Heterogeneous Large-Scale Group Decision Making Using Fuzzy Cluster Analysis and Its Application to Emergency Response Plan Selection

Guangxu Li, Gang Kou, and Yi Peng

Abstract—As the number of people involved in a decision-making problem increases, the complexity of the group decision-making (GDM) process increases accordingly. The size of participants and the heterogeneous information have important effects on the consensus reaching process in GDM. To deal with these two issues, traditional methods divide large groups into smaller ones to reduce the scale of GDM and translate heterogeneous information into a uniform format to handle the heterogeneity problem. These methods face two challenges: 1) how to determine the appropriate group size? and 2) how to avoid or reduce loss of information during the transformation process? To address these two challenges, this article uses fuzzy cluster analysis to integrate heterogeneous information for large-scale GDM problems. First, a large group is divided into smaller ones using fuzzy cluster analysis and the F-statistic is applied to determine the satisfactory number of clusters. The original information is retained based on the similarity degree. Then, a consensus reaching process is conducted within these small groups to form a unified opinion. A feedback mechanism is developed to adjust the small GDM matrix when any group cannot reach a consensus, and the heterogeneous technique for order preference by similarity to an ideal solution (TOPSIS) is used to select the best alternative. To validate the proposed approach, an experiment study is conducted using a practical example of selecting the best rescue plan in an emergency situation. The result shows that the proposed approach helps to choose the best rescue plan faster.

Index Terms—Consensus reaching process, emergency decision, fuzzy cluster analysis, heterogeneous information, large-scale group decision making (LSGDM).

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I. INTRODUCTION

IN GROUP decision making (GDM), a group of decision makers (DMs) works together to analyze problems, evaluate alternatives, and choose an agreed solution from a collection of alternatives [1]–[7]. Large-scale GDM (LSGDM) is a special type of GDM problem, in which a large number of DMs participate in the decision-making process [8]–[13]. Usually, when the number of DMs exceeds 11, the GDM process is considered as an LSGDM [14]. Due to the diverse educational backgrounds, experiences, and levels of knowledge, DMs express their preferences using heterogeneous information, which has multiple meanings [15]: 1) different preference representation formats: DMs express their opinions using different preference relations, such as preference orderings, utility functions, multiplicative preference relations, and fuzzy preference relations and 2) different types of number formats: the values of preferences consist of not only crisp information but also interval numbers, fuzzy numbers, and linguistic data.

How to deal with the heterogeneous information provided by DMs is an important research topic in heterogeneous GDM. Some works focused on the heterogeneous preference relations. Herrera-Viedma *et al.* [16] proposed a consensus method for heterogeneous GDM problems in which the preference information were given by different preference structures. This consensus method can automatically support the consensus reaching process (CRP) without a moderator, thus avoiding the subjectivity that a moderator may introduce to the CRP. Herrera *et al.* [17] developed an aggregation process to integrate heterogeneous preference relations. Fan *et al.* [18] presented a goal programming approach to solve heterogeneous GDM problems in which the opinions on alternatives provided by experts were represented in two preferences formats.

Some works dealt with different types of number formats used by DMs [19]–[21]. Wang and Cai [22] built a generalized distance-based VIKOR model to integrate different number formats (crisp number, interval number, intuitionistic fuzzy number, and hesitant fuzzy linguistic value) and applied the model to the emergency supplier selection. Martínez *et al.* [23] investigated a decision-making process, and used different fuzzy transformation functions to deal with numerical, linguistic, interval-valued formats. Wan and Li [24] proposed a new Atanassov's intuitionistic fuzzy programming method to solve heterogeneous

multiattribute GDM problem in which the attribute values were given by Atanassov's intuitionistic fuzzy (A-IF) sets, trapezoidal fuzzy numbers, intervals, and real numbers. Ölçer and Odabaşi [25] developed a fuzzy GDM methodology to integrate different types of attributes (crisp numbers, fuzzy numbers, and linguistic information) by transforming heterogeneous information in the fuzzy environment. Morente-Molinera *et al.* [26] proposed multigranular fuzzy linguistic methods to manage heterogeneous linguistic information and make the information homogeneous using the transformation functions. To avoid the transformation step during the heterogeneous information processing, Lourenzutti and Krohling [27] presented a generalized technique for order preference by similarity to an ideal solution (TOPSIS) method for heterogeneous GDM to manage the different types of information. However, this method did not consider CRP of the opinions among DMs in heterogeneous GDM. Li *et al.* [28] proposed a GDM model to integrate crisp numbers, interval numbers, triangular fuzzy numbers, and trapezoidal fuzzy numbers by a weighted-power average operator and consider the CRP before the final solution was obtained.

The increase of technological and societal demands has given rise to large-scale GDM problems [29]. When the number of DMs increases, it is difficult to adjust inconsistent elements and reach a consensus [30]–[35]. The research works on LSGDM that are closely related to this study include CRP and the cluster analysis. Pérez *et al.* [15] presented a consensus model to detect and manage the heterogeneity criterion in LSGDM. It can be used as an automatic system to compute and send customized advice to experts if the desired consensus level has not been reached. Quesada *et al.* [36] presented an expert weighting methodology for large-scale CRP. In this method, experts' weights can be adjusted based on their overall behavior. Palomares *et al.* [37] proposed a graphical monitoring tool to visualize the evolution of experts' preferences in each consensus round in LSGDM.

Cluster analysis has been used to increase the efficiency of LSGDM. Cluster analysis includes various methods to group data into smaller and simpler subunits [38]–[42]. Zhang *et al.* [43] proposed a linguistic distribution assessment-based clustering algorithm to cluster DMs in LSGDM. Lourenzutti and Krohling [27] proposed a consensus model that incorporates a fuzzy *c*-means cluster to manage individual and subgroup noncooperative behaviors in LSGDM. But they did not consider heterogeneous information. Zhu *et al.* [44] investigated group clustering problems with double information in heterogeneous LSGDM (HLSGDM), in which the heterogeneous information contained the preference information expressed in a judgment matrix and the reference information obtained from the actual data or survey results. However, this method did not consider the CRP and the selection process of alternatives. Zhang *et al.* [45] presented a CRP for HLSGDM with individual concerns and satisfactions. They calculated the heterogeneous preference information using similarity degrees and clustered DMs using preference cluster and an aggregation method.

Although a large number of HLSGDM related works have been reported in the literature, some important issues are still unsolved: 1) how to deal with different types of number formats in HLSGDM and avoid the loss of information in CRP? and 2) how to determine the number of groups when clustering a large number of DMs into smaller groups in HLSGDM?

The purpose of this article is to develop an HLSGDM approach to obtain a satisfactory number of clusters using the fuzzy cluster analysis. The approach is designed for decision-making problems that require fast and accurate decisions, such as emergency response system (ERS). In emergency decision-making situations, the complicated decision-making environment and the limited knowledge of DMs often lead to ambiguity in DMs' preferences and make it difficult for some attributes to be accurately estimated. Therefore, we choose fuzzy cluster analysis to cluster DMs and alternatives. Moreover, the satisfactory number of groups is determined through the F-statistic method in the fuzzy cluster analysis process. The CRP and the feedback mechanism are also developed based on the satisfactory number of groups. To avoid information loss, the proposed approach uses the similarity measure, which utilizes the degree of deviation to determine the similarity degrees between different DMs. The arithmetic average operator is applied to aggregate information with the same attribute type without converting heterogeneous information into a uniform format. The proposed approach can manage crisp numbers, interval numbers, and triangular fuzzy numbers. Finally, the fuzzy TOPSIS [46] is used to rank the alternatives according to the satisfactory clustering result.

The main innovations of this article are: 1) using fuzzy cluster analysis to divide a large number of DMs into smaller groups and utilizing the F-statistic to determine the satisfactory number of clusters in HLSGDM and 2) to avoid information loss, we process heterogeneous information using the similarity degree, rather than transforming them into a single form.

The remainder of this article is organized as follows. Section II introduces some definitions and notations used in this study. Section III presents the fuzzy cluster analysis method and the F-statistic in detail. Section IV describes the HLSGDM process. Section V uses a numerical example about selecting the best rescue plan for ERS to validate the proposed approach. Section VI compares the proposed approach with other methods, and Section VII concludes this article.

II. DEFINITIONS AND NOTATIONS

This section reviews some basic definitions, notations, and properties of fuzzy numbers [47], [48]. The basic notations and definitions below will be used throughout this article until otherwise stated.

Let $\tilde{A}$ be a fuzzy set in the universe of discourse X, $F(\tilde{A})$ be a power set, which is a collection of all fuzzy sets, and $\mu_{\tilde{A}}(x)$ be the membership function, where the value of $\mu_{\tilde{A}}(x)$ is called the membership value of x in $\tilde{A}$ and represents the degree of truth that x is an element of the fuzzy set $\tilde{A}$. It is assumed that $\mu_{\tilde{A}}(x) \in [0, 1]$, where $\mu_{\tilde{A}}(x) = 0$ indicates that x does not belong to the fuzzy set $\tilde{A}$, and $\mu_{\tilde{A}}(x) = 1$ means

that x completely belongs to the fuzzy set $\tilde{A}$

$$\tilde{A} = \left\{ (x, \mu_{\tilde{A}}(x)) | x \in X \right\}$$

where $\mu_{\tilde{A}}(x)$ is the membership function, and x is an element of the fuzzy set $\tilde{A}$.

Definition 1: Let $\tilde{A} = [a, b] = \{x | 0 \leq a \leq x \leq b\}$, then $\tilde{A}$ is called an interval number. Especially, $\tilde{A}$ is a nonnegative real number, if $a = b$.

Definition 2: A triangular fuzzy number $\tilde{A}$ is defined as $\tilde{A} = (a, b, c)$, $0 \leq a \leq b \leq c$, if the membership function $\mu_{\tilde{A}} : \mathbb{R} \rightarrow [0, 1]$ is defined as follows:

$$\mu_{\tilde{A}} = \begin{cases} (x - a)/(b - a), & a \leq x \leq b \\ (c - x)/(c - b), & b \leq x \leq c \\ 0, & \text{others.} \end{cases} \tag{1}$$

Proposition 1: Let $\tilde{A}$ and $\tilde{B}$ be two fuzzy numbers in the universe of discourse X, and $\& \in \{\oplus, -, \otimes, \div, \cdot, \vee, \wedge\}$. Then, the result of operation $\tilde{A}\&\tilde{B}$ is still a fuzzy number, specifically, if $\&$ is $\div$, then $\tilde{B}$ is a nonzero fuzzy number.

Property 1: Given two fuzzy numbers $\tilde{A} = (a_1, a_2, \ldots, a_n)$ and $\tilde{B} = (b_1, b_2, \ldots, b_n)$, which satisfy the condition of the fuzzy set definition, and a positive real number λ, some main operations of the fuzzy numbers and the distance between $\tilde{A}$ and $\tilde{B}$ can be expressed as follows.

1) $\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, \ldots, a_n + b_n)$.
2) $\tilde{A} \otimes \tilde{B} = (a_1 b_1, a_2 b_2, \ldots, a_n b_n)$.
3) $\lambda \cdot \tilde{A} = (\lambda a_1, \lambda a_2, \ldots, \lambda a_n)$.
4) $\tilde{A} \div \tilde{B} = (a_1/b_n, a_2/b_{n-1}, \ldots, a_n/b_1)$.
5) $d(\tilde{A}, \tilde{B}) = \sqrt{\sum_{i=1}^{n}(a_i - b_i)^2}$.

Definition 3 [48]: A matrix $R = (r_{ij})_{m \times n}$ is defined as a fuzzy matrix, if $r_{ij} \in [0, 1]$ $(i = 1, 2, \ldots, m, j = 1, 2, \ldots, n)$.

Definition 4 [48]: Let $R = (r_{ij})_{m \times m}$ $(i = 1, 2, \ldots, m, j = 1, 2, \ldots, m)$ be a fuzzy matrix. $R = (r_{ij})_{m \times m}$ is defined as a fuzzy similar matrix, if $r_{ii} = 1$ (reflexivity) and $r_{ij} = r_{ji}$ (symmetry).

Definition 5 [48]: Let $R = (r_{ij})_{m \times m}$ $(i = 1, 2, \ldots, m, j = 1, 2, \ldots, m)$ be a fuzzy similar matrix. $R = (r_{ij})_{m \times m}$ is defined as a fuzzy equivalent matrix, if $r_{ij} \geq \vee_{k=1}^{m}(r_{ik} \wedge r_{kj})$ (transitivity), where $\vee$ and $\wedge$ stand for max and min, respectively.

Theorem 1 [48]: Let $R = (r_{ij})_{m \times m}$ $(i = 1, 2, \ldots, m, j = 1, 2, \ldots, m)$ be a fuzzy similar matrix and its transitive closure $t(R)$ be $t(R) = R^k = R \circ R \circ \cdots \circ R$, where $R \circ R = (o_{ij})_{m \times m}$ and $o_{ij} = \vee_{k=1}^{m}(r_{ik} \wedge r_{kj})$. If there are two integers k and l, $\forall l > k$, we obtain $R^l = R^k$, then the transitive closure $t(R)$ is a fuzzy equivalent matrix.

In a decision-making process, the alternatives are expressed as $x = \{x_1, x_2, \ldots, x_m\}$ and the evaluation attributes are expressed as $c = \{c_1, c_2, \ldots, c_n\}$. The attributes are additively independent. x_{ij} is the assessed value of the attribute c_j of the alternative x_i. The different values of x_{ij} can be represented by the matrix $V = (x_{ij})_{m \times n}$, which is called a decision-making matrix. The attributes can be benefit criteria or cost criteria. Therefore, the difference of the attribute index on the dimension should be eliminated. For the attribute value, the normalized processes are given by

$$\tilde{x}_{ij} = \begin{cases} x_{ij} / \sum_{i=1}^{m} x_{ij}, & \forall j \in I_1 \\ (1/x_{ij}) / (\sum_{i=1}^{m}(1/x_{ij})), & \forall j \in I_2 \end{cases} \tag{2}$$

where I_1 is associated with a set of benefit criteria, and I_2 is associated with a set of cost criteria. $\tilde{V} = (\tilde{x}_{ij})_{m \times n}$ represents the normalized decision-making matrix.

Definition 6 [28]: Let $\tilde{V}^k = (\tilde{x}_{ij}^k)_{m \times n}$ and $\tilde{V}^l = (\tilde{x}_{ij}^l)_{m \times n}$ be two normalized decision-making matrices, which are given by DM k and DM l, respectively. Then, the degree of deviation between $\tilde{V}^k = (\tilde{x}_{ij}^k)_{m \times n}$ and $\tilde{V}^l = (\tilde{x}_{ij}^l)_{m \times n}$ is denoted as follows:

$$D\left(\tilde{V}^k, \tilde{V}^l\right) = \frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} d\left(\tilde{x}_{ij}^k, \tilde{x}_{ij}^l\right) \tag{3}$$

where $D(\tilde{V}^k, \tilde{V}^l) \geq 0$.

Based on Definition 6, the following definition defines the similarity degree between the normalized decision-making matrixes.

Definition 7 [28]: Let $\tilde{V}^k = (\tilde{x}_{ij}^k)_{m \times n}$ and $\tilde{V}^l = (\tilde{x}_{ij}^l)_{m \times n}$ be the normalized decision-making matrices, which are given by DM k and DM l, respectively. Then, the similarity degree between $\tilde{V}^k$ and $\tilde{V}^l$ is given by

$$\text{sim}\left(\tilde{V}^k, \tilde{V}^l\right) = \frac{1}{1 + D\left(\tilde{V}^k, \tilde{V}^l\right)} \tag{4}$$

where $\tilde{x}_{ij}^k$ and $\tilde{x}_{ij}^l$ could be given by crisp numbers, interval numbers, or triangular fuzzy numbers.

The similarity degree $\text{sim}(\tilde{V}^k, \tilde{V}^l)$ has the following properties: 1) $0 \leq \text{sim}(\tilde{V}^k, \tilde{V}^l) \leq 1$, and $\text{sim}(\tilde{V}^k, \tilde{V}^l) = 1$ if and only if $\tilde{x}_{ij}^k$ in $\tilde{V}^k$ and $\tilde{x}_{ij}^l$ in $\tilde{V}^l$ are completely equal. The similarity measure is 0 when the two decision matrices are completely independent or the distance between these two decision matrices is infinite and 2) $\text{sim}(\tilde{V}^k, \tilde{V}^l) = \text{sim}(\tilde{V}^l, \tilde{V}^k)$.

These definitions and properties are used in the fuzzy cluster analysis to divide a large-scale group into small groups in the HLSGDM.

III. FUZZY CLUSTER ANALYSIS

The cluster analysis includes various methods offered by statistics and machine learning in order to group data into simpler subunits [38]. One of its main advantages is that it does not assume any specific distribution of the data. Fuzzy cluster analysis is suitable for situations when the boundaries of clusters are not obvious. It is applied to solve these problems based on the fuzzy similar matrix and fuzzy equivalent matrix [39]. In emergency response plan selection, some attributes cannot be accurately estimated and are manifested as unclear boundaries in cluster analysis. Based on these, we used the fuzzy cluster analysis in this article. The fuzzy cluster process is carried out as follows.

Step 1 (Collect and Normalize the Original Data Matrix):
Assume that $X = \{x_1, x_2, \ldots, x_m\}$ is the classified alternatives and each alternative has n units of attributes, which may be benefit criteria or cost criteria. In order to eliminate the effect of physical dimensions of different forms of attributes, we carry out the process of normalization. The original data matrix can be normalized in a fuzzy matrix following (2).

The normalized fuzzy matrix is

$$\tilde{X} = \left(\tilde{x}_{ij}\right)_{m \times n} = \begin{pmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2n} \\ \vdots & \vdots & & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \cdots & \tilde{x}_{mn} \end{pmatrix}$$

where $\tilde{x}_i = (\tilde{x}_{i1}, \tilde{x}_{i2}, \cdots, \tilde{x}_{in})$.

Step 2 (Obtain the Fuzzy Similar Matrix): In this article, the similarity measure based on the degree of deviation is applied to obtain the similar degree $r_{ij} = \text{sim}(\tilde{x}_i, \tilde{x}_j)$. Then, we have the fuzzy similar matrix R

$$R = \left(R_{ij}\right)_{m \times m} = \begin{pmatrix} r_{11} & r_{12} & \cdots & r_{1m} \\ r_{21} & r_{22} & \cdots & r_{2m} \\ \vdots & \vdots & & \vdots \\ r_{m1} & r_{m2} & \cdots & r_{mm} \end{pmatrix}$$

where the similar matrix R is the symmetric matrix, and $r_{ij} = r_{ji}$ and $r_{ii} = 1$.

Step 3 (Establish the Fuzzy Equivalent Matrix): Based on Theorem 1, the fuzzy equivalent matrix can be determined based on the transitive closure $t(R)$. The square method is applied to determine $t(R)$. For example, $R \circ R = R^2, R^2 \circ R^2 = R^4, \ldots, R^{2^{i-1}} \circ R^{2^{i-1}} = R^{2^i}$, when the equation $R^k \circ R^k = R^k$ establishes first, R^k is the transitive closure $t(R)$. Letting $R^* = R^k$, R^* is called the fuzzy equivalent matrix and is given as follows:

$$R^* = \begin{pmatrix} r_{11}^* & r_{12}^* & \cdots & r_{1m}^* \\ & r_{22}^* & \cdots & r_{2m}^* \\ & & & \vdots \\ & & & r_{mm}^* \end{pmatrix}$$

where the fuzzy equivalent matrix R^* is also the symmetric matrix, and $r_{ij}^* = r_{ji}^*$ and $r_{ii}^* = 1$.

Step 4 (Generate Dynamic Clustering Results): Based on Zadeh [48], we can take a real number $\lambda \in [0, 1]$, when $r_{ij}^* \geq \lambda$, object x_i and x_j can be clustered into one cluster. Therefore, choosing different thresholds λ can generate dynamic clustering results.

In order to get the satisfactory number of clusters, the F-statistic method is applied to determine the best value of the threshold λ in step 5.

Step 5 (Establish the Satisfactory Number of Clusters Based on the F-Statistic Method): Assume that $X = \{x_1, x_2, \ldots, x_m\}$ is the original classified alternatives, and each alternative has n attributes: $x_i = (x_{i1}, x_{i2}, \ldots, x_{in})$, $(i = 1, 2, \ldots, m)$. $\bar{x}$ is the center of the overall sample, and $\bar{x}_w = 1/m \sum_{i=1}^{m} x_{iw}$, $(w = 1, 2, \ldots, n)$. Meanwhile, let $\{G_1, G_2, \ldots, G_{k(\lambda)}\}$ be the k clusters with different thresholds λ and the number in the cluster $G_j (j = 1, 2, \ldots, k(\lambda))$ be m_j. Assume that the center of cluster G_j is $\bar{x}^{(j)} = (\bar{x}_1^{(j)}, \bar{x}_2^{(j)}, \ldots, \bar{x}_k^{(j)})$, where $\bar{x}_k^{(j)} = 1/m_j \sum_{i=1}^{m_j} x_{ik}^{(j)}$. The F-statistic method can be described as follows:

$$F = \frac{\sum_{j=1}^{k} m_j \left\| \bar{x}^{(j)} - \bar{x} \right\|^2 \Big/ (k-1)}{\sum_{j=1}^{k} \sum_{i=1}^{m_j} \left\| x_i^{(j)} - \bar{x}^{(j)} \right\|^2 \Big/ (m-k)} \quad (5)$$

where $x_i^{(j)}$ is the attribute in cluster G_j and $\left\| \bar{x}^{(j)} - \bar{x} \right\| = \sqrt{\sum_{w=1}^{n} (\bar{x}_w^{(j)} - \bar{x}_w)^2}$ is the distance between $\bar{x}^{(j)}$ and $\bar{x}$. Equation (5) is noted as the F-statistic.

In (5), the numerator represents the distance between clusters, and the denominator represents the distance between attributes in the same cluster. Therefore, the larger the F-value, the larger the distance between the clusters. Furthermore, the clustering result is better when the difference between the clusters is larger.

In addition, the F-statistic is also used to determine whether the clustering results are satisfactory in this study. According to the theory of statistical analysis of variance, the F-statistic obeys the F-distribution whose degrees of freedom are $k - 1$ and $m - k$ [44]. If $F > F_\alpha(k - 1, m - k)(\alpha = 0.05)$, the difference between the two clusters is significant and this result is reasonable. $F_\alpha(k - 1, m - k)(\alpha = 0.05)$ is the value of the F-statistic in the F-distribution table when $\alpha = 0.05$. The F-statistic obeys the F-distribution, whose confidence degree is $1 - \alpha$ and degrees of freedom are $k - 1$ and $m - k$. In this case, we consider the clustering result satisfactory.

Based on the fuzzy cluster analysis, the satisfactory number of clusters can be determined in HLSGDM, and the HLSGDM process would be carried out in the satisfactory clustering result.

IV. HETEROGENEOUS LARGE-SCALE GROUP DECISION-MAKING PROCESS

Traditionally, the number of DMs in an effective GDM process should be less than 7 [50]. However, with the development of technology and the change of social demands, the size of participants in some GDM problems is getting larger. A GDM process can be defined as a large-scale GDM problem when the number of DMs exceeds 11 [14]. When the number of DMs is over 20, it is difficult to reach a unanimous consensus. One solution is to apply the cluster analysis to divide a large number of DMs into smaller groups first. Then, the CRP is carried out within each cluster of DMs, who are in the same cluster and have similar experiences.

Based on the law of large numbers, the evaluation results will become more accurate when the number of DMs increases in LSGDM. Though this article uses only 20 DMs in the numerical example, the proposed method is useful when the number of DMs is larger than 20.

Let $E = \{e_1, e_2, \ldots, e_h\}$ be a group of DMs, $x = \{x_1, x_2, \ldots, x_m\}$ be a set of alternatives, and $C = \{c_1, c_2, \ldots, c_n\}$ be a set of evaluation attributes. x_{ij}^k is the assessed value given by DM e_k for the attribute c_j of the alternative x_i and the decision matrix is given by $V^k = (x_{ij}^k)_{m \times n}$. In this article, the assessed value x_{ij}^k can take three different forms: 1) crisp numbers (S_1); 2) interval numbers (S_2); and 3) triangular fuzzy numbers (S_3). S_i denotes the set of the assessed values, and $S_i \cap S_j = \emptyset (i \neq j)$, where $\emptyset$ is an empty set.

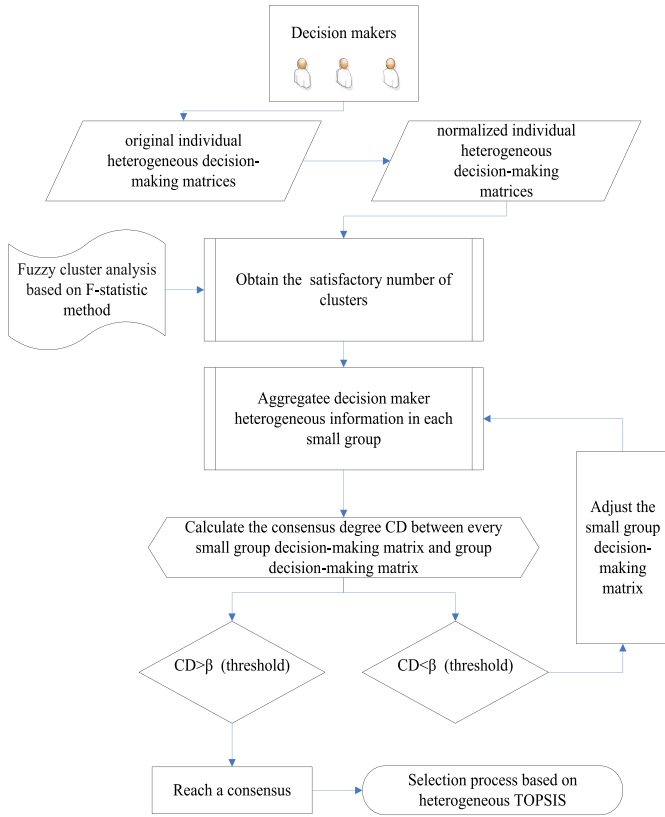The HLSGDM process used in this article is shown in Fig. 1.

Fig. 1. HLSGDM process.

As shown in Fig. 1, the attributes are either benefit criteria or cost criteria in HLSGDM. For any S_i, the normalized process can be calculated by (2).

Based on the normalized decision matrix, the HLSGDM process can be given as follows.

Step 1 (Fuzzy Cluster Analysis): In this step, we obtain the satisfactory number of clusters based on the *F*-statistic method.

Step 2 (Aggregate the Heterogeneous Information in Each Small Group):

Definition 8: Let $a_1, a_2, \ldots, a_m$ be m elements, and $\omega = (\omega_1, \omega_2, \ldots, \omega_m)^T$ be the set of weight of each element. Then, the arithmetic average operator is defined as follows:

$$\text{WAA}(a_1, a_2, \ldots, a_m) = \sum_{i=1}^{m} \omega_i a_i.$$

After the satisfactory number of clusters was obtained, the heterogeneous information can be aggregated by an arithmetic average operator in the small groups, and the small groups can be aggregated by the arithmetic average operator in the satisfactory clustering result. $g\tilde{V}^i$ is a small GDM matrix in which heterogeneous information are aggregated by the arithmetic average operator and $G\tilde{V}$ is a large GDM matrix in which small groups are aggregated by the arithmetic average operator based on the same attributes in the decision-making matrices. In this article, we assume that each DM has the same weight and each attribute has the same weight.

Step 3 (Consensus Reaching Process): A CRP in LSGDM is similar to the CRP in GDM, which is an iterative process

that consists of several rounds of discussion in which DMs adjust their preferences to reach a maximum level of consensus on alternatives. The difference is that LSGDM involves more DMs in the process, which makes it more difficult to reach a group consensus. In this article, an algorithm is designed to calculate the consensus degree. The calculation formula is given by Definition 9. The calculation of the similarity degree is provided by Definitions 6 and 7 [28].

Definition 9: Let $g\tilde{V}^l = (g\tilde{x}^l_{ij})_{m \times n}$ be a small GDM matrix and $G\tilde{V} = (G\tilde{x}_{ij})_{m \times n}$ be a GDM matrix. Then, the consensus degree between $g\tilde{V}^l$ and $G\tilde{V}$ is

$$\text{CD}\big(g\tilde{V}^l, G\tilde{V}\big) = \frac{1}{1 + D\big(g\tilde{V}^l, G\tilde{V}\big)}. \tag{6}$$

The consensus degree $\text{CD}(g\tilde{V}^l, G\tilde{V})$ has the following properties: 1) $0 \leq \text{CD}(g\tilde{V}^l, G\tilde{V}) \leq 1$ and $\text{CD}(g\tilde{V}^l, G\tilde{V}) = 0$ if and only if $g\tilde{V}^l$ and $G\tilde{V}$ are completely dissimilar; 2) $\text{CD}(g\tilde{V}^l, G\tilde{V}) = \text{CD}(G\tilde{V}, g\tilde{V}^l)$; and 3) $\text{CD}(g\tilde{V}^l, G\tilde{V}) = 1$ if and only if $g\tilde{V}^l$ and $G\tilde{V}$ are completely similar ($g\tilde{V}^l = G\tilde{V}$).

If the consensus degree $\text{CD}(g\tilde{V}^i, G\tilde{V}) \geq \beta$, the HLSGDM process reaches a consensus. Otherwise, the feedback mechanism is applied to adjust the small GDM matrix until the decision-making process reaches a consensus. β is a consensus threshold that is used to determine whether each expert reaches a consensus. There is no unified approach to choose a consensus threshold. Generally, the consensus threshold β can be assigned a high value such as $\beta = 0.9$ or a larger value when the decision-making process is very important [51]. In other cases, the consensus threshold β can be chosen a lower value such as $\beta = 0.8$ or a smaller value when the decision time is more urgent and the experts need to select the best alternative quickly, such as in an emergency management process.

Step 4 (Feedback Mechanism): In the feedback process, the nonconsensus small GDM matrix should be modified by a general iterative algorithm with parameters as follows.

Let the GDM matrix be $G\tilde{V} = (G\tilde{x}_{ij})_{m \times n}$ and the nonconsensus matrix be $\tilde{V}^u = (\tilde{x}^u_{ij})_{m \times n}$. Then, the adjusted matrix $I\tilde{V}^u = (I\tilde{x}^u_{ij})_{m \times n}$ between $G\tilde{V} = (G\tilde{x}_{ij})_{m \times n}$ and $\tilde{V}^u = (\tilde{x}^u_{ij})_{m \times n}$ can be calculated as follows:

$$I\tilde{x}^u_{ij} = \eta G\tilde{x}_{ij} + (1 - \eta)\tilde{x}^u_{ij} \tag{7}$$

where η is a parameter and $0 \leq \eta \leq 1$. In LSGDM, the parameter η can be adjusted according to the preferences of DMs in order to meet their needs.

The modified small GDM matrix gets closer to the GDM matrix, so the iterative process could improve the consensus degree.

Step 5 (Selection Process Based on the Heterogeneous Fuzzy TOPSIS [28]): Letting the final group aggregation matrix be $FG\tilde{V} = (fg\tilde{x}_{ij})_{m \times n}$, the selection process is given as follows.

First, we select the heterogeneous positive ideal solution (HPIS) $fg\tilde{x}^+$ and the heterogeneous negative ideal solution (HNIS) $fg\tilde{x}^-$, where $fg\tilde{x}^{s_1+} = \max_i fg\tilde{x}_{ij}$ and $fg\tilde{x}^{s_1-} = \min_i fg\tilde{x}_{ij}$, if $fg\tilde{x}_{ij}$ is a crisp number. $fg\tilde{x}^{s_2+} = [\max_i fg\tilde{x}^l_{ij}, \max_i fg\tilde{x}^r_{ij}]$ and $fg\tilde{x}^{s_2-} = [\min_i fg\tilde{x}^l_{ij}, \min_i fg\tilde{x}^r_{ij}]$, if $fg\tilde{x}_{ij}$ is an interval number. $fg\tilde{x}^{s_3+} = (\max_i fg\tilde{x}^l_{ij}, \max_i fg\tilde{x}^m_{ij}, \max_i$

TABLE I
VALUE OF F AND F_α IN DIFFERENT CLUSTERS

Number of clusters	4	7	9
the value of F	2.3974	1.9996	3.7201
the critical value $F_\alpha\,(\alpha=0.05)$	$F_\alpha(3,16)=8.69$	$F_\alpha(6,13)=3.98$	$F_\alpha(8,11)=3.31$

$fg\tilde{x}_{ij}^r$) and $fg\tilde{x}^{s3-} = (\min_i fg\tilde{x}_{ij}^l, \min_i fg\tilde{x}_{ij}^m, \min_i fg\tilde{x}_{ij}^r)$, if $fg\tilde{x}_{ij}$ is a triangular fuzzy number.

Second, we calculate the distance DP_i between each alternative and the HPIS, and the distance DN_i between each alternative and the HNIS, where

$$DP_i = \sum_{j=1}^n d\left(fg\tilde{x}_{ij}, fg\tilde{x}_j^+\right), i = 1, 2, \ldots, m$$

$$DN_i = \sum_{j=1}^n d\left(fg\tilde{x}_{ij}, fg\tilde{x}_j^-\right), i = 1, 2, \ldots, m.$$

Third, we calculate the degree of similarity between the ideal solutions

$$\tilde{S}_i = \frac{DN_i}{DP_i + DN_i}, i = 1, 2, \ldots, m.$$

Finally, we rank the alternatives according to $\tilde{S}_i$ in descending order, and select the best alternative.

Fuzzy TOPSIS is simple in the computational procedure, easy to represent human preferences, and allow an unlimited number of criteria and explicit tradeoffs between those criteria [52]. The heterogeneous fuzzy TOPSIS is applied to select the best alternative after the consensus is reached.

In HLSGDM, the heterogeneous large-scale group is clustered into small groups using fuzzy cluster analysis and the original information is retained. Based on the characteristics of the proposed HLSGDM, it is suitable for decision-making problems that require fast and accurate group decisions. The next section applies the proposed HLSGDM to a real-life emergency response plan selection problem.

V. NUMERICAL EXAMPLE

Emergency decision-making problems have tight time constraints and high uncertainty. In real-life situations, the emergency decisions normally involve multiple DMs with diverse backgrounds, who often have conflicting options, and need a fast and effective decision-making process [53]. This section uses a real-life emergency rescue plan selection example adapted from [14] to validate the proposed HLSGDM process. This example is about selecting the best rescue plan for an ERS. At 4:30 P.M. on September 23, 2014, a flooding accident of coal mine took place in Xuanwei, Yunnan Province, China. In the accident, eight miners were trapped underground. The emergency command had carried out a preliminary analysis of the incident and invited 20 DMs to select the best rescue plan in a limited time. There were four types of DMs: 1) five emergency officials (marked e_i, $i = 1, 2, \ldots, 5$); 2) five armed police (marked e_i, $i = 6, 7, \ldots, 10$); 3) five geological experts (marked e_i, $i = 11, 12, \ldots, 15$); and 4) five mine representatives (marked e_i, $i = 16, 17, \ldots, 20$). According to the preliminary analysis, five plans were expressed as A_1, A_2, A_3, A_4, and A_5. A_1 planed to use partial blasting and mining machines. A_2 suggested to drain water from the mine using mechanically driven pumps. A_3 proposed to use partial blasting to clear up and then let fire fighters to rescue. A_4 organized armed police and fire fighters to clear obstacles and took mine cars down into the mine. A_5 arranged excavators and deep-hole drilling machines.

Each plan has three cost criteria: 1) C_1 casualty rate; 2) C_2 required rescue time; and 3) C_3 rescue cost. In the actual rescue process, the rescue cost cannot be accurately estimated because the decision-making environment is complex and the DMs have different preferences. Therefore, it is often expressed using interval fuzzy numbers or linguistic information. In this article, the attribute values were adapted from [14]. We first transformed the rescue cost to linguistic information based on the data. Then, the values of rescue cost were transformed from the linguistic information to triangular fuzzy numbers.

After analyzing the five plans, the 20 DMs provided the assessment results of the criteria using different types, such as crisp numbers, interval numbers, and triangular fuzzy numbers. The results were listed in Table VII (see Appendix A). In this article, we used Excel 2010 to conduct the following operations: normalization of decision matrix, CRP, and aggregation of heterogeneous information and fuzzy TOPSIS. We used MATLAB to implement the fuzzy cluster analysis based on the fuzzy matrix and fuzzy c-means.

Based on the data from the original individual decision matrices, the HLSGDM process to select the best rescue plan for the ERS was as follows.

Step 1 (Calculate the Fuzzy Similar Matrix): Based on (2)–(4), the fuzzy similar matrix R was calculated, as shown in Table VIII (see Appendix B).

Step 2 (Determine the Fuzzy Equivalent Matrix R^* Following Definition 5 and Theorem 1): The fuzzy equivalent matrix R^* can be determined by the transitive closure $t(R)$. The square method was used to determine the transitive closure $t(R)$. After four transitive operations, we had $t(R) = R^8 = R^8 \circ R^8$, so $R^* = R^8$. Then, the fuzzy equivalent matrix R^* was obtained, as shown in Table IX (see Appendix C).

Step 3 (Obtain the Clusters): Based on the fuzzy equivalent matrix, we obtained the following clusters using different threshold λ: when $\lambda = 0.8525$, the DMs can be grouped into four clusters

TABLE II
GROUP AGGREGATION MATRIX

G1		C_1	C_2	C_3	G2	C_1	C_2	C_3
	A_1	0.352	[0.104,0.193]	(0.172,0.353,0.802)		0.143	[0.188,0.655]	(0.100,0.261,0.732)
	A_2	0.197	[0.102,0.191]	(0.086,0.150,0.248)		0.201	[0.094,0.218]	(0.075,0.174,0.366)
	A_3	0.197	[0.174,0.405]	(0.078,0.129,0.195)		0.155	[0.063,0.131]	(0.100,0.261,0.732)
	A_4	0.114	[0.173,0.427]	(0.085,0.142,0.220)		0.251	[0.042,0.082]	(0.060,0.130,0.244)
	A_5	0.140	[0.116,0.298]	(0.124,0.225,0.398)		0.251	[0.188,0.655]	(0.075,0.174,0.366)
G3		C_1	C_2	C_3	G	C_1	C_2	C_3
	A_1	0.338	[0.175,0.334]	(0.091,0.227,0.593)		0.316	[0.146,0.319]	(0.127,0.286,0.703)
	A_2	0.159	[0.139,0.237]	(0.107,0.259,0.661)		0.181	[0.117,0.214]	(0.093,0.200,0.442)
	A_3	0.131	[0.170,0.323]	(0.061,0.125,0.223)		0.162	[0.157,0.330]	(0.074,0.146,0.284)
	A_4	0.203	[0.142,0.240]	(0.105,0.259,0.688)		0.171	[0.141,0.298]	(0.090,0.191,0.424)
	A_5	0.170	[0.129,0.210]	(0.061,0.130,0.266)		0.169	[0.132,0.311]	(0.090,0.177,0.337)

TABLE III
NEW GROUP AGGREGATION MATRIX

G2		C_1	C_2	C_3	G	C_1	C_2	C_3
	A_1	0.230	[0.167,0.487]	(0.114,0.273,0.717)		0.329	[0.143,0.295]	(0.129,0.288,0.701)
	A_2	0.191	[0.105,0.216]	(0.084,0.187,0.404)		0.180	[0.118,0.214]	(0.095,0.202,0.447)
	A_3	0.159	[0.110,0.231]	(0.087,0.204,0.508)		0.163	[0.163,0.345]	(0.072,0.138,0.251)
	A_4	0.211	[0.091,0.190]	(0.075,0.161,0.334)		0.166	[0.148,0.313]	(0.092,0.195,0.437)
	A_5	0.210	[0.160,0.483]	(0.083,0.175,0.351)		0.163	[0.128,0.287]	(0.091,0.177,0.335)

$\{e_1, e_2, e_3, e_4, e_5, e_6, e_{10}, e_{11}, e_{12}, e_{13}, e_{14}, e_{15}, e_{16}, e_{18}, e_{20}\}$

$\{e_7, e_8, e_{17}\}, \{e_9\}, \{e_{19}\}$

when $\lambda = 0.8685$, the DMs can be grouped into seven clusters

$\{e_1\}, \{e_2, e_3, e_4, e_5, e_6, e_{10}, e_{12}, e_{13}, e_{14}, e_{15}, e_{16}, e_{18}, e_{20}\}$

$\{e_7, e_8\}, \{e_9\}, \{e_{11}\}, \{e_{17}\}, \{e_{19}\}$

when $\lambda = 0.8709$, the DMs can be grouped into nine clusters

$\{e_1\}, \{e_2, e_3, e_4, e_5, e_6, e_{13}\}, \{e_7, e_8\}, \{e_9\}$

$\{e_{10}, e_{14}, e_{15}, e_{16}, e_{18}, e_{20}\}, \{e_{11}\}, \{e_{12}\}, \{e_{17}\}, \{e_{19}\}.$

To establish the satisfactory number of clusters, the *F*-statistic method was applied to determine the value of *F* based on (5). The larger the *F*-value, the greater the distance between the clusters. Based on the *F*-distribution table, we can find the critical value of F_α in different clusters (Table I). In order to be consistent with the clustering results presented in different selections of λ, a part of the results was listed in Table I. We can determine the satisfactory number of clusters was nine clusters based on Table I.

Step 4 (Consensus Reaching Process): For LSGDM processes that require fast and accurate decisions, such as ERS, it is difficult, if not impossible, to reach unanimous decisions. One alternative is to find the largest group of DMs with similar experiences and only consider their opinions. In other words, the CRP is only carried out in large groups of clusters. Based on the law of large numbers, the evaluation results are more accurate when the number of DMs increases in LSGDM. In this article, the satisfactory number of clusters was nine. To ensure fast and accurate decisions, we considered the opinions of people bearing relatively large weights. Because the number of people in the top three clusters accounted for 70% of the total number, we used the top three small groups with

the largest number of individuals in the satisfactory clustering result to select five alternatives. $\{e_2, e_3, e_4, e_5, e_6, e_{13}\}$ is group 1; $\{e_7, e_8\}$ is group 2; and $\{e_{10}, e_{14}, e_{15}, e_{16}, e_{18}, e_{20}\}$ is group 3.

The DMs had reached the consensus when they were in one group. Based on the original individual decision matrices, we aggregated the heterogeneous information in the three small groups. Normalized small group aggregation matrices and the normalized group aggregation matrix were listed in Table V.

Using (3) and (4), we calculated the degree of consensus between each normalized small group aggregation matrix, and the normalized group aggregation matrix

$$\text{CD}(G1, G) = 0.922, \text{CD}(G2, G) = 0.865$$
$$\text{CD}(G3, G) = 0.930.$$

Because the best rescue plan of ERS is important for the rescuers and the higher consensus degree among the DMs can make the decision result more accurate, the consensus threshold β should be a high value. In [15], the consensus threshold β was set at 0.8. We chose a higher value ($\beta = 0.9$) in the HLSGDM process.

Based on the consensus degree of step 4, the second small group did not achieve consensus. Therefore, the feedback mechanism was applied to adjust the initial normalized small group aggregation matrix in step 5.

Step 5 (Feedback Mechanism): To reach the consensus, the second normalized small GDM matrix should be modified using the general iterative algorithm with parameters. In this evaluation process, we assumed that the group decision matrix and the small group decision matrix were equally important, so we set $\eta = 0.5$. The modified small GDM matrices and the GDM matrix were given in Table II.

TABLE IV
COMPARISON ANALYSIS WITH [14]

		Consensus threshold	Time of iterations	Number of clusters	Ranking results
Method in [14]		0.80	3	6	$A_5 \succ A_4 \succ A_2 \succ A_1 \succ A_3$
The proposed method	Top three categories	0.90	1	9	$A_1 \succ A_4 \succ A_2 \succ A_5 \succ A_3$
		0.80	0	9	$A_1 \succ A_4 \succ A_2 \succ A_5 \succ A_3$
		0.85	0	9	$A_1 \succ A_4 \succ A_2 \succ A_5 \succ A_3$
		0.95	2	9	$A_1 \succ A_4 \succ A_2 \succ A_5 \succ A_3$
	All categories	0.90	3	9	$A_1 \succ A_4 \succ A_2 \succ A_5 \succ A_3$

TABLE V
NUMBER OF ITERATIONS IN DIFFERENT VALUES OF η

value of η	0.1	0.3	0.5	0.7	0.9
Number of iterations	1	1	1	2	4

TABLE VI
COMPARISON ANALYSIS WITH FUZZY C-MEANS

Number of clusters	The proposed method	fuzzy c-means
4	$\{e_1, e_2, e_3, e_4, e_5, e_6, e_{10}, e_{11}, e_{12}, e_{13}, e_{14}, e_{15}, e_{16}, e_{18}, e_{20}\}, \{e_7, e_8, e_{17}\}, \{e_9\}, \{e_{19}\}$	$\{e_1\}, \{e_2, e_3, e_4, e_5, e_6, e_{10}, e_{11}, e_{12}, e_{13}, e_{14}, e_{15}, e_{16}, e_{18}, e_{20}\}, \{e_7, e_8\}, \{e_9, e_{17}, e_{19}\}$
7	$\{e_1\}, \{e_2, e_3, e_4, e_5, e_6, e_{10}, e_{12}, e_{13}, e_{14}, e_{15}, e_{16}, e_{18}, e_{20}\}, \{e_7, e_8\}, \{e_9\}, \{e_{11}\}, \{e_{17}\}, \{e_{19}\}$	$\{e_1, e_{19}\}, \{e_2, e_3, e_4, e_5\}, \{e_6, e_{11}, e_{12}, e_{13}, e_{15}, e_{20}\}, \{e_7, e_8\}, \{e_9\}, \{e_{10}, e_{14}, e_{16}, e_{18}\}, \{e_{17}\}$
9	$\{e_1\}, \{e_2, e_3, e_4, e_5, e_6, e_{13}\}, \{e_7, e_8\}, \{e_9\}, \{e_{10}, e_{14}, e_{15}, e_{16}, e_{18}, e_{20}\}, \{e_{11}\}, \{e_{12}\}, \{e_{17}\}, \{e_{19}\}$	$\{e_1, e_{17}, e_{19}\}, \{e_2, e_3, e_4, e_5\}, \{e_6, e_{13}\}, \{e_7, e_8\}, \{e_9\}, \{e_{10}, e_{14}, e_{16}, e_{18}\}, \{e_{11}, e_{12}\}, \{e_{15}\}, \{e_{20}\}$

Following this, the new consensus degrees were obtained

$$NCD(G1, G) = 0.927, NCD(G2, G) = 0.918$$
$$NCD(G3, G) = 0.932.$$

The consensus degree of each small group exceeds the consensus threshold $\beta = 0.9$. In this case, all DMs have reached consensus, and the number of iterations to reach a consensus is 2, which is less than the three rounds of iterations used in [14].

Step 6 (Select the Best Rescue Plan Based on the Heterogeneous TOPSIS): Based on the new group aggregation matrix in Table VI, the HPIS and HNIS were determined as follows:

$$HPIS = (0.329, [0.163, 0.345], (0.129, 0.288, 0.701))$$
$$HNIS = (0.163, [0.118, 0.214], (0.072, 0.138, 0.251)).$$

We calculated the distance DP_i between each alternative and the HPIS as well as the distance DN_i between each alternative and the HNIS

$$DP_1 = 0.053, DP_2 = 0.556, DP_3 = 0.642, DP_4 = 0.480$$
$$DP_5 = 0.618, DN_1 = 0.728, DN_2 = 0.224, DN_3 = 0.138$$
$$DN_4 = 0.301, DN_5 = 0.167.$$

Then, we determined the degree of similarity S_i of the ideal solution based on DP_i and DN_i

$$S_1 = 0.932, S_2 = 0.287, S_3 = 0.177, S_4 = 0.386, S_5 = 0.213.$$

The five alternatives were ranked in descending order according to their corresponding S_i

$$A_1 \succ A_4 \succ A_2 \succ A_5 \succ A_3.$$

The best rescue plan is A_1.

VI. COMPARISON ANALYSES

The section compared the proposed HLSGDM approach with other methods to validate its effectiveness using the numerical example from the previous section.

Emergency decision making demands fast and accurate decision-making results. Compared with the LSGDM model developed in [14], we chose a higher value ($\beta = 0.9$) in the HLSGDM process to balance the importance of events and the speed of selection. We also analyzed the consensus decision results under different consensus thresholds. The comparative results were shown in Table IV. Although the results of clusters of the two approaches were different, there are some similarities. For example, both approaches put e_2, e_3, e_4, and e_5 to one group, e_{14}, e_{16}, and e_{18} to one group, and e_7 and e_8 to one group. The proposed approach focuses on fast rescue and rapid achievement of consensus of DMs. Comparing with the approach proposed in [14], the proposed approach can help the group reached a higher consensus degree using less number of iterations. It showed that the proposed HLSGDM can help DMs to choose the best rescue plan faster.
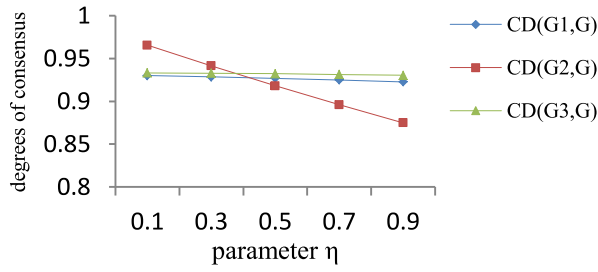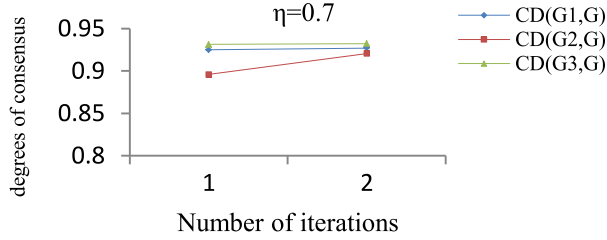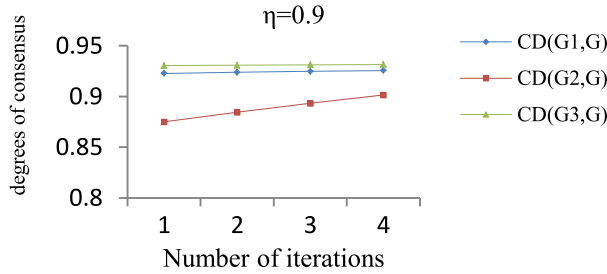
Fig. 2.   Consensus degree under different parameters.

Fig. 3.   Consensus degree under $\eta = 0.7$.

Fig. 4.   Consensus degree under $\eta = 0.9$.

Since we used the top three small groups (group 1, group 2, and group 3) with the largest number of individuals to select the best alternative, we also made a comparison between the results of the top three groups and all the groups. The results were also given in Table IV. The results showed that the ranking results were the same. However, the CRP using the top three groups was faster than the one using all the groups to reach the same decision, which illustrated the effectiveness of the proposed approach.

To illustrate the impact of parameter η on each group, Fig. 2 summarized the consensus changes for each group under different η.

Moreover, we calculated the number of iterations in the CRP when η took different values and the results were summarized in Table V. This showed that when the group opinion was judged to be more important, the nonconsensus opinions needed to be adjusted more times to reach a consensus. Figs. 3 and 4 illustrated the changes of consensus degrees of each group when $\eta = 0.7$ and $\eta = 0.9$, respectively.

According to the previous analysis of parameter η, it can be found that this general iterative algorithm can make the modified small group decision matrix more and more close to the group decision matrix.

Table VI showed the comparison results of the proposed approach and the fuzzy *c*-means. The clustering results

obtained by the proposed approach are different from those obtained by the fuzzy *c*-means. Because the clustering results of fuzzy *c*-means cannot be measured by the F-statistic, it is difficult to compare the proposed approach and fuzzy *c*-means from this aspect. However, the proposed approach does not need to determine the number of clusters in advance, which is an advantage over the fuzzy *c*-means.

VII. CONCLUSION

The number of DMs and the heterogeneous preference formats cause difficulties in large-scale GDM. A common practice is to divide large groups into smaller ones and translate heterogeneous information into a uniform format. The challenges of this approach include how to determine the satisfactory number of clusters and avoid the loss of original decision information.

This article proposed an HLSGDM approach, which can be applied to select the reasonable decision-making alternative based on the opinions of a large group of DMs. In the proposed approach, the fuzzy cluster analysis was used to cluster large-scale groups into the small groups and the original information was retained because there is no transformation involved. The F-statistic method was applied to determine the satisfactory number of clusters. The consensus degree between each small group and the large group was established based on the similarity degree in the satisfactory clustering result. Moreover, the feedback mechanism was used to adjust the small GDM matrix if any small group cannot reach a consensus. Furthermore, the heterogeneous TOPSIS was used to select the best alternative. A numerical example, which is about selecting the best rescue plan for an ERS, indicated that the proposed approach can choose the best rescue plan faster than the method proposed in [14]. Based on the law of large numbers, the evaluation results will become more accurate when the number of DMs increases in an HLSGDM. Therefore, the proposed approach is useful when the number of DMs is larger than 20.

The main contributions of this article are: 1) using fuzzy cluster analysis to divide large-scale groups into smaller groups and determine the best number of clusters using the F-statistic method; 2) using the similarity degree between different DMs, which is calculated based on the individual-opinion matrixes, to aggregate the heterogeneous information without transforming heterogeneous information into a uniform format. Thus, the original decision information is retained; and 3) developing a CRP and feedback mechanism to increase the efficiency and accuracy for HLSGDM problems.

Past and future decision-making information could be useful in large-scale GDM. For example, when evaluating customers' preferences or assessing consumers' credit risk, current, past, and future information should all be considered. One of our future research directions is to study dynamic HLSGDM approaches based on time-series information. In addition, fuzzy linguistic information, hesitant fuzzy information, and intuitionistic fuzzy information can reflect the preferences of DMs [54]–[56]. How to deal with these types of information in LSGDM is an important future research direction. One of the limitations of this study is that it assumes that DMs are

TABLE VII
ORIGINAL INDIVIDUAL DECISION MATRIX

		C_1	C_2	C_3			C_1	C_2	C_3			C_1	C_2	C_3
e_1	A_1	0.9	[2,3]	(0.6,0.7,0.8)	e_2	A_1	0.1	[5,6]	(0.2,0.3,0.4)	e_3	A_1	0.2	[5,6]	(0.1,0.2,0.3)
	A_2	0.8	[1,2]	(0.3,0.4,0.5)		A_2	0.5	[5,6]	(0.7,0.8,0.9)		A_2	0.5	[4,5]	(0.7,0.8,0.9)
	A_3	0.7	[7,8]	(0.4,0.5,0.6)		A_3	0.5	[2,3]	(0.6,0.7,0.8)		A_3	0.4	[2,3]	(0.5,0.6,0.7)
	A_4	0.6	[1,2]	(0.1,0.2,0.3)		A_4	0.8	[2,3]	(0.6,0.7,0.8)		A_4	0.3	[2,3]	(0.5,0.6,0.7)
	A_5	0.5	[8,9]	(0.3,0.4,0.5)		A_5	0.6	[8,9]	(0.3,0.4,0.5)		A_5	0.5	[7,8]	(0.2,0.3,0.4)
e_4	A_1	0.2	[3,4]	(0.2,0.3,0.4)	e_5	A_1	0.9	[4,5]	(0.2,0.3,0.4)	e_6	A_1	0.4	[3,4]	(0.1,0.2,0.3)
	A_2	0.5	[5,6]	(0.7,0.8,0.9)		A_2	0.1	[5,6]	(0.7,0.8,0.9)		A_2	0.5	[3,4]	(0.2,0.3,0.4)
	A_3	0.5	[2,3]	(0.6,0.7,0.8)		A_3	0.3	[1,2]	(0.7,0.8,0.9)		A_3	0.1	[7,8]	(0.7,0.8,0.9)
	A_4	0.9	[3,4]	(0.5,0.6,0.7)		A_4	0.5	[5,6]	(0.6,0.7,0.8)		A_4	0.4	[1,2]	(0.7,0.8,0.9)
	A_5	0.6	[7,8]	(0.3,0.4,0.5)		A_5	0.6	[4,5]	(0.2,0.3,0.4)		A_5	0.2	[1,2]	(0.3,0.4,0.5)
e_7	A_1	0.7	[1,2]	(0.1,0.2,0.3)	e_8	A_1	0.7	[1,2]	(0.1,0.2,0.3)	e_9	A_1	0.4	[3,4]	(0.2,0.3,0.4)
	A_2	0.5	[3,4]	(0.2,0.3,0.4)		A_2	0.5	[3,4]	(0.2,0.3,0.4)		A_2	0.9	[4,5]	(0.3,0.4,0.5)
	A_3	0.6	[5,6]	(0.1,0.2,0.3)		A_3	0.7	[5,6]	(0.1,0.2,0.3)		A_3	0.9	[7,8]	(0.8,0.9,1.0)
	A_4	0.4	[8,9]	(0.3,0.4,0.5)		A_4	0.4	[8,9]	(0.3,0.4,0.5)		A_4	0.1	[5,6]	(0.2,0.3,0.4)
	A_5	0.4	[1,2]	(0.2,0.3,0.4)		A_5	0.4	[1,2]	(0.2,0.3,0.4)		A_5	0.9	[1,2]	(0.7,0.8,0.9)
e_{10}	A_1	0.4	[1,2]	(0.1,0.2,0.3)	e_{11}	A_1	0.4	[8,9]	(0.3,0.4,0.5)	e_{12}	A_1	0.6	[5,6]	(0.4,0.5,0.6)
	A_2	0.7	[4,5]	(0.1,0.2,0.3)		A_2	0.3	[7,8]	(0.3,0.4,0.5)		A_2	0.7	[5,6]	(0.5,0.6,0.7)
	A_3	0.9	[1,2]	(0.5,0.6,0.7)		A_3	0.7	[7,8]	(0.2,0.3,0.4)		A_3	0.5	[3,4]	(0.3,0.4,0.5)
	A_4	0.6	[2,3]	(0.1,0.2,0.3)		A_4	0.5	[1,2]	(0.1,0.2,0.3)		A_4	0.6	[1,2]	(0.7,0.8,0.9)
	A_5	0.6	[4,5]	(0.7,0.8,0.9)		A_5	0.9	[4,5]	(0.1,0.2,0.3)		A_5	0.4	[2,3]	(0.6,0.7,0.8)
e_{13}	A_1	0.1	[6,7]	(0.1,0.2,0.3)	e_{14}	A_1	0.2	[3,4]	(0.1,0.2,0.3)	e_{15}	A_1	0.3	[8,9]	(0.6,0.7,0.8)
	A_2	0.5	[3,4]	(0.3,0.4,0.5)		A_2	0.5	[7,8]	(0.1,0.2,0.3)		A_2	0.4	[3,4]	(0.2,0.3,0.4)
	A_3	0.7	[3,4]	(0.4,0.5,0.6)		A_3	0.7	[4,5]	(0.3,0.4,0.5)		A_3	0.5	[3,4]	(0.6,0.7,0.8)
	A_4	0.9	[1,2]	(0.3,0.4,0.5)		A_4	0.4	[5,6]	(0.1,0.2,0.3)		A_4	0.4	[3,4]	(0.3,0.4,0.5)
	A_5	0.4	[1,2]	(0.7,0.8,0.9)		A_5	0.4	[7,8]	(0.4,0.5,0.6)		A_5	0.7	[3,4]	(0.6,0.7,0.8)
e_{16}	A_1	0.2	[3,4]	(0.1,0.2,0.3)	e_{17}	A_1	0.5	[5,6]	(0.6,0.7,0.8)	e_{18}	A_1	0.2	[3,4]	(0.1,0.2,0.3)
	A_2	0.5	[7,8]	(0.1,0.2,0.3)		A_2	0.4	[5,6]	(0.1,0.2,0.3)		A_2	0.5	[7,8]	(0.1,0.2,0.3)
	A_3	0.7	[4,5]	(0.3,0.4,0.5)		A_3	0.5	[5,6]	(0.1,0.2,0.3)		A_3	0.7	[4,5]	(0.3,0.4,0.5)
	A_4	0.4	[5,6]	(0.1,0.2,0.3)		A_4	0.6	[8,9]	(0.5,0.6,0.7)		A_4	0.4	[5,6]	(0.1,0.2,0.3)
	A_5	0.4	[7,8]	(0.4,0.5,0.6)		A_5	0.4	[1,2]	(0.3,0.4,0.5)		A_5	0.4	[7,8]	(0.4,0.5,0.6)
e_{19}	A_1	0.9	[4,5]	(0.8,0.9,1.0)	e_{20}	A_1	0.3	[6,7]	(0.2,0.3,0.4)					
	A_2	0.1	[5,6]	(0.1,0.2,0.3)		A_2	0.8	[2,3]	(0.2,0.3,0.4)					
	A_3	0.4	[4,5]	(0.1,0.2,0.3)		A_3	0.6	[5,6]	(0.3,0.4,0.5)					
	A_4	0.6	[6,7]	(0.1,0.2,0.3)		A_4	0.4	[6,7]	(0.1,0.2,0.3)					
	A_5	0.4	[4,5]	(0.5,0.6,0.7)		A_5	0.7	[3,4]	(0.1,0.2,0.3)					

TABLE VIII
FUZZY SIMILAR MATRIX R

1.000	0.799	0.799	0.812	0.765	0.800	0.787	0.786	0.796	0.802	0.843	0.842	0.789	0.819	0.838	0.819	0.770	0.819	0.800	0.853
0.799	1.000	0.924	0.953	0.872	0.825	0.797	0.798	0.802	0.819	0.814	0.851	0.871	0.842	0.843	0.842	0.768	0.842	0.762	0.827
0.799	0.924	1.000	0.922	0.864	0.849	0.811	0.809	0.794	0.828	0.814	0.843	0.861	0.843	0.843	0.843	0.765	0.843	0.753	0.840
0.812	0.953	0.922	1.000	0.888	0.823	0.824	0.823	0.817	0.845	0.813	0.852	0.848	0.869	0.863	0.869	0.783	0.869	0.786	0.842
0.765	0.872	0.864	0.888	1.000	0.798	0.797	0.796	0.794	0.797	0.802	0.817	0.784	0.801	0.815	0.801	0.769	0.801	0.789	0.816
0.800	0.825	0.849	0.823	0.798	1.000	0.838	0.836	0.825	0.791	0.780	0.851	0.883	0.808	0.815	0.808	0.803	0.808	0.751	0.791
0.787	0.797	0.811	0.824	0.797	0.838	1.000	0.997	0.836	0.805	0.776	0.837	0.819	0.821	0.807	0.821	0.855	0.821	0.795	0.812
0.786	0.798	0.809	0.823	0.796	0.836	0.997	1.000	0.837	0.805	0.776	0.836	0.819	0.821	0.806	0.821	0.853	0.821	0.796	0.811
0.796	0.802	0.794	0.817	0.794	0.825	0.836	0.837	1.000	0.807	0.784	0.813	0.834	0.835	0.835	0.835	0.805	0.835	0.786	0.833
0.802	0.819	0.828	0.845	0.797	0.791	0.805	0.805	0.807	1.000	0.785	0.792	0.804	0.926	0.842	0.926	0.768	0.926	0.821	0.813
0.843	0.814	0.814	0.813	0.802	0.780	0.776	0.776	0.784	0.785	1.000	0.856	0.799	0.809	0.836	0.809	0.771	0.809	0.800	0.868
0.842	0.851	0.843	0.852	0.817	0.851	0.837	0.836	0.813	0.792	0.856	1.000	0.852	0.819	0.868	0.819	0.835	0.819	0.800	0.831
0.789	0.871	0.861	0.848	0.784	0.883	0.819	0.819	0.834	0.804	0.799	0.852	1.000	0.835	0.836	0.835	0.780	0.835	0.760	0.812
0.819	0.842	0.843	0.869	0.801	0.808	0.821	0.821	0.835	0.926	0.809	0.819	0.835	1.000	0.873	1.000	0.794	1.000	0.847	0.855
0.838	0.843	0.843	0.863	0.815	0.815	0.807	0.806	0.835	0.842	0.836	0.868	0.836	0.873	1.000	0.873	0.836	0.873	0.849	0.875
0.819	0.842	0.843	0.869	0.801	0.808	0.821	0.821	0.835	0.926	0.809	0.819	0.835	1.000	0.873	1.000	0.794	1.000	0.847	0.855
0.770	0.768	0.765	0.783	0.769	0.803	0.855	0.853	0.805	0.768	0.771	0.835	0.780	0.794	0.836	0.794	1.000	0.794	0.845	0.791
0.819	0.842	0.843	0.869	0.801	0.808	0.821	0.821	0.835	0.926	0.809	0.819	0.835	1.000	0.873	1.000	0.794	1.000	0.847	0.855
0.800	0.762	0.753	0.786	0.789	0.751	0.795	0.796	0.786	0.821	0.800	0.800	0.760	0.847	0.849	0.847	0.845	0.847	1.000	0.812
0.853	0.827	0.840	0.842	0.816	0.791	0.812	0.811	0.833	0.813	0.868	0.831	0.812	0.855	0.875	0.855	0.791	0.855	0.812	1.000

TABLE IX
FUZZY EQUIVALENT MATRIX R^*

1.000	0.853	0.853	0.853	0.853	0.853	0.845	0.845	0.837	0.853	0.853	0.853	0.853	0.853	0.853	0.853	0.845	0.853	0.849	0.853
0.853	1.000	0.924	0.953	0.888	0.871	0.845	0.845	0.837	0.869	0.868	0.868	0.871	0.869	0.869	0.869	0.845	0.869	0.849	0.869
0.853	0.924	1.000	0.924	0.888	0.871	0.845	0.845	0.837	0.869	0.868	0.868	0.871	0.869	0.869	0.869	0.845	0.869	0.849	0.869
0.853	0.953	0.924	1.000	0.888	0.871	0.845	0.845	0.837	0.869	0.868	0.868	0.871	0.869	0.869	0.869	0.845	0.869	0.849	0.869
0.853	0.888	0.888	0.888	1.000	0.871	0.845	0.845	0.837	0.869	0.868	0.868	0.871	0.869	0.869	0.869	0.845	0.869	0.849	0.869
0.853	0.871	0.871	0.871	0.871	1.000	0.845	0.845	0.837	0.869	0.868	0.868	0.883	0.869	0.869	0.869	0.845	0.869	0.849	0.869
0.845	0.845	0.845	0.845	0.845	0.845	1.000	0.997	0.837	0.845	0.845	0.845	0.845	0.845	0.845	0.845	0.855	0.845	0.845	0.845
0.845	0.845	0.845	0.845	0.845	0.845	0.997	1.000	0.837	0.845	0.845	0.845	0.845	0.845	0.845	0.845	0.855	0.845	0.845	0.845
0.837	0.837	0.837	0.837	0.837	0.837	0.837	0.837	1.000	0.837	0.837	0.837	0.837	0.837	0.837	0.837	0.837	0.837	0.837	0.837
0.853	0.869	0.869	0.869	0.869	0.869	0.845	0.845	0.837	1.000	0.868	0.868	0.869	0.926	0.873	0.926	0.845	0.926	0.849	0.873
0.853	0.868	0.868	0.868	0.868	0.868	0.845	0.845	0.837	0.868	1.000	0.868	0.868	0.868	0.868	0.868	0.845	0.868	0.849	0.868
0.853	0.868	0.868	0.868	0.868	0.868	0.845	0.845	0.837	0.868	0.868	1.000	0.868	0.868	0.868	0.868	0.845	0.868	0.849	0.868
0.853	0.871	0.871	0.871	0.871	0.883	0.845	0.845	0.837	0.869	0.868	0.868	1.000	0.869	0.869	0.869	0.845	0.869	0.849	0.873
0.853	0.869	0.869	0.869	0.869	0.869	0.845	0.845	0.837	0.926	0.868	0.868	0.869	1.000	0.873	1.000	0.845	1.000	0.849	0.873
0.853	0.869	0.869	0.869	0.869	0.869	0.845	0.845	0.837	0.873	0.868	0.868	0.869	0.873	1.000	0.873	0.845	0.873	0.849	0.875
0.853	0.869	0.869	0.869	0.869	0.869	0.845	0.845	0.837	0.926	0.868	0.868	0.869	1.000	0.873	1.000	0.845	1.000	0.849	0.873
0.845	0.845	0.845	0.845	0.845	0.845	0.855	0.855	0.837	0.845	0.845	0.845	0.845	0.845	0.845	0.845	1.000	0.845	0.845	0.845
0.853	0.869	0.869	0.869	0.869	0.869	0.845	0.845	0.837	0.926	0.868	0.868	0.869	1.000	0.873	1.000	0.845	1.000	0.849	0.873
0.849	0.849	0.849	0.849	0.849	0.849	0.845	0.845	0.837	0.849	0.849	0.849	0.849	0.849	0.849	0.849	0.845	0.849	1.000	0.849
0.853	0.869	0.869	0.869	0.869	0.869	0.845	0.845	0.837	0.873	0.868	0.868	0.873	0.873	0.875	0.873	0.845	0.873	0.849	1.000

independent and there are no noncooperative behaviors in the CRP. In practice, the interactions between DMs and the noncooperative behaviors in the CRP often exist in HLSGDM. Thus, another future research direction is to develop methods that consider the noncooperative behaviors of DMs.

APPENDIX A

See Table VII.

APPENDIX B

See Table VIII.

APPENDIX C

See Table IX.

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