Dual-objective optimization for lane reservation with residual capacity and budget constraints

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Abstract—**With the increase of transport demands, more pressure and challenges are being imparted into efficient transportation. As a conventional and direct congestion alleviation strategy, constructing new roads and lanes are increasingly restricted by limited land resources and high costs. Thus, making full use of existing transport network via appropriate management is critical to realize the sustainable development of transportation systems. As a flexible management strategy, lane reservation strategy has been widely adopted in real life. The reserved lanes can improve the efficiency of special transports, while they bring negative impact such as travel delay for general-purpose transports. In addition, the setting and operating of reserved lanes require a certain amount of cost. This paper proposes a new dual-objective integer linear programming model for optimally determining reserved lanes on a network for time-guaranteed special transports in order to simultaneously maximize the benefits and minimize the negative impact brought by reserved lanes, which incorporates road residual capacity and limited budget to the actual decision. Moreover, an iterative weighted sum-based method is proposed to solve it, in which a new relax-and-optimize algorithm is developed to exactly solve the single-objective optimization problems. Results of extensive numerical experiments show the effectiveness and efficiency of the proposed model and approach.**

Index Terms—**Lane reservation (LR), modeling and simulation, multiobjective optimization, relax-and-optimize, transportation.**

I. Introduction

EFFICIENT transportation constitutes a crucial component in the sustainable growth of economy, as it supports the movement of personnel and goods. Consequently, improving transportation efficiency has drawn a lot of attention from both researchers and practitioners. Over the past 30 years, a number of transportation management problems have been extensively investigated, such as vehicle routing problem [1]–[3], location routing problem [4]–[6], and vehicle scheduling problem [7]–[9]. However, due to the rapid growth of travel demands, traffic congestion has become one of the major issues faced by many cities around the world, which leads to many additional mobility-related problems, such as inefficient human's daily travel and cargo transportation, traffic accidents, environmental pollution, and energy waste. Besides, special transport needs bring new challenges to traffic managers. For instance, during large-scale sport events the athletes are usually required to be delivered from the athlete village(s) to geographically dispersed stadiums within strict travel deadlines. One example is the Guangzhou Asian Games in 2010, whose organizing committee was required to ship athletes from athlete villages to any stadium within half an hour. However, such special transport demand cannot be easily achieved due to the congested traffic of host city without the development of effective approaches.

The conventional and direct way to congestion alleviation is to expand the transportation network capacity by constructing new road infrastructures. However, the growth rate of network capacity may not meet the continuously growing travel demands. Moreover, constructing new traffic infrastructures is restricted by long duration, high expenditure, and limited geographical space. To make full use of existing transport network via appropriate traffic management becomes increasingly important and critical to realizing the sustainable transportation systems. Recently, as an important traffic management strategy, lane reservation (LR) strategy has been widely used in real life. Its basic concept is to convert part of the lanes on an existing network into reserved ones during certain time periods for the exclusive use of specific transports. With reserved lanes, the travel speeds of special transports would be increased such that the above-mentioned time-guaranteed transportation tasks can be achieved. Actually, the LR strategy has been implemented during the Sydney, Athens, and Beijing Olympic Games in 2000, 2004, and 2008, respectively. Another representative application is the exclusive bus lane, which has been widely used to improve bus transit efficiency.

Clearly, the LR strategy would provide a relatively smooth transport environment for special transports. However, the general-purpose transports on adjacent nonreserved lanes may

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become more congested due to reserved lanes and travel delay may be caused. It is understandable that there is a clear trade-off between the two aspects, since greater positive benefits, for example, the greater reduction in total transport time for the special transport tasks, usually requires to reserve more lanes, while it will result in a larger negative impact to general-purpose transports, and vice versa. Decision-makers may desire to achieve a reasonable balance between the two aspects. Therefore, it is indispensable to carefully reserve lanes so as to maximize their benefits and minimize their negative impact. Moreover, it is necessary to consider the road residual capacity for special transports, because if the residual capacity on a road segment is not enough and this road must be passed by special transports, then the road segment has to be reserved. In addition, the setting and operating of reserved lanes requires a certain amount of cost [10], the budget should also be taken into account.

To address the above-mentioned issues, this paper proposes a new dual-objective integer programming (IP) model to maximize the benefits and minimize the total negative impact brought by reserved lanes, while simultaneously satisfying the limited total budget, time-guaranteed requirements, and road residual capacity constraints. An iterative weighted sum-based method is proposed to solve it. In particular, an efficient exact relax-and-optimize algorithm is developed to solve the corresponding single-objective optimization problems. Numerical experiments are conducted to validate the effectiveness and efficiency of the proposed model and algorithm.

The remainder of this paper is organized as follows. Section II presents the literature review. In Section III, we formally describe the problem and formulated it as a dual-objective IP model. In Section IV, an iterative weighted sum-based method is developed for solving the proposed model, in which a relax-and-optimize algorithm is proposed to exactly solve the corresponding single-objective optimization problems. Numerical experiments are performed in Section V. The conclusion is presented in the last section, along with the discussion for future research.

II. Literature Review

In the literature, transportation planning and management problems, such as vehicle routing, location routing, and vehicle scheduling, have received much attention from researchers. However, limited attention has been paid on optimal lane reservation, although it is one of the important problems in transportation management. Ravi *et al.* [11] proposed an LR system for highways, through which drivers paying certain fees could enjoy congestion-free travel. Wu *et al.* [12] first investigated an LR problem (LRP) motivated by the time-guaranteed transportation requirements arising in the Guangzhou Asian Games in 2010, for which they developed an IP model to minimize the LR negative impact that is considered as the increased time for general-purpose transports. A constructive heuristic algorithm was proposed to obtain a satisfactory solution of a real case involving 22 tasks in a network with 22 nodes. Later, Che *et al.* [13] proved that the LRP addressed in [12] is NP-hard and developed an improved quantum-inspired evolutionary algorithm to solve the problem in large-scale network environments. Instances with up to 50 tasks in the network with 500 nodes were solved.

The LRP proposed in [12] was significantly expanded and a number of variants were investigated. Zhou *et al.* [14] addressed an LRP for hazardous material transportation to minimize both the total transportation risk and negative impact. An ε-constraint and fuzzy-logic-based method was developed for it. Fang *et al.* [15] generalized the LRP in [12] to a capacitated LRP (CLRP) with the residual capacities of non-reserved road segments. Later, Fang *et al.* [16] developed an exact cut-and-solve (CS)-based method for the CLRP. Instances with up to 120 nodes in the network and 40 tasks were solved. Fang *et al.* [17] examined an LRP for automated truck freight transportation, where a fleet of trucks should be operated on dedicated truck routes, denoted by ATP, and proved that the problem is NP-hard. Wu *et al.* [18] developed an efficient two-phase algorithm to solve the ATP to optimality. By taking the link travel time variation into account, Fang *et al.* [19] studied a time-dependent LRP and developed a mixed-integer programming (MIP) model for it. Wu *et al.* [20] investigated a robust LRP by considering the LR robustness and built a bi-objective MIP model for it. Recently, different from the above works, Bai *et al.* [21] incorporated the environmental impact into the optimal LR decision and presented an IP model to minimize total carbon emission index. Besides, Mesbah *et al.* [22] developed a bi-level programming model for exclusive bus lane selection to minimize the total user cost. The model was solved by a genetic algorithm. Miandoabchi *et al.* [23] investigated a road network design problem in which lane reservation is considered as one of the decision variables. The objective is to maximize the consumer surplus and demand share of bus routes. Hybrid meta-heuristics were suggested to solve the model. Khoo *et al.* [24] presented a bi-objective optimization model for bus lane reservation and scheduling problem with the objective of minimizing the travel time of bus and nonbus traffic. A nondominated genetic algorithm was suggested for solving the model. Wu *et al.* [25] addressed an LRP for station arrival-time bus transit and presented an IP model to minimize the total negative impact of reserved lanes.

To the best of our knowledge, this paper differs from the existing works in two perspectives. First, this paper considers maximizing the benefits and minimizing the negative impact generated by the LR decision simultaneously for the first time such that a balance between the two objectives can be studied explicitly. Second, the addressed LRP takes limited budget, time-guaranteed requirements, and road residual capacity into account together, thereby leading to a more general LRP. As such, the existing methods proposed for the LRPs in the literature cannot be directly applied to it. This paper further enriches the lane reservation optimization literature. The main contributions brought by this paper include the following perspectives.

1) A new dual-objective CLRP (DCLRP) is studied and formulated as an IP model, which is further improved by eliminating redundant constraints and adding valid

inequalities. Besides, the DCLRP is shown to be NP-hard.

2) To solve the proposed model, an effective and efficient iterative weighted sum-based method is proposed based on its characteristics. Different from the traditional weighted sum method in which the corresponding single-objective optimization problems are exactly solved by using a commercial solver such as CPLEX, this paper develops a new relax-and-optimize algorithm to solve them more efficiently. Moreover, the proposed relax-and-optimize algorithm is adapted to exactly solve the standard CLRP. Computational results on 330 benchmark instances under various settings confirm the effectiveness and efficiency of the proposed approaches for solving the DCLRP and the standard CLRP comparing with the existing algorithms.

III. Development of the Optimization Model

A. Problem Description and Assumptions

The DCLRP addressed in this paper can be defined on a transportation network which can be represented by a directed graph $G = \{N, A\}$. N and A denote the sets of nodes and arcs, respectively. A node and an arc can be viewed as a road intersection and a road segment, respectively. A set of origin-destination (OD) pairs, denoted by $K, k \in K$, with each one corresponding to a transportation task is given in the network.

The DCLRP consists in optimally deciding which road segments on the transport network to be reserved and design a time-efficient route for each task subject to the road residual capacity and limited budget constraints. The objectives are to maximize the benefits of reserved lanes as well as to minimize the total negative impact due to the lane reservation.

To better define and formulate the problem, this paper makes the basic assumptions. First, at most one lane is allowed to be reserved on a road segment. Second, the capacity of a reserved lane is assumed to be enough and reserved lanes can be shared by the tasks. Third, each non reserved road segment has a residual capacity for the special transports and the budget for reserving lanes is limited. Finally, the travel speed of task vehicles can be increased on reserved lanes.

B. Notations

The indices and sets, parameters, and decision variables used for the formulation are summarized as follows.

1) Indices and Sets:

i, j	Nodes.
N	Set of nodes.
$\lvert N \rvert$	Number of nodes.
(i, j)	Arcs.
A	Set of arcs.
$\lvert A \rvert$	Number of arcs.
k	Tasks.
K	Set of tasks.
$\lvert K \rvert$	Number of tasks.
O	Set of origin nodes of tasks.
D	Set of destination nodes of tasks.
o_k	Origin node of task k.
d_k	Destination node of task k.

2) Parameters:

Bdg	Budget for setup and operating reserved lanes.
T_k	Expected transport time for completing task k.
f_k	Number of vehicles per unit time for task k.
cap_{ij}	Residual capacity on a nonreserved lane on arc (i, j).
t_{ij}	Task vehicle travel time on a reserved lane of arc (i, j).
t'_{ij}	Task vehicle travel time on arc (i, j) without reserved lanes.
l_{ij}	Distance of arc (i, j).
fc_{ij}	Reserved lane setup cost on arc (i, j).
vc_{ij}	Reserved lane operation cost on arc (i, j).
td_{ij}	Negative impact for nontask vehicles resulted by a reserved lane on arc (i, j).

3) Decision Variables:

z_{ij}	Binary variable, equals to one if a lane is reserved on arc $(i, j) \in A$ for special transports, otherwise, equals to zero.
x_{kij}	Binary variable, equals to one if task $k \in K$ passes a reserved lane on arc $(i, j) \in A$, otherwise, equals to zero.
y_{kij}	Binary variable, equals to one if task $k \in K$ passes a nonreserved lane on arc $(i, j) \in A$, otherwise, equals to zero.

C. Basic Model

With the notations defined above, the DCLRP is formulated as a binary IP model

$$(\mathcal{P}_0)\ \min\ f_1 = \sum_{k \in K} \sum_{(i,j) \in A} f_k (t_{ij} x_{kij} + t'_{ij} y_{kij}) \tag{1}$$

$$\min\ f_2 = \sum_{(i,j) \in A} td_{ij} z_{ij} \tag{2}$$

$$\text{s.t.}\ \sum_{(o_k,j) \in A} \left(x_{k o_k j} + y_{k o_k j} \right) = 1\ \forall k \in K \tag{3}$$

$$\sum_{(i,d_k) \in A} \left(x_{k i d_k} + y_{k i d_k} \right) = 1\ \forall k \in K \tag{4}$$

$$\sum_{(i,j) \in A} \left(x_{kij} + y_{kij} \right) = \sum_{(j,i) \in A} \left(x_{kji} + y_{kji} \right)\quad \forall i \in N \backslash \{o_k, d_k\}\ \forall k \in K \tag{5}$$

$$\sum_{(i,j) \in A} (fc_{ij} + vc_{ij} l_{ij}) z_{ij} \le \mathrm{Bdg} \tag{6}$$

$$z_{ij} \ge \sum_{k \in K} x_{kij} / |K|\ \forall (i, j) \in A \tag{7}$$

$$y_{kij} + z_{ij} \le 1\ \forall k \in K\ \forall (i, j) \in A \tag{8}$$

$$\sum_{(i,j) \in A} (t_{ij} x_{kij} + t'_{ij} y_{kij}) \le T_k\ \forall k \in K \tag{9}$$

$$\sum_{k \in K} f_k y_{kij} \le \mathrm{cap}_{ij} (1 - z_{ij})\ \forall (i, j) \in A \tag{10}$$

$$z_{ij} \in \{0, 1\}\ \forall (i, j) \in A \tag{11}$$

$$x_{kij} \in \{0, 1\}\ \forall k \in K\ \forall (i, j) \in A \tag{12}$$

$$y_{kij} \in \{0, 1\}\ \forall k \in K\ \forall (i, j) \in A. \tag{13}$$

Objective (1) is to minimize the total transport time of task vehicles per unit time, with which the total positive benefit brought by reserved lanes for the special transports is maximized, and objective (2) is to minimize the total negative impact due to reserved lanes. Note that the negative impact is considered as the increased time of general-purpose transports due to LR, as is the case in most existing studies. For more details on the estimation of negative impact, we refer readers to the related works [12], [15].

Constraints (3)–(5) are to design a feasible path for each task, where constraint (3) [resp. (4)] ensures that an arc outgoes from (resp. comes into) the origin (resp. destination) of task k and constraint (5) is the flow balance constraint. Constraint (6) represents that the total cost of reserving lanes should not exceed the given budget. Constraint (7) represents that the path of any task can pass a reserved lane on arc (i,j) (i.e., $x_{kij} = 1$) only if this arc is reserved (i.e., $z_{ij} = 1$). Note that constraint (7) can also be formulated as $z_{ij} \geq x_{kij}\ \forall k \in K, (i,j) \in A$ involving $|K||A|$ inequalities, while constraint (7) in its current form contains only $|A|$ inequalities. Constraint (8) states that task k passes a nonreserved lane on arc (i,j) only when this arc is not reserved (i.e., $z_{ij} = 0$). Constraint (9) ensures that the total travel time of any task should not exceed its expected completing time. Constraint (10) ensures that the number of task vehicles per unit time cannot exceed the residual capacity of an arc without reserved lanes. Constraints (11)–(13) are restrictions on decision variables. The following theorem gives the complexity of the DCLRP.

Lemma 1: LRP in [12] and CLRP in [15] are two special cases of DCLRP.

Proof: When the total budget is large enough, i.e., $\mathrm{Bdg} = +\infty$ such that constraint (6) can be relaxed and only f_2 is to be optimized, a DCLRP turns to be a CLRP. It is an LRP if the residual capacity of each road segment is large enough, i.e., $\mathrm{cap}_{ij} = +\infty\ \forall (i,j) \in A$, such that constraint (9) can be relaxed. ■

Theorem 1: The DCLRP is NP-hard.

Proof: LRP and CLRP have been shown to be NP-hard [13], [15] and they are special cases of DCLRP. Consequently, the DCLRP is also NP-hard. ■

D. Further Improvement of the Basic Model

As mentioned above, constraint (8) implies that task k passes a nonreserved lane on arc (i,j) when not reserved, which was formulated for the CLRP [15]. In the following, we show that such constraint is unnecessary for the DCLRP and can be removed. Let us consider a model $\mathcal{P}_0'$ formed by $\mathcal{P}_0$ removing constraint (8) and then we analyze the property of its Pareto optimal solution.

Theorem 2: In a Pareto optimal solution of $\mathcal{P}_0'$, for any arc $(i,j) \in A$, if this arc is reserved, then any task $k \in K$ passing this arc will always pass the corresponding reserved lane, i.e., $y_{kij} + z_{ij} \leq 1$ always holds.

Proof: We prove the theorem by contradiction. Suppose that there exist a Pareto optimal solution such that $y_{kij} + z_{ij} > 1$, i.e., $z_{ij} = 1$ and $y_{kij} = 1$. The fact that $z_{ij} = 1$ means that there is a reserved lane on arc (i,j). On the other hand, $y_{kij} = 1$ means that task k passes a nonreserved lane on arc (i,j). For this reason, we can derive a solution with lower total task completion time due to the fact that $t_{ij} < t_{ij}'$. This contradicts the optimality of the solution. ■

Corollary 1: For any Pareto optimal solution to model $\mathcal{P}_0'$, if $z_{ij} = 0$, then $y_{kij} \leq 1$ and if $z_{ij} = 1$, then $y_{kij} = 0\ \forall (i,j) \in A \forall k \in K$. That is to say, constraint (8) can be relaxed in model $\mathcal{P}_0$.

With Corollary 1, $|K||A|$ constraints in total can be removed from $\mathcal{P}_0$. Take the largest scale instance in [15] as an example, 17 820 constraints can be eliminated. This obviously contributes to reducing the resolution time of $\mathcal{P}_0$. Moreover, it is known that an IP model can often be tightened by adding valid inequalities, as it is usually solved by a branch-and-cut algorithm and valid inequalities can help generate tighter lower bounds in its search. Note that these valid inequalities would not exclude optimal solutions. To further tighten the model, the following valid inequalities are added:

$$\sum_{(i,o_k)\in A} \left(x_{kio_k} + y_{kio_k}\right) = 0\ \forall k \in K \tag{14}$$

$$\sum_{(d_k,j)\in A} \left(x_{kd_kj} + y_{kd_kj}\right) = 0\ \forall k \in K \tag{15}$$

$$\sum_{(i,j)\in A} \left(x_{kij} + y_{kij}\right) \leq 1\ \forall i \in N\backslash\{o_k, d_k\}\ \forall k \in K \tag{16}$$

$$\sum_{(j,i)\in A} \left(x_{kji} + y_{kji}\right) \leq 1\ \forall i \in N\backslash\{o_k, d_k\}\ \forall k \in K \tag{17}$$

where constraints (14) and (15) specify that there are no arcs coming into origin nodes and outgoing from destination nodes, respectively. Inequalities (16) and (17) ensure that any node can be passed by any task $k \in K$ at most once. Then, an improved model is derived as follows:

$$(\mathcal{P}_1)\ \ \min\ f_1 = \sum_{k\in K}\sum_{(i,j)\in A} f_k\Big(t_{ij}x_{kij} + t_{ij}'y_{kij}\Big)$$

$$\min\ f_2 = \sum_{(i,j)\in A} td_{ij}z_{ij}$$

$$\text{s.t.}\ \ (3)-(7)\text{ and }(9)-(17).$$

IV. Solution Approach

In this section, an iterative weighted sum-based method is proposed to solve model $\mathcal{P}_1$. For the sake of convenience, let S denotes the feasible solution set of model $\mathcal{P}$, and $s \in S$. Before proceeding, we first give some basic concepts related to dual-objective optimization as follows [20], [26].

Definition 1: For two solutions $s_1 \in S$ and $s_2 \in S$, we say s_1 dominates s_2, denoted by $s_1 \succ s_2$, if $f_1(s_1) < f_1(s_2)$ and $f_2(s_1) \leq f_2(s_2)$ hold or $f_1(s_1) \leq f_1(s_2)$ and $f_2(s_1) < f_2(s_2)$ hold.

Definition 2: A solution $s^* \in S$ is called a nondominated solution if no other solution $s \in S$ dominates s^*.

Definition 3: All nondominated points in the objective space is called the Pareto front.

A. Objective Function Conversion

When dealing with a dual-objective optimization model, it is usually converted to a single-objective one by using the weighted sum [27]. For the proposed model, to derive a set of nondominated solutions, its two objectives are converted to one. To this aim, we define a coefficient (weight) λ with its value being between 0 and 1 to combine the two objectives. In managerial insight, λ could represent the weight of relative importance between the two objectives. Then, the aggregated objective is written as

$$\min \quad f = \lambda \cdot f_1 + (1 - \lambda) \cdot f_2. \tag{18}$$

In objective (18), the coefficient (weight) λ could be set as 0, 0.1, 0.2, ..., 0.9, 1, i.e., its varying step is set 0.1. This step could be set according to the scale of the investigated problem. By changing the value of λ, different solutions can be obtained. However, note that different objectives in dual-objective models may have different units. For instance, the units of f_1 and f_2 in the proposed model are different. To solve this problem, we first nondimensionalize the two objective functions before the objective conversion. Then, we use the following nondimensionalization method to deal with the considered objective functions:

$$f_i'(s) = \frac{f_i(s) - f_i^{\mathrm{LB}}}{f_i^{\mathrm{UB}} - f_i^{\mathrm{LB}}}, i = 1, 2 \tag{19}$$

where $f_i'(s), i = 1, 2$ represents the objective function after the nondimensionalization; and f_i^{LB} and f_i^{UB} denote the lower and upper bounds of the ith objective function value, respectively. To better fix the lower and upper bounds for both objective function values, we first give the definitions of ideal and Nadir points depicting the precise area of the whole Pareto front as follows [20], [26].

Definition 4: The ideal point of the DCLRP addressed is defined as (f_1^I, f_2^I), where $f_1^I = \min_{s\in S}(f_1(s))$ and $f_2^I = \min_{s\in S}(f_2(s))$, and the Nadir point is defined as (f_1^N, f_2^N), where $f_1^N = \min f_1(s)$, s.t. $s \in S, f_2(s) = f_2^I$, and $f_2^N = \min f_2(s)$, s.t. $s \in S, f_1(s) = f_1^I$.

It is not hard to find that the ideal point is composed of the lowest values of both objective functions, while the Nadir point specifies their upper bounds in the Pareto front. Thus, for i = 1 and 2, f_i^{LB} and f_i^{UB} are set as f_i^I and f_i^N, respectively. Since all the single-objective problems defined in Definition 4 are linear, this paper solves them using CPLEX MIP solver to determine the values of f_i^{LB} and f_i^{UB}. Then, the dual-objective optimization model $\mathcal{P}_1$ is transformed into the following $\mathcal{P}_1(\lambda)$:

$$\begin{aligned} \mathcal{P}_1(\lambda):\ \min\ & \lambda \cdot f_1' + (1 - \lambda) \cdot f_2' \\ \text{s.t.}\ & \text{Constraints (3)–(7) and (9)–(17).} \end{aligned} \tag{20}$$

According to Theorem 1, the following corollary holds.

Corollary 2: The complexity of $\mathcal{P}_1(\lambda)$ is NP-hard.

We set weight λ as 0, 0.1, ..., and 1, a set of nondominated solutions in the Pareto front can be obtained by solving the corresponding single-objective optimization models $\mathcal{P}_1(\lambda)$. Note that different from the traditional weighted sum method in which single-objective problems are solved by using a commercial solver such as CPLEX, a new relax-and-optimize algorithm is developed to more efficiently solve the corresponding single-objective optimization problems. In the next section, the designed relax-and-optimize algorithm is presented in details. The overall weight sum-based method for model $\mathcal{P}_1$ is described in Section IV-C.

B. Relax-and-Optimize Algorithm for $\mathcal{P}_1(\lambda)$

As mentioned above, to obtain a set of Pareto solutions of the DCLRP, a sequence of NP-hard single-objective optimization problems $\mathcal{P}_1(\lambda)$ need to be solved. Thus, the resolution efficiency of $\mathcal{P}_1(\lambda)$ would seriously influence the efficiency of solving the DCLRP. In this paper, in order to accelerate the resolution of our problem, a new efficient method called *relax-and-optimize* is proposed to solve $\mathcal{P}_1(\lambda)$ instead of directly using an optimization software, e.g., CPLEX.

1) Preprocessing: To speed up the resolution of $\mathcal{P}_1(\lambda)$, a preprocessing similar to that in [15] is first conducted. For $\forall k \in K$, we first define set A_k as follows:

$$A_k = \left\{(o_k, i) | t_{o_k i} + \tau_{i,d_k} > T_k \ \forall (o_k, i) \in A\right\} \ \forall k \in K$$
$$A_k' = \left\{(j, d_k) | \tau_{o_k,j} + t_{jd_k} > T_k \ \forall (j, d_k) \in A\right\} \ \forall k \in K$$

where τ_{i,d_k} and $\tau_{o_k,j}$ denote the shortest travel time from i to d_k and o_k to j, respectively, on an entirely reserved network. It can be imagined that if each arc $(i, j) \in A$ in the network is with travel time value of t_{ij}, then τ_{i,d_k} and $\tau_{o_k,j}$ can be easily obtained by using the Dijkstra shortest path algorithm with its complexity $O(|N|^2)$. Meanwhile, it can be found that if arcs in A_k and A_k' are used by task k, then travel duration constraint (9) would be violated. Thus, the corresponding variables can be fixed to 0 and a new model $\mathcal{P}_1'(\lambda)$ is formulated as follows:

$$\mathcal{P}_1'(\lambda):\ \min\ \lambda \cdot f_1' + (1 - \lambda) \cdot f_2'$$
$$\text{s.t.}\ x_{ko_k i} + y_{ko_k i} = 0 \ \forall (o_k, i) \in A_k \ \forall k \in K \tag{21}$$
$$x_{kjd_k} + y_{kjd_k} = 0 \ \forall (j, d_k) \in A_k' \ \forall k \in K \tag{22}$$

and constraints (3)–(7) and (9)–(17).

Compared with $\mathcal{P}_1(\lambda)$, feasible solutions are not excluded from $\mathcal{P}_1'(\lambda)$, hence $\mathcal{P}_1'(\lambda)$ is equivalent to $\mathcal{P}_1(\lambda)$.

2) Relax-and-Optimize: It is known that generally it is much more difficult to solve an IP model than a real linear program due to the integer variables. Inspired by this observation, we design a so-called relax-and-optimize algorithm based on the relaxation of integer variables to solve model $\mathcal{P}_1'(\lambda)$. Its core idea is presented as follows.

We first transform the original integer linear program, i.e., $\mathcal{P}_1'(\lambda)$, into a real linear program by relaxing all integer variables, i.e., z_{ij}, x_{kij}, and y_{kij}. The corresponding real linear program is then exactly solved and an optimal solution is obtained, in which the values of variables z_{ij}, x_{kij}, and y_{kij} either equal to 0 or 1, or belong to (0, 1). Let $\widetilde{z}_{ij}$, $\widetilde{x}_{kij}$, and $\widetilde{y}_{kij}$ denote the values of variables z_{ij}, x_{kij}, and y_{kij}, respectively. For arcs tending to be reserved at first priority, their corresponding $\widetilde{z}_{ij}$ have large probabilities of being one in the optimal solution of the original problem. Implying that these arcs have large probabilities of being fully reserved. On the other hand, part of arcs would never be used, owing to their

Algorithm 1 Relax-and-Optimize Algorithm for $\mathcal{P}_1(\lambda)$

Implement pre-processing and obtain an equivalent integer linear program $\mathcal{P}'_1(\lambda)$;
Initialize $\Omega = \varnothing$, $\Psi = \varnothing$, and $\mathcal{Y} = \varnothing$;
Solve $\mathcal{RP}(\lambda)$ exactly and obtain variables z_{ij} and y_{kij}'s values $\widetilde{z}_{ij}$ and $\widetilde{y}_{kij}$;
Let $\Omega = \{(i,j)|0 < \widetilde{z}_{ij} \leq 1\}$, $\Psi = \{(i,j)|0 < \widetilde{z}_{ij} < 1\}$, and $\mathcal{Y} = \{y_{kij}|0 < \widetilde{y}_{kij} < 1\}$;
while $\Psi \neq \varnothing$ and $\mathcal{Y} \neq \varnothing$
Let all variables $z_{ij}, y_{kij}, \forall(i,j) \in \Omega$ and $\forall y_{kij} \in \mathcal{Y}$ be 0-1 variables;
Solve $\mathcal{RP}(\lambda)$ and obtain variables z_{ij} and y_{kij}'s values $\widetilde{z}_{ij}$ and $\widetilde{y}_{kij}$;
$\Psi = \{(i,j)|0 < \widetilde{z}_{ij} < 1\}$, $\Omega = \Omega \cup \Psi$, and $\mathcal{Y} = \{y_{kij}|0 < \widetilde{y}_{kij} < 1\}$;
end while
Output the solution and its corresponding objective value.

over-high impact. Their corresponding values $\widetilde{z}_{ij}$ remain zero. The remaining variables z_{ij} have values falling into interval (0, 1), implying the partial use of these arcs.

In a relaxed model of $\mathcal{P}'_1(\lambda)$, the objective function value is in proportion to the values of z_{ij}, x_{kij}, and y_{kij}. However, in the original model $\mathcal{P}'_1(\lambda)$, the objective function value is fixed as long as the values of $z_{ij}, x_{kij}, y_{kij} > 0$. To remedy such distortion, those variables taking values between 0 and 1 should be set to binary variables in the next iteration. For those variables taking the value of 1, we set them as binary ones in the next iteration to save the computational time. Because the value of decision variable x_{kij} is directly and strongly influenced by the decision of LR (i.e., z_{ij}) via constraint (7), then all variables x_{kij} keep real, and thus only z_{ij} and y_{kij} are considered in our designed algorithm. The following corollary showing the influence between z_{ij} and x_{kij} holds according to Theorem 2.

Corollary 3: In an optimal solution of $\mathcal{P}'_1(\lambda)$, for any arc $(i,j) \in A$, if this arc is reserved, i.e., $z_{ij} = 1$, then any task $k \in K$ passing this arc will always pass the reserved lane, i.e., $x_{kij} = 1$, always holds; otherwise $x_{kij} = 0$ according to constraint (7).

Let Ω and $\mathcal{Y}$ denote the set $\{(i,j)|0 < \widetilde{z}_{ij} \leq 1\}$ and $\{y_{kij}|0 < \widetilde{y}_{kij} < 1\}$, respectively. Then, the relaxed model is formed as follows:

$$\mathcal{RP}(\lambda): \min \quad \lambda \cdot f'_1 + (1-\lambda) \cdot f'_2 \tag{23}$$

$$\text{s.t.} \quad z_{ij} \in \{0,1\} \ \forall (i,j) \in \Omega \tag{24}$$

$$y_{kij} \in \{0,1\} \ \forall (i,j) \in \Omega, \forall k \in K, \forall y_{kij} \in \mathcal{Y} \tag{25}$$

$$0 \leq z_{ij} \leq 1 \ \forall (i,j) \in A \backslash \Omega \tag{26}$$

$$0 \leq y_{kij} \leq 1 \ \forall k \in K \ \forall (i,j) \in A \backslash \Omega, \forall y_{kij} \notin \mathcal{Y} \tag{27}$$

$$0 \leq x_{kij} \leq 1 \ \forall k \in K \ \forall (i,j) \in A \tag{28}$$

and constraints (3)–(7), (9)–(10), (14)–(17), and (21)–(22).

We note that if all variables' values of the obtained solution are either 0 or 1, i.e., integer, then an optimal solution for $\mathcal{P}'_1(\lambda)$ is derived and the algorithm terminates. Otherwise, a new relaxed model formed and a new iteration repeats.

Algorithm 2 Iterative Weighted Sum-Based Method for DCLRP

Computing f_i^{LB} and f_i^{UB}, i=1 and 2;
Obtain $f'_i(s)$, i=1 and 2, with Eq. (19);
Transform $\mathcal{P}_1$ to $\mathcal{P}_1(\lambda)$ and initialize $\lambda = 0.0$, $\delta = 0.1$, $\mathcal{F} = \varnothing$;
while $\lambda \leq 1$
Solve $\mathcal{P}_1(\lambda)$ calling **Algorithm 1**, and obtain solution s^* and the corresponding objective vector $(f_1(s^*), f_2(s^*))$;
Let $\mathcal{F} = \mathcal{F} \cup \{(f_1(s^*), f_2(s^*))\}$ and $\lambda = \lambda + \delta$;
end while
Output the Pareto solution set $\mathcal{F}$ after removing dominated solutions.

Based on the description above, the overall algorithm for model $\mathcal{P}_1(\lambda)$ is outlined in Algorithm 1, which is shown to be able to obtain an optimal solution of $\mathcal{P}_1(\lambda)$ after termination.

Theorem 3: When Algorithm 1 terminates, the current incumbent solution is optimal.

Proof: Since $\mathcal{RP}(\lambda)$ is a liner relaxed or partial relaxed problem of $\mathcal{P}_1(\lambda)$, the optimal objective value of $\mathcal{RP}(\lambda)$ optimal solution is a lower bound of $\mathcal{P}_1(\lambda)$. If all decision variables of the current incumbent solution take integer values, its objective value is also an upper of $\mathcal{P}_1(\lambda)$. So in this case, the current incumbent solution, the optimal solution of $\mathcal{RP}(\lambda)$ is also an optimal solution of $\mathcal{P}_1(\lambda)$. ■

Although Algorithm 1 is specially developed for solving the addressed problem, the idea of relax and optimize may be adapted to solve other kinds of combinatorial optimization problems of similar structure. For example, facility location problems in which the location decisions would influence other decisions such as assignment decisions. In this paper, we have shown that it can be easily adapted to solve the standard CLRP and is more efficient than the existing algorithm in Section V-B.

C. Iterative Weighted Sum-Based Method for DCLRP

As mentioned above, a set of nondominated solution in the Pareto front can be obtained by solving a series of $\mathcal{P}'_1(\lambda)$ with iteratively varying the value of λ. The iterative weighted sum-based method of the DCLRP can be summarized in Algorithm 2.

V. Experiments and Results

Extensive numerical experiments are conducted to validate the effectiveness and efficiency of the proposed model and algorithm, which are divided into two categories: 1) resolution of the DCLRP and 2) resolution of the standard CLRP [15]. In the first category, we focus on solving the DCLRP by the proposed approach. As the proposed model and algorithm can be easily adapted to solve the CLRP, thus in the second category, we also test the proposed model and algorithm for solving the standard CLRP. All these experiments are performed on a laptop with 2.5 GHz and 2.95 GB RAM running Windows 10. The models and algorithms are implemented by C++ linked with CPLEX 12.6 using the Concert Technology.

TABLE I

COMPARISON RESULTS FOR THE INSTANCES OF DCLRP WITH $|N| = 100$

Set	$\|N\|$	$\|K\|$	$Nb_{Algo2'}$	Nb_{Algo2}	$T_{Algo2'}$	T_{Algo2}	$T_{Algo2}/T_{Algo2'}$
1	100	5	6.0	6.0	51.30	49.53	0.97
2	100	10	8.6	8.6	164.31	99.72	0.61
3	100	15	9.6	9.6	262.91	156.25	0.59
4	100	20	10.2	10.2	349.73	185.96	0.53
5	100	25	9.6	9.6	591.85	270.84	0.46
6	100	30	10.2	10.2	748.56	531.00	0.71
7	100	35	10.4	10.4	2636.33	829.64	0.31
8	100	40	10.6	10.6	4532.87	2279.12	0.50
9	100	45	7.4	11.0	7200.00	2814.42	0.39
Average			9.2	9.6	1837.54	801.83	0.44

The computational time limit is set to 7200 CPU seconds. Note that each value reported in the result table calculates the average value of five instances in an experimental problem set.

A. Results on the DCLRP

Results of the proposed algorithm on two sets of DCLRP instances are presented in this section. Table I contains the results on the instances with 100 fixed number of nodes while Table II reports the results on the instances with varying nodes from 50 to 120. The test problems are constructed based on the benchmark instances of CLRP [15], which comprise 33 problem sets where each involves five instances (i.e., 165 instances in total).

For each instance, the transportation network was generated using the model reported by [28], where the nodes are randomly distributed within a rectangle and the existence of an arc is decided by a probability function. OD pairs are randomly generated from the node set N. t'_{ij} is defined as l_{ij}/v_{ij}, where v_{ij} is the average travel speed on arc (i, j). $t_{ij} = \alpha_{ij} t'_{ij}$ where α_{ij} is a travel time discount via a reserved lane and is generated in $U(0.5, 0.8)$, where $U(a, b)$ denotes a uniform distribution between a and b with $a < b$. Travel delay td_{ij} is estimated as $t'_{ij}/(m_{ij}-1)$, where m_{ij} denotes the number of lanes on arc (i, j). It approximately computes the increased time due to a reserved lane [12], [15]. f_k and c_{ij} are randomly generated in [5, 10] and [20, 30], respectively. T_k is defined as $\tau_k + \beta_k(\tau'_k - \tau_k)$, where τ_k and τ'_k denote the shortest travel time from o_k to d_k via an entirely reserved path and an exclusively nonreserved path, respectively, and β_k is randomly generated in $U(0, 1)$. fc_{ij} and vc_{ij} are set as 1000 Yuan and 10 000 Yuan/km, respectively. Bdg is set as $\sum_{(i,j)\in A}(fc_{ij} + vc_{ij} l_{ij}) * U(0.1, 0.2)$.

To validate the effectiveness and efficiency of the proposed algorithm, we compare it with the traditional weighted sum method based well-known commercial optimization solver CPLEX, denoted by Algorithm 2′, which is defined as Algorithm 2 in which $\mathcal{P}_1(\lambda)$ is solved by using CPLEX MIP solver instead of Algorithm 1. The symbols used to report the results are presented as follows.

1) $Nb_{Algo2'}$: Number of Pareto optimal solutions obtained by Algorithm 2′.
2) Nb_{Algo2}: Number of Pareto optimal solutions obtained by Algorithm 2.
3) $T_{Algo2'}$: The CPU time (s) of Algorithm 2′.
4) T_{Algo2}: The CPU time (s) of Algorithm 2.

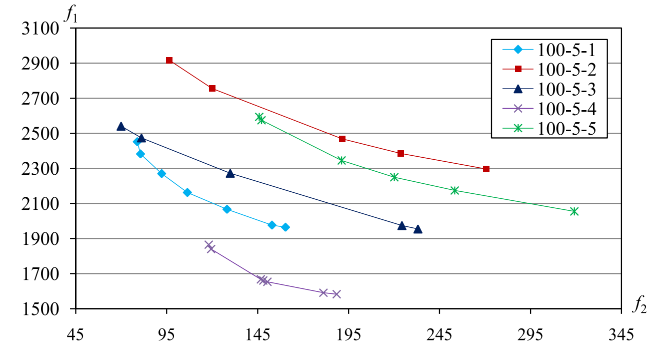

Fig. 1. Pareto points for problem set with $|N| = 100$ and $|K| = 5$.

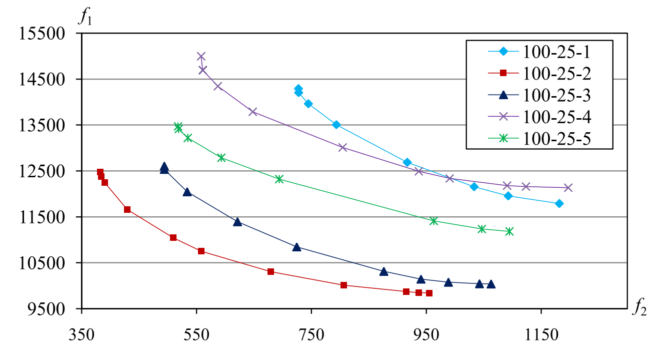

Fig. 2. Pareto points for problem set with $|N| = 100$ and $|K| = 25$.

From Table I, we can see that the proposed algorithm (i.e., Algorithm 2) can obtain the same number of Pareto points as Algorithm 2′ over problem sets 1–8. On average, Algorithm 2 can obtain more Pareto optimal solutions than Algorithm 2′, since for the largest scale instance set 9, Algorithm 2′ cannot find the same number of Pareto optimal solutions as Algorithm 2 within 7200 s. These results show that the proposed algorithm outperforms the weighted sum-based CPLEX method. Furthermore, it can be observed that the average number of Pareto optimal solutions has an increasing trend as the number of tasks $|K|$ increases. This is mainly because that problems with more tasks enjoy larger solution space, implying more Pareto optimal solutions.

On the other hand, Algorithm 2 takes less computational time than Algorithm 2′ on each set, especially for large scale ones. We can observe that the CPU time of Algorithm 2′ increases sharply with the number of tasks $|K|$, while that of Algorithm 2 increases relatively slightly. $T_{\text{Algo2}'}$ increases from 51.30 to 7200 s when $|K|$ increases from 5 to 45, while T_{Algo2} increases from 49.53 to 2814.42 s. Moreover, it can be seen that the value of $T_{\text{Algo2}}/T_{\text{Algo2}'}$ has a decreasing trend as $|K|$ increases. On average, the proposed algorithm only spends 44% CPU time of the weighted sum-based CPLEX method. These results indicate that Algorithm 2 outperforms Algorithm 2′ in terms of computational efficiency and the proposed relax-and-optimize Algorithm 1 is more efficient than CPLEX in solving single-objective problem $\mathcal{P}_1(\lambda)$. In summary, the proposed algorithm is able to efficiently solve the considered DCLRP with fixed number of nodes and varying number of tasks.

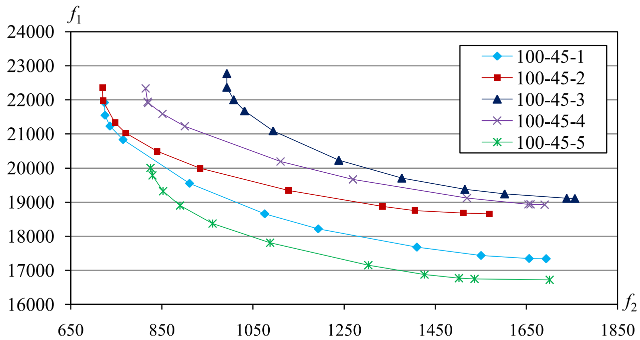

Fig. 3. Pareto points for problem set with $|N| = 100$ and $|K| = 45$.

In order to clearly illustrate the tradeoff between the two objectives of the DCLRP, the Pareto fronts for the instances of three representative sets, i.e., the small-size set 1, medium-size set 5, and large-size set 9 are shown in Figs. 1–3. It can be observed from these figures that for the instance in the larger-size set, there exist more Pareto optimal solution and we can see that the obtained solutions are well-distributed in its corresponding objective space. Decision-makers can choose their desired solution from these alternatives according to their specified requirements.

To further test the proposed method, we also solve instances with nodes varying from 50 to 120. From Table II, we can see that Algorithm 2 can derive the same number of nondominated points as Algorithm $2'$. This again indicates that the proposed algorithm is able to obtain Pareto solutions of high quality when solving the DCLRP. The Pareto solutions for the instances of each set are not presented here due to the space limitation, but we note that the images constituted by them are similar to those in Figs. 1–3. We again note that the average computational time of Algorithm 2 is less than that of Algorithm $2'$ for each problem set. Meanwhile, the computational time of the proposed method increases more slightly than that of Algorithm $2'$. The above numerical results indicate that the proposed algorithm is effective and efficient in solving the DCLRP with varying number of nodes.

B. Results on the CLRP

Fang *et al.* [15] have proved that the standard CLRP is NP-hard and showed that problem instances with 100 nodes and 45 tasks cannot be solved by their proposed algorithm within acceptable time. As mentioned above, the proposed relax-and-optimize algorithm, i.e., Algorithm 1, can be easily adapted to solve the standard CLRP. Moreover, the existing model for the CLRP can be further improved. For the sake of clarity, the existing model, improved model, and proposed algorithm for the standard CLRP are stated in the Appendix.

To further validate the performance of the improved model and relax-and-optimize algorithm, we also test 165 benchmark instances of the standard CLRP. The following notations are used to report the results.

1) *OB:* The objective value obtained by the proposed relax-and-optimize algorithm.
2) T_F: The computational time (s) of CPLEX for solving the existing model of CLRP.
3) T_W: The computational time (s) of CPLEX for solving the improved model of CLRP.
4) T_{cs}: The computational time (s) of the existing algorithm [16] for solving CLRP.
5) T_{Algo3}: The computational time (s) of the relax-and-optimize algorithm, i.e., Algorithm 3 detailed in Appendix, for solving CLRP.

TABLE II
COMPARISON RESULTS FOR THE INSTANCES OF DCLRP WITH $|K| = 20, 25, 30$

Set	$\|N\|$	$\|K\|$	$Nb_{Algo2'}$	Nb_{Algo2}	$T_{Algo2'}$	T_{Algo2}	$T_{Algo2}/T_{Algo2'}$
10	50	20	8.8	8.8	53.70	52.72	0.98
11	60	20	8.6	8.6	189.94	148.35	0.78
12	70	20	8.8	8.8	318.82	134.98	0.42
13	80	20	9.2	9.2	431.19	287.65	0.67
14	90	20	8.4	8.4	114.53	117.59	1.03
15	100	20	8.8	8.8	168.20	125.58	0.75
16	110	20	9.4	9.4	522.18	290.55	0.56
17	120	20	9.4	9.4	611.59	432.69	0.71
Average			8.9	8.9	301.27	198.76	0.66
18	50	25	9.0	9.0	138.16	132.13	0.96
19	60	25	8.8	8.8	174.06	169.70	0.97
20	70	25	9.2	9.2	259.67	175.75	0.68
21	80	25	10.2	10.2	425.64	305.85	0.72
22	90	25	10.2	10.2	578.84	297.26	0.51
23	100	25	9.8	9.8	723.36	487.67	0.67
24	110	25	9.2	9.2	903.12	664.59	0.74
25	120	25	9.8	9.8	1418.68	722.92	0.51
Average			9.5	9.5	577.69	369.48	0.64
26	50	30	9.4	9.4	182.70	194.50	1.06
27	60	30	9.4	9.4	153.98	132.22	0.86
28	70	30	10.2	10.2	496.86	426.83	0.86
29	80	30	9.8	9.8	360.43	339.65	0.94
30	90	30	10.2	10.2	350.76	332.87	0.95
31	100	30	10.0	10.0	793.14	353.38	0.45
32	110	30	10.4	10.4	938.59	532.88	0.57
33	120	30	9.8	9.8	2490.90	1101.87	0.44
Average			9.9	9.9	720.92	426.78	0.59

TABLE III
COMPARISON RESULTS FOR THE INSTANCES OF CLRP WITH $|N| = 100$

Set	$\|N\|$	$\|K\|$	OB	T_F	T_W	T^*_{cs}	T_{Algo3}
34	100	5	101.82	1.76	1.50	1.17	6.43
35	100	10	217.68	7.53	6.90	5.36	13.29
36	100	15	376.81	25.49	21.84	13.24	20.68
37	100	20	411.05	34.59	30.85	35.00	25.29
38	100	25	535.92	79.92	68.77	65.30	42.32
39	100	30	606.51	138.48	133.88	123.13	82.17
40	100	35	657.39	712.73	398.27	931.41	108.82
41	100	40	737.51	806.60	551.34	1137.52	350.73
Average				225.89	151.67	289.02	81.22
42	100	45	814.99	-	2097.64	-	335.08

* Results on 3.00GHz computer with 4.00 GB RAM with CPLEX 12.1.

Table III reports the computational results for problems sets with 100 fixed nodes while the number of tasks increases from 5 to 45. From Table III, we can conclude the following.

1) The improved model is more efficient than the existing one as T_W is less than T_F over all problem sets.

TABLE IV
COMPARISON RESULTS FOR THE INSTANCES OF CLRP WITH $|K| = 20, 25, 30$

Set	$\|N\|$	$\|K\|$	OB	T_F	T_W	T^*_{cs}	T_{Algo3}
43	50	20	377.52	6.79	5.35	6.87	7.54
44	60	20	465.88	15.38	13.19	11.11	10.57
45	70	20	429.37	49.27	46.62	28.86	19.72
46	80	20	358.2	41.86	39.88	31.24	26.91
47	90	20	406.74	49.26	30.74	26.22	24.02
48	100	20	599.99	89.68	64.81	56.24	49.37
49	110	20	612.71	73.06	58.79	44.54	39.69
50	120	20	617.99	95.38	66.30	63.47	56.79
51	50	25	480.07	9.50	9.02	10.3	10.08
52	60	25	493.86	13.52	9.90	10.54	12.48
53	70	25	557.57	22.63	18.15	24.29	13.16
54	80	25	540.2	60.50	33.08	31.71	30.77
55	90	25	441.48	65.48	42.86	37.72	32.41
56	100	25	634.21	84.78	63.62	98.1	56.29
57	110	25	816.23	151.54	90.27	117.67	84.96
58	120	25	735.01	277.53	174.53	156.53	119.97
59	50	30	537.18	22.05	18.43	19.45	30.21
60	60	30	553.21	55.34	41.00	35.33	37.63
61	70	30	588.56	69.56	52.32	51.07	46.43
62	80	30	552.65	81.31	55.42	59.46	42.52
63	90	30	732.86	80.00	64.56	64.22	55.12
64	100	30	789.64	469.48	254.03	168.31	158.6
65	110	30	986.2	266.55	113.55	277.85	91.98
66	120	30	1017.09	566.42	245.44	310.44	173.64
Average				113.20	67.16	72.56	51.29

* Results on 3.00GHz computer with 4.00 GB RAM with CPLEX 12.1.

Moreover, with the improved model the largest scale problem set 42 can be solved to optimality within 2097.64 s while the existing model fails within 7200 s.

2) We can see that T_{Algo3} varies from 6.43 to 350.73 s while T_W and T_{cs} increase from 1.50 to 2097.64 s and 1.17 to over 7200 s, respectively, and T_{Algo3} is less than T_W and T_{cs} over all sets except the smallest sized sets 34–36. These results show that our relax-and-optimize algorithm outperforms the existing algorithm [16] and commercial software CPLEX in terms of computational efficiency.

Table IV summarizes the results of instances with $|N|$ increasing from 50 to 120 and $|K|$ being 20, 25, and 30. From Table IV, we can observe the following.

1) T_W is less than T_F over all problem sets 43–66. T_F varies from 6.79 to 566.42 s with its average value being 113.20 s while T_W varies from 5.35 to 245.44 s with its average value being 67.16 s. The improved model only takes an average 59.33% (67.16/113.20) computational time of the existing one. This indicates that the proposed model is more efficient than the existing one.

2) The average value of T_{Algo3} is smaller than that of T_W and the proposed algorithm spends 76.37% (51.29/67.16) computational time of CPLEX. In addition, our algorithm spends less average computational time than the existing method. This demonstrates that the proposed algorithm outperforms CPLEX and the existing algorithm in terms of solution efficiency.

Algorithm 3 Relax-and-Optimize Algorithm for the CLRP

Implement pre-processing for Wu-model and obtain an equivalent model;
Initialize $\Omega = \varnothing$, $\Psi = \varnothing$, and $\mathcal{Y} = \varnothing$;
Solve the equivalent model and obtain variables z_{ij} and y_{kij}'s values $\widetilde{z}_{ij}$ and $\widetilde{y}_{kij}$;
Let $\Omega = \{(i,j)|0 < \widetilde{z}_{ij} \leq 1\}$, $\Psi = \{(i,j)|0 < \widetilde{z}_{ij} < 1\}$ and $\mathcal{Y} = \{y_{kij}|0 < \widetilde{y}_{kij} < 1\}$;
while $\Psi \neq \varnothing$ and $\mathcal{Y} \neq \varnothing$
  Let all variables $z_{ij}, y_{kij}, \forall(i,j) \in \Omega$ and $\forall y_{kij} \in \mathcal{Y}$ be 0-1;
  Solve the model again and obtain variables z_{ij} and y_{kij}'s values;
  $\Psi = \{(i,j)|0 < \widetilde{z}_{ij} < 1\}$, $\Omega = \Omega \cup \Psi$, and $\mathcal{Y} = \{y_{kij}|0 < \widetilde{y}_{kij} < 1\}$;
end while
Output the solution and its corresponding objective value.

VI. CONCLUSION

This paper addresses a dual-objective optimization issue for lane reservation in transportation network with limited budget, time-efficient transport requirements, and road residual capacity constraints for the first time. Its objective is to simultaneously maximize the LR benefit achieved by minimizing the total task transportation time and minimize negative impact due to reserved lanes. We first formulate a dual-objective IP model for the problem and it is further enhanced by adding valid inequalities and eliminating redundant constraints. Its complexity is analyzed. Since it is NP-hard, an iterative weighted sum-based method is proposed to efficiently derive Pareto optimal solutions. Computational results for 165 instances under various parameters settings show that the proposed algorithm is efficient in generating Pareto solutions of high quality for the DCLRP. The results can be used to guide decision-makers in making reasonable tradeoff between the LR benefit and the total negative impact caused. Moreover, the comparison results on the standard CLRP show that the proposed improved model and algorithm are more efficient than the existing ones.

The proposed method is shown to be effective and efficient for solving the DCLRP, but it also has some limitations. On one hand, we observe that there exist some redundant iterations in the computational procedure of our method, which are a bit time-consuming for large-size instances. To design effective redundant iterations avoidance mechanisms to further improve the proposed method would be an interesting future research direction. On the other hand, the computational time of the proposed method still increases with the problem size due to the NP-hardness of the DCLRP. More effective and efficient algorithms [29]–[33] for solving larger-size problems should be tested in the future. Moreover, the proposed model may be further expanded by taking into account other practical requirements, i.e., dynamic link travel time, giving stochastic or/and time-dependent models.

Appendix

We first state the IP model for the standard CLRP proposed by Fang *et al.* [15], denoted by Fang-model, and it is then improved by eliminating constraints and adding valid inequalities. Fang-model is laid out below

$$\text{(Fang-model)} \quad \min \sum_{(i,j)\in A} td_{ij}z_{ij} \tag{29}$$

$$\text{s.t.} \quad x_{kij} \leq z_{ij} \ \forall k \in K \ \forall (i,j) \in A \tag{30}$$

and constraints (3)–(5) and (8)–(13).

We show that constraint (8) is redundant and can be relaxed by the following theorem.

Theorem 4: For an optimal solution of the CLRP, constraint (8) is redundant and can be relaxed from Fang-model.

Proof (Case 1): $z_{ij} = 0$, in Fang-model, constraint (8) becomes $y_{ij}^k \leq 1, \ \forall k \in K \ \forall (i,j) \in A$. We note that such restriction has been included in constraint (13). Hence, it is not necessary to add constraint (8) in this case.

Case 2: $z_{ij} = 1$, then constraint (8) requires that $y_{ij}^k = 0, \ \forall k \in K, \ \forall (i,j) \in A$. However, we show that removing it would not exclude optimal solutions as follows.

Let R-Fang-model be Fang-model excluding constraint (8), i.e., R-Fang-model is a relaxed model of Fang-model. Assume that $\mathbf{s} = (\vec{z}, \vec{x}, \vec{y})$ is an optimal solution of R-Fang-model and there exist for some arcs and tasks, $z_{ij} = 1$ but $y_{ij}^k = 1$. In the following, we show that a new solution $\mathbf{s}' = (\vec{z}, \vec{x}', \vec{y}')$, where $\vec{x}'$ and $\vec{y}'$ is formed from $\vec{x}$ and $\vec{y}$ by respectively changing $x_{ij}^k = 0$ to $x_{ij}^k = 1$ and $y_{ij}^k = 1$ to $y_{ij}^k = 0$ is optimal for Fang-model.

It is not hard to find that $\mathbf{s}'$ still satisfies the related constraint (9) of R-Fang-model due to the fact that $t_{ij} < t'_{ij}$, hence $\mathbf{s}'$ is optimal for R-Fang-model as well. To this step, $\mathbf{s}'$ satisfies constraint (8) and thus it is optimal for Fang-model.

Hence, the above proof shows that constraint (8) is redundant for Fang-model as an optimal solution of R-Fang-model can be easily adjusted to be optimal for Fang-model. ■

Similarly, valid inequalities (14)–(17) can be used to tighten the existing model. Then, an improved model for the standard CLRP, denoted by Wu-model, is presented as follows:

$$\text{(Wu-model)} \quad \min \sum_{(i,j)\in A} td_{ij}z_{ij}$$

$$\text{s.t.} \quad \text{Constraints } (3)-(5),\ (7),\ \text{and } (9)-(17).$$

In addition, the proposed relax-and-optimize algorithm for $\mathcal{P}_1(\lambda)$ is adapted to solve the CLRP, whose procedure is shown in Algorithm 3.

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