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# Global EDF Schedulability Analysis for Parallel Tasks on Multi-core Platforms

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**Abstract**—With the widespread adoption of multi-core architectures, it is becoming more important to develop software in ways that takes advantage of such parallel architectures. This particularly entails a shift in programming paradigms towards fine-grained, thread-parallel computing. Many parallel programming models have been introduced for targeting such intra-task thread-level parallelism. However, most successful results on traditional multi-core real-time scheduling are focused on sequential programming models. For example, thread-level parallelism is not properly captured into the concept of *interference*, which is key to many schedulability analysis techniques. Thereby, most interference-based analysis techniques are not directly applicable to parallel programming models. Motivated by this, we extend the notion of interference to capture thread-level parallelism more accurately. We then leverage the proposed notion of parallelism-aware interference to derive efficient EDF schedulability tests that are directly applicable to parallel task models, including DAG models, on multi-core platforms, without knowing an optimal schedule. Our evaluation results indicate that the proposed analysis significantly advances the state-of-the-art in global EDF schedulability analysis for parallel tasks. In particular, we identify that our proposed schedulability tests are adaptive to different degrees of thread-level parallelism and scalable to the number of processors, resulting in substantial improvement of schedulability for parallel tasks on multi-core platforms.

**Index Terms**—Real-time scheduling, Parallel task, Global EDF, Interference

## 1 INTRODUCTION

WITH the advance of semiconductor technology, multi-/many-core architectures are widely used to better manage trade-offs between performance, power efficiency, and reliability in deep submicron technology. As the size of a CMOS transistor continuously shrinks, more cores are getting integrated on a single die. For example, Intel introduced the Intel Xeon Phi coprocessor with around 60 cores [1], and Cavium designed ARM-based processors that scale up to 48 cores [2]. Given the increasing emphasis on multi-/many-core chip design, software parallelism is likely to be one of the greatest constraints on computer performance. This inherently entails a shift in programming paradigms towards fine-grained thread-parallel computing, rather than relatively coarse-grained application-level parallelism.

A popular technique to achieve fine-grained, thread-level parallelism operates on the principle of *divide-and-conquer*. It breaks down a larger task into many smaller subtasks, runs those subtasks in parallel, and synchronizes them to merge the results once each subtask completes computation. Many parallel programming models have been proposed to support such a principle for parallel computation, including OpenMP [3], Cilk/Cilk++ [4], Intel Threading Building Blocks [5], Wool [6], and Chapel [7]. Those parallel programming models share a common scenario that some subtasks (threads) in a program (task) can

run in parallel to produce partial results individually and certain threads should synchronize to integrate the partial results<sup>1</sup>. Many parallel task models have been considered to capture those two important aspects: *thread-level parallelism* and *synchronization*. One popular parallel task model is the synchronous parallel task model, where a task consists of a sequence of parallel regions, called *segments*, and each segment includes one or more threads. All the threads belonging to the same segment are released at the same time and at most  $m$  threads are able to run simultaneously, where  $m$  is the number of available cores. Two consecutive segments are subject to synchronization (precedence constraint); all the threads belonging to one segment must complete their own execution in order to move onward to the next segment. Recently, a growing research attention has been given to a more general parallel task model, the DAG (Directed Acyclic Graph) model, where a node represents a thread and an edge describes a precedence dependence between two threads. In the DAG model, the granularity of precedence constraint becomes smaller to the level of thread to offer a more flexible way of describing thread-level parallelism and synchronization than the synchronous parallel task model.

A shift from uni-core to multi-core processors allows *inter-task parallelism*, where several tasks can execute simultaneously on multi-core processors. In addition, a shift from single-thread to multi-thread tasks allows *intra-task parallelism*, where even a single task can have multiple threads running simultaneously to take full advantage of multi-core processing. Despite the growing importance of intra-task parallelism, most real-time multi-core scheduling

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1. In terminology of some programming models, such as OpenMP, Intel Threading Building Blocks, and Chapel, the term “task” is used to represent a unit of parallel execution referred to as a “thread” in this paper.

TABLE 1

Schedulability analysis methods for parallel tasks: <sup>†</sup> indicates a case where the proposed analysis method is applicable only for partitioned scheduling.

	Comparative	Independent
Fork-join task model	[21] <sup>†</sup>	
Synchronous parallel task model	[22] [24] [26]	[23] [25] [27] <sup>†</sup>
DAG task model	[28] [30] [29]	[28] [29] [31] [32] <b>This paper</b>

studies are focused on inter-task parallelism of sequential tasks, and relatively much less attention has been paid to understanding intra-task parallelism towards the schedulability analysis of parallel tasks. For example, a large number of studies extensively investigated the schedulability analysis of sequential tasks under EDF (earliest-deadline-first) and fixed-priority global scheduling [8], producing many influential results. Such results include the concept of problem window and interference [9], [10], interference bounding techniques [11], [12], response time computation methods [10], [13], and optimal priority assignment [14]. Many other scheduling algorithms have been proposed for sequential models in order to take advantage of multi-core more effectively, including optimal algorithms such as pfair [15], DP-Fair [16], RUN [17], and U-EDF [18]. In addition, some approaches [19], [20] have been also proposed for scheduling tasks with pipeline precedence constraints in distributed real-time systems. However, the insights behind those successful results are not directly applicable to parallel tasks, due to the unique characteristics of thread-level parallelism.

For example, the notion of interference has been well defined in the sequential task case and serves as the basis for many schedulability analysis methods [9], [10], [11]. However, the current notion of interference does not capture thread-level parallelism because it assumes that each task has only a single thread to run at any time instant. Hence, those analysis methods are not directly applicable or easily extensible to parallel tasks.

Recently, a few schedulability analysis methods have been proposed for parallel task systems (see Table 1). Those methods can broadly fall into two types: *comparative* and *independent*. The analysis methods in the literature can be considered as *comparative*, if they derive resource augmentation bounds; the resource augmentation bound indicates how well a scheduler performs relatively to an optimal scheduler. Thus, such resource augmentation bounds serve as good measures to assess the performance of a scheduler and compare different schedulers. However, they can be hardly used to determine the schedulability of a set of given parallel tasks when no optimal schedule is known. In particular, [33] proved that it is impossible to find an optimal online multi-core scheduler for sporadic task systems. On the other hand, analysis methods can be said

to be *independent*, if they can serve as schedulability tests directly even without having to find any optimal schedule. Motivated by this, the goal of this paper is to develop an efficient, independent schedulability test that can determine the schedulability of parallel tasks, including DAG tasks, directly in connection to no optimal schedule.

**Contribution.** The main contributions of this paper can be summarized as follows:

- We identify a chain of threads, called a *critical thread*, in a DAG task that makes the most significant impact to a deadline miss, if exists. We then derive interference-based schedulability analysis of global EDF scheduling for sporadic DAG task systems with novel notions of *critical interference* and *p-depth critical interference* in order to capture thread-level parallelism accurately (see Section 4).
- We develop a polynomial-time workload-based schedulability test. We identify the worst-case interference scenario for DAG tasks considering the structure of a DAG in detail and derive tight upper bounds on interference based on workload (see Section 5).
- We also develop a pseudo-polynomial-time slack-based iterative schedulability test to reduce pessimism effectively in bounding interference (see Section 6).
- We present simulation results, showing that the proposed schedulability analysis methods significantly outperforms the state-of-the-art methods available for DAG tasks even if no optimal schedule is known (see Section 7).

In our earlier work [23], we presented interference-based schedulability tests for a synchronous parallel task set under global EDF scheduling. In this paper, we extend this initial study towards a more expressive parallel task model, DAG task model, with further improvements. In the synchronous parallel task model, each task consists of a sequence of segments with synchronization points at the end of each segment. The DAG task model refines the granularity of synchronization from segment-level to thread-level, allowing to describe more flexible way of thread-level parallelism and synchronization. In order to generalize the schedulability analysis techniques in [23] for the DAG task model, this paper has the following new technical contributions in addition to re-organizing and re-writing the entire paper for better and/or concise presentation:

- The notions of critical thread, critical interference, and p-depth critical interference are extended accordingly with the DAG task model. Building upon those new notions, we extend the workload-based schedulability test in [23] towards the DAG task model, including a new way of calculating the worst-case workload for DAG tasks.
- We additionally propose an improved schedulability test by the use of slack values.
- We include more evaluation results, including a comparison with other related works for DAG tasks.

## 2 RELATED WORK

Many scheduling approaches have been introduced for intra-task thread-level parallelism in the hard real-time context. The schedulability tests developed in the literature can be largely categorized into two types: (1) *comparative* ones that determine the schedulability of a scheduler relatively to an ideal scheduler, and (2) *independent* ones that determine the schedulability of a scheduler independently in connection to no other scheduler. For example, according to [28], resource and capacity augmentation bounds<sup>2</sup> serve as comparative and independent tests, respectively. Table 1 summarizes the scheduling approaches in the literature by schedulability test types and parallel task models.

**Fork-join task model.** One of the widely used parallel task models is the *fork-join* model [21]. A fork-join task consists of an alternate sequence of sequential and parallel regions, called *segments*, and all the threads within each segment should synchronize in order to proceed to the next segment. Under the assumption that each parallel segment can have at most as many threads as the number of processors, [21] presented a resource augmentation bound of 3.42 for partitioned DM (deadline-monotonic) scheduling of periodic fork-join tasks with implicit deadlines.

**Synchronous parallel task model.** Relaxing the restriction that sequential and parallel segments alternate, several studies have considered a more general synchronous parallel task model that allows each segment to have any arbitrary number of threads. [22] presented a resource augmentation bound of 4 for global EDF scheduling and 5 for partitioned DM scheduling of periodic tasks with implicit deadlines. Building upon this work, [34] presented a prototype scheduling service for their *RT-OpenMP* concurrency platform. [26] also showed a resource augmentation bound of 2 for a certain class of global scheduling algorithms, such as PD<sup>2</sup> [35], LLREF [36], DP-Wrap [16], or U-EDF [18] to schedule sporadic tasks with constrained deadlines.

Those studies [21], [22], [26] share a common principle of task decomposition for schedulability analysis. They decompose a single synchronous parallel task into multiple independent sequential sub-tasks through intermediate deadline assignment. This approach is safe — satisfying the intermediate deadlines of all sub-tasks leads to meeting the deadlines of their aggregate synchronous parallel tasks. They then employ existing schedulability analysis for those sequential sub-tasks. [22] decomposes a parallel task into a set of sequential sub-tasks such that the density of each segment is upper bounded by some value, and [26] decomposes a parallel task such that the maximum density among all segments in a parallel task is minimized. However, such an

2. The resource augmentation bound  $r$  of a scheduler  $S$  has the property that if a task set is feasible on  $m$  unit-speed processors, then the task set is schedulable under  $S$  on  $m$  processors of speed  $r$ . For a scheduler  $S$  and its corresponding schedulability condition  $X$ , their capacity augmentation bound  $c$  has the property that if the given condition  $X$  is satisfied with a task set, the task set is schedulable by  $S$  on  $m$  processors of speed  $c$ . Since the resource augmentation bound is connected to an ideal optimal schedule, it is hard (if not impossible) to use it as a schedulability test due to the difficulty of finding an optimal schedule in many multi-core scheduling domains. On the other hand, the capacity augmentation bound has nothing to do with an optimal schedule, and this allows it to serve as an easy schedulability test (see [28] more details).

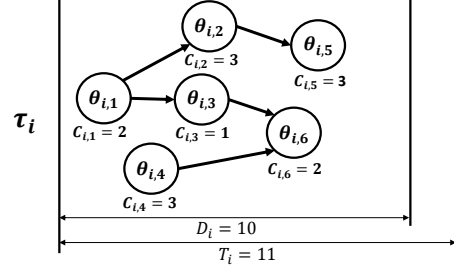


Fig. 1. A DAG task  $\tau_i$

indirect analysis via task decomposition can be pessimistic, because task decomposition can incur non-trivial overheads. Furthermore, it requires modifications to existing operating systems to support task decomposition [34].

Recently, some studies [23], [24], [25], [27] developed direct schedulability analysis without task decomposition for synchronous parallel tasks. [24] showed a resource augmentation bound of  $2 - 1/m$  for sporadic tasks with constrained deadlines under global EDF scheduling. [23] introduced an interference-based analysis for global EDF scheduling of sporadic tasks with constrained deadlines. [25] and [27] presented a response-time analysis (RTA) for sporadic tasks under global fixed-priority scheduling and partitioned fixed-priority scheduling, respectively.

**DAG task model.** Refining the granularity of synchronization from segment-level to thread-level, a DAG (Directed Acyclic Graph) task model is considered, where a node represents a thread and an edge specifies a precedence dependency between nodes. A thread can execute only after all of its predecessors have been executed. [30] showed a resource augmentation bound of 2 for a single DAG task with arbitrary deadlines under global EDF scheduling. For a set of DAG tasks, a resource augmentation bound of  $2 - 1/m$  was presented for global EDF scheduling [28], [29]. [29] also derived a  $3 - 1/m$  resource augmentation bound for global DM scheduling. In addition to those resource augmentation bounds, which serve as comparative schedulability tests, [28] introduced capacity augmentation bounds that can work as independent schedulability tests for sporadic DAG tasks with arbitrary deadlines under global EDF and RM (rate-monotonic) scheduling. [31] later improved the capacity augmentation bounds for global EDF and RM scheduling and proposed a new scheduling policy, called federated scheduling, for DAG tasks. [29] also presented a simple polynomial-time independent schedulability test for global EDF scheduling, and this work was later extended in [32] to yield an improved pseudo-polynomial time independent schedulability test. This paper proposes a new interference-based, independent schedulability test for a DAG task set under global EDF scheduling, significantly improving the schedulability compared to the existing techniques.

## 3 SYSTEM MODEL

We consider a multi-core platform, where sporadic DAG (Directed Acyclic Graph) tasks run over  $m$  identical processors under global EDF scheduling. A set of tasks is denoted by  $\tau$ . In the sporadic DAG task model, a task  $\tau_i \in \tau$  is specified by  $(G_i, D_i, T_i)$ , where  $G_i$  is a directed acyclic

graph as shown in Figure 1,  $D_i$  is the relative deadline, and  $T_i$  is the minimum separation. The DAG  $G_i$  is specified as  $G_i = (V_i, E_i)$ , where  $V_i$  is a set of nodes and  $E_i$  is a set of directed edges between two nodes. Each node  $\theta_{i,u} \in V_i$  represents a sequential operation (a “thread”), and is characterized by the worst-case execution time requirement (WCET)  $C_{i,u}$ . The number of threads in  $\tau_i$  is denoted by  $N_i$ . A directed edge  $(\theta_{i,u}, \theta_{i,v}) \in E_i$  represents the precedence dependency that  $\theta_{i,v}$  cannot start execution unless  $\theta_{i,u}$  has finished execution. A thread  $\theta_{i,u}$  becomes ready for execution as soon as all of its predecessors have completed their execution.

A *path* in the sporadic DAG task  $\tau_i$  is a sequence of threads  $\theta_{i,1}, \theta_{i,2}, \dots, \theta_{i,f}$  such that  $(\theta_{i,j}, \theta_{i,j+1})$  is an edge in  $G_i$  for  $1 \leq j < f$ . The length of this path is defined as the sum of the WCETs of all its threads:  $\sum_{j=1}^f C_{i,j}$ . The length of a longest path, denoted by  $LC_i$  can be computed in linear time in the number of nodes and the number of edges in  $G_i$ , when its nodes are sorted and processed in a topological order. In Figure 1, the longest path of  $\tau_i$  is the sequence of  $\theta_{i,1}$ ,  $\theta_{i,2}$  and  $\theta_{i,5}$ , with the longest path length  $LC_i$  equal to  $2 + 3 + 3 = 8$ . We also define  $C_i$  as the total WCET of  $\tau_i$ , and it is presented as

$$C_i = \sum_{\theta_{i,j} \in V_i} C_{i,j}. \quad (1)$$

Note that  $LC_i$  is the minimum amount of time needed to execute all threads in  $\tau_i$  assuming that it can use as many processors as possible for its execution, and  $C_i$  is the maximum amount of time to complete the execution of all the threads in  $\tau_i$  on a single core.

A sporadic DAG task  $\tau_i$  invokes a series of jobs, and successive jobs are released with a duration of at least  $T_i$  time units apart. If a job of  $\tau_i$  is released at time instant  $t$  then all  $|V_i|$  threads  $\theta_{i,u} \in V_i$  are released at time instant  $t$  and must complete execution by the absolute deadline  $t + D_i$ . We consider a constrained deadline  $D_i$  such that  $D_i \leq T_i$ . It should be  $LC_i \leq D_i$  but not necessarily  $C_i \leq D_i$ . Let  $U_i$  denote the utilization of  $\tau_i$  and be defined as  $U_i = C_i/T_i$ . We denote the  $l$ -th job of a task  $\tau_i$  with  $J_i^l$ . We will omit the superscript in the notation for simplicity when no ambiguity arises. For a job  $J_i^l$ ,  $r_i^l$  and  $d_i^l$  are its release time and deadline. The *execution window* of a job  $J_i^l$  is then defined as interval  $[r_i^l, d_i^l]$ .

In EDF scheduling, threads are assigned priorities according to their absolute deadline: the earlier the deadline of a thread, the higher its priority. Thereby, threads in different jobs may have different priorities, but all threads within a single job have the same priority since they share the same absolute deadline. With global EDF, each thread ready to execute is placed in a system-wide queue, ordered by non-decreasing absolute deadline, and the first  $m$  threads are extracted from the queue to execute on the available processors at every time instant. In this paper, we assume quantum-based time and without loss of generality, let one time unit denote the quantum length. All task parameters are assumed to be specified as multiples of this quantum length.

We summarize the notation used throughout the paper in the supplement, available online.

## 4 SCHEDULABILITY ANALYSIS FOR DAG TASKS

In this section, we derive schedulability analysis of global EDF scheduling for sporadic DAG task systems with constrained deadlines. To this end, we extend the concept of interference towards the DAG task model and introduce a new concept called *critical interference*. With the notion of critical interference, we investigate what happens if there is a deadline miss and identify a necessary condition for a job to miss its deadline, which serves as a basis for schedulability analysis. In this section, a run-time schedule of a task set is assumed to be known. However, a worst-case situation where a task suffers the worst possible interference is generally unknown for global scheduling of sporadic task systems. This prevents calculation of the exact interference without knowledge of run-time schedule. Therefore, in later sections, we derive a safe upper bound on the interference and transform the necessary condition into an efficient schedulability test.

### 4.1 The concept of critical interference

In the real-time scheduling literature, the notion of interference has been employed in many schedulability analysis methods [10], [11], [37], [38], [39], using the following definitions:

- Interference  $I_k(a, b)$ : the sum of all intervals in which  $\tau_k$  is ready for execution but cannot execute due to other higher-priority tasks in  $[a, b]$ .
- Interference  $I_{i,k}(a, b)$ : the sum of all intervals in which  $\tau_i$  is executing and  $\tau_k$  is ready to execute but not executing in  $[a, b]$ .

With the above definitions, the relation between  $I_k(a, b)$  and  $I_{i,k}(a, b)$  serves as an important basis for deriving schedulability analysis. In the single-thread task case, it is intuitive to construct such a relation on  $m$  processors as follows [10]:

$$I_k(a, b) = \frac{1}{m} \sum_{\tau_i \in \tau} I_{i,k}(a, b). \quad (2)$$

However, it is not straightforward to build such a relation in the multi-thread task case, as illustrated in the following example.

**Example 4.1.** As an example, suppose that two threads of higher-priority task  $\tau_i$  and one thread of lower-priority task  $\tau_k$  are ready for execution on two processors at time  $t$ . Then, the two threads of  $\tau_i$  will run on two processors in  $[t, t + 1)$ , delaying the execution of  $\tau_k$ . According to the above definitions,  $\tau_i$  imposes interference on  $\tau_k$  in  $[t, t + 1)$ , yielding  $I_k(t, t + 1) = 1$  and  $I_{i,k}(t, t + 1) = 1$ . However, Eq. (2) no longer supports such definitions.

The above example suggests a need for extending the concept of interference for the parallel task model, and this raises three problems: (i) how to calculate the interference on  $\tau_k$  when only some (but not all) threads of  $\tau_k$  are interfered, (ii) how to calculate the interference of  $\tau_i$  on  $\tau_k$  when only some (but not all) threads of  $\tau_i$  interfere with  $\tau_k$ , and (iii) how to calculate the intra-task interference of threads of  $\tau_k$  on other threads of the same task  $\tau_k$ .

To address problem (i), we represent the concept of interference of a task  $\tau_k$  by considering the interference on a

chain<sup>3</sup> of its threads. This chain is called a *critical chain*, and threads belonging to a critical chain are called *critical threads*. The detailed description of a critical chain is provided in Section 4.2. With the notion of critical threads, we can now extend the traditional definition of interference towards the DAG task model and introduce a new concept called *critical interference* as follows:

- Critical interference  $\mathcal{I}_k(a, b)$ : the sum of all intervals in which **a critical thread of  $\tau_k$**  is ready for execution but cannot execute due to other higher-priority threads in  $[a, b)$ .
- Critical interference  $\mathcal{I}_{i,k}(a, b)$ : the sum of all intervals in which **at least one thread of  $\tau_i$**  is executing and **the critical thread of  $\tau_k$**  is ready to execute but not executing in  $[a, b)$ .

Note that when all tasks have a single thread, then the single thread is equal to the critical thread and our definition is the same as the traditional definition of interference.

To address problem (ii), we introduce a new concept called *p-depth critical interference*. The *p*-depth critical interference of a task  $\tau_i$  on  $\tau_k$  characterizes not only the length of the delay  $\tau_i$  causes to  $\tau_k$  but also the number of threads of  $\tau_i$  that cause the delay.

To address problem (iii), we incorporate the notion of intra-task interference into both the critical interference and the *p*-depth critical interference such that they include interference on a critical thread by other non-critical threads of the same task.

In the remainder of this section, we describe the notion of critical threads and address the problems raised the above in detail.

## 4.2 Necessary condition for the first deadline miss of a DAG task

In this section we formally define a critical chain of a job. We first seek to identify a necessary condition for any DAG task to miss a deadline on  $m$  processors. Let a *last-completing thread* of a job be a thread that completes last among the job's threads. A job misses deadline if its last-completing thread misses deadline. A thread is said to be a *last-completing predecessor* of  $\theta_{k,u}$  if it finishes last among all of the predecessors of  $\theta_{k,u}$ .<sup>4</sup> A thread can only be ready when its last-completing predecessor completes. If we recursively track all the concatenating last completing predecessors from a last completing thread until there is no predecessor, we can construct a *critical chain*  $\lambda_k^l$  of job  $J_k^l$ . Each thread in the critical chain is defined as a *critical thread*, and the length of the critical chain of  $J_k^l$ , denoted by  $CP_k^l$ , is defined as the sum of the WCETs of all its critical threads. A DAG task is then considered as *complete* as soon as its critical threads complete execution.

We define interference on a critical thread  $\theta_{k,u} \in \lambda_k$  over interval  $[a, b)$  (denoted as  $\mathcal{I}_{<k,u>(a, b)$ ) as the cumulative length of all intervals in which the critical thread  $\theta_{k,u}$  is

3. A chain indicates a particular path at run-time.

4. We note that if there are multiple threads that finish last among all of the predecessors of  $\theta_{k,u}$  at the same time, we can choose any of them. Without loss of generality, we choose the one that has the lowest index among them.

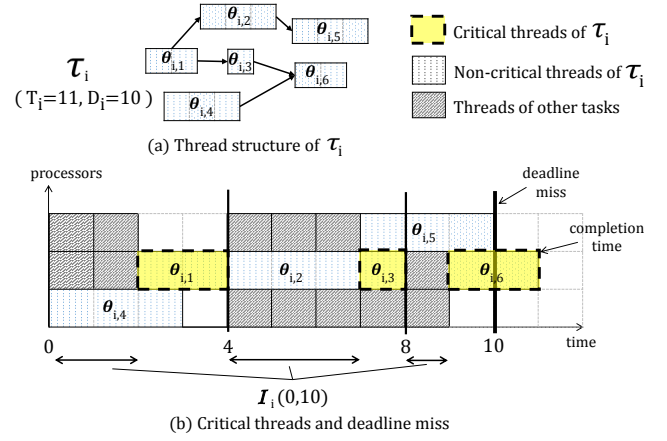


Fig. 2. An example of DAG task  $\tau_i$  on 3 processors. (a) Task  $\tau_i$  consists of 6 threads with  $T_i = 11$  and  $D_i = 10$ . (b) A job  $J_i^l$  of task  $\tau_i$  misses a deadline at 10. Here, critical threads are  $\theta_{i,1}$ ,  $\theta_{i,3}$ , and  $\theta_{i,6}$ , and those critical threads were blocked in 3 time intervals (i.e.,  $[0,2)$ ,  $[4,7)$ , and  $[8,9)$ ), respectively. This yields  $\mathcal{I}_i(0,10) > D_i - CP_i^l$ , where  $CP_i^l$  is the sum of the execution times of all critical threads in job  $J_i^l$ .

ready to execute but not executing due to the execution of higher-priority threads that belong to not only other tasks but also the same task. To avoid any confusion, it is worth noting that  $\mathcal{I}_{<k,u>(a, b)$  includes intra-task interference that a critical thread  $\theta_{k,u} \in \lambda_k$  receives from other threads  $\theta_{k,v} \notin \lambda_k$  of the same task  $\tau_k$ . According to our definition,  $\mathcal{I}_k(a, b)$  is a total interference imposed collectively on all the critical threads of  $\tau_k$ , i.e.,

$$\mathcal{I}_k(a, b) = \sum_{\theta_{k,u} \in \lambda_k} \mathcal{I}_{<k,u>(a, b). \quad (3)$$

In order to derive schedulability analysis using the concept of critical interference, we investigate what happens when the “first” deadline miss occurs and identify necessary conditions for a job to miss its deadline. Generally, a deadline miss happens since there is a large amount of higher priority execution that blocks the remaining execution of critical threads of a job until its deadline. We consider any legal sequence of job requests of task set  $\tau$ , on which EDF misses a deadline. Suppose that a job of task  $\tau_k$ , denoted by  $J_k^*$ , is the first job to miss a deadline among all the jobs of all tasks. Then, by definition, all the jobs of earlier deadlines than the deadline of  $J_k^*$  complete execution before their deadlines, and the task system remains underloaded until the first deadline miss. For  $J_k^*$ ,  $r_k^*$  and  $d_k^*$  are its release time and deadline. Then, one can see that at least one critical thread  $\theta_{k,u} \in \lambda_k^*$  of  $J_k^*$  must execute for less than  $C_{k,u}$  time units, and the total execution time taken by all critical threads must be less than  $CP_k^*$  time units. In order for all critical threads of  $J_k^*$  to execute for strictly less than  $CP_k^*$  time units over  $[r_k^*, d_k^*)$ , it is necessary that its critical interference  $\mathcal{I}_k(r_k^*, d_k^*)$  be strictly more than  $(D_k - CP_k^*)$  time units. This observation yields a necessary condition for job  $J_k^*$  to miss a deadline, i.e.,

$$\mathcal{I}_k(r_k^*, d_k^*) > D_k - CP_k^*. \quad (4)$$

Figure 2 illustrates a situation where a job of DAG task  $\tau_i$  misses a deadline. Figure 2(a) shows the thread structure of task  $\tau_i$  and its parameters that we will consider throughout

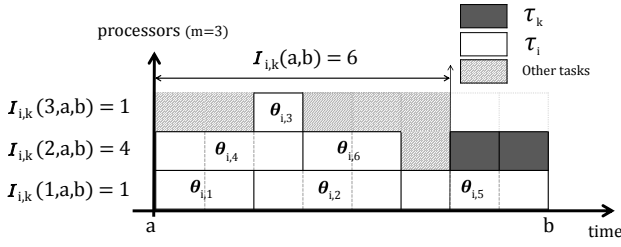


Fig. 3. The notion of  $p$ -depth critical interference. Suppose that task  $\tau_k$  has a lower priority than  $\tau_i$ . A job of  $\tau_k$  is released at time instant  $a$ , but it cannot execute in  $[a, a+6)$  due to the execution of other higher priority tasks. In this example,  $\tau_i$  executes a single thread in interval  $[a+5, a+6)$ , which corresponds to 1-depth critical interference on  $\tau_k$ . This yields  $\mathcal{I}_{i,k}(1, a, b) = 1$ .  $\tau_i$  executes two threads in intervals  $[a, a+2)$  and  $[a+3, a+5)$ , leading to  $\mathcal{I}_{i,k}(2, a, b) = 4$ . And  $\mathcal{I}_{i,k}(3, a, b) = 1$ .

this paper. In Figure 2(b), a job  $J_i^l$  of  $\tau_i$  is released at 0 with a deadline of 10. Thread  $\theta_{i,6}$  is the last completion thread in  $\tau_i$ , and it is released when  $\theta_{i,3}$  has finished at time instant 8 (i.e.,  $\theta_{i,3}$  is a last completing predecessor of  $\theta_{i,6}$ ).  $\theta_{i,1}$  is a last completing predecessor of  $\theta_{i,3}$ . Since  $\theta_{i,1}$  has no predecessor, critical threads of  $J_i^l$  are  $\theta_{i,1}$ ,  $\theta_{i,3}$ , and  $\theta_{i,6}$ . The execution of those critical threads were delayed in interval  $[0,2)$ ,  $[4,7)$ , and  $[8,9)$ , leading to  $\mathcal{I}_{<i,1>}(0,10) = 2$ ,  $\mathcal{I}_{<i,3>}(0,10) = 3$ , and  $\mathcal{I}_{<i,6>}(0,10) = 1$ . Then, the critical interference  $\mathcal{I}_k(0,10)$  on  $\tau_i$  during interval  $[0,10)$  is  $2 + 3 + 1 = 6$ , resulting in  $\mathcal{I}_k(0,10) > D_i - CP_i^l$ . This makes it infeasible for the last thread  $\theta_{i,6}$  to fully execute for  $C_{i,6}$  time units before the deadline of 10. This leads to the deadline miss of task  $\tau_i$ .

As shown in Example 4.1, it is not as straightforward as Eq. (2) to build the relation between  $\mathcal{I}_k(a, b)$  and  $\mathcal{I}_{i,k}(a, b)$ . This is mainly because  $\mathcal{I}_{i,k}(a, b)$  does not capture how many threads of  $\tau_i$  interfere with the critical threads of  $\tau_k$ . We thereby introduce a new concept of  $p$ -depth critical interference that characterizes the number of interfering threads, and this new notion will bridge  $\mathcal{I}_k(a, b)$  and  $\mathcal{I}_{i,k}(a, b)$  effectively for DAG tasks. Let us define the  $p$ -depth critical interference  $\mathcal{I}_{i,k}(p, a, b)$  of task  $\tau_i$  on task  $\tau_k$  during interval  $[a, b)$  as the cumulative length of all intervals in which (1) a critical thread of  $\tau_k$  is ready to execute but does not execute and (2) exactly  $p$  number of threads of  $\tau_i$  are executing (see Figure 3). It is worth noting that when it comes to the intra-task interference case,  $\mathcal{I}_{k,k}(p, a, b)$  corresponds to a case where a critical thread of  $\tau_k$  is not executing while exactly  $p$  number of other non-critical threads of  $\tau_k$  are executing. The  $p$ -depth critical interference enables to represent the behavior of parallel execution in more detail, allowing to figure out exactly how many threads of a task  $\tau_i$  are executing simultaneously when  $\tau_i$  delays the execution of another task  $\tau_k$ . A total critical interference  $\mathcal{I}_{i,k}(a, b)$  can be decomposed into individual  $p$ -depth critical interferences as follows:

$$\mathcal{I}_{i,k}(a, b) = \sum_{p=1}^m \mathcal{I}_{i,k}(p, a, b). \quad (5)$$

The  $p$ -depth critical interference also makes it easy to constitute a total interference  $\mathcal{I}_k(a, b)$  out of individual interferences of each task on task  $\tau_k$  on  $m$  processors as follows.

**Lemma 1.** For any work-conserving algorithm, the total critical interference  $\mathcal{I}_k(a, b)$  imposed on task  $\tau_k$  in interval  $[a, b)$  is equal to the total amount of contribution of individual threads to the interference on each critical thread divided by the number of processors, i.e.,

$$\mathcal{I}_k(a, b) = \frac{1}{m} \sum_{\tau_i \in \tau} \sum_{p=1}^m \mathcal{I}_{i,k}(p, a, b) \times p. \quad (6)$$

*Proof:* Since the scheduling algorithm is work-conserving, in the time instants in each of which a critical thread of a task is ready but not executing, each processor must be occupied by all the other threads of another task and including itself. The total amount of the contribution to the critical interference on  $\tau_k$  is  $\sum_{\tau_i \in \tau} \sum_{p=1}^m \mathcal{I}_{i,k}(p, a, b) \times p$ . If it is divided by the number of processors, we get exactly the length of cumulative intervals in which a critical thread of  $\tau_k$  is ready to execute but cannot in an interval  $[a, b)$ .  $\square$

Building upon the notion of  $p$ -depth critical interference and Lemma 1, the necessary condition for task  $\tau_k$  to miss a deadline (presented in Eq. (4)) can be rewritten as follows:

$$\frac{1}{m} \sum_{\tau_i \in \tau} \sum_{p=1}^m \mathcal{I}_{i,k}(p, r_k^*, d_k^*) \times p + CP_k^* > D_k. \quad (7)$$

Conversely, in order for a task to be schedulable, it is sufficient to demonstrate that for all of its jobs, Eq. (7) cannot be satisfied. Hence to show that task set  $\tau$  is schedulable under global EDF scheduling, this condition must be checked for each task in  $\tau$ , which serves as a basis of a schedulability condition. In order to leverage the condition for schedulability analysis properly, we need to calculate the terms in the left-hand side (LHS) of Eq. (7) accurately. Unfortunately, it is hard to compute those terms precisely without knowledge of run-time schedule. Thereby, we wish to derive safe but tight upper bounds on the terms and transform the necessary condition into schedulability tests based on those upper bounds.

## 5 WORKLOAD-BASED EDF SCHEDULABILITY TEST

This section derives a polynomial-time schedulability test of global EDF scheduling for DAG tasks. We note that any sporadic task system has infinitely many different legal job arrival sequences. Hence, checking Eq. (7) for all such sequences is computationally intractable. Moreover, it is generally difficult to calculate the interference terms of  $\mathcal{I}_{i,k}(p, r_k^*, d_k^*) \times p$  and the critical chain term of  $CP_k^*$ , since they are decided at run-time according to the schedules of other tasks that interfere with the job. Therefore, we instead seek to derive upper bounds on the LHS of Eq. (7) for each task.

To this end, we first consider a job  $J_k$  having a particular length of its critical chain and derive an upper bound on the interference term  $\mathcal{I}_{i,k}(p, r_k, d_k)$  imposed on the job. By definition, a critical chain of a job in  $\tau_k$  is determined as one of all possible paths in  $G_k$ , and its length ranges between the shortest path length in  $G_k$  (denoted by  $SC_k$ ) and the longest path length in  $G_k$  (i.e.,  $LC_k$ ). When we derive an upper bound on interference, we use the concept of *workload* that has been widely used in the literature [10], [12], [38], [39],



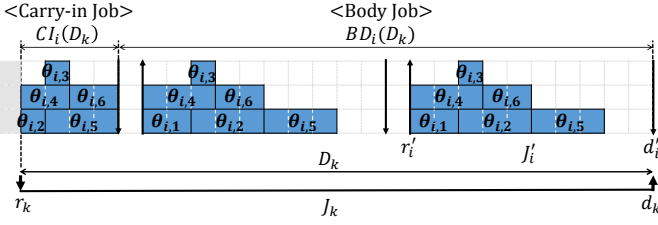


Fig. 4. A worst-case release pattern in which  $\widehat{W}_{i,k}$  is maximized.

[40]: the workload  $W_i(a, b)$  of  $\tau_i$  is the sum of all intervals in which  $\tau_i$  is executing in interval  $[a, b)$ . The interference imposed on a job can be divided into inter-task interference received from threads of other tasks and intra-task interference received from threads of the same task  $\tau_k$ . We note that bounding inter-task interference is independent of the length of a critical chain, but bounding intra-task interference is dependent on it. We derive upper bounds on inter-task and intra-task interferences, respectively, based on workload. Then, in order to ensure that there exists no job that causes the first deadline miss, we need to check Eq. (7) for all possible lengths of critical chains that a task can have. To this end, we will show that our proposed schedulability test needs to be conducted for only one case, rather than exploring all the possible cases, to get the upper bound of the LHS of Eq. (7) for all jobs.

## 5.1 Bounding interference

We define the workload  $W_{<i,v>}(a, b)$  of thread  $\theta_{i,v}$  in  $\tau_i$  is the sum of all intervals in which  $\theta_{i,v}$  is executing in interval  $[a, b)$ . Then, the following inequality holds for any  $J_k$  of  $\tau_k$ :

$$\sum_{p=1}^m \mathcal{I}_{i,k}(p, r_k, d_k) \times p \leq \sum_{\theta_{i,v} \in hp_i(J_k)} W_{<i,v>}(r_k, d_k) \quad (8)$$

$$\stackrel{\text{def.}}{=} \widehat{W}_{i,k},$$

where  $hp_i(J_k)$  is a set of threads in  $\tau_i$  that have a priority higher than or equal to a critical thread of  $J_k$ .

According to Inequality (8), an upper-bound on the interference term can be obtained by finding an upper-bound on  $\widehat{W}_{i,k}$  in any scheduling window of a job of  $\tau_k$ . Note that when the upper-bound of  $\widehat{W}_{i,k}$  is calculated, we assume that there is no deadline miss. This is because our schedulability test aims to derive necessary conditions for the first deadline miss. Therefore, we will assume this for derivation of the upper-bound of  $\widehat{W}_{i,k}$  in the rest of the paper. In addition, we are not relying on any particular execution behavior of a task as well as the number of processors when deriving the upper bound on  $\widehat{W}_{i,k}$ . We next consider two cases to discuss a worst-case release pattern for maximizing  $\widehat{W}_{i,k}$ : inter-task ( $i \neq k$ ) and intra-task cases ( $i = k$ ).

### 5.1.1 Bounding inter-task workload

To simplify the presentation, we use the following terms. A job is said to be a *carry-in* job of an interval  $[a, b)$  if it is released before  $a$  but has a deadline within  $[a, b)$  or a *body* job if its release time and deadline are both within  $[a, b)$ .

**Worst-case release pattern.** Figure 4 shows a worst-case release pattern, in which task  $\tau_i$  has the maximum amount

of  $\widehat{W}_{i,k}$  that interferes with job  $J_k$  over interval  $[r_k, d_k)$  under global EDF scheduling. As shown in the figure, all the jobs of  $\tau_i$  are released periodically, and its last body job ( $J'_i$ ) of the interval  $[r_k, d_k)$  has a deadline equal to that of  $J_k$  (i.e.,  $d'_i = d_k$ ). All individual threads in  $[r_k, d_k)$  have a priority higher than or equal to  $J_k$  and then execute as long as their WCETs. For the carry-in job, we consider a worst-case situation in which all threads of the carry-in job are executed as late as possible subject to satisfying the deadline of the carry-in job. With this release pattern we can include the largest number of threads of  $\tau_i$  having higher priority than  $J_k$  in the interval  $[r_k, d_k)$ , thus bounding the value of  $\widehat{W}_{i,k}$ . We recapitulate the above result in the following lemma:

**Lemma 2.** For any task  $\tau_i$  under the assumption that all jobs meet their deadlines as long as all threads in a job execute at most their WCETs, a release pattern of  $\tau_i$  that maximizes  $\widehat{W}_{i,k}$  is: (a)  $\tau_i$  releases jobs with a minimum inter-arrival time of  $T_i$  time units, (b) the deadline of a job of  $\tau_i$  aligns with the deadline of the job of  $\tau_k$ , and (c) all threads of the carry-in job of  $\tau_i$  are executed as late as possible (right before the deadline of the carry-in job).

*Proof:* Figure 4 illustrates a worst-case release pattern in which task  $\tau_i$  satisfies requirements (a), (b) and (c).

First of all, the contribution of jobs to  $\widehat{W}_{i,k}$  cannot be larger than when the release times of jobs are exactly periodic. That is, moving the release times of some jobs farther apart cannot increase  $\widehat{W}_{i,k}$ .

In Figure 4, we then consider what happens to the contribution if we simultaneously shift all the release times and deadlines of  $\tau_i$  earlier or later. The maximum shift we need to consider in either direction is  $T_i$ , since for longer shifts the effect occurs periodically. From Figure 4, if we shift their deadlines earlier, the contribution of carry-in and body jobs to  $\widehat{W}_{i,k}$  cannot increase. The job having its deadline after  $d_k$  cannot achieve higher priority than  $J_k$ , so it cannot contribute to  $\widehat{W}_{i,k}$ . Therefore, shifting their deadline earlier cannot increase the maximum contribution to  $\widehat{W}_{i,k}$ . If we shift their deadlines later, the absolute deadline of the last body job  $J'_i$  becomes later than the one of  $J_k$ , so it cannot contribute to  $\widehat{W}_{i,k}$ . Thereby,  $\widehat{W}_{i,k}$  is decreased by  $C_i$ . The shift may increase the contribution of the carry-in job, but by at most  $C_i$ . Therefore, shifting their deadline later cannot increase the maximum contribution to  $\widehat{W}_{i,k}$ . From the two cases, we can see that the maximum contribution of the body jobs is achieved when a deadline of a job of  $\tau_i$  aligns with the deadline of the job of  $\tau_k$ , and the interval of a carry-in job in  $[r_k, d_k)$  is maximized.

For the carry-in job of  $\tau_i$ , only threads executing in  $[r_k, d_k)$  can contribute to  $\widehat{W}_{i,k}$ . If all threads of the carry-in job are executed as late as possible right before the deadline, it can maximize the number of threads executing in  $[r_k, d_k)$ . It is worthy noting that we assume that all jobs meet their deadlines. Thus, each thread must complete execution so that all of its successive threads are guaranteed to finish before the deadline of the carry-in job although all threads execute for their WCETs. The maximum contribution of the carry-in job is achieved with the release pattern shown in Figure 4.  $\square$



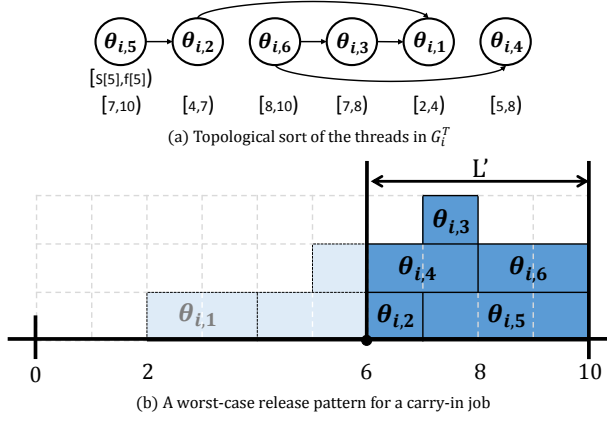


Fig. 5. Calculation of an upper bound on carry-in job workload under a worst-case release pattern.

### Calculating an upper bound on worst-case workload.

By using a worst-case release pattern, we can now determine the interval of length  $D_k$  which maximizes  $\widehat{W}_{i,k}$ . Define  $\mathcal{W}_{< i, v >}(D_k)$  as the value of  $W_{< i, v >}(r_k, d_k)$  in such an interval. Under a worst-case release pattern, there exist only body and carry-in jobs in any interval of length  $D_k$ , and all of the threads in those jobs have higher priority than  $\tau_k$ .

The interval  $[r_k, d_k]$  of length  $D_k$  can be partitioned into body and carry-in intervals, and the length of the intervals are denoted as  $BD_i(D_k)$  and  $CI_i(D_k)$ , respectively, and described as

$$BD_i(D_k) = \left\lfloor \frac{D_k}{T_i} \right\rfloor \cdot T_i, \quad (9)$$

$$CI_i(D_k) = D_k - BD_i(D_k). \quad (10)$$

Let us consider a bound  $\mathcal{W}_{< i, v >}^{BD}(D_k)$  on the body job workload in any interval of length  $D_k$ . The maximum number of body jobs of  $\tau_i$  over an interval of length  $D_k$  is  $\left\lfloor \frac{D_k}{T_i} \right\rfloor$ , and each thread  $\theta_{i,v}$  in a body job can fully execute for its WCET. Then,  $\mathcal{W}_{< i, v >}^{BD}(D_k)$  is calculated as

$$\mathcal{W}_{< i, v >}^{BD}(D_k) = \left\lfloor \frac{D_k}{T_i} \right\rfloor \cdot C_{i,v}. \quad (11)$$

Now, we consider a bound  $\mathcal{W}_{< i, v >}^{CI}(L')$  on the carry-in job workload in any carry-in interval of length  $L'$ . For the carry-in job case, only some of threads can contribute to  $\mathcal{W}_{< i, v >}^{CI}(L')$  in an interval of length  $L'$ , and their execution can partially fit into the carry-in interval due to the precedence relation. Thereby, we need to figure out the execution interval of each thread under the worst-case release pattern. Under a worst-case release pattern, all threads of the carry-in job are executed as late as possible. If we compare each thread's starting time and finishing time with the carry-in interval, we can calculate the contribution on  $\mathcal{W}_{< i, v >}^{CI}(L')$  for each thread  $\theta_{i,v}$  (lines 14-24). In the example shown in Figure 5(a), the threads in  $\tau_i$  are topologically sorted according to  $G_i^T$ . The finishing time  $f[1]$  of  $\theta_{i,1}$  is determined as 4, which is the starting time of  $\theta_{i,2}$ , and the finishing time  $s[1]$  is determined as 2, which is  $f[1] - C_{i,1}$ . In Figure 5(b), each thread in  $\tau_i$  executes its WCET from its starting time to its finishing time.

### Algorithm 1 Calculate-CarryinJob-Workload ( $\tau_i, L'$ )

```

1: for each thread  $\theta_{i,u} \in V_i$  do
2:    $s[u] \leftarrow D_i, f[u] \leftarrow D_i$ 
3: end for
4:  $G_i^T \leftarrow (V_i, E_i^T)$ 
5: topologically sort the threads in  $G_i^T$ 
6: for each thread  $\theta_{i,u}$ , taken in topologically sorted order do
7:   for each thread  $\theta_{i,v} \in \text{Parent}[u]$  do
8:     if  $s[v] < f[u]$  then
9:        $f[u] \leftarrow s[v]$ 
10:    end if
11:  end for
12:  $s[u] \leftarrow f[u] - C_{i,u}$ 
13: end for
14: for each thread  $\theta_{i,u}$  in  $G_i^T$  do
15:   if  $s[u] \geq D_i - L'$  then
16:      $\mathcal{W}_{< i, u >}^{CI}(L') = C_{i,u}$ 
17:   else
18:     if  $s[u] < D_i - L'$  and  $f[u] > D_i - L'$  then
19:        $\mathcal{W}_{< i, u >}^{CI}(L') = f[u] - (D_i - L')$ 
20:     end if
21:   else
22:      $\mathcal{W}_{< i, u >}^{CI}(L') = 0$ 
23:   end if
24: end for

```

Algorithm 1 shows how to calculate  $\mathcal{W}_{< i, v >}^{CI}(L')$  for a carry-in job under the worst-case release pattern shown in Figure 5(b). It computes each thread's starting time and finishing time under the worst-case release pattern. At first, the starting and finishing time of each thread are initialized to  $D_i$  (lines 1-3). We reverse the direction of all edges in  $E_i$  (denoted by  $E_i^T$ ) and define DAG  $G_i^T$  as  $G_i^T = (V_i, E_i^T)$  (line 4). Algorithm 1 then performs a topological sort of  $G_i^T$  (line 5). The topological sort is a linear ordering of all threads such that if  $G_i^T$  contains a directed edge from  $\theta_{i,u}$  to  $\theta_{i,v}$ , then  $\theta_{i,u}$  appears before  $\theta_{i,v}$  in the ordering. It traverses each thread in topologically sorted order and calculates its starting and finishing time (lines 6-13). The finishing time  $f[u]$  of thread  $\theta_{i,u}$  is determined as the earliest one among the starting times of the threads that have the edge to  $\theta_{i,u}$  in  $G_i^T$ . The starting time  $s[u]$  of thread  $\theta_{i,u}$  is then determined as its finishing time minus its WCET. We note that this presents the worst-case release pattern for a carry-in job in which all threads of the carry-in job are executed as late as possible. If we compare each thread's starting time and finishing time with the carry-in interval, we can calculate the contribution on  $\mathcal{W}_{< i, v >}^{CI}(L')$  for each thread  $\theta_{i,v}$  (lines 14-24). In the example shown in Figure 5(a), the threads in  $\tau_i$  are topologically sorted according to  $G_i^T$ . The finishing time  $f[1]$  of  $\theta_{i,1}$  is determined as 4, which is the starting time of  $\theta_{i,2}$ , and the finishing time  $s[1]$  is determined as 2, which is  $f[1] - C_{i,1}$ . In Figure 5(b), each thread in  $\tau_i$  executes its WCET from its starting time to its finishing time.

A bound on the workload  $\mathcal{W}_{< i, v >}(r_k, d_k)$  that will contribute to the worst case as shown in Figure 4, for  $i \neq k$ , is expressed as follows:

$$\mathcal{W}_{< i, v >}(D_k) = \mathcal{W}_{< i, v >}^{CI}(CI_i(D_k)) + \mathcal{W}_{< i, v >}^{BD}(BD_i(D_k)). \quad (12)$$

We note that both  $\mathcal{W}_{< i, v >}^{CI}(CI_i(D_k))$  and  $\mathcal{W}_{< i, v >}^{BD}(BD_i(D_k))$  in Eq. (12) are independent of a critical chain of a job of  $\tau_k$ . This means that we always get the

same result of  $\mathcal{W}_{<i,v>}^{CI}(CI_i(D_k))$  and  $\mathcal{W}_{<i,v>}^{BD}(BD_i(D_k))$ , no matter what a critical chain of length is. We can then compute bounds on the amount of inter-task interference on any job of  $\tau_k$  as follows:

$$\widehat{W}_{i,k} \leq \sum_{\theta_{i,v} \in V_i} \mathcal{W}_{<i,v>}(D_k). \quad (13)$$

### 5.1.2 Bounding intra-task workload

The critical threads in a critical chain of a job of  $\tau_k$  can get interference from the other threads belonging to the same job. For the intra-task interference, the worst-case release pattern is already determined as the execution window of a job of  $\tau_k$ . Then, all threads within a single job except the critical threads can interfere on the critical threads, and it is clear that the interference of a single thread  $\theta_{k,v}$  on the critical threads is upper bounded by  $C_{k,v}$ . Thereby, the sum of WCETs of all threads within a single job except the ones of critical threads can contribute on  $\widehat{W}_{k,k}$ .

Unlike inter-task workload, intra-task workload is dependent on the length of a critical chain of a job. We define  $\mathcal{W}_k(x)$  as a bound on intra-task workload of a job with a critical chain of length  $x$ . The sum of WCETs of all threads within a single job of  $\tau_k$  except the ones of the critical threads can contribute to  $\mathcal{W}_k(x)$ . We recall that the critical chain length  $x$  is the sum of WCETs of the critical threads, so  $\mathcal{W}_k(x)$  is computed as

$$\mathcal{W}_k(x) = C_k - x. \quad (14)$$

We can then compute bounds on the amount of intra-task interference on a job with a critical chain of length  $x$  as

$$\widehat{W}_{k,k} \leq \mathcal{W}_k(x). \quad (15)$$

## 5.2 Deriving a schedulability test

Putting together inter-task and intra-task workloads as an upper-bound on the interference on a job of  $\tau_k$  with a critical chain of length  $x$  ( $SC_k \leq x \leq LC_k$ ), we can check the schedulability of the job for a given task set under global EDF scheduling as follows.

**Lemma 3.** Suppose that a task set  $\tau$  is scheduled by global EDF scheduling on  $m$  identical processors. Then, a job invoked by  $\tau_k \in \tau$  having a critical chain length  $x$  ( $SC_k \leq x \leq LC_k$ ) does not cause the first deadline miss, if the following inequality holds:

$$\frac{1}{m} \left( \sum_{i \neq k} \sum_{\theta_{i,v} \in V_i} \mathcal{W}_{<i,v>}(D_k) + \mathcal{W}_k(x) \right) + x \leq D_k. \quad (16)$$

*Proof:* From Lemma 2 and Inequality (8), the following inequality holds for a job of  $\tau_k$  having a critical chain length of  $x$ :

$$\begin{aligned} & \frac{1}{m} \sum_{\tau_i \in \tau} \sum_{p=1}^m \mathcal{I}_{i,k}(p, r_k, d_k) \times p + x \\ & \leq \frac{1}{m} \left( \sum_{i \neq k} \sum_{\theta_{i,v} \in V_i} \mathcal{W}_{<i,v>}(D_k) + \mathcal{W}_k(x) \right) + x. \end{aligned} \quad (17)$$

Then, if Eq. (16) is satisfied for the job, the job fails to satisfy the necessary condition of Eq. (7) that triggers the first deadline miss.  $\square$

In order to derive a schedulability test for a task set, it should be guaranteed that all individual jobs satisfy Lemma 3. Then, we need to check Eq. (16) for all possible critical chain lengths of each task  $\tau_k$  ranging between  $SC_k$  and  $LC_k$ . We claim that our workload-based schedulability test needs to be conducted for only one critical chain length, which is the longest path length (i.e.,  $LC_k$ ).

**Lemma 4.** For a given task set  $\tau$ , a task  $\tau_k$  satisfies the followings:

$$\begin{aligned} & \frac{1}{m} \left( \sum_{i \neq k} \sum_{\theta_{i,v} \in V_i} \mathcal{W}_{<i,v>}(D_k) + \mathcal{W}_k(x) \right) + x \\ & \leq \frac{1}{m} \left( \sum_{i \neq k} \sum_{\theta_{i,v} \in V_i} \mathcal{W}_{<i,v>}(D_k) + \mathcal{W}_k(LC_k) \right) + LC_k. \end{aligned} \quad (18)$$

*Proof:* The term  $\frac{1}{m} \sum_{i \neq k} \sum_{\theta_{i,v} \in V_i} \mathcal{W}_{<i,v>}(D_k)$  is included in both LHS and RHS of inequality (18). Therefore, we show that  $\frac{1}{m} \mathcal{W}_k(x) + x \leq \frac{1}{m} \mathcal{W}_k(LC_k) + LC_k$  as follows.

$$\begin{aligned} & \frac{1}{m} \mathcal{W}_k(x) + x \\ & = \frac{1}{m} (C_k - x + mx) \\ & \quad (\text{From Eq. (14)}) \\ & = \frac{1}{m} (C_k - x + mx + LC_k - LC_k + mLC_k - mLC_k) \\ & = \frac{1}{m} (C_k - LC_k + mLC_k + (LC_k - x) - m(LC_k - x)) \\ & \quad (\text{Re-arrange the terms}) \\ & \leq \frac{1}{m} (C_k - LC_k + mLC_k) \\ & \quad (\because (LC_k - x) \leq m(LC_k - x)) \\ & \quad (\because \text{By definition, } x \leq LC_k \text{ and } m > 0) \\ & = \frac{1}{m} \mathcal{W}_k(LC_k) + LC_k \\ & \quad (\text{From Eq. (14)}) \end{aligned}$$

$\square$

Finally, we develop a workload-based schedulability test for DAG tasks under global EDF scheduling.

**Theorem 1.** A task set  $\tau$  is schedulable under global EDF scheduling on  $m$  identical processors if for each task  $\tau_k \in \tau$ , the following inequality holds:

$$\sum_{i \neq k} \sum_{\theta_{i,v} \in V_i} \mathcal{W}_{<i,v>}(D_k) + \mathcal{W}_k(LC_k) \leq m(D_k - LC_k). \quad (19)$$

*Proof:* From Lemmas 3 and 4,  $\frac{1}{m} \left( \sum_{i \neq k} \sum_{\theta_{i,v} \in V_i} \mathcal{W}_{<i,v>}(D_k) + \mathcal{W}_k(LC_k) \right) + LC_k$  is an upper bound on the LHS of Eq. (7) for all jobs of a task  $\tau_k$ . Then, if Eq. (19) is satisfied for all tasks in a task set  $\tau$ , no job can trigger the first deadline miss, and the task set is schedulable under global EDF scheduling on  $m$  identical processors.  $\square$

**Complexity.** We denote the number of tasks in a task set by  $n$ . For each task  $\tau_i$ , Algorithm 1 is performed in  $O(|V_i| + |E_i|)$  time to calculate an upper bound on carry-in workload. The longest path length  $LC_i$  can be calculated in  $O(|V_i| + |E_i|)$  time. This repeats  $n$  times for all tasks in a task set to calculate  $LC_i$ . Independently, it requires  $O(n)$

time to calculate Eq. (19) for a given  $\tau_k$ , and it repeats  $n$  times for all tasks in a task set to check schedulability of a task set. Therefore, the schedulability test in Theorem 1 requires  $\max\{O(n(|V_{max}| + |E_{max}|)), O(n^2)\}$  time for a task set, where  $|V_{max}|$  and  $|E_{max}|$  represents the largest  $|V_i|$  and  $|E_i|$  among all tasks  $\tau_i \in \tau$ .

## 6 SLACK-BASED ITERATIVE SCHEDULABILITY TEST

In general, the computation of workload bounds in Section 5 involves much pessimism due to the overestimation on calculating the worst-case workload of a carry-in job. In particular, the worst-case release pattern for a carry-in job shown in Figure 5(b) does not consider the fact that a task can finish its execution earlier than its deadline. Once we identify such an early completion of a task's execution, we can reduce the amount of workload of a carry-in job using slack values, where the slack  $S_k$  of task  $\tau_k$  is defined as a length of the minimum time interval between finishing time and deadline of a job of task  $\tau_k$ . The idea of exploiting slack values is introduced to reduce such pessimism effectively for the sequential task model [11], [37]. In this section, building upon our workload-based schedulability test, we derive an improved schedulability test by taking advantage of slack values for DAG tasks. To this end, we first introduce how to calculate slack values for each task in the following lemma.

**Lemma 5.** The slack of task  $\tau_k$  is given by

$$S_k = D_k - CP_k^* - \left\lceil \frac{\sum_{\tau_i \in \tau} \sum_{p=1}^m \mathcal{I}_{i,k}(p, r_k^*, d_k^*) \times p}{m} \right\rceil, \quad \text{if } S_k > 0. \quad (20)$$

*Proof:* We re-arrange Eq. (20) as follows:

$$\begin{aligned} & \left\lceil \frac{\sum_{\tau_i \in \tau} \sum_{p=1}^m \mathcal{I}_{i,k}(p, r_k^*, d_k^*) \times p}{m} \right\rceil = D_k - CP_k^* - S_k \\ \Rightarrow & \frac{\sum_{\tau_i \in \tau} \sum_{p=1}^m \mathcal{I}_{i,k}(p, r_k^*, d_k^*) \times p}{m} \leq D_k - CP_k^* - S_k \\ \Leftrightarrow & \mathcal{I}_k(r_k^*, d_k^*) \leq D_k - CP_k^* - S_k \\ & (\because \text{By Lemma 1}) \\ \Leftrightarrow & \mathcal{I}_k(r_k^*, d_k^*) + CP_k^* \leq D_k - S_k \end{aligned} \quad (21)$$

According to Eq. (21), the sum of the critical interference and total execution time taken by all critical threads is less than or equal to  $D_k - S_k$ . Since we assume that  $S_k > 0$ , task  $\tau_k$  has already finished its execution at  $S_k$  time units ahead of its deadline.  $\square$

By exploiting the bounds on the interference derived in Section 5, a lower bound  $S_k^{lb}$  on the slack  $S_k$  of a task  $\tau_k$  under global EDF scheduling is then given by

$$S_k^{lb} = D_k - LC_k - \left\lceil \frac{1}{m} \left( \sum_{i \neq k} \sum_{\theta_i, v \in V_i} \mathcal{W}_{< i, v >}(D_k) + \mathcal{W}_k(LC_k) \right) \right\rceil, \quad (22)$$

when this term is positive.

We now exploit slack values for reducing the pessimism calculating the worst-case carry-in job workload of  $\tau_k$ . When a lower bound on the slack of a task is available, it is possible

### Algorithm 2 Slack-based Iterative Schedulability Test ( $\tau$ )

---

```

1: Updated  $\leftarrow$  true, Nround  $\leftarrow$  0
2: for each task  $\tau_k \in \tau$  do
3:    $S_k^{lb} \leftarrow 0$ 
4: end for
5: repeat
6:   Feasible  $\leftarrow$  true
7:   Updated  $\leftarrow$  false
8:   for  $k \leftarrow 1$  to  $|\tau|$  do
9:     NewBound  $\leftarrow$  Eq. (23)
10:    if NewBound  $< 0$  then
11:      Feasible  $\leftarrow$  false
12:    else
13:      if NewBound  $> S_k^{lb}$  then
14:         $S_k^{lb} \leftarrow$  NewBound
15:        Updated  $\leftarrow$  true
16:      end if
17:    end for
18:  end if
19:  Nround  $++$ 
20:  if Feasible is true then
21:    return schedulable
22:  end if
23: until Updated is true and Nround  $\leq$  NroundLimit
24: return unschedulable

```

---

to give a tighter upper bound on interference. If the value of  $S_k^{lb}$  is positive, every job of  $\tau_k$  will complete at least  $S_k^{lb}$  time units before its deadline. Using this information, we can reduce pessimism on calculating the worst-case carry-in job workload of  $\tau_k$  shown in Algorithm 1. It can be easily incorporated by replacing the initial starting and finishing times of each thread (line 2 in Algorithm 1) with  $s[u] \leftarrow D_i - S_i^{lb}$  and  $f[u] \leftarrow D_i - S_i^{lb}$ . Then, a worst-case workload can be calculated in a similar way to Eq. (12), and we denote the workload incorporating a lower bound on the slack of task  $\tau_i$  by  $\mathcal{W}_{< i, v >}(D_k, S_i^{lb})$ . Note that when a lower bound on  $S_i^{lb}$  is not known, we can simply use  $S_i^{lb} = 0$ . Finally, a lower bound on the slack of a task  $\tau_k$  under global EDF scheduling on  $m$  identical processors is given by

$$S_k^{lb} = D_k - LC_k - \left\lceil \frac{1}{m} \left( \sum_{i \neq k} \sum_{\theta_i, v \in V_i} \mathcal{W}_{< i, v >}(D_k, S_i^{lb}) + \mathcal{W}_k(LC_k) \right) \right\rceil, \quad (23)$$

when this term is positive.

Then, the approaches [11], [37] of exploiting a slack value can be adopted into our workload-based schedulability test presented in Theorem 1. Therefore, we derive a slack-based iterative schedulability test of a task set under global EDF scheduling, and it is presented in Algorithm 2. As shown in Algorithm 2, for every task in a task set, a lower bound value on the slack of the task is initialized to zero (line 3). Eq. (23) is used to compute a new value of the lower bound on the slack of a task (line 9). If the computed value is negative, the task is considered to be unschedulable (line 11). If it is positive, the lower bound is accordingly updated (lines 13-15). It is repeated for every task in the task set until the lower bound on the slack for every task is not updated (lines 5-23). If no computed value is negative, the task set is deemed schedulable. If the iteration stops, the task set is deemed unschedulable (line 24).

**Complexity.** Similar to the workload-based schedulability test in Section 4, the slack-based iterative schedulability

test requires  $O(n(|V_{max}| + |E_{max}|))$  time to calculate  $LC_i$  for all tasks. Additionally, the complexity of the iterative test depends on the number of iterations of slack updates. A single iteration of slack updates requires  $O(n^2)$  time. The total number of iterations of the *repeat* cycle at line 5 is upper-bounded by  $O(n \cdot \max_{\tau_i \in \tau} D_i)$  [37]. Therefore, the overall time complexity is  $\max\{O(n(|V_{max}| + |E_{max}|)), O(n^3 \cdot \max_{\tau_i \in \tau} D_i)\}$ . We note that the complexity can be significantly reduced if the test is stopped after a finite number  $NroundLimit$  of iterations. If this is the case, the overall complexity becomes  $\max\{O(n(|V_{max}| + |E_{max}|)), O(n^2 \cdot NroundLimit)\}$ . However, limiting the number of iterations may reduce the number of schedulable task sets, rejecting some feasible task set that could be deemed schedulable after a few more iterations. We examine such schedulability loss for different values of  $NroundLimit$  in Section 7.

## 7 EVALUATION

In this section, we present simulation results to evaluate our global EDF schedulability analysis that is directly applicable to a set of parallel tasks.

### 7.1 Simulation environment

We generate DAG tasks mainly based on the method used in [41]. For a DAG task  $\tau_i$ , its parameters are determined as follows. Period and deadline of  $\tau_i$  ( $T_i = D_i$ )<sup>5</sup> are uniformly chosen in  $[100, 1000]$ . The number of nodes (threads)  $N_i$  is uniformly chosen in  $[1, 30]$ . For each pair of nodes, an edge is generated with the probability of  $Pr$ ; for each edge, its orientation is chosen to ensure acyclicity. For individual threads  $\theta_{i,u}$ , the WCET ( $C_{i,u}$ ) is randomly selected in the range of  $[1, T_i/N_i]$ .

In order to understand how our proposed approaches perform with DAG tasks, we experiment by varying the following parameters: the degree of parallelism in a DAG task and the number of processors. In addition, we examine the effect of  $NroundLimit$  in our slack-based iterative schedulability test presented in Section 6. Other experiments when varying the number of threads and variation of WCETs among threads in a DAG task are reported in the supplement, available online.

In each experiment, we compare our proposed schedulability tests with some related methods (shown in Table 1) for the DAG task model under global EDF scheduling. More specifically, we consider the following schedulability tests:

- our workload-based schedulability test in Theorem 1 (denoted by OUR)
- our slack-based iterative schedulability test allowing the maximum number of iterations in Algorithm 2 (denoted by OUR-I)
- the EDF schedulability test in [32] (denoted by BAR).
- the capacity augmentation bound (i.e., Theorem 4) in [31] (denoted by LCA)
- the EDF schedulability test (i.e., Theorem 21) in [29] (denoted by BMS).

5. In this section, we only show the results of implicit deadline DAG tasks because some of related works consider the implicit deadline task case only, but our proposed analysis shows similar behaviors in constrained deadline DAG tasks compared to those of implicit ones.

As mentioned in Section 2, the five schedulability tests shown in Figure 6 are classified as independent schedulability analysis. OUR-I and BAR are of pseudo-polynomial time complexity, while OUR, LCA, and BMS are of polynomial time complexity.

We note that other related analysis techniques are not included in our evaluation, because the one in [30] is applicable only to the single DAG task case, while the multiple parallel task case is of our interest. The capacity augmentation bound in [28] is excluded in our evaluation, because LCA is the best known capacity augmentation bound of EDF. In [29], a pseudo-polynomial time schedulability test was also proposed in addition to BMS, but it is not included in this comparison because BAR strictly dominates the pseudo-polynomial test. We also note that the resource augmentation bounds in [28], [29] are not included in this comparison, because those bounds can serve as schedulability tests only when an optimal schedule is known. However, no optimal schedule for parallel tasks has been developed so far, and therefore, it is difficult (if not impossible) to check the feasibility of a task set through simulation.

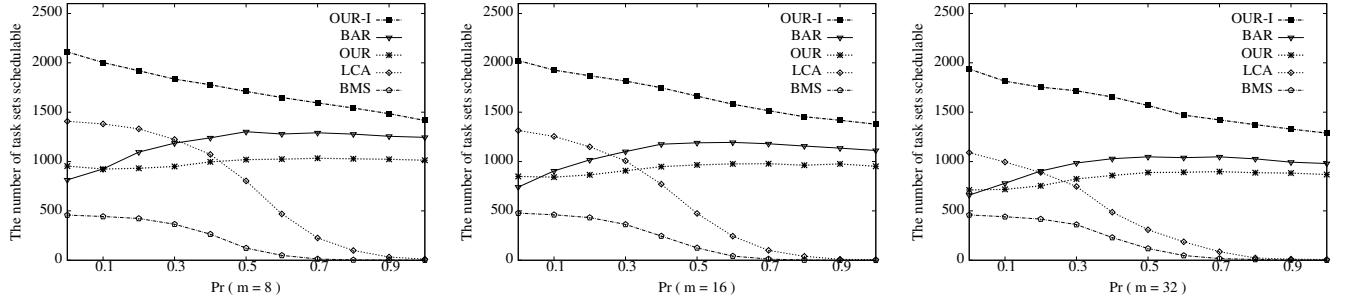
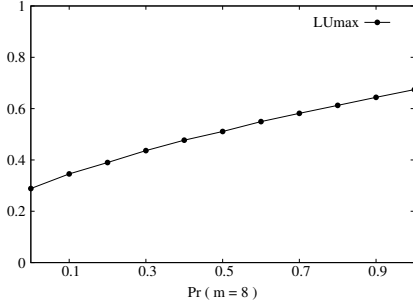
### 7.2 Simulation results

**Effect of the degree of parallelism.** Our first simulations were performed to evaluate our approaches with different degrees of intra-task parallelism. We generate 4,000 task sets for  $m = 8, 16$ , and  $32$ , where  $m$  is the number of processors, yet leaving  $Pr$  undetermined, as follows.

- S1 We first generate a seed task set with two tasks with the parameters determined as described above.
- S2 If the system utilization  $U_{sys}$  (i.e.,  $U_{sys} = \sum_{\tau_i \in \tau} U_i$ ) of the seed task set is greater than  $m$ , we discard this seed set and go to Step S1.
- S3 We include this seed set for simulation. We then add  $m/4$  more task into the seed set and go to Step S2 until 4,000 task sets are generated.

We now consider constructing edges between nodes (i.e., precedence dependency between threads) with the probability parameter  $0 \leq Pr \leq 1$ . When  $Pr = 0$ , there is no edge and thereby no thread has predecessors, maximizing the degree of intra-task parallelism. In contrast, with  $Pr = 1$ , each node is fully connected to all the other nodes, representing no single thread can execute in parallel with any other threads in the same task. An increasing value of  $Pr$  generates a growing number of edges in each DAG task, leading to a greater degree of precedence constraints between nodes but a smaller degree of intra-task parallelism. Thereby, as  $Pr$  increases, each task is highly likely to have a larger longest path length  $LC_i$ . We define  $LU_{max}$  as the maximum  $LC_i/T_i$  among the tasks  $\tau_i \in \tau$ , and Figure 7 shows a tendency for  $LU_{max}$  as  $Pr$  increases. In order to run simulation for different degrees of intra-task parallelism, we perform simulation with 4,000 task sets in 11 different cases in terms of  $Pr$ , where we increase  $Pr$  from 0.0 to 1.0 in the step of 0.1, resulting in 44,000 simulations.

Figure 6 compares different schedulability tests (OUR, OUR-I, BAR, LCA, and BMS) in terms of the number of task sets deemed schedulable with different values of  $Pr$  for  $m = 8, 16$ , and  $32$ . The figure shows that OUR-I outperforms

Fig. 6. Schedulability with different values of  $Pr$  for DAG tasksFig. 7.  $LU_{max}$  as  $Pr$  changes

the other existing methods throughout all the values of  $Pr$ . In cases of  $m = 8, 16$ , and  $32$ , OUR-I finds 47%, 54%, and 65% more schedulable task sets than BAR does, and OUR-I is also shown to improve schedulability, compared to OUR, by 75%, 80%, and 89% more, respectively. OUR finds 35%, 60%, and 90% additional task sets, which are deemed unschedulable by LCA for  $m = 8, 16$ , and  $32$ , respectively. We note that OUR-I finds almost all task sets deemed schedulable by the other tests except only 124, 12, and 1 task sets for  $m = 8, 16$ , and  $32$ , respectively. We also note that OUR-I shows better performance compared to other tests for cases with a larger number of processors.<sup>6</sup>

Looking at pseudo-polynomial-time schedulability tests (OUR-I and BAR), it is interesting to see that the performance gap between OUR-I and BAR becomes larger with a smaller value of  $Pr$ . Looking at polynomial-time schedulability tests (OUR, LCA, and BMS), when  $Pr$  increases the two methods of LCA and BMS are shown to perform worse, while OUR is relatively much insensitive to the value of  $Pr$ . Thereby, the performance gap between OUR and those two methods (LCA and BMS) becomes larger as the degree of precedence constraint increases.

To understand such performance results of OUR and OUR-I as  $Pr$  varies, we examine the effect of the degree of parallelism on schedulability with respect to our interference-based analysis. (i) In the perspective of a task  $\tau_k$  receiving interference from other tasks in a task set,  $LC_k$  generally decreases when the degree of parallelism increases (i.e., when  $Pr$  decreases). This gives task  $\tau_k$  more room to accommodate larger interference from other tasks, likely leading to better schedulability. (ii) On the other hand, in

the perspective of a task  $\tau_i$  imposing interference on  $\tau_k$ , the larger number of threads of  $\tau_i$  has a chance to delay execution of  $\tau_k$  at the same time when the degree of parallelism increases, leading to an increase in interference on  $\tau_k$ . Moreover, to derive a safe upper bound of the interference suffered by  $\tau_k$ , OUR assumes that every higher priority task  $\tau_i$  has carry-in. This is an over-pessimistic assumption, since in a real scheduling sequence, it may be the case that some task  $\tau_i$ 's carry-in job has finished before the beginning of the execution window of  $\tau_k$ , thus it actually does not contribute any carry-in to the interference on  $\tau_k$ . Such pessimism on bounding interference increases when the degree of parallelism increases. Statements (i) and (ii) are conflicting with each other in terms of schedulability. Figure 6 shows that the performance of OUR stays insensitive to  $Pr$  with Statements (i) and (ii) having comparable impacts on schedulability.

The figure also shows that OUR-I can compensate for the loss of performance from the pessimism involved in bounding interference, and the benefit of slack-based iterative method is getting bigger with a smaller value of  $Pr$ . Based on statement (i), it is highly likely to have larger slack values of  $\tau_k$  when the degree of parallelism increases. Then, those slack values effectively reduce the pessimism associated with the estimation of the carry-in of an interfering task according to statement (ii), which leads to better schedulability. Such an effect makes a larger performance gap between OUR-I and BAR with a smaller value of  $Pr$ .

The two existing polynomial-time tests (LCA and BMS) are sensitive to  $LU_{max}$  due to their schedulability analysis, since those two methods share in common that their schedulability tests check whether  $LU_{max}$  is smaller than or equal to some threshold (e.g.,  $(3+\sqrt{5})/2$  in LCA,  $1/3$  in BMS). A larger value of  $Pr$  generally increases the maximum  $LU_{max}$  for a task set (as shown in Figure 7), leading to worse schedulability.

**Effect of the number of processors.** Our second simulations were performed to show schedulability with a different number of processors. We generate 4,000 task sets whose utilization  $U_{sys}$  is in  $[3.9, 4.1]$ . Each task set is obtained by repeatedly adding tasks until the system utilization is in  $[3.9, 4.1]$  while each individual task is generated with the parameters described in Section 7.1 and the value of  $Pr$  fixed as 0.5.<sup>7</sup>

6. We also conducted simulations for  $m = 48$ , but the results are shown in the supplement, available online, since the trends are similar.

7. If the system utilization exceeds the limit in generating a task set, we discard the lastly added task, add a new task, and repeat the procedure until the system utilization is in the desired range.

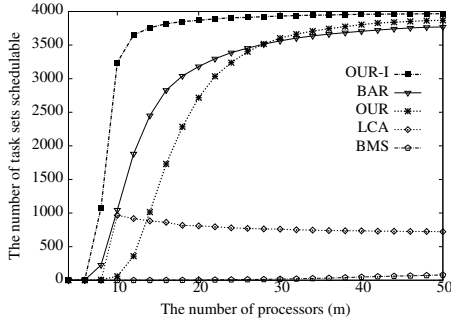


Fig. 8. Schedulability with different number of processors

Figure 8 shows the number of task sets deemed schedulable when the number of processors ( $m$ ) is varied from 4 to 50. OUR-I significantly outperforms the other tests. OUR-I requires a much lower number of processors (around 12) to schedule 90% of the task sets while OUR and BAR require more than 30 processors to do so. LCA and BMS behave even worse in the sense that they cannot admit a large portion of the generated task sets even with a very large number of processors. This is because their schedulability analysis is highly dependent on the value of  $LU_{max}$  irrelevant to the number of processors, as discussed in the first simulation results. We note that the running time of BAR increases as  $m$  increases by reflecting its analytical procedure shown in [32] while the running time of other tests including OUR-I and OUR is relatively stable to the number of processors. In particular, the average running time of OUR-I was 0.5 ms for all values of  $m$ , while the average running time of BAR was increased from 1 to 121 ms when changing  $m$  from 4 to 50.

TABLE 2  
Schedulability ratio of OUR-I with different values of  $NroundLimit$  relative to the maximum value of  $NroundLimit$

	$m$	Iteration limit ( $NroundLimit$ )				
		1	2	4	8	16
Schedulability ratio (%)	8	73.9	97.7	99.9	100	100
	16	70.0	97.7	99.9	100	100
	32	66.4	97.3	99.9	99.9	100

**Effect of  $NroundLimit$  in OUR-I.** Our third simulations were performed to investigate the schedulability loss of OUR-I for different values of  $NroundLimit$ : 1, 2, 4, 8, 16. We ran simulations on the same task sets used for Figure 6 with  $m = 8, 16$ , and 32, and Table 2 shows the schedulability ratio of OUR-I with different values of  $NroundLimit$  relative to the case of the maximum value of  $NroundLimit$  (i.e.,  $NroundLimit = n \cdot \max_{\tau_i \in \tau} D_i$ ). For example, for  $NroundLimit = 1$ , OUR-I finds a solution 73.9%, 70.0%, and 66.4% close to the case of the maximum value of  $NroundLimit$  when  $m = 8, 16$ , and 32, respectively. When two slack updates for each task are allowed (i.e.,  $NroundLimit = 2$ ), the number of task sets deemed schedulable by OUR-I increases rapidly. With  $NroundLimit = 16$ , OUR-I finds every task set that can be detected using an unbounded  $NroundLimit$  for all  $m = 8, 16$ , and 32. We note that the average running time of OUR-I with  $NroundLimit = 16$  was 0.4 ms to check the

schedulability of a task set, while the average running time of BAR was 11.1 ms when  $m = 8$ .

**Summary.** In summary, OUR outperforms the other existing polynomial-time schedulability tests. In addition, OUR-I significantly improves the schedulability of EDF with good efficiency and thus shows the best performance compared to the state-of-the-art independent schedulability tests available for DAG tasks. We identify that our proposed schedulability tests are adaptive to different degrees of intra-task parallelism and scalable to the number of processors in terms of both performance and complexity.

## 8 CONCLUSION

The motivation for our work was the desire to understand the thread-level parallelism of DAG tasks in the context of hard real-time multi-core scheduling. In this paper, we extended the notion of interference formalizing it at a finer-grained thread level and building a connection to the notion at a task level. We then generalized interference-based analysis methods according to the new proposed notion of interference, introducing global EDF schedulability conditions that are directly applicable to a set of DAG tasks. Our evaluation results showed that it significantly improves the state-of-the-art analysis techniques available for parallel tasks.

This paper incorporated thread-level parallelism directly into schedulability analysis focusing on the global EDF algorithm. However, we believe the schedulability of parallel tasks can be advanced much more significantly if thread-level parallelism is directly reflected into scheduling algorithms as well. Hence, a direction of our future work includes developing new real-time scheduling algorithms that support intra-task parallelism and synchronization directly.

## ACKNOWLEDGMENT

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