Sampled-Data Fuzzy Controller for Time-Delay Nonlinear Systems: Fuzzy-Model-Based LMI Approach

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***Abstract*—This paper presents the stability analysis and performance design for a sampled-data fuzzy control system with time delay, which is formed by a nonlinear plant with time delay and a sampled-data fuzzy controller connected in a closed loop. As the sampled-data fuzzy controller can be implemented by a microcontroller or a digital computer, the implementation time and cost can be reduced. However, the sampling activity and time delay, which are potential causes of system instability, will complicate the system dynamics and make the stability analysis much more difficult than that for a pure continuous-time fuzzy control system. In this paper, a sampled-data fuzzy controller with enhanced nonlinearity compensation ability is proposed. Based on the fuzzy-model-based control approach, linear matrix inequality (LMI)-based stability conditions are derived to guarantee the system stability. By using a descriptor representation, the complexity of the sampled-data fuzzy control system with time delay can be reduced to ease the stability analysis, which effectively leads to a smaller number of LMI-stability conditions. Information of the membership functions of both the fuzzy plant model and fuzzy controller are considered, which allows arbitrary matrices to be introduced, to ease the satisfaction of the stability conditions. An application example will be given to show the merits and design procedure of the proposed approach. Furthermore, LMI-based performance conditions are derived to aid the design of a well-performed sampled-data fuzzy controller.**

***Index Terms*—Fuzzy control, performance, sampled-data control, stability, time delay.**

I. Introduction

The FUZZY control approach offers a systematic way to deal with nonlinear systems. During the past two decades, various fuzzy control approaches have been proposed and fruitful results have been achieved, particularly in the issue of system stability. To study the stability of fuzzy control systems, the Takagi–Sugeno (T–S) fuzzy-model-based approach is most commonly adopted. By employing a T–S fuzzy model [1], [2], a nonlinear system can be represented as an averaged sum of some weighted linear subsystems. The T–S fuzzy model effectively extracts the linear and nonlinear elements of the nonlinear system. The semilinear characteristic of the T–S fuzzy model allows some linear control approaches or theories to be further developed to facilitate the stability analysis and controller synthesis. In [3] and [4], basic stability conditions were derived to guarantee the system stability. In [4], the parallel distribution compensation (PDC) technique was proposed to design the feedback gains of the fuzzy controller. Based on the PDC design, some relaxed stability results were obtained [5]–[10]. In [5]–[10], linear matrix inequality (LMI)-based controller design techniques were reported. The feedback gains of the fuzzy controller are expressed as some decision variables of LMI-stability conditions [5]–[10] which can be solved numerically and effectively using some convex programming techniques [11].

In [3]–[10], LMI-based stability conditions for a continuous-time fuzzy control system were reported. However, the time delay, which appears in many real-life engineering processes, was not considered. In other words, when a time delay is present in the system, the theories reported in [3]–[10] cannot be applied. More importantly, a time delay can be an element to cause system instability. It is important to develop theories to handle control problems with time delay and successfully put the fuzzy controller into practice.

Recently, the stability of time-delay nonlinear systems has been studied based on the fuzzy-model-based approach. A modified T–S fuzzy model [12]–[19] was proposed to represent the nonlinear plant with time delay. To handle nonlinear systems with time delay using the fuzzy control technique, two approaches can be found in the literature, namely the delay-independent [12]–[14] and delay-dependent approaches [15]–[19]. Delay-independent stability conditions for time-delay fuzzy control systems were derived in [12]–[14] based on the Lyapunov–Krasovksii or Lyapunov–Razumikhin approaches. For the delay-independent approach, the stability conditions are not related to the time-delay information. Once the time-delay fuzzy control system is guaranteed to be stable, it is stable for any value of time delay. As the information of time delay is not considered during the stability analysis, only conservative stability analysis results are usually obtained. In [15]–[19], delay-dependent stability conditions were derived based on the Lyapunov–Krasovksii approach. During the stability analysis, the time-delay information is considered, which leads to a complicated analysis procedure compared with that of the delay-independent approach. However, less

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conservative stability conditions are produced. In the delay-dependent stability conditions, the time-delay information is one of the elements to determine the system stability.

Owing to the rapid growth of the digital circuit technologies, powerful microcontrollers and digital computers can be made available at low cost. Hence, controllers for some domestic or industrial applications are implemented using microcontrollers or digital computers to reduce the implementation cost and time. However, in such a case, the overall control system becomes a sampled-data system of which the control signals are kept constant during the sampling period and are allowed to change only at the sampling instant. As a result, the control signals are stepwise, which introduce discontinuities and make the system dynamics more complicated. Although the sampling period can be regarded as a time-varying delay [20], as a result of the discontinuous control signals, the stability analysis methods used in [12]–[19] cannot be applied to sampled-data nonlinear systems. In [20], a linear sampled-data system was investigated. However, the analysis will become very complicated when a nonlinear system is considered. In [21]–[24], the fuzzy-model-based control approach was employed to study the stability of sampled-data nonlinear systems. An equivalent jump system was proposed to represent the dynamics of the sampled-data fuzzy control systems at the sampling instant. The closed-loop system is guaranteed to be stable if both the sampled-data fuzzy control system governing the system dynamics during the sampling period and the jump system governing the system dynamics at the sampling instant are both stable subject to a common time-varying solution to a number of Lyapunov inequalities. In [25], the linear analysis approach in [20] was employed and extended to analyze the stability of nonlinear sampled-data control systems. However, the information of the system nonlinearity was not utilized to produce less conservative stability conditions. In [26]–[28], an intelligent digital redesign approach was proposed. The idea is to approximate the nonlinear plant by a discrete-time fuzzy model. Based on the discrete-time fuzzy model, a discrete-time fuzzy controller is then proposed to close the feedback loop. However, the discretization error due to the discrete-time fuzzy model may become a source to cause the system instability.

In this paper, the system stability of a time-delay nonlinear system with a sampled-data fuzzy controller is investigated. It can be seen from [12]–[19] that the system dynamics are complicated by the existence of time delay, making the stability analysis more difficult [5]–[10]. The sampled-data controller will cause the problem to be even more challenging. To facilitate the stability analysis, the T–S fuzzy model with time delay is employed to represent the nonlinear plant with time delay. The proposed sampled-data fuzzy controller exhibits an enhanced nonlinearity, as compared with that of the fuzzy controllers obtained by applying the PDC design technique, to compensate the unstable elements of the nonlinear system with time delay. The nonlinearity of the proposed sampled-data fuzzy controller is enriched by employing extra fuzzy rules, which will cause the number of stability conditions to increase. To alleviate this problem, a descriptor representation is employed to reduce the number of stability conditions. Under the sampled-data case, the advantage of simplifying the stability analysis brought by sharing the same premise membership functions [5]–[10], [12]–[19] between the fuzzy plant model and fuzzy controller vanishes. To produce a less conservative stability analysis result, the information of the membership functions of both the fuzzy plant model and the fuzzy controller are considered. Some free arbitrary matrices are introduced to ease the satisfaction of the LMI stability conditions. Furthermore, LMI performance conditions are derived subject to a scalar performance function, which quantitatively measures the system performance, to aid the design of stable and well-performed sampled-data fuzzy-model-based control systems with time delay.

This paper is organized as follows. In Section II, the fuzzy plant model and the fuzzy controller are presented. In Section III, LMI-based stability and performance conditions are derived for the sampled-data fuzzy-model-based control systems with time delay. In Section IV, an application example is given to illustrate the merits and design procedure of the proposed sampled-data approach. A conclusion is drawn in Section V.

II. Fuzzy Plant Model and Sampled-Data Fuzzy Controller

A sampled-data time-delay fuzzy-model-based control system is formed by a fuzzy plant model with time delay and a sampled-data fuzzy controller connected in closed loop.

A. Fuzzy Plant Model

In some engineering process, a time delay is an inevitable source causing system instability. For example, time delays usually appear in some mechanical systems [13], chemical systems [14], and long transmission systems. Let p be the number of fuzzy rules describing the nonlinear plant with time delay. The ith rule is of the following format [12]–[19]:

$$\begin{aligned}\text{Rule } i: &\text{IF } f_1(\mathbf{x}(t)) \text{ is } M_1^i \text{ AND} \ldots \text{AND } f_\Psi(\mathbf{x}(t)) \text{ is } M_\Psi^i\\ &\text{THEN } \dot{\mathbf{x}}(t) = \mathbf{A}_i\mathbf{x}(t) + \mathbf{A}_{di}\mathbf{x}(t-\tau_d) + \mathbf{B}_i\mathbf{u}(t)\end{aligned} \tag{1}$$

where M_α^i is a fuzzy term of rule i corresponding to the scalar function $f_\alpha(\mathbf{x}(t))$, $\alpha = 1, 2, \ldots, \Psi$; $i = 1, 2, \ldots, p$; Ψ is a positive integer; $\mathbf{A}_i \in \Re^{n\times n}$ and $\mathbf{A}_{di} \in \Re^{n\times n}$ are known constant system matrices; $\mathbf{B}_i \in \Re^{n\times m}$ is a constant input matrix; $\mathbf{x}(t) \in \Re^{n\times 1}$ is the system state vector and $\mathbf{u}(t) \in \Re^{m\times 1}$ is the input vector; $h_d \geq \tau_d \geq 0$ denotes a constant time delay, h_d is a nonzero positive scalar which denotes the upper bound of the time delay the system can persist subject to the consideration of system stability. The overall system dynamics are described by

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^{p} w_i(\mathbf{x}(t))\left(\mathbf{A}_i\mathbf{x}(t) + \mathbf{A}_{di}\mathbf{x}(t-\tau_d) + \mathbf{B}_i\mathbf{u}(t)\right) \tag{2}$$

where

$$\sum_{i=1}^{p} w_i(\mathbf{x}(t)) = 1, \quad w_i(\mathbf{x}(t)) \in [0 \quad 1] \text{ for all } i \tag{3}$$

$$w_i(\mathbf{x}(t)) = \frac{\mu_{M_1^i}(f_1(\mathbf{x}(t))) \times \mu_{M_2^i}(f_2(\mathbf{x}(t))) \times \cdots \times \mu_{M_\Psi^i}(f_\Psi(\mathbf{x}(t)))}{\sum_{k=1}^{p}\left(\mu_{M_1^k}(f_1(\mathbf{x}(t))) \times \mu_{M_2^k}(f_2(\mathbf{x}(t))) \times \cdots \times \mu_{M_\Psi^k}(f_\Psi(\mathbf{x}(t)))\right)} \tag{4}$$

is a scalar nonlinear function of $\mathbf{x}(t)$ and $\mu_{M_\alpha^i}(f_\alpha(\mathbf{x}(t)))$, $\alpha = 1, 2, \ldots, \Psi$, are the grades of membership corresponding to the fuzzy term M_α^i. The symbol "$\times$" denotes the multiplication operation. It is assumed that $\mathbf{x}(t) = \boldsymbol{\varphi}(t)$ for $t \in [-\max(h_d, h_s) \ \ 0]$, where $\boldsymbol{\varphi}(t)$ denotes the initial system state condition of $\mathbf{x}(t)$ and h_s denotes the constant sampling period.

B. Sampled-Data Fuzzy Controller

A sampled-data fuzzy controller with p^2 fuzzy rules is designed based on the fuzzy model of the nonlinear plant. The jth rule of the sampled-data fuzzy controller is of the following format [3]–[10]:

$$\text{Rule } j: \text{ IF } \overline{g}_1(\mathbf{x}(t_\gamma)) \text{ is } \overline{N}_1^j \text{ AND} \ldots \text{AND } \overline{g}_\Omega(\mathbf{x}(t_\gamma)) \text{ is } \overline{N}_\Omega^j$$
$$\text{THEN} \quad \mathbf{u}_j(t) = \overline{\mathbf{G}}_j \mathbf{x}(t_\gamma), \qquad t_\gamma < t \le t_{\gamma+1} \tag{5}$$

where $\overline{N}_\beta^j$ is a fuzzy term of rule j corresponding to the scalar function $\overline{g}_\beta(\mathbf{x}(t))$, $\beta = 1, 2, \ldots, \Omega; j = 1, 2, \ldots, p^2$; Ω is a positive integer; $\overline{\mathbf{G}}_j \in \Re^{m \times n}$ is the feedback gain of rule j to be designed; $t\gamma = \gamma h_s, \gamma = 0, 1, 2, \ldots, = \infty$, denotes the sampling instant; $h_s = t_{\gamma+1} - t_\gamma$ denotes the constant sampling period; $\mathbf{u}_j(t) \in \Re^{m \times 1}$ is the input vector of rule j. The inferred output of the sampled-data fuzzy controller is given by

$$\mathbf{u}(t) = \sum_{j=1}^{p^2} \overline{m}_j(\mathbf{x}(t_\gamma)) \overline{\mathbf{G}}_j \mathbf{x}(t_\gamma), \qquad t_\gamma < t \le t_{\gamma+1} \tag{6}$$

where $\overline{m}_j$ of (8), shown at the bottom of the page, is a scalar nonlinear function of $\mathbf{x}(t_\gamma)$ and $\mu_{\overline{N}_\beta^j}(\overline{g}_\beta(\mathbf{x}(t_\gamma)))$ is the known grade of membership corresponding to the fuzzy term $\overline{N}_\beta^j$. It can be seen from (6) that $\mathbf{u}(t) = \mathbf{u}(t_\gamma)$, which holds constant value for $t_\gamma < t \le t_{\gamma+1}$. The sampled-data fuzzy controller of (6) can be represented as

$$\begin{aligned}\mathbf{u}(t) &= \sum_{j=1}^{p}\sum_{k=1}^{p} m_j(\mathbf{x}(t_\gamma)) m_k(\mathbf{x}(t_\gamma)) \mathbf{G}_{jk}\mathbf{x}(t_\gamma) \\ &= \sum_{j=1}^{p}\sum_{k=1}^{p} m_j(\mathbf{x}(t_\gamma)) m_k(\mathbf{x}(t_\gamma)) \mathbf{G}_{jk}\mathbf{x}(t - \tau_s(t))\end{aligned} \tag{9}$$

where $\tau_s(t) = t - t_\gamma \le h_s$ for $t_\gamma < t \le t_{\gamma+1}$. Let $m_j(\mathbf{x}(t_\gamma))$ be given by (10), shown at the bottom of the page, where $\sum_{j=1}^{p} m_j(\mathbf{x}(t_\gamma)) = 1$, $m_j(\mathbf{x}(t_\gamma)) \in [0 \quad 1]$, $j = 1, 2, \ldots, p$; $g_\beta(\mathbf{x}(t_\gamma))$ is a scalar function and $\mu_{N_\beta^j}(g_\beta(\mathbf{x}(t_\gamma)))$, $j = 1, 2, \ldots, p$, are the grades of membership. The sampled-data fuzzy controller in the form of (10) can make the analysis simpler and reduce the number of stability conditions. The sampling period and time delay of a system are illustrated in Fig. 1 of which the signal of system state $x_1(t)$ is shown. The sampling period is a fixed time period denoted by h_s. At time instant t_γ, $\gamma = 1, 2, \ldots, \infty, x_1(t)$ is sampled and denoted by $x_1(t\gamma)$. The system state $x_1(t)$ retarded by time delay τ_d at time t is denoted by $x_1(t - \tau_d)$.

Remark 1: The sampled-data fuzzy controllers of (6) and (9) are equivalent if the membership functions of the sampled-data fuzzy controller of (6), i.e., $\overline{m}_j(\mathbf{x}(t_\gamma))$, are designed properly. An example is given as follows to show the design of the memberships of the sampled data fuzzy controller such that (6) and (9) are equivalent. In this example,

$$\sum_{j=1}^{p^2} \overline{m}_j(\mathbf{x}(t_\gamma)) = 1, \qquad \overline{m}_j(\mathbf{x}(t_\gamma)) \in [0 \ \ 1] \text{ for all } j \tag{7}$$

$$\overline{m}_j(\mathbf{x}(t_\gamma)) = \frac{\mu_{\overline{N}_1^j}(\overline{g}_1(\mathbf{x}(t_\gamma))) \times \mu_{\overline{N}_2^j}(\overline{g}_2(\mathbf{x}(t_\gamma))) \times \cdots \times \mu_{\overline{N}_\Omega^j}(\overline{g}_\Omega(\mathbf{x}(t_\gamma)))}{\sum_{k=1}^{p^2}\left(\mu_{\overline{N}_1^k}(g_1(\mathbf{x}(t_\gamma))) \times \mu_{\overline{N}_2^k}(\overline{g}_2(\mathbf{x}(t_\gamma))) \times \cdots \times \mu_{\overline{N}_\Omega^k}(\overline{g}_\Omega(\mathbf{x}(t_\gamma)))\right)} \tag{8}$$

$$m_j(\mathbf{x}(t_\gamma)) = \frac{\mu_{N_1^j}(g_1(\mathbf{x}(t_\gamma))) \times \mu_{N_2^j}(g_2(\mathbf{x}(t_\gamma))) \times \cdots \times \mu_{N_\Omega^j}(g_\Omega(\mathbf{x}(t_\gamma)))}{\sum_{k=1}^{p}\left(\mu_{N_1^k}(g_1(\mathbf{x}(t_\gamma))) \times \mu_{N_2^k}(g_2(\mathbf{x}(t_\gamma))) \times \cdots \times \mu_{N_\Omega^k}(g_\Omega(\mathbf{x}(t_\gamma)))\right)} \tag{10}$$

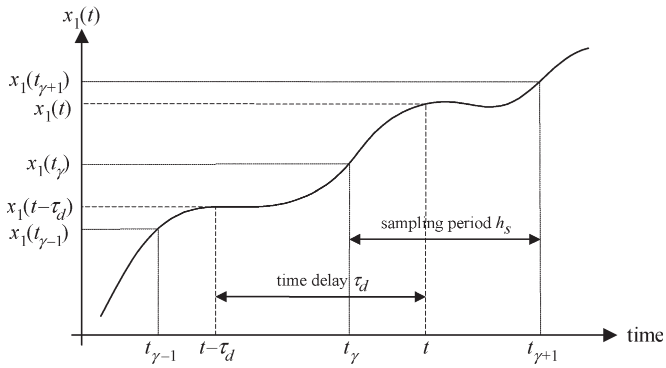

Fig. 1. Sampling period and time delay.

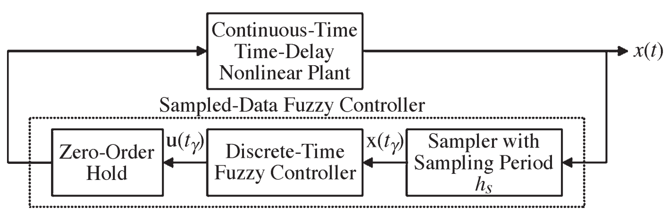

Fig. 2. Block diagram of a sampled-data time-delay fuzzy-model-based control system.

$\mu_{\overline{N}_\beta^l}(\overline{g}_\beta(\mathbf{x}(t_\gamma)))$ and $\mu_{N_\beta^j}(g_\beta(\mathbf{x}(t_\gamma)))$ are written as $\mu_{\overline{N}_\beta^l}$ and $\mu_{N_\beta^j}$, respectively, for simplicity. Considering $p = \Omega = 2$, from (8), we have $\overline{m}_l(\mathbf{x}(t_\gamma)) = (\mu_{\overline{N}_1^1}\mu_{\overline{N}_2^1})/(\mu_{\overline{N}_1^1}\mu_{\overline{N}_2^1} + \mu_{\overline{N}_1^2}\mu_{\overline{N}_2^2} + \mu_{\overline{N}_1^3}\mu_{\overline{N}_2^3} + \mu_{\overline{N}_1^4}\mu_{\overline{N}_2^4})$, $l = 1, 2, 3, 4$, for the sampled-data fuzzy controller of (6). Similarly, from (10), we have the expression for $m_j(\mathbf{x}(t_\gamma))m_k(\mathbf{x}(t_\gamma))$, shown at the bottom of the page, for the sampled-data fuzzy controller of (9). It can be seen in this case that (6) is equivalent to (9) when $\overline{m}_l(\mathbf{x}(t_\gamma)) = m_j(\mathbf{x}(t_\gamma))m_k(\mathbf{x}(t_\gamma))$ for $l = 2(j-1)+k$. By comparing their denominators, we design the membership functions of the sampled-data fuzzy controller of (6) as $\mu_{\overline{N}_i^1} = \mu^2_{N_i^1}$, $\mu_{\overline{N}_i^2} = \mu_{\overline{N}_i^3} = \mu_{N_i^1}\mu_{N_i^2}$, $\mu_{\overline{N}_i^4} = \mu^2_{N_i^2}$, $i = 1, 2$, and the feedback gains as $\overline{\mathbf{G}}_1 = \mathbf{G}_{11}$, $\overline{\mathbf{G}}_2 = \mathbf{G}_{12}$, $\overline{\mathbf{G}}_3 = \mathbf{G}_{21}$, and $\overline{\mathbf{G}}_4 = \mathbf{G}_{22}$. Under such a design, the sampled-data fuzzy controller of (6) with $p = \Omega = 2$ can be written in the form of (9), i.e., $\mathbf{u}(t) = \sum_{j=1}^4 \overline{m}_j(\mathbf{x}(t_\gamma))\overline{\mathbf{G}}_j\mathbf{x}(t_\gamma) = \sum_{j=1}^2\sum_{k=1}^2 m_j(\mathbf{x}(t_\gamma))m_k(\mathbf{x}(t_\gamma))\mathbf{G}_{jk}\mathbf{x}(t_\gamma)$.

C. Sampled-Data Time-Delay Fuzzy-Model-Based Control System

The sampled-data time-delay fuzzy-model-based control system is formed by the time-delay nonlinear plant represented by the fuzzy model of (2) and the sampled-data fuzzy controller of (9) connected in closed loop as shown in Fig. 2. In the following analysis, $w_i(\mathbf{x}(t))$ and $m_j(\mathbf{x}(t_\gamma))$ are denoted as w_i and m_j, respectively, for simplicity. From (2) to (9), and with the property that $\sum_{i=1}^p w_i = \sum_{j=1}^p m_j = \sum_{i=1}^p\sum_{j=1}^p\sum_{k=1}^p w_i m_j m_k = 1$, we have

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \sum_{i=1}^p w_i\Bigg(\mathbf{A}_i\mathbf{x}(t) + \mathbf{A}_{di}\mathbf{x}(t-\tau_d)\\ &\qquad + \mathbf{B}_i\left(\sum_{j=1}^p\sum_{k=1}^p m_j m_k \mathbf{G}_{jk}\mathbf{x}(t_\gamma)\right)\Bigg)\\ &= \sum_{i=1}^p\sum_{j=1}^p\sum_{k=1}^p w_i m_j m_k\left(\mathbf{A}_i\mathbf{x}(t) + \mathbf{A}_{di}\mathbf{x}(t-\tau_d)\right.\\ &\qquad \left.+ \mathbf{B}_i\mathbf{G}_{jk}\mathbf{x}\left(t-\tau_s(t)\right)\right). \qquad (11)\end{aligned}$$

III. STABILITY ANALYSIS AND PERFORMANCE DESIGN

The system stability and performance of the sampled-data time-delay fuzzy-model-based control system in the form of (11) is investigated in this section. Based on the Lyapunov method, LMI-based stability conditions will be derived to guarantee the system stability. Furthermore, LMI-based performance conditions will also be derived subject to a scalar performance index [30] to guarantee the system performance. The LMI-based stability and performance conditions are then used to design stable and well-performed sampled-data fuzzy-mode-based control systems with time delay. In [25], the system stability of the sampled-data neural-network-based control systems was investigated. Its analysis approach is extended in this paper to investigate the stability of fuzzy-model-based control systems subject to time delay, which makes the system dynamics to be even more complicated. The information of the membership functions of both the fuzzy plant model and fuzzy controller is employed to reduce the conservativeness of the stability analysis results. The LMI-based stability and performance conditions are summarized in the following theorem.

Theorem 1: The sampled-data time-delay fuzzy-model-based control system in the form of (11) is asymptotically stable if the membership functions are designed such that $w_i(\mathbf{x}(t)) - \rho m_i(\mathbf{x}(t_\gamma)) > 0$, $i = 1, 2, \ldots, p$, for all $\mathbf{x}(t)$ and $\mathbf{x}(t_\gamma)$, and there exist nonzero positive scalars, h_d, h_s, ρ, ζ and η, and matrices $\mathbf{X}_1 = \mathbf{X}_1^{\mathrm{T}} \in \Re^{n\times n}$, $\mathbf{X}_2 \in \Re^{m\times n}$, $\mathbf{X}_3 \in \Re^{m\times n}$, $\mathbf{X}_4 \in \Re^{m\times m}$, $\mathbf{X}_5 \in \Re^{n\times n}$, $\mathbf{X}_6 \in \Re^{n\times n}$, $\mathbf{X}_7 \in \Re^{n\times m}$, $\mathbf{X}_8 \in \Re^{n\times n}$, $\mathbf{J}_1 = \mathbf{J}_1^{\mathrm{T}} \in \Re^{n\times n} > 0$, $\mathbf{J}_2 = \mathbf{J}_2^{\mathrm{T}} \in \Re^{n\times n} > 0$ and $\mathbf{J}_3 = \mathbf{J}_3^{\mathrm{T}} \in \Re^{m\times m} > 0$, $\mathbf{K}_i \in \Re^{m\times n}$, $\mathbf{\Lambda}_i = \mathbf{\Lambda}_i^{\mathrm{T}} \in \Re^{(4n+m)\times(4n+m)}$, $\mathbf{M}_i = \mathbf{M}_i^{\mathrm{T}} \in \Re^{(3n+m)\times(3n+m)}$, $\mathbf{N}_i \in \Re^{n\times n}$, $\mathbf{T}_i = \mathbf{T}_i^{\mathrm{T}} \in \Re^{(3n+m)\times(3n+m)}$, $i = 1, 2, \ldots, p$ such that the following LMI-based stability and performance conditions are satisfied.

$$m_j\left(\mathbf{x}(t_\gamma)\right)m_k\left(\mathbf{x}(t_\gamma)\right)$$
$$= \frac{\left(\mu_{N_1^j}\mu_{N_2^j}\right)\left(\mu_{N_1^k}\mu_{N_2^k}\right)}{\left(\mu_{N_1^1}\mu_{N_2^1}\right)\left(\mu_{N_1^1}\mu_{N_2^1}\right) + \left(\mu_{N_1^1}\mu_{N_2^1}\right)\left(\mu_{N_1^2}\mu_{N_2^2}\right) + \left(\mu_{N_1^2}\mu_{N_2^2}\right)\left(\mu_{N_1^1}\mu_{N_2^1}\right) + \left(\mu_{N_1^2}\mu_{N_2^2}\right)\left(\mu_{N_1^2}\mu_{N_2^2}\right)}, \qquad j, k, = 1, 2$$

A. Stability Conditions

$$\mathbf{X}_1 > 0;\ \overline{\mathbf{Q}}_{ii} - \frac{(1-\rho)}{\rho}\Lambda_i < 0, \qquad i = 1,2,\ldots,p$$
$$\overline{\mathbf{Q}}_{ij} + \Lambda_j < 0, \qquad i,j = 1,2,\ldots,p$$
$$\begin{bmatrix} \mathbf{M}_i & * \\ \mathbf{X}_1\mathbf{Y}_i^{\mathrm{T}}\mathbf{X} & 2\zeta\mathbf{X}_1 - \zeta^2\mathbf{M} \end{bmatrix} \geq 0, \qquad i = 1,2,\ldots,p$$
$$\begin{bmatrix} \mathbf{T}_i & * \\ \mathbf{X}_1\hat{\mathbf{Y}}_i^{\mathrm{T}}\mathbf{X} & 2\zeta\mathbf{X}_1 - \zeta^2\mathbf{M} \end{bmatrix} \geq 0, \qquad i = 1,2,\ldots,p$$

where the expressions for $\overline{\mathbf{Q}}_{ij}$ and $\mathbf{X}$, are given as shown at the bottom of the page and the feedback gains are designed as $\mathbf{G}_{jk} = \mathbf{F}_j\mathbf{G}_k$, $\mathbf{F}_j = \mathbf{K}_j\mathbf{X}_1^{-1}$, and $\mathbf{G}_k = \mathbf{N}_k\mathbf{X}_1^{-1}$, $j,\ k = 1,2,\ldots,p$.

B. Performance Conditions

$$\begin{bmatrix} \mathbf{W}^{(11)} & * \\ \mathbf{W}_i^{(21)} & \mathbf{W}^{(22)} \end{bmatrix} < 0, \qquad i = 1,2,\ldots,p$$

where

$$\mathbf{W}^{(11)} = \begin{bmatrix} -\eta\mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\eta\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\eta\mathbf{I} \end{bmatrix}$$
$$\mathbf{W}_i^{(21)} = \begin{bmatrix} \mathbf{X}_1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{N}_i & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{K}_i \end{bmatrix}, \qquad i = 1,2,\ldots,p$$

and

$$\mathbf{W}^{(22)} = \begin{bmatrix} -\mathbf{J}_1^{-1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\mathbf{J}_2^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\mathbf{J}_3^{-1} \end{bmatrix}.$$

In Theorem 1, the symbol "$*$" denotes the transposed element of the matrix at the corresponding position. It should be noted that the system performance is measured by a scalar performance index defined in (B1) and the weighting matrices $\mathbf{J}_1$, $\mathbf{J}_2$, and $\mathbf{J}_3$ are needed to be determined by the designer prior to applying Theorem 1. The proofs for the LMI-based stability and performance conditions are shown in Appendices A and B, respectively.

IV. Application Example

The proposed sampled-data fuzzy controller is employed to deal with a truck-trailer with time delay [13].

Step 1) It was reported in [13] that the dynamics of the truck trailer with time delay are defined as follows [13]:

$$\dot{x}_1(t) = -a\frac{\nu\bar{t}}{Lt_o}x_1(t) - (1-a)\frac{\nu\bar{t}}{Lt_o}x_1(t-t_d) + \frac{\nu\bar{t}}{lt_o}u(t) \tag{12}$$
$$\dot{x}_2(t) = a\frac{\nu\bar{t}}{Lt_o}x_1(t) + (1-a)\frac{\nu\bar{t}}{Lt_o}x_1(t-t_d) \tag{13}$$
$$\dot{x}_3(t) = \frac{\nu\bar{t}}{Lt_o}\sin\left(f_1\left(\mathbf{x}(t), \mathbf{x}(t-t_d)\right)\right) \tag{14}$$

where $\mathbf{x}(t) = [\,x_1(t) \quad x_2(t) \quad x_3(t)\,]^{\mathrm{T}}$, $x_1(t)$ is the angle difference between the truck and the trailer, $x_2(t)$ is the angle of the trailer, $x_3(t)$ is the vertical position of the rear end of the trailer, $f_1(\mathbf{x}(t), \mathbf{x}(t-t_d)) = x_2(t) + a(\nu\bar{t}/2L)x_1(t) + (1-a)(\nu\bar{t}/2L)x_1(t-t_d)$, $l = 2.8$ is the length of the truck, $L = 5.5$ is the length of the trailer, $\nu = -1.0$ is the constant backward speed, $a = 0.7$ is the retarded coefficient, $\bar{t} = 2.0$, $t_o = 0.5$. The system is assumed to be operating in the operating domain of $x_1(t) \in [\,-\pi/2 \quad \pi/2\,]$, $\dot{x}_1(t) \in [\,-3 \quad 3\,]$, $x_2(t) \in [\,-\pi/2 \quad \pi/2\,]$, and $\dot{x}_2(t) \in [\,-2 \quad 2\,]$. The control objective is to backward move the truck-trailer along a straight line, i.e., $\mathbf{x}(t) = \mathbf{0}$, using the sampled-data fuzzy controller. It is reported in [13] that the truck trailer can be represented by a fuzzy model with the following two rules.

Rule1 : **IF** $f_1\left(\mathbf{x}(t), \mathbf{x}(t-\tau_d)\right)$ is about 0

THEN $\dot{\mathbf{x}}(t) = \mathbf{A}_1\mathbf{x}(t) + \mathbf{A}_{d1}\mathbf{x}(t-\tau_d) + \mathbf{B}_1u(t)$ (15)

Rule2 : **IF** $f_1\left(\mathbf{x}(t), \mathbf{x}(t-\tau_d)\right)$ is about π or $-\pi$

THEN $\dot{\mathbf{x}}(t) = \mathbf{A}_2\mathbf{x}(t) + \mathbf{A}_{d2}\mathbf{x}(t-\tau_d) + \mathbf{B}_2u(t)$ (16)

$$\overline{\mathbf{Q}}_{ij} = \begin{bmatrix} \mathbf{X}_5 + \mathbf{X}_5^{\mathrm{T}} & * & * & * & * \\ \mathbf{N}_j - \mathbf{X}_1 + \mathbf{X}_6^{\mathrm{T}} & -\mathbf{X}_1 - \mathbf{X}_1^{\mathrm{T}} & * & * & * \\ \mathbf{K}_j - \mathbf{X}_2 + \mathbf{X}_7^{\mathrm{T}} & \mathbf{K}_j - \mathbf{X}_3 & -\mathbf{X}_4 - \mathbf{X}_4^{\mathrm{T}} & * & * \\ (\mathbf{A}_i + \mathbf{A}_{di})\mathbf{X}_1 + \mathbf{B}_i\mathbf{X}_2 - \mathbf{X}_5 + \mathbf{X}_8^{\mathrm{T}} & \mathbf{B}_i\mathbf{X}_3 - \mathbf{X}_6 & \mathbf{B}_i\mathbf{X}_4 - \mathbf{X}_7 & -\mathbf{X}_8 - \mathbf{X}_8^{\mathrm{T}} & * \\ (h_d + h_s)\mathbf{X}_5 & (h_d + h_s)\mathbf{X}_6 & (h_d + h_s)\mathbf{X}_7 & (h_d + h_s)\mathbf{X}_8 & -(h_d + h_s)\mathbf{M} \end{bmatrix} + h_d\begin{bmatrix} \mathbf{M}_i & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} + h_s\begin{bmatrix} \mathbf{T}_j & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$$

$$\mathbf{X} = \begin{bmatrix} \mathbf{X}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{X}_1 & \mathbf{X}_1 & \mathbf{0} & \mathbf{0} \\ \mathbf{X}_2 & \mathbf{X}_3 & \mathbf{X}_4 & \mathbf{0} \\ \mathbf{X}_5 & \mathbf{X}_6 & \mathbf{X}_7 & \mathbf{X}_8 \end{bmatrix} \qquad \mathbf{Y}_i = \mathbf{X}^{\mathrm{T}^{-1}}\begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{A}_{di} \end{bmatrix} \qquad \hat{\mathbf{Y}}_i = \mathbf{X}^{\mathrm{T}^{-1}}\begin{bmatrix} \mathbf{0} \\ \mathbf{G}_i \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}$$

where

$$\mathbf{A}_1 = \begin{bmatrix} -a\frac{\nu\bar{t}}{Lt_o} & 0 & 0 \\ a\frac{\nu\bar{t}}{Lt_o} & 0 & 0 \\ a\frac{\nu^2\bar{t}^2}{2Lt_o} & \frac{\nu\bar{t}}{t_o} & 0 \end{bmatrix}$$

$$\mathbf{A}_2 = \begin{bmatrix} -a\frac{\nu\bar{t}}{Lt_o} & 0 & 0 \\ a\frac{\nu\bar{t}}{Lt_o} & 0 & 0 \\ a\frac{d\nu^2\bar{t}^2}{2Lt_o} & \frac{d\nu\bar{t}}{t_o} & 0 \end{bmatrix}$$

$$\mathbf{A}_{d1} = \begin{bmatrix} -(1-a)\frac{\nu\bar{t}}{Lt_o} & 0 & 0 \\ (1-a)\frac{\nu\bar{t}}{Lt_o} & 0 & 0 \\ (1-a)\frac{\nu^2\bar{t}^2}{2Lt_o} & 0 & 0 \end{bmatrix}$$

$$\mathbf{A}_{d2} = \begin{bmatrix} -(1-a)\frac{\nu\bar{t}}{Lt_o} & 0 & 0 \\ (1-a)\frac{\nu\bar{t}}{Lt_o} & 0 & 0 \\ (1-a)\frac{d\nu^2\bar{t}^2}{2Lt_o} & 0 & 0 \end{bmatrix}$$

$$\mathbf{B}_1 = \mathbf{B}_2 = \begin{bmatrix} \frac{\nu\bar{t}}{lt_o} \\ 0 \\ 0 \end{bmatrix}$$

and $d = 10t_o/\pi$. The overall system is described by (2) in which the membership functions are defined as follows:

$$\begin{aligned} &w_1(\mathbf{x}(t), \mathbf{x}(t-\tau_d)) \\ &= \left(1 - \frac{1}{1+\exp\left(-3\left(f_1\left(\mathbf{x}(t), \mathbf{x}(t-\tau_d)\right) - 0.5\pi\right)\right)}\right) \\ &\quad \times \left(\frac{1}{1+\exp\left(-3\left(f_1\left(\mathbf{x}(t), \mathbf{x}(t-\tau_d)\right) + 0.5\pi\right)\right)}\right) \end{aligned}$$

and

$$w_2\left(\mathbf{x}(t), \mathbf{x}(t-\tau_d)\right) = 1 - w_1\left(\mathbf{x}(t), \mathbf{x}(t-\tau_d)\right)$$

Step 2) A four-rule sampled-data fuzzy controller is employed to stabilize the truck trailer with time delay. Referring to (9), the sampled-data fuzzy controller is designed as follows:

$$u(t) = \sum_{j=1}^{2}\sum_{k=1}^{2} m_j\left(\mathbf{x}\left(t_\gamma\right)\right) m_k\left(\mathbf{x}\left(t_\gamma\right)\right) \mathbf{G}_{jk}\mathbf{x}(t_\gamma) \qquad (17)$$

where $\mathbf{G}_{jk} = \mathbf{F}_j\mathbf{G}_k$, j, $k = 1, 2$. It is chosen that $m_j(\mathbf{x}(t_\gamma)) = w_j(\mathbf{x}(t_\gamma))$, $j = 1, 2$.

Step 3) Based on Theorem 1, with $h_d = \tau_d = 1s$, $h_s = 0.04s$, $\rho = 0.725$, $\zeta = 2$, $\eta = 10^{-2}$, $\mathbf{J}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $\mathbf{J}_2 = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}$, and $\mathbf{J}_3 = 0.1$, we have $\mathbf{G}_{11} = [1.4781 \quad -0.1771 \quad 0.0040]$, $\mathbf{G}_{12} = [1.4784 \quad -0.1805 \quad 0.0040]$, $\mathbf{G}_{21} = [1.4793 \quad -0.1811 \quad 0.0041]$, and $\mathbf{G}_{22} = [1.4798 \quad -0.1844 \quad 0.0040]$. In this example, the nonlinear plant is assumed to operate in the domain characterized by $\dot{x}_1(t) \in [-3 \quad 3]$ and $\dot{x}_2(t) \in [-2 \quad 2]$. With this information and considering $t_\gamma \le t \le t_\gamma + h_s$, we have $x_1(t) = x_1(t_\gamma) + \int_{t_\gamma}^{t} \dot{x}_1(t)dt$ which offers the lower and upper bounds as $x_1(t_\gamma) - 3\int_{t_\gamma}^{t_\gamma+h_s} dt = x_1(t_\gamma) - 3h_s = x_1(t_\gamma) - 0.12$ and $x_1(t_\gamma) + 3\int_{t_\gamma}^{t_\gamma+h_s} dt = x_1(t_\gamma) + 3h_s = x_1(t_\gamma) + 0.12$, respectively. Similarly, we have $x_2(t) = x_2(t_\gamma) + \int_{t_\gamma}^{t} \dot{x}_2(t)dt$ which offers the lower and upper bounds as $x_2(t_\gamma) - 2\int_{t_\gamma}^{t_\gamma+h_s} dt = x_2(t_\gamma) - 2h_s = x_2(t_\gamma) - 0.08$ and $x_2(t_\gamma) + 2\int_{t_\gamma}^{t_\gamma+h_s} dt = x_2(t_\gamma) + 2h_s = x_2(t_\gamma) + 0.08$, respectively. Consequently, for any sampling instant t_γ, the value of $x_1(t)$ and $x_2(t)$ on or before the next sampling instant are in the range of $x_1(t_\gamma) - 0.12 \le x_1(t) \le x_1(t_\gamma) + 0.12$ and $x_2(t_\gamma) - 0.08 \le x_2(t) \le x_2(t_\gamma) + 0.08$, respectively, for $t_\gamma \le t \le t_\gamma + h_s$. For the system operating in its operating domain, it can be shown that $w_j(\mathbf{x}(t)) - 0.725m_j(\mathbf{x}(t_\gamma)) \ge 0$ for $\mathbf{x}(t)$, $\mathbf{x}(t_\gamma)$ and all j.

To illustrate the effectiveness of the performance conditions, another set of feedback gains are obtained with the same design parameter values except $\mathbf{J}_2 = \begin{bmatrix} 100 & 0 & 0 \\ 0 & 100 & 0 \\ 0 & 0 & 100 \end{bmatrix}$. By applying Theorem 1, $\mathbf{G}_{11} = [1.4775 \quad -0.1766 \quad 0.0040]$, $\mathbf{G}_{12} = [1.4780 \quad -0.1801 \quad 0.0040]$, $\mathbf{G}_{21} = [1.4785 \quad -0.1806 \quad 0.0041]$, and $\mathbf{G}_{22} = [1.4792 \quad -0.1839 \quad 0.0040]$ are obtained.

The sampled-data fuzzy controllers, with the two sets of feedback gains, are employed to control the truck-trailer. Fig. 3 shows the system state responses and the control signals of the sampled-data time-delay fuzzy-model-based control systems under the initial state condition of $\mathbf{x}(t) = [1.5 \quad -2 \quad 5]^{\mathrm{T}}$. The initial system state function is defined as $\varphi(t) = [1.5 \quad -2 \quad 5]^{\mathrm{T}}$ for $t \in [-1 \quad 0]$. Referring to Fig. 3, it can be seen that both sampled-data fuzzy controllers can stabilize the nonlinear system with time delay. A sampled-data fuzzy controller with $\mathbf{J}_2 = \begin{bmatrix} 100 & 0 & 0 \\ 0 & 100 & 0 \\ 0 & 0 & 100 \end{bmatrix}$ puts heavy weights to the control signal. It can be seen from Fig. 3 that the time integral of control signal is suppressed and its effect is also reflected in the system states $x_1(t)$ to $x_3(t)$. Furthermore, it can be seen that the control signals are stepwise which are kept constant during the sampling period.

For comparison purpose, the sampled-data fuzzy controller of (17) is reduced to $u(t) = \sum_{j=1}^{2} m_j(\mathbf{x}(t_\gamma))\mathbf{F}_j\mathbf{x}(t_\gamma)$ by choosing $\mathbf{G}_k = \mathbf{I}$, where $\mathbf{I}$ is the identity matrix. This form of fuzzy controller is generally used in many continuous-time fuzzy systems designed through the PDC technique [5]–[9], [31]. It can be seen that this PDC sampled-data fuzzy controller provides a simpler controller structure. Under the PDC approach, as $\mathbf{G}_k = \mathbf{N}_k\mathbf{X}_1^{-1}$, we have to set $\mathbf{N}_k = \mathbf{X}^{-1}$ to make $\mathbf{G}_k = \mathbf{I}$. Hence, by replacing $\mathbf{N}_k$ with $\mathbf{X}^{-1}$, Theorem 1

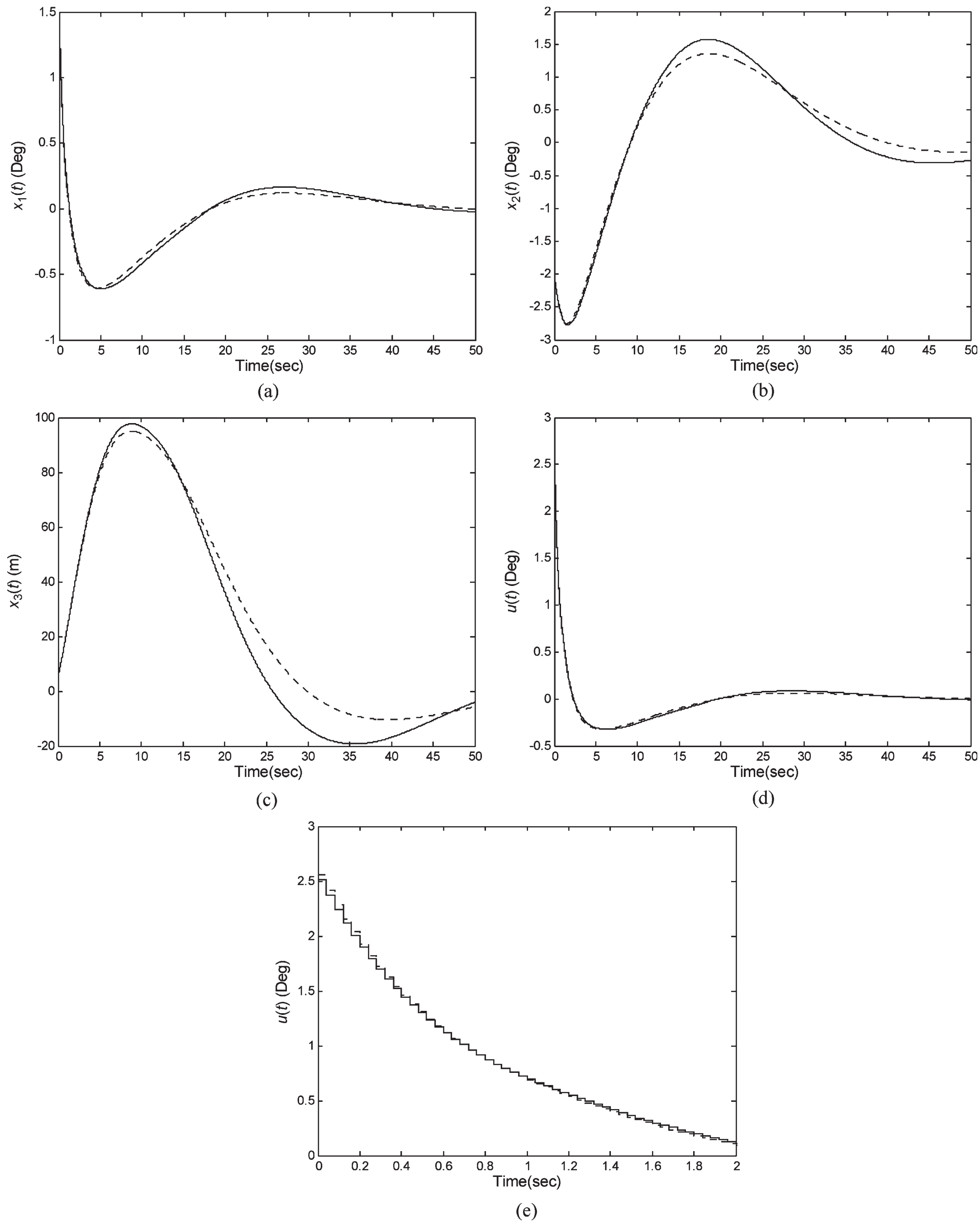

Fig. 3. System state responses and control signals of the truck-trailer with time delay under the sampled-data fuzzy controller with $\mathbf{J}_2 = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}$ (solid lines) and $\mathbf{J}_2 = \begin{bmatrix} 100 & 0 & 0 \\ 0 & 100 & 0 \\ 0 & 0 & 100 \end{bmatrix}$ (dotted lines). (a) $x_1(t)$. (b) $x_2(t)$. (c) $x_3(t)$. (d) $u(t)$ for 0s $\leq t \leq$ 50s. (e) $u(t)$ for 0s $\leq t \leq$ 2s.

can be modified to verify the stability of the closed-loop system with the PDC sampled-data fuzzy controller. Under the PDC case, it can be seen that no feasible solution can be obtained. In the proposed sampled-data fuzzy controller, the introduction of the feedback gains $\mathbf{G}_k$ effectively enhances the nonlinearity to compensate the unstable elements of the nonlinear system. Consequently, an enhanced stabilization ability of the proposed sampled-data fuzzy controller is offered as compared with that of the PDC one.

V. Conclusion

A sampled-data fuzzy controller with enhanced nonlinearity compensation ability has been proposed to deal with nonlinear systems with time delay. The sampled-data fuzzy controller can be implemented by a microcontroller or a digital computer to reduce the implementation time and cost. LMI-stability conditions have been derived based on Lyapunov method to guarantee the system stability. A descriptor representation has been employed to facilitate the stability analysis. The complexity introduced by the sampling actions and the number of LMI-based stability conditions have been reduced. Furthermore, the information of the membership functions of both the fuzzy plant model and fuzzy controller has been used to ease the satisfaction of the stability conditions. LMI-based performance conditions have been proposed to help design a well-performed sampled-data fuzzy controller. An application example has been given to show the effectiveness of the proposed approach.

Appendix A

The proof of the LMI-based stability conditions in Theorem 1 is given in this appendix. The stability of the sampled-data time-delay fuzzy-model-based control system of (11) is studied based on the descriptor representation [20]. From (2), the fuzzy model with time delay can be written as follows:

$$\dot{\mathbf{x}}(t) = \mathbf{y}(t) \tag{A1}$$

$$\begin{aligned}\mathbf{y}(t) &= \sum_{i=1}^{p} w_i \left(\mathbf{A}_i\mathbf{x}(t) + \mathbf{A}_{di}\mathbf{x}(t-\tau_d) + \mathbf{B}_i\mathbf{u}(t)\right)\\ &= \sum_{i=1}^{p} w_i \left((\mathbf{A}_i + \mathbf{A}_{di})\mathbf{x}(t) + \mathbf{B}_i\mathbf{u}(t)\right)\\ &\quad - \sum_{i=1}^{p} w_i\mathbf{A}_{di}\int_{t-\tau_d}^{t}\mathbf{y}(\varphi)d\varphi. \end{aligned} \tag{A2}$$

To facilitate the stability analysis, the feedback gains are defined as $\mathbf{G}_{ij} = \mathbf{F}_i\mathbf{G}_j$ where $\mathbf{F}_i \in \Re^{m\times n}$ and $\mathbf{G}_j \in \Re^{n\times n}$. From (9), the sampled-data fuzzy controller can be written as follows:

$$\mathbf{u}(t) = \sum_{i=1}^{p} m_i\mathbf{F}_i\mathbf{s}(t) \tag{A3}$$

$$\begin{aligned}\mathbf{s}(t) &= \sum_{i=1}^{p} m_i\mathbf{G}_i\mathbf{x}\left(t-\tau_s(t)\right)\\ &= \sum_{i=1}^{p} m_i\mathbf{G}_i\mathbf{x}(t) - \sum_{i=1}^{p} m_i\mathbf{G}_i\int_{t-\tau_s(t)}^{t}\mathbf{y}(\varphi)d\varphi. \end{aligned} \tag{A4}$$

Considering (A2), we have the following property which will be used during the analysis:

$$\sum_{i=1}^{p} w_i \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{A}_i+\mathbf{A}_{di} & \mathbf{0} & \mathbf{B}_i & -\mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{x}(t)\\ \mathbf{s}(t)\\ \mathbf{u}(t)\\ \mathbf{y}(t)\end{bmatrix} - \sum_{i=1}^{p} w_i\begin{bmatrix}\mathbf{0}\\ \mathbf{0}\\ \mathbf{0}\\ \mathbf{A}_{di}\end{bmatrix}\int_{t-\tau_d}^{t}\mathbf{y}(\varphi)d\varphi = \begin{bmatrix}\mathbf{0}\\ \mathbf{0}\\ \mathbf{0}\\ \mathbf{0}\end{bmatrix}. \tag{A5}$$

Similarly, from (A3) and (A4), we have the following properties:

$$\sum_{i=1}^{p} m_i \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{F}_i & -\mathbf{I} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{x}(t)\\ \mathbf{s}(t)\\ \mathbf{u}(t)\\ \mathbf{y}(t)\end{bmatrix} = \begin{bmatrix}\mathbf{0}\\ \mathbf{0}\\ \mathbf{0}\\ \mathbf{0}\end{bmatrix} \tag{A6}$$

$$\sum_{i=1}^{p} m_i \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{G}_i & -\mathbf{I} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{x}(t)\\ \mathbf{s}(t)\\ \mathbf{u}(t)\\ \mathbf{y}(t)\end{bmatrix} - \sum_{i=1}^{p} m_i\begin{bmatrix}\mathbf{0}\\ \mathbf{G}_i\\ \mathbf{0}\\ \mathbf{0}\end{bmatrix}\int_{t-\tau_s(t)}^{t}\mathbf{y}(\varphi)d\varphi = \begin{bmatrix}\mathbf{0}\\ \mathbf{0}\\ \mathbf{0}\\ \mathbf{0}\end{bmatrix}. \tag{A7}$$

To investigate the system stability, the following Lyapunov function candidate is considered:

$$V(t) = V_1(t) + V_2(t) \tag{A8}$$

where

$$V_1(t) = \mathbf{x}(t)^{\mathrm{T}}\mathbf{P}_1\mathbf{x}(t) \tag{A9}$$

$$\begin{aligned}V_2(t) = &\int_{-h_d}^{0}\int_{t+\sigma}^{t}\mathbf{y}(\varphi)^{\mathrm{T}}\mathbf{R}\mathbf{y}(\varphi)d\varphi d\sigma\\ &+\int_{-h_s}^{0}\int_{t+\sigma}^{t}\mathbf{y}(\varphi)^{\mathrm{T}}\mathbf{R}\mathbf{y}(\varphi)d\varphi d\sigma\end{aligned} \tag{A10}$$

where $\mathbf{P}_1 = \mathbf{P}_1^{\mathrm{T}} \in \Re^{n\times n} > 0$ and $\mathbf{R} = \mathbf{R}^{\mathrm{T}} \in \Re^{n\times n} > 0$. It will be shown that $\dot{V}(t) \le 0$ (equality holds when $\mathbf{x}(t) = \mathbf{y}(t) = \mathbf{0}$) which implies asymptotic stability of the system. From (A1) to (A4), and with the property that $\sum_{i=1}^{p} w_i = \sum_{j=1}^{p} m_j = \sum_{i=1}^{p}\sum_{j=1}^{p} w_i m_j = 1$, we have

$$\begin{aligned}\dot{V}_1(t) &= \mathbf{x}(t)^{\mathrm{T}}\mathbf{P}_1\dot{\mathbf{x}}(t) + \dot{\mathbf{x}}(t)^{\mathrm{T}}\mathbf{P}_1\mathbf{x}(t)\\ &= \begin{bmatrix}\mathbf{x}(t)\\ \mathbf{s}(t)\\ \mathbf{u}(t)\\ \mathbf{y}(t)\end{bmatrix}^{\mathrm{T}}\left(\mathbf{P}^{\mathrm{T}}\begin{bmatrix}\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\end{bmatrix} + \begin{bmatrix}\mathbf{0} & \mathbf{0} & v\mathbf{0} & \mathbf{I}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\end{bmatrix}^{\mathrm{T}}\mathbf{P}\right)\\ &\quad\times\begin{bmatrix}\mathbf{x}(t)\\ \mathbf{s}(t)\\ \mathbf{u}(t)\\ \mathbf{y}(t)\end{bmatrix}\end{aligned} \tag{A11}$$

where

$$\mathbf{P} = \begin{bmatrix}\mathbf{P}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{P}_2 & \mathbf{P}_3 & \mathbf{0} & \mathbf{0}\\ \mathbf{P}_4 & \mathbf{P}_5 & \mathbf{P}_6 & \mathbf{0}\\ \mathbf{P}_7 & \mathbf{P}_8 & \mathbf{P}_9 & \mathbf{P}_{10}\end{bmatrix}$$

and $\mathbf{P}_2 \in \Re^{n\times n}$, $\mathbf{P}_3 \in \Re^{n\times n}$, $\mathbf{P}_4 \in \Re^{m\times n}$, $\mathbf{P}_5 \in \Re^{m\times n}$, $\mathbf{P}_6 \in \Re^{m\times m}$, $\mathbf{P}_7 \in \Re^{n\times n}$, $\mathbf{P}_8 \in \Re^{n\times n}$, $\mathbf{P}_9 \in \Re^{n\times m}$, and $\mathbf{P}_{10} \in \Re^{n\times n}$. From (A5)–(A7) and (A11), we have

$$\dot{V}_1(t) = \sum_{i=1}^{p}\sum_{j=1}^{p} w_i m_j \begin{bmatrix}\mathbf{x}(t)\\ \mathbf{s}(t)\\ \mathbf{u}(t)\\ \mathbf{y}(t)\end{bmatrix}^{\mathrm{T}} \times \left(\mathbf{P}^{\mathrm{T}}\begin{bmatrix}\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}\\ \mathbf{G}_j & -\mathbf{I} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{F}_j & -\mathbf{I} & \mathbf{0}\\ \mathbf{A}_i+\mathbf{A}_{di} & \mathbf{0} & \mathbf{B}_i & -\mathbf{I}\end{bmatrix} + \begin{bmatrix}\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}\\ \mathbf{G}_j & -\mathbf{I} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{F}_j & -\mathbf{I} & \mathbf{0}\\ \mathbf{A}_i+\mathbf{A}_{di} & \mathbf{0} & \mathbf{B}_i & -\mathbf{I}\end{bmatrix}^{\mathrm{T}}\mathbf{P}\right)\begin{bmatrix}\mathbf{x}(t)\\ \mathbf{s}(t)\\ \mathbf{u}(t)\\ \mathbf{y}(t)\end{bmatrix} - 2\sum_{i=1}^{p} w_i \begin{bmatrix}\mathbf{x}(t)\\ \mathbf{s}(t)\\ \mathbf{u}(t)\\ \mathbf{y}(t)\end{bmatrix}^{\mathrm{T}} \mathbf{P}^{\mathrm{T}}\begin{bmatrix}\mathbf{0}\\ \mathbf{0}\\ \mathbf{0}\\ \mathbf{A}_{di}\end{bmatrix}\int_{t-\tau_d}^{t}\mathbf{y}(\varphi)d\varphi - 2\sum_{i=1}^{p} m_i \begin{bmatrix}\mathbf{x}(t)\\ \mathbf{s}(t)\\ \mathbf{u}(t)\\ \mathbf{y}(t)\end{bmatrix}^{\mathrm{T}} \mathbf{P}^{\mathrm{T}}\begin{bmatrix}\mathbf{0}\\ \mathbf{G}_i\\ \mathbf{0}\\ \mathbf{0}\end{bmatrix}\int_{t-\tau_s(t)}^{t}\mathbf{y}(\varphi)d\varphi. \tag{A12}$$

In the following, based on the property [29] that $-2\mathbf{a}(t)^{\mathrm{T}}\hat{\mathbf{N}}_i\mathbf{y}(\varphi) \le \begin{bmatrix}\mathbf{a}(t)\\ \mathbf{y}(\varphi)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\mathbf{R}_i & \mathbf{Y}_i-\hat{\mathbf{N}}_i\\ \mathbf{Y}_i^{\mathrm{T}}-\hat{\mathbf{N}}_i^{\mathrm{T}} & \mathbf{R}\end{bmatrix}\begin{bmatrix}\mathbf{a}(t)\\ \mathbf{y}(\varphi)\end{bmatrix}$ where $\mathbf{R}_i = \mathbf{R}_i^{\mathrm{T}} \in \Re^{4n\times 4n}$ and $\begin{bmatrix}\mathbf{R}_i & \mathbf{Y}_i\\ \mathbf{Y}_i^{\mathrm{T}} & \mathbf{R}\end{bmatrix} \ge 0$, $i = 1, 2, \ldots, p$, the last two integral terms of (A12) can be handled. Considering the second last integral term, i.e., $-2\sum_{i=1}^{p} w_i \begin{bmatrix}\mathbf{x}(t)\\ \mathbf{s}(t)\\ \mathbf{u}(t)\\ \mathbf{y}(t)\end{bmatrix}^{\mathrm{T}} \mathbf{P}^{\mathrm{T}}\begin{bmatrix}\mathbf{0}\\ \mathbf{0}\\ \mathbf{0}\\ \mathbf{A}_{di}\end{bmatrix}\int_{t-\tau_d}^{t}\mathbf{y}(\varphi)d\varphi$ and let $\mathbf{a}(t) = \begin{bmatrix}\mathbf{x}(t)\\ \mathbf{s}(t)\\ \mathbf{u}(t)\\ \mathbf{y}(t)\end{bmatrix}$ and $\hat{\mathbf{N}}_i = \mathbf{Y}_i = \mathbf{P}^{\mathrm{T}}\begin{bmatrix}\mathbf{0}\\ \mathbf{0}\\ \mathbf{0}\\ \mathbf{A}_{di}\end{bmatrix}$, and using the property of $\tau_d \le h_d$, we have

$$-2\sum_{i=1}^{p} w_i \begin{bmatrix}\mathbf{x}(t)\\ \mathbf{s}(t)\\ \mathbf{u}(t)\\ \mathbf{y}(t)\end{bmatrix}^{\mathrm{T}} \mathbf{P}^{\mathrm{T}}\begin{bmatrix}\mathbf{0}\\ \mathbf{0}\\ \mathbf{0}\\ \mathbf{A}_{di}\end{bmatrix}\int_{t-\tau_d}^{t}\mathbf{y}(\varphi)d\varphi \le h_d\sum_{i=1}^{p} w_i \begin{bmatrix}\mathbf{x}(t)\\ \mathbf{s}(t)\\ \mathbf{u}(t)\\ \mathbf{y}(t)\end{bmatrix}^{\mathrm{T}} \times \mathbf{R}_i \begin{bmatrix}\mathbf{x}(t)\\ \mathbf{s}(t)\\ \mathbf{u}(t)\\ \mathbf{y}(t)\end{bmatrix} + \int_{t-\tau_d}^{t}\mathbf{y}(\varphi)^{\mathrm{T}}\mathbf{R}\mathbf{y}(\varphi)d\varphi. \tag{A13}$$

Similarly, considering the last integral term in the right-hand side of (A13), i.e., $-2\sum_{i=1}^{p} m_i \begin{bmatrix}\mathbf{x}(t)\\ \mathbf{s}(t)\\ \mathbf{u}(t)\\ \mathbf{y}(t)\end{bmatrix}^{\mathrm{T}} \mathbf{P}^{\mathrm{T}}\begin{bmatrix}\mathbf{0}\\ \mathbf{G}_i\\ \mathbf{0}\\ \mathbf{0}\end{bmatrix}\mathbf{y}(\varphi)$ and using the properties of $-2\mathbf{a}(t)^{\mathrm{T}}\hat{\mathbf{M}}_i\mathbf{y}(\varphi) \le \begin{bmatrix}\mathbf{a}(t)\\ \mathbf{y}(\varphi)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\mathbf{S}_i & \hat{\mathbf{Y}}_i-\hat{\mathbf{M}}_i\\ \hat{\mathbf{Y}}_i^{\mathrm{T}}-\hat{\mathbf{M}}_i^{\mathrm{T}} & \mathbf{R}\end{bmatrix}\begin{bmatrix}\mathbf{a}(t)\\ \mathbf{y}(\varphi)\end{bmatrix}$ and $\tau_s(t) \le h_s$ where $\hat{\mathbf{M}}_i = \hat{\mathbf{Y}}_i = \mathbf{P}^{\mathrm{T}}\begin{bmatrix}\mathbf{0}\\ \mathbf{G}_i\\ \mathbf{0}\\ \mathbf{0}\end{bmatrix}$, $\mathbf{S}_i = \mathbf{S}_i^{\mathrm{T}} \in \Re^{4n\times 4n}$ and $\begin{bmatrix}\mathbf{S}_i & \hat{\mathbf{Y}}_i\\ \hat{\mathbf{Y}}_i^{\mathrm{T}} & \mathbf{R}\end{bmatrix} \ge 0$, $i = 1, 2, \ldots, p$, we have

$$-2\sum_{i=1}^{p} m_i \begin{bmatrix}\mathbf{x}(t)\\ \mathbf{s}(t)\\ \mathbf{u}(t)\\ \mathbf{y}(t)\end{bmatrix}^{\mathrm{T}} \mathbf{P}^{\mathrm{T}}\begin{bmatrix}\mathbf{0}\\ \mathbf{G}_i\\ \mathbf{0}\\ \mathbf{0}\end{bmatrix}\int_{t-\tau_s(t)}^{t}\mathbf{y}(\varphi)d\varphi \le h_s\sum_{i=1}^{p} m_i \begin{bmatrix}\mathbf{x}(t)\\ \mathbf{s}(t)\\ \mathbf{u}(t)\\ \mathbf{y}(t)\end{bmatrix}^{\mathrm{T}} \times \mathbf{S}_i \begin{bmatrix}\mathbf{x}(t)\\ \mathbf{s}(t)\\ \mathbf{u}(t)\\ \mathbf{y}(t)\end{bmatrix} + \int_{t-\tau_s(t)}^{t}\mathbf{y}(\varphi)^{\mathrm{T}}\mathbf{R}\mathbf{y}(\varphi)d\varphi. \tag{A14}$$

From (A12)–(A14), we have

$$\dot{V}_1(t) \le \sum_{i=1}^{p}\sum_{j=1}^{p} w_i m_j \begin{bmatrix}\mathbf{x}(t)\\ \mathbf{s}(t)\\ \mathbf{u}(t)\\ \mathbf{y}(t)\end{bmatrix}^{\mathrm{T}} \times \left(\mathbf{P}^{\mathrm{T}}\begin{bmatrix}\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}\\ \mathbf{G}_j & -\mathbf{I} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{F}_j & -\mathbf{I} & \mathbf{0}\\ \mathbf{A}_i+\mathbf{A}_{di} & \mathbf{0} & \mathbf{B}_i & -\mathbf{I}\end{bmatrix} + \begin{bmatrix}\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}\\ \mathbf{G}_j & -\mathbf{I} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{F}_j & -\mathbf{I} & \mathbf{0}\\ \mathbf{A}_i+\mathbf{A}_{di} & \mathbf{0} & \mathbf{B}_i & -\mathbf{I}\end{bmatrix}^{\mathrm{T}}\mathbf{P} + h_d\mathbf{R}_i + h_s\mathbf{S}_j\right) \times \begin{bmatrix}\mathbf{x}(t)\\ \mathbf{s}(t)\\ \mathbf{u}(t)\\ \mathbf{y}(t)\end{bmatrix} + \int_{t-\tau_d}^{t}\mathbf{y}(\varphi)^{\mathrm{T}}\mathbf{R}\mathbf{y}(\varphi)d\varphi + \int_{t-\tau_s(t)}^{t}\mathbf{y}(\varphi)^{\mathrm{T}}\mathbf{R}\mathbf{y}(\varphi)d\varphi. \tag{A15}$$

From (A10), we have

$$\begin{aligned}\dot{V}_2(t) &= h_d\mathbf{y}(t)^{\mathrm{T}}\mathbf{R}\mathbf{y}(t) - \int_{t-h_d}^{t} \mathbf{y}(\varphi)^{\mathrm{T}}\mathbf{R}\mathbf{y}(\varphi)d\varphi \\ &\quad + h_s\mathbf{y}(t)^{\mathrm{T}}\mathbf{R}\mathbf{y}(t) - \int_{t-h_s}^{t} \mathbf{y}(\varphi)^{\mathrm{T}}\mathbf{R}\mathbf{y}(\varphi)d\varphi \\ &= \begin{bmatrix}\mathbf{x}(t)\\ \mathbf{s}(t)\\ \mathbf{u}(t)\\ \mathbf{y}(t)\end{bmatrix}^{\mathrm{T}} \begin{bmatrix}\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & (h_d+h_s)\mathbf{R}\end{bmatrix} \begin{bmatrix}\mathbf{x}(t)\\ \mathbf{s}(t)\\ \mathbf{u}(t)\\ \mathbf{y}(t)\end{bmatrix} \\ &\quad - \int_{t-h_d}^{t} \mathbf{y}(\varphi)^{\mathrm{T}}\mathbf{R}\mathbf{y}(\varphi)d\varphi - \int_{t-h_s}^{t} \mathbf{y}(\varphi)^{\mathrm{T}}\mathbf{R}\mathbf{y}(\varphi)d\varphi. \end{aligned} \tag{A16}$$

With the facts that $h_d \geq \tau_d$ and $h_s \geq \tau_s(t)$, they lead to $\int_{t-h_d}^{t} \mathbf{y}(\varphi)^{\mathrm{T}}\mathbf{R}\mathbf{y}(\varphi)d\varphi \geq \int_{t-\tau_d}^{t} \mathbf{y}(\varphi)^{\mathrm{T}}\mathbf{R}\mathbf{y}(\varphi)d\varphi$ and $\int_{t-h_s}^{t} \mathbf{y}(\varphi)^{\mathrm{T}}\mathbf{R}\mathbf{y}(\varphi)d\varphi \geq \int_{t-\tau_s(t)}^{t} \mathbf{y}(\varphi)^{\mathrm{T}}\mathbf{R}\mathbf{y}(\varphi)d\varphi$. From (A8), (A15), and (A16), we have (A17), shown at the bottom of the page, where $\mathbf{z}(t) = \mathbf{X}^{-1}\begin{bmatrix}\mathbf{x}(t)\\ \mathbf{s}(t)\\ \mathbf{u}(t)\\ \mathbf{y}(t)\end{bmatrix}$, $\mathbf{X} = \mathbf{P}^{-1} = \begin{bmatrix}\mathbf{X}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{X}_1 & \mathbf{X}_1 & \mathbf{0} & \mathbf{0}\\ \mathbf{X}_2 & \mathbf{X}_3 & \mathbf{X}_4 & \mathbf{0}\\ \mathbf{X}_5 & \mathbf{X}_6 & \mathbf{X}_7 & \mathbf{X}_8\end{bmatrix}$, $\mathbf{X}_1 = \mathbf{X}_1^{\mathrm{T}} = \mathbf{P}_1^{-1} \in \Re^{n\times n}$, $\mathbf{X}_2 \in \Re^{m\times n}$, $\mathbf{X}_3 \in \Re^{m\times n}$, $\mathbf{X}_4 \in \Re^{m\times m}$, $\mathbf{X}_5 \in \Re^{n\times n}$, $\mathbf{X}_6 \in \Re^{n\times n}$, $\mathbf{X}_7 \in \Re^{n\times m}$, $\mathbf{X}_8 \in \Re^{n\times n}$, and

$$\begin{aligned}\mathbf{Q}_{ij} &= \mathbf{P}^{\mathrm{T}}\begin{bmatrix}\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}\\ \mathbf{G}_j & -\mathbf{I} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{F}_j & -\mathbf{I} & \mathbf{0}\\ \mathbf{A}_i+\mathbf{A}_{di} & \mathbf{0} & \mathbf{B}_i & -\mathbf{I}\end{bmatrix} + \begin{bmatrix}\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}\\ \mathbf{G}_j & -\mathbf{I} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{F}_j & -\mathbf{I} & \mathbf{0}\\ \mathbf{A}_i+\mathbf{A}_{di} & \mathbf{0} & \mathbf{B}_i & -\mathbf{I}\end{bmatrix}^{\mathrm{T}}\mathbf{P} \\ &\quad + \begin{bmatrix}\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & (h_d+h_s)\mathbf{R}\end{bmatrix} + h_d\mathbf{R}_i + h_s\mathbf{S}_j.\end{aligned}$$

Let $\mathbf{X}^{\mathrm{T}}\mathbf{R}_i\mathbf{X} = \mathbf{M}_i = \mathbf{M}_i^{\mathrm{T}} \in \Re^{(3n+m)\times(3n+m)}$, $\mathbf{X}^{\mathrm{T}}\mathbf{S}_i\mathbf{X} = \mathbf{T}_i = \mathbf{T}_i^{\mathrm{T}} \in \Re^{(3n+m)\times(3n+m)}$, $\mathbf{G}_i = \mathbf{N}_i\mathbf{X}_1^{-1}$, and $\mathbf{F}_i = \mathbf{K}_i\mathbf{X}_1^{-1}$ where $\mathbf{N}_i \in \Re^{n\times n}$ and $\mathbf{K}_i \in \Re^{m\times n}$, $i = 1, 2, \ldots, p$. From (A17), we consider the term in the right-hand side of the last equation which is restated in (A18), shown at the bottom of the next page.

When $\mathbf{Q} = \sum_{i=1}^{p}\sum_{j=1}^{p} w_i m_j \mathbf{X}^{\mathrm{T}}\mathbf{Q}_{ij}\mathbf{X} < 0$, it can be seen from (A17) that, $\dot{V}(t) \leq 0$ which implies the asymptotic stability of (11). From (A18), by Schur complement, $\mathbf{Q} < 0$ is equivalent to the following condition:

$$\overline{\mathbf{Q}} = \sum_{i=1}^{p}\sum_{j=1}^{p} w_i m_j \overline{\mathbf{Q}}_{ij} < 0 \tag{A19}$$

where $\mathbf{M} = \mathbf{R}^{-1} \in \Re^{n\times n}$ and $\overline{\mathbf{Q}}_{ij}$ is defined as shown at the bottom of the next page.

The symbol "$*$" denotes the transposed element of the matrix at the corresponding position. From (A19), let $\rho > 0$

$$\begin{aligned}\dot{V}(t) &\leq \sum_{i=1}^{p}\sum_{j=1}^{p} w_i m_j \begin{bmatrix}\mathbf{x}(t)\\ \mathbf{s}(t)\\ \mathbf{u}(t)\\ \mathbf{y}(t)\end{bmatrix}^{\mathrm{T}} \left(\mathbf{P}^{\mathrm{T}}\begin{bmatrix}\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}\\ \mathbf{G}_j & -\mathbf{I} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{F}_j & -\mathbf{I} & \mathbf{0}\\ \mathbf{A}_i+\mathbf{A}_{di} & \mathbf{0} & \mathbf{B}_i & -\mathbf{I}\end{bmatrix} + \begin{bmatrix}\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}\\ \mathbf{G}_j & -\mathbf{I} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{F}_j & -\mathbf{I} & \mathbf{0}\\ \mathbf{A}_i+\mathbf{A}_{di} & \mathbf{0} & \mathbf{B}_i & -\mathbf{I}\end{bmatrix}^{\mathrm{T}}\mathbf{P}\right. \\ &\qquad \left. + h_d\mathbf{R}_i + h_s\mathbf{S}_j + \begin{bmatrix}\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & (h_d+h_s)\mathbf{R}\end{bmatrix}\right)\begin{bmatrix}\mathbf{x}(t)\\ \mathbf{s}(t)\\ \mathbf{u}(t)\\ \mathbf{y}(t)\end{bmatrix} \\ &\quad + \int_{t-\tau_d}^{t} \mathbf{y}(\varphi)^{\mathrm{T}}\mathbf{R}\mathbf{y}(\varphi)d\varphi + \int_{t-\tau_s(t)}^{t} \mathbf{y}(\varphi)^{\mathrm{T}}\mathbf{R}\mathbf{y}(\varphi)d\varphi - \int_{t-h_d}^{t} \mathbf{y}(\varphi)^{\mathrm{T}}\mathbf{R}\mathbf{y}(\varphi)d\varphi - \int_{t-h_s}^{t} \mathbf{y}(\varphi)^{\mathrm{T}}\mathbf{R}\mathbf{y}(\varphi)d\varphi \\ &\leq \sum_{i=1}^{p}\sum_{j=1}^{p} w_i m_j \mathbf{z}(t)^{\mathrm{T}}\mathbf{X}^{\mathrm{T}}\mathbf{Q}_{ij}\mathbf{X}\mathbf{z}(t)\end{aligned} \tag{A17}$$

which is chosen such that $w_i - \rho m_i > 0$, $i = 1, 2, \ldots, p$, for all $\mathbf{x}(t)$ and $\mathbf{x}(t_\gamma)$, and using the property of $\sum_{j=1}^{p} m_j = \sum_{i=1}^{p}\sum_{j=1}^{p} w_i m_j = 1$, we have

$$\begin{aligned}\overline{\mathbf{Q}} &= \sum_{i=1}^{p}\sum_{j=1}^{p}(w_i - \rho m_i + \rho m_i)m_j\overline{\mathbf{Q}}_{ij}\\ &= \rho\sum_{i=1}^{p} m_i\overline{\mathbf{Q}}_{ii} + \sum_{i=1}^{p}\sum_{j=1}^{p}(w_i - \rho m_i)m_j\overline{\mathbf{Q}}_{ij}\\ &\quad + \sum_{i=1}^{p}\sum_{j=1}^{p}(w_i - \rho m_i)m_j(\mathbf{\Lambda}_j - \mathbf{\Lambda}_j)\\ &= \rho\sum_{i=1}^{p} m_i\overline{\mathbf{Q}}_{ii} + \sum_{i=1}^{p}\sum_{j=1}^{p}(w_i - \rho m_i)m_j\overline{\mathbf{Q}}_{ij}\\ &\quad + \sum_{i=1}^{p}\sum_{j=1}^{p}(w_i - \rho m_i)m_j\mathbf{\Lambda}_j - \rho\sum_{j=1}^{p} m_j\frac{(1-\rho)}{\rho}\mathbf{\Lambda}_j\\ &= \rho\sum_{i=1}^{p} m_i\left(\overline{\mathbf{Q}}_{ii} - \frac{(1-\rho)}{\rho}\mathbf{\Lambda}_i\right)\\ &\quad + \sum_{i=1}^{p}\sum_{j=1}^{p}(w_i - \rho m_i)m_j(\overline{\mathbf{Q}}_{ij} + \mathbf{\Lambda}_j) \end{aligned} \tag{A20}$$

where $\mathbf{\Lambda}_j = \mathbf{\Lambda}_j^{\mathrm{T}} \in \Re^{(4n+m)\times(4n+m)}$, $j = 1, 2, \ldots, p$, are arbitrary matrices. From (20), it can be seen that $\mathbf{Q} < 0$ if the stability conditions of $\overline{\mathbf{Q}}_{ii} - ((1-\rho)/\rho)\mathbf{\Lambda}_i < 0$ and $\overline{\mathbf{Q}}_{ij} + \mathbf{\Lambda}_j < 0$ for all i and j are satisfied. The introduction of $\mathbf{\Lambda}j$ is to share the unstable elements between the two matrices inside the summation terms in order to reduce the conservativeness of the stability conditions.

It should be noted that the holding of the inequality of (A13) requires $\begin{bmatrix}\mathbf{R}_i & \mathbf{Y}_i\\ \mathbf{Y}_i^{\mathrm{T}} & \mathbf{R}\end{bmatrix} \geq 0$ for all i. Postmultiply $\begin{bmatrix}\mathbf{X} & \mathbf{0}\\ \mathbf{0} & \mathbf{X}_1\end{bmatrix}^{\mathrm{T}}$ and premultiply $\begin{bmatrix}\mathbf{X} & \mathbf{0}\\ \mathbf{0} & \mathbf{X}_1\end{bmatrix}$ to $\begin{bmatrix}\mathbf{R}_i & \mathbf{Y}_i\\ \mathbf{Y}_i^{\mathrm{T}} & \mathbf{R}\end{bmatrix}$, we have

$$\begin{aligned}&\begin{bmatrix}\mathbf{X} & \mathbf{0}\\ \mathbf{0} & \mathbf{X}_1\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\mathbf{R}_i & \mathbf{Y}_i\\ \mathbf{Y}_i^{\mathrm{T}} & \mathbf{R}\end{bmatrix}\begin{bmatrix}\mathbf{X} & \mathbf{0}\\ \mathbf{0} & \mathbf{X}_1\end{bmatrix}\\ &= \begin{bmatrix}\mathbf{X}^{\mathrm{T}}\mathbf{R}_i\mathbf{X} & \mathbf{X}^{\mathrm{T}}\mathbf{Y}_i\mathbf{X}_1\\ \mathbf{X}_1\mathbf{Y}_i^{\mathrm{T}}\mathbf{X} & \mathbf{X}_1\mathbf{R}\mathbf{X}_1\end{bmatrix}\\ &= \begin{bmatrix}\mathbf{M}_i & *\\ \mathbf{X}_1\mathbf{Y}_i^{\mathrm{T}}\mathbf{X} & \mathbf{X}_1\mathbf{M}^{-1}\mathbf{X}_1\end{bmatrix}\\ &\geq 0, \qquad i =, 1, 2, \ldots, p. \end{aligned} \tag{A21}$$

It should be noted that (A21) is not an LMI due to the existence of the term $\mathbf{X}\mathbf{M}^{-1}\mathbf{X}$. With the property that $\mathbf{M} = \mathbf{M}^{\mathbf{T}} > 0$, we consider the following inequality:

$$\begin{aligned}&(\mathbf{X}_1 - \zeta\mathbf{M})^{\mathrm{T}}\mathbf{M}^{-1}(\mathbf{X}_1 - \zeta\mathbf{M})\\ &\quad = \mathbf{X}_1^{\mathrm{T}}\mathbf{M}^{-1}\mathbf{X}_1 - \zeta\mathbf{X}_1^{\mathrm{T}} - \zeta\mathbf{X}_1 + \zeta^2\mathbf{M}\\ &\quad > 0 \Rightarrow \mathbf{X}_1\mathbf{M}^{-1}\mathbf{X}_1 > 2\zeta\mathbf{X}_1 - \zeta^2\mathbf{M} \end{aligned} \tag{A22}$$

$$\begin{aligned}\mathbf{Q} &= \sum_{i=1}^{p}\sum_{j=1}^{p} w_i m_j \mathbf{X}^{\mathrm{T}}\mathbf{Q}_{ij}\mathbf{X}\\ &= \sum_{i=1}^{p}\sum_{j=1}^{p} w_i m_j\left(\begin{bmatrix}\mathbf{X}_5 + \mathbf{X}_5^{\mathrm{T}} & * & * & *\\ \mathbf{N}_j - \mathbf{X}_1 + \mathbf{X}_6^{\mathrm{T}} & -\mathbf{X}_1 - \mathbf{X}_1^{\mathrm{T}} & * & *\\ \mathbf{K}_j - \mathbf{X}_2 + \mathbf{X}_7^{\mathrm{T}} & \mathbf{K}_j - \mathbf{X}_3 & -\mathbf{X}_4 - \mathbf{X}_4^{\mathrm{T}} & *\\ (\mathbf{A}_i + \mathbf{A}_{di})\mathbf{X}_1 + \mathbf{B}_i\mathbf{X}_2 - \mathbf{X}_5 + \mathbf{X}_8^{\mathrm{T}} & \mathbf{B}_i\mathbf{X}_3 - \mathbf{X}_6 & \mathbf{B}_i\mathbf{X}_4 - \mathbf{X}_7 & -\mathbf{X}_8 - \mathbf{X}_8^{\mathrm{T}}\end{bmatrix}\right.\\ &\qquad \left. + h_d\mathbf{M}_i + h_s\mathbf{T}_j + (h_d + h_s)\begin{bmatrix}\mathbf{X}_5^{\mathrm{T}}\\ \mathbf{X}_6^{\mathrm{T}}\\ \mathbf{X}_7^{\mathrm{T}}\\ \mathbf{X}_8^{\mathrm{T}}\end{bmatrix}\mathbf{R}\begin{bmatrix}\mathbf{X}_5^{\mathrm{T}}\\ \mathbf{X}_6^{\mathrm{T}}\\ \mathbf{X}_7^{\mathrm{T}}\\ \mathbf{X}_8^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}\right) \end{aligned} \tag{A18}$$

$$\begin{aligned}\overline{\mathbf{Q}}_{ij} &= \begin{bmatrix}\mathbf{X}_5 + \mathbf{X}_5^{\mathrm{T}} & * & * & * & *\\ \mathbf{N}_j - \mathbf{X}_1 + \mathbf{X}_6^{\mathrm{T}} & -\mathbf{X}_1 - \mathbf{X}_1^{\mathrm{T}} & * & * & *\\ \mathbf{K}_j - \mathbf{X}_2 + \mathbf{X}_7^{\mathrm{T}} & \mathbf{K}_j - \mathbf{X}_3 & -\mathbf{X}_4 - \mathbf{X}_4^{\mathrm{T}} & * & *\\ (\mathbf{A}_i + \mathbf{A}_{di})\mathbf{X}_1 + \mathbf{B}_i\mathbf{X}_2 - \mathbf{X}_5 + \mathbf{X}_8^{\mathrm{T}} & \mathbf{B}_i\mathbf{X}_3 - \mathbf{X}_6 & \mathbf{B}_i\mathbf{X}_4 - \mathbf{X}_7 & -\mathbf{X}_8 - \mathbf{X}_8^{\mathrm{T}} & *\\ (h_d + h_s)\mathbf{X}_5 & (h_d + h_s)\mathbf{X}_6 & (h_d + h_s)\mathbf{X}_7 & (h_d + h_s)\mathbf{X}_8 & -(h_d + h_s)\mathbf{M}\end{bmatrix}\\ &\quad + h_d\begin{bmatrix}\mathbf{M}_i & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix} + h_s\begin{bmatrix}\mathbf{T}_j & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\end{aligned}$$

where ζ is a nonzero positive scalar. From (A21) and (A22), it can be seen that the holding of the following LMIs implies the holding of (A21):

$$\begin{bmatrix} \mathbf{M}_i & * \\ \mathbf{X}_1\mathbf{Y}_i^{\mathrm{T}}\mathbf{X} & 2\zeta\mathbf{X}_1 - \zeta^2\mathbf{M} \end{bmatrix} \geq 0, \qquad i = 1, 2, \ldots, p. \tag{A23}$$

Similarly, the holding of the inequalities of (A14) requires $\begin{bmatrix} \mathbf{S}_i & \hat{\mathbf{Y}}_i \\ \hat{\mathbf{Y}}_i^{\mathrm{T}} & \mathbf{R} \end{bmatrix} \geq 0$ for all i which hold when the following LMIs hold:

$$\begin{bmatrix} \mathbf{T}_i & * \\ \mathbf{X}_1\hat{\mathbf{Y}}_i^{\mathrm{T}}\mathbf{X} & 2\zeta\mathbf{X}_1 - \zeta^2\mathbf{M} \end{bmatrix} \geq 0, \qquad i = 1, 2, \ldots, p. \tag{A24}$$

The sampled-data time-delay fuzzy-model-based control system of (11) is asymptotically stable if $w_i - \rho m_i > 0$, $\overline{\mathbf{Q}}_{ii} - ((1-\rho)/\rho)\mathbf{\Lambda}_i < 0$ and $\overline{\mathbf{Q}}_{ij} + \mathbf{\Lambda}_j < 0$ for all i and j and the LMI-based stability conditions of (A23) and (A24) hold. Furthermore, it can be seen that if there exists a solution to the stability conditions. It implies that $\mathbf{X}_1 = \mathbf{X}_1^{\mathrm{T}} > 0$, $\mathbf{X}_4 + \mathbf{X}_4^{\mathrm{T}} > 0$ and $\mathbf{X}_8 + \mathbf{X}_8^{\mathrm{T}} > 0$ which are sufficient conditions for $\mathbf{X}$ to be a nonsingular matrix to guarantee the existence of the inverse of $\mathbf{X} = \mathbf{P}^{-1}$. **Q.E.D.**

APPENDIX B

The proof of the LMI-based performance conditions in Theorem 1 is given in this appendix. The system performance is quantitatively measured by the following scalar performance index [30]:

$$J = \int_0^\infty \begin{bmatrix} \mathbf{x}(t_\gamma) \\ \mathbf{s}(t_\gamma) \\ \mathbf{u}(t_\gamma) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \mathbf{J}_1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{J}_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{J}_3 \end{bmatrix} \begin{bmatrix} \mathbf{x}(t_\gamma) \\ \mathbf{s}(t_\gamma) \\ \mathbf{u}(t_\gamma) \end{bmatrix} dt_\gamma \tag{B1}$$

where $\mathbf{J}_1 = \mathbf{J}_1^{\mathrm{T}} \in \Re^{n\times n} > 0$, $\mathbf{J}_2 = \mathbf{J}_2^{\mathrm{T}} \in \Re^{n\times n} > 0$ and $\mathbf{J}_3 = \mathbf{J}_3^{\mathrm{T}} \in \Re^{m\times m} > 0$. It can be seen from the performance index of (B1) that J is regarded as the integral of the energy of system states and control signals. The contribution of each term is governed by the corresponding weighting matrix $\mathbf{J}_1$, $\mathbf{J}_2$, or $\mathbf{J}_3$ determined by the designer. From (A3), (A4), and (B1), we have

$$J = \int_0^\infty \begin{bmatrix} \mathbf{x}(t_\gamma) \\ \mathbf{x}(t_\gamma) \\ \mathbf{s}(t_\gamma) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \sum_{i=1}^p m_i\mathbf{G}_i^{\mathrm{T}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \sum_{i=1}^p m_i\mathbf{F}_i^{\mathrm{T}} \end{bmatrix} \times \begin{bmatrix} \mathbf{J}_1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{J}_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{J}_3 \end{bmatrix} \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \sum_{i=1}^p m_i\mathbf{G}_i & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \sum_{i=1}^p m_i\mathbf{F}_i \end{bmatrix} \times \begin{bmatrix} \mathbf{x}(t_\gamma) \\ \mathbf{x}(t_\gamma) \\ \mathbf{s}(t_\gamma) \end{bmatrix} dt_\gamma. \tag{B2}$$

Let $J < \eta \int_{\tau_0}^{\tau_1} \begin{bmatrix} \mathbf{x}(t_\gamma) \\ \mathbf{x}(t_\gamma) \\ \mathbf{s}(t_\gamma) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \mathbf{X}_1^{-1}\mathbf{X}_1^{-1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{X}_1^{-1}\mathbf{X}_1^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{X}_1^{-1}\mathbf{X}_1^{-1} \end{bmatrix} \times \begin{bmatrix} \mathbf{x}(t_\gamma) \\ \mathbf{x}(t_\gamma) \\ \mathbf{s}(t_\gamma) \end{bmatrix} dt_\gamma$, where η is a nonzero positive scalar. The scalar performance index can be attenuated to a prescribed level governed by the value of η. Based on this condition and from (B2), we have (B3), shown at the bottom of the page.

From (B3) and recalling that $\mathbf{G}_i = \mathbf{N}_i\mathbf{X}_1^{-1}$ and $\mathbf{F}_i = \mathbf{K}_i\mathbf{X}_1^{-1}$, $i = 1, 2, \ldots, p$, we have

$$\int_{\tau_0}^{\tau_1} \begin{bmatrix} \mathbf{x}(t_\gamma) \\ \mathbf{x}(t_\gamma) \\ \mathbf{s}(t_\gamma) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \mathbf{X}_1^{-1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{X}_1^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{X}_1^{-1} \end{bmatrix} \times \mathbf{W} \begin{bmatrix} \mathbf{X}_1^{-1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{X}_1^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{X}_1^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{x}(t_\gamma) \\ \mathbf{x}(t_\gamma) \\ \mathbf{s}(t_\gamma) \end{bmatrix} dt_\gamma < 0 \tag{B4}$$

where

$$\mathbf{W} = \sum_{i=1}^p m_i \mathbf{W}_i^{(21)^{\mathrm{T}}} \left(-\mathbf{W}^{(22)^{-1}}\right) \sum_{j=1}^p m_j \mathbf{W}_j^{(21)} + \mathbf{W}^{(11)} \tag{B5}$$

$$\int_{\tau_0}^{\tau_1} \begin{bmatrix} \mathbf{x}(t_\gamma) \\ \mathbf{x}(t_\gamma) \\ \mathbf{s}(t_\gamma) \end{bmatrix}^{\mathrm{T}} \left(\begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \sum_{i=1}^p m_i\mathbf{G}_i^{\mathrm{T}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \sum_{i=1}^p m_i\mathbf{F}_i^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} \mathbf{J}_1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{J}_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{J}_3 \end{bmatrix} \times \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \sum_{i=1}^p m_i\mathbf{G}_i & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \sum_{i=1}^p m_i\mathbf{F}_i \end{bmatrix} - \eta \begin{bmatrix} \mathbf{X}_1^{-1}\mathbf{X}_1^{-1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{X}_1^{-1}\mathbf{X}_1^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{X}_1^{-1}\mathbf{X}_1^{-1} \end{bmatrix} \right) \begin{bmatrix} \mathbf{x}(t_\gamma) \\ \mathbf{x}(t_\gamma) \\ \mathbf{s}(t_\gamma) \end{bmatrix} dt_\gamma < 0 \tag{B3}$$

where $\mathbf{W}^{(11)} = \begin{bmatrix} -\eta\mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\eta\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\eta\mathbf{I} \end{bmatrix}$, $\mathbf{W}_i^{(21)} = \begin{bmatrix} \mathbf{X}_1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{N}_i & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{K}_i \end{bmatrix}$, $i = 1, 2, \ldots, p$, and $\mathbf{W}^{(22)} = \begin{bmatrix} -\mathbf{J}_1^{-1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\mathbf{J}_2^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\mathbf{J}_3^{-1} \end{bmatrix}$. It can be seen that the inequality of (B4) holds when $\mathbf{W} < 0$. From (B5) and by Schur complement, $\mathbf{W} < 0$ is equivalent to the following inequality:

$$\overline{\mathbf{W}} = \sum_{i=1}^{p} m_i \mathbf{W}_i < 0 \tag{B6}$$

where $\mathbf{W}_i = \begin{bmatrix} \mathbf{W}^{(11)} & * \\ \mathbf{W}_i^{(21)} & \mathbf{W}^{(22)} \end{bmatrix}$, $i = 1, 2, \ldots, p$.

It can be seen that inequality of (B6) holds when $\mathbf{W}_i < 0$, $i = 1, 2, \ldots, p$, which are the LMI-based performance conditions summarized in Theorem 1. **Q.E.D.**

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