# Filter-And-Forward Distributed Beamforming in Relay Networks with Frequency Selective Fading

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#### Abstract

A new approach to distributed cooperative beamforming in relay networks with frequency selective fading is proposed. It is assumed that all the relay nodes are equipped with finite impulse response (FIR) filters and use a filter-and-forward (FF) strategy to compensate for the transmitter-to-relay and relay-to-destination channels.

Three relevant half-duplex distributed beamforming problems are considered. The first problem amounts to minimizing the total relay transmitted power subject to the destination quality-of-service (QoS) constraint. In the second and third problems, the destination QoS is maximized subject to the total and individual relay transmitted power constraints, respectively. For the first and second problems, closed-form solutions are obtained, whereas the third problem is solved using convex optimization. The latter convex optimization technique can be also directly extended to the case when the individual and total power constraints should be jointly taken into account. Simulation results demonstrate that in the frequency selective fading case, the proposed FF approach provides substantial performance improvements as compared to the commonly used amplify-and-forward (AF) relay beamforming strategy.

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This work was supported in parts by the European Research Council (ERC) Advanced Investigator Grants program under Grant 227477-ROSE, German Research Foundation (DFG) under Grant GE 1881/1-1, and National Science and Engineering Research Council of Canada (NSERC) under Discovery Grants program. The results of this paper have been presented in part at IEEE/ITG Workshop on Smart Antennas, Berlin, February 2009 and *ICASSP'09*, Taipei, Taiwan, April 2009.

## I. INTRODUCTION

Recently, cooperative wireless communication techniques gained much interest in the literature as they can exploit cooperative diversity without any need of having multiple antennas at each user [1]-[9]. In such user-cooperative schemes, different users share their communication resources to assist each other in transmitting the information throughout the network by means of relaying messages from the source to destination through multiple independent paths.

Different relaying strategies have been proposed to achieve cooperative diversity. Two most popular relaying strategies are the amplify-and-forward (AF) and decode-and-forward (DF) approaches [1], [4], [6]-[11]. In the AF scheme, relays simply retransmit properly scaled and phase-shifted versions of their received signals, while in the DF scheme the relay nodes decode and then re-encode their received messages prior to retransmitting them. Due to its low complexity, the AF relaying strategy is of especial interest [12]-[18].

When the channel state information (CSI) is not available at the relay nodes, distributed space-time coding can be used to obtain the cooperative diversity gain [10]-[13]. However, with available CSI, distributed network beamforming can provide better performance [14], [15].

Recently, several distributed AF beamforming techniques for relay networks with flat fading channels have been developed [14]-[18]. The approaches of [14]-[16] optimize the receiver quality-of-service (QoS) subject to the individual and/or total power constraints under the assumption that the instantaneous CSI is perfectly known at the destination or relay nodes. In these approaches, the QoS is measured in terms of the signal-to-noise ratio (SNR) at the receiver. In [17], the problem of AF relay beamforming is considered under the assumption that the second-order statistics of the source-to-relay and relay-to-destination channels are available. Based on the latter assumption, several distributed beamforming algorithms are developed in [17]. In the first technique of [17], the total relay transmit power is minimized subject to the total or individual relay power constraints. In [18], the approach of [17] has been extended to multiple source-destination pairs. Recently, the problem of using an imperfect (e.g., quantized) CSI feedback in distributed beamforming has been also considered [19].

Although some extensions of distributed space time-coding techniques to the frequency selective fading case are known in the literature [20], the problem of distributed beamforming in frequency selective environments has not been addressed so far. In particular, all the techniques of [14]-[19] assume the transmitter-to-relay and relay-to-destination channels to be frequency flat. However, in practical scenarios

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these channels are likely to be frequency selective. In the latter case, there is a significant amount of inter-symbol interference (ISI) which makes it difficult to directly extend the techniques of [14]-[19] to frequency selective fading channel scenarios.

In this paper, we consider a relay network of one transmitter, one destination, and multiple relay nodes under the assumption of frequency selective, finite impulse response (FIR) transmitter-to-relay and relay-to-destination channels. To compensate for the effect of such channels, a new filter-and-forward (FF) relaying protocol is proposed. According to the FF strategy, all the relay nodes are equipped with finite impulse response (FIR) filters that are used to compensate for the transmitter-to-relay and relay-to-destination channels.

Three relevant distributed beamforming problems are considered under the assumption that the instantaneous CSI is available at the receiver or at the relay nodes. Similar to the techniques of [14]-[18], the receiver is assumed to use a perfect source-to-relay and relay-to-destination CSI to compute the relay weight coefficients and feed them back to the relay nodes using a low-rate receiver-to-relays feedback link. Alternatively, the relays can directly compute their weight coefficients, provided that the CSI is available at the relay nodes.

Our first distributed beamforming problem amounts to minimizing the total relay transmitted power subject to the destination QoS constraint. As in the frequency selective case the major performance limiting factor is ISI rather than noise, the QoS is measured in terms of the receiver signal-to-interference-plus-noise ratio (SINR), in contrast to the techniques of [14]-[17] that use the SNR as a measure of QoS in the flat fading case. In our second and third problems, the destination QoS is maximized subject to the total and individual relay transmitted power constraints, respectively. For the first and second problems, closed-form solutions are obtained, whereas the third problem is solved using convex optimization. The latter convex optimization technique can be also directly extended to the case when the individual and total power constraints should be jointly taken into account.

It is shown that in the flat fading case, the proposed FF network beamforming techniques reduce to the AF network beamformers of [14]-[17] which are particular cases of our techniques.

In frequency selective channel scenarios, our simulation results demonstrate that the proposed FF approach provides substantial performance improvements as compared to the AF relay beamforming strategy.

# II. RELAY NETWORK MODEL

## A. Filter-And-Forward Relaying Protocol

Let us consider a half-duplex relay network with one single-antenna transmitting source, one singleantenna receiver (destination) node and *R* single-antenna relay nodes. Similar to [14] and [16]-[18], it is assumed that there is no direct link between the transmitter and destination nodes and that the network is perfectly synchronized. Each transmission is assumed to consist of two stages. In the first stage, the transmitting source broadcasts its data to the relays. The signals received at the relay nodes are then passed through the relay FIR filters to compensate for the effects of the transmitter-to-relay and relayto-destination frequency selective channels. This type of relay processing corresponds to our proposed FF relaying protocol; see Fig. 1. As FIR filters have been commonly used for channel equalization in point-to-point communication systems, the FF strategy appears to be a very natural extension of the AF protocol to frequency selective relay channels. However, an important difference between these two cases is that in relay networks, it is meaningful to use a separate FIR filter at each relay node, while in the traditional point-to-point case, such a filter is commonly employed at the receiver.

In the second transmission stage, the outputs of each relay filter are sent to the destination that is assumed to have the full instantaneous CSI. Using this knowledge, the receiver determines the filter weight coefficients of each relay according to a certain beamforming criterion. It is also assumed that there is a low-rate feedback link from the destination to each relay node that is used to inform the relays about their optimal weight coefficients. Alternatively, if the full instantaneous CSI is available at the relays rather than the destination, each relay node can determine its own weight coefficients independently.<sup>1</sup> In the latter case, no extra receiver-to-relay feedback is needed. Note that quite similar assumptions have been used in [14]-[18] in the frequency flat fading case.

## B. Signal Model

Let us model the transmitter-to-relay and relay-to-destination channels as linear FIR filters

$$\mathbf{f}(\omega) = \sum_{l=0}^{L_f - 1} \mathbf{f}_l e^{-j\omega l}, \qquad \mathbf{g}(\omega) = \sum_{l=0}^{L_g - 1} \mathbf{g}_l e^{-j\omega l}$$
(1)

<sup>1</sup>Note that network beamforming is commonly referred to as "distributed" because it is assumed that no relay can share its received signals with any other relay.

where

$$\mathbf{f}_l = [f_{l,1}, \dots, f_{l,R}]^T \tag{2}$$

$$\mathbf{g}_l = [g_{l,1}, \dots, g_{l,R}]^T \tag{3}$$

are the  $R \times 1$  channel impulse response vectors corresponding to the *l*th effective tap of the transmitterto-relay and relay-to-destination channels, respectively. Here,  $\mathbf{f}(\omega)$  and  $\mathbf{g}(\omega)$  are the  $R \times 1$  vectors of channel frequency responses, and  $L_f$  and  $L_g$  are the corresponding channel lengths, respectively. The  $R \times 1$  vector  $\mathbf{r}(n) = [r_1(n), \dots, r_R(n)]^T$  of the signals received by the relay nodes in the *n*th channel use can be modeled as

$$\mathbf{r}(n) = \sum_{l=0}^{L_f - 1} \mathbf{f}_l s(n-l) + \boldsymbol{\eta}(n)$$
(4)

where s(n) is the signal transmitted by the source,  $\eta(n) = [\eta_1(n), \dots, \eta_R(n)]^T$  is the  $R \times 1$  vector of relay noise, and  $(\cdot)^T$  denotes the transpose. Introducing the notations

$$\mathbf{F} \triangleq [\mathbf{f}_0, \cdots, \mathbf{f}_{L_f-1}]$$
$$\mathbf{s}(n) \triangleq [s(n), s(n-1), \cdots, s(n-L_f+1)]^T$$

we can write (4) as

$$\mathbf{r}(n) = \mathbf{Fs}(n) + \boldsymbol{\eta}(n). \tag{5}$$

The signal vector  $\mathbf{t}(n) = [t_1(n), \cdots, t_R(n)]^T$  sent from the relays to the destination can be expressed as

$$\mathbf{t}(n) = \sum_{l=0}^{L_w-1} \mathbf{W}_l^H \mathbf{r}(n-l)$$
(6)

where

$$\mathbf{W}_{l} \triangleq \operatorname{diag}\{w_{l,1}, \cdots, w_{l,R}\}$$

is the diagonal matrix of the relay filter impulse responses corresponding to the *l*th effective filter tap of each relay,  $L_w$  is the length of the relay FIR filters,  $(\cdot)^H$  denotes the Hermitian transpose, and for any vector  $\boldsymbol{x}$ , the operator diag $\{\boldsymbol{x}\}$  forms the diagonal matrix containing the entries of  $\boldsymbol{x}$  on its main diagonal. Correspondingly, for any square matrix  $\boldsymbol{X}$ , the operator diag $\{\boldsymbol{X}\}$  forms a vector whose elements are the diagonal entries of  $\boldsymbol{X}$ . Note that if  $L_w = 1$ , then the FF transmission in (6) reduces to the AF one.

Inserting (5) into (6), we have

$$\mathbf{t}(n) = \sum_{l=0}^{L_w - 1} \mathbf{W}_l^H (\mathbf{Fs}(n-l) + \boldsymbol{\eta}(n-l)).$$
(7)

Let us define

$$\tilde{\mathbf{s}}(n) \triangleq [s(n), s(n-1), \cdots, s(n-L_f-L_w+2)]^T.$$

It can be seen that the vector  $\mathbf{s}(n-l)$  is a subvector of  $\tilde{\mathbf{s}}(n)$ . Using this observation, (7) can be rewritten as

$$\mathbf{t}(n) = \sum_{l=0}^{L_w - 1} \mathbf{W}_l^H(\mathbf{F}_l \tilde{\mathbf{s}}(n) + \boldsymbol{\eta}(n-l))$$
(8)

where

$$\mathbf{F}_{l} \triangleq [\overbrace{\mathbf{0}_{R \times 1}, \cdots, \mathbf{0}_{R \times 1}}^{l \text{ columns}}, \mathbf{F}, \overbrace{\mathbf{0}_{R \times 1}, \cdots, \mathbf{0}_{R \times 1}}^{(L_{w}-1-l) \text{ columns}}], \quad l = 0, \cdots, L_{w}-1.$$

Let us also define

$$\mathbf{W} \triangleq [\mathbf{W}_{0}, \cdots, \mathbf{W}_{L_{w}-1}]^{T}$$

$$\mathcal{F} \triangleq \begin{bmatrix} \mathbf{F}_{0} \\ \mathbf{F}_{1} \\ \vdots \\ \mathbf{F}_{L_{w}-1} \end{bmatrix} = \begin{bmatrix} \mathbf{f}_{0} & \mathbf{f}_{1} & \cdots & \mathbf{f}_{L_{f}-1} & \mathbf{0}_{R\times 1} & \cdots & \mathbf{0}_{R\times 1} \\ \mathbf{0}_{R\times 1} & \mathbf{f}_{0} & \mathbf{f}_{1} & \cdots & \mathbf{f}_{L_{f}-1} & \cdots & \mathbf{0}_{R\times 1} \\ & \ddots & \ddots & \ddots & & \ddots \\ \mathbf{0}_{R\times 1} & \mathbf{0}_{R\times 1} & \cdots & \mathbf{f}_{0} & \mathbf{f}_{1} & \cdots & \mathbf{f}_{L_{f}-1} \end{bmatrix}$$

$$\tilde{\boldsymbol{\eta}}(n) \triangleq [\boldsymbol{\eta}^{T}(n), \boldsymbol{\eta}^{T}(n-1), \cdots, \boldsymbol{\eta}^{T}(n-L_{w}+1)]^{T}$$

where  $\mathbf{0}_{N \times M}$  is the  $N \times M$  matrix of zeros. Using these notations, we obtain that

$$\sum_{l=0}^{L_w-1} \mathbf{W}_l^H \mathbf{F}_l = \mathbf{W}^H oldsymbol{\mathcal{F}}$$

and, therefore, (8) can be expressed as

$$\mathbf{t}(n) = \mathbf{W}^H \boldsymbol{\mathcal{F}} \tilde{\mathbf{s}}(n) + \mathbf{W}^H \tilde{\boldsymbol{\eta}}(n).$$
(9)

The received signal at the destination can be written as

$$y(n) = \sum_{l=0}^{L_g-1} \mathbf{g}_l^T \mathbf{t}(n-l) + \upsilon(n)$$
(10)

where v(n) is the receiver noise waveform and  $g_l$  is the channel impulse response vector defined in (3).

Using (9), we can rewrite (10) as

$$y(n) = \sum_{l=0}^{L_g-1} \mathbf{g}_l^T \mathbf{W}^H \mathcal{F} \tilde{\mathbf{s}}(n-l) + \sum_{l=0}^{L_g-1} \mathbf{g}_l^T \mathbf{W}^H \tilde{\boldsymbol{\eta}}(n-l) + \upsilon(n).$$
(11)

Taking into account that the matrices  $W_l$  are all diagonal and using the properties of the Kronecker matrix product, we obtain that

$$\mathbf{g}_{l}^{T} \mathbf{W}^{H} = [\mathbf{g}_{l}^{T} \mathbf{W}_{0}^{H}, \cdots, \mathbf{g}_{l}^{T} \mathbf{W}_{L_{w}-1}^{H}]$$
$$= [\mathbf{w}_{0}^{H} \mathbf{G}_{l}, \cdots, \mathbf{w}_{L_{w}-1}^{H} \mathbf{G}_{l}]$$
$$= \mathbf{w}^{H} (\mathbf{I}_{L_{w}} \otimes \mathbf{G}_{l})$$
(12)

where

$$\mathbf{w} \triangleq [\mathbf{w}_0^T, \cdots, \mathbf{w}_{L_w-1}^T]^T$$
$$\mathbf{w}_l \triangleq \operatorname{diag}\{\mathbf{W}_l\}$$
$$\mathbf{G}_l \triangleq \operatorname{diag}\{\mathbf{g}_l\}$$

 $I_N$  is the  $N \times N$  identity matrix, and  $\otimes$  denotes the Kronecker product. Using (12), we can further rewrite (11) as

$$y(n) = \sum_{l=0}^{L_g-1} \mathbf{w}^H (\mathbf{I}_{L_w} \otimes \mathbf{G}_l) \mathcal{F} \tilde{\mathbf{s}}(n-l) + \sum_{l=0}^{L_g-1} \mathbf{w}^H (\mathbf{I}_{L_w} \otimes \mathbf{G}_l) \tilde{\boldsymbol{\eta}}(n-l) + \upsilon(n).$$
(13)

Defining

$$\mathbf{\breve{s}}(n) \triangleq [s(n), s(n-1), \cdots, s(n-L_f - L_w - L_g + 3)]^T$$
$$\mathbf{\breve{\eta}}(n) \triangleq [\mathbf{\eta}^T(n), \mathbf{\eta}^T(n-1), \cdots, \mathbf{\eta}^T(n-L_w - L_g + 2)]^T$$

we notice that  $\tilde{s}(n-l)$  and  $\tilde{\eta}(n-l)$  are subvectors of  $\breve{s}(n)$  and  $\breve{\eta}(n)$ , respectively. Therefore, we can express (13) as

$$y(n) = \sum_{l=0}^{L_g-1} \mathbf{w}^H (\mathbf{I}_{L_w} \otimes \mathbf{G}_l) \mathcal{F}_l \breve{\mathbf{s}}(n) + \sum_{l=0}^{L_g-1} \mathbf{w}^H (\mathbf{I}_{L_w} \otimes \mathbf{G}_l) \breve{\mathbf{I}}_l \breve{\boldsymbol{\eta}}(n) + \upsilon(n)$$
(14)

where

$$\mathcal{F}_{l} \triangleq [\overbrace{\mathbf{0}_{RL_{w} \times 1}, \cdots, \mathbf{0}_{RL_{w} \times 1}}^{l \text{ columns}}, \mathcal{F}, \overbrace{\mathbf{0}_{RL_{w} \times 1}, \cdots, \mathbf{0}_{RL_{w} \times 1}}^{(L_{g}-1-l) \text{ columns}}], \quad l = 0, \cdots, L_{g}-1$$

$$\widecheck{\mathbf{I}}_{l} \triangleq [\overbrace{\mathbf{0}_{RL_{w} \times R}, \cdots, \mathbf{0}_{RL_{w} \times R}}^{l \text{ blocks}}, \mathbf{I}_{RL_{w}}, \overbrace{\mathbf{0}_{RL_{w} \times R}, \cdots, \mathbf{0}_{RL_{w} \times R}}^{(L_{g}-1-l) \text{ blocks}}], \quad l = 0, \cdots, L_{g}-1.$$

To express (14) in a more compact form, we further define

$$oldsymbol{\mathcal{G}} \triangleq egin{bmatrix} \mathbf{I}_{L_w} \otimes \mathbf{G}_0, \cdots, \mathbf{I}_{L_w} \otimes \mathbf{G}_{L_g-1} \end{bmatrix} \ egin{matrix} \mathbf{ar{F}} \triangleq egin{matrix} \mathbf{\mathcal{F}}_0^T, \cdots, \mathbf{\mathcal{F}}_{L_g-1}^T \end{bmatrix}^T \ egin{matrix} \tilde{\mathbf{I}} \triangleq egin{matrix} \mathbf{I}_0^T, \cdots, \mathbf{ar{I}}_{L_g-1}^T \end{bmatrix}^T \end{split}$$

and note that

$$\sum_{l=0}^{L_g-1} (\mathbf{I}_{L_w} \otimes \mathbf{G}_l) \mathcal{F}_l = \mathcal{G} \breve{\mathbf{F}} \ \sum_{l=0}^{L_g-1} (\mathbf{I}_{L_w} \otimes \mathbf{G}_l) \breve{\mathbf{I}}_l = \mathcal{G} \widetilde{\mathbf{I}}.$$

Using the latter two equations, (14) can be expressed as

$$y(n) = \mathbf{w}^{H} \mathcal{G} \breve{\mathbf{F}} \breve{\mathbf{s}}(n) + \mathbf{w}^{H} \mathcal{G} \widetilde{\mathbf{I}} \breve{\boldsymbol{\eta}}(n) + \upsilon(n).$$
(15)

Let  $\bar{\mathbf{f}}$  and  $\bar{\mathbf{F}}$  denote the first column and the residue of  $\breve{\mathbf{F}}$ , respectively, so that  $\breve{\mathbf{F}} = [\bar{\mathbf{f}}, \bar{\mathbf{F}}]$ . Then, (15) yields

$$y(n) = \mathbf{w}^{H} \mathcal{G}[\bar{\mathbf{f}}, \bar{\mathbf{F}}] \begin{bmatrix} s(n) \\ \bar{\mathbf{s}}(n) \end{bmatrix} + \mathbf{w}^{H} \mathcal{G} \tilde{\mathbf{I}} \check{\boldsymbol{\eta}}(n) + v(n)$$
$$= \underbrace{\mathbf{w}^{H} \mathcal{G} \bar{\mathbf{f}} s(n)}_{\text{signal}} + \underbrace{\mathbf{w}^{H} \mathcal{G} \bar{\mathbf{F}} \bar{\mathbf{s}}(n)}_{\text{ISI}} + \underbrace{\mathbf{w}^{H} \mathcal{G} \tilde{\mathbf{I}} \check{\boldsymbol{\eta}}(n) + v(n)}_{\text{noise}}$$
(16)

where

$$\mathbf{\bar{s}}(n) \triangleq [s(n-1), \cdots, s(n-L_f-L_w-L_g+3)]^T.$$

In (16), we can identify the three components

$$y_s(n) \triangleq \mathbf{w}^H \mathcal{G} \bar{\mathbf{f}} s(n) \tag{17}$$

$$y_i(n) \triangleq \mathbf{w}^H \mathcal{G} \bar{\mathbf{F}} \bar{\mathbf{s}}(n)$$
 (18)

$$y_n(n) \triangleq \mathbf{w}^H \mathcal{G} \tilde{\mathbf{I}} \check{\boldsymbol{\eta}}(n) + \upsilon(n)$$
 (19)

as the destination signal, ISI, and noise components, respectively. Note that for the sake of computational simplicity of our techniques developed in the next section, *block processing* is not considered here, that is, the signal copies delayed by multipath are not coherently combined.

The signal component in (17) can be expressed as

$$y_{s}(n) = \mathbf{w}_{0}^{H} \mathbf{G}_{0} \mathbf{f}_{0} s(n)$$
$$= \mathbf{w}_{0}^{H} (\mathbf{g}_{0} \odot \mathbf{f}_{0}) s(n)$$
$$= \mathbf{w}_{0}^{H} \mathbf{h}_{0} s(n)$$
(20)

where

$$\mathbf{h}_0 \triangleq \mathbf{g}_0 \odot \mathbf{f}_0 \tag{21}$$

and  $\odot$  denotes the Schur-Hadamard (elementwise) matrix product.

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## III. FILTER-AND-FORWARD RELAY BEAMFORMING

In this section, we develop three distributed FF beamforming approaches that utilize several alternative criteria. Our first FF beamforming technique is based on minimizing the total relay transmitted power subject to the destination QoS constraint, while our second and third approaches are based on maximizing the destination QoS subject to the total and individual relay transmitted power constraints, respectively. A useful modification of our third approach is also discussed, that enables to combine the later two types of constraints.

## A. Minimization of the Total Relay Power Under the QoS Constraint

We first consider the distributed FF beamforming problem that obtains the relay filter weights by minimizing the total relay transmitted power P subject to the destination QoS constraint. As mentioned above, the destination QoS is given by the receiver SINR value<sup>2</sup> and, therefore, the latter problem can be written as

$$\min_{\mathbf{w}} P \quad \text{s.t. SINR} \ge \gamma \tag{22}$$

where  $\gamma$  is the minimal required SINR at the destination.

Let us use the following two common assumptions

$$\mathbf{E}\{\tilde{\mathbf{s}}(n)\tilde{\mathbf{s}}^{H}(n)\} = P_{s}\mathbf{I}_{L_{f}+L_{w}-1}, \quad \mathbf{E}\{\tilde{\boldsymbol{\eta}}(n)\tilde{\boldsymbol{\eta}}^{H}(n)\} = \sigma_{\eta}^{2}\mathbf{I}_{RL_{w}}$$
(23)

on statistical independence of the signal and noise waveforms, respectively. Here,  $P_s$  is the source transmitted power and  $\sigma_{\eta}^2$  is the relay noise variance. Using (9) and (23), the transmitted power of the *m*th relay can be written as

$$p_{m} = \mathrm{E}\{|t_{m}(n)|^{2}\}$$

$$= \mathrm{E}\{\mathbf{e}_{m}^{T}\mathbf{W}^{H}\mathcal{F}\tilde{\mathbf{s}}(n)\tilde{\mathbf{s}}^{H}(n)\mathcal{F}^{H}\mathbf{W}\mathbf{e}_{m}\} + \mathrm{E}\{\mathbf{e}_{m}^{T}\mathbf{W}^{H}\tilde{\boldsymbol{\eta}}(n)\tilde{\boldsymbol{\eta}}^{H}(n)\mathbf{W}\mathbf{e}_{m}\}$$

$$= P_{s}\mathbf{e}_{m}^{T}\mathbf{W}^{H}\mathcal{F}\mathcal{F}^{H}\mathbf{W}\mathbf{e}_{m} + \sigma_{\eta}^{2}\mathbf{e}_{m}^{T}\mathbf{W}^{H}\mathbf{W}\mathbf{e}_{m}$$
(24)

where  $\mathbf{e}_m$  is the *m*th column of the identity matrix.

Using  $\mathbf{E}_m \triangleq \operatorname{diag}\{\mathbf{e}_m\}$  and the properties of the Kronecker product, (24) can be rewritten as

$$p_{m} = P_{s} \mathbf{w}^{H} \left( \mathbf{I}_{L_{w}} \otimes \mathbf{E}_{m} \right) \mathcal{F} \mathcal{F}^{H} \left( \mathbf{I}_{L_{w}} \otimes \mathbf{E}_{m} \right)^{H} \mathbf{w} + \sigma_{\eta}^{2} \mathbf{w}^{H} \left( \mathbf{I}_{L_{w}} \otimes \mathbf{E}_{m} \right) \left( \mathbf{I}_{L_{w}} \otimes \mathbf{E}_{m} \right)^{H} \mathbf{w}.$$
(25)

<sup>2</sup>This is true because the processing at the destination is rather simple; in particular, no block processing is used.

The total relay transmitted power can be then expressed as

$$P = \sum_{m=1}^{R} p_m = \mathbf{w}^H \left(\sum_{m=1}^{R} \mathbf{D}_m\right) \mathbf{w} = \mathbf{w}^H \mathbf{D} \mathbf{w}$$
(26)

where

$$\mathbf{D}_{m} \triangleq P_{s} \left( \mathbf{I}_{L_{w}} \otimes \mathbf{E}_{m} \right) \mathcal{F} \mathcal{F}^{H} \left( \mathbf{I}_{L_{w}} \otimes \mathbf{E}_{m} \right)^{H} + \sigma_{\eta}^{2} \left( \mathbf{I}_{L_{w}} \otimes \mathbf{E}_{m} \right) \left( \mathbf{I}_{L_{w}} \otimes \mathbf{E}_{m} \right)^{H}$$
$$\mathbf{D} \triangleq \sum_{m=1}^{R} \mathbf{D}_{m} = P_{s} \sum_{m=1}^{R} \left( \mathbf{I}_{L_{w}} \otimes \mathbf{E}_{m} \right) \mathcal{F} \mathcal{F}^{H} \left( \mathbf{I}_{L_{w}} \otimes \mathbf{E}_{m} \right)^{H} + \sigma_{\eta}^{2} \mathbf{I}_{RL_{w}}.$$

The SINR at the destination can be written as

SINR = 
$$\frac{\mathrm{E}\{|y_s(n)|^2\}}{\mathrm{E}\{|y_i(n)|^2\} + \mathrm{E}\{|y_n(n)|^2\}}.$$
 (27)

Using (20), we obtain that

$$E\{|y_s(n)|^2\} = E\{|\mathbf{w}_0^H \mathbf{h}_0 s(n)|^2\}$$
$$= P_s \mathbf{w}_0^H \mathbf{h}_0 \mathbf{h}_0^H \mathbf{w}_0$$
$$= P_s \mathbf{w}^H \mathbf{A}^H \mathbf{h}_0 \mathbf{h}_0^H \mathbf{A} \mathbf{w}$$
$$= \mathbf{w}^H \mathbf{Q}_s \mathbf{w}$$
(28)

where

$$\mathbf{A} \triangleq [\mathbf{I}_R, \mathbf{0}_{R \times (L_w - 1)R}]$$
$$\mathbf{Q}_s \triangleq P_s \mathbf{A}^H \mathbf{h}_0 \mathbf{h}_0^H \mathbf{A}.$$

Using (18), we have

$$E\{|y_i(n)|^2\} = E\{\mathbf{w}^H \mathcal{G} \bar{\mathbf{F}} \bar{\mathbf{s}}(n) \bar{\mathbf{s}}^H(n) \bar{\mathbf{F}}^H \mathcal{G}^H \mathbf{w}\}$$
$$= P_s \mathbf{w}^H \mathcal{G} \bar{\mathbf{F}} \bar{\mathbf{F}}^H \mathcal{G}^H \mathbf{w}$$
$$= \mathbf{w}^H \mathbf{Q}_i \mathbf{w}$$
(29)

where

$$\mathbf{Q}_i \triangleq P_s \mathcal{G} \bar{\mathbf{F}} \bar{\mathbf{F}}^H \mathcal{G}^H.$$

Making use of (19), we also obtain that

$$E\{|y_n(n)|^2\} = E\{\mathbf{w}^H \mathcal{G} \tilde{\mathbf{I}} \tilde{\boldsymbol{\eta}}(n) \tilde{\boldsymbol{\eta}}^H(n) \tilde{\mathbf{I}}^H \mathcal{G}^H \mathbf{w}\} + \sigma_v^2$$
$$= \sigma_\eta^2 \mathbf{w}^H \mathcal{G} \tilde{\mathbf{I}} \tilde{\mathbf{I}}^H \mathcal{G}^H \mathbf{w} + \sigma_v^2$$
$$= \mathbf{w}^H \mathbf{Q}_n \mathbf{w} + \sigma_v^2$$
(30)

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where

$$\mathbf{Q}_n \triangleq \sigma_n^2 \mathcal{G} \tilde{\mathbf{I}} \, \tilde{\mathbf{I}}^H \mathcal{G}^H.$$

Using (26) and (28)-(30), the problem in (22) can be rewritten in the following form:

$$\min_{\mathbf{w}} \mathbf{w}^{H} \mathbf{D} \mathbf{w} \quad \text{s.t.} \quad \frac{\mathbf{w}^{H} \mathbf{Q}_{s} \mathbf{w}}{\mathbf{w}^{H} \mathbf{Q}_{i} \mathbf{w} + \mathbf{w}^{H} \mathbf{Q}_{n} \mathbf{w} + \sigma_{v}^{2}} \geq \gamma.$$
(31)

Introducing

$$\tilde{\mathbf{w}} \triangleq \mathbf{D}^{1/2} \mathbf{w}, \quad \mathbf{Q} \triangleq \mathbf{D}^{-1/2} (\mathbf{Q}_s - \gamma \mathbf{Q}_i - \gamma \mathbf{Q}_n) \mathbf{D}^{-1/2}$$
 (32)

we can reformulate the problem in (31) as

$$\min_{\tilde{\mathbf{w}}} \|\tilde{\mathbf{w}}\|^2 \quad \text{s.t.} \quad \tilde{\mathbf{w}}^H \mathbf{Q} \tilde{\mathbf{w}} \ge \gamma \sigma_v^2.$$
(33)

The constraint function in (33) can be used for checking the feasibility of the problem for any given value of  $\gamma$ . In particular, for all the values of  $\gamma$  that lead to *negative semidefinite*  $\mathbf{Q}$ , the problem in (33) is infeasible. It can be also easily proved that the constraint in (33) can be replaced by the equality constraint  $\tilde{\mathbf{w}}^H \mathbf{Q} \tilde{\mathbf{w}} = \gamma \sigma_v^2$ . Hence, the problem (33) is equivalent to

$$\min_{\tilde{\mathbf{w}}} \|\tilde{\mathbf{w}}\|^2 \quad \text{s.t.} \quad \tilde{\mathbf{w}}^H \mathbf{Q} \tilde{\mathbf{w}} = \gamma \sigma_v^2.$$
(34)

The solution of (34) can be found by means of the Lagrange multiplier method. Let us minimize the Lagrangian

$$H(\tilde{\mathbf{w}},\lambda) = \tilde{\mathbf{w}}^H \tilde{\mathbf{w}} + \lambda (\gamma \sigma_v^2 - \tilde{\mathbf{w}}^H \mathbf{Q} \tilde{\mathbf{w}})$$
(35)

where  $\lambda$  is a Lagrange multiplier. Taking gradient of (35) and equating it to zero, we obtain that the solution is equal to that of the following eigenvalue problem:

$$\mathbf{Q}\tilde{\mathbf{w}} = \frac{1}{\lambda}\tilde{\mathbf{w}}.$$
(36)

Multiplying both sides of (36) with  $\lambda \tilde{\mathbf{w}}^H$  yields

$$\|\tilde{\mathbf{w}}\|^2 = \tilde{\mathbf{w}}^H \tilde{\mathbf{w}} = \lambda \tilde{\mathbf{w}}^H \mathbf{Q} \tilde{\mathbf{w}} = \lambda \gamma \sigma_v^2.$$
(37)

It can be seen from (37) that minimizing  $\|\tilde{\mathbf{w}}\|^2$  leads to the smallest positive  $\lambda$ , which is equivalent to the largest  $1/\lambda$  in (36). Using the latter fact, we conclude that the optimal solution to (33) can be written as

$$\tilde{\mathbf{w}}_{\text{opt}} = \beta \, \mathcal{P}\{\mathbf{Q}\} \tag{38}$$

where  $\mathcal{P}\{\cdot\}$  denotes the normalized principal eigenvector of a matrix and

$$\beta = \left(\frac{\gamma \sigma_v^2}{\mathcal{P}\{\mathbf{Q}\}^H \mathbf{Q} \,\mathcal{P}\{\mathbf{Q}\}}\right)^{1/2}.\tag{39}$$

Therefore, the optimal beamformer weight vector and the minimum total relay transmitted power can be expressed as

$$\mathbf{w}_{\rm opt} = \beta \, \mathbf{D}^{-1/2} \mathcal{P}\{\mathbf{Q}\} \tag{40}$$

$$P_{\min} = \gamma \sigma_v^2 / \mathcal{L}_{\max} \{ \mathbf{Q} \}$$
(41)

respectively, where  $\mathcal{L}_{max}\{\cdot\}$  denotes the largest (principal) eigenvalue of a matrix.

Hence, the FF distributed beamforming problem (22) enjoys a simple closed-form solution based on the principal eigenvector of the matrix  $\mathbf{Q}$ .

# B. QoS Maximization Under the Total Relay Power Constraint

Now, let us consider another useful distributed beamforming problem. Let us maximize the receiver SINR under the constraint that the total relay transmitted power does not exceed some maximal value  $P_{\text{max}}$ . This problem can be written as

$$\max_{\mathbf{w}} \text{SINR} \quad \text{s.t.} \quad P \le P_{\max}. \tag{42}$$

Using (26) and (28)-(30), the latter problem can be expressed as

$$\max_{\mathbf{w}} \ \frac{\mathbf{w}^H \mathbf{Q}_s \mathbf{w}}{\mathbf{w}^H \mathbf{Q}_i \mathbf{w} + \mathbf{w}^H \mathbf{Q}_n \mathbf{w} + \sigma_v^2} \quad \text{s.t.} \quad \mathbf{w}^H \mathbf{D} \mathbf{w} \le P_{\max}.$$
(43)

Introducing

$$\tilde{\mathbf{Q}}_s \triangleq \mathbf{D}^{-1/2} \mathbf{Q}_s \mathbf{D}^{-1/2}, \quad \tilde{\mathbf{Q}}_{i+n} \triangleq \mathbf{D}^{-1/2} (\mathbf{Q}_i + \mathbf{Q}_n) \mathbf{D}^{-1/2}$$

we obtain that the problem (43) can be rewritten as

$$\max_{\tilde{\mathbf{w}}} \frac{\tilde{\mathbf{w}}^H \tilde{\mathbf{Q}}_s \tilde{\mathbf{w}}}{\tilde{\mathbf{w}}^H \tilde{\mathbf{Q}}_{i+n} \tilde{\mathbf{w}} + \sigma_v^2} \qquad \text{s.t.} \quad \|\tilde{\mathbf{w}}\|^2 \le P_{\max}$$
(44)

where, as before,  $\tilde{\mathbf{w}} \triangleq \mathbf{D}^{1/2}\mathbf{w}$ . It can be easily proved that the objective function in (44) achieves its maximum when the constraint is satisfied with equality (i.e.,  $\|\tilde{\mathbf{w}}\|^2 = P_{\text{max}}$ ). Therefore, the problem (44) can be rewritten as

$$\max_{\tilde{\mathbf{w}}} \frac{\tilde{\mathbf{w}}^H \tilde{\mathbf{Q}}_s \tilde{\mathbf{w}}}{\tilde{\mathbf{w}}^H (\tilde{\mathbf{Q}}_{i+n} + (\sigma_v^2 / P_{\max}) \mathbf{I}) \tilde{\mathbf{w}}} \quad \text{s.t.} \quad \|\tilde{\mathbf{w}}\|^2 = P_{\max}.$$
(45)

In contrast to the problem of Section III-A, the problem (45) is always feasible because for any positive  $P_{\text{max}}$ , its feasible set is nonempty. Using the results of [21] (where a mathematically similar problem

has been discussed in a different context), we conclude that the objective function in (45) is maximized when  $\tilde{\mathbf{w}}$  is chosen as the normalized principal eigenvector of the matrix  $(\tilde{\mathbf{Q}}_{i+n} + \sigma_v^2/P_{\max}\mathbf{I})^{-1}\tilde{\mathbf{Q}}_s$ . Note here that any arbitrary scaling of  $\tilde{\mathbf{w}}$  does not change the value of the objective function in (45). However, the so-obtained vector  $\tilde{\mathbf{w}}$  have to be properly scaled to satisfy the power constraint  $\|\tilde{\mathbf{w}}\|^2 = P_{\max}$ . Then, the solution to (45) can be written as

$$\tilde{\mathbf{w}}_{\text{opt}} = \sqrt{P_{\text{max}}} \mathcal{P}\left\{ (\tilde{\mathbf{Q}}_{i+n} + (\sigma_v^2 / P_{\text{max}})\mathbf{I})^{-1} \tilde{\mathbf{Q}}_s \right\}$$
(46)

and, therefore, the optimal beamforming weight vector and the maximum SINR at the destination can be written as

$$\mathbf{w}_{\text{opt}} = \sqrt{P_{\text{max}}} \mathbf{D}^{-1/2} \mathcal{P} \left\{ (\tilde{\mathbf{Q}}_{i+n} + (\sigma_v^2 / P_{\text{max}}) \mathbf{I})^{-1} \tilde{\mathbf{Q}}_s \right\}$$
(47)

$$\operatorname{SINR}_{\max} = \mathcal{L}_{\max} \left\{ (\tilde{\mathbf{Q}}_{i+n} + (\sigma_v^2 / P_{\max}) \mathbf{I})^{-1} \tilde{\mathbf{Q}}_s \right\}$$
(48)

respectively.

## C. QoS Maximization Under the Individual Relay Power Constraints

Now, let us consider another relevant distributed beamforming problem which differs from (42) is that the individual relay power constraints are used instead of the total relay power constraints. This problem can be written as

$$\max_{\mathbf{w}} \text{ SINR s.t. } p_m \le p_{m,\max}, \quad m = 1, \cdots, R$$
(49)

where  $p_{m,\max}$  denotes the maximal transmitted power of the *m*th relay. Using (25) and (28)-(30), and introducing a new auxiliary variable  $\tau > 0$  [22], the problem (49) can be transformed to

$$\max_{\mathbf{w},\tau} \quad \tau$$
s.t. 
$$\frac{\mathbf{w}^{H}\mathbf{Q}_{s}\mathbf{w}}{\mathbf{w}^{H}\mathbf{Q}_{i}\mathbf{w} + \mathbf{w}^{H}\mathbf{Q}_{n}\mathbf{w} + \sigma_{v}^{2}} \geq \tau^{2}$$

$$\mathbf{w}^{H}\mathbf{D}_{m}\mathbf{w} \leq p_{m,\max}, \qquad m = 1, \cdots, R.$$
(50)

The first constraint in (50) can be rewritten as

$$\sqrt{P_s} |\mathbf{w}^H \mathbf{h}| \ge \tau \sqrt{\mathbf{w}^H \mathbf{Q}_i \mathbf{w} + \mathbf{w}^H \mathbf{Q}_n \mathbf{w} + \sigma_v^2}$$
(51)

where  $\mathbf{h} \triangleq \mathbf{A}^H \mathbf{h}_0$ . We observe that any arbitrary phase rotation of  $\mathbf{w}$  will not change the value of the objective function in (50). Using a proper phase rotation, we have that the constraint (51) is equivalent

to

$$\sqrt{P_s} \mathbf{w}^H \mathbf{h} \ge \tau \sqrt{\mathbf{w}^H \mathbf{Q}_i \mathbf{w} + \mathbf{w}^H \mathbf{Q}_n \mathbf{w} + \sigma_v^2}$$
(52)

$$\operatorname{Im}\{\mathbf{w}^{H}\mathbf{h}\}=0\tag{53}$$

where  $\text{Im}\{\cdot\}$  denotes the imaginary part of a complex value. Note, however, that (53) can be omitted as it is automatically taken into account in (52) by the fact that the right-hand side of (52) is non-negative. Then, the problem (50) can be rewritten as

$$\max_{\mathbf{w},\tau} \quad \tau$$
s.t.  $\sqrt{P_s} \mathbf{w}^H \mathbf{h} \ge \tau \sqrt{\mathbf{w}^H \mathbf{Q}_i \mathbf{w} + \mathbf{w}^H \mathbf{Q}_n \mathbf{w} + \sigma_v^2}$ 

$$\mathbf{w}^H \mathbf{D}_m \mathbf{w} \le p_{m,\max}, \quad m = 1, \cdots, R.$$
(54)

Let

$$\mathbf{B} \triangleq \begin{bmatrix} \sigma_{v}^{2} & \mathbf{0}_{RL_{w}\times 1}^{T} \\ \mathbf{0}_{RL_{w}\times 1} & \mathbf{Q}_{i} + \mathbf{Q}_{n} \end{bmatrix} = \mathbf{U}^{H}\mathbf{U}$$
(55)

$$\mathbf{D}_m = \mathbf{V}_m^H \mathbf{V}_m, \quad m = 1, \cdots, R \tag{56}$$

be the Cholesky factorizations of the matrices  $\mathbf{B}$  and  $\mathbf{D}_m$ , respectively. Introducing new notations

$$\breve{\mathbf{w}} \triangleq [1, \mathbf{w}^T]^T, \quad \breve{\mathbf{V}}_m \triangleq [\mathbf{0}_{RL_w \times 1}, \mathbf{V}_m], \quad \breve{\mathbf{h}} \triangleq [0, \mathbf{h}^T]^T$$
(57)

we can further rewrite the problem (54) as

$$\max_{\breve{\mathbf{w}},\tau} \quad \tau 
\text{s.t.} \quad \sqrt{P_s} \breve{\mathbf{w}}^H \breve{\mathbf{h}} \ge \tau \| \mathbf{U} \breve{\mathbf{w}} \| 
\| \breve{\mathbf{V}}_m \breve{\mathbf{w}} \| \le \sqrt{p_{m,\max}}, \quad m = 1, \cdots, R 
\quad \breve{\mathbf{w}}^H \mathbf{e}_1 = 1.$$
(58)

In contrast to the problem of Section III-A, the problem (58) is always feasible. This can be directly seen from its equivalent formulation (49) whose feasible set is always nonempty. Moreover, the problem (58) is *quasi-convex* [22], because for any value of  $\tau$ , it reduces to the following second-order cone

programming (SOCP) feasibility problem:

find 
$$\mathbf{\breve{w}}$$
  
s.t.  $\sqrt{P_s}\mathbf{\breve{w}}^H\mathbf{\widetilde{h}} \ge \tau \|\mathbf{U}\mathbf{\breve{w}}\|$   
 $\|\mathbf{\breve{V}}_m\mathbf{\breve{w}}\| \le \sqrt{p_{m,\max}}, \quad m = 1, \cdots, R$  (59)  
 $\mathbf{\breve{w}}^H\mathbf{e}_1 = 1.$ 

Let  $\tau_*$  be the optimal value of  $\tau$  in (58). Then, for any  $\tau > \tau_*$ , the problem (59) is infeasible. On the contrary, if (59) is feasible, then we conclude that  $\tau \le \tau_*$ . Hence, the optimum  $\tau_*$  and the optimal weight vector  $\breve{w}_*$  can be found using the bisection search technique discussed in [17]. Assuming that  $\tau_*$  lies in the interval  $[\tau_l, \tau_u]$ , the bisection search procedure to solve (58) can be summarized as the following sequence of steps:

- 1)  $\tau := (\tau_l + \tau_u)/2.$
- 2) Solve the convex feasibility problem (59). If (59) is feasible, then  $\tau_l := \tau$ , otherwise  $\tau_u := \tau$ .
- 3) If  $(\tau_u \tau_l) < \varepsilon$  then stop. Otherwise, go to Step 1.

Here,  $\varepsilon$  is the error tolerance value in  $\tau$ .

Note that the feasibility problem (59) is a standard SOCP problem, which can be efficiently solved using interior point methods [23] with the worst-case complexity of  $\mathcal{O}((RL_w)^{3.5})$ . The initial interval for the bisection search can be selected as  $[\tau_l, \tau_u] = [0, \sqrt{\text{SINR}_{\max}(P_{\max})}]$ , where  $\text{SINR}_{\max}(P_{\max})$  can be obtained from (48) by choosing  $P_{\max} = \sum_{m=1}^{R} p_{m,\max}$ . This particular choice is motivated by the fact that the optimal SINR of (42) always upper bounds the optimal SINR of (49).

*Remark:* It is worth noting that the total power constraint can be easily added to (59) just as one more second-order cone constraint

$$\|\mathbf{V}\mathbf{w}\| \le \sqrt{P_{\max}}$$

where  $\mathbf{V}^{H}\mathbf{V}$  is the Cholesky factorization of **D**. This allows us to directly extend the approach of (59) to a practically important case when both the individual and the total power constraints have to be taken into account [16].

#### D. Relationships Between the Proposed Methods and Earlier Techniques in the Flat Fading Case

Let us now explore the relationship between the proposed three methods and the techniques of [14], [16] and [17] in the specific case when all the channels are frequency flat and each relay filter is just a complex coefficient ( $L_f = L_g = L_w = 1$ ). In the latter case, the transmitted power of the *m*th relay in (25) can be simplified to

$$p_m = P_s \mathbf{w}_0^H \mathbf{E}_m \mathbf{f}_0 \mathbf{f}_0^H \mathbf{E}_m^H \mathbf{w}_0 + \sigma_\eta^2 \mathbf{w}_0^H \mathbf{E}_m \mathbf{E}_m^H \mathbf{w}_0$$
$$= \sigma_{f,m}^2 |w_{0,m}|^2 + \sigma_\eta^2 |w_{0,m}|^2$$
(60)

where  $\sigma_{f,m}^2 \triangleq P_s |f_{0,m}|^2$ . Then, the total relay transmitted power can be expressed as

$$P = \sum_{m=1}^{R} p_m = \mathbf{w}_0^H \mathbf{D}_0 \mathbf{w}_0$$
(61)

where

$$\mathbf{D}_0 \triangleq \operatorname{diag} \{ \sigma_{f,1}^2, \cdots, \sigma_{f,R}^2 \} + \sigma_{\eta}^2 \mathbf{I}_R.$$

The received signal at the destination (15) can be simplified to

$$y(n) = \underbrace{\mathbf{w}_{0}^{H}(\mathbf{f}_{0} \odot \mathbf{g}_{0})s(n)}_{\text{signal}} + \underbrace{\mathbf{w}_{0}^{H}(\mathbf{g}_{0} \odot \boldsymbol{\eta}(n)) + \upsilon(n)}_{\text{noise}}.$$
(62)

As the channel is frequency flat, there is no ISI term in (62). Hence, the SINR reduces to SNR, and it can be written as

$$SNR = \frac{\mathbf{w}_0^H \mathbf{Q}_{s0} \mathbf{w}_0}{\mathbf{w}_0^H \mathbf{Q}_{n0} \mathbf{w}_0 + \sigma_v^2}$$
(63)

where

$$\begin{split} \mathbf{Q}_{s0} &\triangleq P_s(\mathbf{f}_0 \odot \mathbf{g}_0) (\mathbf{f}_0 \odot \mathbf{g}_0)^H = P_s \mathbf{h}_0 \mathbf{h}_0^H \\ \mathbf{Q}_{n0} &\triangleq \sigma_\eta^2 \mathrm{diag}\{|g_{0,1}|^2, \cdots, |g_{0,R}|^2\}. \end{split}$$

Introducing the variables  $0 \le \alpha_m \le 1$   $(m = 1, \dots, R)$  and using them to express the relay powers as

$$p_m = \alpha_m^2 p_{m,\max} \tag{64}$$

we obtain from (60) and (64) that

$$|w_{0,m}| = \alpha_m \sqrt{\frac{p_{m,\max}}{\sigma_\eta^2 + P_s |f_{0,m}|^2}}. \label{eq:w_0,m}$$

In [14], it is proposed to compensate the phases caused by the transmitter-to-relay and relay-to-destination channels by a proper choice of the phase of each  $w_{0,m}$ . This gives

$$w_{0,m} = \alpha_m \sqrt{\frac{p_{m,\max}}{\sigma_\eta^2 + P_s |f_{0,m}|^2}} e^{j\theta_m}$$
(65)

where  $\theta_m = \arg f_{0,m} + \arg g_{0,m}$ . Inserting (65) into (63), we have

$$SNR = \frac{P_s \left( \sum_{m=1}^R \alpha_m |f_{0,m}g_{0,m}| \sqrt{\frac{p_{m,\max}}{\sigma_\eta^2 + P_s |f_{0,m}|^2}} \right)^2}{\sigma_v^2 + \sum_{m=1}^R \frac{\alpha_m^2 p_{m,\max} |g_{0,m}|^2 \sigma_\eta^2}{\sigma_\eta^2 + P_s |f_{0,m}|^2}}.$$
(66)

From (66), it can be seen that our problem (49) in the considered particular case is equivalent to

$$\max_{\alpha_{1},\dots,\alpha_{R}} \quad \frac{P_{s}\left(\sum_{m=1}^{R} \alpha_{m} |f_{0,m}g_{0,m}| \sqrt{\frac{p_{m,\max}}{\sigma_{\eta}^{2} + P_{s}|f_{0,m}|^{2}}}\right)^{2}}{\sigma_{v}^{2} + \sum_{m=1}^{R} \frac{\alpha_{m}^{2} p_{m,\max}|g_{0,m}|^{2} \sigma_{\eta}^{2}}{\sigma_{\eta}^{2} + P_{s}|f_{0,m}|^{2}}}$$
  
s.t.  $0 \le \alpha_{1}, \dots, \alpha_{R} \le 1$ . (67)

It can be readily verified that (67) and the problem in [14] are identical. Moreover, the problem of [16] extends that of [14] to the case when both the individual and the total power constraints are used. Therefore, in the flat fading case where the AF strategy is used instead of the FF one, our approach of Section III-C reduces to that of [14] and, with the additional total power constraint added to (59), it reduces to that of [16].

To understand the relationship of our three approaches of Section III and the techniques of [17], we note that in the flat fading AF case the only difference between the problem formulations in [17] and our problem formulations is that an extra statistical expectation over all the random transmitter-to-relay and relay-to-destination channels has been used in [17]. Hence, in the flat fading case, the problem formulations of [17] transfer to our problems (22), (42) and (49) when the instantaneous instead of the second-order CSI is used in the methods of [17] and when the FF strategy is replaced by the AF one in our techniques.

#### **IV. SIMULATION RESULTS**

In our simulations, we consider a relay network with R = 10 relays and quasi-static frequency selective transmitter-to-relay and relay-to-destination channels with the lengths  $L_f = L_g = 5$ . The transmitter uses the binary phase shift keying (BPSK) modulation. The channel impulse response coefficients are modeled as zero-mean complex Gaussian random variables with an exponential power delay profile [24]

$$p(t) = \frac{P_R}{\sigma_t} \sum_{l=0}^{L_x} e^{-t/\sigma_t} \delta(t - lT_s)$$
(68)

where  $L_x \in \{L_f, L_g\}$ ,  $T_s$  is the symbol duration,  $\delta(\cdot)$  is the Dirac delta function,  $P_R$  is the average power of the multipath components, and  $\sigma_t$  characterizes the delay spread. In our simulations,  $P_R = 1$ and  $\sigma_t = 2T_s$  are used. The relay and destination noises are assumed to have the same powers, and the source transmitted power is 10 dB higher than the noise power. To obtain the bit error rate (BER) curves, it has been assumed that the symbol-by-symbol maximum likelihood (ML) decoder is used at the receiver.

In the first example, we test the approach of (40) which is based on minimizing the total transmit power subject to the QoS constraint. Fig. 2 displays the total relay transmitted power versus the minimum required SINR at the destination for different lengths of the relay filters. As for randomly generated signal, noise and channel values the feasibility of (33) is a random event, this problem can be infeasible for some percentage of simulation runs. To deal with this fact in our first example, we call the problem *ergodically infeasible* when the number of simulation runs leading to infeasibility of (33) is larger than the half of the total number of simulation runs; otherwise, this problem is classified as *ergodically feasible*. If the problem is ergodically infeasible, the corresponding points are dropped from the figures displaying the behaviour of the total transmitted power. In the case of ergodic feasibility, the corresponding points are computed by averaging over the "feasible" runs and displayed in these figures. For example, there are several dropped points in Fig. 2 at high SINR values that correspond to the case of ergodic infeasibility of (33). To further illustrate the effects of the required SINR and  $L_w$  on the feasibility of the problem (33), the probability that this problem is feasible is displayed in Fig. 3. The latter probability is referred to as *feasibility probability*.

It can be seen from Fig. 2 that using the FF strategy at the relays, one can substantially reduce the total relay transmitted power as compared to the AF approach. Also, from Figs. 2 and 3 is is clear that the FF strategy significantly improves the SINR feasibility range of the considered distributed beamforming problem. These improvements are monotonic in  $L_w$ : for example, according to Fig. 2, for SINR = 12 dB this problem is ergodically infeasible for the relay filter lengths  $L_w = 1, 2$ , but it becomes ergodically feasible for any  $L_w \ge 3$ . These observations are further supported by Fig. 3 that demonstrates that the feasibility probability can be substantially improved by increasing the relay filter length  $L_w$ .

Figs. 4 and 5 depict the total relay transmitted power and feasibility probability versus the relay filter length  $L_w$  for different values of the required SINR at the destination. Similarly to the previous two figures, Figs. 4 and 5 clearly demonstrate that the performance (in terms of the relay transmitted power) and feasibility of the QoS constraint can be substantially improved by using the FF approach in lieu of the AF strategy, and these improvements become more pronounced when increasing the relay filter length. Note that theoretically, to fully compensate the effect of frequency selective source-to-relay and relayto-destination channels, it is required that  $L_w \ge L_f + L_g - 1$ . However, in the exponential power delay profile case (where these channels are mainly determined by several first taps), they are well compensated even with  $L_w < L_f + L_g - 1$ . As follows from Fig. 4, depending on the SINR value, the filter lengths  $L_w = 2$  to  $L_w = 5$  appear to be sufficient.

In the second example, we test the approach of (47) which maximizes the QoS subject to the total power constraint. Note that the problem (45) is always feasible and, therefore, there is no infeasibility issue in this example. Fig. 6 shows the achieved SINR versus the total relay transmitted power  $P_{\text{max}}$  for different lengths of the relay filters. Fig. 7 depicts the SINR versus the relay filter length  $L_w$  for different values of the total relay transmitted power  $P_{\text{max}}$ . It can be seen from these figures that the QoS can be substantially improved by increasing the filter length  $L_w$ .

To illustrate the receiver error probability performance of the FF relaying approach based on the particular example of the distributed beamformer (47), in Figs. 8 and 9 we display the receiver BERs versus the total transmitted power  $P_{\text{max}}$  and the relay filter length  $L_w$ , respectively. It can be observed from these two figures that increasing the filter length, we can substantially decrease the receiver error probability.

In our last example, the approach of Section III-C is tested, which maximizes the QoS under the individual relay power constraints using a combination of (59) and bisection search. As in the previous example, the underlying problem (58) is always feasible and, therefore, there is no infeasibility issue here. It is assumed that all the relays have the same maximal allowed transmitted power  $p_{\text{max}}$ . Fig. 10 displays the SINR versus  $p_{\text{max}}$  for different lengths of the relay filters. Fig. 11 shows the SINR versus the relay filter length  $L_w$  for different values of  $p_{\text{max}}$ .

The conclusions following from Figs. 10 and 11 are quite similar to that following from Figs. 6 and 7. As the individual power constraints are tighter than the total one, the SINRs achieved for any value of the total transmitted power in Figs. 10 and 11 are a few dB's lower than that achieved in Figs. 6 and 7 for the same value of  $Rp_{max}$ .

Summarizing, all our examples clearly verify that the proposed FF strategy substantially outperforms the AF approach in the frequency selective fading case.

#### V. CONCLUSION

The problem of distributed network beamforming has been addressed in the case when the transmitterto-relay and relay-to-destination channels are frequency selective. To compensate for the effects of these channels, a novel filter-and-forward relay beamforming strategy has been proposed as an extension of the traditional amplify-and-forward protocol. According to the former strategy, FIR filters have to be used at the relay nodes to remove ISI. Three relevant half-duplex filter-and-forward beamforming problems have been formulated and solved. Our first technique minimizes the total relay transmitted power subject to the destination QoS constraint, whereas the second and third methods maximize the destination QoS subject to the total and individual relay transmitted power constraints, respectively. For the first and second approaches, closed-form beamformers have been obtained, whereas the third beamformer is computed using convex optimization, via a combination of bisection search and second-order cone programming. It has been also shown that the latter convex optimization-based relay beamformer can be easily extended to the practically important case when the individual and total power constraints should be jointly taken into account.

Our simulation results demonstrate that in the frequency selective fading case, the proposed filterand-forward beamforming strategy provides substantial performance improvements as compared to the commonly used amplify-and-forward relay beamforming approach.

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Fig. 1. Filter-and-forward relay network.



Fig. 2. Total relay transmitted power versus required SINR; first example.



Fig. 3. Feasibility probability of the problem (33) versus required SINR; first example.



Fig. 4. Total relay transmitted power versus relay filter length  $L_w$ ; first example.

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Fig. 5. Feasibility probability of the problem (33) versus relay filter length  $L_w$ ; first example.



Fig. 6. SINR versus the maximal total relay transmitted power  $P_{\text{max}}$ ; second example.



Fig. 7. SINR versus relay filter length  $L_w$ ; second example.



Fig. 8. SER versus the maximal total relay transmitted power  $P_{\max}$ ; second example.



Fig. 9. SER versus relay filter length  $L_w$ ; second example.



Fig. 10. SINR versus the maximal individual relay transmitted power  $p_{\max}$ ; third example.



Fig. 11. SINR versus relay filter length  $L_w$ ; third example.

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