The Application of Robustness Analysis to the Conflict With Incomplete Information

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***Abstract*—When players with different interests try to achieve a better state, conflicts among players arise. Conflicts may arise also among public players. For example, a local government may insist on the interest of the region while the national government represents the interests of the whole country.**

Conflict analysis is one of the methods to model such conflicts mathematically. Its stability analysis specifies stable states based on the ordinal information on players' preferences. However, if the preference of a player is private, stability of states is not known. In such a case, players or third parties have to collect additional information on other players' preference. It is necessary to specify the minimum information to collect.

In this paper, graph model for conflict resolution (GMCR) is extended for the cases with incomplete information. Then, the generalized robustness analysis is proposed to specify the minimum conditions for stability of states. Finally, robustness analysis is applied to the conflict on water resources development.

***Index Terms*—Conflict with incomplete information, graph model for conflict resolution (GMCR), robustness analysis, two-player conflict.**

I. Introduction

FRASER and Hipel [1] proposed conflict analysis based on metagame theory [2]. Conflict analysis defines stability of states and specifies stable states. Nash equilibrium in game theory is one of the concepts of stability. Fang *et al.* [3] extended the methodology and proposed graph model for conflict resolution (GMCR). In GMCR, agents' moves between states are extended to include common and irreversible moves.

In order to analyze the stability of states, it is critical to know the preferences of players. However, it is often difficult to obtain complete information. That is why minimum conditions for stability should be specified before the inspection of preferences. Okada *et al.* [4] proposed robustness analysis to identify the minimum conditions on players' preferences in the 2-player conflicts where one player's preference is not known to another player. On the other hand, the third party may not have enough information on both players' preferences.

In this paper, robustness analysis is extended to apply to the graph model for 2-player conflicts where both players' preferences are not known each other. In Section II, GMCR is extended for the cases with incomplete preference information. In Section III, generalized robustness analysis is proposed. In Section IV, the methodology of robustness analysis is applied to the conflict on water resources development.

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Player 2 / Player 1	Confess	Not Confess
Confess	1	2
Not Confess	3	4

(The numbers in the matrix mean states.)

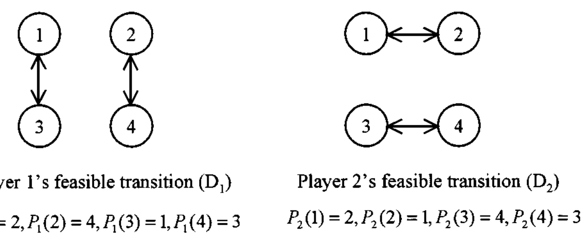

Fig. 1. Strategic form (upper) and graph form (below) of "prisoners' dilemma."

II. Graph Model for Conflict Resolution With Incomplete Information

A. Graph Model for Conflict Resolution (GMCR)

Fang *et al.* [3] proposed the GMCR as the representation of conflicts among players. Let $N = \{1, 2, \ldots, n\}$ be the set of players and $K = \{1, 2, \ldots k\}$ be the set of states of the conflict. We also define N-tuple $\{D_i\} (i = 1, 2, \ldots, n)$ as the set of directed graphs that $D_i = (K, V_i)$. The set of arcs V_i means player i's possible move between states. Let $k_1 k_2$ be the arc from the state k_1 to the state k_2. If $k_1 k_2 \in V_i$, player i can move from the state k_1 to the state k_2 unilaterally. We also need to define the payoff function P_i: $K \rightarrow R$ (R: the set of real numbers). Payoff function determines players' evaluations of the state in K, and specifies players' preference orders. If $P_i(k_1)$ is larger than $P_i(k_2)$, player i prefers the state k_1 to the state k_2. GMCR is represented by 4-tuple $\{N, K, V, P\}$, where $N = \{1, 2, \ldots n\}$, $K = \{1, 2, \ldots k\}$, $V = \{V_1, V_2, \ldots, V_n\}$, and $P = \{P_i | K \rightarrow R, i \in N\}$. Fig. 1 shows both strategic form and graph form of the "prisoner's dilemma." In this case, $P_1(1) = 2$, $P_1(2) = 4$, $P_1(3) = 1$, $P_1(4) = 3$ and $P_2(1) = 2$, $P_2(2) = 1$, $P_2(3) = 4$, $P_2(4) = 3$, respectively.

The following are other definitions used in GMCR.

Player 1 \ Player 2	Not Cooperate	Cooperate
Not Cooperate	1	2
Cooperate	3	4

(The numbers in the matrix mean states.)

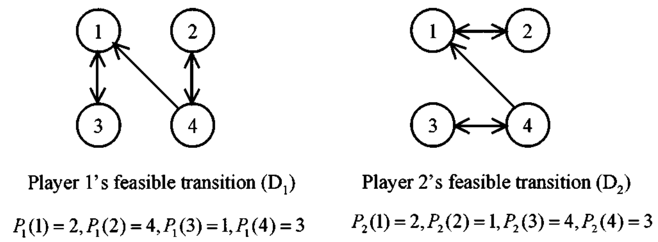

Fig. 2. Strategic form (upper) and graph form (below) of formulation of cooperation.

a) State k's reachable list $S_i(k)$
$k_1 \in S_i(k)$ if player i can move unilaterally from the state k to the state k_1 ($kk_1 \in V_i$).

b) State k's unilateral improvement (UI) $S_i^+(k)$
$k_1 \in S_i^+(k)$ if $k_1 \in S_i(k)$ and $P_i(k_1) > P_i(k)$.

Compared with strategic form, the major advantage of graph form is that it can represent common and irreversible moves. The definitions of common and irreversible moves are as follows.

Definition:

- The move k_1k_2 is common move if $k_1k_2 \in V_i$ and $k_1k_2 \in V_j$ for $i, j \in N$ and $i \neq j$.
- The move k_1k_2 is irreversible move if $k_1k_2 \in V_i$ and $k_2k_1 \notin V_i$ for $i \in N$.

In strategic form, states are defined as the combination of players' strategies. As a result, the move between two states is possible only when one player changes its strategy. However, in real conflicts, plural players may be able to move to the same states, and the choice such as "nuclear attack" in military conflicts, leads a conflict to the irreversible result, which cannot be returned. GMCR can incorporate common and irreversible moves into the model.

Fig. 2 shows the game on the formulation of cooperation between two players. The difference from prisoner's dilemma in Fig. 1 is the existence of common and irreversible moves from the state 4 to the state 1. It is assumed that both players have two alternatives, "collaborate" and "not collaborate" in strategic form. To realize cooperation, both players need to take the alternative "collaborate." However, cooperation collapses even if one player changes its alternative from "collaborate" to "not collaborate." Once cooperation is collapsed by the common move, players cannot return the same path to cooperative state directly. They have to change their strategies one by one again.

B. Definition of Stability and Equilibrium in Two-Player Conflict

Based on Fang *et al.* [3], some definitions on stability are shown as follows. Here we assume a 2-player game. The following are the solution concepts used in graph model for conflict [3].

Definition:

- Nash Stability
  The state k is Nash stable for player i if, and only if, i cannot improve its payoff by changing his own strategies. In other words, $S_i^+(k) = \{\varnothing\}$.
- Sequential Stability [4]
  The state k is sequentially stable for player i if, and only if, for every $k_1 \in S_i^+(k)$, there exists $k_2 \in S_j^+(k_1)$ with $P_i(k) \geq P_i(k_2)$. (Here, k_2 is called the sanction for player is UI, k_1.)
- General Metarationality (GMR)
  The state k is general metarational for player $i$if, and only if, for every $k_1 \in S_i^+(k)$, there exists $k_2 \in S_j(k_1)$ with $P_i(k) \geq P_i(k_2)$.

We use these solution concepts also in the following robustness analysis.

C. Extension of Graph Model for Conflict Resolution for the Case With Incomplete Preference Information

Potential users of GMCR are players themselves, consultants advising players, third parties analyzing conflicts, mediator, etc. (Fang *et al.* [3]). Let us call such users "analysts." In some cases, an analyst does not have complete information on players' preferences. When one player is an analyst, it may not know about its counterparts' preferences. Consultants, third parties, or mediator may have only limited information on all players' preferences.

Before presenting the GMCR with incomplete information, we introduce a binary description for representing ordinal preference. $k_1 \succ_i k_2$ means that player i (strictly) prefers k_1 to k_2, and $k_1 \succeq_i k_2$ means that player i strictly or equally prefers k_1 to k_2. On the other hand, $k_1 \sim_i k_2$ means that k_1 and k_2 are indifferent for player i.

Let the pattern of players' preference orders and the set of patterns be ω and Ω, respectively. When k states are strictly ordered (there are no indifferent states), the number of each player's preference orders amounts to $k!$. Consequently, the number of patterns of n players' preference orders becomes $(k!)^n$ and $|\Omega| = (k!)^n$.

If an analyst has complete information, it can recognize ω exactly. However, if an analyst has only limited knowledge, it only recognizes that the pattern of preference orders is included in the subset of Ω. Let π be the subset of Ω representing an analyst's knowledge on players' preferences. Although an analyst knows that the true pattern of preference orders is certainly included in the set π, it does not know which is a true pattern in π.

Using information set π, we propose GMCR with incomplete information. GMCR under analyst's information set π is represented by 4-tuple $\{N, K, V, \pi\}$, where $N = \{1, 2, \ldots n\}$, $K = \{1, 2, \ldots k\}$, $V = \{V_1, V_2, \ldots, V_n\}$, and $\pi \subseteq \Omega$. When π is a singleton set, an analyst has complete information.

Here we define the sets that represent analyst's knowledge on preferences. Ordered sets on analyst's information set π $[\Phi_i^+(k, \pi), \Phi_i^-(k, \pi), \Phi_i^\wedge(k, \pi)$, and $\Phi_i^*(k, \pi)]$:

- The state k_1 belongs to $\Phi_i^+(k, \pi)$ if k_1 is preferred to k by player i at every pattern of preference orders in information set π ($k_1 \succ_i k \ \ \forall \omega \in \pi$).
- The state k_1 belongs to $\Phi_i^\wedge(k, \pi)$ if k_1 is equally preferred to k by player i at every pattern of preference orders in information set π ($k_1 \sim_i k \ \ \forall \omega \in \pi$).
- The state k_1 belongs to $\Phi_i^-(k, \pi)$ if k_1 is less preferred to k by player i at every pattern of preference orders in information set π ($k_1 \prec_i k \ \ \forall \omega \in \pi$).
- The state k_1 belongs to $\Phi_i^*(k, \pi)$ if k_1 does not belong to either $\Phi_i^+(k, \pi)$, $\Phi_i^\wedge(k, \pi)$, or $\Phi_i^-(k, \pi)$. In other words, an analyst does not know if player i prefers k_1 to k or not with information set π.

The product of reachable list and ordered set is defined as "ordered reachable list." That is

$$S_i^+(k, \pi) = S_i(k) \cap \Phi_i^+(k, \pi) \tag{1}$$

$$S_i^\wedge(k, \pi) = S_i(k) \cap \Phi_i^\wedge(k, \pi) \tag{2}$$

$$S_i^-(k, \pi) = S_i(k) \cap \Phi_i^-(k, \pi) \tag{3}$$

$$S_i^*(k, \pi) = S_i(k) \cap \Phi_i^*(k, \pi). \tag{4}$$

When π is a singleton set, $S_i^+(k, \pi) = S_i^+(k)$ and $S_i^*(k, \pi) = \{\emptyset\}$.

D. Stability in GMCR With Incomplete Information

Even if the information on players' preferences is incomplete for the analyst, stability analysis for some states can be carried out. For 2-player conflict, definitions for Nash stability, sequential stability, and GMR based on analyst's knowledge can be shown as follows.

Nash Stability Based on Analyst's Knowledge: The state k is Nash stable for player i based on analyst's information set if, and only if, the following condition is satisfied:

$$S_i^+(k, \pi) \cap S_i^*(k, \pi) = \{\emptyset\}. \tag{5}$$

Sequential Stability Based on Analyst's Knowledge: The state k is sequentially stable for player i based on analyst's knowledge if, and only if, the following condition is satisfied:

$$k_1 \in \Phi_i^-(k, \pi)d \text{ or } S_j^+(k_1, \pi) \cap \{\Phi_i^-(k, \pi) \cup \Phi_i^\wedge(k, \pi)\} \neq \{\emptyset\} \qquad \forall\, k_1 \in S_i(k). \tag{6}$$

General Metarationality Based on Analyst's Knowledge: The state k is general metarational for player i based on analyst's knowledge if the following condition is satisfied:

$$k_1 \in \Phi_i^-(k, \pi) \text{ or } S_j(k_1, \pi) \cap \{\Phi_i^-(k, \pi) \cup \Phi_i^\wedge(k, \pi)\} \neq \{\emptyset\} \qquad \forall\, k_1 \in S_i(k). \tag{7}$$

Nash stability is decided without any information on other player's preference. If a player does not have any UIs, the state is Nash stable for the player. On the other hand, sequential stability and GMR depends on the player's knowledge on its counterpart's preference. If a player knows that its counterpart has a UI (in sequential stability) that reduces its payoff, the player gives up moving from the current state.

In the next section, robustness analysis that specifies the necessary and sufficient conditions for the stability of states is shown.

III. Generalization of Robustness Analysis

A. Robustness Analysis

Stability analysis in conflict analysis decides if the corresponding state is stable or not, based on the solution concepts. However, if an analyst does not know players' preferences, it cannot judge the stability of the state.

In the actual situation, an analyst does not necessarily have the complete information at the beginning of the analysis. Therefore, when an analyst needs to know about the stability, it has to collect information and renew its knowledge. Okada *et al.* [4] proposed robustness analysis to specify the minimum conditions that is necessary to judge the stability of the corresponding state. robustness analysis is a kind of an inverse problem of stability analysis.

Okada *et al.* [4] proposed robustness analysis for 2-player conflict in which the preference of one player is not known. In this paper, we generalize the methodology to apply 2-player conflict under arbitrary information set of an analyst. The generalized robustness analysis can be applied to the conflict where both players' preferences are not known. Analysts can use the result of robustness analysis as follows.

Players: Although a player knows its own preference, it may not have enough information on its counterpart's preference. Robustness analysis can provide the minimum information guaranteeing that a state can become a resolution of a conflict.

Mediator: Mediator tries to find the state that can be accepted as a compromise by both players. The stable state has high possibility to be accepted by players. However, in many cases, mediator has incomplete information on both players' preferences. If a mediator can confirm the condition for stability that is shown by robustness analysis, it can present the state as a proposal for agreement with conviction.

In the following parts, robustness analysis is generalized.

B. The Conditions for Stability in 2-Player Conflicts

Here we show the conditions that are necessary for the third party to judge if the corresponding state is stable or not. Since these conditions are specified based on the third party's knowledge, the information sets on the third party's knowledge are used. The sufficient condition for stability of a state is represented by some inequalities on players' preference. We call the set of these conditions "condition set." In the following subsection, conditions for Nash stability, sequential stability, and GMR are formulated.

1) The Condition for Nash Stability: The state k is Nash stable for player i if the UI from state k by player i is empty set [$S_i^+(k) = \{\emptyset\}$]. In GMCR under the information set π, $S_i^+(k, \pi)$ is the subset of $S_i^+(k)$, and $S_i^+(k)$ is included in $S_i^+(k, \pi) \cup S_i^*(k, \pi)$ (see Fig. 3). Consequently, the condition that the state k is Nash stable for player i is shown as follows.

(Presumption) $S_i^+(k, \pi) = \{\emptyset\}$: For every $k_1 \in S_i^*(k, \pi)$

$$k \succ_i k_1. \tag{8}$$

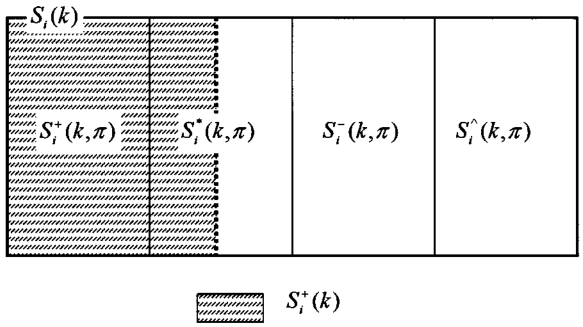

Fig. 3. Relationship between sets.

The number of conditions included in a sufficient condition set becomes $|S_i^*(k,\, \pi)|$. The number of sufficient condition sets for player i is one.

2) The Condition for Sequential Stability: The stake k is sequentially stable for player i if sanction exists for every state included in $S_i^+(k)$. Since $S_i^+(k,\, \pi)$ is the subset of $S_i^+(k)$, the existence of sanction for player i's movement to the state in $S_i^+(k,\, \pi)$ is necessary to guarantee sequential stability of the stake k. That is

- For every $k_1 \in S_i^+(k,\, \pi)$

  ① $k_2 \succ_j k_1$ and $k \succeq_i k_2$

$$\exists\, k_2 \in S_j^*(k_1,\, \pi) \tag{9}$$

  ② $k \succeq_i k_2$

$$\exists\, k_2 \in S_j^+(k_1,\, \pi). \tag{10}$$

Here, $j = 2$ if $i = 1$, and $j = 1$ if $i = 2$. ① or ② is the condition that player i does not move to the state which is known as UI for player i.

For the state in $S_i^*(k,\, \pi)$, it is not known if a state is included in $S_i^+(k)$ or not. If an analyst obtains information that shows that player i prefers k to the state in $S_i^*(k,\, \pi)$, it needs to find information showing existence of sanction. The flowchart for specifying conditions is shown in Fig. 4 and in the following inequalities.

- For every $k_1 \in S_i^*(k,\, \pi)$

  ③

$$k \succ_i k_1 \tag{11}$$

  ④ $k_2 \succ_j k_1$ and $k \succeq_i k_2$

$$\exists\, k_2 \in S_j^*(k_1,\, \pi) \tag{12}$$

  ⑤ $k \succeq_i k_2$

$$\exists\, k_2 \in S_j^+(k_1,\, \pi). \tag{13}$$

  ③ is the condition that k_1 is not UI for player i. ④ and ⑤ are the condition for the existence of the sanction for k_1.

The condition for sequential stability of the state k includes one of ① or ② for every $k_1 \in S_i^+(k,\, \pi)$ and ③–⑤ for every $k_1 \in S_i^*(k,\, \pi)$.

3) The Condition for General Metarationality: In the case of GMR, the condition for the existence of sanction is not necessary. Therefore, we need to assume a very conservative player when we use the solution concept of GMR.

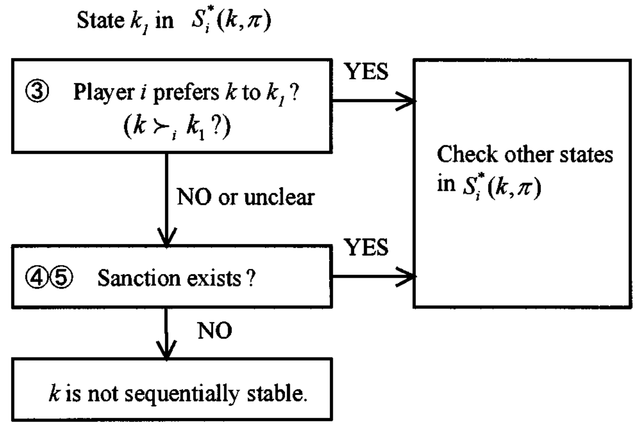

Fig. 4. Flowchart for specifying conditions for not moving to the state in $S_i^*(k,\, \pi)$ (sequential stability).

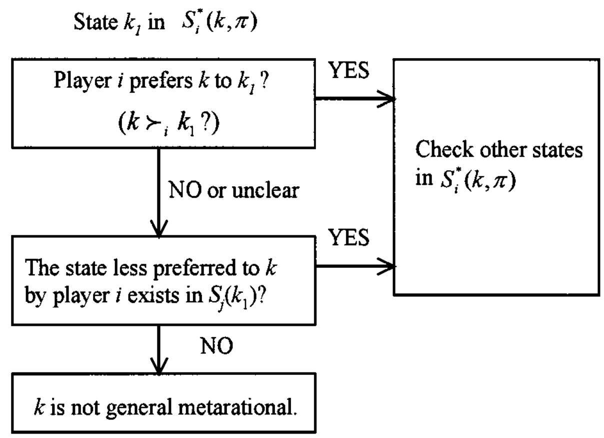

Fig. 5. Flowchart for specifying conditions for not moving to the state in $S_i^*(k,\, \pi)$ (general metarationality).

- For every $k_1 \in S_i^+(k,\, \pi)$

$$k \succeq_i k_2 \qquad \exists\, k_2 \in S_j(k_1). \tag{14}$$

If $S_j(k_1) \cap \Phi_i^-(k,\, \pi) \neq \{\varnothing\}$, k_2 can always be found.

In case of the states in $S_i^*(k,\, \pi)$, the process for checking possibility of player is move is similar to the process for sequential stability. First, an analyst checks if player i prefers k to the state in $S_i^*(k,\, \pi)$. If an analyst obtains information that shows that player i prefers k to the state in $S_i^*(k,\, \pi)$, it needs to find the information showing existence of sanction (see Fig. 5).

- For every $k_1 \in S_i^*(k,\, \pi)$,

$$k \succ_i k_1 \tag{15}$$

or

$$k \succeq_i k_2 \qquad \exists\, k_2 \in S_j(k_1). \tag{16}$$

If $S_j(k_1) \cap \Phi_i^-(k,\, \pi) \neq \{\varnothing\}$, k_2 can always be found.

The sets of Nash stable states and sequentially stable state are the subsets of general metarational states [3]. Therefore, the conditions that the state is general metarational (14)–(16) are the necessary conditions for other solution concepts. That implies that the state which satisfies the conditions for Nash stability or sequential stability is more robust than the state which only satisfies the conditions for GMR.

C. Example of Robustness Analysis

As an example, robustness analysis for the conflict shown in Fig. 6 is carried out. Since there are four states in the conflict, $|\Omega| = (4!)^2 = 576$. Let us assume that an analyst's information set π consists of the following eight pairs of preference orders:

$$\begin{array}{lcl} A \succ_1 B \succ_1 C \succ_1 D & \text{and} & B \succ_2 C \succ_2 A \succ_2 D \\ A \succ_1 B \succ_1 C \succ_1 D & \text{and} & C \succ_2 A \succ_2 B \succ_2 D \\ A \succ_1 C \succ_1 B \succ_1 D & \text{and} & B \succ_2 C \succ_2 A \succ_2 D \\ A \succ_1 C \succ_1 B \succ_1 D & \text{and} & C \succ_2 A \succ_2 B \succ_2 D \\ C \succ_1 A \succ_1 B \succ_1 D & \text{and} & B \succ_2 C \succ_2 A \succ_2 D \\ C \succ_1 A \succ_1 B \succ_1 D & \text{and} & C \succ_2 A \succ_2 B \succ_2 D \\ C \succ_1 A \succ_1 D \succ_1 B & \text{and} & B \succ_2 C \succ_2 A \succ_2 D \\ C \succ_1 A \succ_1 D \succ_1 B & \text{and} & C \succ_2 A \succ_2 B \succ_2 D. \end{array} \tag{17}$$

Ordered sets and ordered reachable lists are shown as follows.

Ordered Sets:

$$\begin{array}{l} \Phi_1^+(A, \pi) = \{\varnothing\}, \quad \Phi_1^-(A, \pi) = \{B, D\} \\ \Phi_1^\wedge(A, \pi) = \{\varnothing\}, \quad \Phi_1^*(A, \pi) = \{C\} \\ \Phi_1^+(B, \pi) = \{A\}, \quad \Phi_1^-(B, \pi) = \{\varnothing\} \\ \Phi_1^\wedge(B, \pi) = \{\varnothing\}, \quad \Phi_1^*(B, \pi) = \{C, D\} \\ \Phi_1^+(C, \pi) = \{\varnothing\}, \quad \Phi_1^-(C, \pi) = \{D\} \\ \Phi_1^\wedge(C, \pi) = \{\varnothing\}, \quad \Phi_1^*(C, \pi) = \{A, B\} \\ \Phi_1^+(D, \pi) = \{A, C\}, \quad \Phi_1^-(D, \pi) = \{\varnothing\} \\ \Phi_1^\wedge(D, \pi) = \{\varnothing\}, \quad \Phi_1^*(D, \pi) = \{B\} \\ \Phi_2^+(A, \pi) = \{C\}, \quad \Phi_2^-(A, \pi) = \{D\} \\ \Phi_2^\wedge(A, \pi) = \{\varnothing\}, \quad \Phi_2^*(A, \pi) = \{B\} \\ \Phi_2^+(B, \pi) = \{\varnothing\}, \quad \Phi_2^-(B, \pi) = \{D\} \\ \Phi_2^\wedge(B, \pi) = \{\varnothing\}, \quad \Phi_2^*(B, \pi) = \{A, C\} \\ \Phi_2^+(C, \pi) = \{\varnothing\}, \quad \Phi_2^-(C, \pi) = \{A, D\} \\ \Phi_2^\wedge(C, \pi) = \{\varnothing\}, \quad \Phi_2^*(C, \pi) = \{B\} \\ \Phi_2^+(D, \pi) = \{A, B, C\}, \quad \Phi_2^-(D, \pi) = \{\varnothing\} \\ \Phi_2^\wedge(D, \pi) = \{\varnothing\}, \quad \Phi_2^*(D, \pi) = \{\varnothing\} \end{array} \tag{18}$$

$$\begin{array}{l} S_1^+(A, \pi) = \{\varnothing\}, \quad S_1^-(\varnothing\ \pi) = \{\varnothing\}, \quad S_1^\wedge(A, \pi) = \{\varnothing\} \\ S_1^*(A, \pi) = \{C\} \quad S_1^+(B, \pi) = \{\varnothing\}, \quad S_1^-(B, \pi) = \{\varnothing\} \\ S_1^\wedge(B, \pi) = \{\varnothing\}, \quad S_1^*(B, \pi) = \{D\} \quad S_1^+(C, \pi) = \{\varnothing\} \\ S_1^-(C, \pi) = \{\varnothing\}, \quad S_1^\wedge(C, \pi) = \{\varnothing\}, \quad S_1^*(C, \pi) = \{A\} \\ S_1^+(D, \pi) = \{\varnothing\}, \quad S_1^-(D, \pi) = \{\varnothing\}, \quad S_1^\wedge(D, \pi) = \{\varnothing\} \\ S_1^*(D, \pi) = \{B\} \quad S_2^+(A, \pi) = \{\varnothing\}, \quad S_2^-(A, \pi) = \{\varnothing\} \\ S_2^\wedge(A, \pi) = \{\varnothing\}, \quad S_2^*(A, \pi) = \{B\} \quad S_2^+(B, \pi) = \{\varnothing\} \\ S_2^-(B, \pi) = \{\varnothing\}, \quad S_2^\wedge(B, \pi) = \{\varnothing\}, \quad S_2^*(B, \pi) = \{A\} \\ S_2^+(C, \pi) = \{\varnothing\}, \quad S_2^-(C, \pi) = \{D\}, \quad S_2^\wedge(C, \pi) = \{\varnothing\} \\ S_2^*(C, \pi) = \{\varnothing\} \quad S_2^+(D, \pi) = \{C\}, \quad S_2^-(D, \pi) = \{\varnothing\} \\ S_2^\wedge(D, \pi) = \{\varnothing\}, \quad S_2^*(D, \pi) = \{\varnothing\}. \end{array} \tag{19}$$

Let us think about the stability of the state B. Using the conditions shown in **C**, the state B is stable if the following relationships are satisfied.

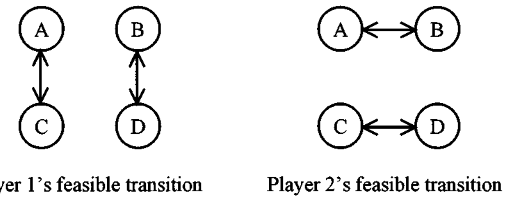

Fig. 6. Conflict consisting of four states.

Nash Stability:

Player 1

$$B \succ_1 D \tag{20}$$

Player 2

$$B \succ_2 A. \tag{21}$$

Sequential Stability:

Player 1

(a) $$B \succ_1 D \tag{22}$$

(b) $$B \succeq_1 C \tag{23}$$

Player 2

(a) $$B \succ_2 A \tag{24}$$

(b) $$C \succ_1 A \quad \text{and} \quad B \succeq_2 C. \tag{25}$$

General Metarationality:

Player 1

(a) $$B \succ_1 D \tag{26}$$

(b) $$B \succeq_1 C \tag{27}$$

Player 2

(a) $$B \succ_2 A \tag{28}$$

(b) $$B \succeq_2 C. \tag{29}$$

When two conditions [(a) and (b)] exist, the state B is stable if at least one of the two conditions is satisfied. The result of robustness analysis shows that limited number of preference relationships needs to be specified to guarantee the stability of a state.

IV. Application of Robustness Analysis

A. Planning Conflict

Environmental problem often involves conflicts between stakeholders. For example, Fang *et al.* [3] discuss the Garrison diversion unit conflict [5], which involves the governments of the United States and Canada, the government of Manitoba, environmentalists, etc. In the conflicts on environment, the alternative for mediating interests on development and environment is necessary. Therefore, it is important to guarantee the stability of a state.

In this section, the methodology of robustness analysis is applied to the conflict on hydropower generation and a river environment [6]–[8] (Fig. 7). There exist some reservoirs only for hydropower generation in a river basin. For the purpose of

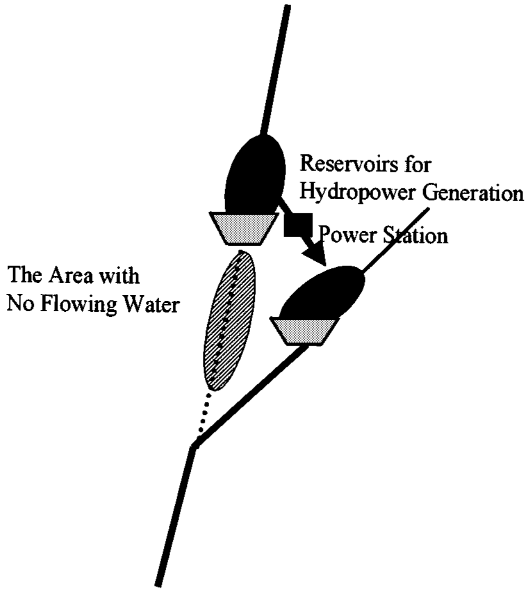

Fig. 7. Effects of hydropower generation on downstream river environment.

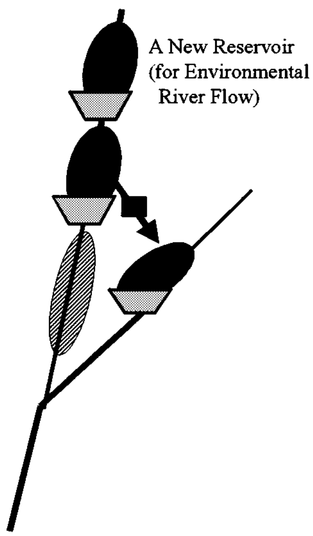

Fig. 8. Constructing a new reservoir for maintaining environmental river flow.

efficient use of potential energy, the water stored in reservoirs is often sent to the other reservoir directly. Bypassing water by pipelines may reduce a large amount of water downstream. That results in a shortage of river flow downstream. This shortage affects ecosystems, leisure, landscapes, sightseeing, and groundwater.

Several countermeasures (alternatives) to increase the flow of water are available. For example, local government can construct a new reservoir to obtain a storage capacity for environmental flow (see Fig. 8). In this case, local government can make a decision independently of existing user (hydropower generation). However, the resolution may result in an inefficient use of sites. Local government may not be able to find an appropriate site to construct an additional reservoir.

On the other hand, the power generation company may be able to reduce the level of hydropower generation (see Fig. 9). When this alternative is taken, the company has to generate electricity by other means (other reservoirs, thermal power generation, etc.) If the scale of hydropower generation is reduced, the benefit to the power generation company is reduced. Therefore, the power generation company may ask the local government to compensate the loss. The transfer of benefit can be interpreted as net benefit reallocation of the project.

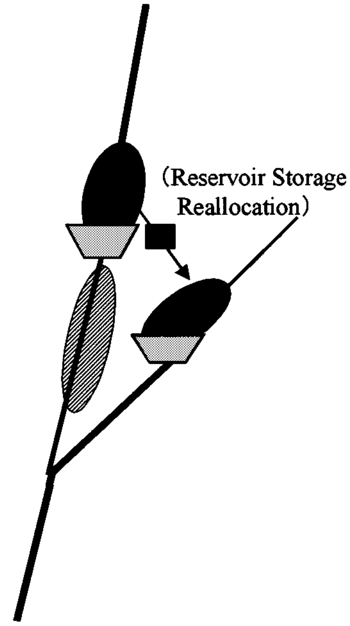

Fig. 9. Reservoir renewal (one example).

In this case, the power generation company represents the vested interests, while the local government represents the new interests of the river environment. A critical difference between them is that the power generation company (an existing user) can continue the hydropower generation in a status quo, while the local government cannot recover the river flow without taking any actions.

We assume two players. One is a local government and the other is a power generation company. In this conflict, the local government (Player 1) represents societal needs for better river environment. The purpose of the player is to conserve (or recover) the river environment by increasing discharge from a reservoir. We assume that the local government does not care about the benefit of a power generation company. The purpose of power generation company (Player 2) is to generate and sell electricity. Player 2 hopes to maintain the current level of hydropower generation. However, if it can achieve the cooperation with Player 1 (local government), it may have an incentive to change the current situation. Player 2's incentive depends on the net benefit which Player 2 can obtain.

Player 1 can maintain the status quo or construct a new reservoir by itself. These alternatives are called S and N, respectively. Player 2 has the alternative that it stays in the status quo. The alternative is called S. Redevelopment solutions are not realized if at least one player takes these strategies.

We assume the case where several types of redevelopment solutions are assumed. Redevelopment solutions are represented by the combination of the alternatives on net benefit allocation and structural measures. This is assumed as follows.

1) Player 1 can select the alternatives on net benefit allocation.
2) Player 2 can select the alternatives on structural measures.

Player 1's alternatives for redevelopment solutions (R_k):

- Low payment level (R_L).
- Medium payment level (R_M).
- High payment level (R_H).

Player 2's alternatives for redevelopment solutions (A_l):

- Using other power generation means (A_O).
- Adding hydropower generation in another reservoir (A_D).
- Upgrading the dam (A_U).

Redevelopment solutions are represented by R_kA_l. The redevelopment solution R_kA_l indicates that Player 1 takes strategy R_k on net benefit allocation and Player 2 takes strategy A_l on structural measure.

In order to realize redevelopment solutions, both players have to take the strategies compensating the loss (Player 1) and changing the status quo (Player 2), respectively. However, if Player 1 gives up compensating and preferred noncooperative solution (NS), the state is moved from a cooperative solution (R_kA_l) to a noncooperative solution (NS). Similarly, if Player 2 gives up taking a structural measure for redevelopment, the state is moved from R_iA_j to NS. On the other hand, the transition from NS to R_kA_l could not happen unilaterally. All of this shows that each player can cause the collapse of a cooperative solution unilaterally and the transition is an irreversible move.

Figs. 10 and 11 show D_1 and D_2 in the graph model for the planning conflict. The number of the feasible states is 20. The collapse of cooperation is represented by irreversible and common moves.

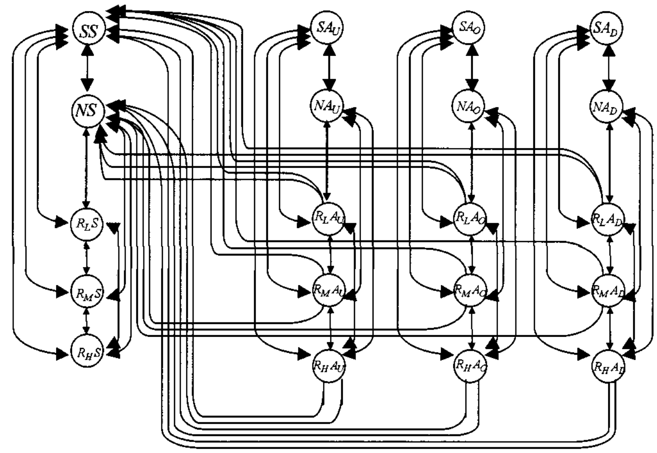

Fig. 10. Player 1's feasible transition between states.

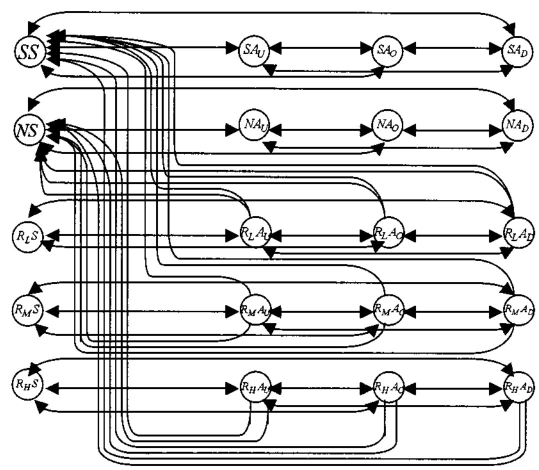

Fig. 11. Player 2's feasible transition between states.

B. Information Structure in Planning Conflict

In this planning conflict, we assume the following information structure:

$$
\begin{aligned}
R_mS &\in \{\Phi_1^-(R_kA_l,\pi)\cap\Phi_1^-(NS,\pi)\cap\Phi_1^-(SS,\pi)\}\\
&\qquad (m=O,D,U \quad k=O.D,U \quad l=L,M,H)\\
NA_n &\in \{\Phi_1^-(R_kA_l,\pi)\cap\Phi_1^-(NS,\pi)\cap\Phi_1^-(SS,\pi)\}\\
&\qquad (n=L,M,H \quad k=O.D,U \quad l=L,M,H)\\
SA_n &\in \{\Phi_1^-(R_kA_l,\pi)\cap\Phi_1^-(NS,\pi)\cap\Phi_1^-(SS,\pi)\}\\
&\qquad (n=L,M,H \quad k=O.D,U \quad l=L,M,H)\\
R_mS &\in \{\Phi_2^-(R_kA_l,\pi)\cap\Phi_2^-(NS,\pi)\cap\Phi_2^-(SS,\pi)\}\\
&\qquad (m=O,D,U \quad k=O.D,U \quad l=L,M,H)\\
NA_n &\in \{\Phi_2^-(R_kA_l,\pi)\cap\Phi_2^-(NS,\pi)\cap\Phi_2^-(SS,\pi)\}\\
&\qquad (n=L,M,H \quad k=O.D,U \quad l=L,M,H)\\
SA_n &\in \{\Phi_2^-(R_kA_l,\pi)\cap\Phi_2^-(NS,\pi)\cap\Phi_2^-(SS,\pi)\}\\
&\qquad (n=L,M,Hk=O.D,Ul=L,M,H). \qquad (30)
\end{aligned}
$$

Equation (30) reveals that temporary states are not preferred to the solutions which could become final solutions. We call R_kA_l, NS, and SS "real solutions." This information enables us to classify states into two groups.

On the other hand, it is also obvious that both players prefer the states in which its own payment is lower. That is

$$
\begin{gathered}
R_LA_l \succ_1 R_MA_l \succ_1 R_HA_l\\
R_LA_l \prec_2 R_MA_l \prec_2 R_HA_l\\
(A_l \text{ means Player 2s alternative}).
\end{gathered} \qquad (31)
$$

Using this information, we can make the order between these states. Table I shows the ordered sets $\Phi_i^+(R_MA_O,\pi)$, $\Phi_i^-(R_MA_O,\pi)$, and $\Phi_i^*(R_MA_O,\pi)$ in the planning conflict.

C. Players' Preferences and Social Efficiency

As regards players' preferences, the following relationships exist.

- If $R_kA_l \succ_1 NS$, Player 1 prefers the redevelopment solution R_kA_l to constructing a new reservoir by themselves.
- If $R_kA_l \prec_1 NS$, Player 1 prefers constructing a new reservoir by themselves to the redevelopment solution R_kA_l.
- If $R_kA_l \succ_1 SS$, Player 1 prefers the redevelopment solution R_kA_l to maintaining the status quo.
- If $R_kA_l \prec_1 SS$, Player 1 prefers maintaining the status quo to the redevelopment solution R_kA_l.

TABLE I
PLAYER 1S ORDERED SETS IN THE PLANNING CONFLICT [$\Phi_i^+(R_M A_O, \pi)$, $\Phi_i^*(R_M A_O, \pi)$, AND $\Phi_i^-(R_M A_O, \pi)$]

$\Phi_i^+(R_M A_O,\pi)$	$\Phi_i^*(R_M A_O,\pi)$	$\Phi_i^-(R_M A_O,\pi)$
$R_L A_O$	$R_L A_U$, $R_M A_U$, $R_H A_U$, $R_L A_D$, $R_M A_D$, $R_H A_D$, NS, SS	$R_H A_O$, $R_L S$, $R_M S$, $R_H S$, NA_O, NA_U, NA_D SA_O, SA_U, SA_D

- If $R_k A_l \succ_2 NS$, Player 2 prefers the redevelopment solution $R_k A_l$ to maintaining the status quo.
- If $R_k A_l \prec_2 NS$, Player 2 prefers maintaining the status quo to the redevelopment solution $R_k A_l$.

On the other hand, redevelopment solution $R_k A_l$ is better than the noncooperative solution for the community consisting of both players if the following conditions (32) are satisfied:

$$\begin{aligned} &R_k A_l \succ_1 NS, \quad R_k A_l \succ_2 NS \\ &R_k A_l \succ_1 SS, \quad R_k A_l \succ_2 SS. \end{aligned} \tag{32}$$

If (32) is satisfied, redevelopment option $R_k A_l$ should be implemented from the viewpoint of a community, because NS and SS are Pareto-dominated by $R_k A_l$. However, in a planning conflict, if the profit obtained by a player can be improved by moving from $R_k A_l$ to another states, the player does not have an incentive to stay at $R_k A_l$.

D. Application Results

Table II shows the conditions that state $R_M A_O$ (Player 1 pays the cost at medium level and uses other power generation means) is Nash stable or sequentially stable for Player 1 (A) and Player 2 (B). The inequalities in Table II are the condition sets which the planning authority has to detect to confirm that $R_M A_O$ is a stable renewal alternative.

From $R_M A_O$, Player 1 can always move to low payment level. In other words, it is common knowledge that Player 1 has a UI from $R_M A_O$ to $R_L A_O$. Therefore, it is obvious that $R_M A_O$ is not Nash stable for Player 1.

From Table II, we can obtain the following properties.

1) $R_M A_O$ is stable for Player 1 if a) Player 1 prefers $R_M A_O$ to NS and SS, and b) Player 2 can improve its payoff from $R_L A_O$ by changing its option from A_O (using other power generation means) to A_D (adding hydropower generation in other reservoir) or A_U (upgrading the dam) and Player 1 prefers $R_M A_O$ to the resulting state ($R_L A_D$ or $R_L A_U$) (b-1) or Player 2 prefers NS or SS to $R_L A_O$ (b-2).

If $R_M A_O$ is stable for Player 1, {(a) and (b-1)} or {(a) and (b-2)} must be satisfied.

2) $R_M A_O$ is stable for Player 2 if Player 2 prefers $R_M A_O$ to NS or SS (c), and Player 2 prefers $R_M A_O$ to $R_M A_D$ or $R_M A_U$ (d), Player 2 prefers $R_M A_O$ to $R_L A_D$ or $R_L A_U$ (e-1) or Player 1 prefers NS or SS to $R_M A_D$ or $R_M A_U$ (e-2).

If $R_M A_O$ is stable for Player 2, {(c) and (d)}, {(c) and (e-1)}, or {(c) and (e-2)} must be satisfied.

Sequential stability needs player's ability to forecast counterpart's sanction. In order to lead players to the stable state, the planning authority needs not only to inspect players' preferences, but also to hold the information jointly with players. That is, specified preference order should be made common knowledge.

E. Case With More Information

Now we assume another situation where the planning authority can obtain additional information on players' preference. In this case, the number of conditions included in sufficient condition sets can be reduced. Table III shows sufficient condition sets for $R_M A_O$s stability with following additional information.

1) Player 1 prefers arbitrary cooperative solution (reservoir renewal) $R_k A_l$ to both the status quo SS and the noncooperative solution NS

$$R_k A_l \succ_1 SS \quad \text{and} \quad R_k A_l \succ_1 NS \qquad (k = L, M, H, l = O, U, D). \tag{33}$$

2) Player 2 always prefers the structural measure A_O (using other power generation means) to A_D (adding hydropower generation in another reservoir) and prefers A_U (upgrading the dam) to A_O

$$R_k A_O \succ_2 R_k A_D \quad \text{and} \quad R_k A_U \succ_2 R_k A_O \qquad (k = L, M, H). \tag{34}$$

In this case, R_M is the second largest payment for Player 1 and A_O is the second preferred alternative for Player 2. $R_M A_O$ can be regarded as the compromising alternative.

Table III shows that $R_M A_O$s stability depends on the tradeoffs of both players' preference between structural alternatives and payment. That is, the critical conditions for stability are that both players prefer $R_M A_O$ to $R_L A_U$.

V. CONCLUSION

In this paper, we proposed the application of robustness analysis to the conflicts where the information on players' preferences is incomplete. Robustness analysis is generalized to apply the situation where preferences of both players are not known by the third party. Then, the methodology was applied to the planning conflict between two players representing hydropower generation and river environment.

If the third party has only incomplete information on preferences of stakeholders, it is necessary to use a different methodology for coordination, which is different from one where complete information is available. The robustness analysis can become a useful approach to detect the stable and better alternatives.

TABLE II
(a) RESULTS OF ROBUSTNESS ANALYSIS (STATE $R_M A_O$).
(b) CONDITION SETS FOR PLAYER 2S STABILITY

Condition Set 1	$SS \prec_1 R_M A_O$ $R_L A_D \preceq_1 R_M A_O$	$NS \prec_1 R_M A_O$ $R_L A_O \prec_2 R_L A_D$
Condition Set 2	$SS \prec_1 R_M A_O$ $R_L A_U \preceq_1 R_M A_O$	$NS \prec_1 R_M A_O$ $R_L A_O \prec_2 R_L A_U$
Condition Set 3	$SS \prec_1 R_M A_O$ $R_L A_O \prec_2 SS$	$NS \prec_1 R_M A_O$
Condition Set 4	$SS \prec_1 R_M A_O$ $R_L A_O \prec_2 NS$	$NS \prec_1 R_M A_O$

(a)

Condition Set 1	$SS \prec_2 R_M A_O$ or $NS \prec_2 R_M A_O$ $R_M A_D \prec_2 R_M A_O$	 $R_M A_U \prec_2 R_M A_O$
Condition Set 2	$SS \prec_2 R_M A_O$ or $NS \prec_2 R_M A_O$ $R_L A_D \preceq_2 R_M A_O$	 $R_M A_U \prec_2 R_M A_O$
Condition Set 3	$SS \prec_2 R_M A_O$ or $NS \prec_2 R_M A_O$ $R_M A_D \prec_1 SS$	 $R_M A_U \prec_2 R_M A_O$
Condition Set 4	$SS \prec_2 R_M A_O$ or $NS \prec_2 R_M A_O$ $R_M A_D \prec_1 NS$	 $R_M A_U \prec_2 R_M A_O$
Condition Set 5	$SS \prec_2 R_M A_O$ or $NS \prec_2 R_M A_O$ $R_M A_D \prec_2 R_M A_O$	 $R_M A_U \prec_1 SS$
Condition Set 6	$SS \prec_2 R_M A_O$ or $NS \prec_2 R_M A_O$ $R_M A_D \prec_2 R_M A_O$	 $R_M A_U \prec_1 NS$
Condition Set 7	$SS \prec_2 R_M A_O$ or $NS \prec_2 R_M A_O$ $R_M A_D \prec_2 R_M A_O$	 $R_L A_U \preceq_2 R_M A_O$
Condition Set 8	$SS \prec_2 R_M A_O$ or $NS \prec_2 R_M A_O$ $R_M A_D \prec_1 SS$	 $R_M A_U \prec_1 SS$
Condition Set 9	$SS \prec_2 R_M A_O$ or $NS \prec_2 R_M A_O$ $R_M A_D \prec_1 NS$	 $R_M A_U \prec_1 NS$
Condition Set 10	$SS \prec_2 R_M A_O$ or $NS \prec_2 R_M A_O$ $R_M A_D \prec_1 SS$	 $R_M A_U \prec_1 NS$
Condition Set 11	$SS \prec_2 R_M A_O$ or $NS \prec_2 R_M A_O$ $R_M A_D \prec_1 NS$	 $R_M A_U \prec_1 SS$
Condition Set 12	$SS \prec_2 R_M A_O$ or $NS \prec_2 R_M A_O$ $R_M A_D \prec_1 SS$	 $R_L A_U \preceq_2 R_M A_O$
Condition Set 13	$SS \prec_2 R_M A_O$ or $NS \prec_2 R_M A_O$ $R_L A_D \preceq_2 R_M A_O$	 $R_M A_U \prec_1 SS$
Condition Set 14	$SS \prec_2 R_M A_O$ or $NS \prec_2 R_M A_O$ $R_M A_D \prec_1 NS$	 $R_L A_U \preceq_2 R_M A_O$
Condition Set 15	$SS \prec_2 R_M A_O$ or $NS \prec_2 R_M A_O$ $R_L A_D \preceq_2 R_M A_O$	 $R_M A_U \prec_1 NS$
Condition Set 16	$SS \prec_2 R_M A_O$ or $NS \prec_2 R_M A_O$ $R_L A_D \preceq_2 R_M A_O$	 $R_L A_U \preceq_2 R_M A_O$

(b)

TABLE III-A
(a) RESULTS OF ROBUSTNESS ANALYSIS (STATE $R_M A_O$) WITH ADDITIONAL INFORMATION. (b) CONDITION SETS FOR PLAYER 1S STABILITY.
(b) SUFFICIENT CONDITION SETS FOR PLAYER 2S STABILITY

Condition Set 1	$R_L A_U \preceq_1 R_M A_O$
Condition Set 2	$R_L A_O \prec_2 SS$
Condition Set 3	$R_L A_O \prec_2 NS$

(a)

Condition Set 1	$SS \prec_2 R_M A_O$ or $NS \prec_2 R_M A_O$ $R_L A_U \preceq_2 R_M A_O$

(b)

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