Distributed Output-Feedback Adaptive Fuzzy Leader-Following Consensus of Stochastic Nonlinear Interconnected Multiagent Systems

Yafeng Li, Ju H. Park, *Senior Member, IEEE*, Libing Wu, and Qingkai Kong

***Abstract*—In this article, the distributed output-feedback adaptive fuzzy leader-following consensus is investigated for a class of high-order stochastic nonlinear multiagent systems (MASs) with unknown control gains and uncertain interactions from other agents. First, a novel dynamic gain filter is constructed to compensate unmeasured states and compared with the existing results, the number of dynamic variables is greatly reduced. Then, the tuning function method is used to construct adaptive laws to compensate unknown parameters. The uncertain interaction functions are decomposed and approximated by the fuzzy logic systems. Next, using the backstepping method, the dynamic gain filter-based distributed adaptive fuzzy leader-following protocol is designed, which only needs the dynamic variables' information of neighbors, and the problem on computing the mutually dependent inputs in some existing results is completely avoided. Finally, the numerical simulation results are given to illustrate the effectiveness of the proposed method.**

***Index Terms*—Adaptive fuzzy control, distributed control, leader following, multiagent systems (MASs).**

I. INTRODUCTION

LEADER-FOLLOWING consensus as one of the synchronization control problems of multiagent systems (MASs) has attracted many researchers' attention due to its potential application value [1]–[4] and the references therein, where there is usually a signal generator as the leader via the communication graph all the other agents (followers) receive the neighbors' information to achieve synchronization ultimately. In the past decades, lots of results on the distributed control of MASs have been proposed, including linear MASs [4]–[9] and nonlinear MASs [10]–[23]. In [4], the consensus problem for multivehicle systems described by single integral systems was investigated. Further, the scholars studied the distributed state-feedback control [5], distributed output-feedback control [6], distributed control under switching communication graphs [7], distributed event-triggered control [8], distributed finite-time consensus control [9], and so on, for the high-order linear MASs. However, in practical systems, the nonlinearity, stochastic disturbances, and even uncertain nonlinear interactions in MASs exist generally. Hence, compared with the linear MASs, the nonlinear ones are more general. Distributed leader-following protocols on the nonlinear MASs in the existing literature can be easily divided into two classes: 1) distributed state-feedback protocol and 2) distributed output-feedback protocol.

For the state-feedback control of the MASs, assume that all the states information of the agents can be measured. Via the communication graph and using the neighbors' states information, many distributed state-feedback control protocols are proposed in the existing literature [10]–[20], where combined with the techniques, such as adaptive control [10], backstepping technique [11], finite-time control [12], [13], dynamic surface control [14], fuzzy logic systems (FLSs) [15], neural networks [16], [17], and so on, many distributed control problems on the nonlinear MASs have been addressed. In addition, the uncertain interactions were also considered in [15], which often encountered in the practical systems, such as mechanical systems, power electrical systems, and FLSs were used to approximate these unknown functions. In [19] and [20], the stochastic disturbances were further considered, and based on the stability theory of stochastic systems, the output consensus problems of a class of stochastic nonlinear MASs were addressed.

However, in many practical application systems, the states information is difficult to measure and even cannot be measured directly. Based on this, many scholars have paid much attention to the distributed output-feedback control of the MASs [22]–[26]. In [22], based on the constructed distributed observer, the distributed output-feedback consensus was investigated, where the nonlinear terms should satisfy the Lipschitz condition and the diffusion term should be bounded. The work [23] studied the fuzzy observed-based adaptive consensus tracking control for second-order MASs with heterogeneous nonlinear dynamics. The work [25] studied the output-feedback full state consensus problem using the

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dynamic gain method. In [26], the unknown control gains were further considered, and based on the constructed k-filter, the distributed output-feedback protocol was given.

Inspired by the aforementioned introductions, this article investigates the distributed output-feedback adaptive fuzzy leader-following control for a class of stochastic nonlinear MASs with interactions and unknown control gains. Based on the constructed dynamic gain filter, the distributed output-feedback controller is designed by the backstepping method to make the outputs achieve the consensus. The contributions of this article are listed as follows.

1) The dynamic gain filter is constructed to compensate unmeasured states. Compared with the designed filter in [26]–[28], the number of dynamic variables of the constructed filter is greatly reduced to further reduce the computation burden, and the nonlinear terms in the dynamic systems can be more general. Compared with the observer in [22] and [23], the unknown control gains' problem can also be addressed.
2) The interactions existing not only in the drift terms but also the diffusion terms are considered and the condition on these interaction terms are more general than the ones in [26] where the interactions are described by $\sum_{j=1}^{N} f_{ij}(t, y_j)$ satisfying $\|f_{ij}(t, y_j)\| \leq \bar{\gamma}_{ij}|y_j|$, and $\bar{\gamma}_{ij}$ is a positive constant. The interaction terms in this article are uncertain smooth functions on the outputs of agents, and beyond that, there are no other restrictions.
3) The proposed control protocol only needs the dynamic variables information of itself and its neighbors. The case, that the interactions of heterogeneous MASs contain all the states information and the control gains of each agent are different, is also considered, which makes the proposed results be applicable for a wider class of MASs.

II. Preliminaries and Problem Formulation

A. Graph Theory and Notations

In this article, R^n denotes the n-dimensional space. $\|X\| = (\mathrm{Tr}\{X^T X\})^{(1/2)}$ is the norm of a matrix X. The MASs studied in this article include N followers and one leader, whose communication graph is depicted by G. Its connectivity matrix is $A = [a_{ij}] \in R^{n\times n}$, the in-degree matrix is $D = \mathrm{diag}\{d_1, \ldots, d_N\}$, the Laplacian matrix is $L = D - A$, and the pinning matrix is $B = \mathrm{diag}\{b_1, \ldots, b_N\}$. The set of neighbors of a node i is N_i. For saving space, the details are not presented here, further referring to [20].

B. Problem Statement

Consider the stochastic nonlinear MASs, in which the dynamic of the ith agent is described as

$$\begin{cases} dx_{i1} = \{x_{i2} + f_{i1}(y)\}dt + g_{i1}^T(y)dw \\ dx_{\mathrm{is}} = \left\{x_{i(s+1)} + f_{is}(y) + \sum_{m=2}^{s} \phi_{ism}(y_i)x_{im}\right\}dt \\ \qquad\quad + g_{is}^T(y)dw, \ \ s = 2, \ldots, n-1 \\ dx_{\mathrm{in}} = \left\{\rho u_i + f_{\mathrm{in}}(y) + \sum_{m=2}^{n} \phi_{inm}(y_i)x_{im}\right\}dt \\ \qquad\quad + g_{\mathrm{in}}^T(y)dw \\ y_i = x_{i1} \end{cases} \tag{1}$$

where for $i = 0, \ldots, N$, u_i and y_i are the input and output of the ith agent, respectively. $y := [y_1, \ldots, y_N]^T$. $\rho > 0$ is the unknown positive constant. $w(t) \in R^m$ is an independent standard Wiener process defined on a complete probability space. $\phi_{ism}(y_i)$ is the known nonlinear function. $f_{is}(y)$ and $g_{is}(y) \in R^m$ represent the uncertain interactions from the other subsystems.

The objectives of this article are listed as follows.

1) Construct the dynamic gain filter to compensate the unmeasured states.
2) Design the dynamic gain filter-based distributed output-feedback adaptive fuzzy protocol for the stochastic nonlinear MASs (1) to achieve the leader-following consensus.

The following assumptions will be used.

Assumption 1: The leader's output $y_0 \in R$ and its derivative $\dot{y}_0$ are bounded, and y_0 is available for the ith follower satisfying $0 \in N_i$, $i = 1, \ldots, n$.

Assumption 2: The graph G constructed by N followers is undirected, and for each follower, there exists at least one directed path from the leader to the followers.

Remark 1: The studied MASs (1) is more general than the ones studied in some existing results, such as [26], i.e., the interaction terms are more general and the stochastic disturbances are further considered. In addition, since the diffusion terms $g_{is}^T(y)$ are different, each agent may encounter different disturbances. $w(t) \in R^m$ is just a collection of all kinds of possible disturbances. Assumption 1 is also a general condition and has been used in many literature [14], [19], [26]. Assumption 2 indicates that the matrix $(L + B)$ is positive definite, referring to [20] and [25]. In fact, combined with the dynamic surface control method [14] or the hierarchical decomposition method [26], the results of this article can be easily extended to the directed graph case, and at the same time, other restrictions will be required.

C. Fuzzy Logic Systems

The FLSs has been used as an effective tool to approximate the uncertain functions in many literature [29]–[37]. In this article, we consider an FLS that consists of the singleton fuzzifier, center average defuzzifier, product-inference rule, and the Gaussian membership function.

First, the fuzzy rule base consists of fuzzy IF-THEN rules

$$R_l: \ \text{If } x_1 \text{ is } F_1^l \text{ and } \cdots \text{ and } x_{\varkappa} \text{ is } F_{\varkappa}^l, \text{ then } y \text{ is } G^l \tag{2}$$

where $l = 1, 2, \ldots, M$ is the number of the rules. $x = [x_1, \ldots, x_{\varkappa}]^T \in R^{\varkappa}$ and $y \in R$ are the inputs and output of FLSs, respectively. F_s^l and G^l are the fuzzy sets, whose fuzzy membership functions are $\mu_{F_s^l}(x_s)$ and $\mu_{G^l}(y)$.

Define the fuzzy basis functions as

$$\varphi_l = \frac{\prod_{s=1}^{\varkappa} \mu_{F_s^l}(x_s)}{\sum_{l=1}^{M}\left(\prod_{s=1}^{\varkappa} \mu_{F_s^l}(x_s)\right)}. \tag{3}$$

Then, FLSs can be represented as follows referring to [37]:

$$f(x) = \frac{\sum_{l=1}^{M} \theta_l\left(\prod_{s=1}^{\varkappa} \mu_{F_s^l}(x_s)\right)}{\sum_{l=1}^{M}\left(\prod_{s=1}^{\varkappa} \mu_{F_s^l}(x_s)\right)} = \theta^T \varphi \tag{4}$$

in which the parameter vector $\theta = [\theta_1, \ldots, \theta_M]^T$ is the collection of the point where $\mu_{G^l}(y)$ achieves its maximum value.

The following lemmas are useful for the controller design.

Lemma 1 [37]: Let $F(x)$ be a continuous function defined on a compact set Ω. Then, for any constant $\varepsilon^* > 0$, there exists an FLS $f(x)$ such that $\sup_{x\in\Omega} |F(x) - f(x)| < \varepsilon^*$.

Lemma 2 [38]: Let c and d be positive real numbers and $\gamma(x, y) > 0$ be a real-valued function. Then, $|x|^c|y|^d \le \big[c\gamma(x, y)|x|^{c+d}\big]/(c + d) + \big[d\gamma^{-c/d}(x, y)|y|^{c+d}\big]/(c + d)$.

Lemma 3 [39]: Let $x \in R^n$, $y \in R^m$, and $f : R^n \times R^m \to R$ be a continuous function. Then, there are smooth scalar functions $a(x) \ge 0$, $b(y) \ge 0$, such that $\|f(x, y)\| \le a(x) + b(y)$.

Lemma 4: If for $x = [x_1, \ldots, x_n]^T$, the C^1 function $f(x)$ satisfies $f(0) = 0$, then there exist positive smooth functions $b_1(x_1),\ldots, b_n(x_n)$ such that $|f(x)| \le \sum_{i=1}^n |x_i|b_i(x_i)$.

Proof: By the mean value theorem, we have

$$f(x) - f(0) = \int_0^1 df(\theta x) = R(x)x \tag{5}$$

where $R(x) = \int_0^1 (\partial f/\partial\beta)|_{\beta=\theta x} d\theta$ is a $1 \times n$ covector.

Due to $f(0) = 0$, there exists a continuous function $\bar{f}(x)$ such that

$$|f(x)| \le \|R(x)\|\|x\| \le \bar{f}(x)(\|x_1\| + \cdots + \|x_n\|). \tag{6}$$

Using Lemma 3, we have

$$\begin{aligned}
&\bar{f}(x)(\|x_1\| + \cdots + \|x_n\|)\\
&\le \|x_1\|\big(\bar{b}_1(x_1) + \cdots + \bar{b}_n(x_n)\big)\\
&\quad + \ldots + \|x_n\|\big(\bar{b}_1(x_1) + \cdots + \bar{b}_n(x_n)\big)\\
&\le \|x_1\|\big(\bar{b}_1(x_1) - \bar{b}_1(0) + \cdots + \bar{b}_n(x_n) - \bar{b}_n(0)\big)\\
&\quad + \|x_1\|\big(\bar{b}_1(0) + \cdots + \bar{b}_n(0)\big) + \cdots\\
&\quad + \|x_n\|\big(\bar{b}_1(x_1) - \bar{b}_1(0) + \cdots + \bar{b}_n(x_n) - \bar{b}_n(0)\big)\\
&\quad + \|x_n\|\big(\bar{b}_1(0) + \cdots + \bar{b}_n(0)\big).
\end{aligned} \tag{7}$$

By the smoothness, there exists a smooth function $\tilde{b}_j(x_j)$ such that $\bar{b}_j(x_j) - \bar{b}_j(0) = x_j\tilde{b}_j(x_j)$, so we have

$$\begin{aligned}
\|x_i\|\big(\bar{b}_j(x_j) - \bar{b}_j(0)\big) &\le \frac{1}{2}x_i^2 + \frac{1}{2}x_j^2\tilde{b}_j^2(x_j)\\
&\le \frac{1}{2}|x_i|\Big(1 + x_i^2\Big) + \frac{1}{2}|x_j|\Big(1 + x_j^2\Big)\tilde{b}_j^2(x_j).
\end{aligned} \tag{8}$$

Combined with (7), the result of Lemma 4 can be easily obtained. The proof is completed.

Remark 2: From Lemma 4, it is easy to obtain that for any smooth function $\bar{\alpha}(x)$, there exist positive smooth functions $\bar{b}_i(x_i)$ such that $\bar{\alpha}(x) = (\bar{\alpha}(x) - \bar{\alpha}(0)) + \bar{\alpha}(0) \le \sum_{i=1}^n |x_i|\bar{b}_i(x_i) + \bar{\alpha}(0)$, which is very useful for designing the distributed protocol in the sequel. Lemma 4 is used to decompose the uncertain interactions, which will be approximated by the FLSs. Hence, in (5), $R(x)$ need not be calculated in the controller design process.

III. Distributed Leader-Following Consensus

In this section, the dynamic gain filter is constructed to compensate unmeasured states. Then, by the backstepping method, the filter-based distributed output-feedback adaptive fuzzy controller is designed.

A. Dynamic Gain Filter Design

Construct the dynamic gain filter as follows:

$$d\zeta_i = \big\{\big(A - l_iL_{i0}qc^T\big)\zeta_i + l_iL_{i0}qy_i + \phi_i(y_i)\zeta_i\big\}dt \tag{9}$$

$$d\nu_i = \big\{\big(A - l_iL_{i0}qc^T\big)\nu_i + Eu_i + \phi_i(y_i)\nu_i\big\}dt \tag{10}$$

where $\nu_i = [\nu_{i1}, \ldots, \nu_{\text{in}}]^T$ and $\zeta_i = [\zeta_{i1}, \ldots, \zeta_{\text{in}}]^T$. $l_i \ge 1$ is the designed dynamic gain satisfying (23). $L_{i0} := \text{diag}\{1, l_i, \ldots, l_i^{n-1}\}$. $E = [0, \ldots, 0, 1]^T \in R^n$. $c = [1, 0, \ldots, 0]^T \in R^n$. $q = [q_1, \ldots, q_n]^T$ is to be designed satisfying (16). The matrix A and $\phi_i(y_i)$ are as follows:

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0\\ 0 & 0 & 1 & \cdots & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & 0 & \cdots & 1\\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \in R^{n\times n} \tag{11}$$

$$\phi_i(y_i) = \begin{bmatrix} 0 & 0 & \cdots & 0\\ 0 & \phi_{i22} & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & \phi_{in2} & \cdots & \phi_{inn} \end{bmatrix}. \tag{12}$$

Defining the virtual estimate of x_i as $\hat{x}_i = [\hat{x}_{i1}, \ldots, \hat{x}_{\text{in}}]^T = \zeta_i + \rho\nu_i$, and the estimate error as $e_i = [e_{i1}, \ldots, e_{\text{in}}]^T = x_i - \hat{x}_i$, from (1), (9), and (10), one has

$$\begin{aligned}
de_i = &\big\{\big(A - l_iL_{i0}qc^T\big)e_i + f_i(y) + \phi_i(y_i)e_i\big\}dt\\
&+ g_i^T(y)dw
\end{aligned} \tag{13}$$

where $f_i(y) = [f_{i1}(y), f_{i2}(y), \ldots, f_{\text{in}}(y)]^T$ and $g_i^T(y) = [g_{i1}(y), g_{i2}(y), \ldots, g_{\text{in}}(y)]^T$.

Defining $\epsilon_i = [\epsilon_{i1}, \ldots, \epsilon_{\text{in}}]^T = l_i^{-\mu}L_{i0}^{-1}e_i$ and $\mu > 0$, then we have

$$\begin{aligned}
d\epsilon_i = &\Big\{l_i^{-\mu}L_{i0}^{-1}\big(A - l_iL_{i0}qc^T\big)e_i + l_i^{-\mu}L_{i0}^{-1}f_i(y)\\
&\quad + l_i^{-\mu}L_{i0}^{-1}\phi_i(y_i)e_i - \dot{l}_il_i^{-1}(\mu I_n + D_i)l_i^{-\mu}L_{i0}^{-1}e_i\Big\}dt\\
&+ l_i^{-\mu}L_{i0}^{-1}g_i^T(y)dw\\
= &\Big\{l_i\big(A - qc^T\big)\epsilon_i + l_i^{-\mu}L_{i0}^{-1}f_i(y) + L_{i0}^{-1}\phi_i(y_i)L_{i0}\epsilon_i\\
&\quad - \dot{l}_il_i^{-1}(\mu I_n + D)\epsilon_i\Big\}dt + l_i^{-\mu}L_{i0}^{-1}g_i^T(y)dw
\end{aligned} \tag{14}$$

where $D = \text{diag}\{0, 1, \ldots, n - 1\}$.

Choose the Lyapunov function candidate as

$$V_{ei} = \epsilon_i^TP\epsilon_i \tag{15}$$

where the positive-definite matrix P is chosen such that

$$P\big(A - qc^T\big) + \big(A - qc^T\big)^TP \le -I_n. \tag{16}$$

From (14) and (15), the differential operator of V_{ei} is

$$\begin{aligned}\ell V_{ei} &= l_i 2\epsilon_i^T P\big(A - qc^T\big)\epsilon_i + 2\epsilon_i^T P l_i^{-\mu} L_{i0}^{-1} f_i(y)\\ &\quad + 2\epsilon_i^T P L_{i0}^{-1}\phi_i(y_i)L_{i0}\epsilon_i - 2\epsilon_i^T P \dot{l}_i l_i^{-1}\\ &\quad \times (\mu I_n + D_i)\epsilon_i\\ &\quad + \mathrm{Tr}\left\{\left[l_i^{-\mu} L_{i0}^{-1} g_i^T(y)\right]^T P\left[l_i^{-\mu} L_{i0}^{-1} g_i^T(y)\right]\right\}. \end{aligned} \tag{17}$$

From (16), we have

$$\begin{aligned} l_i 2\epsilon_i^T P\big(A - qc^T\big)\epsilon_i &= l_i\epsilon_i^T\Big(P\big(A - qc^T\big) + \big(A - qc^T\big)^T P\Big)\epsilon_i\\ &\le -l_i\epsilon_i^T\epsilon_i. \end{aligned} \tag{18}$$

Due to the special structure of $\phi_i(y_i)$ as (12), we have

$$\begin{aligned} &L_{i0}^{-1}\phi_i(y_i)L_{i0}\\ &= \begin{bmatrix} 0 & 0 & \cdots & 0 & 0\\ 0 & \phi_{i22} & \cdots & 0 & 0\\ 0 & l_i^{-1}\phi_{i32} & \cdots & 0 & 0\\ \vdots & \vdots & \ddots & \vdots & \vdots\\ 0 & l_i^{2-n}\phi_{in2} & \cdots & l_i^{-1}\phi_{\mathrm{in}(n-1)} & \phi_{\mathrm{in}n} \end{bmatrix}. \end{aligned} \tag{19}$$

Combined with the fact $l_i \ge 1$, i.e., $l_i^{-1} \le 1$, $l_i^{-2} \le 1, \ldots$, there exists a positive smooth function $\Phi_i(y_i)$, such that

$$2\epsilon_i^T P L_{i0}^{-1}\phi_i(y_i)L_{i0}\epsilon_i \le \Phi_i(y_i)\epsilon_i^T\epsilon_i \tag{20}$$

where $\Phi_i(y_i)$ is determined by P and $\phi_{ism}(y_i)$. Similarly, there exist positive smooth functions $F_i(y)$ and $G_i(y)$, and positive constant γ_{i1}, such that

$$\begin{aligned} &2\epsilon_i^T P l_i^{-\mu} L_{i0}^{-1} f_i(y)\\ &\le 2\big\|\epsilon_i^T P\big\|\,\|f_i(y)\|\\ &\le \gamma_{i1}\epsilon_i^T\epsilon_i + l_i^{-2\mu}F_i^2(y) \end{aligned} \tag{21}$$

$$\mathrm{Tr}\left\{\left[l_i^{-\mu} L_{i0}^{-1} g_i^T(y)\right]^T P\left[l_i^{-\mu} L_{i0}^{-1} g_i^T(y)\right]\right\} \le l_i^{-2\mu}G_i^2(y). \tag{22}$$

Design $\dot{l}_i$ as

$$\dot{l}_i = l_i(-\eta_{i1} l_i + \eta_{i2} + \bar{\varphi}_i(y_i)),\ \ l_i(0) = \frac{\eta_{i2}}{\eta_{i1}} \ge 1 \tag{23}$$

where the designed positive constants η_{i2} and η_{i1}, and the designed non-negative smooth function $\bar{\varphi}_i(y_i)$ satisfy (27). Since P is a symmetric positive-definite matrix, we can choose a sufficiently large μ such that $2\mu P + PD + DP$ is a positive definite. Then, there will exist positive constants μ_1 and μ_2 such that

$$\mu_1 I_n \le 2\mu P + PD + DP \le \mu_2 I_n. \tag{24}$$

From (23) and (24), we have

$$\begin{aligned} &-\frac{\dot{l}_i}{l_i}\epsilon_i^T(2\mu P + PD + DP)\epsilon_i\\ &\quad \le \mu_2\eta_{i1} l_i\epsilon_i^T\epsilon_i - \mu_1(\eta_{i2} + \bar{\varphi}_i(y_i))\epsilon_i^T\epsilon_i. \end{aligned} \tag{25}$$

From (17)–(25), we have

$$\begin{aligned} \ell V_{ei} \le &-\epsilon_i^T\epsilon_i(l_i(1 - \mu_2\eta_{i1}) - \gamma_{i1}\\ &+ \mu_1(\eta_{i2} + \bar{\varphi}_i(y_i)) - \Phi_i(y_i))\\ &+ l_i^{-2\mu}F_i^2(y) + l_i^{-2\mu}G_i^2(y). \end{aligned} \tag{26}$$

Designing μ_2, η_{i1}, and η_{i2}, $\bar{\varphi}_i(y_i)$ satisfies

$$\begin{cases} 1 - \mu_2\eta_{i1} \ge \frac{1}{2}, \quad \eta_{i2} \ge \eta_{i1} > 0\\ \mu_1(\eta_{i2} + \bar{\varphi}_i(y_i)) - \Phi_i(y_i) - \gamma_{i1} \ge 0. \end{cases} \tag{27}$$

Then, we have

$$\ell V_{ei} \le -\frac{1}{2} l_i\epsilon_i^T\epsilon_i + l_i^{-2\mu}F_i^2(y) + l_i^{-2\mu}G_i^2(y). \tag{28}$$

Remark 3: In (1), if $f_{is}(y)$ can be modeled as $f_{is}(y) = \bar{\theta}_i^T f_{0is}(y)$, where $\bar{\theta}_i \in R^{\varrho}$ is an unknown constant vector and $f_{0is}(y) \in R^{\varrho}$ is a known smooth function, then with the method in [27] and [28], $(2 + \varrho)n$ dynamic variables should be introduced. In (9) and (10), we only introduce $2n$ dynamic variables to compensate the unmeasured states and we only assume the uncertain function $f_{is}(y)$ is smooth. Compared with [26], the nonlinear terms in (1) are more general.

Remark 4: In this section, by introducing the dynamic gain (23), the term $\phi_{ism}(\cdot)$ can be nonlinear function about the output y_i, which makes the filter be applicable for a wider class of nonlinear systems. Especially, if $\phi_{ism}(\cdot)$ is a known constant, we can choose constant gain l_i to construct the filter. In (27), μ_2 and η_{i1} are chosen such that $1 - \mu_2\eta_{i1} \ge (1/2)$, where the constant $(1/2)$ can be chosen as an other positive value greater than zero and less than 1. For convenience, we chose $(1/2)$.

B. Distributed Output-Feedback Adaptive Fuzzy Leader-Following Controller Design

First, define local neighborhood consensus error as

$$z_{i1} = \sum_{j=1}^{N} a_{ij}\big(y_i - y_j\big) + b_i(y_i - y_0). \tag{29}$$

Then, we have

$$\bar{z}_1 = (L + B)(y - \mathbf{1}_N y_0) \tag{30}$$

where $\bar{z}_1 = [z_{11}, z_{21}, \ldots, z_{N1}]^T$, $y = [y_1, y_2, \ldots, y_N]^T$, $\mathbf{1}_N \in \mathbf{R}^N$ is a unit vector, and

$$\begin{aligned} d\bar{z}_1 = &(L + B)(\bar{x}_2 + \mathbf{f}_1(y) - \mathbf{1}_N\dot{y}_0)dt\\ &+ (L + B)\mathbf{g}_1^T(y)dw \end{aligned} \tag{31}$$

where $\bar{x}_2 := [x_{12}, \ldots, x_{N2}]^T$, $\mathbf{f}_1(y) = [f_{11}(y), \ldots, f_{N1}(y)]^T$, and $\mathbf{g}_1^T(y) = [g_{11}(y), \ldots, g_{N1}(y)]^T$. For $s = 2, \ldots, n$, define the corresponding transformation as

$$z_{is} = v_{is} - \alpha_{i(s-1)} \tag{32}$$

where $\alpha_{i(s-1)}$ is the designed virtual controller.

Choose the Lyapunov function as

$$\begin{cases} \bar{V} = \sum_{s=1}^{n} V_s + V_\theta + V_\beta + V_\rho\\ V_1 = \frac{1}{2}\bar{z}_1^T(L + B)^{-1}\bar{z}_1\\ V_s = \frac{1}{4}\sum_{i=1}^{N} z_{is}^4, \quad s = 2, \ldots, n\\ V_\beta = \frac{1}{2}\sum_{i=1}^{N}\rho\tilde{\beta}_i^2, \quad V_\rho = \frac{1}{2}\sum_{i=1}^{N}\tilde{\rho}_i^2\\ V_\theta = \frac{1}{2}\sum_{i=1}^{N}\tilde{\theta}_i^T\Gamma_i^{-1}\tilde{\theta}_i \end{cases} \tag{33}$$

where $\tilde{\beta}_i := \rho^{-1} - \hat{\beta}_i$, $\tilde{\rho}_i := \rho - \hat{\rho}_i$, and $\tilde{\theta}_i := \theta_i - \hat{\theta}_i$. θ_i is the ideal constant weight of FLSs defined in (41). Γ_i is the designed positive-definite matrix.

Step 1: Define $\bar{\zeta}_2 := [\zeta_{12}, \ldots, \zeta_{N2}]^T$, $\bar{z}_2 := [z_{12}, \ldots, z_{N2}]^T$, $\bar{\alpha}_1 := [\alpha_{11}, \ldots, \alpha_{N1}]^T$, and $\bar{e}_2 := [e_{12}, \ldots, e_{N2}]^T$. Due to $\hat{x}_i = \zeta_i + \rho\nu_i$ and $e_i = x_i - \hat{x}_i$, combined with (32), the differential operator of V_1 is

$$\begin{aligned}\ell V_1 = &\bar{z}_1^T\big(\bar{\zeta}_2 + \rho(\bar{z}_2 + \bar{\alpha}_1) + \bar{e}_2 + \mathbf{f}_1(y) - \mathbf{1}_N\dot{y}_0\big)\\ &+ \frac{1}{2}\mathrm{tr}\big\{\mathbf{g}_1(y)(L+B)^T\mathbf{g}_1^T(y)\big\}. \end{aligned}\tag{34}$$

Using Lemma 2 and the transformation $\epsilon_{i2} = l_i^{-1-\mu}e_{i2}$, we have

$$\rho\bar{z}_1^T\bar{z}_2 \le \frac{3}{4}\rho^{\frac{4}{3}}\sum_{i=1}^N z_{i1}^{\frac{4}{3}} + \frac{1}{4}\sum_{i=1}^N z_{i2}^4 \tag{35}$$

$$\begin{aligned}\bar{z}_1^T\bar{e}_2 &= \bar{z}_1^T\Big[l_1^{1+\mu}\epsilon_{12}, \ldots, l_N^{1+\mu}\epsilon_{N2}\Big]^T\\ &\le \frac{1}{2}\sum_{i=1}^N l_i^{2\mu+2}z_{i1}^2 + \frac{1}{2}\sum_{i=1}^N \epsilon_{i2}^2\end{aligned} \tag{36}$$

$$-\bar{z}_1^T\mathbf{1}_N\dot{y}_0 \le \frac{1}{2}\sum_{i=1}^N z_{i1}^2 + \frac{1}{2}N\dot{y}_0^2. \tag{37}$$

By Lemma 4, Assumption 1, and (30), there exist smooth positive functions $\bar{f}_{i1}(z_{i1})$ such that

$$\begin{aligned}\bar{z}_1^T\mathbf{f}_1(y) &\le \sum_{i=1}^N |z_{i1}|\Big|f_{i1}\Big((L+B)^{-1}\bar{z}_1 + \mathbf{1}_N y_0\Big) - f_{i1}(0) + f_{i1}(0)\Big|\\ &\le \sum_{i=1}^N z_{i1}^2\bar{f}_{i1}(z_{i1}) + \Phi_{11}\end{aligned} \tag{38}$$

where Φ_{11} is a bounded positive constant.

Similar with (38), there exist smooth positive function $\bar{g}_{i1}(z_{i1})$ and bounded positive constant Φ_{12} such that

$$\frac{1}{2}\mathrm{tr}\big\{\mathbf{g}_1(y)(L+B)^T\mathbf{g}_1^T(y)\big\} \le \sum_{i=1}^N z_{i1}^2\bar{g}_{i1}(z_{i1}) + \Phi_{12}. \tag{39}$$

From (34)–(39), we have

$$\begin{aligned}\ell V_1 \le &\sum_{i=1}^N z_{i1}\bigg(\zeta_{i2} + \rho\alpha_{i1} + \frac{1}{2}l_i^{2\mu+2}z_{i1} + \frac{1}{2}z_{i1} + \Psi_i(z_{i1})\\ &- \Psi_i(z_{i1}) + z_{i1}\big(\bar{f}_{i1}(z_{i1}) + \bar{g}_{i1}(z_{i1})\big) + \frac{3}{4}\rho^{\frac{4}{3}}z_{i1}^{\frac{1}{3}}\bigg)\\ &+\sum_{i=1}^N\frac{\epsilon_{i2}^2}{2} + \sum_{i=1}^N\frac{z_{i2}^4}{4} + \frac{1}{2}N\dot{y}_0^2 + \Phi_{11} + \Phi_{12}\end{aligned} \tag{40}$$

where $\Psi_i(z_{i1})$ is the uncertain function defined in (69).

Remark 5: The structure of $\Psi_i(z_{i1})$ is only related with the interaction terms which will be decomposed in each step of the backstepping method. The detail form of $\Psi_i(z_{i1})$ is determined in (69). What we emphasize is that since $\Psi_i(z_{i1})$ only contains the outputs information, it does not hinder the design of each virtual controller α_{is}. If the interaction terms $f_{is}(y)$ and $g_{is}^T(y)$ are known, then $\Psi_i(z_{i1})$ can be further adjusted and dealt with in α_{i1}. In this article, we use FLSs to approximate this uncertain function.

According to Lemma 1, the uncertain function $\Psi_i(z_{i1})$ can be approximated by the FLSs $\theta_i^T\varphi_i(z_{i1})$. The ideal constant weight vector θ_i is defined as

$$\theta_i = \arg\min_{\theta_i^*\in U_i}\left[\sup_{z_{i1}\in\Omega_i}\left|\hat{\theta}_i^T\varphi_i(z_{i1}) - \Psi_i(z_{i1})\right|\right] \tag{41}$$

where U_i and Ω_i are compact sets for θ_i and z_{i1}, respectively.

Define the minimum fuzzy approximation error as

$$\varepsilon_{i1}(z_{i1}) = \Psi_i(z_{i1}) - \theta_i^T\varphi_i(z_{i1}) \tag{42}$$

where $\varepsilon_{i1}(z_{i1}) \le \varepsilon_{i1}^*$, and ε_{i1}^* is any positive constant.

Design the virtual controller α_{i1} and the adaptive laws as

$$\alpha_{i1} = -\hat{\beta}_i\bar{\alpha}_{i1} \tag{43}$$

$$\dot{\hat{\beta}}_i = z_{i1}\bar{\alpha}_{i1} - \sigma\hat{\beta}_i \tag{44}$$

$$\dot{\hat{\theta}}_i = \Gamma_i z_{i1}\varphi_i(z_{i1}) - \sigma\Gamma_i\hat{\theta}_i \tag{45}$$

where $\bar{\alpha}_{i1} = \zeta_{i2} + (1/2)l_i^{2\mu+2}z_{i1} + z_{i1} + \hat{\theta}_i^T\varphi_i(z_{i1}) + kz_{i1}$. α_{i1} only contains the states information of the ith agent and its neighbors. k and σ are designed positive constants.

From (33) and (40)–(45), we have

$$\begin{aligned}&\ell V_1 + \ell V_\theta + \ell V_\beta\\ &\le -\sum_{i=1}^N kz_{i1}^2 + \sum_{i=1}^N z_{i1}(\Psi_{i1}(z_{i1}) - \Psi_i(z_{i1}))\\ &\quad+\sum_{i=1}^N\frac{\epsilon_{i2}^2}{2} + \sum_{i=1}^N\frac{z_{i2}^4}{4} + \sigma\sum_{i=1}^N\hat{\theta}_i^T\tilde{\theta}_i + \sigma\sum_{i=1}^N\rho\hat{\beta}_i\tilde{\beta}_i\\ &\quad+\frac{1}{2}\sum_{i=1}^N\varepsilon_{i1}^{*2} + \frac{1}{2}N\dot{y}_0^2 + \Phi_{11} + \Phi_{12}\end{aligned} \tag{46}$$

where $\Psi_{i1}(z_{i1}) := z_{i1}(\bar{f}_{i1}(z_{i1}) + \bar{g}_{i1}(z_{i1})) + (3/4)\rho^{(4/3)}z_{i1}^{(1/3)}$.

Step 2: Calculate the differential operator of V_2 as

$$\begin{aligned}&\ell V_2\\ &= \sum_{i=1}^N z_{i2}^3\Big(\nu_{i3} - l_i^2 q_2\nu_{i1} + \phi_{i22}(y_i)\nu_{i2} - \ell\alpha_{i1}\Big) + \sum_{i=1}^N\frac{3z_{i2}^2}{2}\\ &\quad\times\mathrm{tr}\left\{\left[\sum_{j\in\{N_i,i\}}\frac{\partial\alpha_{i1}}{\partial x_{j1}}g_{j1}^T(y)\right]^T\left[\sum_{k\in\{N_i,i\}}\frac{\partial\alpha_{i1}}{\partial x_{k1}}g_{k1}^T(y)\right]\right\}\\ &= \sum_{i=1}^N z_{i2}^3\Bigg(z_{i3} + \alpha_{i2} - l_i^2 q_2\nu_{i1} + \phi_{i22}(y_i)\nu_{i2} + \Xi_{i2}\\ &\qquad - \frac{\partial\alpha_{i1}}{\partial y_0}\dot{y}_0 - \sum_{j\in\{N_i,i\}}\frac{\partial\alpha_{i1}}{\partial x_{j1}}\big(\rho\nu_{j2} + e_{j2} + f_{j1}(y)\big)\\ &\qquad - \frac{1}{2}\sum_{j,k\in\{N_i,i\}}\frac{\partial^2\alpha_{i1}}{\partial x_{j1}\partial x_{k1}}g_{j1}^T(y)g_{k1}(y)\Bigg)\\ &\quad+\sum_{i=1}^N\frac{3z_{i2}^2}{2}\mathrm{tr}\left\{\sum_{j,k\in\{N_i,i\}}\frac{\partial\alpha_{i1}}{\partial x_{j1}}\frac{\partial\alpha_{i1}}{\partial x_{k1}}g_{j1}(y)g_{k1}^T(y)\right\}\end{aligned} \tag{47}$$

where Ξ_{i2} denotes all the other known terms in $-\ell\alpha_{i1}$ and only contains the ith agent's and its neighbors' information. Using

Lemma 2, Lemma 4, and Assumption 1, there exist smooth positive functions $\bar{f}_{i2}(z_{i1})$ and $\bar{g}_{i2}(z_{i1})$, and bound positive constants Φ_{21} and Φ_{22} such that

$$\sum_{i=1}^{N} z_{i2}^3 z_{i3} \leq \frac{3}{4}\sum_{i=1}^{N} z_{i2}^4 + \frac{1}{4}\sum_{i=1}^{N} z_{i3}^4 \tag{48}$$

$$\begin{aligned}&-\sum_{i=1}^{N}\sum_{j\in\{N_i,i\}} z_{i2}^3 \frac{\partial\alpha_{i1}}{\partial x_{j1}} e_{j2} - \sum_{i=1}^{N} z_{i2}^3 \frac{\partial\alpha_{i1}}{\partial y_0}\dot{y}_0 \\ &\leq \frac{1}{2}\sum_{i=1}^{N}\sum_{j\in\{N_i,i\}} l_j^{2\mu+2} z_{i2}^6 \left(\frac{\partial\alpha_{i1}}{\partial x_{j1}}\right)^2 + \frac{N}{2}\sum_{i=1}^{N}\epsilon_{i2}^2 \\ &\quad + \frac{3}{4}\sum_{i=1}^{N} z_{i2}^4\left(\left(\frac{\partial\alpha_{i1}}{\partial y_0}\right)^2 + 1\right) + \frac{N}{4}\dot{y}_0^4\end{aligned} \tag{49}$$

$$\begin{aligned}&-\sum_{i=1}^{N} z_{i2}^3\left(\sum_{j\in\{N_i,i\}}\frac{\partial\alpha_{i1}}{\partial x_{j1}} f_{j1} + \sum_{j,k\in\{N_i,i\}}\frac{\partial^2\alpha_{i1}}{\partial x_{j1}\partial x_{k1}}\frac{g_{j1}^T g_{k1}}{2}\right) \\ &\leq \sum_{i=1}^{N} z_{i1}^4 \bar{f}_{i2}(z_{i1}) + \Phi_{21} \\ &\quad + \sum_{i=1}^{N} z_{i2}^4\left(\sum_{j\in\{N_i,i\}}\left(\frac{\partial\alpha_{i1}}{\partial x_{j1}}\right)^{\frac{4}{3}} + \sum_{j,k\in\{N_i,i\}}\left(\frac{\partial^2\alpha_{i1}}{\partial x_{j1}\partial x_{k1}}\right)^{\frac{4}{3}} + 1\right) \\ &\leq \sum_{i=1}^{N} z_{i1}^4 \bar{f}_{i2}(z_{i1}) + \Phi_{21} \\ &\quad + \sum_{i=1}^{N} z_{i2}^4\left(\sum_{j\in\{N_i,i\}}\left(\frac{\partial\alpha_{i1}}{\partial x_{j1}}\right)^2 \right. \\ &\qquad\qquad \left. + N + \sum_{j,k\in\{N_i,i\}}\left(\frac{\partial^2\alpha_{i1}}{\partial x_{j1}\partial x_{k1}}\right)^2 + N^2 + 1\right)\end{aligned} \tag{50}$$

$$\begin{aligned}&\sum_{i=1}^{N}\frac{3z_{i2}^2}{2}\mathrm{tr}\left\{\sum_{j,k\in\{N_i,i\}}\frac{\partial\alpha_{i1}}{\partial x_{j1}}\frac{\partial\alpha_{i1}}{\partial x_{k1}} g_{j1}(y) g_{k1}^T(y)\right\} \\ &\leq \sum_{i=1}^{N} z_{i2}^4\left(\sum_{j,k\in\{N_i,i\}}\left(\frac{\partial\alpha_{i1}}{\partial x_{j1}}\frac{\partial\alpha_{i1}}{\partial x_{k1}}\right)^2 + 1\right) \\ &\quad + \sum_{i=1}^{N} z_{i1}^4 \bar{g}_{i2}(z_{i1}) + \Phi_{22}.\end{aligned} \tag{51}$$

It follows from (47)–(51) that:

$$\begin{aligned}&\ell V_2 \\ &= \sum_{i=1}^{N} z_{i2}^3\left(\alpha_{i2} - l_i^2 q_2 \nu_{i1} + \phi_{i22}(y_i)\nu_{i2}\right. \\ &\quad + \frac{3}{4}z_{i2} + \Xi_{i2} + \frac{3}{4}z_{i2}\left(\left(\frac{\partial\alpha_{i1}}{\partial y_0}\right)^2 + 1\right) \\ &\quad - \sum_{j\in\{N_i,i\}}\frac{\partial\alpha_{i1}}{\partial x_{j1}}\rho\nu_{j2} + \frac{1}{2}\sum_{j\in\{N_i,i\}} l_j^{2\mu+2} z_{i2}^3\left(\frac{\partial\alpha_{i1}}{\partial x_{j1}}\right)^2 \\ &\quad + z_{i2}\left(\sum_{j\in\{N_i,i\}}\left(\frac{\partial\alpha_{i1}}{\partial x_{j1}}\right)^2 + \sum_{j,k\in\{N_i,i\}}\left(\frac{\partial^2\alpha_{i1}}{\partial x_{j1}\partial x_{k1}}\right)^2\right) \\ &\quad \left. + z_{i2}\left(N^2+N+2\right) + z_{i2}\sum_{j,k\in\{N_i,i\}}\left(\frac{\partial\alpha_{i1}}{\partial x_{j1}}\frac{\partial\alpha_{i1}}{\partial x_{k1}}\right)^2\right) \\ &\quad + \frac{1}{4}\sum_{i=1}^{N} z_{i3}^4 + \sum_{i=1}^{N} z_{i1}^4 \bar{f}_{i2}(z_{i1}) + \sum_{i=1}^{N} z_{i1}^4 \bar{g}_{i2}(z_{i1}) \\ &\quad + \frac{N}{2}\sum_{i=1}^{N}\epsilon_{i2}^2 + \frac{N}{4}\dot{y}_0^4 + \Phi_{21} + \Phi_{22}.\end{aligned} \tag{52}$$

Design the virtual controller α_{i2} as

$$\begin{aligned}\alpha_{i2} = -&\left(k z_{i2} - l_i^2 q_2 \nu_{i1} + \phi_{i22}(y_i)\nu_{i2} + z_{i2} + \Xi_{i2} + \frac{3}{4}z_{i2}\right. \\ &\times\left(\left(\frac{\partial\alpha_{i1}}{\partial y_0}\right)^2 + 1\right) + \frac{1}{2}\sum_{j\in\{N_i,i\}} l_j^{2\mu+2} z_{i2}^3\left(\frac{\partial\alpha_{i1}}{\partial x_{j1}}\right)^2 \\ &- \sum_{j\in\{N_i,i\}}\frac{\partial\alpha_{i1}}{\partial x_{j1}}\hat{\rho}_i\nu_{j2} + z_{i2}\sum_{j\in\{N_i,i\}}\left(\frac{\partial\alpha_{i1}}{\partial x_{j1}}\right)^2 \\ &+ z_{i2}\sum_{j,k\in\{N_i,i\}}\left(\frac{\partial^2\alpha_{i1}}{\partial x_{j1}\partial x_{k1}}\right)^2 + z_{i2}\left(N^2 + N + 2\right) \\ &\left. + z_{i2}\sum_{j,k\in\{N_i,i\}}\left(\frac{\partial\alpha_{i1}}{\partial x_{j1}}\frac{\partial\alpha_{i1}}{\partial x_{k1}}\right)^2\right)\end{aligned} \tag{53}$$

where the term $(\partial\alpha_{i1}/\partial y_0)$ is used. Obviously, if the ith agent cannot receive the leader's information, then α_{i1} will not contain the information y_0 and this term will be zero. Hence, α_{i2} only contains the ith agent's and its neighbors' information. From (33), (52), and (53), we have

$$\begin{aligned}&\ell V_2 + \ell V_\rho \\ &\leq -\sum_{i=1}^{N} k z_{i2}^4 + \sum_{i=1}^{N}\left(\frac{z_{i3}^4}{4} - \frac{z_{i2}^4}{4}\right) + \sum_{i=1}^{N} z_{i1}\Psi_{i2}(z_{i1}) + \Phi_{21} \\ &\quad + \sum_{i=1}^{N}\frac{N\epsilon_{i2}^2}{2} + \sum_{i=1}^{N}\tilde{\rho}_i\left(\tau_{i1} - \dot{\hat{\rho}}_i\right) + \frac{N}{4}\dot{y}_0^4 + \Phi_{22}\end{aligned} \tag{54}$$

where $\Psi_{i2}(z_{i1}) := z_{i1}^3(\bar{f}_{i2}(z_{i1}) + \bar{g}_{i2}(z_{i1}))$ and $\tau_{i1} = -z_{i2}^3\sum_{j\in\{N_i,i\}}(\partial\alpha_{i1}/\partial x_{j1})\nu_{j2}$.

Combined with (46), we have

$$\begin{aligned}&\ell V_1 + \ell V_2 + \ell V_\theta + \ell V_\beta + \ell V_\rho \\ &\leq -\sum_{i=1}^{N} k\left(z_{i1}^2 + z_{i2}^4\right) + \sum_{i=1}^{N} z_{i1}\left(\sum_{j=1}^{2}\Psi_{ij}(z_{i1}) - \Psi_i(z_{i1})\right) \\ &\quad + \sum_{i=1}^{N}\frac{z_{i3}^4}{4} + \sum_{i=1}^{N}\frac{(N+1)\epsilon_{i2}^2}{2} + \sum_{i=1}^{N}\tilde{\rho}_i\left(\tau_{i1} - \dot{\hat{\rho}}_i - \sigma\hat{\rho}_i\right) \\ &\quad + \sigma\sum_{i=1}^{N}\hat{\rho}_i\tilde{\rho}_i + \sigma\sum_{i=1}^{N}\hat{\theta}_i^T\tilde{\theta}_i + \sigma\rho\sum_{i=1}^{N}\hat{\beta}_i\tilde{\beta}_i + \frac{1}{2}\sum_{i=1}^{N}\varepsilon_{i1}^{*2} \\ &\quad + \frac{1}{2}N\dot{y}_0^2 + \frac{N}{4}\dot{y}_0^4 + \sum_{j=1}^{2}\left(\Phi_{j1} + \Phi_{j2}\right)\end{aligned} \tag{55}$$

where the unknown parameter ρ will appear in the following design process, whose adaptive law will be designed by the tuning function method and given in the last step.

Step s ($s = 3, \ldots, n-1$): Assume that at step $(s-1)$, there exist continuous function $\Psi_{i(s-1)}(z_{i1})$, smooth function

$\tau_{i(s-2)} = -\sum_{m=2}^{s-1}\sum_{j\in\{N_i,i\}} z_{im}^3([\partial\alpha_{i(m-1)}]/[\partial x_{j1}])\nu_{j2}$, and virtual controller $\alpha_{i(s-1)}(\cdot)$ associated with variables $\hat{\beta}_i$, $\hat{\theta}_i^T$, $\hat{\rho}_i$, l_j, x_{j1}, $\zeta_{j1},\ldots,\zeta_{js}$, $\nu_{j1},\ldots$, and $\nu_{j(s-1)}$, for $j \in \{N_i, i\}$, such that

$$\begin{aligned}\ell\sum_{j=1}^{s-1} V_j &+ \ell V_\theta + \ell V_\beta + \ell V_\rho \\ \leq &-\sum_{i=1}^N k\left(z_{i1}^2 + \sum_{j=2}^{s-1} z_{ij}^4\right) + \sum_{i=1}^N z_{i1}\left(\sum_{j=1}^{s-1}\Psi_{ij} - \Psi_i\right) + \sum_{i=1}^N \frac{z_{is}^4}{4} \\ &+ \sum_{i=1}^N \frac{(s-2)N+1}{2}\epsilon_{i2}^2 + \sigma\sum_{i=1}^N \hat{\theta}_i^T\tilde{\theta}_i + \sigma\sum_{i=1}^N\left(\rho\hat{\beta}_i\tilde{\beta}_i + \hat{\rho}_i\tilde{\rho}_i\right) \\ &+ \sum_{i=1}^N\left(\tilde{\rho}_i + \sum_{m=3}^{s-1} z_{im}^3\frac{\partial\alpha_{i(m-1)}}{\partial\hat{\rho}_i}\right)\left(\tau_{i(s-2)} - \dot{\hat{\rho}}_i - \sigma\hat{\rho}_i\right) \\ &+ \sum_{i=1}^N \frac{\varepsilon_{i1}^{*2}}{2} + \frac{N}{2}\dot{y}_0^2 + \frac{(s-2)N}{4}\dot{y}_0^4 + \sum_{j=1}^{s-1}\left(\Phi_{j1} + \Phi_{j2}\right). \end{aligned} \tag{56}$$

Then, calculate the differential operator of V_s as

$$\begin{aligned}\ell V_s &\\ = &\sum_{i=1}^N z_{is}^3\left(\nu_{i(s+1)} - l_i^2 q_s \nu_{i1} + \sum_{j=2}^s \phi_{isj}(y_i)x_{ij}\nu_{ij} - \ell\alpha_{i(s-1)}\right) \\ &+ \sum_{i=1}^N \frac{3z_{is}^2}{2}\mathrm{tr}\left\{\sum_{j,k\in\{N_i,i\}}\frac{\partial\alpha_{i(s-1)}}{\partial x_{j1}}\frac{\partial\alpha_{i(s-1)}}{\partial x_{k1}}g_{j1}(y)g_{k1}^T(y)\right\} \\ = &\sum_{i=1}^N z_{is}^3\left(z_{i(s+1)} + \alpha_{is} - l_i^2 q_s\nu_{i1} + \sum_{j=2}^s\phi_{isj}(y_i)x_{ij}\nu_{ij} + \Xi_{is}\right. \\ &\quad - \frac{\partial\alpha_{i(s-1)}}{\partial\hat{\rho}_i}\dot{\hat{\rho}}_i - \sum_{j\in\{N_i,i\}}\frac{\partial\alpha_{i(s-1)}}{\partial x_{j1}}\left(\rho\nu_{j2} + e_{j2} + f_{j1}(y)\right) \\ &\quad\left. - \frac{1}{2}\sum_{j,k\in\{N_i,i\}}\frac{\partial^2\alpha_{i(s-1)}}{\partial x_{j1}\partial x_{k1}}g_{j1}^T g_{k1} - \frac{\partial\alpha_{i(s-1)}}{\partial y_0}\dot{y}_0\right)dt \\ &+ \sum_{i=1}^N\frac{3z_{is}^2}{2}\mathrm{tr}\left\{\sum_{j\in\{N_i,i\}}\frac{\partial\alpha_{i(s-1)}}{\partial x_{j1}}\frac{\partial\alpha_{i(s-1)}}{\partial x_{k1}}g_{j1}(y)g_{k1}^T(y)\right\} \end{aligned} \tag{57}$$

where Ξ_{is} denotes all the other known terms in $-\ell\alpha_{i(s-1)}$ and only contains the ith agent's and its neighbors' information.

Using Lemmas 2 and 4, and Assumption 1, there exist smooth positive functions $\bar{f}_{is}(z_{i1})$ and $\bar{g}_{is}(z_{i1})$, and bound positive constants Φ_{s1} and Φ_{s2} such that

$$\sum_{i=1}^N z_{is}^3 z_{i(s+1)} \leq \frac{3}{4}\sum_{i=1}^N z_{is}^4 + \frac{1}{4}\sum_{i=1}^N z_{i(s+1)}^4 \tag{58}$$

$$\begin{aligned} -\sum_{i=1}^N\sum_{j\in\{N_i,i\}} z_{is}^3\frac{\partial\alpha_{i(s-1)}}{\partial x_{j1}}e_{j2} &- \sum_{i=1}^N z_{is}^3\frac{\partial\alpha_{i(s-1)}}{\partial y_0}\dot{y}_0 \\ \leq &\frac{1}{2}\sum_{i=1}^N\sum_{j\in\{N_i,i\}} l_j^{2\mu+2}z_{is}^6\left(\frac{\partial\alpha_{i(s-1)}}{\partial x_{j1}}\right)^2 + \frac{N}{2}\sum_{i=1}^N\epsilon_{i2}^2 \\ &+ \frac{3}{4}\sum_{i=1}^N z_{is}^4\left(\left(\frac{\partial\alpha_{i(s-1)}}{\partial y_0}\right)^2 + 1\right) + \frac{N}{4}\dot{y}_0^4 \end{aligned} \tag{59}$$

$$\begin{aligned} -\sum_{i=1}^N z_{is}^3 &\begin{pmatrix} \sum_{j\in\{N_i,i\}}\frac{\partial\alpha_{i(s-1)}}{\partial x_{j1}}f_{j1} \\ + \sum_{j,k\in\{N_i,i\}}\frac{\partial^2\alpha_{i(s-1)}}{\partial x_{j1}\partial x_{k1}}\frac{g_{j1}^T g_{k1}}{2}\end{pmatrix} \\ \leq &\sum_{i=1}^N z_{i1}^4\bar{f}_{is}(z_{i1}) + \Phi_{s1} \\ &+ \sum_{i=1}^N z_{is}^4\left(\sum_{j\in\{N_i,i\}}\left(\frac{\partial\alpha_{i(s-1)}}{\partial x_{j1}}\right)^2\right. \\ &\quad\left. + \sum_{j,k\in\{N_i,i\}}\left(\frac{\partial^2\alpha_{i(s-1)}}{\partial x_{j1}\partial x_{k1}}\right)^2 + N + N^2 + 1\right) \end{aligned} \tag{60}$$

$$\begin{aligned} \sum_{i=1}^N \frac{3z_{is}^2}{2}&\mathrm{tr}\left\{\sum_{j,k\in\{N_i,i\}}\frac{\partial\alpha_{i(s-1)}}{\partial x_{j1}}\frac{\partial\alpha_{i(s-1)}}{\partial x_{k1}}g_{j1}(y)g_{k1}^T(y)\right\} \\ \leq &\sum_{i=1}^N z_{is}^4\left(\sum_{j,k\in\{N_i,i\}}\left(\frac{\partial\alpha_{i(s-1)}}{\partial x_{j1}}\frac{\partial\alpha_{i(s-1)}}{\partial x_{k1}}\right)^2 + 1\right) \\ &+ \sum_{i=1}^N z_{i1}^4\bar{g}_{is}(z_{i1}) + \Phi_{s2}. \end{aligned} \tag{61}$$

It follows from (57)–(61) that:

$$\begin{aligned}\ell V_s &\\ \leq &\sum_{i=1}^N z_{is}^3\left(\alpha_{is} - l_i^2 q_s\nu_{i1} + \sum_{j=2}^s\phi_{isj}(y_i)x_{ij}\nu_{ij}\right. \\ &\quad + \Xi_{is} + \frac{3}{4}z_{is}^3 + \sum_{j\in\{N_i,i\}} l_j^{2\mu+2}\frac{z_{is}^3}{2}\left(\frac{\partial\alpha_{i(s-1)}}{\partial x_{j1}}\right)^2 \\ &\quad + z_{is}\sum_{j\in\{N_i,i\}}\left(\frac{\partial\alpha_{i(s-1)}}{\partial x_{j1}}\right)^2 + \frac{3}{4}z_{is}\left(\left(\frac{\partial\alpha_{i(s-1)}}{\partial y_0}\right)^2 + 1\right) \\ &\quad + z_{is}\sum_{j,k\in\{N_i,i\}}\left(\frac{\partial^2\alpha_{i(s-1)}}{\partial x_{j1}\partial x_{k1}}\right)^2 + z_{is}\left(N + N^2 + 2\right) \\ &\quad + z_{is}\sum_{j,k\in\{N_i,i\}}\left(\frac{\partial\alpha_{i(s-1)}}{\partial x_{j1}}\frac{\partial\alpha_{i(s-1)}}{\partial x_{k1}}\right)^2 \\ &\quad\left. - \frac{\partial\alpha_{i(s-1)}}{\partial\hat{\rho}_i}\dot{\hat{\rho}}_i - \sum_{j\in\{N_i,i\}}\frac{\partial\alpha_{i(s-1)}}{\partial x_{j1}}\rho\nu_{j2}\right) \\ &+ \frac{1}{4}\sum_{i=1}^N z_{i(s+1)}^4 \\ &+ \frac{N}{2}\sum_{i=1}^N\epsilon_{i2}^2 + \sum_{i=1}^N z_{i1}^4\bar{f}_{is}(z_{i1}) + \sum_{i=1}^N z_{i1}^4\bar{g}_{is}(z_{i1}) \\ &+ \frac{N}{4}\dot{y}_0^4 + \Phi_{s1} + \Phi_{s2}. \end{aligned} \tag{62}$$

Design the virtual controller α_{is} as

$$\alpha_{is} = -\Bigg(kz_{is} - l_i^2 q_s \nu_{i1} + \sum_{j=2}^{s} \phi_{isj}(y_i)x_{ij}\nu_{ij} + z_{is}^3 + \Xi_{is} + \frac{z_{is}}{4} \times \left(\left(\frac{\partial \alpha_{i(s-1)}}{\partial y_0}\right)^2 + 1\right) + \sum_{j\in\{N_i,i\}} l_j^{2\mu+2}\frac{z_{is}^3}{2}\left(\frac{\partial \alpha_{i(s-1)}}{\partial x_{j1}}\right)^2 + z_{is}\sum_{j\in\{N_i,i\}}\left(\frac{\partial \alpha_{i(s-1)}}{\partial x_{j1}}\right)^2 + z_{is}\sum_{j,k\in\{N_i,i\}}\left(\frac{\partial^2 \alpha_{i(s-1)}}{\partial x_{j1}\partial x_{k1}}\right)^2 + z_{is}\left(N+N^2+2\right) + z_{is}\sum_{j,k\in\{N_i,i\}}\left(\frac{\partial \alpha_{i(s-1)}}{\partial x_{j1}}\frac{\partial \alpha_{i(s-1)}}{\partial x_{k1}}\right)^2 - \sum_{j\in\{N_i,i\}}\frac{\partial \alpha_{i(s-1)}}{\partial x_{j1}}\hat{\rho}_i\nu_{j2} + \frac{\partial \alpha_{i(s-1)}}{\partial \hat{\rho}_i}\left(\tau_{i(s-1)} - \sigma\hat{\rho}_i\right) - \sum_{j\in\{N_i,i\}}\frac{\partial \alpha_{i(s-1)}}{\partial x_{j1}}\nu_{j2}\left(\sum_{m=3}^{s-1} z_{im}\frac{\partial \alpha_{i(m-1)}}{\partial \hat{\rho}_i}\right)\Bigg) \tag{63}$$

where $\tau_{i(s-1)} = -\sum_{m=2}^{s}\sum_{j\in\{N_i,i\}} z_{im}^3([\partial\alpha_{i(m-1)}]/[\partial x_{j1}])\nu_{j2}$, and same with the analysis of α_{i2} below (53), α_{is} only contains the ith agent's and its neighbors' information.

It follows from (56)–(63) that:

$$\begin{aligned}
&\ell\sum_{j=1}^{s} V_j + \ell V_\theta + \ell V_\beta + \ell V_\rho \\
&\le \sum_{i=1}^{N} z_{i1}\left(\sum_{j=1}^{s}\Psi_{ij}(z_{i1}) - \Psi_i(z_{i1})\right) - \sum_{i=1}^{N} k\left(z_{i1}^2 + \sum_{j=2}^{s} z_{ij}^4\right) \\
&\quad + \sum_{i=1}^{N}\frac{z_{i(s+1)}^4}{4} + \sum_{i=1}^{N}\frac{((s-1)N+1)\epsilon_{i2}^2}{2} + \sigma\sum_{i=1}^{N}\hat{\theta}_i^T\tilde{\theta}_i \\
&\quad + \sigma\rho\sum_{i=1}^{N}\hat{\beta}_i\tilde{\beta}_i + \sum_{i=1}^{N}\left(\tilde{\rho}_i + \sum_{m=3}^{s} z_{im}^3\frac{\partial\alpha_{i(m-1)}}{\partial\hat{\rho}_i}\right) \\
&\quad \times\left(\tau_{i(s-1)} - \dot{\hat{\rho}}_i - \sigma\hat{\rho}_i\right) + \sigma\sum_{i=1}^{N}\hat{\rho}_i\tilde{\rho}_i + \frac{1}{2}\sum_{i=1}^{N}\varepsilon_{i1}^{*2} \\
&\quad + \frac{1}{2}N\dot{y}_0^2 + \frac{(s-1)N}{4}\dot{y}_0^4 + \sum_{j=1}^{s}\left(\Phi_{j1} + \Phi_{j2}\right)
\end{aligned} \tag{64}$$

where $\Psi_{is}(z_{i1}) = z_{i1}^3\bar{f}_{is}(z_{i1}) + z_{i1}^3\bar{g}_{is}(z_{i1})$.

Step n: Through the above inductive design method, we can design u_i and the adaptive law $\dot{\hat{\rho}}_i$ as follows:

$$u_i = \alpha_{\text{in}} \tag{65}$$

$$\dot{\hat{\rho}}_i = \tau_{i(n-1)} - \sigma\hat{\rho}_i \tag{66}$$

where α_{in} can be calculated as *step s*, and $\tau_{i(n-1)} = -\sum_{m=2}^{n}\sum_{j\in\{N_i,i\}} z_{im}^3([\partial\alpha_{i(m-1)}]/[\partial x_{j1}])\nu_{j2}$. Then, the differential operator of $\ell\sum_{j=1}^{n} V_j + \ell V_\theta + \ell V_\beta + \ell V_\rho$ is

$$\begin{aligned}
&\ell\sum_{j=1}^{n} V_j + \ell V_\theta + \ell V_\beta + \ell V_\rho \\
&\le -\sum_{i=1}^{N} k\left(z_{i1}^2 + \sum_{j=2}^{n} z_{ij}^4\right) + \sum_{i=1}^{N} z_{i1}\left(\sum_{j=1}^{n}\Psi_{ij}(z_{i1}) - \Psi_i\right) \\
&\quad + \sum_{i=1}^{N}\frac{((n-1)N+1)\epsilon_{i2}^2}{2} + \sigma\sum_{i=1}^{N}\hat{\theta}_i^T\tilde{\theta}_i + \sigma\rho\sum_{i=1}^{N}\hat{\beta}_i\tilde{\beta}_i \\
&\quad + \sigma\sum_{i=1}^{N}\hat{\rho}_i\tilde{\rho}_i + \frac{1}{2}\sum_{i=1}^{N}\varepsilon_{i1}^{*2} + \frac{1}{2}N\dot{y}_0^2 \\
&\quad + \frac{(n-1)N}{4}\dot{y}_0^4 + \sum_{j=1}^{n}\left(\Phi_{j1} + \Phi_{j2}\right).
\end{aligned} \tag{67}$$

Now, we give the main result of this article.

Theorem 1: For the high-order stochastic nonlinear MASs (1) satisfying Assumptions 1 and 2, based on the dynamic gain filter (9) and (10), the distributed output-feedback adaptive fuzzy controller (65) with the adaptive laws (44), (45), and (66) can render all the expectations of output tracking errors $(y_1 - y_0), (y_2 - y_0), \ldots, (y_N - y_0)$ to be semiglobally uniformly ultimately bounded and converge to an adjustable set.

Proof: Since $l_i \ge 1$, based on Assumption 1, Lemma 4, and (30), there exist positive smooth functions $\Psi_{ei}(z_{i1})$ and bounded positive constant Φ_{ei} such that

$$\begin{aligned}
&\sum_{i=1}^{N}((n-1)N+2)l_i^{-2\mu}\left(F_i^2(y) + G_i^2(y)\right) \\
&\le \sum_{i=1}^{N} z_{i1}^2\Psi_{ei}(z_{i1}) + \sum_{i=1}^{N} l_i^{-2\mu}\Phi_{ei}.
\end{aligned} \tag{68}$$

Using Young's inequality, we have

$$\begin{aligned}
&\sigma\hat{\theta}_i^T\tilde{\theta}_i = \sigma\left(\theta_i^T - \tilde{\theta}_i^T\right)\tilde{\theta}_i \le -\frac{1}{2}\sigma\left\|\tilde{\theta}_i\right\|^2 + \frac{1}{2}\sigma\|\theta_i\|^2 \\
&\sigma\rho\hat{\beta}_i\tilde{\beta}_i = \sigma\rho\left(\rho^{-1} - \tilde{\beta}_i\right)\tilde{\beta}_i \le \sigma\rho\rho^{-1}\tilde{\beta}_i - \sigma\rho\tilde{\beta}_i^2 \\
&\le -\frac{1}{2}\sigma\rho\tilde{\beta}_i^2 + \frac{1}{2}\sigma\rho^{-1} \\
&\sigma\hat{\rho}_i\tilde{\rho}_i = \sigma(\rho_i - \tilde{\rho}_i)\tilde{\rho}_i \le -\frac{1}{2}\sigma\tilde{\rho}_i^2 + \frac{1}{2}\sigma\rho_i^2.
\end{aligned}$$

Combined with (28), (33), (67), and (68), and letting

$$\Psi_i(z_{i1}) = \sum_{j=1}^{n}\Psi_{ij}(z_{i1}) + z_{i1}\Psi_{ei}(z_{i1}) \tag{69}$$

where $\Psi_{ij}(z_{i1}) = z_{i1}^3\bar{f}_{ij}(z_{i1}) + z_{i1}^3\bar{g}_{ij}(z_{i1})$, we have

$$\begin{aligned}
\ell V &= \ell\bar{V} + \sum_{i=1}^{N}((n-1)N+2)\ell V_{ei} \\
&\le -\sum_{i=1}^{N}\left(k\left(z_{i1}^2 + \sum_{j=2}^{n} z_{ij}^4\right) + \frac{1}{2}l_i\epsilon_i^T\epsilon_i\right) \\
&\quad -\frac{\sigma}{2}\left(\tilde{\theta}_i^T\tilde{\theta}_i + \rho\tilde{\beta}_i^2 + \tilde{\rho}_i^2\right) \\
&\quad + \frac{1}{2}\sigma\|\theta_i\|^2 + \frac{1}{2}\sigma\rho^{-1} + \frac{1}{2}\sigma\rho_i^2 \\
&\quad + \sum_{i=1}^{N}\frac{\varepsilon_{i1}^{*2}}{2} + \frac{N\dot{y}_0^2}{2} + \frac{(n-1)N}{4}\dot{y}_0^4 \\
&\quad + \sum_{j=1}^{n}\left(\Phi_{j1} + \Phi_{j2}\right) + \sum_{i=1}^{N} l_i^{-2\mu}\Phi_{ei}.
\end{aligned} \tag{70}$$

Let $\alpha = \min\{([l_i(0)]/[2((n-1)N+2)\max(P)]), \sigma, (2k/[\max\{(L+B)^{-1}\}]), 4k, [\sigma\rho/(\max\{\Gamma_i^{-1}\})], \text{ for } i = 1,\ldots,N\}$ and $\varepsilon_1 = (1/2)\sigma\|\theta_i\|^2 + (1/2)\sigma\rho^{-1} + (1/2)\sigma\rho_i^2$, $\varepsilon_2 \geq \sum_{i=1}^{N}[(\varepsilon_{i1}^{*2})/2] + [(N\dot{y}_0^2)/2] + [(n-1)N/4]\dot{y}_0^4 + \sum_{j=1}^{n}(\Phi_{j1}+\Phi_{j2}) + \sum_{i=1}^{N} l_i^{-2\mu}\Phi_{ei}$, where α, ε_1, and ε_2 are positive constants.

Combined with (70), we have

$$\ell V \leq -\alpha V + \varepsilon_1 + \varepsilon_2. \tag{71}$$

Further, one can obtain

$$E[V(t)] \leq e^{-\alpha t}E[V(0)] + \frac{\varepsilon_1+\varepsilon_2}{\alpha}\left(1-e^{-\alpha t}\right) \tag{72}$$

where (ε_2/α) can be adjusted to be small enough, and the bound of (ε_1/α) is associated with θ_i and ρ.

Since $E[(1/2)\bar{z}_1^T(L+B)^{-1}\bar{z}_1] \leq E[V(t)]$ and

$$\begin{aligned}\bar{z}_1^T(L+B)^{-1}\bar{z}_1 &= \left[y-1_n y_0\right]^T(L+B)\left[y-1_n y_0\right]\\ &\geq \|y-1_n y_0\|^2\lambda_{\min}(L+B)\end{aligned} \tag{73}$$

we have $(E[|y_i-y_0|])^2 \leq E[|y_i-y_0|^2] \leq E[\|y-1_n y_0\|^2] \leq ([2E[V(t)]]/[\lambda_{\min}(L+B)])$. Hence, the expectations of output tracking errors are bounded and can converge to an adjustable set. In addition, based on Assumption 1 and (71), y_i, ϵ_i, z_{is}, $\hat{\beta}_i$, $\hat{\rho}_i$, and $\hat{\theta}_i$ are bounded almost surely (a.s.). Hence, l_i is bounded a.s. From (9), we have

$$\begin{aligned}&d\left(l_i^{-\mu}L_{i0}^{-1}\zeta_i\right)\\ &= \Big\{l_i\left(A-qc^T\right)\left(l_i^{-\mu}L_{i0}^{-1}\zeta_i\right) + l_i^{-\mu}L_{i0}^{-1}\phi_i(y_i)\zeta_i\\ &\quad -\dot{l}_i l_i^{-1}(\mu I_n + D)\left(l_i^{-\mu}L_{i0}^{-1}\zeta_i\right) + l_i^{1-\mu}qy_i\Big\}dt.\end{aligned} \tag{74}$$

Similar with (14)–(28), choose the Lyapunov function $V_{i\zeta} = (l_i^{-\mu}L_{i0}^{-1}\zeta_i)^T P(l_i^{-\mu}L_{i0}^{-1}\zeta_i)$, whose derivative satisfies

$$\begin{aligned}&\dot{V}_{i\zeta}\\ &\leq -\frac{1}{2}l_i\left(l_i^{-\mu}L_{i0}^{-1}\zeta_i\right)^T\left(l_i^{-\mu}L_{i0}^{-1}\zeta_i\right) + 2\left(l_i^{-\mu}L_{i0}^{-1}\zeta_i\right)^T P l_i^{1-\mu}qy_i\\ &\leq -\frac{1}{4}l_i\left(l_i^{-\mu}L_{i0}^{-1}\zeta_i\right)^T\left(l_i^{-\mu}L_{i0}^{-1}\zeta_i\right) + 4\left\|Pl_i^{1-\mu}qy_i\right\|^2.\end{aligned} \tag{75}$$

Due to that y_i and l_i are bounded a.s., from (75), ζ_i is bounded a.s. Combined with (43)–(45), (53), (63), and (66), ν_{is} is bounded a.s. So the stability of the filter can be guaranteed. From $\hat{x}_i = \zeta_i + \rho\nu_i$ and $e_i = x_i - \hat{x}_i$, x_i and $\hat{x}_i$ are bounded a.s. So all of the dynamic variables are bounded a.s. The proof is completed.

In Theorem 1, the dynamic gain filter-based distributed output-feedback adaptive fuzzy consensus proposal is proposed for the stochastic MASs, where the interactions only contain the outputs information of each agent and the unknown control gains in each agent are same. The results can be further extended to the following MASs:

$$\begin{cases} dx_{i1} = \{x_{i2} + f_{i1}(x)\}dt + g_{i1}^T(x)dw \\ dx_{\text{is}} = \left\{x_{i(s+1)} + f_{is}(x) + \sum_{m=2}^{s}\phi_{ism}(y_i)x_{im}\right\}dt \\ \qquad + g_{is}^T(x)dw,\ s = 2,\ldots,n-1 \\ dx_{\text{in}} = \left\{\rho_i u_i + f_{\text{in}}(x) + \sum_{m=2}^{n}\phi_{inm}(y_i)x_{im}\right\}dt \\ \qquad + g_{\text{in}}^T(x)dw \\ y_i \quad = x_{i1} \end{cases} \tag{76}$$

where different from the MASs (1), for $i = 0,\ldots,N$, the unknown constant gain ρ_i in each agent can be different. $x := [x_{11},\ldots,x_{Nn}]^T$. $f_{is}(x)$ and $g_{is}(x) \in R^m$ represent the uncertain interactions from the other subsystems.

Assumption 3: For $i = 1,\ldots,N$ and $s = 1,2,\ldots,n$, the drift term $f_{is}(x)$ and diffusion term $g_{is}(x)$ satisfy the following conditions, respectively:

$$|f_{is}(x)| \leq \bar{f}_{is}(y) \tag{77}$$

$$\|g_{is}(x)\| \leq \bar{g}_{is}(y) \tag{78}$$

where $\bar{f}_{is}(y)$ and $\bar{g}_{is}(y)$ are known positive smooth functions.

Theorem 2: For the high-order stochastic nonlinear MASs (76) satisfying Assumptions 1–3, the distributed adaptive proposal (65) can extend to this case to solve the output leader-following consensus problem.

Proof: In the filter design process, the estimator can be chosen as $\hat{x}_i = [\hat{x}_{i1},\ldots,\hat{x}_{\text{in}}]^T = \zeta_i + \rho_i\nu_i$. In the distributed protocol design process, we can design the adaptive law $\hat{\rho}_{ij}$ for $j \in \{N_i, i\}$ to compensate ρ_j for $j \in \{N_i, i\}$ in the ith agent. The rest of the proof is similar to the one of Theorem 1 and omitted.

Remark 6: In (76), the interactions in the ith agent can contain all the states information of all agents, which makes the interactions be more general and the proposed method be applicable for a wider class of MASs. There are many nonlinear functions satisfy Assumption 3, such as $y_i^2\cos(x_{i2}+x_{j2})$, $[(y_i^2)/(1+x_{i2}^2)]$ and so on. In addition, the unknown control gain ρ_i in each agent can be different, and we need to design more adaptive laws to compensate these constant gains.

IV. Numerical Example

To illustrate the effectiveness of the proposed methods, consider the heterogeneous MASs with one leader (agent 0) and four followers (agents 1–4) under the communication graph Fig. 1. The agent 1 is the parallel active suspension system, referring to [28] and [40] for details. The dynamic of the suspension system with a random noise can be written as

$$\text{Agent 1:}\ \begin{cases} dx_{11} = \left\{\frac{1}{A}x_{12}\right\}dt + g_{11}dw \\ dx_{12} = \left\{-c_f x_{12} + \rho i_v\right\}dt + g_{12}dw \\ y_1 \quad = x_{11} \end{cases} \tag{79}$$

where x_{11} is the suspension travel, x_{12} is the fluid flow into the hydraulic actuator, A is the effective surface of piston, c_f and ρ are some positive constants, and $i_v := u_1$ is the current input adjusting the opening of the current-controlled solenoid value that controls the fluid flow. Set $A = 1$, $c_f = 2$, $g_{11} = 0$, and $g_{12} = x_{11}^2 + 0.5x_{11}\sin(x_{12})$, and the parameter ρ is assumed to be unknown and chosen as $\rho = 5$. If the unknown parameters in each agent are different, Theorem 2 can be used. For simplicity, we assume the unknown parameters are same. The dynamic of the ith ($i = 2,3,4$) agent is as follows:

$$\text{Agent } i:\ \begin{cases} dx_{i1} = \{x_{i2} + f_{i1}(y)\}dt + g_{i1}^T(y)dw \\ dx_{i2} = \{\rho u_i + f_{i2}(y) + \phi_{i22}(y_i)x_{i2}\}dt \\ \qquad + g_{i2}^T(y)dw \\ y_i \quad = x_{i1} \end{cases} \tag{80}$$

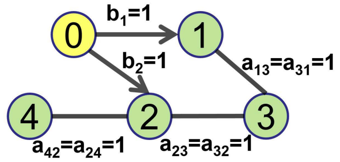

Fig. 1. Communication topology.

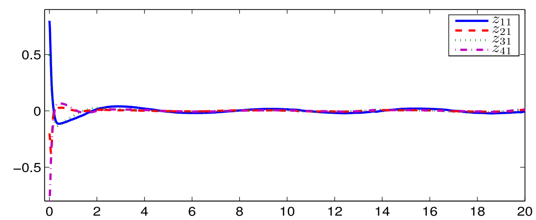

Fig. 2. Responses of consensus errors.

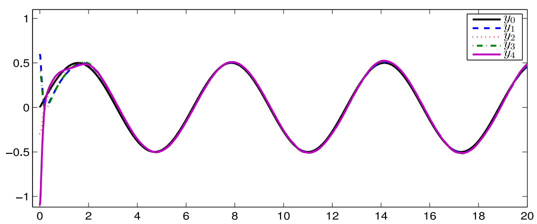

Fig. 3. Responses of outputs.

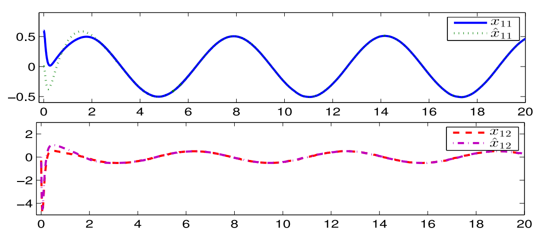

Fig. 4. Responses of state variables x_{11}, x_{12}, $\hat{x}_{11}$, and $\hat{x}_{12}$.

where $f_{21}(y) = y_2^2 - y_3^2$, $f_{22}(y) = y_2^2 + y_4$, $g_{21}(y) = 0$, $g_{22}(y) = y_2^2 - y_4$, $\phi_{222}(y_2) = 0.4y_2^2$, $f_{31}(y) = y_3^2 - y_2^2$, $f_{32}(y) = y_3^2 + y_2$, $g_{31}(y) = 0$, $g_{32}(y) = y_3^2 - y_2$, $\phi_{322}(y_3) = 0.4y_3^2$, $f_{41}(y) = y_4^2 - y_2^2$, $f_{42}(y) = y_4^2$, $g_{41}(y) = 0$, $g_{42}(y) = y_4^2$, and $\phi_{422}(y_4) = 0.4y_4^2$. Design the dynamic gain k-filter as (9) and (10), where using the LMI tool box in MATLAB to solve (16), we obtain that $q^T = [1.1498\ 1.3900]$ and $P = \begin{bmatrix} 1.4750 & -0.5003 \\ -0.5003 & 1.4750 \end{bmatrix}$. Choosing $\mu = 0.5$, based on (24), we have $\mu_1 = 1.1677$ and $\mu_2 = 4.7323$. From (20), choose $\Phi_1(\cdot) = 0.5003 * 2$, and for $i = 2, 3, 4$, $\Phi_i(\cdot) = 3.4503\phi_{i22}(y_i)$. Based on (27), set $\gamma_{11} = 0.2$, $\gamma_{21} = 0.7$, $\gamma_{31} = 0.6$, and $\gamma_{41} = 0.4$. For $i = 1, 2, 3, 4$, $\eta_{i1} = \eta_{i2} = 0.1057$, and $\bar{\varphi}_i(y_i) = \mu_1^{-1}(\Phi_i(y_i) + \gamma_{i1})$. Then, the dynamic gains are designed as (23).

In the sequel, based on (43)–(45), the virtual controller α_{i1} and adaptive laws $\dot{\hat{\theta}}_i$ and $\dot{\hat{\beta}}_i$ are designed as

$$\begin{cases} \alpha_{i1} = -\hat{\beta}_i \bar{\alpha}_{i1} \\ \dot{\hat{\beta}}_i = z_{i1}\bar{\alpha}_{i1} - 0.2\hat{\beta}_i, \quad \dot{\hat{\theta}}_i = z_{i1}\varphi_i(z_{i1}) - 0.2\hat{\theta}_i \end{cases} \tag{81}$$

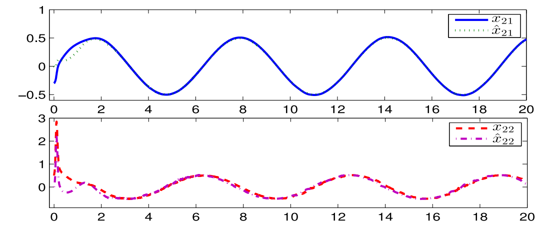

Fig. 5. Responses of state variables x_{21}, x_{22}, $\hat{x}_{21}$, and $\hat{x}_{22}$.

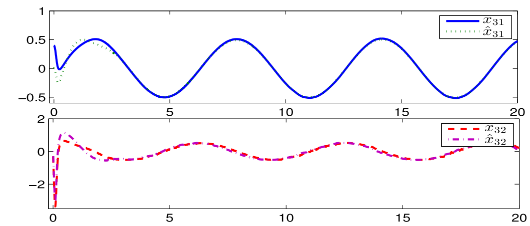

Fig. 6. Responses of state variables x_{31}, x_{32}, $\hat{x}_{31}$, and $\hat{x}_{32}$.

where $\bar{\alpha}_{i1} = \zeta_{i2} + (1/2)l_i^3 z_{i1} + \hat{\theta}_i^T \varphi_i(z_{i1}) + 7z_{i1}$. Choose the fuzzy membership functions as $\mu_{F^{i1}} = \exp(-0.5(z_{i1}+2)^2)$, $\mu_{F^{i2}} = \exp(-0.5(z_{i1}+1)^2)$, $\mu_{F^{i3}} = \exp(-0.5z_{i1}^2)$, $\mu_{F^{i4}} = \exp(-0.5(z_{i1}-1)^2)$, and $\mu_{F^{i5}} = \exp(-0.5(z_{i1}-2)^2)$, and define fuzzy basis functions as $\varphi_{ij}(z_{i1}) = [(\mu_{F^{ij}})/(\sum_{j=1}^{5}\mu_{F^{ij}})]$.

Then, based on (53) and (66), the controller u_i and adaptive law $\dot{\hat{\rho}}_i$ are designed as

$$\begin{aligned} u_i = -\Bigg(& 11z_{i2} - l_i^2 q_2 \nu_{i1} + \phi_{i22}(y_i)\nu_{i2} - \frac{\partial \alpha_{i1}}{\partial \hat{\beta}_i}\dot{\hat{\beta}}_i \\ & - \sum_{j\in\{N_i,i\}} \frac{\partial \alpha_{i1}}{\partial x_{j1}}\zeta_{j2} - \frac{\partial \alpha_{i1}}{\partial \zeta_{i2}}\dot{\zeta}_{i2} - \frac{\partial \alpha_{i1}}{\partial l_i}\dot{l}_i \\ & - \frac{\partial \alpha_{i1}}{\partial \hat{\theta}_i}\dot{\hat{\theta}}_i + \frac{3}{4}z_{i2}\left(\left(\frac{\partial \alpha_{i1}}{\partial y_0}\right)^2 + 1\right) \\ & + \sum_{j\in\{N_i,i\}} \frac{z_{i2}^3}{2} l_j^{2\mu+2}\left(\frac{\partial \alpha_{i1}}{\partial x_{j1}}\right)^2 - \sum_{j\in\{N_i,i\}} \frac{\partial \alpha_{i1}}{\partial x_{j1}}\hat{\rho}_i \nu_{j2} \\ & + z_{i2}\sum_{j\in\{N_i,i\}}\left(\frac{\partial \alpha_{i1}}{\partial x_{j1}}\right)^2 + 7z_{i2} \Bigg) \end{aligned} \tag{82}$$

$$\dot{\hat{\rho}}_i = -z_{i2}^3 \sum_{j\in\{N_i,i\}} \frac{\partial \alpha_{i1}}{\partial x_{j1}}\nu_{j2} - 0.2\hat{\rho}_i \tag{83}$$

where based on (81), $[(\partial \alpha_{i1})/(\partial \hat{\beta}_i)] = -\bar{\alpha}_{i1}$, $[(\partial \alpha_{i1})/(\partial x_{j1})] = -a_{ij}[(\partial \alpha_{i1})/(\partial z_{i1})]$, $[(\partial \alpha_{i1})/(\partial \zeta_{i2})] = -\hat{\beta}_i$, $[(\partial \alpha_{i1})/(\partial l_i)] = -\hat{\beta}_i(3/2)l_i^2 z_{i1}$, $[(\partial \alpha_{i1})/(\partial \hat{\theta}_i)] = -\hat{\beta}_i[\varphi_{i1}\ \varphi_{i2}\ \varphi_{i3}\ \varphi_{i4}\ \varphi_{i5}]$, and $[(\partial \alpha_{i1})/(\partial y_0)] = -b_i[(\partial \alpha_{i1})/(\partial z_{i1})]$.

Choose the initial values as $x_{11}(0) = 0.6$, $x_{12}(0) = -0.3$, $x_{21}(0) = -0.3$, $x_{22}(0) = 0.5$, $x_{31}(0) = 0.4$, $x_{32}(0) = -0.3$, $x_{41}(0) = -1.1$, $x_{42}(0) = 0.4$, $l_1(0) = 1$, $l_2(0) = 2$, $l_3(0) = 1.2$, $l_4(0) = 1.5$, $\hat{\rho}_1(0) = 0.3$, $\hat{\rho}_2(0) = 0.1$, $\hat{\rho}_3(0) = 0.2$, and $\hat{\rho}_4(0) = 0.5$, and the initial values of the other dynamic variables are chosen as 0. Set the reference signal $y_0(t) = 0.5\sin(t)$. Simulation results are shown in Figs. 2–12. Fig. 2 illustrates that the local neighborhood consensus errors are bounded. Fig. 3 illustrates that the outputs of

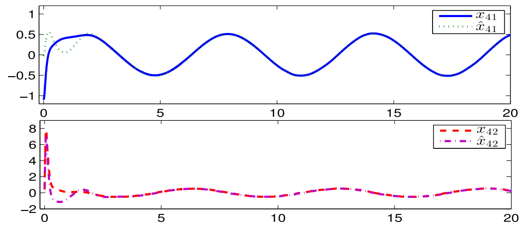

Fig. 7. Responses of state variables x_{41}, x_{42}, $\hat{x}_{41}$, and $\hat{x}_{42}$.

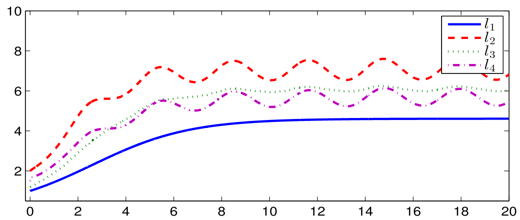

Fig. 8. Responses of dynamic gains.

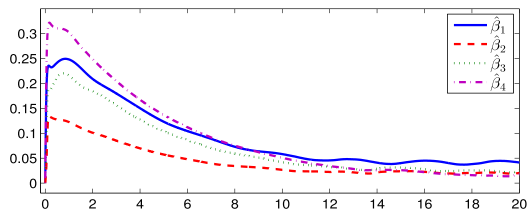

Fig. 9. Responses of adaptive laws $\hat{\beta}_i$.

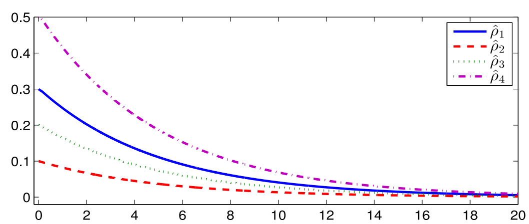

Fig. 10. Responses of adaptive laws $\hat{\rho}_i$.

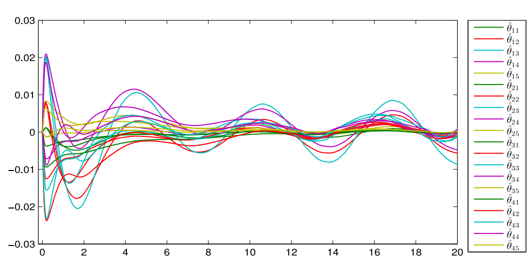

Fig. 11. Responses of adaptive laws $\hat{\theta}_i$.

the followers can track the reference signal $y_0(t)$ effectively based on the proposed adaptive fuzzy output-feedback controllers. Figs. 4–7 show the effectiveness of the constructed dynamic gain filter. Fig. 8 shows that the designed dynamic gains are always greater than 1. Figs. 9–11 show the responses of adaptive laws. Fig. 12 shows the response of the control inputs of each follower.

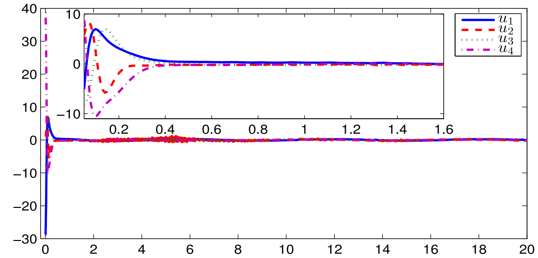

Fig. 12. Responses of control input u_i.

V. Conclusion

In this article, we have studied the distributed output-feedback leader-following control problem for high-order stochastic heterogeneous nonlinear MASs with interactions and unknown control gains. The existence of unknown control gains makes the traditional observer method unusable. At the same time, to reduce the number of the introduced dynamic variables, a novel dynamic gain filter is constructed to compensate the unmeasured states. Based on this filter and by the backstepping method, the distributed protocol is further designed. The restrictions on the uncertain interactions existing not only in the drift terms but also the diffusion terms are very loose. It has been proved that the proposed method can render all the expectations of output tracking errors to be uniformly ultimately bounded. In our future work, we will further consider the output consensus problem for MASs (1) under fixed and switching graphs, which is more challenging, and extending the results of this article to MASs with multiple control inputs will be also considered.

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