N}]$ , we use:

$$\begin{aligned} \text{Cap}'A(\gamma, \mathbf{t}, \mathbf{p}'_{sd}, \mathbf{p}_{sd}, \mathbf{p}_s, \mathbf{p}_d) &= \{(R'_1 = R_1 - \gamma_1 p_{s_1 d_1}, \dots, R'_N = R_N - \gamma_N p_{s_N d_N}) : \\ &\text{The rates } R_1, \dots, R_N \text{ satisfy the linear programming} \\ &\text{constraints with OpR when } d_l \text{ can overhear } s_l\}. \end{aligned}$$

For example, when  $N = 2$ , any  $(R'_1, R'_2)$  that satisfies the following constraints belongs to

$\text{Cap}'A(\gamma, \mathbf{t}, \mathbf{p}'_{sd}, \mathbf{p}_{sd}, \mathbf{p}_s, \mathbf{p}_d)$ , such that  $A$  is the optimal capacity region

$$\begin{aligned} R'_1 &\leq \min(Y_1, t(r)p_{rd_1} - (R_2 - Z_1)^+), \\ R'_2 &\leq \min\left(Y_2, t(r)p_{rd_2} - (R_1 - Z_2)^+ \frac{p_{rd_2}}{p_{rd_1}}\right), \end{aligned}$$

where  $Y_1, Y_2, Z_1, Z_2$  satisfy the following:

$$\begin{aligned} Y_1 &\leq t(s_1)(p_{s_1r} + p_{s_1d_1} - p_{s_1d_1}p_{s_1r}) - \gamma_1 p_{s_1d_1} \\ Y_1 &\leq t(s_1)p_{s_1r} \\ Y_2 &\leq t(s_2)(p_{s_2r} + p_{s_2d_2} - p_{s_2d_2}p_{s_2r}) - \gamma_2 p_{s_2d_2} \\ Y_2 &\leq t(s_2)p_{s_2r} \\ Z_1 &\leq t(s_2)(p_{s_2d_1} + p_{s_2d_2} - p_{s_2d_1}p_{s_2d_2}) \\ Z_1 &\leq t(s_1)(p_{s_1d_2} + p_{s_1d_1} - p_{s_1d_2}p_{s_1d_1}). \end{aligned}$$

When using random network coding, and when considering the symbols that have been directly received from  $s_l$  by any  $d_m$  or  $r$ , any two symbols related to two different received packets are linearly independent. Therefore, by using the feedback, the relay will be able to know the coefficients related to the received packets by its next-hop nodes to generate packets with coefficients in their complementary space. The following linear equations represent an achievable rate region that uses joint IANC and IRNC:

$$\begin{aligned} y_{U_1(u)u} + y_{U_2(u)u} + y'_{U_1(u)u} \\ - y_{uV_1(u)} - y_{uV_2(u)} - y'_{uV_1(u)} \leq \begin{cases} -R_i & u = s_i \\ 0 & \text{Else,} \end{cases} \end{aligned} \quad (7)$$

$$\forall i, \forall u \in E \setminus d_i,$$

$$y_{uv} = \gamma_{uv} p_{uv} \leq t(u, i) p_{uv}, \forall i, \quad \forall u, v \in \mathbb{P}(i), \quad (8)$$

$$\begin{aligned} y_{uV_1(u)} + y_{uV_2(u)} \leq t(u, i)(p_{uV_1(u)} + p_{uV_2(u)} \\ - p_{uV_1(u)}p_{uV_2(u)}), \quad \forall u \in \mathbb{P}(i) \end{aligned} \quad (9)$$

$$\begin{aligned} (y'_{uV_1(u,1)}(1), \dots, y'_{uV_1(u,k)}(k)) \in \\ \text{Cap}'A(\gamma^u, \mathbf{t}^u, \mathbf{p}'_{sd}{}^u, \mathbf{p}_{sd}{}^u, \mathbf{p}_s{}^u, \mathbf{p}_d{}^u). \end{aligned} \quad (10)$$

Here,  $k$  is the number of sessions intersecting at node  $u$ . To avoid the complex notations, we assume that these sessions are  $1, \dots, k$ . Also, we have

$$\begin{aligned} \gamma^u &\triangleq [\gamma_{U_1(u,1)V_1(u,1)}(1), \dots, \gamma_{U_1(u,k)V_1(u,k)}(k)], \\ \mathbf{t}^u &\triangleq [t'(u), t(U_1(u,1), 1), \dots, t(U_1(u,k), k)] \\ \mathbf{p}'_{sd}{}^u &\triangleq [p_{U_1(u,1)V_1(u,1)}, \dots, p_{U_1(u,k)V_1(u,k)}], \\ \mathbf{p}_{sd}{}^u &\triangleq [p_{U_1(u,l)V_1(u,l)}, \dots, p_{U_1(u,l)V_1(u,l-1)}, \\ &p_{U_1(u,l)V_1(u,l+1)}, \dots, p_{U_1(u,l)V_1(u,k)}], \mathbf{p}_{sd}{}^u \triangleq [\mathbf{p}_{s_1d}{}^u, \dots, \mathbf{p}_{s_kd}{}^u], \\ \mathbf{p}_s{}^u &\triangleq [p_{U_1(u,1)u}, \dots, p_{U_1(u,k)u}], \text{ and} \\ \mathbf{p}_d{}^u &\triangleq [p_{uV_1(u,1)}, \dots, p_{uV_1(u,k)}]. \end{aligned}$$

For session  $i$ , any node  $u$  has three different kinds of incoming packets and three different kinds of outgoing packets. The incoming packet types are IANC packets, received from a previous hop with rate  $y_{U_1(u)u}$ , IANC packets, overheard from a two-hop away node with rate  $y_{U_2(u)u}$ , and joint IANC and IRNC packets, received from a previous hop with rate  $y'_{U_1(u)u}$ . Note that, due to the restriction we have, joint IANC and IRNC packets that are overheard from two-hop away nodes are dropped. The outgoing packets can also be classified as joint IRNC and IANC packets with rate  $y'_{uV_1(u)}$ , IANC packets that are received and used by the next hop with rate  $y_{uV_1(u)}$ , and

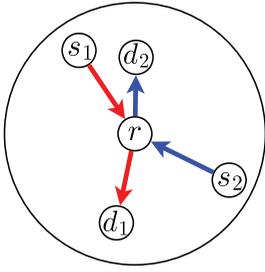


Fig. 6. Simulation settings.

IANC packets that are overheard and used by the next two-hop away nodes with rate  $y_{uV_2(u)}$ .

The constraints in (7) state: At every node, and for every session, the total incoming traffic at a node should be equal to the total outgoing traffic. The constraints (8)-(9) are for IANC and are the same as in the previous section. Constraints in (10) specify the joint IANC and IRNC rate at node  $u$  by treating it as a relay node in a two-hop relay network. Due to the restriction we have, at node  $u$ , only the incoming IANC traffic from a previous hop can be used for joint IANC and IRNC at node  $u$ . This is reflected in the formulation by using  $t^u$  as the second argument of  $\text{Cap}'A$ , which only contains the IANC scheduling frequency of the previous-hop nodes of node  $u$ . Since  $\gamma_{U_1(u,l)V_1(u,l)}(l)p_{U_1(u,l)V_1(u,l)}$  is the rate of the IANC packets for session  $l$  that are sent by node  $U_1(u,l)$ , overheard by the node  $V_1(u,l)$ , and used by that node, the first argument in  $\text{Cap}'A$  states that joint IANC and IRNC is performed in the complementary space of the symbols related to these packets.

## 7 SIMULATIONS

In this section, we present several simulation results to show the effectiveness of our approximation scheme in two-hop relay networks, and we show the improvement that joint IANC and IRNC schemes can provide for the multihop case.

We start from a unit circle with the relay  $r$  placed at the center. We then uniformly at random place  $N$  source nodes  $s_i$  and  $N$  destination nodes  $d_i$  in the circle (see Fig. 6). The only condition we impose is that for each  $(s_i, d_i)$  pair,  $d_i$  must be in the 90-degree pie area opposite to  $s_i$  (see Fig. 6). For each randomly constructed network, we use the euclidean distance between each node to determine the overhearing probability. More explicitly, for any two nodes separated by distance  $D$ , we use the Rayleigh fading model to decide the overhearing probability  $p = \int_{T^*}^{\infty} \frac{2x}{\sigma^2} e^{-\frac{x}{\sigma^2}} dx$ , where we choose  $\sigma^2 \triangleq \frac{1}{(4\pi)^2 D^\alpha}$ , the path loss order  $\alpha = 2.5$ , and the decodable SNR threshold  $T^* = 0.06$ .

We assume that the overhearing event is independent among different receivers. For each randomly generated network, we compute the overhearing probabilities and use the corresponding linear constraints on the time-sharing variables  $ts$  and the rate variables  $Rs$  to compute the achievable rate of each scheme. Given a randomly generated network, the achievable sum rates are computed for all of the schemes. We then repeat this computation for 1,000 randomly generated networks. Let  $\zeta_{\text{scheme},k}^*$  denote the achievable sum rate of the given scheme for the  $k$ th

randomly chosen topology. We are interested in the following two performance metrics: The average sum rate over 1,000 topologies  $\frac{1}{1,000} \sum_{k=1}^{1,000} \zeta_{\text{scheme},k}^*$  and the

$$\text{per topology improvement} \triangleq \frac{\zeta_{\text{scheme},k}^* - \zeta_{\text{baseline},k}^*}{\zeta_{\text{baseline},k}^*}.$$

In our two-hop relay network simulations, we change three different setting parameters. These are: 1) The use of OpR where we consider OpR or no OpR. 2) Node scheduling method, where we use round robin scheduling or include the weight of scheduling as a new variable in the optimization problem. If the weights of scheduling are included in the optimization problem, we call such a scheme the *optimal* scheduling scheme, and 3) The objective function, where we consider three objective functions. The objective functions that we use are: a) Proportional fairness, such that the weight of session  $i$  is the available bandwidth for that session when no other sessions share the network, b) strict fairness that requires the throughput of all of the sessions to be the same, and c) maximizing the sum of the throughput of the sessions.

Fig. 4 in the supplement document, available online, shows the average throughput achieved by our approximation scheme and the optimal scheme when neither OpR nor round robin scheduling is used. As shown in the figure, our scheme achieves 65-95 percent of the optimal solution, depending on the number of sessions. Also, our scheme performs similarly to the *pairwise scheme* [19], which requires coordination among different nodes and has a complexity of  $O(N^2)$ . Figs. 5, 6, and 7 in the supplement document, available online, show similar results when using OpR with optimal scheduling.

Fig. 9 in the supplement document, available online, compares the achievable throughput of our approximation scheme with other XOR-based schemes. These schemes are COPE [9], CLONE [17], and the capacity achieving scheme with XOR coding [23]. The results in the figure show that our scheme performs comparably to the best XOR-based scheme, while the best XOR-based scheme has an exponential complexity. The figure also shows that our scheme improves the throughput 1.5-3.5 fold compared to COPE, the state-of-the-art XOR coding scheme, and by 25 percent over CLONE, the state-of-the-art XOR coding scheme optimized to work with lossy links. The figure shows that even when the objective is to achieve fairness, our scheme still improves the throughput over CLONE by 10-12 percent.

Fig. 10 in the supplement document, available online, shows the CDF function of the per topology improvement of our schemes compared to COPE when  $N = 6$ . The results in the figure show that there are topologies where the throughput improvement is by four fold. Also, for half of the topologies, the improvement is over 2.7 fold. The results also show that even when the objective is fairness, the throughput improvement is large. Fig. 11 in the supplement document, available online, shows the CDF function for the per topology improvement over CLONE-binary for 1,000 topologies when  $N = 6$ . The results show that there are topologies where the throughput improvement witnessed by our scheme is over 60 percent. Also, when the objective is to achieve fairness, more than 90 percent of the topologies show throughput improvement over CLONE

binary. To show the benefits of jointly using IANC and IRNC, we use randomly generated topologies of 15 nodes located in a  $6 \times 6$  unit square area. For simplicity, we assume that the channels are orthogonal, and every node can be scheduled in every time slot. This can be achieved by equipping the nodes with multiple interfaces and using multichannel assignment protocol similar to [24], [25], [26]. For each possible source and sink pair, we find the path that minimizes the ETX metric, as defined in [27]. For each source, we mark the longest path among the found paths that minimize the ETX metric. We randomly select  $N$  paths from the marked ones to perform the simulation. For each value of  $N$ , we simulated 200 different topologies. Fig. 8 in the supplement document, available online, represents the average gain of joint IANC and IRNC over using the optimal IANC solution with two different objectives. As the number of sessions increases, the gain increases due to the fact that we have more sessions going through a node, which increases the coding opportunities. We perform another set of simulations on the topology in Fig. 5. We used IEEE 802.11 scheduling and varied the delivery rates of the links randomly. This generates 1,000 different topologies. Fig. 12 in the supplement document, available online, represents the empirical CDF function of the per topology throughput improvement of using the joint IANC and IRNC compared to using IANC. As can be seen from the figure, there is an improvement in all of the topologies. The improvement is about 12.2 percent on average. There is a topology where the improvement is 53.5 percent.

## 8 CONCLUSION

In this paper, we have studied the problem of joint IANC and IRNC in two-hop relay networks. As the optimal solution has a high complexity, we provide a linear time approximation algorithm and characterize its performance using linear constraints. Our approximation algorithm uses RLNC and carefully mixes IANC with IRNC. We provided evaluation comparisons through extensive simulation experiments between our proposed schemes and the related state-of-the-art schemes and showed the effectiveness of our scheme in achieving the desirable performance metrics with lower complexity. For the achievable rate region in multi-hop wireless networks, we provide a linear programming formulation and show the improvements through simulations. For the two sessions case, we provided a scheme that achieves the desirable capacity without requiring any knowledge about the channel conditions.

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