A Systematically Empirical Evaluation of Vulnerability Discovery Models: a Study on Browsers' Vulnerabilities

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Abstract—A precise vulnerability discovery model (VDM) will provide a useful insight to assess software security, and could be a good prediction instrument for both software vendors and users to understand security trends and plan ahead patching schedule accordingly. Thus far, several models have been proposed and validated. Yet, no systematically independent validation by somebody other than the author exists. Furthermore, there are a number of issues that might bias previous studies in the field. In this work, we fill in the gap by introducing an empirical methodology that systematically evaluates the performance of a VDM in two aspects: quality and predictability. We further apply this methodology to assess existing VDMs. The results show that some models should be rejected outright, while some others might be adequate to capture the discovery process of vulnerabilities. We also consider different usage scenarios of VDMs and find that the simplest linear model is the most appropriate choice in terms of both quality and predictability when browsers are young. Otherwise, logistics-based models are better choices.

Index Terms—Software Security, Empirical Validation, Vulnerability Discovery Model, Vulnerability Analysis

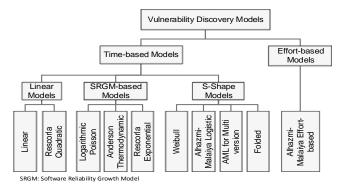
1 INTRODUCTION

VULNERABILITY discovery models (VDMs) operate on known vulnerability data to estimate the total number of vulnerabilities that will be reported after the release of a software. Successful models can be useful instruments for both software vendors and users to understand security trends, plan patching schedules, decide updates and forecast security investments in general.

A VDM is a parametric mathematical function counting the number of cumulative vulnerabilities of a software at an arbitrary time *t*. For example, if $\Omega(t)$ is the cumulative number of vulnerabilities at time *t*, the function of the linear model (LN) is $\Omega(t) = At + B$ where *A*, *B* are two parameters of LN, which are calculated from the historical vulnerability data.

Fig. 1 sketches a taxonomy of the major VDMs. It includes Anderson's Thermodynamic (AT) model [5], Rescorla's Quadratic (RQ) and Rescorla's Exponential (RE) models [19], Alhazmi & Malaiya's Logistic (AML) model [1], AML for Multi-version [11], Weibull model (JW) [10], and Folded model (YF) [24]. The *goodness-of-fit* of these models, *i.e.* how well a model could fit the numbers of discovered vulnerabilities, is normally evaluated in each paper on a specific vulnerability data set, except AML which has been validated for different types of application (*i.e.* operating system [4], [3], browsers [22], web servers [2], [23]). Yet, no independent validation by somebody other than the authors exists. Furthermore, a number of issues might bias the results of previous studies.

• Firstly, many studies did not clearly define what a vulnerability is. Indeed different definitions



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Fig. 1. Taxonomy of Vulnerability Discovery Models.

of vulnerability might lead to different counted numbers of vulnerabilities, and consequently, different conclusions.

- Secondly, all versions of a software were considered as a single "entity". Even though there is a large amount of shared code, they are still different by a non-negligible amount of code.
- Thirdly, the goodness-of-fit of the models was often evaluated at a single time point (of writing their papers) and not used as a predictor *e.g.*, to forecast data for the next quarter for instance.

A detail discussion about these issues is available later in section §3.

In this paper we want to address these shortcomings and derive a methodology that can answer two basic questions concerning VDM: "Are VDMs adequate to capture the discovery process of vulnerabilities?", and "which VDM is the best?".

Table 1 Performance summary of VDMs.

Model	Performance
AT, RQ	should be rejected due to low quality.
LN	is <i>the best model</i> for first 12 months ^(*) .
AML	is <i>the best model</i> for 13^{th} to 36^{th} month ^(*) .
RE, LP	may be adequate for first 12 months ^(**) .
JW, YF	may be adequate for 13^{th} to 36^{th} month ^(*) .

(*): in terms of quality and predictability for next 3/6/12 months.
 (**): in terms of quality and predictability for next 3 months.

1.1 Contributions of This Paper

The contributions of this work are detailed below:

- We proposed an experimental methodology to assess the performance of a VDM based on its *goodness-of-fit quality* and *predictability*.
- We demonstrated the methodology by conducting an experiment analyzing eight VDMs, including AML, AT, JW, RQ, RE, LP, LN, and YF on 30 major releases of four popular browsers Internet Explorer (IE), Firefox (FF), Chrome and Safari.
- We presented an empirical evidence for the adequacy of the VDMs in terms of quality and predictability. The AT and RQ models are not adequate; whereas all other models may be adequate when software is young (12 months). However only s-shape models (AML, JW, YF) should be considered when software is middle age (36 months) or older.
- We compared these VDMs (except AT and RQ) in different usage scenarios in terms of predictability and quality. The simplest model, LN, is more appropriate than other complex models when software is young and the prediction time span is not too long (12 months or less). Otherwise, the AML model is superior. These results are summarized in Table 1.

The rest of the paper is organized as follows. Section $\S2$ presents terminology in our work. The research questions are presented in section $\S3$ and the proposed methodology is described in section $\S4$. Next, we apply the methodology to analyze the empirical performance of VDMs in all data sets in section $\S5$. Then in section $\S6$, we discuss the threats to the validity of our work. Finally, we review related work in section \$7, and conclude in section \$8.

2 TERMINOLOGY

• A vulnerability is "an instance of a [human] mistake in the specification, development, or configuration of software such that its execution can [implicitly or explicitly] violate the security policy"[12], later revised by [18]. The definition covers all aspects of vulnerabilities discussed in [6], [7], [8], [20], see also [18] for a discussion.

- *A data set* is a collection of vulnerability data extracted from one or more data sources.
- *A release* refers to a particular version of an application *e.g.*, Firefox v1.0.
- *A horizon* is a specific time interval sample. It is measured by the number of months since the released date, *e.g.*, from month 1 to 12 months after the release.
- An observed vulnerability sample (or observed sample, for short) is a time series of monthly cumulative vulnerabilities of a major release since the first month after release to a particular horizon.
- An evaluated sample is a tuple of an observed sample, a VDM model fitted to this sample (or another observed sample), and the goodness-of-fit of this model to this sample.

3 RESEARCH QUESTIONS AND METHODOL-OGY OVERVIEW

In this work, we address the following two questions: **RQ1** *How to evaluate the performance of a VDM?*

 $\mathbf{RQ2}$ How to compare between two or more VDMs?

We propose a methodology to answer these questions. The proposed methodology identifies data collection steps and mathematical analyses to empirically assess different performance aspects of a VDM. The methodology is summarized in Table 2.

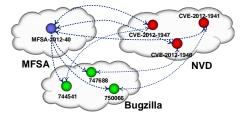
In order to satisfactorily answer the questions above, we must address some biases that potentially affected the validity of previous studies.

The vulnerability definition bias may affect the vulnerability data collection process. Indeed all previous studies reported their data sources, but none clearly mentioned what a vulnerability is, and how to count it. A vulnerability could be either an advisory reported by a software vendor such as Mozilla Foundation Security Advisory – MFSA, or a security bug causing software to be exploited (reported in Mozilla Bugzilla), or an entry in third-party vulnerability databases (e.g., National Vulnerability Database - NVD). Some entries may be classified differently by different entities: a third-party database might report vulnerabilities, but the security bulletin of vendors may not classify them as such. Consequently, the counted number of vulnerabilities could be widely different depending on the different definitions.

Example 1 Fig. 2 exemplifies this issue. A security flaw concerning the buffer overflow and use-after-free of Firefox v13.0 is reported in three databases with different number of entries: one MFSA entry (MFSA-2012-40), three Bugzilla entries (744541, 747688, and 750066), and three NVD entries (CVE-2012-1947, CVE-2012-1940, and CVE-2012-1941). The cross references among these entries are illustrated as directional connections. This figure raises a question "how many vulnerabilities should we count in this case?".

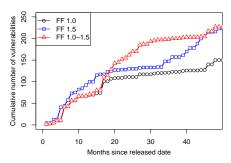
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A buffer-overflow flaw is reported in one entry of MFSA, three entries of Bugzilla, and three entries of NVD. There are cross-references among entries (arrow-headed dotted lines). So, how many vulnerabilities should we count?

Fig. 2. The problem of counting vulnerabilities in Firefox.



This shows the cumulative vulnerabilities of Firefox v1.0, v1.5, and v1.0-v1.5 as a single entity. The "global" entity exhibits a trend that is not present in the "individual" versions.

Fig. 3. Global vs individual vulnerability trends for Firefox.

The *multi-version software* bias affects the count of vulnerability across releases. Some studies (*e.g.*, [22], [23]) considered all versions of software as a single entity, and counted vulnerabilities for this entity. Our previous study [13] has shown that each Firefox version has its own code base, which may differ by 30% or more from the immediately preceding one. Therefore, as time goes by, we can no longer claim that we are counting the vulnerabilities of the same application.

Example 2 Fig. 3 visualizes this problem in a plot of the cumulative vulnerabilities of Firefox v1.0, Firefox v1.5, and Firefox v1.0-1.5 as a single entity. Clearly, the function of the "global" version should be different from the functions of the individual versions.

The *overfitting* bias, as name suggested, concerns the ability of a VDM to explain history in hindsight. Previous studies took a snapshot of vulnerability data, and fitted this entire snapshot to a VDM. This made a brittle claim of fitness: the claim was only valid at the time vulnerabilities were collected. It explained history but did not tell us anything about the future. Meanwhile, we are interested in the ability of a VDM to be a good "law of nature" that is valid across releases and time and to some predict extent the future.

4 METHODOLOGY DETAILS

This section discusses the details of our methodology to evaluate the performance of a VDM.

4.1 Step 1: Acquire the Vulnerability Data

The acquisition of vulnerability data consists of two sub steps: *Data set collection*, and *Data sample extraction*.

During *Data set collection*, we identify the data sources to be used for the study (as they may not equally fit to the task). We can classify them as follows:

- *Third-party advisory* (TADV): is a vulnerability database maintained by a third-party organization (not software vendors) *e.g.*, NVD, Open Source Vulnerability Database (OSVDB).
- *Vendor advisory* (ADV): is a vulnerability database maintained by a software vendor, *e.g.*, MFSA, Microsoft Security Bulletin. Vulnerability information in this database could be announced from third-party, but it is always validated before being announced as an advisory.
- *Vendor bug tracker* (BUG): is a bug-tracking database, usually maintained by vendors.

For our purposes, the following features of a vulnerability are interesting and must be provided:

- *Identifier* (id): is the identifier of a vulnerability within a data source.
- *Disclosure date* (date): refers to the date when a vulnerability is reported to the database¹.
- *Vulnerable Releases* (R): is a list of releases affected by a vulnerability.
- *References* (refs): is a list of reference links to other data sources.

Not every feature is available from all data sources. To obtain missing features, we can use id and refs to link across data sources and extract the expected features from secondary data sources.

Example 3 Vulnerabilities of Firefox are reported in three data sources: NVD², MFSA, and Mozilla Bugzilla. Neither MFSA nor Bugzilla provides the *Vulnerable Releases* feature, but NVD does. Each MFSA entry has one or more links to NVD and Bugzilla. Therefore, we could to combine MFSA and NVD, Bugzilla and NVD to obtain the missing data.

We address the *vulnerability definition* bias by taking into account different definitions of vulnerability. Particularly, we collected different vulnerability data sets with respect to these definitions. We also address the *multi-version* issue by collecting vulnerability data

^{1.} The actual discovery date might be significantly earlier than that.

^{2.} Other third party data sources (*e.g.*, OSVDB, Bugtraq, IBM XForce) also report Firefox's vulnerabilities, but most of them refer to NVD by the CVE-ID. Therefore, we consider NVD as a representative of third-party data sources.

Table 3 Formal definition of data sets.

Data set	Definition
NVD(r) NVD.Bug(r) NVD.Advice(r) NVD.NBug(r) Advice.NBug(r)	$ \begin{cases} nvd \in NVD r \in R_{nvd} \} \\ \{nvd \in NVD \exists b \in BUG : r \in R_{nvd} \land id_{b} \in refs_{nvd} \} \\ \{nvd \in NVD \exists a \in ADV : r \in R_{nvd} \land id_{a} \in refs_{nvd} \} \\ \{b \in BUG \exists nvd \in NVD : r \in R_{nvd} \land id_{b} \in refs_{nvd} \} \\ \{b \in BUG \exists a \in ADV, \exists nvd \in NVD : r \in R_{nvd} \\ \{b \in BUG \exists a \in ADV, \exists nvd \in NVD : r \in R_{nvd} \\ \land id_{b} \in refs_{a} \land id_{nvd} \in refs_{a} \land clustera_{a}(id_{b}, id_{nvd}) \end{cases} \end{cases} $

Note: R_{nvd} , $refs_{nvd}$ denote the vulnerable releases and references of an entry nvd, respectively. id_a , id_b , id_{nvd} denote the identifier of a, b, and nvd. $cluster_a(id_b, id_{nvd})$ is a predicate checking whether id_b and id_{nvd} are located next together in the advisory a.

for individual releases. Table 3 shows different data sets that we have considered in our study. They are combinations of three types of data sources : third-party (*i.e.* NVD as a representative), vendor advisory, and vendor bug tracker. The English descriptions of these data sets for a *release* r are as follows:

- NVD(*r*): a set of NVD entries which claim *r* is vulnerable.
- NVD.Bug(*r*): a set of NVD entries which are confirmed by at least a vendor bug report, and claim *r* is vulnerable.
- NVD.Advice(*r*): a set of NVD entries which are confirmed by at least a vendor advisory, and claim *r* is vulnerable. Notice that the advisory report might *not* mention *r*, but later releases.
- NVD.Nbug(r): a set of vendor bug reports confirmed by NVD, and r is claimed vulnerable by NVD.
- Advice.NBug(*r*): a set of bug reports mentioned in an advisory report of a vendor. The advisory report also refers to at least an NVD entry that claims *r* is vulnerable.

For *Data sample extraction*, we extract observed samples from collected data sets. An *observed sample* is a time series of (monthly) cumulative vulnerabilities of a release. It starts from the first month since release to the end month, called *horizon*. A month is an appropriate granularity for sampling because week and day are too short intervals and are subject to random fluctuation. Additionally, this granularity was used in the literature.

Let R be the set of analyzed releases and DS be the set of data sets, an observed sample (denoted as os) is a time series defined as follows:

$$os = \mathsf{TS}(r, ds, \tau) \tag{1}$$

where:

- $r \in R$ is a release in the evaluation;
- $ds \in DS$ is the data set where samples are extracted;
- $\tau \in T_r = [\tau_{min}^r, \tau_{max}^r]$ is the horizon of the observed sample, in which T_r is the *horizon range* of release r.

In the horizon range of release r, the minimum value of horizon τ_{min}^r of r depends on the starting time of the first observed sample of r. Here we choose $\tau_{min}^r = 6$ for all releases so that all observed samples have enough data points for fitting all VDMs. The maximum value of horizon τ_{max}^r depends on how long the data collection period is for each release.

Example 4 IE v4.0 was released in September, 1997³. The first month was on 31 October, 1997. The first observed sample of IE v4.0 is a time series of 6 numbers of cumulative vulnerabilities for the $1^{st}, 2^{nd}, \ldots, 6^{th}$ months. Since the date of data collection is on 01 July 2012, IE v4.0 have been released for 182 months, and therefore has 177 observed samples. Hence the maximum value of horizon $(\tau_{max}^{\text{IEv4.0}})$ is 182.

4.2 Step 2: Fit a VDM to Observed Samples

We estimate the parameters of the VDM formula by a regression method so that the VDM curve fits an observed sample as much as possible. We denote the fitted curve (or fitted model) as:

$$vdm_{\mathsf{TS}(r,ds,\tau)}$$
 (2)

where vdm is the VDM being fitted; $os = TS(r, ds, \tau)$ is an observed sample from which the vdm's parameters are estimated. (2) could be shortly written as vdm_{os} .

Example 5 Fitting the AML model to the NVD data set of Firefox v3.0 at the 30^{th} month, *i.e.* the observed sample os = TS(FF3.0, NVD, 30), generates the curve:

$$AML_{\mathsf{TS}(\mathsf{FF3.0,NVD},30)} = \frac{183}{183 \cdot 0.078 \cdot e^{-0.001 \cdot 183 \cdot t} + 1}$$

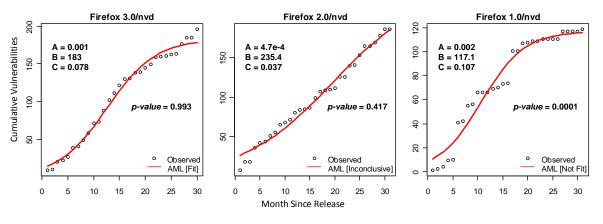
Fig. 4 illustrates the plots of three curves $AML_{TS(r,NVD,30)}$, where *r* is FF3.0, FF2.0, and FF1.0. The X-axis is the number of months since release, and the Y-axis is the cumulative number of vulnerabilities. Circles represent observed vulnerabilities. The solid line indicates the fitted AML curve.

In Fig. 4, the distances of the circles to the curve are used to estimate the goodness-of-fit of the model. The goodness-of-fit is measured by the Pearson's Chi-Square (χ^2) test, which is a common test in the literature. In this test, we measure the χ^2 statistic value of the curve by using the following formula:

$$\chi^2 = \sum_{t=1}^{\tau} \frac{(O_t - E_t)^2}{E_t}$$
(3)

where O_t is the observed cumulative number of vulnerabilities at time t (*i.e.* t^{th} value of the observed sample); E_t denotes the expected cumulative number of vulnerabilities which is the value of the curve at time t. The χ^2 value is proportional to the differences

3. Wikipedia, http://en.wikipedia.org/wiki/Internet_Explorer, visited on 24 June 2012.



A,B,C are three parameters in the formula of the AML model: $\Omega(t) = \frac{B}{BCe^{-ABt}+1}$ (see also Table 4)

Fig. 4. Fitting the AML model to the NVD data sets for Firefox v3.0, v2.0, and v1.0.

between the observed values and the expected values. Hence, the larger χ^2 , the smaller goodness-of-fit. If the χ^2 value is large enough, we can safely reject the model. In other words, the model statistically does not fit the observed data set. The χ^2 test requires all expected values should be at least 5 to ensure the validity of the test [17, Chap. 1]. If there is any expected value is less than 5, we need to combine some first months to increase the expected value *i.e.* increase the starting value of *t* in (3) until $E_t \geq 5$.

The conclusion about whether a VDM curve statistically fits an observed sample relies on the *p-value* of the test, which is derived from χ^2 value and the degrees of freedom (*i.e.* the number of months minus one). Semantically, the *p-value* is the probability that we falsely reject the *null hypothesis* when it is true (*i.e.* error Type I: false positive). The null hypothesis here is: "there is no statistical difference between observed and expected values." which means that the model fits the observed sample. Therefore, if the *p-value* is less than the significance level α of 0.05, we can reject a VDM because there is less than 5% chances that this fitted model would generate the observed sample.

In contrast, to accept a VDM, we exploit the power of the χ^2 test which is the probability of rejecting the null hypothesis when it is false. Normally, 'an 80% power is considered desirable' [15, Chap. 8]. Hence we accept a VDM if the *p*-value is greater than or equal to 0.80. We have more than 80% chances of generating the observed sample from the fitted curve. In all other cases, we should neither accept nor reject the model (inconclusive fit).

The criteria CR2 in Table 2 summarizes the way by which we assess the goodness-of-fit of a fitted model based on the *p*-value of the χ^2 test.

In the sequel, we use the term *evaluated sample* to denote the triplet composed by an observed sample, a fitted model, and the *p*-value of the χ^2 test.

Example 6 In Fig. 4, the first plot shows the AML

model with a *Good Fit* (*p*-value = 0.993 > 0.80), the second plot exhibits the AML model with an *Inconclusive Fit* (0.05 < p-value = 0.417 < 0.80), and the last one denotes the AML model with a *Not Fit* (*p*-value = 0.0001 < 0.05).

There are also other statistic tests for goodness-offit, for instance the Kolmogorov-Smirnov (K-S) test, and the Anderson-Darling (A-D) test. The K-S test is an exact test; it, however, only applies to continuous distributions. An important assumption is that the parameters of the distribution cannot be estimated from the data. Hence, we cannot apply it to perform the goodness-of-fit test for a VDM. The A-D test is a modification of the K-S test that works for some distributions [17, Chap. 1] (*i.e.* normal, log-normal, exponential, Weibull, extreme value type I, and logistic distribution), but some VDMs violate this assumption.

4.3 Step 3: Perform Goodness-of-Fit Quality Analysis

To address the *overfitting* bias, we introduce the *goodness-of-fit quality* (or *quality*, for short) that measures the overall number of *Good Fits* and *Inconclusive Fits* among different samples. In contrast, previous studies considered only one observed sample which is the one with the largest horizon in their experiment.

Let $OS = \{TS(r, ds, \tau) | r \in R \land ds \in DS \land \tau \in T_r\}$ be the set of observed samples, the *overall quality* of a model *vdm* is defined as the weighted ratio of the number of *Good Fit* and *Inconclusive Fit* evaluated samples over the total ones, as shown bellow:

$$Q_{\omega} = \frac{|GES| + \omega \cdot |IES|}{|ES|} \tag{4}$$

where:

ES = {⟨*os*, *vdm*_{os}, *p*⟩ |*os* ∈ *OS*} is the set of evaluated samples generated by fitting *vdm* to observed samples;

- GES = {⟨os, vdm_{os}, p⟩ ∈ ES | p ≥ 0.80} is the set of Good Fit evaluated samples;
- *IES* = {⟨os, vdm_{os}, p⟩ ∈ *ES*|0.05 ≤ p < 0.80} is the set of *Inconclusive Fit* evaluated samples;
- $\omega \in [0..1]$ is the *inconclusiveness contribution* factor denoting that an *Inconclusive Fit* is ω times less important than a *Good Fit*.

Example 7 If we fit the AML model to 3, 895 observed samples of the four browsers IE, Firefox, Chrome, and Safari. For 1,526 times AML is a *Good Fit*, and for 1,463 times AML is an *Inconclusive Fit*. The overall quality of AML is:

$$Q_{\omega=0} = \frac{1,526}{3,895} = 0.39$$
$$Q_{\omega=1} = \frac{1,526 + 1,463}{3,895} = 0.77$$
$$Q_{\omega=0.5} = \frac{1,526 + 0.5 \cdot 1,463}{3,895} = 0.58$$

To calculate the χ^2 test we refit the model each and every time. So this means that we have 1,526 different parameters A, B and C for each good fit curve (see Fig. 4).

The overall quality metric ranges between 0 and 1. The quality of 0 indicates a completely inappropriate model, whereas the quality of 1 indicates a perfect one. This metric is a very optimistic measure as we are essentially "refitting" the model as more data become available. Hence, it is the upper bound value of the VDM quality.

The factor ω denotes the contribution of an inconclusive fit to the overall quality. A skeptical analyst would expect $\omega = 0$, which means only *Good Fits* are meaningful. Meanwhile an optimistic analyst would set $\omega = 1$, which mean an *Inconclusive Fit* is as good as a *Good Fit*. The optimistic choice $\omega = 1$ is usually adopted by the proposers of each model in previous studies in the field while assessing the VDM quality. The effect of the ω factor on the overall quality metrics is illustrated in Fig. 5 showing the variation of the overall quality of two models AML and AT with respect to ω . We do not know whether an *Inconclusive* Fit is good or not because the observed samples do not provide enough evidence. Hence, the choice of $\omega = 0.5$ may be considered a good balance. During our analysis we use $\omega = 0.5$; any exception will be explicitly noted.

The overall quality metric is sensitive to brittle performance in time. A VDM could produce a lot of *Good Fits* evaluated samples for the first 6 months, but almost *Not Fits* at other horizons. Unfortunately, the metric did not address this phenomenon.

To avoid this unwanted effect, we introduce the *temporal quality* metric which represents the evolution of the overall quality over time. The temporal quality $Q_{\omega}(\tau)$ is the weighted ratio of the *Good Fit*

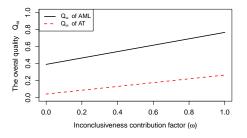


Fig. 5. The variation of the overall quality Q_{ω} with respect to the ω factor.

and *Inconclusive Fit* evaluated samples over total ones at the particular horizon τ . The temporal quality is formulated in the following equation:

$$Q_{\omega}(\tau) = \frac{|GES(\tau)| + \omega \cdot |IES(\tau)|}{|ES(\tau)|}$$
(5)

where:

- τ ∈ T is the horizon that we observe samples, in which T ⊆ ⋃_{r∈R} T_r is the subset of the union of the horizon ranges of all releases r in evaluation;
- ES(τ) = {⟨os, vdm_{os}, p⟩ |os ∈ OS(τ)} is the set of evaluated samples at the horizon τ; where OS(τ) is the set of observed samples at the horizon τ of all releases;
- GES(τ) ⊆ ES(τ) is the set of Good Fit evaluated samples at the horizon τ;
- $IES(\tau) \subseteq ES(\tau)$ is the set of *Inconclusive Fit* evaluated samples at the horizon τ ;
- ω is the same as for the overall quality Q_{ω} .

To study the trend of the temporal quality $Q_{\omega}(\tau)$, we employ the *moving average* technique which is commonly used in time series analysis to smooth out short-term fluctuations and highlight longer-term trends. Intuitively each point in the moving average is the average of some adjacent points in the original series. The moving average of the temporal quality is defined as follows:

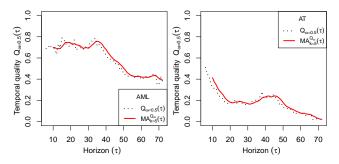
$$MA_{k}^{Q_{\omega}}(\tau) = \frac{1}{k} \sum_{i=1}^{k} Q_{\omega}(\tau - i + 1)$$
(6)

where k is the *window size*. The choice of k changes the spike-smoothening effect: higher k, smoother spikes. Additionally, k should be an odd number so that variations in the mean are aligned with variations in the data rather than being shifted in time.

Example 8 Fig. 6 depicts the moving average for the temporal quality of AML and AT models. In this example, we choose a window size k = 5 because the minimum horizon is six ($\tau_{min}^r = 6$), so k should be less than this horizon ($k < \tau_{min}^r$); and k = 3 is too small to smooth out the spikes.

4.4 Step 4: Perform Predictability Analysis

The predictability of a VDM measures the capability of predicting future trends of vulnerabilities. This



Dotted lines are the temporal quality with $\omega = 0.5$, solid lines are the moving average of the temporal quality with the window size k = 5.

Fig. 6. An example about the moving average of the temporal quality of AML and AT models.

essentially makes a VDM applicable in practice. The calculation of the predictability of a VDM has two phases, the *learning phase* and the *prediction phase*. In the learning phase, we fit a VDM to an observed sample at a certain horizon. In the prediction phase, we evaluate the qualities of the fitted model on observed samples in future horizons.

We extend (5) to calculate the prediction quality. Let $vdm_{\mathsf{TS}(r,ds,\tau)}$ be a fitted model at horizon τ . The prediction quality of this model in the next δ months is calculated as follows:

$$Q_{\omega}^{*}(\tau,\delta) = \frac{|GES^{*}(\tau,\delta)| + \omega \cdot |IES^{*}(\tau,\delta)|}{|ES^{*}(\tau,\delta)|}$$
(7)

where:

- $ES^*(\tau, \delta) = \{ \langle \mathsf{TS}(r, ds, \tau + \delta), vdm_{\mathsf{TS}(r, ds, \tau)}, p \rangle \}$ is the set of evaluated samples at the horizon $\tau + \delta$ in which we evaluate the quality of the model fitted at horizon τ ($vdm_{\mathsf{TS}(r, ds, \tau)}$) on observed samples at the future horizon $\tau + \delta$. We refer to $ES^*(\tau, \delta)$ as set of evaluated samples of prediction;
- $GES^*(\tau, \delta) \subseteq ES^*(\tau, \delta)$ is the set of *Good Fit* evaluated samples of prediction at the horizon $\tau + \delta$;
- *IES*^{*}(τ, δ) ⊆ *ES*^{*}(τ, δ) is the set of *Inconclusive Fit* evaluated samples of prediction at the horizon τ + δ.
- *ω* is the same as for the overall quality *Q_ω*.

Example 9 Fig. 7 illustrates the prediction qualities of two models AML and AT starting from the horizon of 12^{th} month ($\tau = 12$, left) and 24^{th} month ($\tau = 24$, right), and predicting the value for next 12 months ($\delta = 0...12$). White circles are prediction qualities of AML, and red (gray) circles are those of AT.

As we can see from Fig. 7 the ability of a model to predict data decreases with time. It is therefore useful to identify some interesting prediction time spans (such as next 6 months) that can be used for pairwise comparison between VDMs. To this extent, we identify some different scenarios by which we

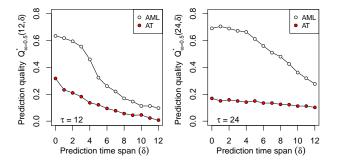


Fig. 7. The prediction qualities of the AML and AT model at some horizons.

specify the duration of data observation and the prediction time span. Other scenarios may be identified depending on the application or the readers' interest:

- *Plan for short-term support*: the data observation period may vary from 6 months to the whole lifetime. We are looking for the ability to predict the trend in next quarter (*i.e.* 3 months) to plan the short-term support activities *e.g.*, allocating resources for fixing vulnerabilities.
- *Plan for long-term support*: we would like to predict a realistic expectation for bug reports in the next 1 year to plan the long-term activities.
- *Upgrade or keep*: the data observation period is short (at from 6 to 12 months). We are looking on what is going to happen in next 6 months. For example to decide whether to keep the current system or to go over the hassle of updating it.
- *Historic analysis*: the data observation period is long (2 to 3 years), we are considering what happens for extra support in the next 1 year.

We should assess the predictability of a VDM not only along the prediction time span, but also along the horizon to ensure that the VDM is able to consistently predict the vulnerability data in an expected period. To facilitate such assessment we introduce the *predictability* metric which is the average of prediction qualities at a certain horizon.

The predictability of the curve vdm_{os} at the horizon τ in a time span of Δ months is defined as the average of the prediction quality of vdm_{os} at the horizon τ and its Δ consecutive horizons $\tau + 1, \tau + 2, ..., \tau + \Delta$, as the following equation shows:

$$Predict_{\omega}(\tau, \Delta) = \sqrt[\Delta+1]{\prod_{\delta=0}^{\Delta} Q_{\omega}^{*}(\tau, \delta)}$$
(8)

where Δ is the prediction time span.

In (8), we use the geometric mean instead of the arithmetic mean. The temporal quality is a normalized measure so using the arithmetic mean to average such values might produce a meaningless result, whereas the geometric mean behaves correctly [9].

4.5 Step 5: Compare VDM

This section addresses the second research question RQ2. The comparison is based on the quality and the predictability of VDMs. The base line for the comparison is that: *the better model is a better one in forecasting changes*. Hereafter, we discuss how to compare VDM:

Given two models vdm_1 and vdm_2 , the comparison between vdm_1 and vdm_2 could be done as below:

We compare the predictability of vdm_1 and that of vdm_2 . Let ρ_1, ρ_2 be the predictability of vdm_1 and vdm_2 , respectively.

$$\rho_1 = \{ Predict_{\omega=0.5}(\tau, \Delta) | \tau = 6..\tau_{max}, vdm_1 \}$$

$$\rho_2 = \{ Predict_{\omega=0.5}(\tau, \Delta) | \tau = 6..\tau_{max}, vdm_2 \}$$
(9)

where the prediction time span Δ could follow the criteria CR4; $\tau_{max} = \min(72, \max_{r \in R} \tau_{max}^r)$. We employ the one-sided Wilcoxon rank-sum test to compare ρ_1, ρ_2 . If the returned *p*-value is less than the significance level $\alpha = 0.05$, the predictability of vdm_1 is stochastically greater than that of vdm_2 . It also means that vdm_1 is better than vdm_2 . If *p*-value $\geq 1 - \alpha$, we conclude the opposite *i.e.* vdm_2 is better than vdm_1 . Otherwise we have not enough evidence either way.

If the previous comparison is inconclusive, we retry the comparison using the value of temporal quality of the VDMs instead of the predictability. We just replace $Q_{\omega=0.5}(\tau)$ for *Predict*_{\omega=0.5}(\tau, \Delta) in the equation (9), and repeat the above activities.

When we compare models *i.e.* we run several hypothesis tests, we should pay attention on the familywise error rate which is the probability of making one or more type I errors. To avoid such problem, we should apply an appropriate controlling procedure such as the Bonferroni correction. In the case above, the significance level by which we conclude a model is better than another one is divided by the number of tests performed.

Example 10 When we compare one model against other seven models, the Bonferroni-corrected significance level is: $\alpha = {}^{0.05} / _7 \approx 0.007$.

The above comparison activities are summarized in the criteria CR5 (see Table 2).

5 AN ASSESSMENT ON EXISTING VDMs

We apply the above methodology to assess the performance of eight existing VDMs (see also Table 4). The experiment evaluates these VDMs on 30 releases of the four popular web browsers: IE, Firefox, Chrome, and Safari. Here, only the formulae of these models are provided. More detail discussion about these models as well as the meaning of their parameters are referred to their corresponding original work.

Table 4 The VDMs in evaluation and their equation.

This table presents the list of VDMs and their equation in the alphabetical order. The meaning of each parameter should be found in original work of each model.

Model	Equation
Alhazmi-Malaiya Logistic (AML)	$\Omega(t) = \frac{B}{BCe^{-ABt} + 1}$
Anderson Thermodynamic (AT)	$\Omega(t) = \frac{k}{\gamma} \ln(t) + C$
Joh Weibull (JW)	$\Omega(t) = \gamma (1 - e^{-\left(\frac{t}{\beta}\right)^{\alpha}})$
Linear (LN)	$\Omega(t) = At + B$
Logistic Poisson (LP)	$\Omega(t) = \beta_0 \ln(1 + \beta_1 t)$
Rescorla Exponential (RE)	$\Omega(t) = N(1 - e^{-\lambda t})$
Rescorla Quadratic (RQ)	$\Omega(t) = \frac{At^2}{2} + Bt$
Younis Folded (YF)	$\Omega(t) = \frac{\gamma}{2} \left[\operatorname{erf}\left(\frac{t-\tau}{\sqrt{2\sigma}}\right) + \operatorname{erf}\left(\frac{t+\tau}{\sqrt{2\sigma}}\right) \right]$

Note: erf() is the error function, $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$

Table 5 Vulnerability data sources of browsers.

Data Source	Category	Apply for
National Vulnerability Database (NVD) Mozilla Foundation Security Advisory (MFSA) Mozilla Bugzilla (MBug) Microsoft Security Bulletin (MSB) Apple Knowledge Base (AKB)	TADV ADV BUG ADV ADV	All browsers Firefox Firefox IE Safari
Chrome Issue Tracker (CIT)	BUG	Chrome

5.1 Data Acquisition

Table 5 presents the availability of vulnerability data sources for the browsers in our study. For each data source, the table reports the name, the category (see also $\S4.1$), and the browser that the data source maintains vulnerability data. We use NVD as a representative third-party data source due to its popularity in past studies. This makes our work comparable with previous ones.

Table 6 reports data sets collected for this experiment (see also Table 3 for the classification). In total, we collected 96 data sets for 30 major releases. In the table, we use the bullet (\bullet) to indicate the availability of data sets. In these collected data sets, we extracted a total of 4,063 observed samples.

Table 6 Collected data sets.

Bullets (\bullet) indicate available data sets. Dashes (-) mean there is no data sources available to collect the data sets.

	Releases	NVD	NVD.Bug	NVD.Advice	NVD.NBug	Advice.Nbug	Total Datasets
Chrome	12(v1.0-v12.0)	•	•	_	•	_	36
Firefox	8(v1.0-v5.0)	•	•	•	•	•	40
IE	5(v4.0-v8.0)	•	_	•	_	_	10
Safari	5(v1.0–v5.0)	•	_	•	—	—	10
Total	30						96

5.2 The Applicability of VDMs

We ran model fitting algorithms for these observed samples by using R v2.13. Model fitting took about 82 minutes on a dual-core 2.73GHz Windows machine with 6GB of RAM yielding 32,504 curves in total.

5.2.1 Goodness-of-Fit Analysis for VDMs

Table 7 reports the goodness-of-fit of existing VDMs on the largest horizons of browser releases, using the NVD data sets. In other words, we use following observed samples to evaluate all models:

$$OS_{\mathsf{NVD}} = \{\mathsf{TS}(r,\mathsf{NVD},\tau_{max}^r) | r \in R\}$$

where R is the set of all releases mentioned in Table 6. Table 7 provides a view that previous studies often used to report the goodness-of-fit for their proposed models. To improve readability, we report the categorized goodness-of-fit based on the *p-value* (see CR2) instead of the raw *p*-values. In this table, we use a check mark (\checkmark), a blank, and a cross (\times) to respectively indicate a Good Fit, an Inconclusive Fit, and a Not Fit. Cells are shaded accordingly to improve the visualization effect. The table shows that two models AT and RQ have a very high ratio of *Not Fit* (0.9 and 0.7, respectively); whereas, all other models have their ratio of Not Fit less than 0.5. We should observe that this is a very large time interval and some systems have long gone into retirement. For example, FF v2.0 vulnerabilities are no longer sought by researchers. They are a byproduct of research on later versions.

To have a more realistic picture, we also study the temporal quality. The inconclusiveness contribution factor ω is set to 0.5 as described in CR3. Fig. 8 exhibits the moving average (windows size k = 5) of the $Q_{\omega}(\tau)$. The dotted vertical lines marks horizon 12 (when software is young), and 36 (when software is middle-age). We cut the temporal quality at horizon 72 though we have more data for some systems (*e.g.*, IE v4, FF v1.0). It is because that after 6 years software is very old, the vulnerability data reported for such releases might be not reliable, and might overfit the VDMs. The dotted horizon line at 0.5 is used as a base line to assess VDMs.

Clearly from the temporal quality trends in Fig. 8 both AT and RQ models should be rejected since their temporal quality always sinks below the base line. Other models may be adequate when software is young (before 12 months). The AML and LN models look better than other models in this respect.

When software is middle-age (between 12 and 36 months), the AML model is still relatively good. JW and YF improve when approaching month 36^{th} though JW get worse after month 12^{th} . The quality of both LN and LP worsen after month 12^{th} , and sink below the base line when approaching month 36^{th} . RE is almost below the base line after month 15^{th} . Hence, in the middle-age period, AML, JW, and YF models

Table 7 A potentially misleading results of overfitting VDMs in the largest horizon of browser releases, using NVD data sets

The goodness of fit of a VDM is based on *p*-value in the χ^2 test. *p*-value < 0.05: not fit (×), *p*-value \geq 0.80: good fit (\checkmark), and inconclusive fit (blank) otherwise. It is calculated over the entire lifetime.

			F	ire	efo	ĸ							C	Chi	ror	ne							IE				Sa	afa	ri	
	1	1.5	2	3	3.5	3.6	4	5	1	2	3	4	5	6	7	8	9	10	11	12	4	5	6	7	8	1	2	3	4	5
AML	×	×		×	\checkmark				√		×				×		\checkmark			×	\checkmark		×		\checkmark	×				~
AT	×		×		×	×			×	×		×	×	×		×	×	×	×		×	×		×	Х		×	×	×	
JW	×		\checkmark	\checkmark	\checkmark	\checkmark	1								\checkmark	√						\checkmark	×	\checkmark						×
LN	×			×			1													×		х		\checkmark					×	
LP	×		\checkmark	$\overline{\checkmark}$	\checkmark	\checkmark	·						×				×					\checkmark	\times	\times	\checkmark				×	
RE	×		\checkmark	\checkmark	\checkmark	\checkmark	\checkmark															\checkmark	\times		\checkmark				×	
RQ	×		×	Х	Х		1									×		×	×	×		х			\checkmark					×
YF	×		\checkmark	\checkmark	\checkmark	\checkmark	1		\checkmark	\checkmark	\checkmark	\checkmark	√	\checkmark	\checkmark						\checkmark	~	\checkmark	×	\checkmark				×	

may turn to be adequate; LN and LP are deteriorating but might be still considered adequate; whereas RE should clearly be rejected.

When software is old (36+ months), AML, JW, and YF deteriorate and go below the base line at month 48^{th} (approximately); other models also collapse below the base line.

Fig. 9 summarizes the distribution of VDM temporal quality in three period: software is young (before 12 moths), software is middle-age (13 to 36 months), and software is old (37 to 72 months). The red horizonal line at 0.5 is the base line. We additionally colour these box plots according to the comparison between the corresponding distribution and the base line as follows:

- white: the distribution is significantly greater than the base line;
- dark gray: the distribution is significantly less than the base line (we should reject the models outright);
- gray: the distribution is not statistically different from the base line.

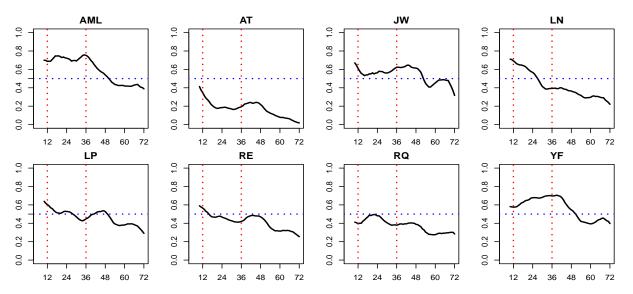
The box plots clearly confirm our observation in Fig. 8. Both AT and RE models are all significantly below the base line. AML, JW, and YF modes are significantly above the base line when software is young and middle age, and not statistically different from the base line when software is old. LN and LP models are significantly greater than the base line when software is young, but they deteriorate for middle-age software, and significantly collapse below the base line for old software.

In summary, our quality analysis shows that:

- AT and RQ models should be rejected.
- All other models may be adequate when software is young. Only s-shape models (*i.e.* AML, YW, YF) might be adequate when software is middle-age.
- No model is good when the software is too old.

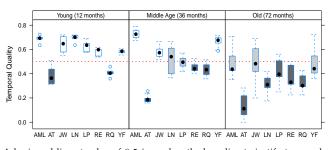
5.2.2 Predictability Analysis for VDMs

From the previous quality analysis, AT and RQ models are low quality, and they should not be considered



The X-axis is the number of months since release (*i.e.* horizon τ). The Y-axis is the value of temporal quality. The solid lines are the moving average of $Q_{\omega=0.5}(\tau)$ with window size k = 5. The dotted horizontal line at 0.5 is the base line to assess VDM. Vertical lines are the marks of the horizons of 12^{th} and 36^{th} month.





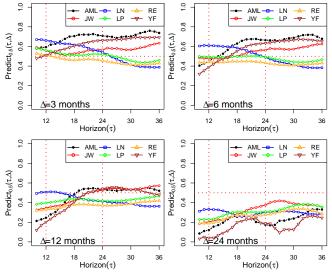
A horizonal line at value of 0.5 is used as the base line to justify temporal quality. Box plots are coloured with respect to the comparison between the corresponding distribution and the base line: white - significantly above the base line, gray - no statistical difference, dark gray - significantly below the base line (*i.e.* rejected).

Fig. 9. The temporal quality distribution of each VDM in different periods of software lifetime.

for all periods of software lifetime. Hence, we exclude these models from the predictability analysis. Furthermore, since no model is good when software is too old, we analyze the predictability of these models only for the first 36 months since the release of a software. This period is still a large time if we consider that most recent releases live less than a year.

Fig. 10 reports the moving average (windows size equals 5) for the trends of VDMs' predictability along horizons in different prediction time spans. The dotted horizonal line at value of 0.5 is the base line to assess the predictability of VDMs (as same as the temporal quality of VDMs).

When the prediction time span is short (3 months), the predictability of LN, AML, JW, and LP models is above the base line for young software (12 months). When software is approaching month 24^{th} , though



A horizonal line at value of 0.5 is the base line to assess the predictability.

Fig. 10. The predictability of VDM in different prediction time spans (Δ).

decreasing the predictability of LN is still above the base line, but goes below the base line after month 24^{th} . The LP model is no different with the base line before month 24^{th} , but then also goes below the base line. In contrast, the predictability of AML, YF and JW are improving with age. They are all above the base line until the end of the study period (month 36^{th} . Therefore, only s-shape models (AML, YF, and JW) may be adequate for middle-age software.

For the medium prediction time span of 6 months, only the LN model may be adequate (above the base line) when software is young, but becomes inadequate (below the base line) after month 24^{th} . In the meanwhile S-shape models are inadequate for young software, but are improving quickly later. They may be all adequate after month 18^{th} and keep this performance until the end of the study period.

When the prediction time span is long (*i.e.* 12 months), all models (except LN) sink below the base line for young software. The LN model is not significantly different from the base line. In other words, no model could be adequate for young software in this prediction time span. After month 18^{th} , the AML model goes above the base line, and after month 24^{th} , all s-shape models are above the base line. Hence they may be all adequate. Their performances are somewhat unchanged for the remain period.

When the prediction time span is very long (*i.e.* 24 months) no model is good enough as all models sink below the base line.

In summary, our predictability analysis shows that:

- For a short prediction time span (*i.e.* 3 months), the predictability of LN, AML, and LP models may be adequate for young software. Hence they could be considered for the scenario *Plan for short-term support*. When software is approaching middle-age, s-shape models (AML, JW, YF) are better than others.
- For a medium (*i.e.* 6 months) and long (*i.e.* 12 month) prediction time spans, only the predictability of the LN model may be adequate for young software. And therefore this model could be appropriate the purpose of the scenarios *Upgrade of keep* and *Plan for long-term support*. When software is approaching middle-age, only s-shape models (AML, JW, YF) may be adequate and might be considered for planning the long-term support and for studying historical trends (*i.e.* scenario *Historic analysis*).
- For a very long prediction time span (*i.e.* 24 months), no model has a good enough predictability.

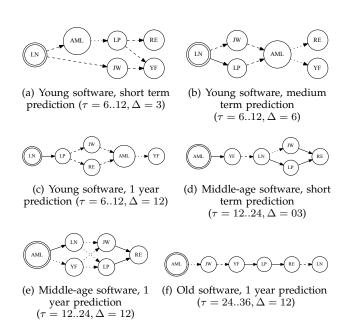
5.3 Comparison of Existing VDMs

The comparison between VDMs follows Step 5. Instead of reporting tables of *p*-values, we visualize the comparison result in terms of directed graphs where nodes represent models, and connections represent the order relationship between models.

Fig. 11 summarizes the comparison results between models in different settings of horizons (τ) and prediction time spans (Δ). A directed connection from two models determines that the source model is better than the target model in terms of either predictability, or quality, or both. The line style of the connection depended on the following rules:

• *Solid line*: the predictability and quality of the source is significantly better than the target's.





A directed connection from two nodes determines that the source model is better than the target one with respect to their predictability (dashed line), or their quality (dotted line), or both (solid line). A double cirlce marks the best model. RQ and AT are not shown as they are the worst models.

Fig. 11. The comparison results among VDMs in some usage scenarios.

Table 8 Suggested models for different usage scenarios.

Scenario	Observation Period	Prediction Time Span	Model(s)
Plan for short-term support	6–12	3	LN
* *	12-24	3	AML
Plan for long-term support	6-12	12	LN
0 11	12-24	12	AML
Upgrade or keep	6-12	6	LN
Historic analysis	24–36	12	AML

Note: the unit is month.

- *Dashed line*: the predictability of the source is significantly better than the target.
- *Dotted line*: the quality of the source is significantly better than the target.

By term *significantly*, we means the *p-value* of the corresponding one-side Wilcoxon rank-sum test is less than the significance level. We apply the Bonferroni correction to control the multi comparison problem, hence the significance level is: $\alpha = 0.05/5 = 0.01$.

According to the figure, Table 8 suggests model(s) for different usage scenario described in the criteria CR4 (see Table 2).

In short, when software is young, the LN model is the most appropriate choice. This is because the vulnerability discovery is linear. When software is approaching middleage, the AML model becomes superior.

6 THREATS TO VALIDITY

Construct validity includes threats affecting the way

we collect vulnerability data and the way we generate VDM curves with respect to the collected data. Following threats in this category are identified:

- **Bugs in data collector.** Most of vulnerability data are available in HTML pages. We have developed a web crawler to extract interesting feature from HTML page, and also XML data. The employed technique is as same as the one discussed in [14]. The crawler might be buggy and could generate errors in data collection. To minimize such impact, we have tested the crawler many times before collecting the data. Then by randomly checking the collected data, when an error is found we corrected the corresponding bug in the crawler and recollected the data.
- Bias in *bug-to-nvd* linking scheme. While collecting data for Advice.Nbug, we apply some heuristic rules to link a bug to an NVD entry based on the relative position in the MFSA report. We manually checked many links for the relevant connection between bug reports and NVD entries. All checked links were found to be consistent. Some errors might still creep in this case.
- Bias in *bug-affects-version* identification. We do not have a complete assurance that a security bug affects to which versions. Consequently, we assume that a bug affects all versions mentioned in the linked NVD. This might overestimate the number of bugs in each version. To mitigate the problem, we estimate the latest release that a bug might impact, and filter all vulnerable releases after this latest. Such estimation is done thank to the mining technique discussed in [21]. We further discuss these types of errors in NVD in [16]. These errors only affect the fitness of models over the long term so only valuations after the 24 or 36 months might be affected.
- Error in curve fitting. From the collected vulnerability, we estimate the parameters of VDMs by using the Nonlinear Least-Square technique implemented in R (nls() function). This might not produce the most optimal solution and may impact the goodness-of-fit of VDMs. To mitigate this issue, we additionally employed a commercial tool *i.e.* CurveExpert Pro⁴ to cross check the goodness-of-fit in many cases. The results have shown that there is no difference between R and CurveExpert.

Internal validity concerns the causal relationship between the collected data and the conclusion drawn in our study. Here, we have identified the following threats that might bias our conclusion.

- Bias in statistics tests. Our conclusions are based on statistics tests. These tests have their own assumptions. Choosing tests whose assumptions
- 4. http://www.curveexpert.net/, site visited on 16 Sep, 2011

are violated might end up with wrong conclusions. To reduce the risk we carefully analyzed the assumptions of the tests to make sure no unwarranted assumption was present. We did not apply any tests with normality assumptions since the distribution of vulnerabilities is not normal.

External validity is the extent to which our conclusion could be generalized to other scenarios. Our experiment is based on the vulnerability data of some major releases of the four most popular browsers covering almost all market shares. Therefore we can be quite confident about our conclusion for browsers in general. However, it does not mean that our conclusion is valid for other types of application such as operating systems. Such validity requires extra experiments.

7 RELATED WORK

Anderson [5] proposed a VDM (a.k.a. Anderson Thermodynamic, AT) based on reliability growth models, in which the probability of a security failure at time t, when n bugs have been removed, is in inverse ratio to t for alpha testers. This probability is even harder for beta testers, λ times more than alpha testers. However, he did not conduct any experiment to validate the proposed model. Our results show that this model is not appropriate. This is a first evidence that reliability and security obey different laws.

Rescorla [19] also proposed two mathematical models, called *Linear model* (a.k.a Rescorla Quadratic, RQ) and *Exponential model* (a.k.a Rescorla Exponential, RE). He has performed an experiment on four versions of different operation systems (*i.e.* Windows NT 4.0, Solaris 2.5.1, FreeBSD 4.0 and RedHat 7.0). In all cases, the goodness-of-fit of these two models were inconclusive since their *p-value* ranged from 0.167 to 0.589. Rescorla discussed many shortcomings of NVD, but his study heavily relied on it nonetheless.

Alhazmi and Malaiya [1] proposed another VDM inspired by s-shape logistic model, called Alhazmi Malaiya Logistic (AML). The intuition behind the model is to divide the discovery process into three phases: learning phase, linear phase and saturation phase. In the first phase, people need some time to study the software, so less vulnerabilities are discovered. In the second phase, when people get deeper knowledge of the software, much more vulnerabilities are found. In the final phase, since the software is out of date, people may lose interest in finding new vulnerabilities. So cumulative vulnerabilities tend to stable. In [1], the authors validated their proposal against several versions of Windows (i.e. Win 95/98/NT4.0/2K) and Linux (i.e. RedHat Linux 6.1, 7.1). Their model fitted Win 95 very well (*p-value* \approx 1), and Win NT4.0 (*pvalue* = 0.923). For other versions, their own validation showed that the AML model was inconclusive (*i.e.* the *p*-value ranged from 0.054 to 0.317).

In another work, Alhazmi and Malaiya [3] compared their proposed model with Rescorla's [19] (RE, RQ) and Anderson's [5] (AT) on Windows 95/XP and Linux RedHat Linux 6.2, Fedora. The result shows that their logistic model has a better goodness-of-fit than others. For Windows 95 and Linux 6.2, as the vulnerabilities distribute along s-shape-like curves, only AML is able to fit it (*p*-value=1), whereas all other models fail to match the data (*p*-value ≤ 0.05). For Windows XP, the story is different. RQ turns to be the best one with *p*-value=0.97, while AML poorly match the data (*p*-value=0.147).

Woo *et al* [22] carried out an experiment with AML model on three browsers IE, Firefox and Mozilla. However, it is unclear which versions of these browsers were analyzed. Most likely, they did not distinguish between versions. As discussed in section §3 (*e.g.*, Example 2), this could largely bias their final result. In their experiment, IE has not been fitted, Firefox was fairly fitted, and Mozilla was good fitted. From this result, we could not conclude any thing about the performance of AML. In another experiment, Woo *et al* [23] validated AML against two web servers: Apache and IIS. Also, they did not distinguish between versions of Apache and IIS. In this experiment, AML has demonstrated a very good performance on vulnerability data (*p-value* = 1).

Kim *et al* [11] introduced the Multiple-Version Discovery Model (MVDM) which is the generalization of AML. The MVDM separated the cumulative vulnerabilities of a version into several fragments where the first fragment captured the vulnerabilities affecting this version and past versions, and the other fragments are the shared vulnerabilities of this version and future versions. The MVDM basically is the weighted aggregation of individual AML model in these fragments. The weights are determined by the ratios of shared code between this version and future ones. The goodness-of-fit of MVDM has been compared with AML in two versions of Apache and two version of MySQL. As the result, both AML and MVDM were well fitted against the data (*p*-value \geq 0.99). MVDM might be better but the difference was quite negligible.

Joh *et al* [10] proposed a VDM based on the Weibull distribution. The proposed model was also compared with the AML model in two versions of Windows (XP, Server 2007) and two versions of Linux (RedHat Linux and RedHat Enterprise Linux). In that evaluation, the goodness-of-fit of the proposed model was compared with the AML model.

Younis *et al* [24] exploited the Folded distribution to model the discovery of vulnerabilities. The authors also compared the proposed model with the AML model in different types of application (Windows 7, OSX 5.0, Apache 2.0.x, and IE8). The reported results showed that the new model is better than the AML in the cases when the learning phase is not present.

8 CONCLUSION

Vulnerability discovery models have the potential to help us in predicting future vulnerability trends. Such predictions could help individuals and companies to adapt their software upgrade and patching schedule. However, we have not seen any method to systematically assess these models. Hence, in this work we have proposed an empirical methodology for VDM validation. The methodology is built upon the analyses on the goodness-of-fit, and the predictability of VDM at several time points during the software lifetime. These analyses rely on two quantitative metrics: *quality* and *predictability*.

We have applied this methodology to conduct an empirical experiment to assess eight VDMs (*i.e.* AML, AT, LN, JW, LP, RE, RQ, and YF) based on the vulnerability data of 30 major releases of four web browsers: IE, Firefox, Chrome, and Safari. Our experiment has revealed that:

- AT and RQ models should be rejected since their quality is not good enough.
- For young software, the quality of all other models may be adequate. Only the predictability of LN is good enough for short (*i.e.* 3 months and medium (*i.e.* 6 months) prediction time spans, other models however is not good enough for latter time span.
- For middle-age software, only s-shape models (*i.e.* AML, JW, and YF) may be adequate in terms of both quality and predictability.
- For old software, no model is good enough.
- No model is good enough for predicting results for a very long period (*i.e.* 24 months in the future).

In conclusion, for young releases of browsers (6 - 12 months old) it is better to use a linear model to estimate the vulnerabilities in the next 3 - 6 months. For middle age browsers (12 - 24 months) it is better to use an s-shape logistic model.

In future, it is interesting to replicate our experiment in other kinds of software, for instance operating systems and server-side applications. Based on that, a more comprehensive assessment about the VDMs will be more solid.

REFERENCES

- Omar Alhazmi and Yashwant Malaiya. Modeling the vulnerability discovery process. In *Proceedings of the 16th IEEE International Symposium on Software Reliability Engineering (ISSRE'05)*, pages 129–138, 2005.
- [2] Omar Alhazmi and Yashwant Malaiya. Measuring and enhancing prediction capabilities of vulnerability discovery models for Apache and IIS HTTP servers. In *Proceedings of the 17th IEEE International Symposium on Software Reliability Engineering (ISSRE'06)*, pages 343–352, 2006.

- [3] Omar Alhazmi and Yashwant Malaiya. Application of vulnerability discovery models to major operating systems. *IEEE Transactions on Reliability*, 57(1):14–22, 2008.
- [4] Omar Alhazmi, Yashwant Malaiya, and Indrajit Ray. Security vulnerabilities in software systems: A quantitative perspective. In Sushil Jajodia and Duminda Wijesekera, editors, *Data and Applications Security XIX*, volume 3654 of *LNCS*, pages 281– 294. 2005.
- [5] Ross Anderson. Security in open versus closed systems the dance of Boltzmann, Coase and Moore. In *Proceedings of Open Source Software: Economics, Law and Policy*, 2002.
- [6] William A. Arbaugh, William L. Fithen, and John McHugh. Windows of vulnerability: A case study analysis. *IEEE Computer*, 33(12):52–59, 2000.
- [7] Algirdas Avizienis, Jean-Claude Laprie, Brian Randell, and Carl Landwehr. Basic concepts and taxonomy of dependable and secure computing. *IEEE Transactions on Dependable and Secure Computing*, 1(1):11–33, 2004.
- [8] Mark Dowd, John McDonald, and Justin Schuh. The art of software security assessment. Addision-Wesley publications, 2007.
- [9] Philip J. Fleming and John J. Wallace. How not to lie with statistics: the correct way to summarize benchmark results. *Communication of the ACM*, 29(3):218–221, 1986.
- [10] HyunChul Joh, Jinyoo Kim, and Yashwant Malaiya. Vulnerability discovery modeling using Weibull distribution. In Proceedings of the 19th IEEE International Symposium on Software Reliability Engineering (ISSRE'08), pages 299–300, 2008.
- [11] Jinyoo Kim, Yashwant Malaiya, and Indrajit Ray. Vulnerability discovery in multi-version software systems. In Proceeding of the 10th IEEE International Symposium on High Assurance Systems Engineering, pages 141–148, 2007.
- [12] Ivan Victor Krsul. Software Vulnerability Analysis. PhD thesis, Purdue University, 1998.
- [13] Fabio Massacci, Stephan Neuhaus, and Viet Hung Nguyen. After-life vulnerabilities: A study on firefox evolution, its vulnerabilities and fixes. In *Proceedings of the 2011 Engineering Secure Software and Systems Conference (ESSoS'11)*, 2011.
- [14] Fabio Massacci and Viet Hung Nguyen. Which is the right source for vulnerabilities studies? an empirical analysis on mozilla firefox. In Proceedings of the International ACM Workshop on Security Measurement and Metrics (MetriSec'10), 2010.
- [15] Steve McKillup. Statistics Explained: An Introductory Guide for Life Scientists. Cambridge University Press, 2005.
- [16] Viet Hung Nguyen and Fabio Massacci. The (un) reliability of nvd vulnerable versions data: an empirical experiment on google chrome vulnerabilities. In *Proceeding of the 8th* ACM Symposium on Information, Computer and Communications Security (ASIACCS'13), 2013.

- [17] NIST/SEMATECH. e-Handbook of Statistical Methods, 2012. http://www.itl.nist.gov/div898/handbook/.
- [18] Andy Ozment. Improving vulnerability discovery models: Problems with definitions and assumptions. In *Proceedings of the 3rd Workshop on Quality of Protection*, 2007.
- [19] Eric Rescorla. Is finding security holes a good idea? IEEE Security and Privacy, 3(1):14–19, 2005.
- [20] Fred B. Schneider. Trust in cyberspace. National Academy Press, 1991.
- [21] Jacek Sliwerski, Thomas Zimmermann, and Andreas Zeller. When do changes induce fixes? In Proceedings of the 2nd International Working Conference on Mining Software Repositories MSR('05), pages 24–28, May 2005.
- [22] Sung-Whan Woo, Omar Alhazmi, and Yashwant Malaiya. An analysis of the vulnerability discovery process in web browsers. In Proceedings of the 10th IASTED International Conferences Software Engineering and Applications, 2006.
- [23] Sung-Whan Woo, HyunChul Joh, Omar Alhazmi, and Yashwant Malaiya. Modeling vulnerability discovery process in Apache and IIS HTTP servers. *Computer & Security*, 30(1):50 – 62, 2011.
- [24] Awad Younis, HyunChul Joh, and Yashwant Malaiya. Modeling learningless vulnerability discovery using a folded distribution. In *Proceeding of the Internaltional Conference Security* and Management (SAM'11), pages 617–623, 2011.

APPENDIX

A replication guide of this work could be found online at https://wiki.science.unitn.it/security/doku. php?id=vulnerability_discovery_models. Also, you can find all required materials (*e.g.*, tools, scripts, and data) to rerun the experiment.

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