Coordination of a Class of Underactuated Systems via Sampled-Data-Based Event-Triggered Schemes

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***Abstract*—This article investigates the coordination of a class of underactuated systems subject to limited energy supply and channel bandwidth and aims to stabilize system states and exclude Zeno behaviors simultaneously. First, by means of event-triggered (E-T) and quantized techniques, several novel quantized sampled-data-based E-T schemes are constructed, which only require discrete-time controller updates and partial quantized states, and thus efficiently mitigate the control and communication workloads. Then, in order to further lower the communication consumptions, several new triggered sampled-data-based communication rules under fixed and switched networks are established, where the communications are performed only at some specific instants and, thus, the ideally continuous-time signal transmission among neighbors can be avoided. Note that sufficient criteria for achieving the coordination of the underactuated systems are derived in terms of the Lyapunov–Krasovskii functional method. Finally, numerous simulations are carried out to demonstrate the effectiveness of the theoretical results.**

***Index Terms*—Event-triggered (E-T) control, quantized control, sampled-data control, switched networks, underactuated systems.**

I. Introduction

THE RESEARCH of underactuated systems has attracted a large amount of interests due to their favorable merits of lower energy consumption and higher operation flexibility comparing with fully actuated systems [1], [2]. Up to now, its various applications have been extensively investigated, for instance, aircrafts, robotic manipulators, mobile, and underwater vehicles [3], [4]. While, because of its intrinsic couplings of passive and active states, it cannot be directly synthesized by the approaches of fully actuated ones, and thus gives rise to great challenges in terms of analysis and control, especially for achieving coordination of a large group of agents. Additionally, in most real-world applications, the communication bandwidth is usually limited in bounded regions and the total energy supply cannot be arbitrarily large, which significantly determines the working efficiency and the performance of control systems. Therefore, it naturally puts forward the question: is there an effective and energy-saving scheme to be constructed for accomplishing the coordination of multiple underactuated systems?

By all accounts, the quantized control has the advantage of digitizing continuous real-valued inputs to piecewise quantized outputs, so that only the channels with finite-bits capacity are needed for the signal transmission. Accordingly, some attempts studying quantized coordination problems have arisen to mitigate control costs, for instance, [5]–[7] introduced logarithmic quantizers for Markov jump systems to release control burdens. In [8], the stability of an uncertain system was considered by using super-twisting control schemes with uniform and logarithmic quantizers. With the help of a dynamic quantizer, the work [9] concentrated on the feedback control of a class of uncertain linear discrete-time systems. Based on mismatched quantization channels, the $\mathcal{H}_2$ control of linear systems with polytopic uncertainties was investigated in [10]. However, the quantized methods mentioned-above are only involved in fully actuated systems with single- or double-integrator dynamics, which are not suitable for the underactuated Lagrangian systems.

On the other hand, event-triggered (E-T) schemes have recently emerged with the development of embedded microprocessors, where the predefined event detectors collect information from neighbors and produce discrete-time control inputs only when the undesirable performance appears. As a result, it is greatly efficient to provide a high-efficiency tradeoff between the control cost and the system performance, and so far, the researchers have shown a large amount of interest for E-T control, and some recently remarkable results can refer to the literature [11]–[18]. Besides, the authors in [19]–[22] were concerned with $\mathcal{H}_\infty$ coordination of T-S fuzzy systems via E-T mechanisms. You *et al.* [23], Hua *et al.* [24], and Li *et al.* [25] addressed the E-T leader-following consensus for nonlinear systems by using a dynamic output-feedback method. The technical notes [26], [27] designed E-T algorithms for networked systems with stochastic uncertainties and distributed channel delays. Qi *et al.* [28] studied the

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E-T and self-triggered $\mathcal{H}_\infty$ control for uncertain switched linear systems with exogenous disturbances. In [29], the finite-time coordination of single-integrator systems was explored by means of an event-driven control protocol over fixed topology. However, it should be noteworthy that only the frequency of controller updates in the above results has been reduced, but a large number of computation and communication exchanges still exist. Therefore, it is always too expensive or quite difficult or even unavailable to transmit signals with high frequency and no errors, particularly for long-distance communication with wireless channels.

One efficient technique to overcome the aforementioned drawback is to construct sampled-data-based E-T schemes, which can both save the communication and control costs [30]. However, although some results have been reported, for instance, [31] and [32] put forward discrete-time sampling E-T schemes for linear systems, and [23] and [33] contributed to establish output-feedback E-T strategies for nonlinear systems without continuous communications, the E-T coordination control, especially for underactuated systems, via sampled-data communications remains an open problem. Moreover, existing sampled-data-based E-T results are derived with the knowledge of exactly real-value full-state feedbacks, which cannot be easily available in some practical cases due to the measuring limitations of commercially available sensors and high power consumptions of other hardware.

With the aforementioned literature review, this article involves the following several challenges.

1) The coordination of multiple underactuated systems is rarely investigated, and the coupled active and passive actuators are much more sensitive to uncertainties and disturbances, thus how to realize coordination and overcome the negative effects make the analysis and control quite challenging.
2) The research of sampled-data-based E-T schemes and comparisons, especially for underactuated systems, have not been comprehensively studied yet, such that how to construct optimal schemes to efficiently reduce communication and control costs is another challenge.
3) Neighbors' full-state information is always assumed to be completely known for system control. To this end, how to construct proper sampled-data-based E-T schemes with only partial states of neighbors for further saving energy resources is an urgent challenge.

In pursuit of overcoming the above challenges, this article explores the feasibility of constructing some novel schemes for the coordination problem. In summary, the contributions and innovations are threefold.

1) The considered mathematical models are more practical since the multiple underactuated systems with Lagrangian dynamics, uncertainties, and disturbances are taken into account.
2) Two types of sampled-data-based E-T schemes are constructed with only using neighbors' positions, where the high-performance E-T schemes with uniform- and logarithmic-quantization sampled-data communications layers are first established, and then, the sampled-data communication layers driven by the E-T mechanism are novelly designed. Note that the communication and control costs can be significantly saved, and the service lifetime of the equipped facilities, especially with limited total energies, can be greatly prolonged.
3) New concepts of a communication rate and trigger rate are established as performance indexes to characterize the communication and control efficiencies of the constructed schemes. In conclusion, to the best of our knowledge, this is the first time to investigate underactuated systems by constructing different sampled-data-based E-T schemes without neighbors' velocities in a unified framework.

This remainder is arranged as follows. The system formulation and control objective are provided in Section II. The quantized and triggered sampled-data-based E-T schemes are, respectively, constructed in Sections III and IV. The simulation and conclusion are drawn in Sections V and VI, respectively.

Notations: The following standard notations will be used throughout this article. $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the minimum and maximum eigenvalues, and $\mathbb{R}^N$ and $\mathbb{R}^{n\times m}$ are $N\times 1$ and $n\times m$ real matrices; I_n and $\mathrm{diag}(\cdot)$ are $n\times n$ identity and block-diagonal matrices, and 0_n and 1_n are the $n\times 1$ matrices with all 0 and 1, and $\mathbf{0}$ in bold is the zero vector or matrix with desired dimension; $\otimes$ and $\mathbb{N}$ denote the Kronecker product and natural number, and $\sup(\cdot)$ and $\|\cdot\|$ denote the supremum and Euclidean norm.

II. Preliminaries

A. System Formulation

Consider the multiple underactuated systems with Lagrangian dynamics [34]

$$\begin{bmatrix} H_{ipp} & H_{ipa} \\ H_{iap} & H_{iaa} \end{bmatrix}\begin{bmatrix} \ddot{x}_{ip} \\ \ddot{x}_{ia} \end{bmatrix} + \begin{bmatrix} C_{ipp} & C_{ipa} \\ C_{iap} & C_{iaa} \end{bmatrix}\begin{bmatrix} \dot{x}_{ip} \\ \dot{x}_{ia} \end{bmatrix} + \begin{bmatrix} C_{idp} & \mathbf{0} \\ \mathbf{0} & C_{ida} \end{bmatrix}\begin{bmatrix} \dot{x}_{ip} \\ \dot{x}_{ia} \end{bmatrix} + \begin{bmatrix} g_{ip} \\ g_{ia} \end{bmatrix} + \begin{bmatrix} d_{ip} \\ d_{ia} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \tau_{ia} \end{bmatrix} \quad (1)$$

where $i \in \{1,\ldots,N\}$, $t \in [t_0,\infty)$, $x_{ip}, \dot{x}_{ip}, \ddot{x}_{ip} \in \mathbb{R}^{n_p}$ are, respectively, the position, velocity, and acceleration of passive actuators, and $x_{ia}, \dot{x}_{ia}, \ddot{x}_{ia} \in \mathbb{R}^{n_a}$ are those of active ones; $H_{ipp} \in \mathbb{R}^{n_p\times n_p}$, $H_{ipa} \in \mathbb{R}^{n_p\times n_a}$, $H_{iap} \in \mathbb{R}^{n_a\times n_p}$, and $H_{iaa} \in \mathbb{R}^{n_a\times n_a}$ are the elements of the positive inertial matrix, and $C_{ipp} \in \mathbb{R}^{n_p\times n_p}$, $C_{ipa} \in \mathbb{R}^{n_p\times n_a}$, $C_{iap} \in \mathbb{R}^{n_a\times n_p}$, and $C_{iaa} \in \mathbb{R}^{n_a\times n_a}$ are the elements of the Coriolis–centrifugal matrix; $C_{idp} \in \mathbb{R}^{n_p\times n_p}$ and $C_{ida} \in \mathbb{R}^{n_a\times n_a}$ are elements of the damping-friction matrix, and $g_{ip} \in \mathbb{R}^{n_p}$ and $g_{ia} \in \mathbb{R}^{n_a}$ are the elements of the gravitational matrix; $d_{ip} \in \mathbb{R}^{n_p}$ and $d_{ia} \in \mathbb{R}^{n_a}$ are the disturbances, and $\tau_{ia} \in \mathbb{R}^{n_a}$ is the control input. Note that $n = n_p + n_a$, and dynamics (1) has the following universal properties [3], [34].

Property 1: The arbitrary vector $z_i = [z_{ip}^T, z_{ia}^T]^T \in \mathbb{R}^n$ satisfies

$$\begin{bmatrix} H_{ipp} & H_{ipa} \\ H_{iap} & H_{iaa} \end{bmatrix}\dot{z}_i + \begin{bmatrix} C_{ipp} & C_{ipa} \\ C_{iap} & C_{iaa} \end{bmatrix}z_i + \begin{bmatrix} C_{idp} & \mathbf{0} \\ \mathbf{0} & C_{ida} \end{bmatrix}z_i + \begin{bmatrix} g_{ip} \\ g_{ia} \end{bmatrix} = Y_i\vartheta_i$$

$$\begin{bmatrix}\hat{H}_{ipp} & \hat{H}_{ipa}\\ \hat{H}_{iap} & \hat{H}_{iaa}\end{bmatrix}\dot{z}_i+\begin{bmatrix}\hat{C}_{ipp} & \hat{C}_{ipa}\\ \hat{C}_{iap} & \hat{C}_{iaa}\end{bmatrix}z_i +\begin{bmatrix}\hat{C}_{idp} & \mathbf{0}\\ \mathbf{0} & \hat{C}_{ida}\end{bmatrix}z_i+\begin{bmatrix}\hat{g}_{ip}\\ \hat{g}_{ia}\end{bmatrix}=Y_i\hat{\vartheta}_i$$

where $\hat{\cdot}$ denote the parameter uncertainties, and $Y_i = [Y_{ip}^T(x_i,\dot{x}_i,z_i,\dot{z}_i), Y_{ia}^T(x_i,\dot{x}_i,z_i,\dot{z}_i)]^T$ is the known regressor matrix, and ϑ_i and $\hat{\vartheta}_i$ are known and unknown vectors of dynamics, respectively.

Property 2: The inertial matrix is symmetric positive definite, and its derivative has the following relationship with respect to the Coriolis–centrifugal matrix:

$$\begin{bmatrix}\dot{H}_{ipp} & \dot{H}_{ipa}\\ \dot{H}_{iap} & \dot{H}_{iaa}\end{bmatrix}=\begin{bmatrix}C_{ipp} & C_{ipa}\\ C_{iap} & C_{iaa}\end{bmatrix}+\begin{bmatrix}C_{ipp} & C_{ipa}\\ C_{iap} & C_{iaa}\end{bmatrix}^T.$$

Property 3: The dynamic terms are physically bounded, i.e., there exist positive constants π_1, π_2, π_3, π_4, and π_5 satisfying $0 < \|[\begin{smallmatrix}H_{ipp} & H_{ipa}\\ H_{iap} & H_{iaa}\end{smallmatrix}]\| \le \pi_1$, $0 < \|[\begin{smallmatrix}C_{ipp} & C_{ipa}\\ C_{iap} & C_{iaa}\end{smallmatrix}][\begin{smallmatrix}\dot{x}_{ip}\\ \dot{x}_{ia}\end{smallmatrix}]\| \le \pi_2(\dot{x}_{ip}^2+\dot{x}_{ia}^2)$, $0 < \|[\begin{smallmatrix}C_{idp} & \mathbf{0}\\ \mathbf{0} & C_{ida}\end{smallmatrix}][\begin{smallmatrix}\dot{x}_{ip}\\ \dot{x}_{ia}\end{smallmatrix}]\| \le \pi_3(\dot{x}_{ip}^2+\dot{x}_{ia}^2)$, $0 < \|[\begin{smallmatrix}g_{ip}\\ g_{ia}\end{smallmatrix}]\| \le \pi_4$, and $0 < \|[\begin{smallmatrix}d_{ip}\\ d_{ia}\end{smallmatrix}]\| \le \pi_5$.

B. Mathematical Preparations

For underactuated system (1), the individuals and interactions can be, respectively, regarded as the node set $\mathcal{V} = \{1,\ldots,N\}$ and the edge set $\mathcal{E} = \{\mathcal{E}_{j\to i}|i,j\in\mathcal{V}, i\neq j\}$. Thus, the digraph $\mathcal{G} = \{\mathcal{V},\mathcal{E},\mathcal{A}\}$ is employed to describe the system, where $\mathcal{A} = [a_{ij}]_{N\times N}$ is the adjacency matrix with $a_{ij} > 0$ if $\mathcal{E}_{j\to i}$ is valid, and $a_{ij} = 0$ otherwise. Besides, $\mathcal{L} = [l_{ij}]_{N\times N}$ is the Laplacian matrix with $l_{ii} = \sum_{j=1}^N a_{ij}$ and $l_{ij,i\neq j} = -a_{ij}$. Furthermore, digraph $\mathcal{G}$ has a spanning tree if matrix $\mathcal{L}$ has a single zero eigenvalue and the others have positive real parts [35]. Some corresponding assumptions, definitions, and lemmas throughout this article are given as follows.

Assumption 1: Digraph $\mathcal{G}$ has a spanning tree, such that: 1) $\mathcal{L}$ is a semi-positive-definite matrix with eigenvalues $0 = \lambda_1 \le \ldots \le \lambda_N = \lambda_{max}$ in an ascending order and 2) $\mu^T\mathcal{L} = 0$, and $\sum_{i=1}^N \mu_i \sum_{j=1}^N a_{ij} = \sum_{i=1}^N\sum_{j=1}^N \mu_j a_{ji}$, for $\mu = [\mu_1,\ldots,\mu_N]^T$, $\sum_1^N \mu_i = 1$.

Assumption 2: The trigger time intervals $h_i^k = t_i^{k+1} - t_i^k$ in the hereinafter E-T schemes are bounded by $h_i^k \le b_3$, $b_3 > 0$, $k\in\mathbb{N}$ $\forall i\in\mathcal{V}$.

Assumption 3: The damping-friction matrix is diagonal positive definite, satisfying $x_{ip}^T C_{idp} x_{ip} \ge b_1 \ge \|C_{ipp}\| x_{ip}^2$, $x_{ia}^T C_{ida} x_{ia} \ge b_2$, $\exists b_1 > 0$, $\exists b_2 > 0$.

Definition 1: The coordination of underactuated system (1) is accomplished if the system states satisfy

$$\begin{cases}\lim\limits_{t\to\infty}\left\|x_{ia}(t)-x_{ja}(t)\right\| = 0, & \lim\limits_{t\to\infty}\|\dot{x}_{ia}(t)\| = 0\\ \lim\limits_{t\ge t_0}\dot{x}_{ip}(t)\in\mathbb{L}_\infty \quad \forall\ i, & j\in\mathcal{V}.\end{cases} \tag{2}$$

Definition 2 [36]: A solution of the Filippov's system $x = f(t,x)$, $x(t_0) = x_0$, $x\in\mathbb{R}^n$, $t\in[t_0,\infty)$ is an absolutely continuous function $x(t)$, $t\in[t_0,t_1]$, and for almost anytime (a.a.) $t\in[t_0,t_1]$, the differential inclusion $\dot{x}\in\Omega(t,x)$ holds, where $\Omega(t,x) = \cap_{\rho>0}\cap_{\sigma(\nu)=0}\overline{\text{Co}}(h(t,B(x,\rho)\backslash\nu))$, $\overline{Co}$ is the convex closure hull, $B(x,\rho)$ is the open ball of center x with radius ρ, and $\nu\in\mathbb{R}^n$, $\sigma(\nu)$ is the Lebesgue measure of set ν.

Lemma 1 [37]: Consider a system $\dot{x}(t) = Ax(t) + Bu(t)$ with constant matrices $A\in\mathbb{R}^{n\times n}$, $B\in\mathbb{R}^{n\times m}$, unknown state $x(t)\in\mathbb{R}^n$, and known input $u(t)\in\mathbb{R}^m$, then, for any initial state $x(t_0)$, the solution of the system over $t\in[t_0,\infty)$ is $x(t) = \exp(A(t-t_0))x(t_0) + \int_{t_0}^t \exp(A(t-\varsigma))\mathrm{Bu}(\varsigma)d\varsigma$.

III. Quantized Sampled-Data-Based E-T Schemes

This section studies the quantized sampled-data-based E-T control of underactuated systems, with the uniform quantizer $Q_u(\cdot)$ and logarithmic quantizer $Q_l(\cdot)$ defined as [8], [38]

$$\begin{aligned}Q_u(x) &= \text{sign}(x)\left\lfloor\frac{|x|}{o_u}+\frac{1}{2}\right\rfloor o_u\\ Q_l(x) &= \begin{cases}0 & x = 0\\ \text{sign}(x)\left(\frac{1-o_l}{1+o_l}\right)^{\left\lfloor \log_{\frac{1-o_l}{1+o_l}}|x|\right\rfloor} & \text{otherwise}\end{cases}\end{aligned} \tag{3}$$

where $o_u > 0$ and $o_l\in(0,1)$ are quantization precisions; $\text{sign}(\cdot)$ is the sign function, and $\lfloor\cdot\rfloor$ is the floor function producing the nearest integers less than or equal to the involved states. Note that (3) gives the following lemma.

Lemma 2 [8]: The uniform and logarithmic quantizers, respectively, satisfy $\|Q_u(x)-x\| \le o_u\sqrt{n}/2$ and $\|Q_l(x)-x\| \le o_l\|x\|$ $\forall x\in\mathbb{R}^n$.

A. Uniform-Quantization Sampled-Data Communication

Based on uniform-quantization communication, the coordination of systems (1) is investigated, and prior to structure the theorem, the partial-state adaption layer for active and passive actuators is first designed as

$$\begin{cases}\dot{\hat{f}}_{ip}(t) = \begin{cases}\eta_{ip}\chi_{ip}^T(t)\chi_{ip}(t) & \hat{f}_{ip}\le b_4\\ \left(1-\frac{\hat{f}_{ip}^2-b_4^2}{\varepsilon^2+b_4^2}\right)\eta_{ip}\chi_{ip}^2(t) & \text{otherwise}\end{cases} & \text{(4a)}\\ \dot{\hat{f}}_{ia}(t) = \begin{cases}\eta_{ia}\chi_{ia}^T(t)\chi_{ia}(t) & \hat{f}_{ia}\le b_5\\ \left(1-\frac{\varepsilon(\hat{f}_{ia}-b_5)}{\exp(-\varepsilon t)}\right)\eta_{ia}\chi_{ia}^2(t) & \text{otherwise}\end{cases} & \text{(4b)}\\ \dot{z}_i(t) = \begin{bmatrix}-\hat{H}_{ipp}^{-1}\left(\hat{d}_{ip}(t)-\hat{f}_{ip}(t)\chi_{ip}(t)+Y_{ip}^0\hat{\vartheta}_i\right)\\ -\alpha_i z_{ia}(t)-\beta_i\left(x_{ia}(t)-\hat{x}_{ia}(t)\right)\end{bmatrix} & \text{(4c)}\\ \dot{\hat{Z}}_i(t) = \begin{bmatrix}\Lambda_{\vartheta i} & \mathbf{0}\\ \mathbf{0} & \Lambda_{di}\end{bmatrix}\begin{bmatrix}-Y_i^T(t)\chi_i(t)\\ -\chi_i(t)\end{bmatrix} & \text{(4d)}\\ \chi_i(t) = \begin{bmatrix}\dot{x}_{ip}(t)-z_{ip}(t)\\ \dot{x}_{ia}(t)-z_{ia}(t)\end{bmatrix} \quad \forall\ i\in\mathcal{V} & \text{(4e)}\end{cases}$$

where $t\in[t_0,\infty)$, η_{ip}, η_{ia}, ε, b_4, b_5, α_i, and β_i are positive constants, and $\Lambda_{\vartheta i}$ and Λ_{di} are symmetric positive-definite matrices, $z_i = [z_{ip}^T, z_{ia}^T]^T$, $\hat{Z}_i = [\hat{\vartheta}_i^T, \hat{d}_i^T]^T$, $d_i = [d_{ip}^T, d_{ia}^T]^T$, $\chi_i = [\chi_{ip}^T, \chi_{ia}^T]^T$, $Y_{ip}^0 = Y_{ip}(x_i,\dot{x}_i,z_{ip},0_{n_p},z_{ia},\dot{z}_{ia})$, $\hat{H}_{ipp}$ is the estimated value with respect to $\hat{\vartheta}_i$, and $\hat{x}_{ia}$ is the digital signals generated by the following uniform-quantization communication layer with positive constant γ_i:

TABLE I
SCHEME 1

Scheme 1: The Uniform-Quantization Sampled-Data-Based E-T Control Scheme
Initialization: initialize $x_i(t_0), \dot{x}_i(t_0), z_i(t_0), \hat{Z}_i(t_0) \in \mathbb{L}_\infty$, $\hat{f}_{ip}(t_0), \hat{f}_{ia}(t_0), \hat{x}_{ia}(t_0) \in \mathbb{L}_\infty, \forall i \in \mathcal{V}$.
For $t \in [t_0, \infty)$ **do**
Partial-state adaptation: obtain $z_i(t), \hat{Z}_i(t), \hat{f}_{ip}(t), \hat{f}_{ia}(t)$ via the output of partial-state adaption layer (4).
Quantized signal transmission: get $\hat{x}_{ia}(t)$ by the output of uniform-quantization communication layer (5).
Control law: design $\tau_{ia}(t)$ as shown in event-triggered controller layer (6) with event detector $E_i(t)$.
If $E_i(t) = 0, \forall t \in \{t_i^k \mid k \in \mathbb{N}\}$
$\tau_{ia}(t) = \tau_{ia}(t_i^k), \forall t \in [t_i^k, t_i^{k+1}), k \in \mathbb{N}$
End if
End for

$$\dot{\hat{x}}_{ia}(t) = -\gamma_i \sum_{j=1}^{N} a_{ij} Q_u\big[\hat{x}_{ia}(t) - \hat{x}_{ja}(t)\big]. \tag{5}$$

Then, the robust E-T controller layer is designed as

$$\begin{cases} \tau_{ia}(t) = \hat{d}_{ia}\left(t_i^k\right) - \hat{f}_{ia}\left(t_i^k\right)\chi_{ia}\left(t_i^k\right) + Y_{ia}\left(t_i^k\right)\hat{\vartheta}_i\left(t_i^k\right) & \text{(6a)} \\ \sum_{j=1}^{3} \left\|\varepsilon_{ij}(t)\right\| = \phi_{i1}\|\chi_{ia}(t)\| + \phi_{i2}\exp(-\omega_i t + \varphi_i) & \text{(6b)} \\ \varepsilon_i(t) = \begin{bmatrix} \hat{d}_{ia}\left(t_i^k\right) \\ \hat{f}_{ia}\left(t_i^k\right)\chi_{ia}\left(t_i^k\right) \\ -Y_{ia}\left(t_i^k\right)\hat{\vartheta}_i\left(t_i^k\right) \end{bmatrix} - \begin{bmatrix} \hat{d}_{ia}(t) \\ \hat{f}_{ia}(t)\chi_{ia}(t) \\ -Y_{ia}(t)\hat{\vartheta}(t) \end{bmatrix} & \text{(6c)} \end{cases}$$

where $i \in \mathcal{V}$, $t \in [t_i^k, t_i^{k+1})$, $k \in \mathbb{N}$, constants $\phi_{i1} \in (0, b_5 - 0.5)$, $\phi_{i2} > 0$, $\omega_i > 0$, $\varphi_i \in \mathbb{R}$, vector $\varepsilon_{ij}(t) \in \mathbb{R}^{n_a}$ for $\forall j \in \{1, 2, 3\}$, $Y_i = [Y_{ip}^T(x_i, \dot{x}_i, z_{ip}, \dot{z}_{ip}, z_{ia}, \dot{z}_{ia}), Y_{ia}^T(x_i, \dot{x}_i, z_{ip}, \dot{z}_{ip}, z_{ia}, \dot{z}_{ia})]^T$.

Remark 1: Design the event detector $E_i(t) = \sum_{j=1}^{3} \varepsilon_{ij}(t) - \phi_{i1}\chi_{ia}(t) - \phi_{i2}\exp(-\omega_i t + \varphi_i)$, which satisfies $E_i \le 0$, and only if $E_i = 0$, the controller layer is allowed to be triggered and updated; otherwise, it remains constant during the period $t \in [t_i^k, t_i^{k+1})$. Note that larger trigger periods of E-T schemes may result in less energy occupation, thus for achieving longer trigger periods, one can design proper control parameters ϕ_{i1}, ϕ_{i2}, ω_i, and φ_i to guarantee an optimal tradeoff between the control cost and performance. Additionally, assume that the vectors and matrices employed in what follows keep appropriate dimensions, unless otherwise stipulated.

Structure the uniform-quantization sampled-data-based E-T scheme in Table I, then by designing $\tilde{f}_{ip} = b_4 - \hat{f}_{ip}$, $\tilde{f}_{ia} = b_5 - \hat{f}_{ia}$, $\tilde{Z}_i = Z_i - \hat{Z}_i$, with a constant vector $Z_i = [\vartheta_i^T, d_i]^T$, substituting Property 1 and the structured scheme (4)–(6) into (1) yields the following cascade closed-loop system:

$$\Delta_{i1}: \begin{cases} \dot{\hat{f}}_{ip} = \begin{cases} -\eta_{ip}\chi_{ip}^T\chi_{ip} & \hat{f}_{ip} \le b_4 \\ \left(\frac{\hat{f}_{ip}^2 - b_4^2}{\varepsilon_{i1}^2 + b_4^2} - 1\right)\eta_{ip}\chi_{ip}^2 & \text{otherwise} \end{cases} \\ \dot{\hat{f}}_{ia} = \begin{cases} -\eta_{ia}\chi_{ia}^T\chi_{ia} & \hat{f}_{ia} \le b_5 \\ \left(\frac{\varepsilon_{i2}(\hat{f}_{ia} - b_5)}{\exp(-\varepsilon_{i3}t)} - 1\right)\eta_{ia}\chi_{ia}^2 & \text{otherwise} \end{cases} \\ \dot{\hat{Z}}_i = \left[\dot{\hat{\vartheta}}_i^T, \dot{\hat{d}}_i^T\right]^T = \begin{bmatrix} \Lambda_{\vartheta i} & \mathbf{0} \\ \mathbf{0} & \Lambda_{di} \end{bmatrix}\begin{bmatrix} Y_i^T\chi_i \\ \chi_i \end{bmatrix} \\ \dot{\chi}_i = \begin{bmatrix} H_{ipp} & H_{ipa} \\ H_{iap} & H_{iaa} \end{bmatrix}^{-1}\left(\begin{bmatrix} -d_{ip} - Y_{ip}\vartheta_i \\ \tau_{ia} - d_{ia} - Y_{ia}\vartheta_i \end{bmatrix} \right. \\ \qquad \left. - \begin{bmatrix} C_{ipp} + C_{idp} & C_{ipa} \\ C_{iap} & C_{iaa} + C_{ida} \end{bmatrix}\begin{bmatrix} \chi_{ip} \\ \chi_{ia} \end{bmatrix}\right) \\ \dot{z}_{ip} = -\hat{H}_{ipp}^{-1}\left(Y_{ip}^0\hat{\vartheta}_i - \hat{f}_{ip}\chi_{ip} - \hat{d}_{ip}\right) \\ \sum_{j=1}^{3}\left\|\varepsilon_{ij}\right\| \le \phi_{i1}\|\chi_{ia}\| + \phi_{i2}\exp(-\omega_i t + \varphi_i) \end{cases} \tag{7}$$

$$\Delta_{i2}: \dot{\hat{x}}_{ia}(t) = -\gamma_i \sum_{j=1}^{N} a_{ij} Q_u\big[\hat{x}_{ia}(t) - \hat{x}_{ja}(t)\big] \tag{8}$$

$$\Delta_{i3}: \begin{cases} \dot{z}_{ia}(t) = -\alpha_i z_{ia}(t) - \beta_i\big(x_{ia}(t) - \hat{x}_{ia}(t)\big) \\ \dot{x}_{ia}(t) = \chi_{ia}(t) + z_{ia}(t). \end{cases} \tag{9}$$

Theorem 1: For underactuated systems (1) with uniform-quantization communication and Assumption 1, the structured control scheme, given in Table I, can accomplish coordination (2) and avoid Zeno behavior synchronously, i.e., $\|x_{ia}(\infty) - x_{ja}(\infty)\| = 0$, $\|\dot{x}_{ia}(\infty)\| = 0$, $\dot{x}_{ip}(t) \in L_\infty$, and $h_i^k = t_i^{k+1} - t_i^k > 0\ \forall i, j \in \mathcal{V}$.

Proof: Based on the cascade closed-loop subsystem Δ_{i1}, Δ_{i2}, and Δ_{i3} for $\forall i \in \mathcal{V}$ in (7)–(9), the theorem can be accomplished through the following four steps.

Step One: The first step is to analyze the convergence of state $\chi_i = [\chi_{ip}^T, \chi_{ia}^T]^T$ of subsystem Δ_{i1} for $\forall i \in \mathcal{V}$ as $t \in [t_0, \infty)$.

Consider the positive Lyapunov–Krasovskii functional candidate $V_1(t) = \sum_{i=1}^{N} V_{i1}(t)$, and V_{i1} with the following form:

$$\begin{aligned} V_{i1}(t) = {} & \begin{bmatrix} \chi_{ip} \\ \chi_{ia} \end{bmatrix}^T \begin{bmatrix} H_{ipp} & H_{ipa} \\ H_{iap} & H_{iaa} \end{bmatrix}\begin{bmatrix} \chi_{ip} \\ \chi_{ia} \end{bmatrix} + \eta_{ip}^{-1}\tilde{f}_{ip}^T\tilde{f}_{ip} \\ & + \begin{bmatrix} \tilde{\vartheta}_i \\ \tilde{d}_i \end{bmatrix}^T \begin{bmatrix} \Lambda_{\vartheta i} & \mathbf{0} \\ \mathbf{0} & \Lambda_{di} \end{bmatrix}\begin{bmatrix} \tilde{\vartheta}_i \\ \tilde{d}_i \end{bmatrix} + \eta_{ia}^{-1}\tilde{f}_{ia}^T\tilde{f}_{ia} \\ & + \frac{\phi_{i2}^2}{2\omega_i}\exp(-2\omega_i t + 2\varphi_i). \end{aligned} \tag{10}$$

Then, by Property 2, Definition 2, $\chi_{ip}^T Y_{ip}\hat{\vartheta}_i = -\chi_{ip}^T\hat{d}_{ip} + \chi_{ip}^T\hat{f}_{ip}\chi_{ip}$ derived from (7), the time derivative of (10) along (7) exists for a.a. $t \in [t_0, \infty)$ and yields

$$\begin{aligned} \dot{V}_{i1}(t) \overset{\text{a.a.}}{\in} {} & \begin{bmatrix} \chi_{ip} \\ \chi_{ia} \end{bmatrix}^T \begin{bmatrix} \dot{H}_{ipp} & \dot{H}_{ipa} \\ \dot{H}_{iap} & \dot{H}_{iaa} \end{bmatrix}\begin{bmatrix} \chi_{ip} \\ \chi_{ia} \end{bmatrix} + 2\begin{bmatrix} \chi_{ip} \\ \chi_{ia} \end{bmatrix}^T \\ & \times \begin{bmatrix} H_{ipp} & H_{ipa} \\ H_{iap} & H_{iaa} \end{bmatrix}\begin{bmatrix} \dot{\chi}_{ip} \\ \dot{\chi}_{ia} \end{bmatrix} + 2\tilde{\vartheta}_i^T\Lambda_{\vartheta i}^{-1}\dot{\tilde{\vartheta}}_i \\ & + 2\tilde{d}_i^T\Lambda_{di}^{-1}\dot{\tilde{d}}_i + 2\eta_{ip}^{-1}\tilde{f}_{ip}^T\dot{\tilde{f}}_{ip} + 2\eta_{ia}^{-1}\tilde{f}_{ia}^T\dot{\tilde{f}}_{ia} \\ & - \phi_{i2}^2\exp(-2\omega_i t + 2\varphi_i) \\ \overset{\text{a.a.}}{\in} {} & -2\chi_{ip}^T C_{idp}\chi_{ip} - 2\chi_{ia}^T C_{ida}\chi_{ia} - 2\chi_{ip}^T\tilde{f}_{ip}\chi_{ip} \end{aligned}$$

$$+2\chi_{ia}^T\Big(\hat{d}_{ia}\Big(t_i^k\Big)-\hat{d}_{ia}(t)\Big)-2\chi_{ia}^T\hat{f}_{ia}\Big(t_i^k\Big)\chi_{ia}\Big(t_i^k\Big)$$
$$+2\chi_{ia}^T\Big(Y_{ia}\Big(t_i^k\Big)\hat{\vartheta}_i\Big(t_i^k\Big)-Y_{ia}(t)\hat{\vartheta}_i(t)\Big)+2\eta_{ip}^{-1}\tilde{f}_{ip}^T\dot{\hat{f}}_{ip}$$
$$+2\eta_{ia}^{-1}\tilde{f}_{ia}^T\dot{\hat{f}}_{ia}-\phi_{i2}^2\exp(-2\omega_i t+2\varphi_i). \tag{11}$$

Note that

$$\eta_{ip}^{-1}\tilde{f}_{ip}^T\dot{\hat{f}}_{ip}=\begin{cases}-\tilde{f}_{ip}^T\chi_{ip}^T\chi_{ip} & \hat{f}_{ip}\le b_4\\ (W_1-1)\tilde{f}_{ip}^T\chi_{ip}^2\le-\tilde{f}_{ip}^T\chi_{ip}^2 & \text{otherwise}\end{cases}$$
$$\eta_{ia}^{-1}\tilde{f}_{ia}^T\dot{\hat{f}}_{ia}=\begin{cases}-\tilde{f}_{ia}^T\chi_{ia}^T\chi_{ia} & \hat{f}_{ia}\le b_5\\ (W_2-1)\tilde{f}_{ia}^T\chi_{ia}^2\le-\tilde{f}_{ia}^T\chi_{ia}^2 & \text{otherwise}\end{cases}$$

where $W_1 = (\hat{f}_{ip}^2-b_4^2/\varepsilon_{i1}^2+b_4^2) > 0$ and $W_2 = \varepsilon_{i2}(\hat{f}_{ia}-b_5)/\exp(-\varepsilon_{i3}t)) > 0$, such that $\eta_{ip}^{-1}\tilde{f}_{ip}^T\dot{\hat{f}}_{ip}+\eta_{ia}^{-1}\tilde{f}_{ia}^T\dot{\hat{f}}_{ia}\le -\tilde{f}_{ip}\chi_{ip}^2-\tilde{f}_{ia}\chi_{ia}^2$. Thus, by Assumption 3 and event detector E_i, one rewrites (11) as

$$\begin{aligned}\dot{V}_{i1}(t)&\le -2b_4\chi_{ip}^T\chi_{ip}-2b_5\chi_{ia}^T\chi_{ia}+2\chi_{ia}^T\Big[\hat{d}_{ia}\Big(t_i^k\Big)-\hat{d}_{ia}(t)\\&\quad+Y_{ia}\Big(t_i^k\Big)\hat{\vartheta}_i\Big(t_i^k\Big)-Y_{ia}(t)\hat{\vartheta}_i(t)-\hat{f}_{ia}\Big(t_i^k\Big)\chi_{ia}\Big(t_i^k\Big)\\&\quad+\hat{f}_{ia}(t)\chi_{ia}(t)\,\Big]-\phi_{i2}^2\exp(-2\omega_i t+2\varphi_i)\\&\le 2\sum_{j=1}^{3}\|\chi_{ia}\|\|\varepsilon_{ij}\|-2b_5\chi_{ia}^2-\phi_{i2}^2\exp(-2\omega_i t+2\varphi_i)\\&\le 2(\phi_{i1}-b_5)\chi_{ia}^2-\phi_{i2}\exp(-2\omega_i t+2\varphi_i)\\&\quad+2\phi_{i2}\|\chi_{ia}\|\exp(-\omega_i t+\varphi_i)\\&=-(\|\chi_{ia}\|-\phi_{i2}\exp(-\omega_i t+\varphi_i))^2\\&\quad+(2\phi_{i1}-2b_5+1)\chi_{ia}^2\end{aligned}$$

which gives $\dot{V}_{i1}(t)\le 0$ for $\phi_{i1}\in(0,b_5-0.5)$, such that $V_{i1}(t)\in\mathbb{L}_\infty$, i.e., $\chi_i,\tilde{\vartheta}_i,\tilde{d}_i,\tilde{f}_{ip},\tilde{f}_{ia}\in\mathbb{L}_\infty$. Then, it gives $\dot{\chi}_i\in\mathbb{L}_\infty$ for bounded the initial values by (7) and Property 3, and thus one concludes $\chi_i(\infty)=0_n$ by Barbalat's lemma.

Step Two: The second step is to analyze the synchronization of state $\hat{x}_{ia}$ and the convergence of state $\dot{\hat{x}}_{ia}$ of subsystem Δ_{i2}. Consider the positive Lyapunov–Krasovskii functional candidate

$$V_2(t)=\sum_{i=1}^{N}\mu_i\hat{x}_{ia}^T(t)\hat{x}_{ia}(t). \tag{12}$$

Similarly, the time derivative of (12) along (8), with Filippov's solution Q_u^F, exists for a.a. $t\in[t_0,\infty)$ and yields

$$\dot{V}_2(t)\overset{\text{a.a.}}{\in}-2\sum_{i=1}^{N}\gamma_i\mu_i\hat{x}_{ia}^T\sum_{j=1}^{N}a_{ij}Q_u^F\big(\hat{x}_{ia}-\hat{x}_{ja}\big). \tag{13}$$

Note that $\sum_{i=1}^N\mu_i\sum_{j=1}^N a_{ij}=\sum_{i=1}^N\sum_{j=1}^N\mu_j a_{ji}$, and $\sum_{i=1}^N\mu_i\sum_{j=1}^N a_{ij}=\sum_{i=1}^N\sum_{j=1}^N\mu_i a_{ij}$, such that by designing $\hat{x}_{jia}=\hat{x}_{ja}-\hat{x}_{ia}$, one obtains

$$\begin{aligned}\dot{V}_2(t)&\overset{\text{a.a.}}{\in}-W_3-\sum_{i=1}^{N}\gamma_i\hat{x}_{ia}^T\sum_{j=1}^{N}\mu_j a_{ji}Q_u^F\big(\hat{x}_{ija}\big)\\&\overset{\text{a.a.}}{\in}-W_3+\sum_{j=1}^{N}\gamma_i\hat{x}_{ja}^T\sum_{i=1}^{N}\mu_i a_{ij}Q_u^F\big(\hat{x}_{ija}\big)\\&\overset{\text{a.a.}}{\in}-W_3+\sum_{i=1}^{N}\sum_{j=1}^{N}\gamma_i\mu_i a_{ij}\hat{x}_{ja}^T Q_u^F\big(\hat{x}_{ija}\big)\\&\overset{\text{a.a.}}{\in}-\sum_{i=1}^{N}\sum_{j=1}^{N}\gamma_i\mu_i a_{ij}\hat{x}_{ija}^T Q_u^F\big(\hat{x}_{ija}\big)\\&\overset{\text{a.a.}}{\in}\sum_{i=1}^{N}\sum_{j=1}^{N}\gamma_i\mu_i a_{ij}\Big(-\big\|\hat{x}_{ija}\big\|^2+W_4\Big)\end{aligned} \tag{14}$$

where $W_3 = \sum_{i=1}^N\sum_{i=1}^N\gamma_i\mu_i a_{ij}\hat{x}_{ia}^T Q_u^F(\hat{x}_{ija})$ and $W_4 = -\hat{x}_{ija}^T(Q_u^F(\hat{x}_{ija}) - \hat{x}_{ija}) \le \|\hat{x}_{ija}\|\|Q_u^F(\hat{x}_{ija})-\hat{x}_{ija}\|$, with $\|Q_u^F(\hat{x}_{ija})-\hat{x}_{ija}\|\le o_u\sqrt{n_a}/2$, $o_u>0$ by Lemma 2. Then, (14) satisfies

$$\dot{V}_2(t)\le-\sum_{i=1}^{N}\sum_{j=1}^{N}\gamma_i\mu_i a_{ij}\big\|\hat{x}_{ija}\big\|\bigg(\big\|\hat{x}_{ija}\big\|-\frac{o_u\sqrt{n_a}}{2}\bigg)$$

which gives $\dot{V}_2(t)\le 0$, i.e., $V_2(t)\in\mathbb{L}_\infty$ if $o_u\le 2\|\hat{x}_{ija}\|/\sqrt{n_a}$, thus $\hat{x}_{ia}\in\mathbb{L}_\infty$, and $\hat{x}_{jia}=\hat{x}_{ja}-\hat{x}_{ia}\in\mathbb{L}_\infty$. Then, one obtains $\dot{\hat{x}}_{jia}\in\mathbb{L}_\infty$ by (8), and thus it concludes $\hat{x}_{ia}(\infty)=\hat{x}_{ja}(\infty)$ and $\dot{\hat{x}}_{ia}(\infty)=0_{n_a}$.

Step Three: The third step is to analyze the synchronization of state x_{ia}, the convergence of state $\dot{x}_{ia}$, and the boundness of states x_{ip} and $\dot{x}_{ip}$ in subsystem Δ_{i3}. Design $\delta_i=[z_{ia}^T,\tilde{x}_{ia}^T]^T$ with $\tilde{x}_{ia}=x_{ia}-\hat{x}_{ia}$, such that rewrite subsystem Δ_{i3} as

$$\dot{\delta}_i=(\Xi_1\otimes I_{n_a})\delta_i+(\Xi_2\otimes I_{n_a})(\chi_{ia}-\dot{\hat{x}}_{ia}) \tag{15}$$

with

$$\Xi_1=\begin{bmatrix}-\alpha_i & -\beta_i\\ 1 & 0\end{bmatrix},\quad \Xi_2=\begin{bmatrix}0\\1\end{bmatrix}.$$

The eigenvalues λ_i of matrix Ξ_1 can be achieved for solving the following characteristic polynomial equation:

$$\det(\lambda_i I_2-\Xi_1)=\lambda_i^2+\alpha_i\lambda_i+\beta_i$$

such that $\lambda_{i,\pm}(\Xi_1)=-\frac{1}{2}\alpha_i\pm\sqrt{\frac{1}{4}\alpha_i^T\alpha_i-\beta_i}<0$ for $\forall i\in\mathcal{V}$ based on positive constants α_i and β_i and, thus, Ξ_1 is a Hurwitz matrix. Based on input-to-sate stability, (15) gives $\delta(\infty)=0_{n_{2a}}$, $\dot{\delta}(\infty)=0_{n_{2a}}$, i.e., $\tilde{x}_{ia}(\infty)=0_{n_a}$, $\dot{x}_{ia}(\infty)=0_{n_a}$, with inputs $\chi_{ia}(\infty)=0_{n_a}$, $\dot{\hat{x}}_{ia}(\infty)=0_{n_a}$. Then, with result $\hat{x}_{ia}(\infty)=\hat{x}_{ja}(\infty)$, it concludes $x_{ia}(\infty)=x_{ja}(\infty)$. Moreover, by Lemma 1, (15) gives

$$\begin{aligned}\delta(t)&=\exp(\Xi_3(t-t_0))\delta(t_0)\\&\quad+\int_{t_0}^{t}\exp(\Xi_3(t-\varsigma))\Xi_4\Big(\chi_{ia}(\varsigma)-\dot{\hat{x}}_{ia}(\varsigma)\Big)d\varsigma\\&\le\exp(\Xi_3(t-t_0))\|\delta(t_0)\|+\int_{t_0}^{t}\exp(\Xi_3(t-\varsigma))W_5 d\varsigma\end{aligned}$$

where $\Xi_3 = \Xi_1\otimes I_{n_a}$, $\Xi_4 = \Xi_2\otimes I_{n_a}$, and $W_5 = \sup_{\varsigma\ge t_0}(\|\chi_{ia}(\varsigma)\|+\|\dot{\hat{x}}_{ia}(\varsigma)\|)$ and, thus, it means $\delta(t),\dot{\delta}(t)\in\mathbb{L}_\infty$, i.e., $x_{ia},\dot{x}_{ia}\in\mathbb{L}_\infty$ for $\hat{x}_{ia},\dot{\hat{x}}_{ia}\in\mathbb{L}_\infty$. By dynamics (1), one obtains $\ddot{x}_{ip}=-H_{ipp}^{-1}(C_{ipp}+C_{idp})\dot{x}_{ip}-H_{ipp}^{-1}(H_{ipa}\ddot{x}_{ia}+C_{ipa}\dot{x}_{ia}+g_{ip}+d_{ip})$, then, by Lemma 1, Assumption 3, and Property 3, with $H_{ipp}^{-1}(H_{ipa}\ddot{x}_{ia}+C_{ipa}\dot{x}_{ia}+g_{ip}+d_{ip})\in\mathbb{L}_\infty$, and $-H_{ipp}^{-1}(C_{ipp}+C_{idp})$ being Hurwitz, it can be concluded that $\dot{x}_{ip}$ is bounded and cannot escape. It thus follows Definition 1 that $\|x_{ia}(\infty)-x_{ja}(\infty)\|=0$, $\|\dot{x}_{ia}(\infty)\|=0$, and $\dot{x}_{ip}(t)\in\mathbb{L}_\infty$.

Step Four: The fourth step is to make sure that the undesirable Zeno behavior, namely, the infinite trigger numbers within a finite periods, is not exhibited. Define $\bar{\delta}(t) = \delta_i(t) - \delta_i(t_i^k)$ as $t \in [t_i^k, t_i^{k+1})$, such that $\dot{\bar{\delta}}_i(t) = \dot{\delta}_i(t)$, and by (15), it yields

$$\|\bar{\delta}_i(t)\| = \left\| \int_{t_i^k}^{t} \dot{\bar{\delta}}_i(\sigma) d\varsigma \right\| < \int_{t_i^k}^{t_i^{k+1}} (W_5 + W_6) d\varsigma$$

where $W_6 = \sup_{\varsigma \geq t_0} \|\lambda_{\min}(\Xi_1)\delta(\varsigma)\|$, and it implies

$$h_i^k = t_i^{k+1} - t_i^k > (W_5 + W_6)\|\bar{\delta}\|$$

which concludes $h^k > 0$, demonstrating that Zeno behavior can be eliminated, and thus finishes the proof. ■

B. Logarithmic-Quantization Sampled-Data Communication

Replace the uniform-quantization communication layer (5) in Theorem 1 with the logarithmic-quantization communication layer (16), and one obtains Theorem 2

$$\Delta_{i2}\colon \dot{\hat{x}}_{ia}(t) = -\gamma_i \sum_{j=1^N} a_{ij} Q_l\left[\hat{x}_{ia}(t) - \hat{x}_{ja}(t)\right]. \tag{16}$$

Theorem 2: For underactuated systems (1) with logarithmic-quantization communication and Assumption 1, the control scheme structured with (4), (6), and (16) can accomplish coordination (2) and avoid Zeno behavior synchronously.

Proof: For cascade closed-loop system Δ_{i1}, Δ_{i2}, and Δ_{i3} in (7), (16), and (9), it gives $\chi_i(\infty) = 0_n$ by the analysis of subsystem Δ_{i1}. Then, consider function (12) for subsystem Δ_{i2} with Filippov's solution Q_l^F and $\|Q_l^F(x) - x\| \leq o_l\|x\|$, $o_l \in (0, 1)$ by Lemma 2, so that for a.a. $t \in [t_0, \infty)$, it gives

$$\begin{aligned} \dot{V}_{i2}(t) &\overset{\text{a.a.}}{\in} -2\sum_{i=1}^{N} \gamma_i \mu_i \hat{x}_{ia}^T \sum_{j=1}^{N} a_{ij} Q_l^F\left(\hat{x}_{ija}\right) \\ &\overset{\text{a.a.}}{\in} \sum_{i=1}^{N}\sum_{j=1}^{N} \gamma_i \mu_i a_{ij}\left(-\|\hat{x}_{ija}\|^2 + W_7\right) \\ &\leq -(1 - o_l)\sum_{i=1}^{N}\sum_{j=1}^{N} \gamma_i \mu_i a_{ij}\|\hat{x}_{ija}\|^2 \\ &\leq 0 \end{aligned} \tag{17}$$

where $W_7 = -\hat{x}_{ija}^T(Q_l^F(\hat{x}_{ija}) - \hat{x}_{ija}) \leq o_l\|\hat{x}_{ija}\|^2$, and likewise, it concludes $\hat{x}_{ia}(\infty) = \hat{x}_{ja}(\infty)$ and $\dot{\hat{x}}_{ia}(\infty) = 0_{n_a}$.

Next, by a similar analysis with respect to Theorem 1, one finally obtains that $\|x_{ia}(\infty) - x_{ja}(\infty)\| = 0$, $\|\dot{x}_{ia}(\infty)\| = 0$, $q_{ip}(t), \dot{q}_{ip}(t) \in \mathbb{L}_\infty$, $h_i^k > 0$, and thus completes the proof. ■

IV. Triggered Sampled-Data-Based E-T Schemes

Note that the above-mentioned schemes depend on fixed networks and continuous-time quantized communications. To further save communication resources, this section studies the triggered sampled-data-based E-T control of underactuated systems with fixed and switched networks, where the communications are discrete time and are linked only at the triggered instants. Some useful lemmas are given as follows in advance.

TABLE II
SCHEME 3

Scheme 3: The E-T Sampled-Data-Based E-T Control Scheme
Initialization: for $\forall i \in \mathcal{V}$, initialize $x_i(t_0), \dot{x}_i(t_0), z_i(t_0)$, $\hat{Z}_i(t_0), \hat{f}_{ip}(t_0), \hat{f}_{ia}(t_0), \hat{x}_{ia}(t_0) \in \mathbb{L}_\infty$.
For $t \in [t_0, \infty)$ **do**
Partial-state adaptation: obtain $z_i(t), \hat{Z}_i(t), \hat{f}_{ip}(t), \hat{f}_{ia}(t)$ by subsystem Δ_{i1} (7).
Quantized signal transmission: get $\hat{x}_{ia}(t)$ by E-T sampled-data communication layer (18).
Controller: following Table I with $E(t, t^k)$ to design $\tau_{ia}(t)$.
End for

Lemma 3 [39]: The matrix $M \in \mathbb{R}^{N\times N}$ is row stochastic if all elements of each row sum equal to one. Moreover, a row stochastic matrix $M \in \mathbb{R}^{N\times N}$ is indecomposable and aperiodic (SIA), if the eigenvalues of matrix M are positive, and its digraph $\mathcal{G}(M)$ contains a spanning tree.

Lemma 4 [40]: If a non-negative matrix $M \in \mathbb{R}^{N\times N}$ has the same positive constant row sums given by $\rho > 0$, then ρ is an eigenvalue of M with an associated eigenvector 1_N. Additionally, the eigenvalue ρ of M has algebraic multiplicity equal to one, if and only if the graph associated with M, i.e., $\mathcal{G}(M)$, contains a spanning tree.

Lemma 5 [41]: Let $M_1, M_2, \ldots, M_k \in \mathbb{R}^N \times \mathbb{R}^N$ be a finite set of SIA matrices with the property that for each sequence $M_{i1}, M_{i2}, \ldots, M_{ij}$ with positive length, the matrix product $M_{i1}M_{i2}\ldots M_{ij}$ is SIA. Then, for each infinite sequence $M_{i1}M_{i2}\ldots M_{ij}\ldots$, there exists a constant column vector $y \in \mathbb{R}^N$ satisfying $\lim_{j\to\infty} M_{i1}M_{i2}\ldots M_{ij} = 1_N y^T$.

A. E-T Sampled-Data Communication

Based on the following E-T sampled-data communication layer and event detector in (18), the coordination of systems (1) is investigated with the E-T sampled-data-based E-T scheme structured in Table II

$$\begin{aligned} \dot{\hat{x}}_{ia}(t) &= -\gamma_i \sum_{j=1}^{N} a_{ij}\left(\hat{x}_{ia}\left(t^k\right) - \hat{x}_{ja}\left(t^k\right)\right) \\ E(t, t^k) &= \left\|\varepsilon\left(t, t^k\right)\right\| - \phi_1\|\chi_a(t)\| - \phi_3 = 0 \end{aligned} \tag{18}$$

where $\gamma_i \in (0, b_3^{-1}\lambda_N^{-1})$, $\lambda_N = \lambda_{max}(\mathcal{L})$ by Assumption 1, $\varepsilon(t, t^k) = [\varepsilon_1^T(t, t^k), \ldots, \varepsilon_N^T(t, t^k)]^T$, $\chi_a = [\chi_{1a}^T, \ldots, \chi_{Na}^T]^T$, $\phi_3 = \phi_2 \exp(-\omega t + \varphi)$, $\phi_1 \in (0, b_5 - 0.5)$, $\phi_2 > 0$, $\omega > 0$, $\varphi \in \mathbb{R}$, $\hat{q}_{ia}(t_i^k)$ and $\hat{q}_{ja}(t_i^k)$ are the E-T sampled-data states.

Theorem 3: For underactuated systems (1) with E-T sampled-data communication and Assumption 1, the control scheme structured in Table II can accomplish coordination (2) and avoid Zeno behavior synchronously.

Proof: By a similar analysis with respect to Theorem 1, one obtains $\chi_i(\infty) = 0_n$ by subsystem Δ_{i1}. Then, designing $\gamma = \text{diag}\{\gamma_1, \ldots, \gamma_N\}$, $\hat{x}_a = [\hat{x}_{1a}^T, \ldots, \hat{x}_{Na}^T]^T$, $\dot{\hat{x}}_a = [\dot{\hat{x}}_{1a}^T, \ldots, \dot{\hat{x}}_{Na}^T]^T$, and $\hat{x}_a(t^k) = [\hat{x}_{1a}^T(t^k), \ldots, \hat{x}_{Na}^T(t^k)]^T$ for (18), one obtains

$$\dot{\hat{x}}_a = -\gamma\left(\mathcal{L} \otimes I_{n_a}\right)\hat{x}_a\left(t^k\right). \tag{19}$$

Integrating both sides of (20) along $t \in [t^k, t^{(k+1)^-}]$, with $(k+1)^- \to (k+1)$, yields

$$\hat{x}_a\left(t^{(k+1)^-}\right) = \left(\left(I_N - h^k\gamma\mathcal{L}\right) \otimes I_{n_a}\right)\hat{x}_a\left(t^k\right).$$

Through a recursive analysis technique, one obtains

$$\hat{x}_a\left(t^{(k+1)^-}\right) = \prod_{\sigma=0}^{k}\left(M_\sigma \otimes I_{n_a}\right)\hat{x}_a\left(t^0\right) \tag{20}$$

where $\hat{x}_a(t^0) = \hat{x}_a(t_0) \in \mathbb{L}_\infty$ and $M_\sigma = I_N - h^\sigma\gamma\mathcal{L}$. $\prod_{\sigma=0}^{k} M_\sigma$ is a row stochastic matrix with positive diagonal elements by Lemma 3 with $\gamma_i b_3 \lambda_N \in (0, 1)$ and $(\prod_{\sigma=0}^{k} M_\sigma)1_N = 1_N$. Note that $\sum_{\sigma=0}^{k} h^\sigma\gamma\mathcal{L}$ has only one eigenvalue as zero by Assumption 1 with $h^\sigma > 0$, $\gamma > 0$. Then, by $(\prod_{\sigma=0}^{k} M_\sigma)1_N = 1_N$, it gives that the eigenvalue 1 of $\prod_{\sigma=0}^{k} M_\sigma$ is of algebraic multiplicity 1 and, thus, it gives that $\mathcal{G}(\prod_{\sigma=0}^{k} M_\sigma)$ contains a spanning tree by Lemma 4. Therefore, we can obtain that $\prod_{\sigma=0}^{k} M_\sigma$ is SIA by Lemma 3. Thus, it concludes $\prod_{\sigma=0}^{k} M_\sigma = 1_N y^T$ for a constant column vector $y \in \mathbb{R}^N$. It thus follows (20) that:

$$\lim_{t\to\infty}\hat{x}_a(t) = \lim_{k\to\infty}\hat{x}_a\left(t^{(k+1)^-}\right) = \left(1_N y^T \otimes I_{n_a}\right)\hat{x}_a(t_0) \tag{21}$$

which gives $\hat{x}_{ia}(\infty) = \hat{x}_{ja}(\infty)$ and $\dot{\hat{x}}_{ia}(\infty) = 0_{n_a}$ for $\forall i, j \in \mathcal{V}$. Then, (2) can be achieved, and the proof can be completed by following the same steps performed in Theorem 1. ∎

B. Switched E-T Sampled-Data Communication

In practical, the communication links are always switched for the unreliability and limitations, thus, this section discusses the coordination problem over switched digraph $\mathcal{G}_\sigma = \{\mathcal{V}, \mathcal{E}, \mathcal{A}_\sigma\}$ and Laplacian matrix $\mathcal{L}_\sigma$, with switching index σ. The communication in Scheme 3 is performed synchronously, while the switched and distributed E-T sampled-data communication layer and event detector are designed as

$$\begin{aligned}\dot{\hat{x}}_{ia} &= -\gamma_i\sum_{j=1}^{N} a_{ij\sigma}\left(\hat{x}_{ia}\left(t_i^k\right) - \hat{x}_{ja}\left(t_j^{k^*}\right)\right)\\ E_i\left(t, t_i^k\right) &= \sum_{j=1}^{3}\left\|\varepsilon_{ij}(t, t_i^k)\right\| - \phi_{i1}\|\chi_{ia}(t)\| - \phi_{i3} = 0\end{aligned} \tag{22}$$

where $\gamma_i \in (0, b_3^{-1}\lambda_N^{-1})$, $a_{ij\sigma}$ is the element of $\mathcal{A}_\sigma$, $k^* = \arg\min_{k\in\mathbb{N}:t\geq t_j^k}\{t - t_j^k\}$, and $\phi_{i3} = \phi_{i2}\exp(-\omega_i t + \varphi_i)$.

Theorem 4: For underactuated systems (1) with switched E-T sampled-data communication, if the union of the switched digraph $\bigcup_{\sigma=b_6}^{b_7}\mathcal{G}_\sigma$, $b_6 \in \mathbb{N}^+$, $b_7 \in \mathbb{N}^+$, has a spanning tree, the control scheme structured with (4), (6), and (22) can accomplish coordination (2) and avoid Zeno behavior.

Proof: Based on Theorems 1 and 3, one obtains $\chi_i(\infty) = 0_n$ and (20) with $\prod_{\sigma=0}^{k} M_\sigma$ being a row stochastic matrix. Then, by [42, Lemma 1] with $k \in \mathbb{N}^+, k \geq 2$ and $M_\sigma = I_N - h^\sigma\gamma\mathcal{L}_\sigma > 0$, it gives

$$\prod_{\sigma=0}^{k} M_\sigma \geq \aleph\sum_{\sigma=0}^{k} M_\sigma \tag{23}$$

where $\aleph > 0$, $\sum_{\sigma=0}^{k} h^\sigma\gamma\mathcal{L}_\sigma$ has only one eigenvalue as zero for $b_6 \in \mathbb{N}^+ > 0$, and $k \geq b_7 \in \mathbb{N}^+$. Then, with $(\sum_{\sigma=0}^{k} M_\sigma)1_N = (k+1)1_N$, it gives that the eigenvalue $k+1$ of $\sum_{\sigma=0}^{k} M_\sigma$ is of algebraic multiplicity 1, such that, by Lemma 4, $\mathcal{G}(\sum_{\sigma=0}^{k} M_\sigma)$ contains a spanning tree. Then, it follows that $\mathcal{G}(\prod_{\sigma=0}^{k} M_\sigma)$ contains a spanning tree by (23). Combining with that $\prod_{\sigma=0}^{k} M_\sigma$ is a row stochastic matrix with positive diagonal elements, we can obtain that $\prod_{\sigma=0}^{k} M_\sigma$ is SIA by Lemma 3. Thus, it concludes $\prod_{\sigma=0}^{k} M_\sigma = 1_N y^T$ for a constant column vector $y \in \mathbb{R}^N$. It thus achieves (21), such that $\hat{x}_{ia}(\infty) = \hat{x}_{ja}(\infty)$, and $\dot{\hat{x}}_{ia}(\infty) = 0_{n_a}$ for $\forall i, j \in \mathcal{V}$. Then, it achieves (2), and completes the proof with the same steps performed in Theorem 1. ∎

C. Further Discussion

As a special E-T mechanism, the time-triggered scheme is structured based on the following switched and distributed time-triggered sampled-data communication layer and time detector with $h_i^k\gamma_i\lambda_N \in (0, 1)$ and modulo function mod$(\cdot)$

$$\begin{aligned}\dot{\hat{x}}_{ia} &= -\gamma_i\textstyle\sum_{j=1}^{N} a_{ij\sigma}\left(\hat{x}_{ia}\left(t_i^k\right) - \hat{x}_{ja}\left(t_j^{k^*}\right)\right)\\ E_i &= -\mathrm{mod}\left(t - t_0, h_i^k\right) = 0, \quad k \in \mathbb{N}.\end{aligned} \tag{24}$$

Corollary 1: For underactuated systems (1) with switched time-triggered sampled-data communication, if the union of switched digraph $\bigcup_{\sigma=b_6}^{b_7}\mathcal{G}_\sigma$, $b_6 \in \mathbb{N}^+$, $b_7 \in \mathbb{N}^+$, has a spanning tree, the scheme structured with (4), (6a), and (24) can accomplish coordination (2) and avoid Zeno behavior.

Proof: The proof of the corollary is similar to that of Theorem 4, and thus is omitted. ∎

Remark 2: In contrast to the control of underactuated systems with conservative fixed communications [1]–[3], this article has comprehensively considered the E-T coordination over both fixed and switched networks. Moreover, the constructed schemes in this article can efficiently decrease the negative effects of disturbances, and extend the ideal cases without disturbances or with only active ones [3].

Remark 3: Considering time delays in the control problem, we take the quantized sampled-data-based E-T schemes, for example, and replace the communication layer (5) with

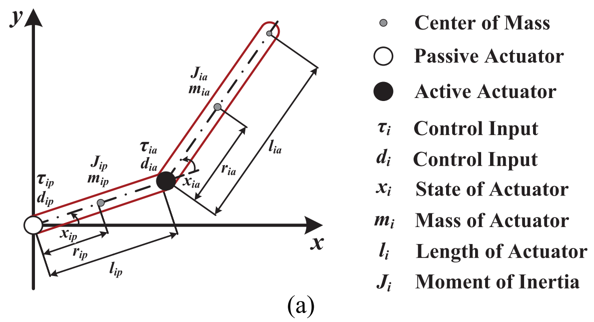
$$\dot{\hat{x}}_{ia}(t) = -\gamma_i\sum_{j=1}^{N} a_{ij}\left(Q_u\left(\hat{x}_{ia}(t)\right) - Q_u\left(\hat{x}_{ja}(t - \tau_{ij})\right)\right)$$

where $\tau_{ij} \in \mathbb{L}_\infty$ $\forall i, j \in \mathcal{V}$, are communication delays, and $-\infty < \dot{\tau}_{ij} < 1$. Then, by a similar argument as [43, Th. 2], and using the quantized states for the adaption layer (4), we can finally obtain the desired control objective.

V. SIMULATION

Based on the 2-DOF planar manipulator, $\forall i \in \mathcal{V} = \{1, 2, 3, 4\}$, with communication digraphs, and physical and control parameters, respectively, given in Fig. 1, and Tables III and IV [34], [44], two types of simulations are carried out.

Simulation 1: With the digraph given in Fig. 1(b), the simulation results of Theorems 1 and 2 are achieved in Figs. 2–4. Fig. 2 provides the evolution of active and passive states



$$\mathcal{L}=\begin{bmatrix}1&0&0&-1\\-1&1&0&0\\-1&0&1&0\\0&-1&-1&2\end{bmatrix}\qquad \mathcal{L}_\sigma\in\left\{0_{4\times4},\begin{bmatrix}0&0&0&0\\-1&1&0&0\\-1&0&1&0\\0&0&0&0\end{bmatrix},\begin{bmatrix}1&0&0&-1\\0&0&0&0\\0&0&0&0\\0&-1&-1&2\end{bmatrix}\right\}$$

(b) (c)

Fig. 1. (a) 2-DOF planar manipulator. (b) Laplacian matrix of fixed digraph. (c) Laplacian matrix of switched digraph.

TABLE III
PHYSICAL PARAMETERS

$m_i=[2,2]^T$, $l_i=[1.5,1.5]^T$, $r_i=[0.75,0.75]^T$, $J_i=[1.5,1.5]^T$
$H_{ipp}=\vartheta_{i1}+2\vartheta_{i2}\cos x_{i2}$, $H_{ipa}=H_{iap}=\vartheta_{i3}+\vartheta_{i2}\cos x_{i2}$
$H_{iaa}=\vartheta_{i3}$, $C_{idpp}=-\vartheta_{i2}\dot{x}_{i2}\sin x_{i2}$, $C_{ipa}=-\vartheta_{i2}\dot{x}_{i12}\sin x_{i2}$
$C_{iap}=\vartheta_{i2}\dot{x}_{i1}\sin x_{i2}$, $C_{idaa}=0$, $d_{ip}=0.5$, $d_{ia}=0.5$
$g_{ip}=9.8(\vartheta_{i4}\cos x_{i1}+\vartheta_{i5}\cos x_{i12})$, $g_{ia}=9.8\vartheta_{i5}\cos x_{i12}$
$\vartheta_{i1}=m_{i1}r_{i1}^2+m_{i2}(l_{i1}^2+r_{i2}^2)+J_{i1}+J_{i2}$, $\vartheta_{i2}=m_{i2}l_{i1}r_{i2}$
$\vartheta_{i3}=m_{i2}r_{i2}^2+J_{i2}$, $\vartheta_{i4}=m_{i1}r_{i1}+m_{i2}l_{i1}$, $\vartheta_{i5}=m_{i2}l_{i2}$
$x_{ip}(t_0)=x_{ia}(t_0)=-\dot{x}_{ip}(t_0)=-\dot{x}_{ia}(t_0)=i-2.5$, $t_0=0$
$x_{i12}=x_{i1}+x_{i2}$, $C_{idpp}=C_{ipp}+C_{idp}$, $C_{idaa}=C_{iaa}+C_{ida}$

TABLE IV
CONTROL PARAMETERS

$\alpha_i=\beta_i=\gamma_i=6$, $\eta_{ip}=\eta_{ia}=20$, $b_4=12$, $b_5=8$
$\phi_{i1}=\phi_1=\phi_{i2}=\phi_2=5$, $\omega_i=\omega=\varphi_i=\varphi=0.2$, $\varepsilon=1$
$\Lambda_{\vartheta i}=3I_5$, $\Lambda_{di}=5I_2$, $\hat{x}_a(0)=[-0.8,-0.4,0.2,0.6]^T$
$\hat{\vartheta}_i(0)=[8.8,1.3,1.4,3.5,2]^T$, $z_i(0)=\hat{d}_i(0)=\hat{f}_i(0)=0_2$

$x_{ia},\dot{x}_{ia},x_{ip},\dot{x}_{ip}$, where x_{ia} and $\dot{x}_{ia}$, respectively, converge to a constant negative value and zero, and $\dot{x}_{ip}$ is bounded in region $[-5,5]$. Fig. 3 provides the evolution of auxiliary variables χ_{ip},χ_{ia} and estimated variable $\hat{x}_{ia}$, where χ_{ip} and χ_{ia} reach to zero, and $\hat{x}_{ia}$ gets to a constant negative value. Fig. 4 provides the evolution of control input τ_{ia} and trigger numbers as event detector $E_i=0$, where τ_{ia} is piecewise constant, and the trigger numbers are (129, 63, 107, 98) and (123, 67, 106, 95), respectively.

Simulation 2: With the digraph given in Fig. 1(b) and (c), the simulation results of Theorems 3 and 4 are achieved in Figs. 5–7. Similarly, Figs. 5 and 6 are, respectively, the evolution of states $x_{ia},\dot{x}_{ia},x_{ip},\dot{x}_{ip}$ and $\chi_{ip},\chi_{ia},\hat{x}_{ia}$, and Fig. 7 is the evolution of τ_{ia} with trigger numbers as (155, 155, 155, 155) and (124, 77, 101, 107).

Discussion: Communication and trigger rates (depicted in Fig. 8) with communication and trigger numbers (depicted in Table V), are designed as performance indexes for performance comparison, and generally speaking, the smaller rates are, the less energy it consumes. Then, one concludes from Figs. 2–8 that: 1) the four schemes achieve nearly the same control performance for $x_{ia},\dot{x}_{ia},x_{ip},\dot{x}_{ip},\chi_{ip},\chi_{ia}$, and the schemes in Theorems 2–4 are preferable for obtaining better convergence of $\hat{x}_{ia}$ and 2) the structured schemes can significantly reduce the controller updates, and the communication

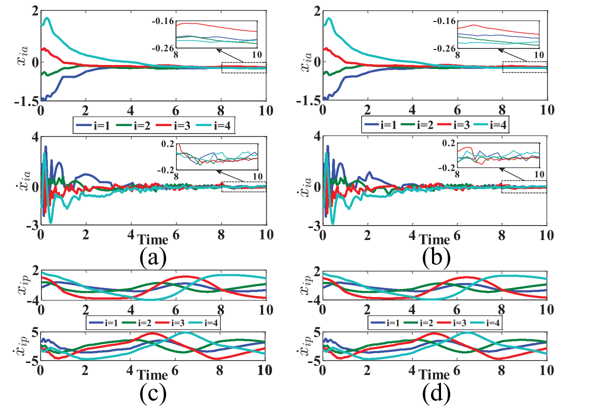


Fig. 2. (a) Evolution of $x_{ia},\dot{x}_{ia}$ in Theorem 1. (b) Evolution of $x_{ia},\dot{x}_{ia}$ in Theorem 2. (c) Evolution of x_{ip}, $\dot{x}_{ip}$ in Theorem 1. (d) Evolution of x_{ip}, $\dot{x}_{ip}$ in Theorem 2.

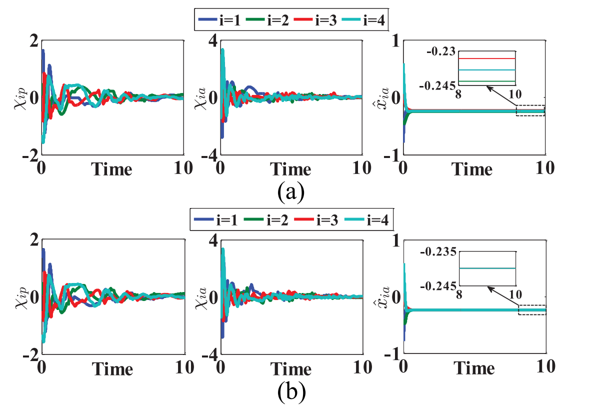


Fig. 3. (a) Evolution of $\chi_{ip},\chi_{ia},\hat{x}_{ia}$ in Theorem 1. (b) Evolution of $\chi_{ip},\chi_{ia},\hat{x}_{ia}$ in Theorem 2.

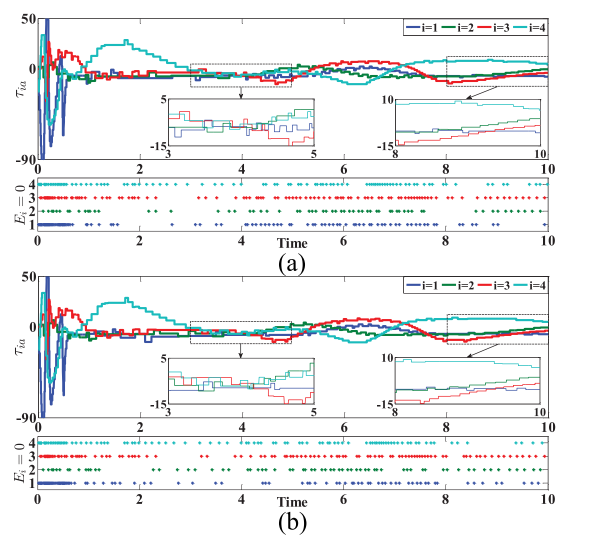


Fig. 4. (a) Evolution of τ_{ia} and trigger numbers in Theorem 1. (b) Evolution of τ_{ia} and trigger numbers in Theorem 2.

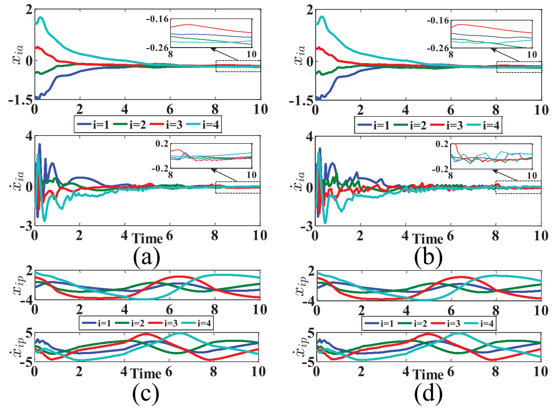


Fig. 5. (a) Evolution of $x_{ia}, \dot{x}_{ia}$ in Theorem 3. (b) Evolution of $x_{ia}, \dot{x}_{ia}$ in Theorem 4. (c) Evolution of $x_{ip}, \dot{x}_{ip}$ in Theorem 3. (d) Evolution of $x_{ip}, \dot{x}_{ip}$ in Theorem 4.

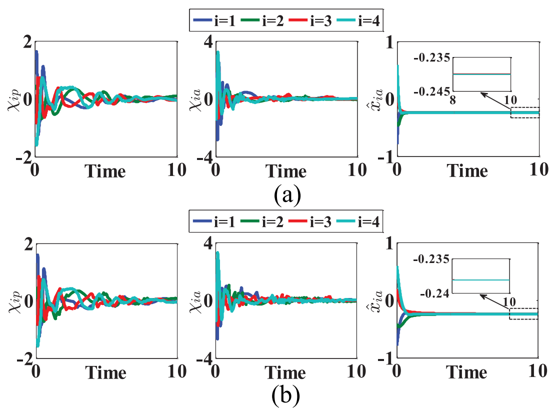


Fig. 6. (a) Evolution of $\chi_{ip}, \chi_{ia}, \hat{x}_{ia}$ in Theorem 3. (b) Evolution of $\chi_{ip}, \chi_{ia}, \hat{x}_{ia}$ in Theorem 4.

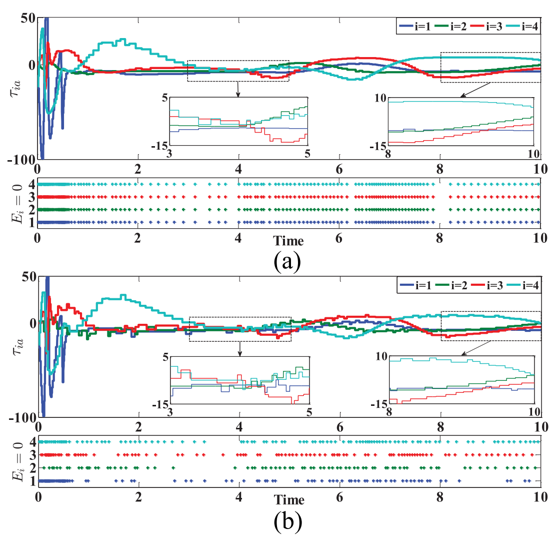


Fig. 7. (a) Evolution of τ_{ia} and trigger numbers in Theorem 3. (b) Evolution of τ_{ia} and trigger numbers in Theorem 4.

consumption of schemes in Theorems 3 and 4 is less than that in Theorems 1 and 2. Moreover, the scheme structured in Theorem 4 is the most efficient for adopting distributed triggered-sampled-data mechanism.

TABLE V
COMMUNICATION AND TRIGGER NUMBERS

Theorem	Communication Numbers	Trigger Numbers
Theorem 1	10000 10000 10000 10000	129 63 107 98
Theorem 2	10000 10000 10000 10000	123 67 106 95
Theorem 3	155 155 155 155	155 155 155 155
Theorem 4	124 77 101 107	124 77 101 107

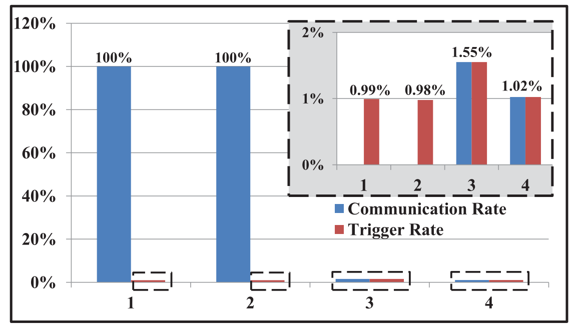


Fig. 8. Communication and trigger rates of the structured theorems.

VI. CONCLUSION

Based on a uniform framework, this article has constructed sampled-data-based E-T schemes for the coordination of a class of multiple underactuated systems with uncertainties and disturbances. First, the E-T schemes over uniform- and logarithmic-quantization sampled-data communications have been discussed, where the communication and control costs can be effectively saved for only using neighbors' quantized positions and piecewise constant control inputs. Then, to further reduce communication consumptions, the E-T schemes over triggered-sampled-data communications have been discussed for both the fixed and switched networks. Finally, two types of simulations with performance comparisons have been provided. Future efforts will be made for the sampled-data-based fault-tolerant control of underactuated systems with Markovian switching networks.

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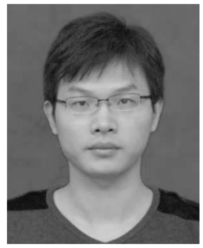

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