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An Operation Model for Distribution Companies Using the Flexibility of Electric Vehicle Aggregators

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Abstract—An operation model for distribution companies (DISCOs) is proposed to reduce their operation costs by fully utilizing the flexibility of electric vehicle aggregators (EVAs). In the proposed model, linear decision rules approximation is adopted to achieve mathematical tractability, and distributionally robust optimization is applied to evaluate costs affected by uncertainties in renewable power outputs and EVA charging demands. Case studies are conducted under various settings. With the proposed model, using EVAs to mitigate uncertainties is achieved and is further classified into delaying uncertainties and eliminating uncertainties. As a result, average penalties for DISCO's deviations from its planned energy portfolio are reduced. Besides, EVA charging demands are shifted to hours with lower energy prices to reduce energy costs of DISCO. Using EVAs to mitigate uncertainties and shifting EVA charging demands are properly coordinated under the proposed model. Moreover, power losses in EVA charging and discharging are utilized to reduce the scale of uncertainties, which decreases average penalties for energy deviations of DISCO.

Index Terms—Distribution company, electric vehicle aggregator, renewable energy, uncertainty, distributionally robust optimization.

NOMENCLATURE

A. Parameters

a_e^t	Energy price in Hour t
$a_{r,def}^{t,i}, a_{r,over}^{t,i}$	Price of reserves for EVA charging deficiency, over-charging at Node i in Hour t
$b_{r,def}^{t,i}, b_{r,over}^{t,i}$	Regular compensation rate for EVA charging deficiency, over-charging at Node i in Hour t
$b_{p,def}^{t,i}, b_{p,over}^{t,i}$	Punitive compensation rate for EVA charging deficiency, over-charging at Node i in Hour t
$b_{p,pos}^t, b_{p,neg}^t$	Penalty coefficients for DISCO's positive, negative deviations from its energy purchase in Hour t
b_d^i	Compensation rate for EVA battery degradation at Node i
v_b	Base voltage
$x_{i,j}, r_{i,j}$	Line reactance, resistance between Node i and j

$P_l^{t,i}, q_l^{t,i}$	Active, reactive load at Node i in Hour t
$P_{RES,f}^{t,i}$	RES power forecast at Node i in Hour t
$P_{EVA,p}^{t,i}$	EVA planned charging demand at Node i in Hour t
λ_i	EVA power factor at Node i
$\eta_{ch}^i, \eta_{dis}^i$	Average EVA charging, discharging efficiency at Node i
$P_{ch,max}^i$	Maximum EVA charging rate at Node i
$P_{dis,max}^i$	Maximum EVA discharging rate at Node i
$e_{def,max}^{t,i}$	Acceptable EVA charging deficiency at Node i in Hour t
$e_{over,max}^{t,i}$	Acceptable EVA over-charging at Node i in Hour t
T	Number of hours in the time horizon
Δt	An hour
N_{sys}	Set of nodes in the distribution system
$N_p(i)$	Parent node of Node i
$N_c(i)$	Set of children nodes of Node i
N_{EVA}	Set of EVA nodes
N_{RES}	Set of RES nodes

B. Uncertainties

$\xi_{EVA}^{t,i}$	Deviation of EVA actual charging demand from EVA planned charging demand at Node i in Hour t
$\xi_{RES}^{t,i}$	Error of RES power forecast at Node i in Hour t
ξ	Vector of EVA and RES uncertainties in all hours
d_ξ	Dimension of ξ
f_ξ	Probability distribution of ξ
$A(\xi)$	Ambiguity set for ξ
μ	Statistical expectation of ξ
Σ	Statistical covariance matrix of ξ

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C. Variables

$r_{\text{def}}^{t,i}, r_{\text{over}}^{t,i}$	Purchased reserves for EVA charging deficiency, over-charging at Node i in Hour t
e_{pur}^t	Purchased energy for Hour t
$\sigma_{t,i}$	EVA status at Node i in Hour t
$p_{\text{sys,EVA}}^{t,i}$	EVA active power supplied by the distribution system at Node i in Hour t
$p_{\text{ch}}^{t,i}, p_{\text{dis}}^{t,i}$	EVA active charging, discharging power at Node i in Hour t
$e_{\text{dist}}^{t,i}$	Cumulative disturbance to EVA at Node i between Hour 1 and t
$f_{\text{p}}^{t,i,j}, f_{\text{q}}^{t,i,j}$	Active, reactive power flow between Node i and j in Hour t
p_{in}^t	Active power imported from the transmission system
$v_{t,i}$	Node i voltage in Hour t
$\alpha_{\text{ch}}^{t,i}, \alpha_{\text{dis}}^{t,i}$	Constant components in real-time operation plans for $p_{\text{ch}}^{t,i}, p_{\text{dis}}^{t,i}$
$f_{\text{ch}}^{t,i}(\xi)$	Function of uncertainties in real-time operation plan for $p_{\text{ch}}^{t,i}$
$f_{\text{dis}}^{t,i}(\xi)$	Function of uncertainties in real-time operation plan for $p_{\text{dis}}^{t,i}$

I. INTRODUCTION

DISCO purchases energy and then delivers to the energy users within the distribution system [1],[2]. As a component of a power system, it may be required to maintain the energy balance of the system by scheduling its energy portfolio in advance. If it defaults on its schedules, it can be penalized [3],[4]. As renewable energy sources (RESs) introduce considerable forecast errors, i.e., uncertainties [5],[6], it is becoming challenging for DISCO to maintain its energy portfolio. Meanwhile, the proliferation of electric vehicles (EVs) introduce additional opportunities for DISCO to reduce its operation costs.

As long as EVs are plugged into power systems for a sufficiently long time, EV charging rates can be adjusted and vehicle-to-grid power flows can be introduced. For example, the flexibility of EVs is used to mitigate wind power forecast errors [7], reduce peak load [8] and improve voltage profile [9]. To directly dispatch EVs as done in [7-9], circumstances of each EV need to be taken into consideration. As EV behaviors are influenced by many factors including human decisions, they involve considerable uncertainties. In terms of modeling EV uncertainties, [10] assumes that EV departure times, trip durations and energy consumptions follow uniform distributions and generates scenarios according to the assumed uniform distributions to represent these EV uncertainties. [11] also uses scenarios to represent EV uncertainties and further combines scenarios that have no difference from the

perspective of system operators. [12] performs more detailed modeling of EV uncertainties by considering charging failures, the tendency of EV users to round time and so on. [13] models EV uncertainties through analyzing EV travel routes and charging of the same EV at different places. However, when a large number of EVs are involved, it will be very difficult to take all these uncertainties into consideration to dispatch each individual EV [14]. Instead, it is more reasonable for system operators to dispatch EVAs and let EVAs control a limited number of EVs. [15] and [16] adopt such schemes, but their models are not designed for DISCO and do not use EVAs to mitigate uncertainties.

EVAs represent equivalent loads that are always connected to the system with time varying charging demands. Because of uncertain factors associated with EV behaviors, there are uncertainties in charging demands of EVAs. Meanwhile, EVAs provide flexibility for system operation as they represent aggregated EV flexibilities. EVA flexibility can be explored by DISCO to reduce penalties for its deviations from the scheduled energy portfolio through mitigating RES and EVA uncertainties and reduce its energy costs through shifting EVA charging demands to hours with lower energy prices. Power losses in EVA charging and discharging can be evaluated through average charging and discharging efficiencies of EVAs [15],[17].

In addition to EVAs, other options which are considered for the provision of flexibility in distribution systems include hourly load curtailments, which can neither shift loads to hours with lower energy prices nor be effective when energy supply is sufficiently available [18],[19]. Other cases considered distributed generators, which can only supply energy, while EVAs can both supply and consume energy [20],[21]. Energy storage is another flexibility option which differs from EVAs in the following aspects [22],[23]. First, energy storage is specifically deployed to facilitate system operation, while EVAs are not. Second, there is no uncertainty related with energy storage, but uncertainties exist in EVA charging demands. Third, operating energy storage incurs costs. In contrast, costs are resulted from disturbing EVA charging. In consideration of these features of EVAs, a tailor-made model is proposed for DISCO to reduce its operation costs by fully utilizing EVA flexibility in this paper.

Using EVA flexibility incurs disturbance to EVAs including over-charging and charging deficiency. Depending on whether recovering the corresponding disturbance to EVAs in the current day, using EVAs to mitigate uncertainties is classified into delaying uncertainties and eliminating uncertainties in this paper. Further discussions will be made in Section III.B.

EVA flexibility has two potential applications for DISCO, namely, mitigating uncertainties and shifting EVA charging demands to hours with lower energy prices. Both EVA applications can introduce cost savings. Besides, they are both constrained by EVA capacity for charging disturbance. If EVAs are used to mitigate uncertainties more extensively, they can shift less charging demands and vice versa. So, the two EVA applications are correlated.

Because of power losses in EVA charging, the power

supplied by the distribution system is greater than that received by EVAs. Similarly, the power received by the distribution system is smaller than that supplied by EVAs when EVAs discharge. Such phenomenon influences the scale of uncertainties from the perspective of DISCO and can be utilized when EVAs charge in some hours and discharge in other hours. Further discussions are made in Section III.C and VI.C.

Major contributions of this paper are the following.

- (1) A comprehensive operation model is proposed for DISCO to reduce its operation costs. In the proposed model, using EVAs to mitigate RES and EVA uncertainties is achieved and is further classified into delaying uncertainties and eliminating uncertainties.
- (2) EVA flexibility is used to shift EVA charging demands to hours with lower energy prices. The two applications of EVA flexibility, i.e., mitigating uncertainties and shifting charging demands, are coordinated in the proposed model to achieve the optimal overall costs.
- (3) Power losses in EVA charging and discharging are utilized to reduce the scale of uncertainties and thus reduce penalties for energy deviations of DISCO.

This paper is organized as follows. Section II gives background settings. Section III discusses applications of EVA flexibility. The proposed model is presented in Section IV and transformed into deterministic forms in Section V. Case studies are conducted in Section VI. Lastly, conclusions are drawn in Section VII.

II. BACKGROUND SETTINGS

The proposed model is based on the perspective of DISCO and focuses on the operation model of DISCO and interactions between DISCO and EVAs. DISCO faces uncertainties in EVA charging demands rather than uncertainties associated with EV behaviors. EVAs need to consider EV uncertainties, and how EVAs operate influences uncertainties in EVA charging demands. But DISCO and EVAs are different entities, and the operation model of EVAs and interactions between EVAs and EVs are not studied here because they are beyond the scope of this paper. It should be noted that although EV uncertainties are not directly considered in the proposed model, they are reflected through uncertainties in EVA charging demands. As EVA charging demands can be observed by DISCO, their uncertainties can be analyzed by using their historical samples.

The proposed model involves the day-ahead and the real-time stage. Actual EVA charging demands and RES outputs are unknown in the day-ahead stage, which means that there are EVA and RES uncertainties. In consideration of real-time operation under possible realizations of EVA and RES uncertainties, the proposed model makes day-ahead decisions and real-time operation plans for DISCO with the aim of minimizing total operation costs of DISCO in the two stages. Real-time operation plans are functions of uncertainties and will give real-time decisions when uncertainties realize. Further discussions about real-time operation plans will be made in Section III and IV.B. The schematic diagram of the proposed model is given in Fig. 1, where TSO stands for transmission system operator.

In the day-ahead stage, DISCO purchases energy that it plans to import from the transmission system. Because of uncertainties in EVA charging demands and RES outputs, the actual energy import of DISCO in the real-time stage may deviate from its energy purchase. DISCO needs to pay penalties for the deviation no matter it is positive or negative.

In the day-ahead stage, EVAs are required to report their planned charging demands to DISCO. The actual EVA charging demands in the real-time stage will be the sum of planned charging demands and EVA uncertainties $\xi_{EVA}^{t,i}$. In the real-time stage, DISCO gives dispatch to EVAs and may cause disturbance to their charging, which is defined in (1). When EVAs receive more energy than their needs, $e_{dist}^{t,i}$ is negative and EVA over-charging happens. When EVAs receive less energy than their needs, $e_{dist}^{t,i}$ is positive and EVA charging deficiency happens. In the day-ahead stage, EVAs report their acceptable amounts for over-charging and charging deficiency to DISCO.

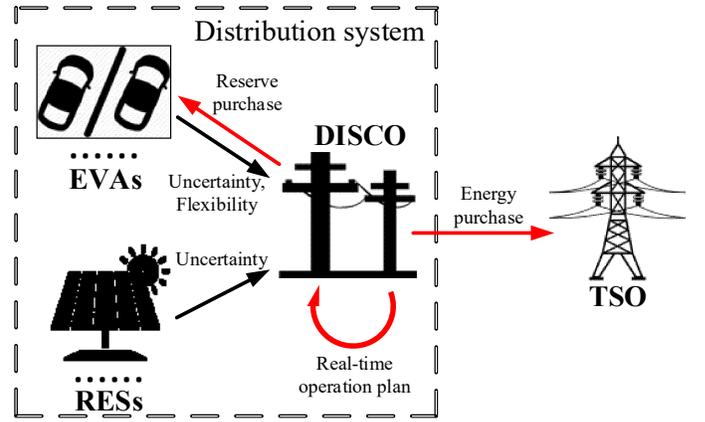


Fig. 1. Schematic diagram of the proposed model

$$e_{dist}^{t,i} = \sum_{\tau=1, \dots, t} \left(\left(p_{EVA,p}^{\tau,i} + \xi_{EVA}^{\tau,i} - p_{ch}^{\tau,i} + p_{dis}^{\tau,i} \right) \Delta t \right) \quad (1)$$

DISCO pays EVAs for disturbance to them in two steps. First, in the day-ahead stage, DISCO reserves certain amounts for EVA over-charging and charging deficiency and pays EVAs for the reserves. As uncertainties realize, disturbance to EVAs is known in the real-time stage. Then, regular compensations are given to EVAs by DISCO for the disturbance within the reserves purchased by DISCO, and punitive compensations that are of higher rates are given for the disturbance beyond the reserves. With such mechanism, EVAs could anticipate the dispatch of DISCO through the reserves that DISCO purchases in the day-ahead stage. The concept of reserves here is not the same with that of reserves from generators in economic dispatch problems. Adjustments of generator outputs are constrained by generator reserves in economic dispatch [24], while disturbance to EVAs can exceed the reserves in this paper and is constrained by the capabilities of EVAs in accepting disturbance. Besides, DISCO compensates the degradation of EVA battery at fixed rates according to EVA discharging power in the real-time stage.

Although it is assumed here that EVAs follow the dispatch of DISCO, the proposed model can also incorporate EVAs that

do not cooperate, which are inflexible loads with uncertainties from the perspective of DISCO. Similar to uncertainties in RES outputs, uncertainties in charging demands of these non-dispatchable EVAs can be mitigated by dispatchable EVAs.

In some literature such as [25], DISCO cooperates with the distribution system operator, which is responsible for network operation. While, in some other literature such as [18] and [26], DISCO manages network operation by itself. The proposed model for DISCO is applicable for both cases as it is able to both consider and not consider network constraints. It should be noted that power losses considered by DISCO in the proposed model are from EVA charging and discharging, and they influence the energy that DISCO needs to purchase and the deviations of DISCO from its decided energy portfolio. Power losses on the distribution network are discussed later in Section IV.A.

III. APPLICATIONS OF EVA FLEXIBILITY

The key of the proposed model lies in applications of EVA flexibility, which are discussed in this section. Fig. 2 gives the schematic diagram of the proposed model's methodology.

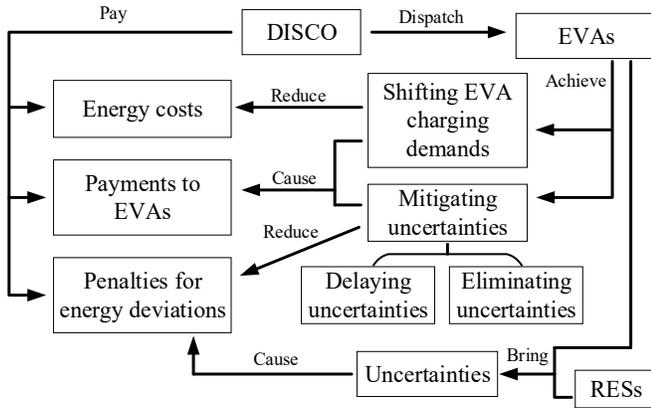


Fig. 2. Schematic diagram of the proposed model's methodology

A. Mitigating uncertainties and shifting EVA charging demands

DISCO's real-time operation plan for EVA charging power is given in (2). If DISCO does not disturb EVAs, EVA charging power will be the sum of EVA planned charging demands and EVA uncertainties, which means $\alpha_{ch}^{t,i} = p_{EVA,p}^{t,i}$ and $f_{ch}^{t,i}(\xi) = \xi_{EVA}^{t,i}$. By choosing proper $f_{ch}^{t,i}(\xi)$, DISCO can use EVAs to mitigate uncertainties. For example, if $f_{ch}^{t,i}(\xi) = 0$, EVA i mitigates its own uncertainty because its charging power is now constant. If $f_{ch}^{t,i}(\xi) = \xi_{EVA}^{t,i} - \xi_{EVA}^{t,j}$, EVA i mitigates the uncertainty of EVA j . Similarly, EVAs can mitigate RES uncertainties. Besides, by setting $\alpha_{ch}^{t,i}$ to proper values, DISCO can shift EVA charging demands to hours with lower energy prices. If EVAs discharge in some hours, more EVA charging demands can be shifted compared with the case when EVAs never discharge. DISCO's operation plan for EVA discharging power is given in (3). Similar to choosing proper $\alpha_{ch}^{t,i}$ and $f_{ch}^{t,i}(\xi)$, choosing proper $\alpha_{dis}^{t,i}$ and $f_{dis}^{t,i}(\xi)$ can have EVAs shift their charging demands and mitigate uncertainties, respectively.

For example, if $f_{dis}^{t,i}(\xi) = -\xi_{EVA}^{t,i}$, uncertainty of EVA i is not mitigated. The negative sign is because EVA i is discharging. If $f_{dis}^{t,i}(\xi) = 0$, EVA i mitigates the uncertainty of itself. If $f_{dis}^{t,i}(\xi) = -\xi_{EVA}^{t,i} + \xi_{EVA}^{t,j}$, EVA i mitigates the uncertainty of EVA j .

$$p_{ch}^{t,i} = \alpha_{ch}^{t,i} + f_{ch}^{t,i}(\xi) \quad (2)$$

$$p_{dis}^{t,i} = \alpha_{dis}^{t,i} + f_{dis}^{t,i}(\xi) \quad (3)$$

Similar to $\alpha_{ch}^{t,i}$ and $\alpha_{dis}^{t,i}$, $f_{ch}^{t,i}(\xi)$ and $f_{dis}^{t,i}(\xi)$ influence power flows between the distribution system and EVAs, and thus can shift EVA charging demands from some hours to other hours. But the effects of $f_{ch}^{t,i}(\xi)$ and $f_{dis}^{t,i}(\xi)$ are different from those of $\alpha_{ch}^{t,i}$ and $\alpha_{dis}^{t,i}$ as follows. First, the effects of $\alpha_{ch}^{t,i}$ and $\alpha_{dis}^{t,i}$ are known once $\alpha_{ch}^{t,i}$ and $\alpha_{dis}^{t,i}$ are determined in the day-ahead stage, while the effects of $f_{ch}^{t,i}(\xi)$ and $f_{dis}^{t,i}(\xi)$ are known only after uncertainties realize in the real-time stage. Besides, proper $\alpha_{ch}^{t,i}$ and $\alpha_{dis}^{t,i}$ reduce the costs of DISCO in purchasing energy imported from the transmission system, while proper $f_{ch}^{t,i}(\xi)$ and $f_{dis}^{t,i}(\xi)$ reduce the penalties for energy deviations of DISCO caused by uncertainties. In consideration of these differences, "shifting EVA charging demands" refers to "choosing proper $\alpha_{ch}^{t,i}$ and $\alpha_{dis}^{t,i}$ ", and "mitigating uncertainties" refers to "choosing proper $f_{ch}^{t,i}(\xi)$ and $f_{dis}^{t,i}(\xi)$ " in the following parts of this paper.

Shifting EVA charging demands and mitigating uncertainties would both disturb EVA charging. When any one of them is carried out, either EVA charging deficiency or EVA over-charging will be incurred. If their disturbance to EVAs is in the **same** direction (EVA charging deficiency or EVA over-charging), their disturbance reinforces each other. While if their disturbance is in **different** directions (EVA charging deficiency and EVA over-charging), their disturbance offsets each other.

B. Delaying uncertainties and eliminating uncertainties

Depending on whether the corresponding disturbance to EVAs is recovered within the day or not, using EVAs to mitigate uncertainties can have different effects. To differentiate these two circumstances, "delaying uncertainties" and "eliminating uncertainties" are used to represent them, respectively. Discussions about the two circumstances are made below.

Delaying uncertainties is illustrated as follows. If the charging power of EVA i in the first hour is $p_{ch}^{1,i} = p_{EVA,p}^{1,i}$, EVA i mitigates its own uncertainty and the disturbance to its charging is $e_{dist}^{1,i} = \xi_{EVA}^{1,i}$. If the disturbance is recovered in the second hour, there is $e_{dist}^{2,i} = 0$, which requires $p_{ch}^{2,i} = p_{EVA,p}^{2,i} + \xi_{EVA}^{1,i} + \xi_{EVA}^{2,i}$. So, $\xi_{EVA}^{1,i}$ causes the variation of EVA charging power and penalties for energy deviations of DISCO in the second hour rather than in the first hour, which means that $\xi_{EVA}^{1,i}$ is **delayed**. $\xi_{EVA}^{1,i}$ and $\xi_{EVA}^{2,i}$ now offset each other in $p_{ch}^{2,i}$ if they are of different signs. To conclude, delaying uncertainties happens when disturbance to EVAs incurred from mitigating

uncertainties is recovered within the day.

Eliminating uncertainties occurs when the disturbance to EVAs incurred from mitigating uncertainties is not fully recovered by the end of the day. The unrecovered disturbance will be merged into EVA planned charging demands when DISCO decides its energy portfolio again in the day-ahead stage of the next day. In other words, the involved uncertainties will become deterministic information in the next day and will not cause energy deviations of DISCO and corresponding penalties anymore, which is like the involved uncertainties being **eliminated**. This can also be interpreted as delaying uncertainties to the next day and thus making them deterministic. To differentiate from the earlier circumstance, this circumstance is called eliminating uncertainties. Operation costs of DISCO in the next day will be influenced by the unrecovered disturbance to EVAs incurred from mitigating uncertainties. But the average influence is tiny because expectations of the considered uncertainties are close to zero.

Delaying uncertainties and eliminating uncertainties cooperate under the proposed model to achieve the optimal overall costs. Their optimal cooperation depends on the parameters and needs to be obtained by solving the proposed model. Further discussions can be found in Section VI.E.

C. Effects of power losses in EVA charging and discharging on uncertainties

Because of power losses in EVA charging and discharging, the EVA active power supplied by the distribution system will be $p_{\text{sys,EVA}}^{t,i} = p_{\text{ch}}^{t,i} / \eta_{\text{ch}}^{t,i}$ when EVAs charge and will be $p_{\text{sys,EVA}}^{t,i} = -p_{\text{dis}}^{t,i} \cdot \eta_{\text{dis}}^{t,i}$ when EVAs discharge. Therefore, $f_{\text{ch}}^{t,i}(\xi)$ and $f_{\text{dis}}^{t,i}(\xi)$ in $p_{\text{ch}}^{t,i}$ and $p_{\text{dis}}^{t,i}$ as shown in (2) and (3) will be magnified or minified by power losses in EVA charging and discharging in $p_{\text{sys,EVA}}^{t,i}$, which directly influences the power that DISCO imports from the transmission system. Because of such phenomenon, uncertainties may be minified for the first time when they are mitigated by EVAs and then be minified for the second time when the corresponding disturbance to EVAs is recovered, which means that the scale of uncertainties is finally reduced from the perspective of DISCO. Detailed illustration is given in Section VI. C.

IV. PROPOSED MODEL FOR DISCO

Compensations to EVAs and penalties for energy deviations of DISCO are influenced by uncertainties. To evaluate costs affected by uncertainties, robust optimization is used in some literature. However, it is often over-conservative because it is worst-case oriented and the worst case rarely happens [27],[28]. Instead of optimizing the costs under the worst uncertainty realization, it is more reasonable to pursue the lowest average costs. In this regard, some literature adopts stochastic optimization, which uses scenarios to represent the stochastic nature of uncertainties. But uncertainties could be poorly represented and thus sub-optimal solutions could be obtained if the number of used scenarios is small [29], and heavy computational burden will be caused if a large number of scenarios are adopted [30],[31]. In consideration of the

drawbacks of robust optimization and stochastic optimization, a more recently developed approach, namely distributionally robust optimization (DRO), is adopted in this paper to evaluate costs affected by uncertainties.

As available information about uncertainties is limited, the exact uncertainty distribution is hard to be acquired. But it is included in the family of distributions satisfying certain known information such as statistical moments. The set constituted by such family of distributions is called the ambiguity set [32],[33]. To utilize available uncertainty information and avoid being over-optimistic, DRO focuses on the worst distribution in the ambiguity set. Compared with robust optimization, DRO is capable of improving the average economic performance. Compared with stochastic optimization, DRO can properly take uncertainties into consideration without causing excessive computational burden. Further discussions about DRO will be made in Section V.B.

A. Formulation of the proposed model

The formulation of the proposed model is given in (4.1)-(4.19). As discussed in Section II, DISCO needs to purchase reserves from EVAs and energy imported from the transmission system in the day-ahead stage. The first and second items in the first line of (4.1) are energy costs and reserve costs, respectively. $E[\cdot]$ is the operator that calculates the expectation. Items within the operator $E[\cdot]$ in the second and third lines of (4.1) are costs of DISCO incurred in the real-time stage and are influenced by uncertainties. The largest expectation of their sum with respect to all distributions in the ambiguity set $A(\xi)$ is calculated through the max operator and considered in (4.1) to hedge against the ambiguity in the uncertainty distribution and avoid over-optimistic solutions [24]. It will be later transformed through DRO in Section V.B. The first item within the operator $E[\cdot]$ in (4.1) is the compensation for disturbance to EVAs, and its explicit expressions are given in (4.2)-(4.5). As discussed in Section II, (4.2)-(4.3) correspond to the case when disturbance to EVAs is within the purchased reserves and there are only regular compensations. (4.4)-(4.5) correspond to the case when disturbance to EVAs is beyond the purchased reserves and there are both regular and punitive compensations. The second item within the operator $E[\cdot]$ in (4.1) is the penalty for DISCO's energy deviations, and its explicit expressions are given in (4.6) and (4.7), which correspond to the cases when DISCO has positive and negative energy deviations, respectively. The last item within the operator $E[\cdot]$ in (4.1) is the compensation for EVA battery degradation caused by EVA discharging. To avoid excessive degradation of EVA battery and reduce operational challenges, the status of EVAs, i.e., charging or discharging, in an hour is required to be fixed whatever uncertainty realizations are, which is achieved through (4.9)-(4.11). Also, EVA charging and discharging power are limited in (4.10) and (4.11), respectively. (4.12) reflects power losses in EVA charging and discharging. Disturbance to EVAs is constrained within their acceptable ranges in (4.13). (4.14) implies that disturbance to EVAs incurred from shifting charging demands should be fully recovered in the end of the day. According to the linearized

power flow model for distribution networks in [34], power balances are ensured through (4.15)-(4.17), and node voltage is given in (4.18) and constrained in (4.19). Power losses on distribution networks are non-linear and much smaller than power flows [34]. Therefore, as done in [34] and [35], they are neglected in the proposed model, which greatly reduces the computational complexity and has very small influence on the result. To incorporate power losses on distribution networks, the piecewise-linearized power flow model from [36] can be adopted at the cost of increased computational complexity.

$$\begin{aligned} \min \quad & \sum_{t=1,\dots,T} a_e^{t,i} e_{\text{pur}}^t + \sum_{t=1,\dots,T} \sum_{i \in N_{\text{EVA}}} \left(a_{r,\text{def}}^{t,i} r_{\text{def}}^{t,i} + a_{r,\text{over}}^{t,i} r_{\text{over}}^{t,i} \right) \\ & + \max_{f_{\xi} \in \mathcal{A}(\xi)} \mathbb{E} \left[\sum_{t=1,\dots,T} \sum_{i \in N_{\text{EVA}}} \max_{k=1,2,3,4} f_k(e_{\text{dist}}^{t,i}, r_{\text{def}}^{t,i}, r_{\text{over}}^{t,i}) + \sum_{t=1,\dots,T} \max_{k=1,2} g_k(p_{\text{in}}^t, e_{\text{pur}}^t) \right. \\ & \quad \left. + \sum_{t=1,\dots,T} \sum_{i \in N_{\text{EVA}}} b_d^i p_{\text{dis}}^{t,i} \Delta t \right] \end{aligned} \quad (4.1)$$

s.t.

$$f_1(e_{\text{dist}}^{t,i}, r_{\text{def}}^{t,i}, r_{\text{over}}^{t,i}) = b_{r,\text{def}}^{t,i} e_{\text{dist}}^{t,i} \quad (4.2)$$

$$f_2(e_{\text{dist}}^{t,i}, r_{\text{def}}^{t,i}, r_{\text{over}}^{t,i}) = -b_{r,\text{over}}^{t,i} e_{\text{dist}}^{t,i} \quad (4.3)$$

$$f_3(e_{\text{dist}}^{t,i}, r_{\text{def}}^{t,i}, r_{\text{over}}^{t,i}) = b_{p,\text{def}}^{t,i} (e_{\text{dist}}^{t,i} - r_{\text{def}}^{t,i}) + b_{r,\text{def}}^{t,i} r_{\text{def}}^{t,i} \quad (4.4)$$

$$f_4(e_{\text{dist}}^{t,i}, r_{\text{def}}^{t,i}, r_{\text{over}}^{t,i}) = b_{p,\text{over}}^{t,i} (-e_{\text{dist}}^{t,i} - r_{\text{over}}^{t,i}) + b_{r,\text{over}}^{t,i} r_{\text{over}}^{t,i} \quad (4.5)$$

$$g_1(p_{\text{in}}^t, e_{\text{pur}}^t) = b_{p,\text{pos}}^t (p_{\text{in}}^t \Delta t - e_{\text{pur}}^t) \quad (4.6)$$

$$g_2(p_{\text{in}}^t, e_{\text{pur}}^t) = b_{p,\text{neg}}^t (e_{\text{pur}}^t - p_{\text{in}}^t \Delta t) \quad (4.7)$$

$$e_{\text{dist}}^{t,i} = \sum_{\bar{t}=1,\dots,t} \left((p_{\text{EVA},p}^{\bar{t},i} + \xi_{\text{EVA}}^{\bar{t},i} - p_{\text{ch}}^{\bar{t},i} + p_{\text{dis}}^{\bar{t},i}) \Delta t \right) \quad \forall \xi, \forall t, \forall i \in N_{\text{EVA}} \quad (4.8)$$

$$\sigma_{t,i} \in \{0,1\}, \forall t, \forall i \in N_{\text{EVA}} \quad (4.9)$$

$$0 \leq p_{\text{ch}}^{t,i} \leq \sigma_{t,i} p_{\text{ch},\text{max}}^i, \forall \xi, \forall t, \forall i \in N_{\text{EVA}} \quad (4.10)$$

$$0 \leq p_{\text{dis}}^{t,i} \leq (1 - \sigma_{t,i}) p_{\text{dis},\text{max}}^i, \forall \xi, \forall t, \forall i \in N_{\text{EVA}} \quad (4.11)$$

$$p_{\text{sys,EVA}}^{t,i} = p_{\text{ch}}^{t,i} / \eta_{\text{ch}}^i - p_{\text{dis}}^{t,i} \cdot \eta_{\text{dis}}^i, \forall \xi, \forall t, \forall i \in N_{\text{EVA}} \quad (4.12)$$

$$-e_{\text{over,max}}^{t,i} \leq e_{\text{dist}}^{t,i} \leq e_{\text{def,max}}^{t,i}, \forall \xi, \forall t, \forall i \in N_{\text{EVA}} \quad (4.13)$$

$$\sum_{t=1,\dots,T} (p_{\text{EVA},p}^{t,i} - p_{\text{ch}}^{t,i} + p_{\text{dis}}^{t,i}) = 0 \text{ if } \xi = \mathbf{0}, \forall i \in N_{\text{EVA}} \quad (4.14)$$

$$f_p^{t,N_p(i),i} = p_1^{t,i} + p_{\text{sys,EVA}}^{t,i} - p_{\text{RES},f}^{t,i} - \xi_{\text{RES}}^{t,i} + \sum_{j \in N_c(i)} f_p^{t,i,j}, \quad \forall \xi, \forall t, \forall i \in N_{\text{sys}} / \{1\} \quad (4.15)$$

$$f_q^{t,N_p(i),i} = q_1^{t,i} + p_{\text{sys,EVA}}^{t,i} \sqrt{1 - \lambda_i^2} / \lambda_i + \sum_{j \in N_c(i)} f_q^{t,i,j}, \quad \forall \xi, \forall t, \forall i \in N_{\text{sys}} / \{1\} \quad (4.16)$$

$$p_{\text{in}}^t = f_p^{t,1,2} \quad (4.17)$$

$$v_{t,i} + \left(r_{N_p(i),i} f_p^{t,N_p(i),i} + x_{N_p(i),i} f_q^{t,N_p(i),i} \right) / v_b = v_{t,N_p(i)}, \quad \forall \xi, \forall t, \forall i \in N_{\text{sys}} / \{1\} \quad (4.18)$$

$$0.95v_b \leq v_{t,i} \leq 1.05v_b, \forall \xi, \forall t, \forall i \in N_{\text{sys}} \quad (4.19)$$

B. Linear decision rules approximation

The proposed model is a multi-period problem involving uncertainties. For such problems, [36] and [37] adopt two-stage models, whose first and second stage are before and after realizations of all uncertainties, respectively. However, the two-stage models in [36] and [37] need to be solved through iterative processes, and non-convex bilinear problems need to be solved in every iteration. As a result, the computational burden of such two-stage models is very heavy. Besides, these two-stage models are problematic in assuming that uncertainties in all periods realize simultaneously. Actually, uncertainties in different periods realize gradually according to their temporal sequences. As shown in [38], such two-stage models could lead to operational infeasibility because of their ignorance of temporal sequences.

To take the temporal sequences into consideration and solve the proposed model within reasonable time, linear decision rules (LDR) approximation is adopted here by assuming that DISCO's real-time operation plans are affine functions of uncertainty realizations in earlier hours [39],[40]. Then, (2) can be rewritten as (5), where $\beta_{t,i}^{\bar{t},j}$ and $\gamma_{t,i}^{\bar{t},j}$ are uncertainty coefficients. As shown in (5), real-time operation plans in any hour only depend on earlier uncertainties but not later ones, which is in accordance of temporal sequences. After replacing real-time operation plans by corresponding affine functions of uncertainties under LDR, the proposed model becomes equivalent to a single-period problem in terms of mathematics and is much easier to solve.

$$p_{\text{ch}}^{t,i} = \alpha_{\text{ch}}^{t,i} + \sum_{\bar{t}=1,\dots,t} \left(\sum_{j \in N_{\text{EVA}}} \beta_{t,i}^{\bar{t},j} \xi_{\text{EVA}}^{\bar{t},j} + \sum_{j \in N_{\text{RES}}} \gamma_{t,i}^{\bar{t},j} \xi_{\text{RES}}^{\bar{t},j} \right) \quad (5)$$

With LDR, the variables that the proposed model needs to solve are day-ahead decisions including the energy portfolio of DISCO and the reserves that DISCO purchases from EVAs, and all constant components and uncertainty coefficients of LDR. Although uncertainty realizations are still unknown in the day-ahead stage, uncertainty coefficients of LDR have already determined how uncertainties will be mitigated in the real-time stage. In other words, using EVAs to mitigate uncertainties can be interpreted as allocating uncertainties to EVAs in advance through uncertainty coefficients of $p_{\text{ch}}^{t,i}$ and $p_{\text{dis}}^{t,i}$ under LDR.

LDR brings no errors when the optimal decisions depend affinely on uncertainties. When the relationship between the optimal decisions and uncertainties gets farther from affine functions, errors brought by LDR become larger. In this paper, uncertainties are mitigated by adjusting EVA charging and discharging power under LDR, which is similar to adjusting generator outputs to compensate uncertainties under LDR as done in [38-41] in terms of mathematics. It has been shown that the errors of LDR are acceptable in [38-41].

V. TRANSFORMATION OF THE PROPOSED MODEL

In this section, the proposed model is transformed into deterministic forms.

A. Robust constraints

The proposed model contains robust equality and inequality constraints, which are both linear under LDR. Robust linear equality constraints can be written in compact forms as (6), where \mathbf{h}' is the transpose of \mathbf{h} . They are satisfied for all possible uncertainty realizations if and only if $\mathbf{h} = \mathbf{0}$ and $g = 0$, and thus can be replaced by deterministic constraints on their constant components and uncertainty coefficients. Uncertainty realizations are assumed to lie in proper polyhedral sets. So, robust linear inequality constraints can be substituted by their deterministic linear counterparts according to robust optimization [42].

$$\mathbf{h}'\xi + g = 0 \quad (6)$$

B. Distributionally robust optimization

The ambiguity set for uncertainty distribution can be constructed differently using uncertainty expectations and variances as in [43], or expectations, mean absolute deviations and standard deviations as in [44]. As discussed in Section III.B, uncertainties from different hours may offset each other when EVAs are used to delay uncertainties, whose possibility depends on uncertainty correlation. Therefore, as shown in (7), the ambiguity set is constructed based on statistical uncertainty expectations and covariance matrix, which can be obtained from historical samples of uncertainties.

$$A(\xi) = \left\{ f_{\xi} \left[\begin{array}{l} \xi \in \mathbb{R}^{d_{\xi}} \\ \mathbb{E}[\xi] = \mu \\ \mathbb{E}[(\xi - \mu)(\xi - \mu)'] = \Sigma \end{array} \right] \right\} \quad (7)$$

The utility function within the operator $\mathbb{E}[\cdot]$ in (4.1) is the sum of several piecewise-linear functions and can be rewritten as a single piecewise-linear function. Then, the worst expectation calculated through the max operator in (4.1) can be transformed into deterministic forms according to DRO techniques in [45]. However, as the number of piecewise-linear functions within the operator $\mathbb{E}[\cdot]$ grows, the number of segments in the rewritten single piecewise-linear function increases exponentially, which would cause great computational difficulties. To solve the proposed model within a reasonable time, an upper bound (8) is used to approximate the original worst expectation in (4.1). [46] shows that errors of such approximation are acceptable. As the compensation for EVA battery degradation depends linearly on uncertainties under LDR, its expectation can be calculated directly from uncertainty expectations. Terms in the first line of (8) can be transformed into deterministic forms by using DRO techniques in [45]. After all transformation in Section V, the proposed model becomes a deterministic mixed-integer second-order conic program and can be solved by off-the-shelf solvers.

$$\sum_{t=1, \dots, T} \sum_{i \in N_{EVA}} \sup_{f_{\xi} \in A(\xi)} \left(\max_{k=1,2,3,4} f_k(e_{int}^{t,i}, r_{def}^{t,i}, r_{over}^{t,i}) \right) + \sum_{t=1, \dots, T} \sup_{f_{\xi} \in A(\xi)} \left(\max_{k=1,2} g_k(p_{in}^t, e_{pur}^t) \right) + \sum_{t=1, \dots, T} \sum_{i \in N_{EVA}} \mathbb{E} \left[b_d^i p_{dis}^{t,i} \Delta t \right] \quad (8)$$

VI. CASE STUDIES AND DISCUSSIONS

Case studies are conducted based on a modified IEEE 33-node system, in which Node 16 and 22 each has an EVA, and Node 13 and 30 each has an RES. Average charging and discharging efficiencies of both EVAs are set to 0.9. To evaluate the performance of obtained decisions, uncertainty realizations are generated according to normal distributions. In Section VI.A to VI.D, the time horizon is assumed to contain only 2 hours to demonstrate the proposed model more clearly. Then in Section VI.E, the time horizon is assumed to contain 24 hours.

In Section VI.A to VI.D, disturbance to EVAs is assumed to be completely recovered at the end of the time horizon, which means that EVAs are used to only delay but not eliminate uncertainties according to the definitions in Section III.B. Prices of reserves for over-charging and charging deficiency of both EVAs are set to $0.2\phi/\text{kWh}$. Regular compensation rates for over-charging and charging deficiency to both EVAs are set to $2\phi/\text{kWh}$. Punitive compensation rates for over-charging and charging deficiency to both EVAs are set to $6\phi/\text{kWh}$. Compensation rates for battery degradation to both EVAs are set to $0.05\phi/\text{kWh}$.

A. Using EVAs to delay uncertainties

In this part, energy prices in both hours are set to $4\phi/\text{kWh}$. Penalty coefficients for positive and negative energy deviations of DISCO are set to $10\phi/\text{kWh}$. As discussed in Section IV.B, using EVAs to mitigate uncertainties under LDR is equivalent to allocating uncertainties to EVAs in advance, which is illustrated here.

TABLE I
Uncertainty coefficients when EVA flexibility is not used

Uncertainty	Variables in the first hour			Variables in the second hour		
	$p_{\text{sys,EVA}}^{1,1}$	$p_{\text{sys,EVA}}^{1,2}$	p_{in}^1	$p_{\text{sys,EVA}}^{2,1}$	$p_{\text{sys,EVA}}^{2,2}$	p_{in}^2
$\xi_{\text{EVA}}^{1,1}$	1.111	0	1.111	0	0	0
$\xi_{\text{EVA}}^{1,2}$	0	1.111	1.111	0	0	0
$\xi_{\text{RES}}^{1,1}$	0	0	-1	0	0	0
$\xi_{\text{RES}}^{1,2}$	0	0	-1	0	0	0
$\xi_{\text{EVA}}^{2,1}$	0	0	0	1.111	0	1.111
$\xi_{\text{EVA}}^{2,2}$	0	0	0	0	1.111	1.111
$\xi_{\text{RES}}^{2,1}$	0	0	0	0	0	-1
$\xi_{\text{RES}}^{2,2}$	0	0	0	0	0	-1

TABLE II
Uncertainty coefficients when EVA flexibility is used

Uncertainty	Variables in the first hour			Variables in the second hour		
	$p_{\text{sys,EVA}}^{1,1}$	$p_{\text{sys,EVA}}^{1,2}$	p_{in}^1	$p_{\text{sys,EVA}}^{2,1}$	$p_{\text{sys,EVA}}^{2,2}$	p_{in}^2
$\xi_{\text{EVA}}^{1,1}$	0.600	-0.600	0	0.512	0.600	1.111
$\xi_{\text{EVA}}^{1,2}$	-0.512	0.512	0	0.512	0.600	1.111
$\xi_{\text{RES}}^{1,1}$	0.460	0.540	0	-0.460	-0.540	-1
$\xi_{\text{RES}}^{1,2}$	0.460	0.540	0	-0.460	-0.540	-1
$\xi_{\text{EVA}}^{2,1}$	0	0	0	1.111	0	1.111
$\xi_{\text{EVA}}^{2,2}$	0	0	0	0	1.111	1.111
$\xi_{\text{RES}}^{2,1}$	0	0	0	0	0	-1
$\xi_{\text{RES}}^{2,2}$	0	0	0	0	0	-1

According to the solution of the proposed model, the active power that the distribution system supplies for EVA 1 in the first hour is $p_{\text{sys,EVA}}^{1,1} = 222.222 + 1.111\xi_{\text{EVA}}^{1,1}$ when EVA

flexibility is not used. So, the coefficient of $p_{\text{sys,EVA}}^{1,1}$ for $\xi_{\text{EVA}}^{1,1}$ is 1.111 and the coefficients of $p_{\text{sys,EVA}}^{1,1}$ for other uncertainties are 0. For clearer illustration, uncertainty coefficients of certain decision variables when EVA flexibility is used and not are given in Table II and I, respectively. As shown in Table I, coefficients for EVA uncertainties are greater than 1 because of power losses in EVA charging. As EVA flexibility is not used, each variable in Table I depends and only depends on uncertainties in its hour. In contrast, when EVA flexibility is used, the active power imported from the transmission system in the first hour, i.e., p_{in}^1 , is not influenced by uncertainties as shown in Table II. In the second hour, because disturbance to EVAs incurred from uncertainty mitigation is recovered, variables depend on uncertainties from both hours. So, uncertainties in the first hour are delayed to the second hour.

Uncertainty-affected costs when EVA flexibility is used and not are recorded in Table III, where the total uncertainty-affected costs are the sum of reserve costs, compensations to EVAs and penalties for energy deviations minus the reduction in energy costs. As uncertainties from different hours can offset each other, average penalties for energy deviations of DISCO decrease when EVA flexibility is used. Meanwhile, corresponding reserve costs and compensations to EVAs are incurred. Overall, the average total uncertainty-affected costs are reduced because of EVA flexibility.

TABLE III
Uncertainty-affected costs when EVA flexibility is used and not

Using EVA flexibility	No	Yes
Reduction in energy costs brought by EVA flexibility (€)	0.0	0.0
Reserve costs (€)	0.0	8.6
Average compensations for disturbance to EVAs (€)	0.0	26.8
Average compensations for EVA battery degradation (€)	0.0	0.0
Average penalties for energy deviations (€)	258.5	182.5
Average total uncertainty-affected costs (€)	258.5	217.9

B. Trade-off between cost savings brought by using EVA flexibility and corresponding payments to EVAs

In Section VI.A, each uncertainty in the first hour is completely delayed to the second hour, which however is not always the case. In this part, case studies are conducted under varying penalty coefficients for energy deviations of DISCO. Energy prices are set to be the same as in Section VI.A. Under all considered penalty coefficients, each uncertainty in the first hour has the same percentage delayed to the second hour. Relevant results are recorded in Table IV. When penalty coefficients are large enough, uncertainties in the first hour are completely delayed to fully utilize EVA flexibility. But when penalty coefficients decrease, using EVAs to mitigate uncertainties becomes less attractive. As a result, EVA flexibility is used less extensively, and thus lower percentage of each uncertainty is delayed to the second hour, resulting in fewer reserve costs and average compensations for disturbance to EVAs. To make the idea of partially delaying uncertainties clearer, an illustration is given here. If the charging power of EVA i in the first hour is $p_{\text{ch}}^{1,i} = p_{\text{EVA,p}}^{1,i} + 0.5 \cdot \xi_{\text{EVA}}^{1,i}$, 50 percent of $\xi_{\text{EVA}}^{1,i}$ is delayed and the charging power of EVA i in the second hour will be $p_{\text{ch}}^{2,i} = p_{\text{EVA,p}}^{2,i} + 0.5 \cdot \xi_{\text{EVA}}^{1,i} + \xi_{\text{EVA}}^{2,i}$. In this

circumstance, $\xi_{\text{EVA}}^{1,i}$ can cause energy deviations of DISCO in both the first and second hour.

TABLE IV
Results under different penalty coefficients

$b_{p,\text{pos}}^1, b_{p,\text{neg}}^1, b_{p,\text{pos}}^2, b_{p,\text{neg}}^2$ (€/kWh)	Percentage of each uncertainty delayed to the second hour (%)	Reserve costs (€)	Average compensations for disturbance to EVAs (€)
11	100	8.6	26.8
10	100	8.6	26.8
9	100	8.6	26.8
8	97.5	8.4	26.2
7	86.7	7.5	23.3
6	74.5	6.4	20.0

C. Effects of power losses in EVA charging and discharging on uncertainties

Case studies are conducted in this part under three settings for energy prices as shown in Table V. Penalty coefficients are set to be the same as in Section VI.A. Under all three settings in Table V, each uncertainty in the first hour is completely delayed to the second hour. Uncertainty coefficients of p_{in}^2 are shown in Fig. 3. Average penalties for energy deviations of DISCO are given in Table VI.

TABLE V
Settings for energy prices

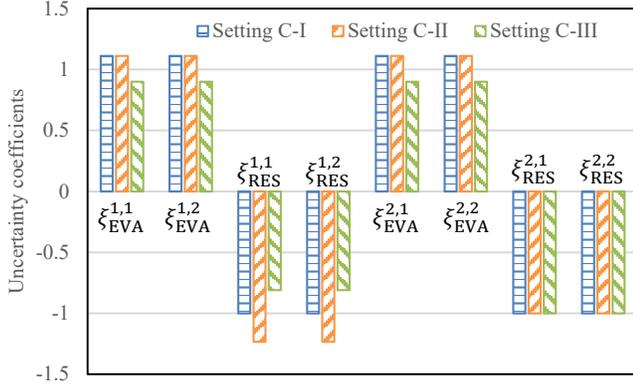
Setting	C-I	C-II	C-III
a_e^1 (€/kWh)	4	7	4
a_e^2 (€/kWh)	4	4	7

As average charging and discharging efficiencies of both EVAs have been set to 0.9, they are all represented by η in the following illustration. Uncertainty coefficients of p_{in}^2 under Setting C-I are the same with those in Table II and can be regarded as the reference for Setting C-II and C-III. Under Setting C-II, because of the significant difference in energy prices in the two hours, EVAs discharge in the first hour and charge in the second hour. To offset RES uncertainties, the total power that the distribution system receives from EVAs in the first hour, i.e., $-p_{\text{sys,EVA}}^{1,1} - p_{\text{sys,EVA}}^{1,2}$, contains $-(\xi_{\text{RES}}^{1,1} + \xi_{\text{RES}}^{1,2})$. Because of power losses in EVA discharging, the total EVA discharging power, i.e., $p_{\text{dis}}^{1,1} + p_{\text{dis}}^{1,2}$, contains $-(\xi_{\text{RES}}^{1,1} + \xi_{\text{RES}}^{1,2})/\eta$. Then, in the second hour, disturbance to EVAs is recovered. As a result, the total EVA charging power, i.e., $p_{\text{ch}}^{2,1} + p_{\text{ch}}^{2,2}$, contains $-(\xi_{\text{RES}}^{1,1} + \xi_{\text{RES}}^{1,2})/\eta$. Because of power losses in EVA charging, $p_{\text{sys,EVA}}^{2,1} + p_{\text{sys,EVA}}^{2,2}$ and p_{in}^2 contains $-(\xi_{\text{RES}}^{1,1} + \xi_{\text{RES}}^{1,2})/\eta^2$. As shown in Fig. 3, coefficients for $\xi_{\text{RES}}^{1,1}$ and $\xi_{\text{RES}}^{1,2}$ have greater absolute values under Setting C-II than under Setting C-I. So, under Setting C-II, these uncertainties are magnified from the perspective of DISCO, and average penalty for energy deviations is higher than that under Setting C-I as shown in Table VI. Under Setting C-III, EVAs charge in the first hour and discharge in the second hour. With similar analysis as for Setting C-II, it can be deduced that p_{in}^2 contains $-(\xi_{\text{RES}}^{1,1} + \xi_{\text{RES}}^{1,2})\eta^2$ and $(\xi_{\text{EVA}}^{1,1} + \xi_{\text{EVA}}^{1,2} + \xi_{\text{EVA}}^{2,1} + \xi_{\text{EVA}}^{2,2})\eta$ under Setting C-III, which matches the results in Fig. 3. So, these uncertainties are minified from the perspective of DISCO, and average penalty for energy deviations is now lower than under Setting C-I.

TABLE VI

Average penalties for DISCO's energy deviations under different settings

Setting	C-I	C-II	C-III
Average penalties for energy deviations (ϵ)	182.5	183.5	148.8

Fig. 3. Uncertainty coefficients of p_{in}^2

D. Interactions between mitigating uncertainties and shifting charging demands

In this part, energy price in the first hour is set to $4\epsilon/\text{kWh}$ and energy price in the second hour, i.e., a_e^2 , is set to varying values. Penalty coefficients are set to be the same as in Section VI.A. As shown in (5), real-time operation plans of DISCO are made up of constant components, that do not depend on uncertainty realizations, and linear functions of uncertainties under LDR. Fig. 4 records the constant components of the total EVA active power supplied by the distribution system, $\sum_{i \in N_{EVA}} p_{sys,EVA}^{t,i}$. Average penalties for energy deviations of DISCO are shown in Fig. 5.

When a_e^2 is $4\epsilon/\text{kWh}$, there is no shifted EVA charging demand, and each uncertainty in the first hour is completely delayed to the second hour. Therefore, there is no penalty for energy deviations in the first hour as shown in Fig. 5. When a_e^2 rises to $5\epsilon/\text{kWh}$, savings in energy costs brought by shifting EVA charging demands are fewer than corresponding payments to EVAs. But as discussed in the last paragraph of Section III.A, shifting EVA charging demands may alleviate the disturbance to EVAs incurred from uncertainty mitigation. For the sake of minimizing overall costs, there is a slight amount of EVA charging demands shifted from the second hour to the first hour as shown in Fig. 4. When a_e^2 equals to $6\epsilon/\text{kWh}$, the difference in energy prices in the two hours becomes large enough and shifted EVA charging demands greatly increase compared with earlier cases. As a_e^2 increases to $6.5\epsilon/\text{kWh}$, shifted EVA charging demands further grow. To guarantee that EVA status is fixed in the second hour, each uncertainty in the first hour is only partially delayed to the second hour. So, the average penalty in the first hour is now positive and the average total penalty in the two hours is higher than those under earlier cases as shown in Fig. 5.

When a_e^2 is $6.7\epsilon/\text{kWh}$, EVA discharging is uneconomical in terms of the sum of energy costs and payments to EVAs. However, EVAs still discharge in the second hour because uncertainties will then be minified as illustrated in Section VI.C and thus lower overall costs will be achieved. To avoid EVA status swinging between discharging and charging in the second

hour, each uncertainty in the first hour is only partially delayed to the second hour, which leads to positive average penalty in the first hour as shown in Fig. 5. When a_e^2 grows to $7\epsilon/\text{kWh}$, EVAs further discharge in the second hour and each uncertainty in the first hour is completely delayed to the second hour. Compared with the cases when EVAs charge in the second hour, the average total penalty is now lower because uncertainties are minified. When a_e^2 increases to $7.5\epsilon/\text{kWh}$, EVA discharging power in the second hour significantly increases and is bounded by EVAs' acceptable amounts for over-charging in the first hour. As a_e^2 keeps increasing to 8 and $8.5\epsilon/\text{kWh}$, each uncertainty in the first hour is only partially delayed to the second hour to have more EVA charging demands shifted from the second hour to the first hour, which causes growth in penalties but achieves optimal overall costs because of savings in energy costs.

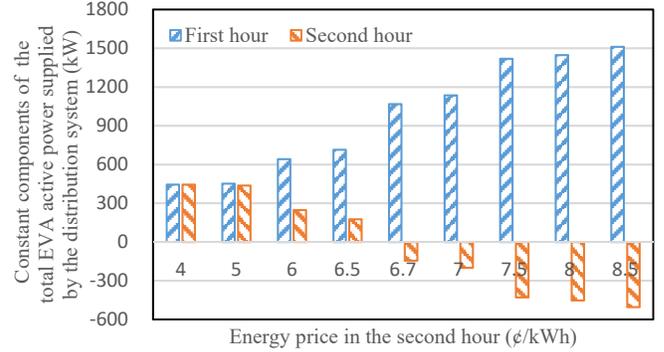


Fig. 4. Constant components of the total EVA active power supplied by the distribution system under varying energy price in the second hour

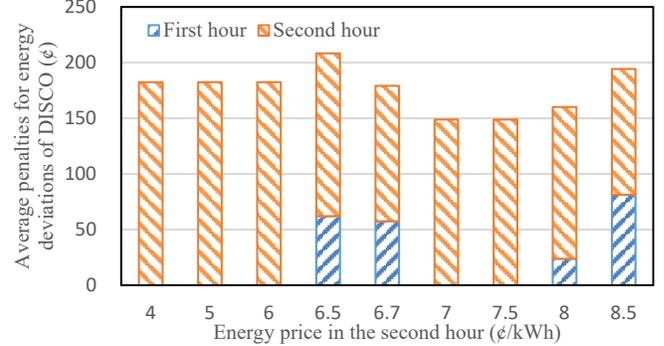


Fig. 5. Average penalties for energy deviations of DISCO under varying energy price in the second hour

E. Delaying uncertainties and eliminating uncertainties

In this part, the time horizon contains 24 hours. Prices of reserves for EVA over-charging and charging deficiency are set to 0.1 time of energy prices. Regular compensation rates for EVA over-charging and charging deficiency are set to be equal to energy prices. Punitive compensations rates for EVA over-charging and charging deficiency are set to 3 times of energy prices. Compensation rates for EVA battery degradation are set to 0.05 time of energy prices. Penalty coefficients for energy deviations of DISCO are set to 3 times of energy prices. As compensations to EVAs will keep increasing as time goes if disturbance to EVAs is not recovered, it is assumed here that disturbance to EVAs incurred from uncertainty mitigation needs to be recovered in 5 hours unless the time horizon ends. Such restriction has tiny influence on the results but can greatly

reduce the computational complexity. Case studies are conducted under three settings as given in Table VII. Average penalties for energy deviations of DISCO are shown in Fig. 6.

The curve of Setting E-I reflects the scale of uncertainties in each hour as EVAs are not used to mitigate uncertainties. Under Setting E-II, average penalties are generally lower than those under Setting E-I, which is because uncertainties are delayed through EVAs and thus can offset uncertainties in later hours. Under Setting E-III, EVAs eliminate or partially eliminate uncertainties in the last several hours. As a result, average penalties in these hours are lower than those under Setting E-II. It should be noted that eliminating uncertainties is not applied on uncertainties in the earlier hours of the day because otherwise the corresponding payments to EVAs will be higher than corresponding savings in penalties for energy deviations. While, for uncertainties in the last several hours, eliminating uncertainties can be more profitable than delaying uncertainties because there are no corresponding penalties for energy deviations.

Uncertainty-affected costs under Setting E-I to E-III are recorded in Table VIII. As shown by the results under Setting E-I and E-II, using EVAs to delay uncertainties brings significant reduction in average penalties and thus achieves lower average total uncertainty-affected costs, which are consistent with the results in Section VI.A. By comparing the results under Setting E-II and E-III, it can be further noticed that having EVAs eliminate uncertainties creates extra savings based on those achieved by using EVAs to delay uncertainties.

TABLE VII
Three settings for uncertainty mitigation

Setting	E-I	E-II	E-III
Using EVAs to delay uncertainties	No	Yes	Yes
Using EVAs to eliminate uncertainties	No	No	Yes

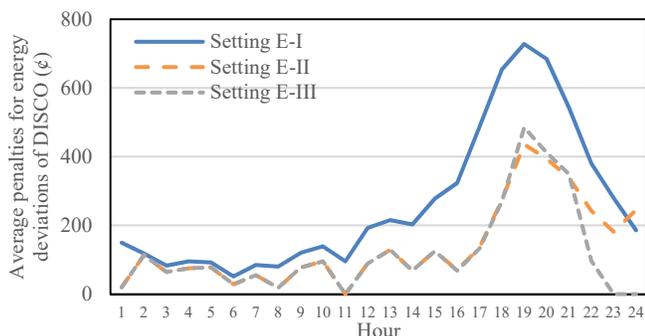


Fig. 6. Average penalties for energy deviations of DISCO under different settings

TABLE VIII
Uncertainty-affected costs under different settings

Setting	E-I	E-II	E-III
Reduction in energy costs brought by EVA flexibility (€)	0.0	5.4	6.7
Reserve costs (€)	0.0	562.1	642.6
Average compensations for disturbance to EVAs (€)	0.0	1369.6	1629.3
Average compensations for EVA battery degradation (€)	0.0	0.0	0.0
Average penalties for energy deviations of DISCO (€)	6265.6	3356.8	2853.2
Average total uncertainty-affected costs (€)	6265.6	5283.2	5118.4

F. Computational efficiency

To comprehensively show the computational efficiency of the proposed model, its computation times under 3 settings are recorded in Table IX. Setting F-I is the same with Setting E-III in Section VI.E. Under Setting F-I, EVAs charge in all hours, which means that all binary variables take the value of 1. Setting F-II is based on Setting F-I but sets the energy price in the 15th hour to 3 times of the original value. Because of the high energy price, EVAs discharge in the 15th hour, and binary variables corresponding to the 15th hour take the value of 0 under Setting F-II. Setting F-III is based on Setting F-I but sets the energy prices in both the 15th and 21st hour to 3 times of the original values. EVAs discharge in both the 15th and 21st hour under Setting F-III. Computation times recorded in this section are obtained by using a desktop with Intel Core i5-9400 CPU. According to Table IX, the computational efficiency of the proposed model is acceptable.

To illustrate the advantage of DRO over stochastic optimization in terms of computational efficiency, case studies are conducted by adopting stochastic optimization rather than DRO in the modified proposed model. The average value of the objective under all considered scenarios is minimized in the modified proposed model, which is a mixed-integer linear program. Table X shows the computation times of the modified proposed model under Setting F-III when different numbers of scenarios are considered. By comparing the results in Table IX and X, it can be noticed that the modified proposed model (using stochastic optimization) already has much longer computation time than the proposed model (using DRO) when it considers 100 scenarios. Uncertainties considered here are from charging demands of two EVAs and outputs of two RESs in 24 hours and thus are 96-dimensional. Obviously, 100 scenarios are not sufficient to properly represent the stochastic nature of the considered uncertainties. But if more scenarios are incorporated, the computational burden brought by stochastic optimization will become even heavier. In contrast, DRO can use as many historical samples of uncertainties as possible to derive more accurate statistical uncertainty moments and thus improve its accuracy without increasing the computational complexity.

TABLE IX

Computation times of the proposed model under different settings

Setting	F-I	F-II	F-III
Computation time (s)	46	393	602

TABLE X

Computation times of the modified proposed model that uses stochastic optimization under Setting F-III when different numbers of scenarios are considered

Number of scenarios	10	100	1000
Computation time (s)	578	2145	29042

VII. CONCLUSION

With the proposed model, flexibility of electric vehicle aggregators (EVAs) is explored by the distribution company (DISCO) to shift EVA charging demands to hours with lower energy prices and mitigate uncertainties in EVA charging demands and renewable power outputs. If the corresponding disturbance to EVAs is recovered within the time horizon, using

EVAs to mitigate uncertainties is equivalent to delaying uncertainties; otherwise, it is equivalent to eliminating uncertainties. It has been found from the case studies that 1) the proposed model is successful in simultaneously utilizing both forms of uncertainty mitigation to reduce average penalties for deviations of DISCO from its planned energy portfolio, 2) the proposed model is effective in coordinating EVA applications in mitigating uncertainties and shifting charging demands to achieve optimal overall costs, and 3) power losses in EVA charging and discharging are used in the proposed model to reduce the scale of uncertainties under certain circumstances.

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