The H infinity fixed-interval smoothing problem for continuous systems

Eric Blanco, Philippe Neveux, Gérard Thomas

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***Abstract*—The H_∞ smoothing problem for continuous systems is treated in a state space representation by means of variational calculus techniques. The smoothing problem is introduced in an H_∞ criterion by means of an artificial discontinuity that splits the problem in term of H_∞ forward and H_∞ backward filtering problems. Hence, the smoother design is realized in three steps. First, a forward filter is developed. Secondly, a backward filter is developed taking into account the backward Markovian model. The third step consists of combining the two previous steps in order to compute the H_∞ smoothed estimate. An example shows the efficiency of this proposed smoother.**

***Index Terms*—H_∞ criterion, Markovian model, robust estimation, signal processing, smoothing, variational calculus.**

I. Introduction

THE most common estimation tool for continuous systems represented in the state space domain is the Kalman filter [7]. Though this filter has proved its efficiency, the filtering operation brings a slight time delay in the estimation due to the causality of the filter. In order to solve this problem, one has to consider a smoothing operation. In the H_2 setting, for systems represented in the state space form, one faces two philosophically different techniques.

1) The Kalman H_2 smoother [3]. It consists of the combination of a forward and a backward filter. In fact, the objective is to realize the weighted sum of the forward estimate and the backward one.
2) The Rauch–Tung–Streibel (RTS) H_2 smoother [13]. It consists of the combination of two forward filters. The first filter is fed with the measured output and the second with the estimate of the first filter.

In both cases, the design of the smoother is directly related to the perfect knowledge of both the model of the system under consideration and the noise statistics. Unfortunately, noise statistics are approximately known in practice. This uncertainty in the noise properties is not handled by H_2 estimators. Consequently, they do not guarantee a constant level of performance when noise statistics vary from the assumed value. Generally, if the level of noise is larger than the assumed one, the performance of the H_2 estimators decreases. In order to solve this problem, numerous developments have been done in the H_∞ setting.

The key idea of the latter design framework is to minimize the estimation error while considering the worst case for noise statistics. The objective is to guaranteey a constant level of performance over the range of variation of the noise statistics. For H_∞ filtering problems, numerous works have been done for both continuous and discrete time systems through different system representations (see [1], [10], [11], and [17] in the state space and [5], [6], and [14] in the transfer function representation, to mention a few). In opposition, the H_∞ smoothing problem for continuous time systems has received poor attention [2], [11].

Blanco [2] has designed a forward–backward H_∞ smoother considering that the smoothed estimate is the combination of two estimates obtained from a forward H_∞ filter and a backward H_∞ filter. The problem is solved considering two distinct H_∞ filtering problems. This approach leads to an upper bound to the H_∞ smoothing criterion. Clearly, this approach appears to be a suboptimal one.

Nagpal [11] has designed an implicit formulation for the H_∞ fixed-interval smoother through a Hamiltonian representation. Even though the structure of the smoother appears to be independent of the H_∞ bound, one can with decoupling efforts find an explicit RTS smoother defined from a classical Riccati equation and a H_∞ Riccati equation. The latter permits one to define a H_∞ bound for the error covariance matrix. As a consequence, the result obtained is rather a mixed H_2/H_∞ smoother than a pure H_∞ smoother.

This paper develops a smoothing technique based on a forward–backward scheme. It differs from Blanco [2] in that the smoother globally minimizes a H_∞ criterion taking into account both the initial and final state estimation error. In this paper, the problem is treated by introducing an artificial discontinuity in the criterion. The latter permits to split the overall smoothing problem in two filtering problems. In [2] and [11], no attention has been paid to the Markovian properties of the state equation especially in the backward filtering problem. In this paper, a backward Markovian model has been used in the backward problem by introducing the result in [15].

This paper is organized as follows. In Section II, the H_∞ smoothing problem is addressed. Notations and assumptions are detailed. In Section III, the main result is developed in three steps: the forward H_∞ filtering, the backward H_∞ filtering, and the resulting H_∞ smoother. An example is presented in Section IV. Concluding remarks are given in Section V.

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II. Statement of the Problem

A. Model and Assumptions

Let consider the MIMO system defined by the following state-space representation:

$$\dot{x}(t) = A(t)x(t) + B(t)w(t) \tag{1}$$
$$y(t) = C(t)x(t) + D(t)v(t) \tag{2}$$
$$z(t) = L(t)x(t) \tag{3}$$

where $x(t) \in \Re^n$ is the state, $y(t) \in \Re^p$ is the measured output, and $z(t) \in \Re^l$ is the signal to estimate. Matrices $A(t)$, $B(t)$, $C(t)$, $D(t)$, and $L(t)$ are piecewise continuous bounded functions of t with appropriate dimensions.

The following assumptions are made.

- The pair $(A(t), C(t))$ is detectable.
- The pair $(A(t), B(t))$ is stabilizable.
- $w(t) \in \Re^m$ and $v(t) \in \Re^p$ are zero-mean uncorrelated white noises such that

$$E\{w'(t)w(\tau)\} = Q_n\delta(t-\tau) \tag{4}$$
$$E\{v'(t)v(\tau)\} = R_n\delta(t-\tau). \tag{5}$$

- The matrices Q_n and R_n are bounded matrices such that $Q_n < Q$ and $R_n < R$.
- The matrix $D(t)$ is invertible and is such that $D(t)B'(t) = 0$.

The following notations will be used in the sequel.

- X' stands for the transpose of the matrix X.
- $E\{\cdot\}$ is the expectation operator.
- The relation $X < Y$ means that the matrix $(X - Y)$ is definite positive.
- $\|x\|_2$ is the L_2-norm and $\|x\|_Y$ is L_2-norm with a weighting matrix Y.

B. The Smoothing Problem

The objective is to define a smoothed estimate $\hat{z}(t)$ of the signal $z(t)$ using the technique of forward–backward smoothing. The principle of the forward–backward smoothing technique is the following.

Given $y(t)$, a measurement signal on the time interval [0,T], the estimate at time $t_1 \in [0, T]$ results from the combination of:

- the forward treatment of $y(t)$ on [0,t_1];
- the backward treatment of $y(t)$ on $[t_1, T]$.

Let introduce the smoothing lemma that is synthesizing this approach (see [3] for a proof).

Lemma 1: Let $\hat{x}_f(t)$ (respectively, $\hat{x}_b(t)$) be the forward estimate (respectively, backward estimate) of the state $x(t)$. Then, the smoothed estimate $\hat{x}(t|T)$ of $x(t)$ is given by the relation

$$\hat{x}(t|T) = P(t|T)\left[P_f^{-1}(t)\hat{x}_f(t) + P_b^{-1}(t)\hat{x}_b(t)\right] \tag{6}$$

with

$$P(t|T) = \left(P_f^{-1}(t) + P_b^{-1}(t)\right)^{-1} \tag{7}$$

and $P_f(t)$ (respectively, $P_b(t)$) is the error covariance propagation matrix of a forward filter (respectively, a backward filter). $\triangle\triangle\triangle$

Remark 1: The relation (6) states that the smoothed estimate of $x(t)$ is always better than or equal to its filtered estimate [4].

In order to solve the problem of smoothing in presence of noise statistic uncertainties, a H_∞ criterion is defined as follows:

$$\sup_{w,v,x_0,x_T} \frac{\|L(x-\hat{x})\|_2^2}{\|w\|_{Q^{-1}}^2 + \|v\|_{R^{-1}}^2 + f(x_0, \hat{x}_0, x_T, \hat{x}_T)} < \gamma^2 \tag{8}$$

where

$$f(x_0, \hat{x}_0, x_T, \hat{x}_T) = \|x_0 - \hat{x}_0\|_{\Theta_0}^2 + \|x_T - \hat{x}_T\|_{\Theta_T}^2 \tag{9}$$

and Θ_0 (respectively, Θ_T) is a weighting matrix which reflects the confidence in the estimate $\hat{x}_0$ (respectively, the estimate $\hat{x}_T$).

Criterion (8) could be written as a min-max optimization problem as follows:

$$\min_{\hat{x}} \max_{w,v,x_0,x_T} J \tag{10}$$

with

$$J = \int_0^T \Phi dt + \Lambda_0 + \Lambda_T \tag{11}$$

where

$$\Phi = (x-\hat{x})'\frac{L'L}{\gamma^2}(x-\hat{x}) - w'Q^{-1}w - v'R^{-1}v \tag{12}$$
$$\Lambda_0 = -\frac{1}{2}(x_0 - \hat{x}_0)'\Theta_0(x_0 - \hat{x}_0) \tag{13}$$
$$\Lambda_T = -\frac{1}{2}(x_T - \hat{x}_T)'\Theta_T(x_T - \hat{x}_T). \tag{14}$$

From the smoothing principle, it follows that the problem should be split into two optimization problems, namely:

- the forward filtering problem characterized by J_f and a state estimate $\hat{x}_f(t)$;
- the backward filtering problem characterized by J_b and a state estimate $\hat{x}_b(t)$.

Thus, (11) becomes

$$J = J_f + J_b \tag{15}$$

with

$$J_f = \int_0^{t_1} \Lambda dt + \Lambda_0 \tag{16}$$
$$J_b = \int_{t_1}^T \Lambda dt + \Lambda_T \tag{17}$$

with Λ the Lagrangian and its associated multiplier Ψ defined as

$$\Lambda = \frac{1}{2}\Phi + \Psi'\Omega \tag{18}$$

where Ω is a constraint on the state $x(t)$ that will be specified for each problem.

In order to simplify the presentation, the dependence in time t will be omitted.

III. Main Results

A. Forward Filter

In this section, an optimal estimate $\hat{z}_f$ of z is sought. The problem under consideration is the minimization of the functional J_f (16). The problem treated is similar to the filtering problem treated by Nagpal [11]. The proof will be given in term of variational calculus, which seems to be an intuitive manner to treat this problem.

Theorem 1 (The H_∞ Forward Filter): The continuous system (1)–(3) admits an H_∞ estimate $\hat{z}_f$ of the signal z if there exists a symmetric definite positive matrix P_f solution to the Riccati equation

$$\dot{P}_f = AP_f + P_f A' - P_f\left(C'\mathcal{R}^{-1}C - \frac{L'L}{\gamma^2}\right)P_f + BQB' \tag{19}$$

with $\mathcal{R}^{-1} = (D^{-1})'R^{-1}D^{-1}$ and $P_f^{-1}(t=0) = \Theta_0$.

The H_∞ estimator minimizing (16) is given by

$$\dot{\hat{x}}_f = A\hat{x}_f + K_f(y - C\hat{x}_f) \tag{20}$$

$$\hat{z}_f = L\hat{x}_f \tag{21}$$

where K_f is the filter gain defined by the relation

$$K_f = P_f C'\mathcal{R}^{-1}. \tag{22}$$

$\triangledown\triangledown\triangledown$

Proof: A variational approach is used to minimize J_f. For that purpose, the Lagrangian multiplier will be denoted as Ψ_f in the sequel. Consider the first variation of J_f [12]

$$\delta J_f = \int_0^{t_1}\left[\tilde{\phi}_f\delta x + \frac{\partial\Lambda}{\partial\Psi_f}\delta\Psi_f + \frac{\partial\Lambda}{\partial w}\delta w\right]dt + F_0\delta x_0 \tag{23}$$

with

$$F_0 = \left(\frac{\partial\Lambda}{\partial x} - \frac{d}{dt}\frac{\partial\Lambda}{\partial\dot{x}}\right)_{t=0}. \tag{24}$$

The optimality condition ($\delta J_f = 0$) entails with Ω in (18) as $\Omega = Ax + Bw - \dot{x}$:

- *Constraint equation*

$$\frac{\partial\Lambda}{\partial\Psi_f} = 0 \Rightarrow \Omega(x,\dot{x},t) = 0 \tag{25}$$

- *Optimality*

$$\frac{\partial\Lambda}{\partial w} = 0 \Rightarrow w^* = QB'\Psi_f \tag{26}$$

- *Transversality condition*

$$F_0 = 0 \Rightarrow x(0) = \hat{x}_f(0) + P_f(0)\Psi_f(0) \tag{27}$$

- *Euler–Lagrange equation*

$$\tilde{\phi}_f = \frac{\partial\Lambda}{\partial x} - \frac{d}{dt}\frac{\partial\Lambda}{\partial\dot{x}} = 0$$
$$\Rightarrow \dot{\Psi}_f = -A'\Psi_f - \frac{L'L}{\gamma^2}(x-\hat{x}_f) - C'\mathcal{R}^{-1}(y-Cx). \tag{28}$$

Using Riccati transformation $x = \hat{x}_f + P_f\Psi_f$, where P_f is a symmetric definite positive matrix, in (1), (26), and (28), one obtains after some manipulations the results presented in Theorem 1. This completes the proof. ■

B. Backward Filter

In this section, an optimal estimate $\hat{z}_b$ of z is sought. The problem under consideration is the minimization of the functional J_b (17). This problem is a H_∞ backward filtering problem.

The state $x(t)$ in (1) is a Markovian process [16]. Hence, there exists a correlation between the value of the state at time T and the driving noise $w(t)$. Consequently, the backward orthogonality condition defined as

$$E\left\{x(T)w'(\tau)\right\} = 0 \tag{29}$$

should be verified. The following lemma defines an equivalent backward Markovian model to the forward model (1) satisfying to this condition (see [15] for a proof).

Lemma 2 (The Markovian Backward Model): Consider the process $x_b(\cdot)$ given by the state model

$$-\frac{dx_b}{dt} = -A_b x_b - Bw_b \tag{30}$$

where

$$A_b = A + BQB'\Pi^{-1} \tag{31}$$

with Π a symmetric definite positive matrix solution to the Riccati equation

$$\dot{\Pi} = A\Pi + \Pi A' + BQB' \tag{32}$$

with $\Pi(t_0) = \Pi_0 = E\{x_0x_0'\}$ and $w_b(\cdot)$ is zero-mean centered process with covariance matrix such that

$$E\left\{w_b(t)w_b'(s)\right\} = Q_n\delta(t-s);\;\; E\left\{w_b(t)x_b'(T)\right\} = 0. \tag{33}$$

If the above-mentioned conditions are verified, then $x_b(\cdot)$ has the same covariance function as $x(\cdot)$. $\triangle\triangle\triangle$

Consequently, minimizing (17) is equivalent to the minimization of the criterion

$$\tilde{J}_b = \int_{t_1}^{T}\tilde{\Lambda}dt + \tilde{\Lambda}_T \tag{34}$$

with $\tilde{\Lambda}$, $\tilde{\Lambda}_T$ defined from (18) and (14) by replacing x, w, and Ψ by x_b, w_b, and Ψ_b, respectively.

Finally, one gets the following result for the definition of the H_∞ backward filter.

Theorem 2 (The H_∞ Backward Filter): The system (1)–(3) admits an H_∞ estimate $\hat{z}_b$ of the signal z if the condition of Lemma 2 is satisfied and if there exists a symmetric definite positive matrix P_b solution to the Riccati equation

$$\frac{dP_b}{dt} = A_b P_b + P_b A_b' + P_b\left(C'\mathcal{R}^{-1}C - \frac{L'L}{\gamma^2}\right)P_b - BQB' \quad (35)$$

with $P_b^{-1}(t=T) = \Theta_T$.

The H_∞ estimator minimizing (34) is given by

$$-\frac{d\hat{x}_b}{dt} = -A_b\hat{x}_b + K_b(y - C\hat{x}_b) \quad (36)$$

$$\hat{z}_b = L\hat{x}_b \quad (37)$$

and the backward filter gain is defined by

$$K_b = P_b C'\mathcal{R}^{-1}. \quad (38)$$

$\nabla\nabla\nabla$

Proof: The same technique used to prove Theorem 1 is employed to prove Theorem 2. It should be noted that the Riccati transform is $x = \hat{x}_b - P_b\Psi_b$ and that the function Ω in (18) is defined as $\Omega = -A_b x_b - Bw_b + \dot{x}_b$. ■

C. H_∞ Smoothing

In Section II-B, the H_∞ criterion has been split in two terms J_f and J_b. In Section III-A and B, the forward and backward filters have been developed using a variational method. From Lemma 1 and the results in Theorems 1 and 2, the expression of the H_∞ smoother is derived as follows.

Theorem 3 (The H_∞ Smoother): The system (1)–(3) admits an H_∞ forward–backward smoother for the signal z, if there exists:

- a symmetric definite positive matrix P_f solution of the Riccati equation

$$-\frac{dP_f^{-1}}{dt} = P_f^{-1}A + A'P_f^{-1} + P_f^{-1}BQB'P_f^{-1} - C'\mathcal{R}^{-1}C + \frac{L'L}{\gamma^2} \quad (39)$$

  with $\lim_{t\to 0} P_f^{-1} = 0$;
- a symmetric definite positive matrix Π solution of the Riccati equation

$$\dot{\Pi} = A\Pi + \Pi A' + BQB' \quad (40)$$

  with $\Pi(t_0) = \Pi_0 = E\{x_0 x_0'\}$;
- a symmetric definite positive matrix P_b solution of the Riccati equation

$$\frac{dP_b^{-1}}{d\tau} = P_b^{-1}A_b + A_b'P_b^{-1} - P_b^{-1}BQB'P_b^{-1} + C'\mathcal{R}^{-1}C - \frac{L'L}{\gamma^2} \quad (41)$$

$$A_b = A + BQB'\Pi^{-1} \quad (42)$$

  with $\lim_{\tau\to 0} P_b^{-1} = 0$.

  Then, the estimate $\hat{z} = L\hat{x}$ is obtained from the relations

$$\hat{x} = P(s_f + s_b) \quad (43)$$

with

a) s_f the solution to

$$\frac{ds_f}{dt} = \mathcal{A}_f s_f + C\mathcal{R}^{-1}y \quad (44)$$

with

$$\mathcal{A}_f = -\left(A' + P_f^{-1}BQB' + \frac{L'L}{\gamma^2}P_f\right) \quad (45)$$

and $s_f(t=0) = 0$;

b) s_b the solution to

$$\frac{ds_b}{d\tau} = \mathcal{A}_b s_b + C\mathcal{R}^{-1}y \quad (46)$$

with

$$\mathcal{A}_b = A_b' - P_b^{-1}BQB' - \frac{L'L}{\gamma^2}P_b \quad (47)$$

and $s_b(\tau = 0) = 0$ where: $t \in [0,T]$ and $\tau = T - t$ $\nabla\nabla\nabla$.

Proof: The problem of the initial value for the matrices P_f and P_b has to be treated. Both values relate to the confidence the designer brings into $\hat{x}_f(t=0)$ and $\hat{x}_b(t=T)$ with regard to $x(t=0)$ and $x(t=T)$, respectively. In order to overcome this problem, the solution is to consider that $P_f(t=0)$ and $P_b(t=T)$ are infinite. Consequently, (19) and (35) have to be rewritten in P_f^{-1} and P_b^{-1} in order to obtain Riccati equations with zero initial value. Hence, the initial value of the new (39) and (41) will be null.

As a consequence, one has to consider the following change of variable:

$$s_f(t) = P_f^{-1}\hat{x}_f(t) \quad (48)$$

$$s_b(t) = P_b^{-1}\hat{x}_b(t). \quad (49)$$

The direct consequence of this operation is that the initial value for this new variables is null. Using Lemma 1 and (20) and (36), one obtains after some mere manipulations (44) and (46). This completes the proof. ■

IV. Illustrative Example

The performances of the proposed smoother will be compared to the performances of the H_2/H_∞ smoother of Nagpal [11]. We consider the following system [14]:

$$A = \mathrm{diag}\{-1; -2; -3\},\ B = 25[1 \quad 1 \quad -1]$$
$$C = [-1 \quad 2 \quad 1],\ L = [1 \quad 1 \quad 1].$$

The covariance of $w(t)$ and $v(t)$ is assumed to be unity. In the problem treated, the real covariance of $v(t)$ is smaller than the assumed value. Hence, we impose ourselves to the worst case situation for the design of the smoother.

Figs. 1 and 2 show two different situations for the same realization of the measurement noise. The plots clearly show the great ability of the proposed smoother to deal with uncertainty on noise properties. In both cases, the dynamic of the restored signal with the H_∞ smoother is close to the signal $z(t)$, whereas

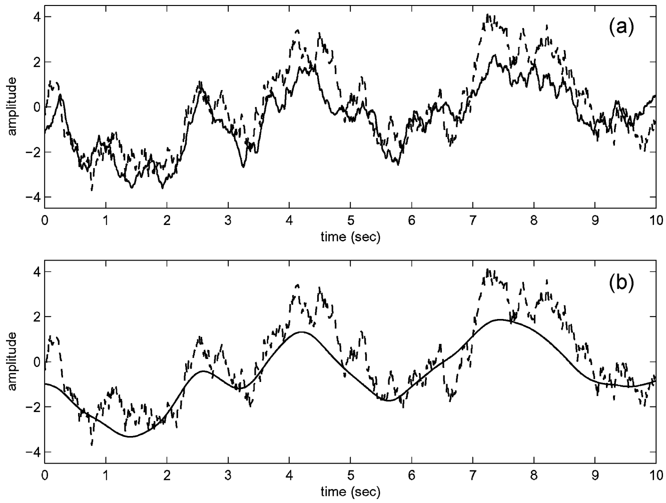

Fig. 1. Comparison of the estimation for (a) the proposed H_∞ smoother and (b) the H_2/H_∞ smoother [11] with SNR $= 10$ dB.

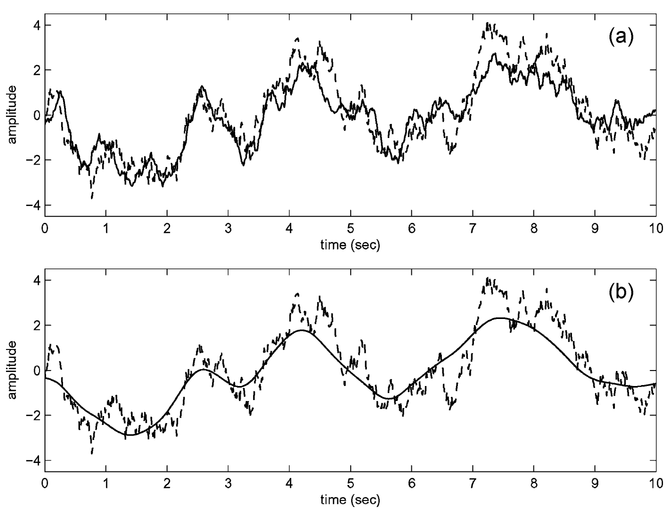

Fig. 2. Comparison of the estimation for (a) the proposed H_∞ smoother and (b) the H_2/H_∞ smoother [11] with SNR $= 30$ dB.

the H_2/H_∞ smoother exhibits a very smoothed restored signal $z(t)$. Hence, the behavior of the H_∞ smoother is very similar to the H_∞ filter in terms of dynamic restoration over noise statistic uncertainty [14].

V. Conclusion

In this paper, the H_∞ smoothing problem has been treated through an efficient variational approach. The key idea of the development is that the optimal smoothed estimate results in the combination of the estimate of an H_∞ forward filter and the estimate of an H_∞ backward filter. In that purpose, the H_∞ criterion has been split in two terms that explicitly pose the smoothing problem in term of forward and backward filtering problems. The result for the forward filter is clearly a standard result. For the backward filter, the problem of correlation between the initial state value and the evolution of the state has been solved using a Markovian model. The approach developed in this paper ensures the optimality of the smoother in the H_∞ setting compared to previous works.

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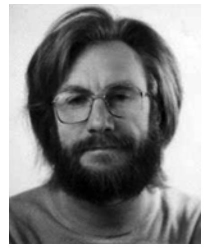

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