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Probabilistic Energy Management for Building Climate Comfort in Smart Thermal Grids with Seasonal Storage Systems

Vahab Rostampour¹ and Tamás Keviczky²

Abstract—This paper presents an energy management framework for building climate comfort (BCC) systems interconnected in a grid via aquifer thermal energy storage (ATES) systems in the presence of two types of uncertainty (private and common). ATES can be used either as a heat source (hot well) or sink (cold well) depending on the season. We consider the uncertain thermal energy demand of individual buildings as a private uncertainty source and the uncertain common resource pool (ATES) between neighbors as a common uncertainty source. We develop a large-scale stochastic hybrid dynamical model to predict the thermal energy imbalance in a network of interconnected BCC systems together with mutual interactions between their local ATES. We formulate a finite-horizon mixed-integer quadratic optimization problem with multiple chance constraints at each sampling time, which is in general a non-convex problem and difficult to solve. We then provide a computationally tractable framework by extending the so-called robust randomized approach and offering a less conservative solution for a problem with multiple chance constraints. A simulation study is provided to compare completely decoupled, centralized and move-blocking centralized solutions. We also present a numerical study using a geohydrological simulation environment (MODFLOW) to illustrate the advantages of our proposed framework.

Index Terms—Smart thermal grids, building climate comfort systems, seasonal storage systems, ATES, multiple chance constraints, probabilistic robustness, robust randomized MPC.

I. INTRODUCTION

GLOBAL energy consumption has significantly increased due to the combined factors of increasing population and economic growth over the past few decades. This increasing consumption highlights the necessity of employing innovative energy saving technologies. Smart Thermal Grids (STGs) can play an important role in the future of the energy sector by ensuring a heating and cooling supply that is more reliable and affordable for thermal energy networks connecting

various households, greenhouses and other buildings, which we refer to as agents. STGs allow for the adaptation to changing circumstances, such as daily, weekly or seasonal variations in supply and demand by facilitating each agent with smart thermal storage technologies.

Aquifer thermal energy storage (ATES) is a less well-known sustainable seasonal storage system that can be used to store large quantities of thermal energy in aquifers. Aquifers are underground porous formations containing water that are suitable for seasonal thermal energy storage. It is especially suitable for climate comfort systems of large buildings such as offices, hospitals, universities, musea and greenhouses, see [1]. Most buildings in moderate climates have a heat shortage in winter and a heat surplus in summer. Where aquifers exist, this temporal discrepancy can be overcome by seasonally storing and extracting thermal energy into and out of the subsurface, enabling the reduction of energy usage and CO₂ emissions of climate comfort systems in buildings.

There are various studies in literature related to buildings integrated into a smart grid [2], [3]. Modeling a building heating system connected to a heat pump can be found in [4], an experimental model with a focus on heating, ventilation, and air conditioning (HVAC) systems in [5], using multi-HVAC systems in [6]. Models for building system dynamics together with HVAC controls are typically linear [7] for obvious computational purposes. For instance resistance and capacitance circuit models, that represent heat transfer and thermodynamical properties of the building, are commonly used for building control studies [8]–[10]. PID controllers for HVAC systems are widely used in many commercial buildings [11]. Model predictive control (MPC), on the other hand, has received a lot of attention [12]–[14], since it can handle large-scale dynamical systems subject to hard constraints, e.g., equipment limitations. Using demand response for smart buildings [15], MPC can be used in building climate comfort (BCC) problems [16], [17]. MPC can overcome BCC problems even in decentralized or distributed setting and it is shown that has several advantages compared to PID controllers [12], [13], [18].

STGs have been studied implicitly in the context of micro combined heat and power systems, see [19], or general smart grids, e.g., [20] and [21]. Building heat demand with a dynamical storage tank was considered in [22], whereas in [23] an adaptive-grid model for dynamic simulation of thermocline thermal energy storage systems was

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developed. A deterministic view on STGs was studied by a few researchers [24]–[26]. STGs with uncertain thermal energy demands have been considered in [27], where a MPC strategy was employed with a heuristic Monte Carlo sampling approach to make the solution robust. A dynamical model of thermal energy imbalance in STGs with a probabilistic view on uncertain thermal energy demands was established in [22], where a stochastic MPC with a theoretical guarantee on the feasibility of the obtained solution was developed.

ATES as a seasonal storage system has not, to the best of our knowledge, been considered in STGs. In [28] and [29], a dynamical model for an ATES system integrated in a BCC system has been developed. Following these studies, the first results toward developing an optimal operational framework to control ATES systems in STGs is presented here. In this framework, uncertain thermal energy demands are considered along with the possible mutual interactions between ATES systems, which may cause limited performance and reduced energy savings. The main contributions of this paper are threefold:

a) We develop a novel large-scale stochastic hybrid dynamical model to predict the dynamics of thermal energy imbalance in STGs consisting of BCC systems with hourly-based operation and ATES as a seasonal energy storage system. Based on our previous work in [28] and [29], we extend an ATES system model to predict the amount of stored water and thermal energy. We first incorporate the ATES model into a BCC problem and then, formulate a large-scale STGs problem by taking into consideration the geographical coupling constraints between ATES systems. Using an MPC paradigm, we formulate a finite-horizon mixed-integer quadratic optimization problem with multiple chance constraints at each sampling time leading to a non-convex problem, which is difficult to solve.

b) We next propose a move-blocking control scheme to enable our stochastic MPC framework to handle long prediction horizons and an hourly-based operation of the BCC systems together with a seasonal variation of desired optimal operation of the ATES system in a unified framework. In practice, the BCC systems have an hourly-based operation and typically day-ahead planning compared to the ATES system that is based on a seasonal operation. Using a fixed prediction horizon length, e.g., least common multiple of these two systems, may turn out to be computationally prohibitive, however also necessary in order to represent ATES interaction dynamics. The time scale discrepancy between the ATES system dynamics and BCC systems are explicitly accounted for in the developed MPC-based optimization formulation. Our proposed control strategy offers a long enough prediction horizon to prevent mutual interactions between ATES systems with much less computational time compared to a fixed prediction horizon that is sampled densely (i.e., every hour).

c) We develop a computationally tractable framework to approximate a solution of our proposed MPC formulation based on our previous work in [22]. In particular, we extend the framework in [22] to cope with multiple chance constraints which provides a more flexible approximation technique compared to the so-called robust randomized approach [30], [31], which is only suitable for a single chance constraint. Our

framework is closely related to, albeit different from, the approach of [32]. In [32], the problem formulation is convex and consists of an objective function with multiple chance constraints, in which the terms in objective and constraints are univariate. In contrast, our problem formulation is mixed-integer and the objective function consists of separable additive components.

It is important to highlight that two major difficulties arising in stochastic hybrid MPC, namely recursive feasibility and stability, are not in the scope of this paper, and they are subject of our ongoing research work. Thus, instead of analyzing the closed-loop asymptotic behavior, in this paper we focus on individual stochastic hybrid MPC problem instances from the optimization point of view and derive probabilistic guarantees for multiple chance constraints fulfillment.

Notations: The following international system of units is used throughout the paper: Kelvin [K] and Celsius [°C] are the units of temperature, Meter [m] is the unit of length, Hour [h] is the unit of time, Kilogram [kg] is the unit of mass, Watt [W] is the unit of power, Joule [J], kiloWatt-hour [kWh], and MegaWatt-hour [MWh] are the units of energy.

$\mathbb{R}, \mathbb{R}_+$ denote the real and positive real numbers, and $\mathbb{N}, \mathbb{N}_+$ the natural and positive natural numbers, respectively. We operate within n -dimensional space $\mathbb{R}^n$ composed by column vectors $u, v \in \mathbb{R}^n$. The Cartesian product over n sets $\mathcal{X}_1, \dots, \mathcal{X}_n$ is given by: $\prod_{i=1}^n \mathcal{X}_i = \mathcal{X}_1 \times \dots \times \mathcal{X}_n = \{(x_1, \dots, x_n) : x_i \in \mathcal{X}_i\}$. The cardinality of a set $\mathcal{A}$ is shown by $|\mathcal{A}| = A$.

Given a metric space Δ , and $\mathbb{P}$ a probability measure defined over Δ , its Borel σ -algebra is denoted by $\mathfrak{B}(\Delta)$. Throughout the paper, measurability always refers to Borel measurability. In a probability space $(\Delta, \mathfrak{B}(\Delta), \mathbb{P})$, we denote the N -Cartesian product set of Δ by Δ^N with the respective product measure by $\mathbb{P}^N$.

II. SYSTEM DYNAMICS MODELING

A. Seasonal Storage Systems

Consider an ATES system consisting of warm and cold wells to store warm water during warm season and cold water during cold season, respectively. Each well can be described as a single thermal energy storage where the amount of stored energy is proportional to the temperature difference between stored water and aquifer ambient water. Stored thermal energy from the last season is going to be used for the current season and so forth. Depending on the season, the operating mode (heating or cooling) of an ATES system changes, by reversing the direction of water flow between wells, see Fig. 1.

We therefore define the states that can describe the ATES system dynamics to be the volume of water, $V_{a,k}^h$ [m³], $V_{a,k}^c$ [m³], and the thermal energy content, $S_{a,k}^h$ [Wh], $S_{a,k}^c$ [Wh], of warm and cold wells. The superscripts “h” and “c” refer to the heating and cooling operating modes of an ATES system, respectively, and the subscript “a” denotes the ATES system variables. Consider the following first-order difference equations as ATES system model dynamics:

$$V_{a,k+1}^h = V_{a,k}^h - \tau(u_{a,k}^h - u_{a,k}^c), \quad (1a)$$

$$V_{a,k+1}^c = V_{a,k}^c + \tau(u_{a,k}^h - u_{a,k}^c), \quad (1b)$$

$$S_{a,k+1}^h = \eta_{a,k} S_{a,k}^h - \tau(h_{a,k}^h - h_{a,k}^c), \quad (1c)$$

$$S_{a,k+1}^c = \eta_{a,k} S_{a,k}^c + \tau(c_{a,k}^h - c_{a,k}^c), \quad (1d)$$

where $\eta_{a,k} \in (0, 1)$ is a lumped coefficient of thermal energy losses in aquifers, $u_{a,k}^h$ [m^3h^{-1}], and $u_{a,k}^c$ [m^3h^{-1}] are control variables corresponding to the pump flow rate of ATES system during heating and cooling modes at each sampling time $k = 1, 2, \dots$, respectively, with τ [h] as the sampling period. $u_{a,k}^h$ circulates water from warm well to cold well, whereas $u_{a,k}^c$ takes water from cold well and injects into warm well of ATES system, during heating modes and cooling modes of the BCC system, respectively. The variables $h_{a,k}^h$ [W], $c_{a,k}^h$ [W] denote the thermal power that is extracted from warm well and injected into cold well of ATES system during heating mode of BCC system, respectively. The variables $c_{a,k}^c$ [W], $h_{a,k}^c$ [W] are the thermal power that is extracted from cold well and injected into warm well of ATES system during cooling mode of BCC system, respectively. These variable are defined by:

$$\begin{cases} h_{a,k}^h = \alpha_{h,k} u_{a,k}^h \\ c_{a,k}^h = \alpha_{c,k} u_{a,k}^h \end{cases}, \quad \begin{cases} c_{a,k}^c = \alpha_{c,k} u_{a,k}^c \\ h_{a,k}^c = \alpha_{h,k} u_{a,k}^c \end{cases}, \quad \begin{cases} h_{a,k} = \alpha_k \tau u_{a,k}^h \\ c_{a,k} = \alpha_k \tau u_{a,k}^c \end{cases},$$

where $\alpha_{h,k} = \rho_w c_{pw} (T_{a,k}^h - T_{a,k}^{\text{amb}})$, $\alpha_{c,k} = \rho_w c_{pw} (T_{a,k}^{\text{amb}} - T_{a,k}^c)$, are the thermal power coefficients of warm and cold wells, respectively, and $\alpha_k = \alpha_{h,k} + \alpha_{c,k}$ is the total thermal power coefficient. The parameters ρ_w [kgm^{-3}], c_{pw} [$\text{Jkg}^{-1}\text{K}^{-1}$] are density and specific heat capacity of water, respectively. $T_{a,k}^h$ [K], $T_{a,k}^c$ [K], and $T_{a,k}^{\text{amb}}$ [K] denote the temperature of water inside warm well, cold well and the ambient aquifer, respectively. $h_{a,k}$ [Wh], and $c_{a,k}$ [Wh] are the amount of thermal energy that can be delivered to the building during heating and cooling modes, respectively. The following assumption is made due to the existing operational practice, and it is not restrictive for our proposed model.

Assumption 1: There is either no operation or only one operating mode active in ATES systems, which leads to either both control variables being zero or only one control variable being nonzero at any time instant.

The dynamics of ATES system in (1) can be also written in a more compact format for each agent $i \in \{1, \dots, N\}$:

$$x_{i,k+1}^a = a_{i,k}^a x_{i,k}^a + b_{i,k}^a u_{i,k}^a, \quad (2)$$

where $x_{i,k}^a = \begin{bmatrix} V_{a,k}^h & V_{a,k}^c & S_{a,k}^h & S_{a,k}^c \end{bmatrix}^\top \in \mathbb{R}^4$ denotes the state vector, $u_{i,k}^a = \begin{bmatrix} u_{a,k}^h & u_{a,k}^c \end{bmatrix}^\top \in \mathbb{R}^2$ is the control vector, and $a_{i,k}^a, b_{i,k}^a$ can be obtained via (1). Note that there are some operational constraints on the ATES control variable as well,

$$u_a^{\min} \leq u_{a,k}^h \leq u_a^{\max}, \quad (3a)$$

$$u_a^{\min} \leq u_{a,k}^c \leq u_a^{\max}, \quad (3b)$$

where $u_a^{\min}, u_a^{\max}$ represent the minimum and maximum pump flow rate of ATES system, respectively.

The proposed model for an ATES system in (2) is a linear time-varying discrete-time system, due to the variation of the temperatures in both wells and the ambient aquifer.

In Section II-C, we will integrate (2) into a BCC system dynamics.

B. Thermal Energy Demand Profile

A dynamical model of building thermal energy demand was developed in our previous work [33] to determine the thermal energy demand of a building at each sampling time k , considering the desired indoor air temperature and the outside weather conditions. We refer to the BCC system that determines the level of thermal energy demand $Q_{d,k}^B$ [Wh] at each sampling time k via

$$Q_{d,k}^B = f_B(p_s^B, T_{\text{des},k}^B, \vartheta_k), \quad (4)$$

where p_s^B corresponds to a parameter vector of the building characteristics, $T_{\text{des},k}^B$ [$^\circ\text{C}$] is the desired indoor air temperature of the building, and $\vartheta_k = [T_{o,k}^B, I_{o,k}, v_{o,k}, Q_{p,k}, Q_{e,k}] \in \mathbb{R}^5$ is a vector of uncertain variables that contains the outside air temperature, the solar radiation, the wind velocity, the thermal energy produced due to occupancy by people, and electrical devices, and lighting inside the building, respectively. This yields the building thermal energy demand that takes into account the overall building effects, e.g., zones, walls, humans and non-human thermal energy sources with the outside uncertain weather conditions. Since we are mainly interested in capturing the variation of thermal energy demand w.r.t. the outside air temperature $T_{o,k}^B$, the uncertain variable ϑ_k , is assigned to $T_{o,k}^B$, and the rest of the variables are fixed to their nominal (forecast) values at each sampling time k .

The operating modes (heating or cooling) of BCC system are determined based on the sign of $Q_{d,k}^B$ at each sampling time k . The variable $Q_{d,k}^B$ with positive and negative signs, represents the thermal energy demand during heating mode and the building surplus thermal energy during cooling mode, respectively. $Q_{d,k}^B$ is zero represents the comfort mode of building, and thus, in such a case no heating or cooling is requested. We also distinguish between the thermal energy demand of building during heating mode $h_{d,k}$, and cooling mode $c_{d,k}$, using the relation: $Q_{d,k}^B = h_{d,k} - c_{d,k}$.

C. Building Climate Comfort Systems

Consider a single agent (i.e., building) $i \in \{1, \dots, N\}$ that is facilitated with a boiler, a heat pump, a storage tank for the heating mode, and a chiller, a storage tank for the cooling mode together with an ATES system that is available for both operating modes (see Fig. 1). We now focus on the modeling of energy balance for the BCC system.

Define two vectors of control variables during heating and cooling modes in each agent i at each sampling time k , to be

$$u_{i,k}^h = [h_{\text{boi},k} \quad h_{\text{im},k}]^\top \in \mathbb{R}^2, u_{i,k}^c = [c_{\text{chi},k} \quad c_{\text{im},k}]^\top \in \mathbb{R}^2.$$

The variables $h_{\text{boi},k}$, $c_{\text{chi},k}$, $h_{\text{im},k}$, and $c_{\text{im},k}$ denote the production of boiler, chiller, the imported energies from external parties during heating and cooling modes, respectively. We consider boiler and chiller operating limits that constrain their production within a certain bound for cost effective maintenance of such equipment. Define $v_{\text{boi},k} \in \{0, 1\}$ and

$v_{chi,k} \in \{0, 1\}$ to be two binary variables to decide about the ON/OFF status boiler and chiller, respectively. Consider now to the following conditional situations:

$$\text{boiler} : \begin{cases} v_{boi,k} = 1 & h_{boi}^{\min} \leq h_{boi,k} \leq h_{boi}^{\max} \\ v_{boi,k} = 0 & \text{otherwise} \end{cases}, \quad (5a)$$

$$\text{chiller} : \begin{cases} v_{chi,k} = 1 & c_{chi}^{\min} \leq c_{chi,k} \leq c_{chi}^{\max} \\ v_{chi,k} = 0 & \text{otherwise} \end{cases}, \quad (5b)$$

where $h_{boi}^{\min}$, $h_{boi}^{\max}$, $c_{chi}^{\min}$, $c_{chi}^{\max}$ denote the minimum and maximum capacity of thermal energy production of boiler and chiller, respectively.

We define two variables to capture the thermal energy imbalance errors during heating mode $x_{i,k}^h \in \mathbb{R}$, and an imbalance error of the cooling mode $x_{i,k}^c \in \mathbb{R}$. They are related to the difference between the level of the storage tank with the forecasted thermal energy demand, $h_{d,k}^f$, $c_{d,k}^f$, during heating and cooling modes, respectively, which are formally defined using the following relations:

$$x_{i,k}^h = h_{s,k} - h_{d,k}^f, \quad (6a)$$

$$x_{i,k}^c = c_{s,k} - c_{d,k}^f. \quad (6b)$$

Herein, $h_{s,k}$, and $c_{s,k}$ represent the level of storage tank during heating and cooling modes, respectively, and obey the following dynamics:

$$h_{s,k+1} = \eta_{s,k}^h x_{i,k}^h + \eta_{s,k}^h (h_{boi,k} + h_{im,k} + \alpha_{hp,k} h_{a,k}), \quad (7)$$

$$c_{s,k+1} = \eta_{s,k}^c x_{i,k}^c + \eta_{s,k}^c (c_{chi,k} + c_{im,k} + c_{a,k}), \quad (8)$$

where $\alpha_{hp,k} = \text{COP}_k(\text{COP}_k - 1)^{-1}$ is related to the effect of the heat pump during heating mode and COP_k stands for the coefficient of performance of heat pump at each sampling time k . The parameters $\eta_{s,k}^h, \eta_{s,k}^c \in (0, 1)$ denote the thermal loss coefficients due to inefficiency of storage tank during heating and cooling modes, respectively. The variables $h_{a,k}$ and $c_{a,k}$ are defined in the previous part and are related to the ATES system model. It is important to note that $h_{a,k}$ and $c_{a,k}$ are dependent on the pump flow rates $u_{a,k}^h$ and $u_{a,k}^c$ of the ATES system during heating and cooling modes of the BCC system, respectively. We now substitute $h_{s,k}$, and $c_{s,k}$ as in (7) into (6) to derive the dynamical behavior of the thermal energy imbalance $x_{i,k}^h$ and $x_{i,k}^c$ that are given by

$$x_{i,k+1}^h = a_{i,k}^h x_{i,k}^h + b_{i,k}^h u_{i,k}^h + b_{i,k}^{a,h} u_{i,k}^a + c_{i,k}^h w_{i,k}^h, \quad (9a)$$

$$x_{i,k+1}^c = a_{i,k}^c x_{i,k}^c + b_{i,k}^c u_{i,k}^c + b_{i,k}^{a,c} u_{i,k}^a + c_{i,k}^c w_{i,k}^c, \quad (9b)$$

where $a_{i,k}^h = \eta_{s,k}^h$, $a_{i,k}^c = \eta_{s,k}^c$, $b_{i,k}^h = [\eta_{s,k}^h \ \eta_{s,k}^h]$, $b_{i,k}^c = [\eta_{s,k}^c \ \eta_{s,k}^c]$, $b_{i,k}^{a,h} = [\eta_{s,k}^h \alpha_{hp,k} \ \alpha_k \ 0]$, $b_{i,k}^{a,c} = [\eta_{s,k}^c \ \alpha_k \ 0]$, $c_{i,k}^h = -1$, and $c_{i,k}^c = -1$. The variables $w_{i,k}^h = h_{d,k+1}^f$ and $w_{i,k}^c = c_{d,k+1}^f$ refer to the forecast of thermal energy demand during heating and cooling modes in the next time step, respectively. The only uncertain variable in each agent i at each sampling time k is considered to be the deviation of actual thermal energy demand from its forecast value as defined in Section II-B, and therefore, $w_{i,k}^h$ and $w_{i,k}^c$ represent uncertain parameters.

Consider now the system dynamics for each agent i by concatenating the thermal energy imbalance errors during heating

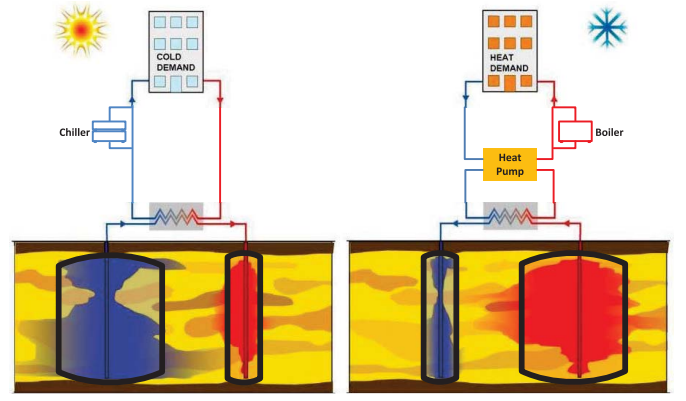


Fig. 1. Heating and cooling operating modes of BCC system with an ATES system during warm (left) and cold (right) seasons.

and cooling modes (9) together with the state vector of the ATES system (2) as follows:

$$x_{i,k+1} = a_{i,k} x_{i,k} + b_{i,k} u_{i,k} + c_{i,k} w_{i,k}, \quad (10)$$

where $x_{i,k} = [x_{i,k}^h \ x_{i,k}^c \ x_{i,k}^a]^T \in \mathbb{R}^6$ denotes the state vector, $u_{i,k} = [u_{i,k}^h \ u_{i,k}^c \ u_{i,k}^a]^T \in \mathbb{R}^6$ is the control vector, and $w_{i,k} = [w_{i,k}^h \ w_{i,k}^c]^T \in \mathcal{W}_{i,k} \subseteq \mathbb{R}^2$ is the uncertainty vector such that $\mathcal{W}_{i,k}$ is an unknown uncertainty set. The system matrices $a_{i,k}$, $b_{i,k}$, $c_{i,k}$ can be readily derived from their definitions and we omit them in the interest of space.

The proposed model for a BCC system in (10) is a stochastic hybrid linear time-varying discrete-time system. It is important to note that the hybrid nature of (10) is due to the fact that each equipment (boiler and chiller) can be either ON or OFF as in (5) depending on heating and cooling modes of the building. This possibility therefore changes the proposed thermal energy imbalance error dynamics (9).

In order to provide a desired thermal comfort for each BCC system in the following section, we will develop a control framework based on the MPC paradigm where (10) is used to predict the thermal energy imbalance error dynamics together with the ATES system dynamics for each agent $i \in \{1, \dots, N\}$, and then, extend this to a network of interconnected BCC systems. Moreover, we will provide a solution method to overcome an important challenge of the network of BCC systems due to the spatial distribution of ATES systems. An important remark is that the variations of system parameters in the proposed dynamical model (10) evolve on a much slower time-scale compared to the system dynamics and, therefore, we consider the system dynamics (10) to be time-invariant in the following parts. It is worth mentioning that our proposed control technique in this paper can be easily extended to cope with time-varying parameters by considering them as multiplicative uncertainty sources, see, e.g., [34].

III. ENERGY MANAGEMENT PROBLEM

A. Energy Balance in Single Agent System

Consider an MPC problem with a finite prediction horizon N_h for each agent $i \in \{1, \dots, N\}$, and introduce the subscript t in our notation to characterize the value of the planning

quantities for a given time $t \in \mathcal{T}$, where the set of predicted time steps is denoted by $\mathcal{T} := \{k, k+1, \dots, k+N_h-1\}$. Using the subscript $t|k$, we refer to the t time step prediction of variables at the simulation time step k .

Define $v_{i,t|k} = [v_{\text{boi},t|k} \ v_{\text{chi},t|k}]^\top \in \{0, 1\}^2$ as a vector of binary variables to decide about the ON/OFF status of boiler and chiller in each agent $i \in \{1, \dots, N\}$. We also take into account the startup cost of boiler and chiller using $c_{i,t|k}^{\text{su}} = [c_{\text{boi},t|k}^{\text{su}} \ c_{\text{chi},t|k}^{\text{su}}]^\top$ and add $c_{i,t|k}^{\text{su}}$ into the control decision variables $u_{i,t|k} = [u_{i,t|k}^{h\top} \ u_{i,t|k}^{c\top} \ u_{i,t|k}^{a\top} \ c_{i,t|k}^{\text{su}\top}]^\top \in \mathbb{R}^8$ for each agent i at each time step $t|k$.

The goal of each agent i is to map the local thermal energy supply of production units to the local thermal energy demand of BCC system. Our goal thus is to formulate an optimization problem to find the control input $u_{i,t|k}$ for each agent i such that the thermal energy imbalance errors stay as small as possible at minimal production cost and to satisfy physical constraints of heating and cooling modes equipment at each sampling time k . We therefore associate a quadratic cost function with each agent i at each prediction time step k as follows:

$$J_i(x_{i,t|k}, u_{i,t|k}) = x_{i,t|k}^\top Q_i x_{i,t|k} + u_{i,t|k}^\top R_i u_{i,t|k}, \quad (11)$$

where $Q_i = \text{diag}([q_i^h \ q_i^c \ \mathbf{0}_{1 \times 4}]) \in \mathbb{R}^{6 \times 6}$ is a weighting matrix coefficient of thermal energy imbalance errors, $R_i = \text{diag}(r_i) \in \mathbb{R}^{8 \times 8}$ indicates a diagonal matrix with the cost vector r_i on its diagonal, and r_i is defined as

$$r_i = [r_{\text{boi}} \ r_{\text{im}}^h \ r_{\text{chi}} \ r_{\text{im}}^c \ r_a^h \ r_a^c \ 1 \ 1]^\top \in \mathbb{R}^8,$$

where r_{boi} (r_{chi}) represents the cost of natural gas that is used by boiler (chiller), r_{im}^h (r_{im}^c) denotes the cost of imported thermal energy from an external party during heating (cooling) mode, and r_a^h (r_a^c) corresponds to the pumping electricity cost of ATEs system to extract the required thermal energy during heating (cooling) modes. The other entries of r_i represent the start-up costs. The proposed cost function consists of two main parts which leads to the regulation of imbalance errors to zero at minimal production cost together with minimum energy balance error of ATEs system in each agent i . The reason for introducing a cost function in this form is that from a computational perspective quadratic cost functions are motivated by convexity and differentiability arguments. Note that the cost function $J_i(\cdot)$ is a random variable due to the uncertain state variables, and thus, we consider $\mathbb{E}[J_i(\cdot)]$ to obtain a deterministic cost function.

We are now in a position to formulate a finite-horizon stochastic hybrid control problem as the local energy management problem for each agent $i \in \{1, \dots, N\}$ using the following chance-constrained mixed-integer optimization problem:

$$\min_{\{u_{i,t|k}, v_{i,t|k}\}_{t \in \mathcal{T}}} \sum_{t \in \mathcal{T}} \mathbb{E}[J_i(x_{i,t|k}, u_{i,t|k})] \quad (12a)$$

$$\text{subject to } c_{i,t|k}^{\text{su}} \geq \Lambda^{\text{su}}(v_{i,t|k} - v_{i,t-1|k}) \geq 0, \forall t \in \mathcal{T} \quad (12b)$$

$$v_{\text{boi},t|k}^{\min} \leq h_{\text{boi},t|k} \leq h_{\text{boi},t|k}^{\max} v_{\text{boi},t|k}, \forall t \in \mathcal{T} \quad (12c)$$

$$v_{\text{chi},t|k}^{\min} \leq c_{\text{chi},t|k} \leq c_{\text{chi},t|k}^{\max} v_{\text{chi},t|k}, \forall t \in \mathcal{T} \quad (12d)$$

$$h_{\text{im},t|k}^{\min} \leq h_{\text{im},t|k} \leq h_{\text{im},t|k}^{\max}, \forall t \in \mathcal{T} \quad (12e)$$

$$c_{\text{im}}^{\min} \leq c_{\text{im},t|k} \leq c_{\text{im},t|k}^{\max}, \forall t \in \mathcal{T} \quad (12f)$$

$$u_a^{\min} \leq u_{a,t|k}^h \leq u_a^{\max}, \forall t \in \mathcal{T} \quad (12g)$$

$$u_a^{\min} \leq u_{a,t|k}^c \leq u_a^{\max}, \forall t \in \mathcal{T} \quad (12h)$$

$$\mathbb{P}\{x_{i,t+1|k} \geq 0 \mid x_{i,t|k}, \forall t \in \mathcal{T}\} \geq 1 - \varepsilon_i, \quad (12i)$$

$$\forall \{w_{i,t|k}\}_{t \in \mathcal{T}} \in \mathcal{W}_i,$$

where Λ^{su} is a diagonal matrix including the startup costs of boiler and chiller on the diagonal, $h_{\text{im}}^{\min}$, $h_{\text{im}}^{\max}$, $c_{\text{im}}^{\min}$, $c_{\text{im}}^{\max}$ are the minimum and maximum capacity of thermal energy production for each external party during heating and cooling modes, respectively. $\varepsilon_i \in (0, 1)$ is the admissible constraint violation parameter. Note that $\mathcal{W}_i$ represents the Cartesian product of $\mathcal{W}_{i,t|k}$ for all $t \in \mathcal{T}$.

In order of appearance, the constraints have the following meaning. Constraint (12b) captures the status change of boiler and chiller (from OFF to ON). Note that the status change from ON to OFF never appears in the cost function due to the positivity constraint of $c_{i,t|k}^{\text{su}} \geq 0$. (12c), (12d), (12e), (12f), (12g), (12h) impose box constraints (capacity limitations) on their variables. In the given lower and upper bounds of both constraints (12c) and (12d), there are multiplications with binary variables which enforce the status change of boiler and chiller, respectively. Constraint (12i) ensures probabilistically feasible trajectories of the thermal energy imbalance errors for in each agent w.r.t all possible realization of the uncertain variables $w_{i,t|k}^h$ and $w_{i,t|k}^c$ for all predicted time step $t \in \mathcal{T}$.

To extend the proposed formulation (12) to the energy management problem of smart thermal grids, we first need to introduce the notation, $\mathbf{x}_i := \{x_{i,t+1|k}\}_{t \in \mathcal{T}} \in \mathbb{R}^{6N_h =: n_x}$, $\mathbf{u}_i := \{u_{i,t|k}\}_{t \in \mathcal{T}} \in \mathbb{R}^{9N_h =: n_u}$, $\mathbf{v}_i := \{v_{i,t|k}\}_{t \in \mathcal{T}} \in \mathbb{R}^{2N_h =: n_v}$, and $\mathbf{w}_i := \{w_{i,t|k}\}_{t \in \mathcal{T}} \in \mathbb{R}^{2N_h =: n_w}$. Given the initial value of the state $x_{i,k}$, one can eliminate the state variables from the dynamics (10) of each agent i :

$$\mathbf{x}_i = A_i \mathbf{x}_{i,k} + B_i \mathbf{u}_i + C_i \mathbf{w}_i, \quad (13)$$

where the exact form of A_i , B_i and C_i matrices are omitted in the interest of space and can be found in [35, Sec. 9.5]. We can now rewrite the total cost function over the prediction horizon in a more compact form as follows:

$$\mathcal{J}_i(\mathbf{x}_i, \mathbf{u}_i) = \mathbf{x}_i^\top \mathbf{Q}_i \mathbf{x}_i + \mathbf{u}_i^\top \mathbf{R}_i \mathbf{u}_i,$$

where $\mathbf{Q}_i$ and $\mathbf{R}_i$ are two block-diagonal matrices with Q_i and R_i on the diagonal for each agent i . Note that the sum $\sum(\cdot)$ and the expectation $\mathbb{E}[\cdot]$ in the cost function (12a) are linear operators and thus, we can change their order without loss of generality. Consider now the reformulation of (12) in a more compact form as follows:

$$\min_{\mathbf{u}_i, \mathbf{v}_i} \mathcal{V}_i(\mathbf{x}_i, \mathbf{u}_i) = \mathbb{E}_{\mathbf{w}_i}[\mathcal{J}_i(\mathbf{x}_i, \mathbf{u}_i)] \quad (14a)$$

$$\text{s.t. } E_i \mathbf{u}_i + F_i \mathbf{v}_i + P_i \leq 0, \quad \forall \mathbf{w}_i \in \mathcal{W}_i \quad (14b)$$

$$\mathbb{P}_{\mathbf{w}_i}[A_i \mathbf{x}_{i,k} + B_i \mathbf{u}_i + C_i \mathbf{w}_i \geq 0] \geq 1 - \varepsilon_i, \quad (14c)$$

where E_i , F_i , P_i are matrices that are built by concatenating all constraints in (12). The index of $\mathbb{E}_{\mathbf{w}_i}$, $\mathbb{P}_{\mathbf{w}_i}$ denotes the dependency of the state trajectory $\mathbf{x}_i$ on the string of random scenarios $\mathbf{w}_i$ for each agent i . The following technical assumption is adopted.

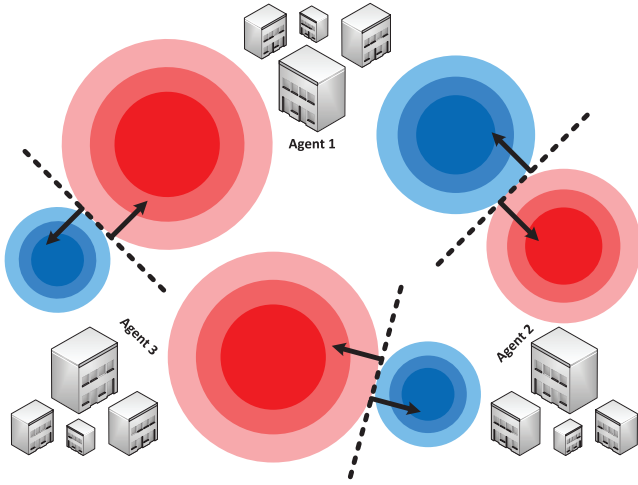


Fig. 2. Three-agent ATEs system in a STG. Each agent has a single ATEs system which consists of a warm and a cold well. Horizontal cross sections of warm and cold wells are shown with red and blue circles. The black dashed lines represent the unwanted mutual interactions between ATEs systems.

Assumption 2: The random variable $\mathbf{w}_i$ is defined on some probability space $(\mathcal{W}_i, \mathfrak{B}(\mathcal{W}_i), \mathbb{P}_{\mathbf{w}_i})$, where $\mathcal{W}_i \subseteq \mathbb{R}^{n_w}$, $\mathfrak{B}(\cdot)$ denotes a Borel σ -algebra, and $\mathbb{P}_{\mathbf{w}_i}$ is a probability measure defined over $\mathcal{W}_i$.

It is worth to mention that for our study we only need a finite number of instances of $\mathbf{w}_i$, and we do not require the probability space $\mathcal{W}_i$ and the probability measure $\mathbb{P}_{\mathbf{w}_i}$ to be known explicitly. The availability of a number of scenarios from the sample space $\mathcal{W}_i$ is enough which will become concrete in later parts of the paper. Such samples can be for instance obtained from historical data.

The proposed optimization problem (14) is a finite-horizon, chance-constrained mixed-integer quadratic program, whose stages are coupled by the binaries (12b), and dynamics of the imbalance error (12i) for each agent i at each sampling time k . It is important to note that the proposed problem (14) is in general a non-convex problem and hard to solve. In the following section, we will develop a tractable framework to obtain a probabilistically feasible solution for each agent i . We refer to the proposed optimization problem (14) as a single agent problem, and whenever all agents solve this problem separately in a receding horizon fashion without any coupling constraints, it is referred to as the *decoupled solution* (DS) in the subsequent parts. We next extend the proposed single agent optimization problem (14) into a STGs setting.

B. ATEs in Smart Thermal Grids

Consider a regional thermal grid consisting of N agents with heterogeneous parameters as it was developed in the previous part. Such a STG setting however can lead to unwanted mutual interactions between ATEs systems as it is illustrated in Fig. 2. We therefore need to introduce a proper coupling constraint between neighboring agents that makes use of the following assumption.

Assumption 3: Each well of an ATEs system is considered as a growing reservoir with respect to the horizontal axis (see black solid line in Fig. 1). We therefore assume to have a

cylindrical reservoir with a fixed height ℓ [m] (filter screen length) and a growing radius $r_{a,k}^h, r_{a,k}^c$ [m] (thermal radius) for each well of an ATEs system.

Using the volume of stored water in each well of ATEs system, one can determine the thermal radius using

$$r_{a,k}^h = \left(\frac{c_{pw} V_{a,k}^h}{c_{aq} \pi \ell} \right)^{0.5}, \quad r_{a,k}^c = \left(\frac{c_{pw} V_{a,k}^c}{c_{aq} \pi \ell} \right)^{0.5}, \quad (15)$$

where $c_{aq} = (1 - n_p)c_{sand} + n_p c_{pw}$ is the aquifer heat capacity. c_{sand} [$\text{Jkg}^{-1}\text{K}^{-1}$] relates to the sand specific heat capacity, and n_p [–] is the porosity of aquifer. Let us now denote the set of neighbors of agent i by

$$\mathcal{N}_i \subseteq \{1, 2, \dots, N\} \setminus \{i\}.$$

We impose a limitation on the thermal radius of warm well $r_{a,k}^h$ and cold well $r_{a,k}^c$ of ATEs system in each agent i , based on the corresponding wells of its neighbor $j \in \mathcal{N}_i$:

$$(r_{a,k}^h)_i + (r_{a,k}^c)_j \leq d_{ij}, \quad j \in \mathcal{N}_i, \quad (16)$$

where d_{ij} is a given distance between agent i and its neighbor $j \in \mathcal{N}_i$. This constraint prevents overlapping between the growing domains of warm and cold wells of ATEs systems in a STG setting. Due to the nonlinear transformation in (15), we propose the following reformulation of this constraint to simplify the problem:

$$(V_{a,k}^h)_i + (V_{a,k}^c)_j \leq V_{ij} - \bar{\delta}_{ij,k}, \quad (17)$$

where $V_{ij} = c_{aq} \pi \ell (d_{ij})^2 / c_{pw}$ denotes the total volume of common resource pool between agent i and its neighbor $j \in \mathcal{N}_i$. The variable $\bar{\delta}_{ij,k} = 2c_{aq} \pi \ell (\bar{r}_{a,k}^h)_i (\bar{r}_{a,k}^c)_j / c_{pw}$ represents a time-varying parameter that captures the mismatch between the linear and nonlinear constraint relations. The following corollary is a direct result of the above reformulation.

Corollary 1: If $(\bar{r}_{a,k}^h)_i$ and $(\bar{r}_{a,k}^c)_j$ represent the current thermal radius of warm and cold wells of ATEs system in agent i and j , respectively, then constraints (16) and (17) are equivalent.

The proof is provided in an online technical report [36].

Definition 1: We define $\delta_{ij,k}$ to be a common uncertainty source between each agent i and its neighboring agent $j \in \mathcal{N}_i$, using the following model:

$$\delta_{ij,k} := \bar{\delta}_{ij,k} (1 \pm 0.1 \zeta), \quad (18)$$

where ζ is a random variable defined on some probability space, $\bar{\delta}_{ij,k}$ is constructed by using two given possible $(\bar{r}_{a,k}^h)_i, (\bar{r}_{a,k}^c)_j$ realizations that can be obtained using historical data in the DS framework. Since the mapping (18) from ζ to $\delta_{ij,k}$ is measurable, one can view $\delta_{ij,k}$ as a random variable on the same probability space as ζ .

C. Problem Formulation in Multi-Agent Network

We now formulate the energy management problem for ATEs systems in STGs as follows:

$$\min_{\{\mathbf{u}_i, \mathbf{v}_i\}_{i=1}^N} \sum_{i=1}^N \mathcal{V}_i(\mathbf{x}_i, \mathbf{u}_i) \quad (19a)$$

$$\text{s.t.} \quad E_i \mathbf{u}_i + F_i \mathbf{v}_i + P_i \leq 0, \quad (19b)$$

$$\mathbb{P}_{\mathbf{w}_i} [A_i x_{i,k} + B_i \mathbf{u}_i + C_i \mathbf{w}_i \geq 0] \geq 1 - \varepsilon_i, \quad (19c)$$

$$\mathbb{P}_{\delta_{ij}} [H_i \mathbf{x}_i + H_j \mathbf{x}_j \leq \bar{V}_{ij} - \delta_{ij}] \geq 1 - \bar{\varepsilon}_{ij}, \quad (19d)$$

$$\forall \mathbf{w}_i \in \mathcal{W}_i, \forall \delta_{ij} \in \Delta_{ij},$$

$$\forall j \in \mathcal{N}_i, \forall i \in \{1, 2, \dots, N\},$$

where H_i, H_j are coefficient matrices of appropriate dimensions, $\bar{V}_{ij} \in \mathbb{R}^{N_h}$ is the upper-bound on the total common resource pool, δ_{ij} is a vector of common uncertainty variables, and $\bar{\varepsilon}_{ij} \in (0, 1)$ denotes the level of admissible coupling constraint violation for each agent i and $\forall j \in \mathcal{N}_i$. $\bar{V}_{ij}$ can be expressed as $\bar{V}_{ij} = \mathbf{1}^{N_h} \otimes V_{ij}$, using the Kronecker product. Notice that the index of $\mathbb{P}_{\delta_{ij}}$ denotes the dependency of the state trajectories on the string of random common scenarios $\delta_{ij} = \{\delta_{ij,t|k}\}_{t \in \mathcal{T}} \subseteq \mathbb{R}^{N_h \times n_s}$.

Assumption 4: The variable δ_{ij} is considered to be a random vector on some probability space $(\Delta_{ij}, \mathfrak{B}(\Delta_{ij}), \mathbb{P}_{\delta_{ij}})$, where $\Delta_{ij} \subseteq \mathbb{R}^{n_s}$, $\mathfrak{B}(\cdot)$ denotes a Borel σ -algebra, and $\mathbb{P}_{\delta_{ij}}$ is a probability measure defined over Δ_{ij} .

Assumption 5: The variables $\mathbf{w}_i \in \mathbb{R}^{n_w}$ and $\delta_{ij} \in \mathbb{R}^{n_s}$ are two vectors of independent random scenarios from two disjoint probability spaces $\mathcal{W}_i$ and Δ_{ij} , respectively.

We refer to the proposed optimization problem (19) as a multi-agent network problem, and whenever the proposed problem (19) is solved in a receding horizon fashion, it is mentioned as the *centralized solution* (CS) in the following parts. The feasible set of (19) is in general non-convex and hard to determine explicitly due to the presence of chance constraints (19c), (19d). In what follows, we will develop a tractable framework to obtain probabilistically feasible solutions for all agents.

D. Move-Blocking Scheme

The proposed system dynamics in (10) for each agent i consists of a BCC system dynamics (4) with typically an hourly-based operation, and an ATEs system (2) that is based on a seasonal variation of desired optimal operation. This leads to a control problem that is sensitive w.r.t. the prediction horizon length, e.g., (14) and (19). Using a fixed prediction horizon length, e.g., least common multiple of these two systems, may turn out to be computationally prohibitive, however, also necessary in order to represent ATEs interaction dynamics. We therefore aim to formulate a move-blocking strategy to reduce the number of control variables.

Consider $\mathcal{T} = \{k, k+1, \dots, k+N_h-1\}$ to be the set of sampling time instances within the full prediction horizon, and $\mathcal{T}_u = \{\tau_1, \tau_2, \dots, \tau_{T_u}\} \subseteq \mathcal{N}_h$ to be the set of sampling instances at which the control input is updated with $T_u = |\mathcal{T}_u|$. We introduce a new vector of multi-rate decision variables $\tilde{\mathbf{u}}_i \in \mathbb{R}^{N_u T_u}$ which are related to the original ones by:

$$\mathbf{u}_i = \Psi \tilde{\mathbf{u}}_i, \quad (20)$$

where $\Psi = [\Psi_1 \ \Psi_2 \ \dots \ \Psi_{T_u}] \in \mathbb{R}^{N_u N_h \times N_u T_u}$ is a linear mapping matrix. For all $m \in \{1, \dots, T_u\}$, we construct

$$\Psi_m = [\psi_{1,m}^\top \ \psi_{2,m}^\top \ \dots \ \psi_{N_h,m}^\top]^\top \in \mathbb{R}^{N_u N_h \times N_u}, \quad (21)$$

where $\psi_{l,m} \in \mathbb{R}^{N_u \times N_u}$ for all $l \in \{1, 2, \dots, N_h\}$ is defined as

$$\psi_{l,m} = \begin{cases} \mathbf{1} & \text{if } k+l-1 = \tau_m, \\ 0 & \text{otherwise} \end{cases}, \quad (22)$$

where $\mathbf{1} \in \mathbb{R}^{N_u \times N_u}$ represents an identity matrix.

We reformulate the optimization problem (19) using the proposed move-blocking scheme (22), and whenever the reformulation of (19) is solved in a receding horizon fashion, it is referred to as the *move-blocking centralized solution* (MCS).

IV. COMPUTATIONALLY TRACTABLE FRAMEWORK

In this section, we provide a framework to approximately solve the mixed-integer chance-constrained optimization problem (19), which is in general difficult to solve using the so-called robust randomized technique [30]. The idea of robust randomized approach is the following. An auxiliary chance-constrained optimization problem is first formulated to determine a probabilistic bounded set of random variable. This yields a bounded set of uncertainty that is a subset of the uncertainty space and contains a portion of the probability mass of the uncertainty with high confidence level. Then a robust version of the initial problem subject to the uncertainty confined in the obtained set is solved. We here extend this approach in order to be able to handle a problem with multiple chance constraints.

Consider $\mathbf{y}_i = (\mathbf{u}_i, \mathbf{v}_i) \in \mathbb{R}^{(n_u+n_v)=n_y}$, $\mathbf{y} = \text{col}(\mathbf{y}_i)_{i=1}^N$, where $\text{col}(\cdot)$ is an operator to stack elements. Define $\mathbf{w} = \text{col}(\mathbf{w}_i)_{i=1}^N \subseteq \mathcal{W}$ to be the private uncertainty sources for a network of agents, $\delta_i = \text{col}(\delta_j)_{j \in \mathcal{N}_i} \subseteq \Delta_i$ to be the common uncertainty sources for each agent, and $\delta = \text{col}(\delta_i)_{i=1}^N \subseteq \Delta$ to be the common uncertainty sources for a multi-agent network, where $\mathcal{W} := \prod_{i=1}^N \mathcal{W}_i$, $\Delta_i := \prod_{j \in \mathcal{N}_i} \Delta_{ij}$, $\Delta := \prod_{i=1}^N \Delta_i$. Consider now the proposed optimization problem in (19) in a more compact format:

$$\min_{\mathbf{y}} \quad \sum_{i=1}^N \mathcal{V}_i(\mathbf{x}_i, \mathbf{u}_i) \quad (23a)$$

$$\text{s.t.} \quad \mathbb{P}_{\mathbf{w}} \left[\mathbf{y} \in \prod_{i=1}^N \mathcal{Y}_i(\mathbf{w}_i) \right] \geq 1 - \varepsilon, \quad \forall \mathbf{w} \in \mathcal{W} \quad (23b)$$

$$\mathbb{P}_{\delta} \left[\mathbf{y} \in \prod_{i=1}^N \bigcap_{j \in \mathcal{N}_i} \check{\mathcal{Y}}_{ij}(\delta_{ij}) \right] \geq 1 - \bar{\varepsilon}, \quad \forall \delta \in \Delta \quad (23c)$$

where $\varepsilon := \sum_{i=1}^N \varepsilon_i \in (0, 1)$, $\bar{\varepsilon} := \sum_{i=1}^N \sum_{j \in \mathcal{N}_i} \bar{\varepsilon}_{ij} \in (0, 1)$. $\mathcal{Y}_i(\mathbf{w}_i) \in \mathbb{R}^{n_y}$ and $\check{\mathcal{Y}}_{ij}(\delta_{ij}) \in \mathbb{R}^{n_y}$ are defined¹ by

$$\mathcal{Y}_i(\mathbf{w}_i) := \left\{ \mathbf{y}_i \in \mathbb{R}^{n_y} : E_i \mathbf{u}_i + F_i \mathbf{v}_i + P_i \leq 0, \right. \\ \left. A_i x_{i,k} + B_i \mathbf{u}_i + C_i \mathbf{w}_i \geq 0 \right\},$$

$$\check{\mathcal{Y}}_{ij}(\delta_{ij}) := \left\{ (\mathbf{y}_i, \mathbf{y}_j) \in \mathbb{R}^{2n_y} : H_i \mathbf{x}_i + H_j \mathbf{x}_j \leq \bar{V}_{ij} - \delta_{ij} \right\}.$$

¹Both sets have a dependency on the initial value of the state $x_{i,k}$ for each agent i at each sampling time k . Given $x_{i,k}$, we here highlight the dependency of these sets on the uncertainties $\mathbf{w}_i$ and δ_{ij} for each agent i at each sampling time k .

It is important to note that $\check{\mathcal{Y}}_{ij}(\delta_{ij}) \in \mathbb{R}^{2n_i N_i}$ represents the cylindrical extension² of $\mathcal{Y}_{ij}(\delta_{ij})$. In the subsequent parts, we refer to the constraint (23b) as the agents' private chance constraints, and to the constraint (23c) as the agents' common chance constraints. The proposed formulation (23) is a mixed-integer quadratic optimization problem with multiple chance constraints, due to the binary variables $\{v_i\}_{i=1}^N$ and the chance constraints (23b), (23c). The index of $\mathbb{P}_w$ and $\mathbb{P}_\delta$ denote the dependency on the string of random scenarios $w \in \mathcal{W}$ and $\delta \in \Delta$, respectively.

Building upon our previous work in [22], we extend the so-called robust randomized approach in [30] and [31] to be able to handle a problem with multiple chance constraints. Problem (23) is a stochastic program with multiple chance constraints, where $\mathbb{P}_w$ and $\mathbb{P}_\delta$ denote two different probability measures for private and common uncertainty sources, respectively.

Define $\mathcal{B}_i, \bar{\mathcal{B}}_{ij}$ to be two bounded sets of private uncertainty source and a bounded set of common uncertainty source for each agent i , respectively. $\mathcal{B}_i, \bar{\mathcal{B}}_{ij}$ are assumed to be axis-aligned hyper-rectangular sets [30, Proposition 1]. This is not restrictive and any convex set with convex volume could have been chosen instead as in [38]. We parametrize $\mathcal{B}_i(\gamma) := [\bar{\gamma}, \underline{\gamma}]$ by $\gamma = (\bar{\gamma}, \underline{\gamma}) \in \mathbb{R}^{2n_w}$, and $\bar{\mathcal{B}}_{ij}(\lambda) := [\bar{\lambda}, \underline{\lambda}]$ by $\lambda = (\bar{\lambda}, \underline{\lambda}) \in \mathbb{R}^{2n_\delta}$, and formulate two chance-constrained problems similarly to [22, Problem 8]. Following the so-called scenario approach in [39], one can determine the number of required uncertainty scenarios to formulate a tractable problem, [22, Problem 9], using $N_s = \frac{2}{\epsilon}(\xi + \ln \frac{1}{\nu})$, where ξ is the dimension of decision vector, ϵ, ν are the level of violation, and the confidence level, respectively. The optimal solutions (γ^*, λ^*) of the proposed tractable problem are probabilistically feasible for the chance-constrained problems, [40, Th. 1]. Moreover, γ^* and λ^* also characterize our desired probabilistic bounded sets $\mathcal{B}_i^*$ and $\bar{\mathcal{B}}_{ij}^*$, respectively. Note that $\mathcal{S}_i$ and $\bar{\mathcal{S}}_{ij}$ are two collections of random scenarios that are i.i.d.

After determining $\mathcal{B}_i^*$ and $\bar{\mathcal{B}}_{ij}^*$ for all agents $i \in \{1, \dots, N\}$, we are now able to reformulate the robust counterpart of the original problem (23) via:

$$\min_y \sum_{i=1}^N \mathcal{V}_i(x_i, u_i) \quad (24a)$$

$$\text{s.t. } y \in \prod_{i=1}^N \bigcap_{w_i \in \{\mathcal{B}_i^* \cap \mathcal{W}_i\}} \mathcal{Y}_i(w_i), \quad (24b)$$

$$y \in \prod_{i=1}^N \bigcap_{j \in \mathcal{N}_i} \bigcap_{\delta_{ij} \in \{\bar{\mathcal{B}}_{ij}^* \cap \Delta_{ij}\}} \check{\mathcal{Y}}_{ij}(\delta_{ij}). \quad (24c)$$

Note that the aforementioned problem is not a randomized program, and instead, the constraints have to be satisfied for all values of the private uncertainty in $\{\mathcal{B}_i^* \cap \mathcal{W}_i\}$, and common uncertainty in $\{\bar{\mathcal{B}}_{ij}^* \cap \Delta_{ij}\}$. The proposed problem (24) is a robust mixed-integer quadratic program. In [41], it was shown that robust problems are tractable [22, Proposition 1], and

remain in the same class as the original problems, e.g., robust mixed-integer programs remain mixed-integer programs, for a certain class of uncertainty sets, such as in our problem (24), the uncertainty is bounded in a convex set. The following theorem quantifies the robustness of solution obtained by (24) w.r.t. the initial problem (23).

Theorem 1: Let $\varepsilon_i, \bar{\varepsilon}_{ij} \in (0, 1)$ and $\beta_i, \bar{\beta}_{ij} \in (0, 1)$ for all $j \in \mathcal{N}_i$, for each $i \in \{1, \dots, N\}$ be chosen such that $\varepsilon = \sum_{i=1}^N \varepsilon_i \in (0, 1)$, $\beta = \sum_{i=1}^N \beta_i \in (0, 1)$, $\bar{\varepsilon} = \sum_{j \in \mathcal{N}_i} \bar{\varepsilon}_{ij} \in (0, 1)$, $\bar{\beta} = \sum_{j \in \mathcal{N}_i} \bar{\beta}_{ij} \in (0, 1)$ and $\bar{\varepsilon} = \sum_{i=1}^N \bar{\varepsilon}_i \in (0, 1)$, $\bar{\beta} = \sum_{i=1}^N \bar{\beta}_i \in (0, 1)$. Determine $\mathcal{B}_i^*$ and $\bar{\mathcal{B}}_{ij}^*$ by constructing $\mathcal{S}_i, \bar{\mathcal{S}}_{ij}$ for all $j \in \mathcal{N}_i$, for each $i \in \{1, \dots, N\}$. If y_s^* is a feasible solution of the problem (24), then y_s^* is also a feasible solution for the chance constraints (23b) and (23c), with the confidence levels of $1 - \beta$ and $1 - \bar{\beta}$, respectively.

The proof is provided in an online technical report [36].

Remark 1: Following the approach in [42], we approximate the objective function empirically for each agent i . $\mathbb{E}_{w_i}[\mathcal{J}_i(\cdot)]$ can be approximated by averaging the value of its argument for some number of different scenarios, which plays a tuning parameter role. To improve the objective value of our proposed formulation, one can employ scenario removal algorithms, leading a trade-off between feasibility and optimality [43].

Remark 2: A tractable decoupled solution (DS) formulation for (14) can be achieved by removing the robust coupling constraint (24c) from (24). Since there is no longer a coupling constraint, each agent i can therefore solve its problem independently.

The solution of (24) is the optimal control input sequence $\{u_{i,k|k}^*, v_{i,k|k}^*, \dots, u_{i,k+N_h-1|k}^*, v_{i,k+N_h-1|k}^*\}_{i=1}^N$. Based on an MPC paradigm, the current input at time step k is implemented in the system dynamics (10) using the first element of optimal solutions as $\{u_{i,k}, v_{i,k}\}_{i=1}^N := \{u_{i,k|k}^*, v_{i,k|k}^*\}_{i=1}^N$ and we proceed in a receding horizon fashion. This means (24) is solved at each time step k by using the current measurement of the state $\{x_{i,k}\}_{i=1}^N$. It is important to highlight that the feasibility guarantees in Theorem 1 are independent from the sampling rate of the real continuous-time system. It is however very important to have a discrete-time system model that can predict the real system behavior as precisely as possible. Once such a suitable discrete-time system model is developed, one can use our proposed tractable frameworks (DS, CS, and MCS), and instead of analyzing the closed-loop asymptotic behavior, achieve the fulfillment of multiple chance constraints from an optimization point of view and have a-priori probabilistic feasibility guarantees via Theorem 1.

V. NUMERICAL STUDY

In this section, we present a simulated case study for a three-agent ATES system in a STG, as it is shown in Fig. 2. We determine the thermal energy demands of three buildings, that had been equipped with ATES systems, modeled using realistic parameters and the actual registered weather data in the city center of Utrecht, The Netherlands, where these buildings are located. We refer interested readers to [44, Appendix A] for the complete detailed parameters of this case study.

²Cylindrical extension replicates the membership degrees from the existing dimensions into the new dimensions [37, Sec. 4].

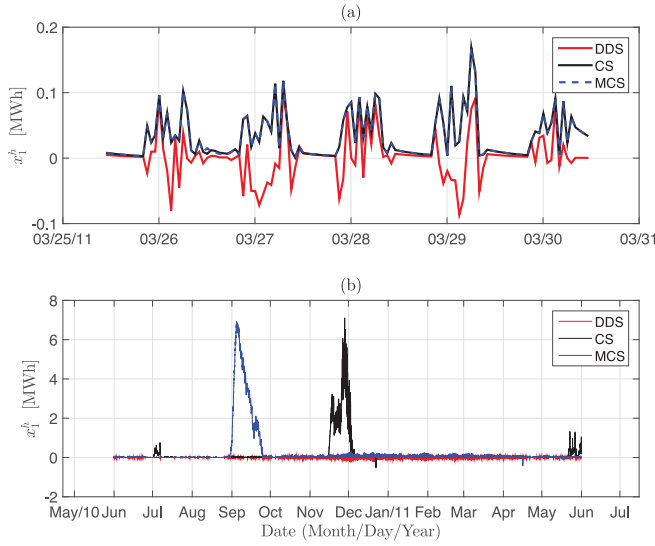


Fig. 3. *A-posteriori* feasibility validation of the obtained results via DDS, CS, and MCS formulations for the imbalance error dynamics in the first building of the three-agent ATEs-STG example. (a) Focuses on a randomly chosen five-day period to allow a better comparison, whereas (b) presents the complete one year results.

We simulate three problem formulations, namely: DS (decoupled solution), CS (centralized solution), and MCS (move-blocking centralized solution), using the proposed tractable framework (24). The simulation time is one year from June 2010 to June 2011 with hourly-based sampling time. The prediction horizon for DS and CS is a day-ahead (24 hours), whereas for MCS is a whole season (3 months). The multi-rate control actions in MCS are considered to be hourly-based during first day, daily-based in the first week, weekly-based within the first month, and monthly-based for the rest of the season. We also simulate a deterministic DS (DDS) for comparison purposes, where the uncertain elements (w_i) are fixed to their forecast value for each agent $i = 1, 2, 3$. In order to generate scenarios from the private uncertainty sources, we use a discrete normal stochastic process, where the thermal energy demand of each building varies within 10% of its actual value at each sampling time. A similar technique is used for the common uncertainty sources. The simulation environment was MATLAB with YALMIP as the interface [45] and Gurobi as a solver.

Fig. 3 and Fig. 4 (a) depict *a-posteriori* feasibility validation of the private chance constraint of agent 1 and the common chance constraint between agent 1 and agent 2. It is important to note that the results obtained for the other two buildings are very similar, and therefore we focus on the results of the first building (agent 1). To illustrate the functionality of our proposed framework to deal with the private chance constraint, in Fig. 3, we present the *a-posteriori* feasibility validation of the obtained results via DDS, CS, and MCS formulations. Fig. 3 (a) shows the obtained results for the last five days in March 2011, and Fig. 3 (b) shows the results for one year simulation from June 2010 until June 2011. In Fig. 3 the “red” color denotes the solution of DDS, “black” color shows the solution of CS, and “blue” presents the solution of MCS.

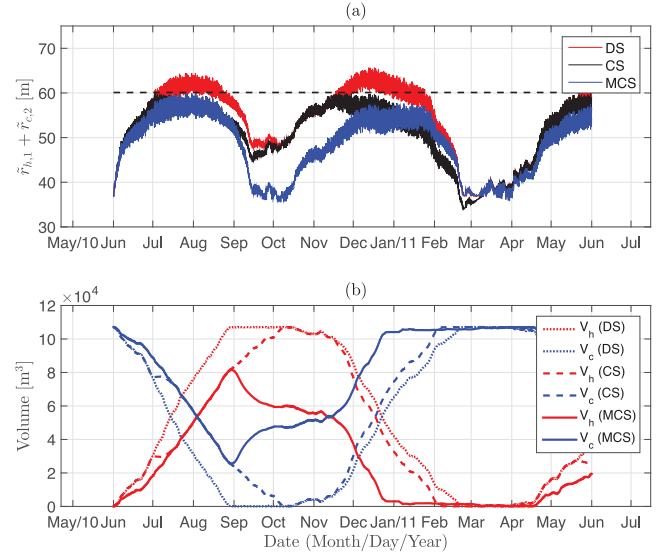


Fig. 4. (a) *A-posteriori* feasibility validation of the obtained results via DS, CS, and MCS formulations for the common coupling constraint between the first and second building of the three-agent ATEs-STG example. (b) ATEs system state trajectories (volume of the stored water in the warm and cold wells of the first building) in the three-agent ATEs-STG example.

Fig. 3 (a) focuses on a randomly chosen five-day period to allow a better comparison between the results of DDS, CS, and MCS. It is clearly shown that the obtained results via CS and MCS, provide a feasible (nonnegative) trajectory of the thermal energy imbalance error during heating mode, whereas the solution of DDS, leads to some violations throughout the simulation time. Notice that all three proposed approaches, namely DS, CS, and MCS, achieved the feasibility of the private chance constraint in a probabilistic sense as it is guaranteed in Theorem 1. We present the results obtained via DDS to highlight such an achievement, whereas the results obtained via DS is omitted to demonstrate the other achievements.

In Fig. 3 (b), the complete one year results of DDS, CS, and MCS are shown. Two important observations are as follows: the obtained results of CS and MCS have very small number of violations, much less than our desired level of violations, throughout the simulation time. This yields a less conservative approach compared to the classical robust control approach (see [35, Ch.14]). As the second observation, in the results of CS and MCS one can see some instances of a large non-zero imbalance error, which is expected: By taking into account the coupling constraints between agents, the solutions of agents are going to extract the stored thermal energy from their ATEs systems to prevent the mutual interactions between their ATEs systems as in Fig. 4(a). Interestingly, the results of MCS show that agent 1 starts to extract the stored thermal energy from its ATEs system sooner due to its longer prediction horizon, compared to CS.

Fig. 4(a) shows the evaluation of our proposed reformulation for the coupling constraint in (17) together with the *a-posteriori* feasibility validation of the common chance constraint between agent 1 and agent 2. We plot the obtained $\tilde{r}_{h,1} + \tilde{r}_{c,2}$ using DS, CS, and MCS formulations. As it is clearly shown DS results are violating the coupling constraint

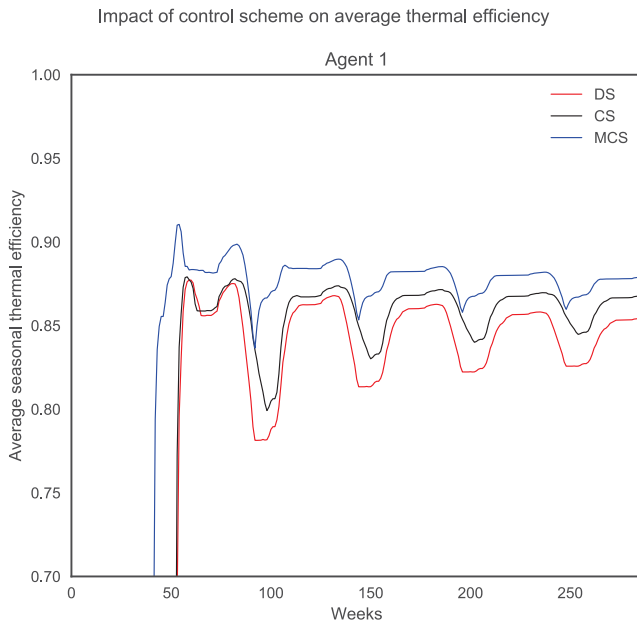


Fig. 5. Impact of DS and CS on average thermal efficiency.

which leads to overlap between the stored water in warm well of ATES system in agent 1 and the stored water in cold well of ATES system in agent 2. This is due to the fact that there are no coupling constraints in the DS framework and each agent works without any information from neighboring agents. It is important to highlight that the results obtained via DDS and DS are the same in terms of the ATES system dynamical behavior. This is due to the fact that the cost parameter associated with the ATES system pump is the same in both DDS and DS formulations, and thus ATES systems participate in the agent energy management in the same way, regardless of the private chance constraints. We also present the evolution of the stored water volume in each well of the ATES system for agent 1 using the obtained results via DS, CS, and MCS formulations in Fig. 4 (b) to illustrate the impact of the different formulations.

It is worth to mention that Fig. 3 and Fig. 4 illustrate all main contributions: 1) having a probabilistically feasible solution for each agent w.r.t. the private uncertainty sources as it is encoded via (23b), 2) respecting the common resource pool between neighboring agents in STGs as it is formulated in (23c) (the first and second outcomes are the direct results of our theoretical guarantee in Theorem 1), and 3) prediction using a longer horizon yields an anticipatory control decision that improves the operation of an ATES system. This is a direct consequence of our proposed move-blocking scheme in (22).

Fig. 5 summarizes the results in terms of average thermal efficiency that we obtained by integrating our control strategy, DS and CS, into Python to build a live-link with MODFLOW, a more realistic aquifer simulation environment³ [46]. Fig. 5 is presented to highlight the impact of considering the proposed coupling constraints, as it is formulated in (23c), versus the

decoupled setting. The impact of our control strategy, DS (red) and CS (blue), on average thermal energy efficiency [47] in each building illustrates that we can store and retrieve the same amount of thermal energy in ATES systems, in a more efficient way due to information exchange between the agents to prevent the mutual interactions between wells using the results of MCS and CS compared to DS.

VI. CONCLUSION

This paper proposed a stochastic MPC framework for an energy management problem in STGs consisting of ATES systems integrated into BCC systems. We developed a large-scale stochastic hybrid model to capture thermal energy imbalance errors in an ATES-STG. In such a framework, we formalized two important practical concerns, namely: 1) the balance between extraction and injection of energy from and into the aquifers within a certain period of time; 2) the unwanted mutual interaction between ATES systems in STGs. Using our developed model, we formulated a finite-horizon mixed-integer quadratic optimization problem with multiple chance constraints. To solve such a problem, we proposed a tractable formulation based on the so-called robust randomized approach. In particular, we extended this approach to handle a problem with multiple chance constraints. We simulated our proposed framework using a three-agent ATES-STG example which confirmed the expected performance improvements.

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³MODFLOW is a modular hydrologic model, and it is considered an international standard for aquifer simulation and predicting groundwater conditions and interactions.

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