Stochastic and Deterministic State-Dependent Social Networks

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Abstract

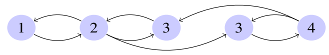
This work addresses the problem of finite-time convergence of, and the determination of the factors that impact on, the final opinion in a social network for a political party or an association, modeled as a distributed iterative system with graph dynamics chosen to mimic how people interact. It is firstly shown that, in this setting, finite-time convergence is achieved only when nodes form a complete network, and that contacting with agents with distinct opinions reduces to a half the required interconnections. Two novel strategies are presented that enable finite-time convergence, even for the case where each node only contacts the two closest neighbors. It is shown that, in a deterministic setting, the final opinion depends on a so-called connectivity parameter, which influences the relative contribution of each agent's initial belief. In the stochastic case, analogous conclusions are drawn, but in terms of expected values. The proposed strategies and results are relevant also in the context of mobile robot networks where the same assumption of having nodes communicating to their closest neighbors is satisfied. In addition, the results obtained are relevant in terms of saving resources while ensuring finite-time consensus. The performance of the proposed strategies is evaluated through simulation, illustrating, in particular, the key nodes that drive the network, as well as the associated rate of convergence.

Network Model

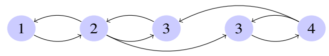
Opinion update based on neighbors opinion

$$x_i(k+1) = \alpha_k \min_{j \in N_i(k)} x_j(k) + (1-\alpha_k) \max_{j \in N_i(k)} x_j(k)$$

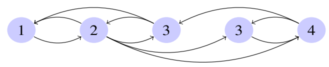
Local interactions produce the analyzed topologies.



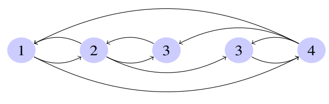
(a) standard network



(b) distinct value

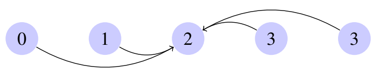


(c) distinct neighbor

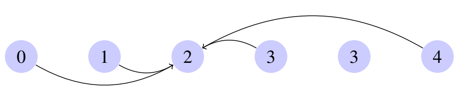


(d) circular value

(detail of a single node η = 2)



(a) standard network



(b) distinct value

Contributions:

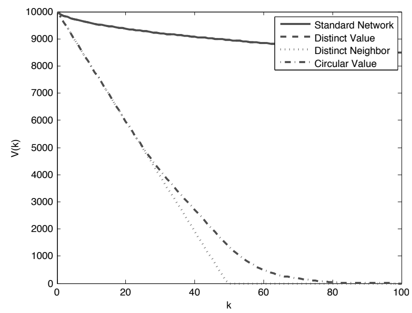
- Model social interactions as iterative distributed algorithms;
- Show conditions for finite-time and asymptotic convergence;
- Proposed network dynamics influence both the convergence time and the final value. Exact expressions for the dependence are provided.

Results

Theorems [Convergence]:

- *Standard Network has finite-time convergence for* $\eta \geq n-1$
- *Distinct Value converges in* $\lceil \log_2 n \rceil$ *steps for* $\eta \geq \frac{n}{2}$
- *Distinct Neighbor converges in* $\lceil \frac{n-(2\eta+1)}{2\eta} \rceil + 1$ *steps for* $\eta \geq 1$
- *Circular Value converges in* $\lceil \frac{n-(2\eta+1)}{2\eta-1} \rceil + 1$ *steps for* $\eta \geq 1$

Comparison of the speed of convergence



Main conclusion:

Given the convergence properties of social networks, the same algorithm can be used for finite-time consensus in robot networks while saving resources as it is only requires 2 neighbors per node.

References

[1] D. Silvestre, P. Rosa, J. P. Hespanha and C. Silvestre, "Finite-time convergence policies in state-dependent social networks," 2015 American Control Conference (ACC), Chicago, IL, 2015, pp. 1041-1046.

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