

Transmission of Information by Orthogonal Functions

Henning F. Harmuth

With 110 Figures



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To my Teacher

Eugen Skudrzyk

Preface

The orthogonality of functions has been exploited in communications since its very beginning. Conscious and extensive use was made of it by KOTEL'NIKOV in theoretical work in 1947. Ten years later a considerable number of people were working in this field rather independently. However, little experimental use could be made of the theoretical results before the arrival of solid state operational amplifiers and integrated circuits.

A theory of communication based on orthogonal functions could have been published many years ago. However, the only useful examples of orthogonal functions at that time were sine-cosine functions and block pulses, and this made the theory appear to be a complicated way to derive known results. It was again the advance of semiconductor technology that produced the first really new, useful example of orthogonal functions: the little-known Walsh functions. In this book emphasis is placed on the Walsh functions, since ample literature is available on sine-cosine functions as well as on block pulses and pulses derived from them.

There are two major reasons why so few orthogonal functions are of practical interest in communications. First, a number of mathematical features other than orthogonality are required, such as completeness or 'good' multiplication and shift theorems. One quickly learns to appreciate the usefulness of multiplication and shift theorems of sine-cosine functions for multiplexing and mobile radio transmission, whenever one tries to duplicate these applications

by other functions. The second reason is that the functions must be easy to produce. The severity of this second requirement is readily comprehended if one tries to think of systems of functions of which a million or more can be actually produced.

Prior to 1960 it was mainly the orthogonality feature that attracted attention in connection with the transmission of digital signals in the presence of noise. But sooner or later the question had to be raised of why the orthogonal system of sine and cosine functions should be treated differently from other systems of orthogonal functions. This question led to the generalization of the concept of frequency and of such concepts derived from it as frequency power spectrum or frequency response of attenuation and phase shift. The Walsh functions made it possible to design practical filters and multiplex equipment based on this generalization of frequency.

Any theory in engineering must offer not only some new understanding, but must lead to new equipment and this equipment must be economically competitive. A considerable variety of equipment using orthogonal functions has been developed, but there is still much controversy about the economic potential. This is due to some extent to problems of compatibility, which always tend to favor previously introduced equipment and methods. In the particular case of Walsh functions, the economic competitiveness is intimately connected to the state of the art in binary digital circuits. It is, e.g., difficult to see why Walsh functions should not be as important for digital filters as sine-cosine functions are for linear, time-invariant networks.

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January 1969

Henning F. Harmuth

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