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Proportional-Integral Observer Design for Uncertain Time-Delay Systems Subject to Deception Attacks: An Outlier-Resistant Approach

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***Abstract*—This paper deals with the proportional-integral observer (PIO) design problem for a class of linear systems with distributed time-delays and randomly occurring parameter uncertainties. The measurement signals, transmitted from the sensors to the observer, might suffer from the randomly occurring deception attacks. The random occurrences of parameter uncertainties and deception attacks are governed by two series of Bernoulli random variables with known probability distributions. An outlier-resistant PIO is developed by introducing an innovation saturation mechanism for the sake of alleviating the adverse effects induced by the deception attacks on the estimation performance. The purpose of the addressed problem is to design a PIO that is capable of guaranteeing the mean-square boundedness of the estimation errors while achieving the desired security level. The desired PIO gain is designed by solving a matrix inequality and the validity of the results obtained is shown by a numerical simulation example.**

***Index Terms*—Proportional-integral observer, outlier-resistant state estimation, randomly occurring deception attacks, randomly occurring parameter uncertainties, distributed time-delays.**

I. Introduction

As early as in the seventies, the proportional-integral observer (PIO) has been constructed in [40] by introducing an extra integral operation with respect to the output estimation error into the traditional Luenberger observer. Hitherto, the PIO has shown great potential in a diverse range of practical domains such as manufacturing processes, network communication systems, power circuit systems and economic systems [3], [5]. Briefly speaking, the PIO consists of the proportional term and the integral term with respect to the output estimation error, thereby reflecting the current and historical information for achieving better performance as compared with the traditional Luenberger observer. Thanks to the utilization of integral action, the PIO has exhibited promising advantages in improving the system robustness, eliminating the steady-state error as well as increasing the observer design freedom. Accordingly, considerable research attention has been paid to the PIO design issue with fruitful results available in the literature, see e.g. [9], [21], [22], [31], [33], [34].

Time-delays, which are known to have substantial impacts on system performance, are often encountered in many real systems (e.g. aerospace systems, industrial control systems, telemedicine system, robot teleoperation system and network communication systems) for a variety of reasons such as equipment aging and complicated structure. In the past few decades, there has been an enormous research effort into the investigation on various kinds of time-delays including constant delays, time-varying delays, random delays, and distributed delays [19], [20], [27], [39], [46]. Apart from the time-delay phenomena, parameter uncertainties serve as another main factor that complicates the system analysis and synthesis [4], [25], [28], [37]. In today's networked world, parameter uncertainties may take place in a random way due probably to network-induced random faults, sudden environmental disturbances and unpredictable fluctuations of network load. Such kind of uncertainties is customarily referred to as randomly occurring parameter uncertainties (ROPUs) which, together with time-delays, may result in undesirable dynamic behaviors such as oscillation, chaos or even divergence. Therefore, it is of great importance to make dedicated efforts in disposing of the influence from time-delays and/or ROPUs on the system analysis and synthesis [26], [45].

With the rapid advancement of networking technologies, networked systems (NSs) has become an emerging research frontier in the past few decades [7], [8], [10], [13], [15], [23], [29]. In practical applications of NSs, it is a normal practice for the system components (such as controllers, sensors, estimators and actuators) to be connected through shared network media, thereby achieving remote, reliable, real-time yet collaborative operation and control. Because of the inherent openness of the shared communication channel, NSs are vulnerable to miscellaneous cyber-threats, and some representative examples found in the real world include Regin malware threat, Operation Aurora, Petya Ransomware, and DigiNotar hacker incident, to name a few. As a result, particular research interest has been gained to develop appropriate estimation/control schemes to improve the system security and relieve the negative effects caused by cyber-attacks on the

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system performance [11], [32], [42]. For example, in [6], [8], [17], [47], the security control problems have been investigated for systems under cyber-attacks, and the security-guaranteed estimation schemes have been developed in [14], [35], [41].

In general, the common cyber-attacks can be classified as denial of service attacks and deception attacks in terms of the ways they are implemented, and the deception attacks are known to be comparatively dangerous due to their behaviors of data hijacking and falsification. When launching deception attacks, the adversaries capture the data packets and inject the false information (or IP address) to deliberately prevent the system operation from normal execution, thus destroying hardware/software and even crashing the whole system. On the other hand, the intended cyber-attacks sent by the adversaries might *not* be always successful on account of various reasons such as 1) deployment of the detection software and protection equipment; 2) inherent bandwidth limit of the shared communication channel; and 3) network-induced phenomena such as packet losses, communication delays and channel fading, and all these have led to the random nature of the deception attacks. As such, it is practically meaningful to model the NS-based deception attacks as random events obeying certain distributions, and such a modeling strategy has been adopted in quite a few recent results, see e.g. [24], [36], [44] where the deception attacks have been assumed to be regulated by Bernoulli/Markov process with known probability distributions.

It is worth noting that, due to effects of the deception attacks, the measurement outputs might undergo abrupt yet large disturbances which, in turn, might induce the so-called *measurement outliers* contributing further to the deterioration of the estimation performance. In other words, it is quite likely that the measurement outliers, if not adequately tackled, would give rise to abnormal changes of the innovation values in the state estimator, thereby jeopardizing the estimation accuracy. As such, it becomes necessary to develop an estimator that is insensitive/invulnerable to the measurement outliers, and such an estimator is termed as the *outlier-resistant* estimator. Recently, the outlier-resistant state estimation issue has begun to arouse initial research interest with some elegant results, see e.g. [2], [12], [30], [38], [48]. For instance, a Kalman filter with saturated output injection has been constructed in [16] to withstand the measurement outliers, and the outlier-resistant state estimation issue has been discussed in [1] for a class of linear time-invariant systems in the presence of measurement outliers. Nonetheless, a thorough literature search has exhibited that the relevant results for the outlier-resistant PIO design issues have been really scattered due primarily to the analytical complexity induced by the integral term in PIO, and this motivates our current investigation.

Inspired by the above discussions, the focus of this paper is on the design of the outlier-resistant PIO for a class of discrete-time delayed systems with ROPUs subject to randomly occurring deception attacks (RODAs). In doing so, we are confronted with the following three fundamental challenges: 1) how to establish a suitable theoretical framework to handle the analytical complexity brought by the random occurrences of the parameter uncertainties and the deception attacks? 2) how to develop a reliable scheme to deal with the measurement outliers induced by the deception attacks? and 3) how to construct an appropriate criteria to quantify the combined impact caused by the RODAs and the stochastic noises on the estimation errors? As such, the objective of this paper is to conquer the three challenges. The main technical contributions lie in that: *1) the PIO design issue is, for the first time, addressed for a kind of discrete-time delayed system subject to RODAs; 2) an innovation-saturation-based mechanism is employed in the PIO design to alleviate the impacts from the deception attacks on the estimation errors; and 3) sufficient conditions are derived to ensure the exponentially mean-square (EMS) boundedness and further achieve the desired security level.*

The rest of this paper is organized as follows. The outlier-resistant PIO design issue is formulated for the discrete-time delayed systems subject to RODAs in Section II. In Section III, the EMS boundedness of the estimation errors is analyzed and a sufficient condition for security is provided. Subsequently, the expected outlier-resistant PIO is designed by using the linear matrix inequality (LMI) method. Section IV presents a numerical example to verify the usefulness and advantage of the proposed PIO design scheme, and some conclusions are drawn in Section V.

Notation. For notational convenience, $\mathbb{R}^{n\times m}$ and $\mathbb{R}^n$ are, respectively, used to denote the set of all $n\times m$ real matrices and the n-dimensional Euclidean space. $X<Y$ (respectively, $X\leq Y$) implies that $Y-X$ is positive definite (respectively, positive semi-definite), where X and Y are real symmetric matrices. I_q stands for a q-dimensional identity matrix and the Kronecker product is denoted by the symbol "$\otimes$". In addition, $\lambda_{\min}(Z)$ and $\lambda_{\max}(Z)$ are, respectively, the minimum and maximum eigenvalues of the symmetric matrix Z.

II. Problem Formulation and Preliminaries

A. The System Model

Consider a class of linear discrete time-delayed systems with ROPUs:

$$\begin{cases} x(s+1) = \big(A+\kappa(s)B(s)\big)x(s)+Mw(s) \\ \qquad\qquad + H\displaystyle\sum_{h=1}^{\hbar}\ell_h x(s-h) \\ y(s) = Cx(s) \\ z(s) = Dx(s) \\ x(\jmath) = \phi(\jmath), \quad \forall \jmath\in\mathfrak{H}\triangleq\{-\hbar,\ldots,-1,0\} \end{cases} \tag{1}$$

where $x(s)\in\mathbb{R}^{n_x}$, $y(s)\in\mathbb{R}^{n_y}$, and $z(s)\in\mathbb{R}^{n_z}$ are, respectively, the state, the measurement output and the output to be estimated; $\hbar$ is a given positive integer and ℓ_h is a positive scalar; $\phi(\jmath)$ is the initial condition sequence; A, C, D, M and H are given matrices with compatible dimensions. $w(s)\in\mathbb{R}$ possesses the following statistical properties:

$$\mathbb{E}\{w(s)\}=0, \quad \mathbb{E}\{w(p)w(q)\}=\begin{cases}\vartheta_w^2, & \text{if } p=q\\ 0, & \text{if } p\neq q\end{cases} \tag{2}$$

where ϑ_w is a known scalar.

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The random variable $\kappa(s)$ is a Bernoulli-distributed sequence satisfying

$$\text{Prob}\{\kappa(s)=1\}=\bar{\kappa}, \quad \text{Prob}\{\kappa(s)=0\}=1-\bar{\kappa}$$

where $\bar{\kappa}\in[0,1)$ is a constant that is known a priori.

The real matrix $B(s)$, which accounts for the parameter uncertainty, meets the following constraint:

$$B(s)=RS(s)T \tag{3}$$

where matrices R and T are known, $S(s)\in\mathbb{R}^{n_s\times n_s}$ is an unknown matrix function satisfying

$$S^T(s)S(s)\leqslant I. \tag{4}$$

B. The Cyber-Attack Model

In the current investigation, the measurement signals are transmitted to the observer via a shared communication network, where the data transmission might be maliciously falsified through the RODAs expressed by

$$\upsilon(s)=y(s)+\vartheta(s)\varpi(s). \tag{5}$$

Here, $\upsilon(s)\in\mathbb{R}^{n_y}$ denotes the input signal of the observer that is corrupted by the attackers, $\varpi(s)\in\mathbb{R}^{n_y}$ denotes the deception attack sent by the hostile attackers and is characterized by

$$\varpi(s)=-y(s)+\delta(s) \tag{6}$$

with $\delta(s)\neq 0$ being an arbitrary signal. $\vartheta(s)$ is a Bernoulli random variable regulating the RODAs with the following probability distribution:

$$\text{Prob}\{\vartheta(s)=1\}=\bar{\vartheta}, \quad \text{Prob}\{\vartheta(s)=0\}=1-\bar{\vartheta}$$

where $\bar{\vartheta}\in[0,1)$ is a known constant. Without loss of generality, $\vartheta(s)$, $\kappa(s)$ and $w(s)$ are assumed to be uncorrelated.

Remark 1: In practical engineering, deception attacks sent by the adversaries cannot be always successful because of the anti-attack countermeasures (e.g. deployment of the defense devices or detectors) and the limited communication capacity. In this sense, from the defender's perspective, the deception attacks take place in a random manner obeying a Bernoulli sequence with certain statistical property. In accordance with (5), if the deception attack is successful, i.e. $\vartheta(s)=1$, the actual signal received by the observer is $\delta(s)$, which means that the measurement outlier is injected to the observer. If the deception attack is unsuccessful, i.e. $\vartheta(s)=0$, the actual signal received by the observer is $y(s)$, which indicates that the normal measurement signals are transmitted to the observer. Note that the probability of successful deception attacks $\bar{\vartheta}$ can be identified *a priori* via some statistical experiments.

C. The Outlier-Resistant PIO

For restraining the estimation performance from being distorted by the RODAs, a saturation function is *purposely* introduced when handling the observer design issue. Specifically, the outlier-resistant PIO is proposed as follows:

$$\begin{cases}\hat{x}(s+1)=A\hat{x}(s)+H\sum_{h=1}^{\hbar}\ell_h\hat{x}(s-h)\\ \qquad\qquad +F_P\Im\big(\upsilon(s)-C\hat{x}(s)\big)+F_I\chi(s)\\ \chi(s+1)=\chi(s)+F\Im\big(\upsilon(s)-C\hat{x}(s)\big)\\ \hat{z}(s)=D\hat{x}(s)\\ \chi(0)=0\\ \hat{x}(\jmath)=0, \quad \forall\jmath\in\mathfrak{H}\end{cases} \tag{7}$$

where $\hat{x}(s)\in\mathbb{R}^{n_x}$ is $x(s)$'s estimation, $\hat{z}(s)\in\mathbb{R}^{n}$ is $z(s)$'s estimation, and $\chi(s)\in\mathbb{R}^{n_\chi}$ describes the integral of the weighted output estimation error. Here, F_P, F_I and F are the observer parameters to be determined.

Define the saturation function $\Im(\cdot):\mathbb{R}^{n_y}\to\mathbb{R}^{n_y}$ as

$$\Im(l)=\begin{bmatrix}\Im(l_1) & \Im(l_2) & \cdots & \Im(l_{n_y})\end{bmatrix}^T, \quad \forall l\in\mathbb{R}^{n_y} \tag{8}$$

where $\Im(l_q)=\text{sign}(l_q)\min\{l_q^M, |l_q|\}$ and l_q^M is the qth entry of saturation level vector l^M ($q\in\mathfrak{Y}\triangleq\{1,2,\ldots,n_y\}$). Moreover, the saturation function $\Im(\cdot)$ is a sector-bounded nonlinear function satisfying:

$$\big(\Im(\alpha_q)-\mu_q\alpha_q\big)^T\big(\Im(\alpha_q)-\alpha_q\big)\leqslant 0, \quad (q\in\mathfrak{Y}) \tag{9}$$

where α_q is a given scalar and μ_q is a positive scalar satisfying $0<\mu_q<1$.

Remark 2: Different from the conventional PIO, the outlier-resistant PIO constructed in (7) contains an intentionally introduced saturation constraint on the innovations and, accordingly, the measurement outliers (induced by the deception attacks) could be adequately tackled. Specifically, the innovation in the outlier-resistant PIO (7) can be constrained within a predetermined range owing to the saturation function (8), where the saturation level l^M is dependent on the prior knowledge (on the range of the innovation and the tolerance level of the detector). In this case, the outlier-resistant PIO would be more effective than the conventional PIO in alleviating the negative effects of the deception attacks on the estimation errors. In fact, the outlier-resistant PIO specializes to the conventional PIO when the saturation level l^M approaches infinity.

Denoting $\tilde{x}(s)\triangleq x(s)-\hat{x}(s)$ and $\tilde{z}(s)\triangleq z(s)-\hat{z}(s)$, the dynamics of the estimation errors can be written as

$$\begin{cases}\tilde{x}(s+1)=A\tilde{x}(s)+H\sum_{h=1}^{\hbar}\ell_h\tilde{x}(s-h)+\kappa(s)B(s)x(s)\\ \qquad\qquad -F_P\Im\big(\upsilon(s)-C\hat{x}(s)\big)-F_I\chi(s)+Mw(s)\\ \chi(s+1)=\chi(s)+F\Im\big(\upsilon(s)-C\hat{x}(s)\big)\\ \tilde{z}(s)=D\tilde{x}(s)\\ \chi(0)=0\\ \tilde{x}(\jmath)=\phi(\jmath), \quad \forall\jmath\in\mathfrak{H}\end{cases} \tag{10}$$

Setting $\psi(s)\triangleq\begin{bmatrix}x^T(s) & \tilde{x}^T(s) & \chi^T(s)\end{bmatrix}^T$, the following augmented system is obtained:

$$\begin{cases} \psi(s+1) = \big(\mathcal{A} + \bar{\kappa}\mathcal{B}(s) + \tilde{\kappa}(s)\mathcal{B}(s)\big)\psi(s) \\ \qquad + \mathcal{H}\sum_{h=1}^{\hbar}\ell_h\psi(s-h) + \mathcal{M}w(s) \\ \qquad + \mathcal{F}\Im\big(\zeta(s)\big) \\ \psi(\jmath) = \varphi(\jmath), \quad \forall \jmath \in \mathfrak{H} \end{cases} \tag{11}$$

where $\varphi(\jmath) \triangleq \begin{bmatrix}\phi^T(\jmath) & \phi^T(\jmath) & 0\end{bmatrix}^T$ and

$$\mathcal{A} \triangleq \begin{bmatrix} A & 0 & 0 \\ 0 & A & -F_I \\ 0 & 0 & I \end{bmatrix}, \ \mathcal{H} \triangleq \begin{bmatrix} H & 0 & 0 \\ 0 & H & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\mathcal{B}(s) \triangleq \begin{bmatrix} B(s) & 0 & 0 \\ B(s) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\mathcal{M} \triangleq \begin{bmatrix} M \\ M \\ 0 \end{bmatrix}, \quad \tilde{\kappa}(s) \triangleq \kappa(s) - \bar{\kappa}$$

$$\mathcal{F} \triangleq \begin{bmatrix} 0 \\ -F_P \\ F \end{bmatrix}, \quad \tilde{\vartheta}(s) \triangleq \vartheta(s) - \bar{\vartheta}$$

$$\zeta(s) \triangleq \big(\mathcal{C}_1 + \tilde{\vartheta}(s)\mathcal{C}_2\big)\psi(s) + \big(\bar{\vartheta} + \tilde{\vartheta}(s)\big)\delta(s)$$

$$\mathcal{C}_1 \triangleq \begin{bmatrix} -\bar{\vartheta}C & C & 0 \end{bmatrix}, \ \mathcal{C}_2 \triangleq \begin{bmatrix} -C & 0 & 0 \end{bmatrix}.$$

For facilitating the subsequent analysis, the definitions of EMS boundedness and mean-square security are given as follows.

Definition 1: The compacted system (11) is exponentially ultimately bounded in mean-square sense if the system dynamics $\psi(s)$ is constrained by

$$\mathbb{E}\{\|\psi(s)\|^2\} \leqslant \epsilon^s\alpha + \nu(s) \text{ and } \lim_{s\to+\infty}\nu(s) = \bar{\nu}. \tag{12}$$

where ϵ, α and $\bar{\nu}$ are constants satisfying $0 < \epsilon < 1$, $\alpha > 0$ and $\bar{\nu} > 0$.

Definition 2: The augmented system (11) is mean-square ς-secure if the following two conditions are satisfied simultaneously: a) the augmented system (11) is exponentially ultimately bounded in mean-square sense; and b) the system dynamics $\psi(s)$ is governed by

$$\mathbb{E}\{\|\psi(s)\|^2\} \leqslant \varsigma, \quad \forall k \geqslant 0, \tag{13}$$

where ς is a given positive scalar denoting the desired security level.

Remark 3: In this paper, the concept of EMS boundedness presented in Definition 1 is employed to depict the joint influence from the RODAs and the stochastic noise on the estimation errors, and the concept of mean-square security presented in Definition 2 is used to characterize the ability of the NSs to tolerate adversaries and recover from cyber-attacks. Obviously, if the augmented system (11) is exponentially ultimately bounded in mean-square sense, then it must be mean-square ς-secure.

The objective of this paper is to design an outlier-resistant PIO (7) for system (1) subject to RODAs.

III. Main Results

In this section, we are going to design an outlier-resistant PIO for the system (1) subject to RODAs. Sufficient conditions are provided to guarantee the EMS boundedness of the estimation errors and the mean-square security is simultaneously satisfied. Then, the desired PIO gains are parameterized in terms of the solution to an LMI.

The following lemmas are given to facilitate the sequel development.

Lemma 1: The saturation function $\Im\big(\zeta(s)\big)$ in (11) satisfies

$$\begin{aligned} &\Im^T\big(\zeta(s)\big)\Im\big(\zeta(s)\big) + \zeta^T(s)U^T\zeta(s) \\ &- \zeta^T(s)(U^T + I)\Im\big(\zeta(s)\big) \leqslant 0 \end{aligned} \tag{14}$$

where $U \triangleq \mathrm{diag}\{\mu_1, \mu_2, \ldots, \mu_q\}$.

Proof: From (9), it is easy to verify that

$$\begin{aligned} &\big(\Im(l) - Ul\big)^T\big(\Im(l) - l\big) \\ =&\sum_{q=1}^{n_y}\Big(\big(\Im(\alpha_q) - \mu_q\alpha_q\big)^T\big(\Im(\alpha_q) - \alpha_q\big)\Big) \\ \leqslant& 0. \end{aligned} \tag{15}$$

By letting $l = \zeta(s)$, we have

$$\Big(\Im(\zeta(s)) - U\zeta(s)\Big)^T\Big(\Im(\zeta(s)) - \zeta(s)\Big) \leqslant 0$$

which implies that inequality (14) holds. ■

Lemma 2: Let $z(s) \in \mathbb{R}^{n_z}$ ($n_z = 2n_x + n_\chi$), $Z \in \mathbb{R}^{n_z\times n_z}$ be a positive semi-definite matrix, $b_h > 0$ ($h = 1, 2, \ldots, \hbar$) be scalar constants, and $\hbar$ be a positive integer. The following relationship is true:

$$\begin{aligned} &\left(\sum_{h=1}^{\hbar}b_hz(s)\right)^T Z\left(\sum_{h=1}^{\hbar}b_hz(s)\right) \\ \leqslant&\left(\sum_{h=1}^{\hbar}b_h\right)\sum_{h=1}^{\hbar}b_hz^T(s)Zz(s). \end{aligned} \tag{16}$$

Lemma 3: Let $X = X^T$, Y and W be real matrices of appropriate dimensions, and $G(s)$ satisfies $G^T(s)G(s) \leqslant I$. Then

$$X + YG(s)W + W^TG^T(s)Y^T < 0 \tag{17}$$

if and only if there exists a positive scalar π such that

$$X + \pi YY^T + \frac{1}{\pi}W^TW < 0 \tag{18}$$

or

$$\begin{bmatrix} X & \pi Y & W^T \\ \pi Y^T & -\pi I & 0 \\ W & 0 & -\pi I \end{bmatrix} < 0. \tag{19}$$

A. Security Analysis

In this subsection, we shall analyze the EMS boundedness of the estimation errors and derive a sufficient condition to guarantee the ς-security of (11).

Theorem 1: Let the observer parameters F_P, F_I and F be given. If there exist positive definite matrices P, Q and positive scalar ι satisfying

$$\Omega = \begin{bmatrix} \Omega_1 & \star \\ \Omega_2 & \Omega_3 \end{bmatrix} < 0 \tag{20}$$

and

$$\frac{\bar{\ell}\lambda_{\max}(Q)\hbar(\varepsilon^{\hbar}-1)+(\hbar+1)\bar{\lambda}}{\lambda_{\min}(P)\varepsilon^{\rho}}\sup_{\jmath\in\mathfrak{H}}\mathbb{E}\{\|\varphi(\jmath)\|^2\} + \frac{\varepsilon}{\lambda_{\min}(P)(\varepsilon-1)}\beta \leqslant \varsigma. \tag{21}$$

where

$$\Omega_1 \triangleq \begin{bmatrix} \Omega_1^{11} & \star & \star & \star \\ 0 & \Omega_1^{22} & \star & \star \\ \Omega_1^{31} & 0 & \Omega_1^{33} & \star \\ \Omega_1^{41} & 0 & \Omega_1^{43} & \Omega_1^{44} \end{bmatrix}$$

$$\Omega_2 \triangleq \begin{bmatrix} \mathcal{A}+\bar{\kappa}\mathcal{B}(s) & \mathcal{H} & \mathcal{F} & 0 \end{bmatrix}, \quad \bar{\ell} \triangleq \sum_{h=1}^{\hbar}\ell_h$$

$$\Omega_3 \triangleq -P^{-1}, \quad \bar{\lambda} \triangleq \max\{\lambda_{\max}(P), \bar{\ell}\lambda_{\max}(Q)\}$$

$$\Omega_1^{11} \triangleq -P+\bar{\ell}Q-\iota\mathcal{C}_1^T U^T\mathcal{C}_1-\iota(\bar{\vartheta}-\bar{\vartheta}^2)\mathcal{C}_2^T U^T\mathcal{C}_2$$

$$\Omega_1^{22} \triangleq -\frac{1}{\bar{\ell}}Q, \quad \Omega_1^{33} \triangleq -\iota I, \quad \Omega_1^{44} \triangleq -\iota\bar{\vartheta}U^T$$

$$\Omega_1^{31} \triangleq \frac{1}{2}\iota(U^T+I)\mathcal{C}_1, \quad \Omega_1^{41} \triangleq -\iota\bar{\vartheta}U^T\big(\mathcal{C}_1+\mathcal{C}_2-\bar{\vartheta}\mathcal{C}_2\big)$$

$$\Omega_1^{43} \triangleq \frac{1}{2}\iota\bar{\vartheta}(U^T+I), \quad \beta \triangleq \lambda_{\max}(\mathcal{M}^T P\mathcal{M})\vartheta_w^2$$

and $\varepsilon > 1$ in (21) is determined by

$$\lambda_{\max}(P)(\varepsilon-1)-\gamma\varepsilon+2\bar{\ell}\lambda_{\max}(Q)\varepsilon(\varepsilon^{\hbar}-1)=0 \tag{22}$$

with

$$\gamma \triangleq \lambda_{\min}\big(-\Omega_1-\Omega_2^T P\Omega_2\big),$$

then the augmented system (11) is ς-secure in the sense of mean-square.

Proof: For examining the EMS boundedness of the augmented system (11), we construct the following Lyapunov-Krasovskii functional:

$$V\big(\psi(s)\big) = V_1\big(\psi(s)\big) + V_2\big(\psi(s)\big) \tag{23}$$

where

$$V_1\big(\psi(s)\big) \triangleq \psi^T(s)P\psi(s)$$

$$V_2\big(\psi(s)\big) \triangleq \sum_{h=1}^{\hbar}\ell_h\sum_{r=s-h}^{s-1}\psi^T(r)Q\psi(r).$$

Then, the difference of $V\big(\chi(s)\big)$ is denoted by

$$\Re V\big(\psi(s)\big) = \Re V_1\big(\psi(s)\big) + \Re V_2\big(\psi(s)\big) \tag{24}$$

where

$$\Re V_1\big(\psi(s)\big) \triangleq \mathbb{E}\{V_1\big(\psi(s+1)\big)|\psi(s)\} - V_1\big(\psi(s)\big)$$

$$\Re V_2\big(\psi(s)\big) \triangleq \mathbb{E}\{V_2\big(\psi(s+1)\big)|\psi(s)\} - V_2\big(\psi(s)\big).$$

The difference of $V_1\big(\psi(s)\big)$ along (11) is calculated by

$$\begin{aligned}
&\mathbb{E}\left\{\Re V_1\big(\chi(s)\big)\right\} \\
=&\mathbb{E}\left\{V_1\big(\psi(s+1)\big)-V_1\big(\psi(s)\big)\right\} \\
=&\mathbb{E}\bigg\{\Big(\big(\mathcal{A}+\bar{\kappa}\mathcal{B}(s)+\tilde{\kappa}(s)\mathcal{B}(s)\big)\psi(s)+\mathcal{F}\Im\big(\zeta(s)\big) \\
&+\mathcal{H}\sum_{h=1}^{\hbar}\ell_h\psi(s-h)+\mathcal{M}w(s)\Big)^T P \\
&\times\Big(\big(\mathcal{A}+\bar{\kappa}\mathcal{B}(s)+\tilde{\kappa}(s)\mathcal{B}(s)\big)\psi(s)+\mathcal{F}\Im\big(\zeta(s)\big) \\
&+\mathcal{H}\sum_{h=1}^{\hbar}\ell_h\psi(s-h)+\mathcal{M}w(s)\Big)-\psi^T(s)P\psi(s)\bigg\} \\
=&\mathbb{E}\bigg\{\psi^T(s)\Big(\mathcal{A}^T P\mathcal{A}-P+\big(\bar{\kappa}+\tilde{\kappa}(s)\big)^2\mathcal{B}^T(s)P\mathcal{B}(s) \\
&+\big(\bar{\kappa}+\tilde{\kappa}(s)\big)\mathcal{A}^T P\mathcal{B}(s)+\big(\bar{\kappa}+\tilde{\kappa}(s)\big)\mathcal{B}^T(s)P\mathcal{A}\Big)\psi(s) \\
&+\Im^T\big(\zeta(s)\big)\mathcal{F}^T P\mathcal{F}\Im\big(\zeta(s)\big)+\left(\sum_{h=1}^{\hbar}\ell_h\psi(s-h)\right)^T \\
&\times\mathcal{H}^T P\mathcal{H}\left(\sum_{h=1}^{\hbar}\ell_h\psi(s-h)\right)+w^T(s)\mathcal{M}^T P\mathcal{M}w(s) \\
&+2\Im^T\big(\zeta(s)\big)\Big(\mathcal{F}^T P\mathcal{A}+\big(\bar{\kappa}+\tilde{\kappa}(s)\big)\mathcal{F}^T P\mathcal{B}(s)\Big)\psi(s) \\
&+2\left(\sum_{h=1}^{\hbar}\ell_h\psi(s-h)\right)^T\Big(\mathcal{H}^T P\mathcal{A}+\big(\bar{\kappa}+\tilde{\kappa}(s)\big)\mathcal{H}^T \\
&\times P\mathcal{B}(s)\Big)\psi(s)+2w^T(s)\Big(\mathcal{M}^T P\mathcal{A}+\big(\bar{\kappa}+\tilde{\kappa}(s)\big) \\
&\times\mathcal{M}^T P\mathcal{B}(s)\Big)\psi(s)+2\left(\sum_{h=1}^{\hbar}\ell_h\psi(s-h)\right)^T\mathcal{H}^T \\
&\times P\mathcal{F}\Im\big(\zeta(s)\big)+2w^T(s)\mathcal{M}^T P\mathcal{F}\Im\big(\zeta(s)\big)+2w^T(s) \\
&\times\mathcal{M}^T P\mathcal{H}\left(\sum_{h=1}^{\hbar}\ell_h\psi(s-h)\right)\bigg\} \\
\leqslant&\mathbb{E}\bigg\{\psi^T(s)\Big(\mathcal{A}^T P\mathcal{A}-P+\bar{\kappa}\mathcal{B}^T(s)P\mathcal{B}(s) \\
&+\sqrt{\bar{\kappa}}\mathcal{A}^T P\mathcal{B}(s)+\sqrt{\bar{\kappa}}\mathcal{B}^T(s)P\mathcal{A}\Big)\psi(s)+\Im^T\big(\zeta(s)\big) \\
&\times\mathcal{F}^T P\mathcal{F}\Im\big(\zeta(s)\big)+\left(\sum_{h=1}^{\hbar}\ell_h\psi(s-h)\right)^T\mathcal{H}^T P\mathcal{H} \\
&\times\left(\sum_{h=1}^{\hbar}\ell_h\psi(s-h)\right)+w^T(s)\mathcal{M}^T P\mathcal{M}w(s) \\
&+2\Im^T\big(\zeta(s)\big)\Big(\mathcal{F}^T P\mathcal{A}+\sqrt{\bar{\kappa}}\mathcal{F}^T P\mathcal{B}(s)\Big)\psi(s) \\
&+2\left(\sum_{h=1}^{\hbar}\ell_h\psi(s-h)\right)^T\Big(\mathcal{H}^T P\mathcal{A}+\sqrt{\bar{\kappa}}\mathcal{H}^T P\mathcal{B}(s)\Big) \\
&\times\psi(s)+2\left(\sum_{h=1}^{\hbar}\ell_h\psi(s-h)\right)^T\mathcal{H}^T P\mathcal{F}\Im\big(\zeta(s)\big)\bigg\}.
\end{aligned} \tag{25}$$

Furthermore, in light of Lemma 2, we compute the difference of $V_2\big(\psi(s)\big)$ as follows:

$$\begin{aligned}
&\mathbb{E}\left\{\Re V_2\big(\psi(s)\big)\right\}\\
=&\mathbb{E}\left\{V_2\big(\psi(s+1)\big)-V_2\big(\psi(s)\big)\right\}\\
=&\mathbb{E}\Bigg\{\sum_{h=1}^{\hbar}\ell_h\sum_{r=s-h+1}^{s}\psi^T(r)Q\psi(r)\\
&-\sum_{h=1}^{\hbar}\ell_h\sum_{r=s-h}^{s-1}\psi^T(r)Q\psi(r)\Bigg\}\\
=&\mathbb{E}\left\{\bar{\ell}\psi^T(s)Q\psi(s)-\sum_{h=1}^{\hbar}\ell_h\psi^T(s-h)Q\psi(s-h)\right\}\\
\leqslant&\mathbb{E}\Bigg\{\bar{\ell}\psi^T(s)Q\psi(s)-\frac{1}{\bar{\ell}}\left(\sum_{h=1}^{\hbar}\ell_h\psi(s-h)\right)^T Q\\
&\times\left(\sum_{h=1}^{\hbar}\ell_h\psi(s-h)\right)\Bigg\}.
\end{aligned}\tag{26}$$

Bearing in mind the statistical characteristics of $w(s)$, we calculate the term $w^T(s)\mathcal{M}^T P\mathcal{M}w(s)$ (contained in (25)) as follows:

$$\begin{aligned}
&\mathbb{E}\{w^T(s)\mathcal{M}^T P\mathcal{M}w(s)\}\\
\leqslant&\lambda_{\max}(\mathcal{M}^T P\mathcal{M})\mathbb{E}\{w^T(s)w(s)\}=\beta.
\end{aligned}\tag{27}$$

Substituting (25)-(27) into (24) leads to

$$\begin{aligned}
&\mathbb{E}\big\{\Re V\big(\psi(s)\big)\big\}\\
=&\mathbb{E}\Big\{\Re V_1\big(\psi(s)\big)+\Re V_2\big(\psi(s)\big)\Big\}\\
\leqslant&\mathbb{E}\Bigg\{\psi^T(s)\Big(\mathcal{A}^T P\mathcal{A}-P+\bar{\kappa}\mathcal{B}^T(s)P\mathcal{B}(s)\\
&+\sqrt{\bar{\kappa}}\mathcal{A}^T P\mathcal{B}(s)+\sqrt{\bar{\kappa}}\mathcal{B}^T(s)P\mathcal{A}+\bar{\ell}Q\Big)\psi(s)\\
&+\Im^T\big(\zeta(s)\big)\mathcal{F}^T P\mathcal{F}\Im\big(\zeta(s)\big)+\left(\sum_{h=1}^{\hbar}\ell_h\psi(s-h)\right)^T\\
&\times\left(\mathcal{H}^T P\mathcal{H}-\frac{1}{\bar{\ell}}Q\right)\left(\sum_{h=1}^{\hbar}\ell_h\psi(s-h)\right)\\
&+2\Im^T\big(\zeta(s)\big)\Big(\mathcal{F}^T P\mathcal{A}+\sqrt{\bar{\kappa}}\mathcal{F}^T P\mathcal{B}(s)\Big)\\
&\times\psi(s)+2\left(\sum_{h=1}^{\hbar}\ell_h\psi(s-h)\right)^T\Big(\mathcal{H}^T P\mathcal{A}\\
&+\sqrt{\bar{\kappa}}\mathcal{H}^T P\mathcal{B}(s)\Big)\psi(s)+2\Im^T\big(\zeta(s)\big)\mathcal{F}^T P\mathcal{H}\\
&\times\left(\sum_{h=1}^{\hbar}\ell_h\psi(s-h)\right)+\beta\Bigg\}\\
=&\Im_1^T(s)\Psi_1\Im_1(s)+\beta
\end{aligned}\tag{28}$$

where

$$\Im_1(s)\triangleq\begin{bmatrix}\psi(s)\\ \sum_{h=1}^{\hbar}\ell_h\psi(s-h)\\ \Im\big(\zeta(s)\big)\end{bmatrix},\ \Psi_1\triangleq\begin{bmatrix}\Psi_1^{11} & \star & \star\\ \Psi_1^{21} & \Psi_1^{22} & \star\\ \Psi_1^{31} & \Psi_1^{32} & \Psi_1^{33}\end{bmatrix}$$

$$\begin{aligned}
\Psi_1^{11}\triangleq&-P+\bar{\ell}Q+\mathcal{A}^T P\mathcal{A}+\bar{\kappa}\mathcal{B}^T(s)P\mathcal{B}(s)\\
&+\sqrt{\bar{\kappa}}\mathcal{A}^T P\mathcal{B}(s)+\sqrt{\bar{\kappa}}\mathcal{B}^T(s)P\mathcal{A}\\
\Psi_1^{22}\triangleq&\mathcal{H}^T P\mathcal{H}-\frac{1}{\bar{\ell}}Q,\quad\Psi_1^{33}\triangleq\mathcal{F}^T P\mathcal{F}\\
\Psi_1^{21}\triangleq&\mathcal{H}^T P\mathcal{A}+\sqrt{\bar{\kappa}}\mathcal{H}^T P\mathcal{B}(s)\\
\Psi_1^{31}\triangleq&\mathcal{F}^T P\mathcal{A}+\sqrt{\bar{\kappa}}\mathcal{F}^T P\mathcal{B}(s)\\
\Psi_1^{32}\triangleq&\mathcal{F}^T P\mathcal{H}.
\end{aligned}$$

Then, it follows from Lemma 1 that

$$\begin{aligned}
&\mathbb{E}\big\{\Re V\big(\psi(s)\big)\big\}\\
\leqslant&\mathbb{E}\big\{\Im_1^T(s)\Psi_1\Im_1(s)\big\}+\beta-\iota\mathbb{E}\big\{\Im^T\big(\zeta(s)\big)\Im\big(\zeta(s)\big)\\
&+\zeta^T(s)U^T\zeta(s)-\zeta^T(s)(U^T+I)\Im\big(\zeta(s)\big)\big\}\\
=&\mathbb{E}\big\{\Im_1^T(s)\Psi_1\Im_1(s)\big\}+\beta-\iota\mathbb{E}\big\{\Im^T\big(\zeta(s)\big)\Im\big(\zeta(s)\big)\big\}\\
&-\iota\mathbb{E}\Big\{\Big(\big(\mathcal{C}_1+\tilde{\vartheta}(s)\mathcal{C}_2\big)\psi(s)+\big(\bar{\vartheta}+\tilde{\vartheta}(s)\big)\delta(s)\Big)^T U^T\\
&+\Big(\big(\mathcal{C}_1+\tilde{\vartheta}(s)\mathcal{C}_2\big)\psi(s)+\big(\bar{\vartheta}+\tilde{\vartheta}(s)\big)\delta(s)\Big)\Big\}\\
&+\iota\mathbb{E}\Big\{\Big(\big(\mathcal{C}_1+\tilde{\vartheta}(s)\mathcal{C}_2\big)\psi(s)+\big(\bar{\vartheta}+\tilde{\vartheta}(s)\big)\delta(s)\Big)^T\\
&\times(U^T+I)\Im\big(\zeta(s)\big)\Big\}\\
=&\mathbb{E}\Big\{\Im_1^T(s)\Psi_1\Im_1(s)\Big\}+\beta-\iota\mathbb{E}\Big\{\Im^T\big(\zeta(s)\big)\Im\big(\zeta(s)\big)\Big\}\\
&-\iota\mathbb{E}\Big\{\psi^T(s)\big(\mathcal{C}_1^T U^T\mathcal{C}_1+(\bar{\vartheta}-\bar{\vartheta}^2)\mathcal{C}_2^T U^T\mathcal{C}_2\big)\psi(s)\\
&+\bar{\vartheta}\delta^T(s)U^T\delta(s)+2\bar{\vartheta}\delta^T(s)U^T\big(\mathcal{C}_1+\mathcal{C}_2\big)\psi(s)\\
&-2\bar{\vartheta}^2\delta^T(s)U^T\mathcal{C}_2\psi(s)\Big\}+\iota\mathbb{E}\Big\{\Im^T\big(\zeta(s)\big)(U^T+I)\\
&\times\mathcal{C}_1\psi(s)+\bar{\vartheta}\delta^T(s)(U^T+I)\Im\big(\zeta(s)\big)\Big\}\\
=&\mathbb{E}\big\{\Im_2^T(s)\Psi_2\Im_2(s)\big\}+\beta
\end{aligned}\tag{29}$$

where

$$\Im_2(s)\triangleq\begin{bmatrix}\psi(s)\\ \sum_{h=1}^{\hbar}\ell_h\psi(s-h)\\ \Im\big(\zeta(s)\big)\\ \delta(s)\end{bmatrix}$$

$$\Psi_2\triangleq\begin{bmatrix}\Psi_2^{11} & \star & \star & \star\\ \Psi_1^{21} & \Psi_1^{22} & \star & \star\\ \Psi_2^{31} & \Psi_1^{32} & \Psi_2^{33} & \star\\ \Omega_1^{41} & 0 & \Omega_1^{43} & \Psi_2^{44}\end{bmatrix}$$

$$\begin{aligned}
\Psi_2^{11}\triangleq&-P+\bar{\ell}Q+\mathcal{A}^T P\mathcal{A}+\bar{\kappa}\mathcal{B}^T(s)P\mathcal{B}(s)\\
&+\sqrt{\bar{\kappa}}\mathcal{A}^T P\mathcal{B}(s)+\sqrt{\bar{\kappa}}\mathcal{B}^T(s)P\mathcal{A}\\
&-\iota\mathcal{C}_1^T U^T\mathcal{C}_1-\iota(\bar{\vartheta}-\bar{\vartheta}^2)\mathcal{C}_2^T U^T\mathcal{C}_2\\
\Psi_2^{33}\triangleq&\mathcal{F}^T P\mathcal{F}-\iota I,\quad\Psi_2^{44}\triangleq-\iota\bar{\vartheta}U^T\\
\Psi_2^{31}\triangleq&\mathcal{F}^T P\mathcal{A}+\sqrt{\bar{\kappa}}\mathcal{F}^T P\mathcal{B}(s)+\frac{1}{2}\iota(U^T+I)\mathcal{C}_1.
\end{aligned}$$

By applying the Schur Complement Lemma, we derive from (20) that

$$\Psi_2=\Omega_1+\Omega_2^T P\Omega_2<0\tag{30}$$

which further implies

$$\mathbb{E}\big\{\Re V\big(\psi(s)\big)\big\}\leqslant-\gamma\mathbb{E}\big\{\|\Im_2(s)\|^2\big\}+\beta.\tag{31}$$

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Next, we shall proceed to estimate the upper bound of $\mathbb{E}\{\|\psi(s)\|^2\}$. Based on the definition of $V(\psi(s))$, one obtains

$$V(\psi(s)) \leqslant \lambda_{\max}(P)\|\psi(s)\|^2 + \bar{\ell}\lambda_{\max}(Q)\sum_{r=s-\hbar}^{s-1}\|\psi(r)\|^2. \tag{32}$$

Furthermore, for any $\tau > 1$, it follows from (31) that

$$\begin{aligned}
&\mathbb{E}\{\tau^{s+1}V(\psi(s+1))\} - \mathbb{E}\{\tau^s V(\psi(s))\}\\
=&\tau^{s+1}\mathbb{E}\{\Re V(\psi(s))\}+\tau^{s+1}\mathbb{E}\{V(\psi(s))\}\\
&-\tau^s\mathbb{E}\{V(\psi(s))\}\\
=&\tau^{s+1}\mathbb{E}\{\Re V(\psi(s))\}+\tau^s(\tau-1)\mathbb{E}\{V(\psi(s))\}\\
\leqslant&\tau^{s+1}\Big(-\gamma\mathbb{E}\{\|\Im_2(s)\|^2\}+\beta\Big)+\tau^s(\tau-1)\\
&\times\Big(\lambda_{\max}(P)\|\psi(s)\|^2+\bar{\ell}\lambda_{\max}(Q)\sum_{r=s-\hbar}^{s-1}\|\psi(r)\|^2\Big)\\
\leqslant&\tau^{s+1}\Big(-\gamma\mathbb{E}\{\|\psi(s)\|^2\}+\beta\Big)+\tau^s(\tau-1)\\
&\times\Big(\lambda_{\max}(P)\|\psi(s)\|^2+\bar{\ell}\lambda_{\max}(Q)\sum_{r=s-\hbar}^{s-1}\|\psi(r)\|^2\Big)\\
\leqslant&\varrho_1(\tau)\tau^s\mathbb{E}\{\|\psi(s)\|^2\}+\varrho_2(\tau)\sum_{r=s-\hbar}^{s-1}\tau^s\mathbb{E}\{\|\psi(r)\|^2\}\\
&+\tau^{s+1}\beta
\end{aligned} \tag{33}$$

where

$$\begin{aligned}
\varrho_1(\tau) &\triangleq -\tau\gamma+(\tau-1)\lambda_{\max}(P)\\
\varrho_2(\tau) &\triangleq (\tau-1)\bar{\ell}\lambda_{\max}(Q).
\end{aligned}$$

For arbitrary positive integer $\rho \geqslant \hbar$, summarizing both sides of (33) from 0 to $\rho-1$ associated with s results in

$$\begin{aligned}
&\mathbb{E}\{\tau^\rho V(\psi(\rho))\}-\mathbb{E}\{V(\psi(0))\}\\
\leqslant&\varrho_1(\tau)\sum_{s=0}^{\rho-1}\tau^s\mathbb{E}\{\|\psi(s)\|^2\}+\frac{\tau(1-\tau^\rho)}{1-\tau}\beta\\
&+\varrho_2(\tau)\sum_{s=0}^{\rho-1}\sum_{r=s-\hbar}^{s-1}\tau^s\mathbb{E}\{\|\psi(r)\|^2\}.
\end{aligned} \tag{34}$$

Besides, the last item in (34) is calculated as

$$\begin{aligned}
&\sum_{s=0}^{\rho-1}\sum_{r=s-\hbar}^{s-1}\tau^s\mathbb{E}\{\|\psi(r)\|^2\}\\
\leqslant&\left(\sum_{r=-\hbar}^{-1}\sum_{s=0}^{r+\hbar}+\sum_{r=0}^{\rho-\hbar-1}\sum_{s=r+1}^{r+\hbar}+\sum_{r=\rho-\hbar}^{\rho-1}\sum_{s=r+1}^{\rho-1}\right)\tau^s\mathbb{E}\{\|\psi(r)\|^2\}\\
\leqslant&\frac{\tau^\hbar-1}{\tau-1}\sum_{r=-\hbar}^{-1}\mathbb{E}\{\|\psi(r)\|^2\}+\frac{\tau(\tau^\hbar-1)}{\tau-1}\sum_{r=0}^{\rho-1}\tau^r\mathbb{E}\{\|\psi(r)\|^2\}\\
&+\frac{\tau(\tau^\hbar-1)}{\tau-1}\sum_{r=0}^{\rho-1}\tau^r\mathbb{E}\{\|\psi(r)\|^2\}.
\end{aligned} \tag{35}$$

From (34) and (35), one has immediately that

$$\begin{aligned}
&\mathbb{E}\{\tau^\rho V(\psi(\rho))\}-\mathbb{E}\{V(\psi(0))\}\\
\leqslant&\varrho_1(\tau)\sum_{s=0}^{\rho-1}\tau^s\mathbb{E}\{\|\psi(s)\|^2\}+\frac{\tau(1-\tau^\rho)}{1-\tau}\beta\\
&+\varrho_2(\tau)\Bigg(\frac{\tau^\hbar-1}{\tau-1}\sum_{r=-\hbar}^{-1}\mathbb{E}\{\|\psi(r)\|^2\}\\
&+\frac{\tau(\tau^\hbar-1)}{\tau-1}\sum_{r=0}^{\rho-1}\tau^r\mathbb{E}\{\|\psi(r)\|^2\}\\
&+\frac{\tau(\tau^\hbar-1)}{\tau-1}\sum_{r=0}^{\rho-1}\tau^r\mathbb{E}\{\|\psi(r)\|^2\}\Bigg)\\
\leqslant&\varrho_3(\tau)\sum_{s=0}^{\rho-1}\tau^s\mathbb{E}\{\|\psi(s)\|^2\}+\frac{\tau(1-\tau^\rho)}{1-\tau}\beta\\
&+\varrho_4(\tau)\sup_{\jmath\in\mathfrak{H}}\mathbb{E}\{\|\varphi(\jmath)\|^2\}.
\end{aligned} \tag{36}$$

where

$$\begin{aligned}
\varrho_3(\tau) &\triangleq \varrho_1(\tau)+\varrho_2(\tau)\frac{2\tau^{\hbar+1}-2\tau}{\tau-1}\\
\varrho_4(\tau) &\triangleq \varrho_2(\tau)\hbar\frac{\tau^\hbar-1}{\tau-1}.
\end{aligned}$$

Noting that $\varrho_3(1) = -\gamma < 0$ and $\lim_{\tau\to\infty}\varrho_3(\tau) = +\infty$, it is readily seen that there exists a scalar $\varepsilon > 1$ such that $\varrho_3(\varepsilon) = 0$, which indicates that

$$\begin{aligned}
&\mathbb{E}\{\varepsilon^\rho V(\psi(\rho))\}-\mathbb{E}\{V(\psi(0))\}\\
\leqslant&\frac{\varepsilon(1-\varepsilon^\rho)}{1-\varepsilon}\beta+\varrho_4(\varepsilon)\sup_{\jmath\in\mathfrak{H}}\mathbb{E}\{\|\varphi(\jmath)\|^2\}.
\end{aligned} \tag{37}$$

It can be observed from (23) that

$$\mathbb{E}\{V(\psi(0))\} \leqslant (\hbar+1)\bar{\lambda}\sup_{\jmath\in\mathfrak{H}}\mathbb{E}\{\|\varphi(\jmath)\|^2\} \tag{38}$$

and

$$\mathbb{E}\{\varepsilon^\rho V(\psi(\rho))\} \geqslant \lambda_{\min}(P)\varepsilon^\rho\mathbb{E}\{\|\psi(\rho)\|^2\}. \tag{39}$$

Furthermore, one has

$$\begin{aligned}
\mathbb{E}\{\|\psi(\rho)\|^2\} \leqslant&\frac{\varrho_4(\varepsilon)+(\hbar+1)\bar{\lambda}}{\lambda_{\min}(P)\varepsilon^\rho}\sup_{\jmath\in\mathfrak{H}}\mathbb{E}\{\|\varphi(\jmath)\|^2\}\\
&+\frac{1-\varepsilon^\rho}{\lambda_{\min}(P)\varepsilon^{\rho-1}(1-\varepsilon)}\beta\\
=&\epsilon^\rho\alpha\sup_{\jmath\in\mathfrak{H}}\mathbb{E}\{\|\varphi(\jmath)\|^2\}+\nu(\rho)
\end{aligned} \tag{40}$$

with

$$\begin{aligned}
&\epsilon \triangleq \frac{1}{\varepsilon}, \quad \alpha \triangleq \frac{\varrho_4(\varepsilon)+(\hbar+1)\bar{\lambda}}{\lambda_{\min}(P)}\\
&\nu(\rho) \triangleq \frac{1-\varepsilon^\rho}{\lambda_{\min}(P)\varepsilon^{\rho-1}(1-\varepsilon)}\beta.
\end{aligned}$$

According to Definition 1, the ultimate upper bound of the estimation error in the mean-square sense can be expressed by:

$$\bar{\nu} = \lim_{\rho\to+\infty}\nu(\rho) = \frac{\varepsilon}{\lambda_{\min}(P)(\varepsilon-1)}\beta. \tag{41}$$

To proceed, we derive from (21), (40) and (41) that

$$\mathbb{E}\{\|\psi(\rho)\|^2\} \leqslant \frac{\varrho_4(\varepsilon)+(\hbar+1)\bar{\lambda}}{\lambda_{\min}(P)\varepsilon^\rho}\sup_{\jmath\in\mathfrak{H}}\mathbb{E}\{\|\varphi(\jmath)\|^2\}$$

$$+\frac{\varepsilon}{\lambda_{\min}(P)(\varepsilon-1)}\beta\leqslant\varsigma. \tag{42}$$

Obviously, it is not difficult to see from Definition 2 that the augmented system (11) is ς-secure in mean-square sense, which ends the proof. ■

B. Outlier-Resistant PIO Design

In this subsection, we are devoted to solving the design problem of the outlier-resistant PIO.

Theorem 2: If there exist positive definite matrices $\acute{P}_1$, $\acute{P}_2$, $\acute{P}_3$, $\acute{Q}_1$, $\acute{Q}_2$ and $\acute{Q}_3$, matrices $\acute{F}_P$, $\acute{F}_I$ and $\acute{F}$, and positive scalar ι and π satisfying

$$\Lambda=\begin{bmatrix}\Omega_1 & \star\\ \Lambda_2 & \Lambda_3\end{bmatrix}<0 \tag{43}$$

and

$$\begin{aligned}&\frac{\bar{\ell}\lambda_{\max}(Q)\hbar(\varepsilon^{\hbar}-1)+(\hbar+1)\bar{\lambda}}{\lambda_{\min}(P)\varepsilon^{\rho}}\sup_{\jmath\in\mathfrak{H}}\mathbb{E}\{\|\varphi(\jmath)\|^2\}\\&+\frac{\varepsilon}{\lambda_{\min}(P)(\varepsilon-1)}\beta\leqslant\varsigma.\end{aligned} \tag{44}$$

where

$$\begin{aligned}\Lambda_2&\triangleq\begin{bmatrix}\acute{\mathcal{A}} & \acute{\mathcal{H}} & \acute{\mathcal{F}} & 0\\ 0 & 0 & 0 & 0\\ \mathcal{T} & 0 & 0 & 0\end{bmatrix}\\
\Lambda_3&\triangleq\begin{bmatrix}-P & \star & \star\\ \bar{\kappa}\mathcal{R}^T & -\pi I & \star\\ 0 & 0 & -\pi I\end{bmatrix}\\
\acute{\mathcal{A}}&\triangleq\begin{bmatrix}\acute{P}_1A & 0 & 0\\ 0 & \acute{P}_2A & -\acute{F}_I\\ 0 & 0 & \acute{P}_3\end{bmatrix}\\
\acute{\mathcal{H}}&\triangleq\begin{bmatrix}\acute{P}_1H & 0 & \\ 0 & \acute{P}_2H & 0\\ 0 & 0 & 0\end{bmatrix}\\
\acute{\mathcal{F}}&\triangleq\begin{bmatrix}0\\ -\acute{F}_P\\ \acute{F}\end{bmatrix},\quad \mathcal{R}\triangleq\begin{bmatrix}R\\ R\\ 0\end{bmatrix}\\
\mathcal{T}&\triangleq\begin{bmatrix}T & 0 & 0\end{bmatrix}\\
P&\triangleq\mathrm{diag}\{\acute{P}_1,\ \acute{P}_2,\ \acute{P}_3\}\\
Q&\triangleq\mathrm{diag}\{\acute{Q}_1,\ \acute{Q}_2,\ \acute{Q}_3\}\end{aligned}$$

and the constant $\varepsilon>1$ in (44) satisfies

$$\lambda_{\max}(P)(\varepsilon-1)-\gamma\varepsilon+2\bar{\ell}\lambda_{\max}(Q)\varepsilon(\varepsilon^{\hbar}-1)=0 \tag{45}$$

with

$$\gamma\triangleq\lambda_{\min}\big(-\Omega_1-\Omega_2^TP\Omega_2\big),$$

then the augmented system (11) is ς-secure in mean-square sense. Furthermore, the gain matrices of the outlier-resistant PIO (7) are calculated by

$$F_P=\acute{P}_1^{-1}\acute{F}_P,\quad F_I=\acute{P}_2^{-1}\acute{F}_I,\quad F=\acute{P}_3^{-1}\acute{F}. \tag{46}$$

Proof: First, we shall deal with the parameter uncertainties by rewriting (20) in the form of (19). It follows from the notations in (11) that

$$\mathcal{B}(s)=\mathcal{R}S(s)\mathcal{T}, \tag{47}$$

and therefore (20) can be rewritten as follows:

$$\Omega=\Theta_1+\Theta_RS(s)\Theta_T+\Theta_T^TS^T(s)\Theta_R^T \tag{48}$$

where

$$\begin{aligned}&\Theta_1\triangleq\begin{bmatrix}\Omega_1 & \star\\ \Theta_2 & \Omega_3\end{bmatrix},\quad \Theta_2\triangleq\begin{bmatrix}\mathcal{A} & \mathcal{H} & \mathcal{F} & 0\end{bmatrix}\\
&\Theta_R\triangleq\begin{bmatrix}0 & 0 & 0 & 0 & \bar{\kappa}\mathcal{R}^T\end{bmatrix}^T,\quad \Theta_T\triangleq\begin{bmatrix}\mathcal{T} & 0 & 0 & 0 & 0\end{bmatrix}.\end{aligned}$$

Then, by using Lemma 3, it is not difficult to see that (48) holds if and only if there exists a positive scalar π such that the following inequality holds:

$$\begin{bmatrix}\Theta_1 & \pi\Theta_R & \Theta_T^T\\ \pi\Theta_R^T & -\pi I & 0\\ \Theta_T & 0 & -\pi I\end{bmatrix}<0. \tag{49}$$

Pre-multiplying and post-multiplying the inequality (49) by $\mathrm{diag}\{I,\ I,\ I,\ I,\ P,\ I,\ I\}$ and its transpose, and utilizing the variable substitution

$$\acute{F}_P=\acute{P}_1F_P,\quad \acute{F}_I=\acute{P}_2F_I,\quad \acute{F}=\acute{P}_3F, \tag{50}$$

we conclude that (49) can be ensured by (43).

According to Theorem 1, one can conclude that, with the outlier-resistant PIO gain matrices F_P, F_P and F given in (46), the augmented system (11) is ς-secure in mean-square sense. The proof is complete. ■

C. Outlier-Resistant Luenberger Observer Design

In this subsection, we shall design an outlier-resistant Luenberger observer for system (1). To begin with, the outlier-resistant Luenberger observer is constructed as follows:

$$\begin{cases}\hat{x}(s+1)=A\hat{x}(s)+H\sum_{h=1}^{\hbar}\ell_h\hat{x}(s-h)\\ \qquad\qquad +L\Im\big(\upsilon(s)-C\hat{x}(s)\big)\\ \hat{x}(\jmath)=0,\quad\forall\jmath\in\mathfrak{H}\end{cases} \tag{51}$$

where L is the observer gain matrix to be designed.

Accordingly, from (1) and (51), the estimation error dynamics can be written as follows:

$$\begin{cases}\tilde{x}(s+1)=A\tilde{x}(s)+H\sum_{h=1}^{\hbar}\ell_h\tilde{x}(s-h)+Mw(s)\\ \qquad\qquad +\kappa(s)B(s)x(s)-L\Im\big(\upsilon(s)-C\hat{x}(s)\big)\\ \tilde{x}(\jmath)=\phi(\jmath),\quad\forall\jmath\in\mathfrak{H}\end{cases} \tag{52}$$

Letting $\breve{\psi}(s)\triangleq\begin{bmatrix}x^T(s) & \tilde{x}^T(s)\end{bmatrix}^T$, the augmented system is characterized as follows:

$$\begin{cases}\breve{\psi}(s+1)=\big(\breve{\mathcal{A}}+\bar{\kappa}\breve{\mathcal{B}}(s)+\tilde{\kappa}(s)\breve{\mathcal{B}}(s)\big)\breve{\psi}(s)\\ \qquad\qquad +\breve{\mathcal{H}}\sum_{h=1}^{\hbar}\ell_h\breve{\psi}(s-h)+\breve{\mathcal{M}}w(s)\\ \qquad\qquad +\breve{\mathcal{F}}\Im\big(\breve{\zeta}(s)\big)\\ \breve{\psi}(\jmath)=\breve{\varphi}(\jmath),\quad\forall\jmath\in\mathfrak{H}\end{cases} \tag{53}$$

where

$$\breve{\mathcal{A}}\triangleq\begin{bmatrix}A & 0\\ 0 & A\end{bmatrix},\quad \breve{\mathcal{H}}\triangleq\begin{bmatrix}H & 0\\ 0 & H\end{bmatrix}$$

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$$\breve{\mathcal{B}}(s) \triangleq \begin{bmatrix} B(s) & 0 \\ B(s) & 0 \end{bmatrix}, \quad \breve{\mathcal{M}} \triangleq \begin{bmatrix} M \\ M \end{bmatrix}$$
$$\breve{\mathcal{F}} \triangleq \begin{bmatrix} 0 \\ -L \end{bmatrix}, \quad \breve{\mathcal{C}}_1 \triangleq \begin{bmatrix} -\bar{\vartheta}C & C \end{bmatrix}$$
$$\breve{\mathcal{C}}_2 \triangleq \begin{bmatrix} -C & 0 \end{bmatrix}, \quad \breve{\varphi}(\jmath) \triangleq \begin{bmatrix} \phi^T(\jmath) & \phi^T(\jmath) \end{bmatrix}^T$$
$$\breve{\zeta}(s) \triangleq \big(\mathcal{C}_1 + \tilde{\vartheta}(s)\mathcal{C}_2\big)\breve{\psi}(s) + \big(\bar{\vartheta} + \tilde{\vartheta}(s)\big)\delta(s).$$

In the following corollary, sufficient conditions are provided to a) ensure that the augmented system (53) is ς-secure in mean-square sense and b) give an explicit form of the gain matrix of outlier-resistant Luenberger observer (51) by means of LMI technique.

Corollary 1: If there exist positive definite matrices $\grave{P}_1$, $\grave{P}_2$, $\grave{Q}_1$ and $\grave{Q}_2$, matrix $\grave{L}$, and positive scalar $\breve{\iota}$ and $\breve{\pi}$ satisfying

$$\Xi = \begin{bmatrix} \Xi_1 & \star \\ \Xi_2 & \Xi_3 \end{bmatrix} < 0 \tag{54}$$

and

$$\frac{\bar{\ell}\lambda_{\max}(\breve{Q})\hbar(\breve{\varepsilon}^{\hbar}-1)+(\hbar+1)\breve{\lambda}}{\lambda_{\min}(\breve{P})\varepsilon^{\rho}}\sup_{\jmath\in\mathfrak{H}}\mathbb{E}\{\|\breve{\varphi}(\jmath)\|^2\} + \frac{\breve{\varepsilon}}{\lambda_{\min}(\breve{P})(\breve{\varepsilon}-1)}\breve{\beta} \leqslant \varsigma. \tag{55}$$

where

$$\Xi_1 \triangleq \begin{bmatrix} \Xi_1^{11} & \star & \star & \star \\ 0 & \Xi_1^{22} & \star & \star \\ \Xi_1^{31} & 0 & \Xi_1^{33} & \star \\ \Xi_1^{41} & 0 & \Xi_1^{43} & \Xi_1^{44} \end{bmatrix}, \ \Xi_2 \triangleq \begin{bmatrix} \grave{\mathcal{A}} & \grave{\mathcal{H}} & \grave{\mathcal{F}} & 0 \\ 0 & 0 & 0 & 0 \\ \breve{\mathcal{T}} & 0 & 0 & 0 \end{bmatrix}$$
$$\Xi_3 \triangleq \begin{bmatrix} -\breve{P} & \star & \star \\ \bar{\kappa}\breve{\mathcal{R}}^T & -\breve{\pi}I & \star \\ 0 & 0 & -\breve{\pi}I \end{bmatrix}, \quad \breve{\beta} \triangleq \lambda_{\max}(\breve{\mathcal{M}}^T\breve{P}\breve{\mathcal{M}})\vartheta_w^2$$
$$\grave{\mathcal{A}} \triangleq \begin{bmatrix} \grave{P}_1 A & 0 \\ 0 & \grave{P}_2 A \end{bmatrix}, \quad \breve{P} \triangleq \text{diag}\{\grave{P}_1,\ \grave{P}_2\}$$
$$\grave{\mathcal{H}} \triangleq \begin{bmatrix} \grave{P}_1 H & 0 \\ 0 & \grave{P}_2 H \end{bmatrix}, \quad \breve{Q} \triangleq \text{diag}\{\grave{Q}_1,\ \grave{Q}_2\}$$
$$\grave{\mathcal{F}} \triangleq \begin{bmatrix} 0 \\ -\grave{L} \end{bmatrix}, \quad \breve{\mathcal{R}} \triangleq \begin{bmatrix} R \\ R \end{bmatrix}, \quad \breve{\mathcal{T}} \triangleq \begin{bmatrix} T & 0 \end{bmatrix}$$
$$\Xi_1^{11} \triangleq -\breve{P} + \bar{\ell}\breve{Q} - \breve{\iota}\breve{\mathcal{C}}_1^T U^T \breve{\mathcal{C}}_1 - \breve{\iota}(\bar{\vartheta} - \bar{\vartheta}^2)\breve{\mathcal{C}}_2^T U^T \breve{\mathcal{C}}_2$$
$$\Xi_1^{22} \triangleq -\frac{1}{\bar{\ell}}\breve{Q}, \quad \Xi_1^{33} \triangleq -\breve{\iota}I, \quad \Xi_1^{44} \triangleq -\breve{\iota}\bar{\vartheta}U^T$$
$$\Xi_1^{31} \triangleq \frac{1}{2}\breve{\iota}(U^T + I)\breve{\mathcal{C}}_1, \quad \Xi_1^{41} \triangleq -\breve{\iota}\bar{\vartheta}U^T\big(\breve{\mathcal{C}}_1 + \breve{\mathcal{C}}_2 - \bar{\vartheta}\breve{\mathcal{C}}_2\big)$$
$$\Xi_1^{43} \triangleq \frac{1}{2}\breve{\iota}\bar{\vartheta}(U^T + I), \quad \breve{\lambda} \triangleq \max\{\lambda_{\max}(\breve{P}),\ \bar{\ell}\lambda_{\max}(\breve{Q})\}$$

and the constant $\breve{\varepsilon} > 1$ in (55) satisfies

$$\lambda_{\max}(\breve{P})(\breve{\varepsilon}-1) - \breve{\gamma}\breve{\varepsilon} + 2\bar{\ell}\lambda_{\max}(\breve{Q})\breve{\varepsilon}(\breve{\varepsilon}^{\hbar}-1) = 0 \tag{56}$$

with

$$\breve{\gamma} \triangleq \lambda_{\min}\big(-\Xi_1 - \Xi_3^T\breve{P}\Xi_3\big)$$
$$\Xi_3 \triangleq \begin{bmatrix} \breve{\mathcal{A}} + \bar{\kappa}\breve{\mathcal{B}}(s) & \breve{\mathcal{H}} & \breve{\mathcal{F}} & 0 \end{bmatrix},$$

then the augmented system (53) is ς-secure in mean-square sense. Furthermore, the gain matrix of the outlier-resistant Luenberger observer (51) is calculated by

$$L = \grave{P}_2^{-1}\grave{L}. \tag{57}$$

Proof: The proof of this corollary is easily obtained only by setting $F_I = 0$ and $F = 0$ in Theorems 1-2, and is therefore omitted here. ∎

Remark 4: Up to now, we have thoroughly investigated the outlier-resistant PIO design issue for a kind of discrete-time delayed system with ROPUs subject to RODAs. With the designed outlier-resistant PIO, the EMS boundedness of the estimation errors has been guaranteed and the security requirement has been met. In addition, the feasibility of the PIO design problem has been transformed into the solvability of an LMI. In comparison with the existing literature, our main results exhibit the following distinctive novelties: 1) the PIO design problem is, for the first time, addressed for discrete time-delayed systems with RODAs; 2) the outlier-resistant PIO design scheme is developed to attenuate the effect of malicious attacks; and 3) a criterion is derived to reveal the influences of the RODAs and the stochastic noise on the estimation performance. Our main results can be extended to more complicated NSs with more comprehensive network-induced phenomena or cyber-attacks [18], [43].

IV. Numerical Simulation

In this section, an illustrative example is provided to show the effectiveness and superiority of the proposed outlier-resistant PIO design scheme.

Consider a linear discrete time-delayed system described by (1) with the following parameters:

$$A = \begin{bmatrix} 0.46 & 0.92 \\ -0.44 & 0.61 \end{bmatrix}, \quad H = \begin{bmatrix} 0.11 & 0.14 \\ 0.15 & -0.13 \end{bmatrix}$$
$$C = \begin{bmatrix} -0.72 & 0.61 \end{bmatrix}, \quad D = \begin{bmatrix} 1.1 & 0.9 \end{bmatrix}, \quad U = 0.35$$
$$M = \begin{bmatrix} -0.06 \\ 0.11 \end{bmatrix}, \ R = \begin{bmatrix} 0.6 \\ 0.5 \end{bmatrix}, \ T = \begin{bmatrix} 0.2 \\ 0.3 \end{bmatrix}^T, \ \bar{\kappa} = 0.3$$
$$\hbar = 3, \quad \ell_h = 2^{-(h+1)}, \quad S(s) = 0.8\sin(s).$$

A. Effectiveness and Superiority of the Proposed Outlier-Resistant PIO Design Approach

In the simulation, the deception signal sent by adversary is denoted by $\delta(s) = 5\cos(s) - \tanh(s)$ and the probability of launching a successful deception attack is given as $\bar{\vartheta} = 0.4$. In addition, the security level is selected as $\varsigma = 0.2$ and the initial condition is taken as $x(0) = \begin{bmatrix} 0.5 & -0.5 \end{bmatrix}^T$.

The solutions to LMI (43) in Theorem 2 are obtained as follows:

$$\acute{P}_1 = \begin{bmatrix} 9.5817 & -1.6386 \\ -1.6386 & 16.8731 \end{bmatrix}, \ \acute{Q}_1 = \begin{bmatrix} 1.6118 & -0.0089 \\ -0.0089 & 1.6029 \end{bmatrix}$$
$$\acute{P}_2 = \begin{bmatrix} 9.3057 & -1.5956 \\ -1.5956 & 20.3154 \end{bmatrix}, \ \acute{Q}_2 = \begin{bmatrix} 1.6421 & -0.0241 \\ -0.0241 & 1.5996 \end{bmatrix}$$
$$\acute{P}_3 = 8.7174, \ \acute{Q}_3 = 2.2466, \ \acute{F} = -0.0090, \ \pi = 1.2797$$
$$\acute{F}_P = \begin{bmatrix} -0.6424 \\ 3.5330 \end{bmatrix}, \quad \acute{F}_I = \begin{bmatrix} -0.3534 \\ 0.5101 \end{bmatrix}, \quad \iota = 1.3848.$$

Accordingly, the outlier-resistant PIO gains can be computed as follows:

$$F_P = \begin{bmatrix} -0.0318 \\ 0.2063 \end{bmatrix}, \quad F_I = \begin{bmatrix} 0.0429 \\ 0.0285 \end{bmatrix}, \quad \acute{F} = -0.0010.$$

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Furthermore, according to Corollary 1, the solutions of LMI (54) and the gain of outlier-resistant Luenberger observer (51) are listed as follows:

$$\grave{P}_1 = \begin{bmatrix} 4.4852 & -0.6864 \\ -0.6864 & 7.1213 \end{bmatrix}, \ \grave{Q}_1 = \begin{bmatrix} 0.6057 & -0.0033 \\ -0.0033 & 0.6071 \end{bmatrix}$$

$$\grave{P}_2 = \begin{bmatrix} 4.3151 & -0.5025 \\ -0.5025 & 7.2158 \end{bmatrix}, \ \grave{Q}_2 = \begin{bmatrix} 0.6057 & -0.0034 \\ -0.0034 & 0.6063 \end{bmatrix}$$

$$\grave{L} = \begin{bmatrix} -0.0275 \\ 2.3519 \end{bmatrix}, \ L = \begin{bmatrix} 0.0318 \\ 0.3282 \end{bmatrix}, \ \breve{\pi} = 7.5951, \ \breve{\iota} = 6.9122.$$

For manifesting the superiority of the outlier-resistant PIO, we make comparisons for evaluating the estimation performance under different observers as follows: 1) estimating with the outlier-resistant PIO; 2) estimating with the outlier-resistant Luenberger observer; 3) estimating with the conventional PIO (which corresponds to $l^M = \infty$).

The estimation results are shown in Figs. 1-2, which depict the system states and their estimates with different observers. From Figs. 1-2, we observe that the outlier-resistant Luenberger observer and the conventional PIO cannot achieve the desired estimation performance whereas the outlier-resistant PIO can. Figs. 3-5 plot the trajectories of estimation error $\tilde{x}(s)$ with the outlier-resistant PIO, the outlier-resistant Luenberger observer, and the conventional PIO, respectively.

From the above simulation results, a conclusion can be drawn that the outlier-resistant PIO is able to mitigate the negative effects of the RODAs and achieve a satisfactory estimation performance. Consequently, the design scheme of the developed outlier-resistant PIO is indeed efficient and performs better than the outlier-resistant Luenberger observer and the conventional PIO.

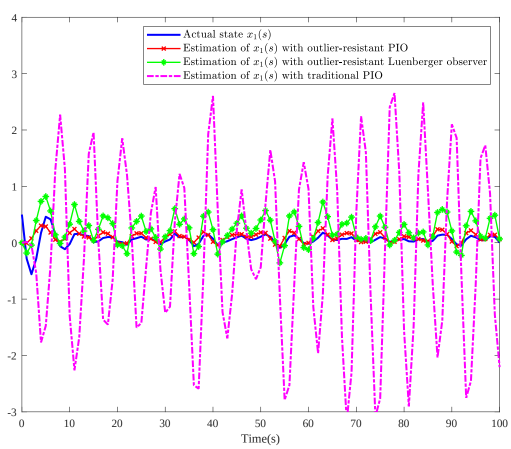

Fig. 1: Trajectories of state $x_1(s)$ and its estimate.

B. Comparisons With Different Attack Probabilities

For the sake of truly revealing the impact from the RODAs on our estimation algorithm, the simulations are repeated 100 times and the comparisons with different attack probabilities are made in this subsection. To facilitate discussion, the

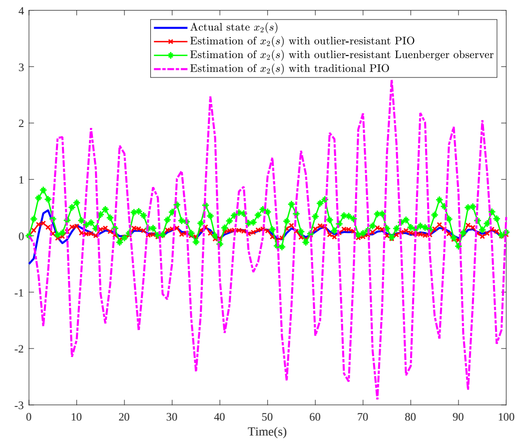

Fig. 2: Trajectories of state $x_2(s)$ and its estimate.

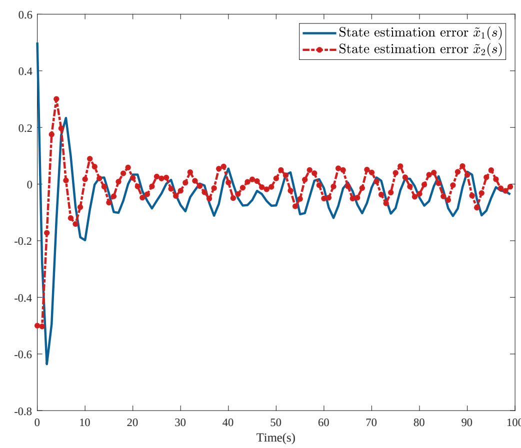

Fig. 3: Trajectories of estimation error $\tilde{x}(s)$ with outlier-resistant PIO.

average mean-square estimation error (AMSEE) with respect to the output is defined by:

$$\mathcal{Z} = \frac{1}{p}\sum_{s=1}^{p}\frac{1}{q}\sum_{\imath=1}^{q}\|z^{\imath}(s) - \hat{z}^{\imath}(s)\|^2$$

where p denotes the number of time instants and q stands for the number of simulation trials.

Fig. 6 depicts the trajectories of measurement output $y(s)$, observer input $\upsilon(s)$, and the time spots when the system suffers from deception attacks with $\bar{\vartheta} = 0.4$. Moreover, the relation between the attack probability $\bar{\vartheta}$ and the AMSEE of the estimated output $\mathcal{Z}$ is given in Table I, which can be observed that the AMSEE of the output increases when the attack probability increases. Therefore, we can naturally draw a conclusion that, with the increase of the attack probability, the estimation performance deteriorates.

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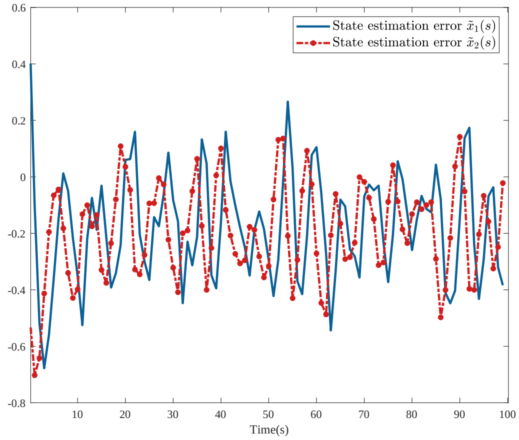

Fig. 4: Trajectories of estimation error $\tilde{x}(s)$ with outlier-resistant Luenberger observer.

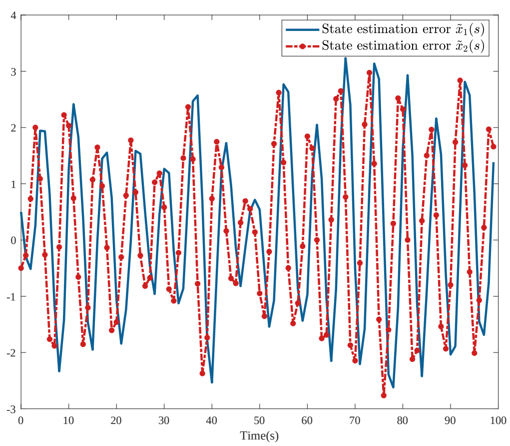

Fig. 5: Trajectories of estimation error $\tilde{x}(s)$ with conventional PIO.

V. Conclusions

In this paper, the PIO design issue has been addressed for a kind of discrete-time delayed system with ROPUs subject to RODAs. A Bernoulli-distributed random variable has been utilized to regulate the random nature of deception attacks initiated by the adversaries. For the purpose of attenuating the impact of the malicious attacks on the estimation performance, an outlier-resistant PIO has been constructed, in which a saturation constraint has been imposed on the innovations. Sufficient conditions have been derived to guarantee the EMS boundedness and achieve the prescribed security level. The explicit forms of the desired PIO parameters have been described in terms of the solutions to an LMI. Finally, the effectiveness and superiority have been validated via an illustrative simulation example.

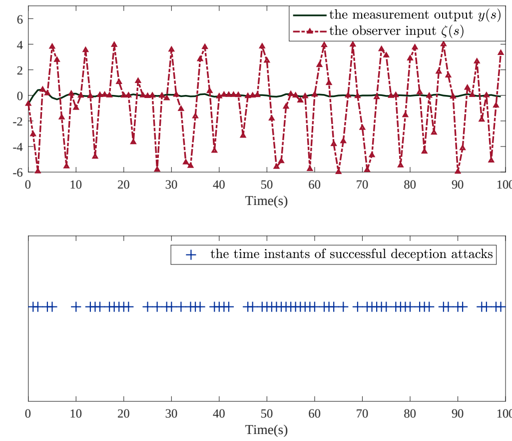

Fig. 6: The trajectories of measurement output $y(s)$ and observer input $v(s)$, and the time instants of successful deception attacks with $\bar{\vartheta} = 0.4$.

TABLE I: AMSEE IN 100 EXPERIMENTS WITH DIFFERENT $\bar{\vartheta}$

$\bar{\vartheta}$	$\mathcal{Z}$
$\bar{\vartheta} = 0.1$	0.0191
$\bar{\vartheta} = 0.2$	0.0205
$\bar{\vartheta} = 0.3$	0.0229
$\bar{\vartheta} = 0.4$	0.0233
$\bar{\vartheta} = 0.5$	0.0255
$\bar{\vartheta} = 0.6$	0.0261
$\bar{\vartheta} = 0.7$	0.0286
$\bar{\vartheta} = 0.8$	0.0326
$\bar{\vartheta} = 0.9$	0.0372

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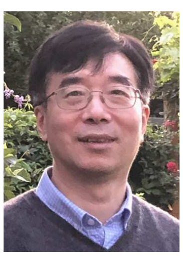

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