

# Improved Linear Transmit Processing for Single-User and Multi-User MIMO Communications Systems

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**Abstract**—In this paper, we propose a novel linear transmit precoding strategy for multiple-input, multiple-output (MIMO) systems employing improper signal constellations. In particular, improved zero-forcing (ZF) and minimum mean square error (MMSE) precoders are derived based on modified cost functions, and are shown to achieve a superior performance without loss of spectrum efficiency compared to the conventional linear and non-linear precoders. The superiority of the proposed precoders over the conventional solutions are verified by both simulation and analytical results. The novel approach to precoding design is also applied to the case of an imperfect channel estimate with a known error covariance as well as to the multi-user scenario where precoding based on the nullspace of channel transmission matrix is employed to decouple multi-user channels. In both cases, the improved precoding schemes yield significant performance gain compared to the conventional counterparts.

## I. INTRODUCTION

Recent studies have shown that the use of multiple antennas in a wireless communication system significantly improves the system's spectral efficiency, enables a growth in transmission rate linear in the minimum number of antennas at either end [1, 2], and improves link reliability and coverage [3]. However, the main problem for transmission over multiple-input, multiple-output (MIMO) channels is the separation or equalization of the parallel data streams. In order to exploit the capacity and performance gains promised by MIMO, we must deal with the channel distortion and the interference. This can be accomplished by transmitter or receiver design, or joint optimization of transmitter and receiver. Different transmit precoding and receive filtering strategies were discussed and compared in [4–6]. It was shown that nonlinear precoding, e.g., Tomlinson-Harashima precoding (THP) outperforms linear precoding as well as nonlinear receiver filtering, such as MIMO decision feedback equalization (DFE). Joint design of precoding at the transmitter and equalization at the receiver for multicarrier MIMO channels under a variety of design criteria was addressed in [7], where the authors formulated the design problem within the framework of convex optimization

theory, in which a number of design criteria can be easily accommodated and efficiently solved. Joint design of optimum linear precoder and decoder for a MIMO channel using a weighted minimum mean square error (MMSE) criterion subject to a transmit power constraint was treated in [8]. Closed form solutions are derived for the optimum precoder and decoder as functions of error weights, transmit power, receiver noise variance, and eigenvalues of the MIMO channel. In [9], Shenouda and Davidson proposed a unified framework for joint transceiver design for MIMO systems implementing interference (pre)subtraction and presented optimal designs for two broad classes of communication objectives, namely those that are Schur-convex and Schur-concave functions of the logarithms of the mean square error (MSE) of each data stream. An equal-diagonal QR decomposition was proposed in [10] to precoded successive-cancellation detection. Joint design of the transmitter-receiver for a block-by-block communication system with decision feedback detection was addressed in [11]. However, joint optimization of transmit and receive filters is feasible only when the receivers and transmitters are fully cooperative, i.e., the transmit signals and the receive signals can be cooperatively pre- and post-transformed, respectively [12]. For non-cooperative systems, which are the focus of this study, one has to resort to transmit or receive processing. Compared to the jointly optimized transmit and receive filters, they also have the advantage of much reduced complexity.

The mobile stations (MSs) are usually restricted to simple algorithms with low computational complexity. One example is the downlink transmission where the base station (BS) with multiple antennas transmits to multiple mobile users. Each user has access only to its received signal and cannot cooperate with the other users. Thus, receive processing is impractical and the system must resort to precoding. Transmit processing (precoding) requires that the channel state information (CSI) is available at the transmitter, which is a feasible assumption when a feedback channel is present or when the transmitter and receiver operate in time division duplex (TDD) mode so that the channel transfer functions are identical for both uplink and downlink.

Both linear and nonlinear transmit processing (precoding) have been proposed in the literature. The nonlinear THP scheme [4, 13, 14] is designed by transferring the feedback part of the DFE to the transmitter in order to

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tackle the problem of error propagation and to reduce the receiver complexity. It was shown in [15, 16] that lossless precoding for cancelling arbitrary interference is theoretically possible if the interference is known a-priori to the transmitter. In [17], the authors discussed how to replace THP with trellis precoding (optimum vector quantizer) which takes into account the noncausal interfering sequence to compensate for the shaping loss. In [18], multidimensional vector quantization based on lattice design was proposed and shown to approach the capacity predicted by Costa's theory [15]. However, the main disadvantage of these nonlinear precoding schemes (especially the trellis-based precoders) is their high computational complexity which is prohibitive for practical implementation.

Motivated by the need for low power consumption and low complexity mobile terminals, we focus on systems where the complex signal processing is performed at the base station, i.e., transmit processing in the downlink. In particular, we investigate simple precoding techniques which only require linear processing at transmitter. Optimization of MIMO linear precoding schemes was proposed in several papers, e.g., [12, 19, 20], where transmit matched filter (MF), zero-forcing (ZF) filter, and minimum mean square error (MMSE) filter were introduced. It was shown in [12] that different transmit filters can be obtained with the same optimization as the respective receive filters, where only a transmit power constraint has to be included. Both the transmit and receive Wiener (MMSE) filters converge to the matched filter and the ZF filter for low and high signal-to-noise ratio (SNR), respectively. The linear precoders that minimize the bit error rate (BER) at moderate-to-high signal-to-noise ratios (SNRs) for block transmission systems with ZF and MMSE equalization were derived in [21] and [22], respectively. They were shown to outperform standard block transmission schemes, such as orthogonal frequency division multiplexing (OFDM). The design was obtained by observing the fact that at high SNR, the expression for the BER is a convex function of the magnitude of the diagonal elements of the equalizer.

In this paper, we propose novel linear precoding schemes to improve the quality of downlink transmission in MIMO systems. The major contributions of this paper are summarized as follows.

- We show that the existing linear precoders are suboptimum for systems employing improper modulation schemes and their performance can be improved by designing the system with modified cost functions and by exploitation of the improperness of signal constellation. The proposed linear schemes are compared to their conventional counterparts in both single-user and multi-user MIMO systems, and are shown to have superior performance without any loss of spectrum efficiency.
- We investigate the design of robust precoders in the presence of imperfect CSI when the channel is modeled as a channel estimate and its estimation

error covariance. Under such circumstances, different precoders are derived based on the expected value of MSEs for both improper and proper signals. The results indicate that imperfect CSI leads to significant performance loss since statistical robust designs do not guarantee any optimality for a particular realization of the random channel. However, the problem is less severe in the proposed systems. By utilizing improperness of the signal constellation, the precoder can be designed to be more robust to imperfect CSI.

- We conduct a theoretical study verifying that the proposed ZF precoder yields a superior error rate performance and an increase in system capacity compared to the conventional ZF precoder.

The remainder of the paper is organized as follows. The improved precoding algorithms are introduced in Section II for single-user MIMO systems. The issue of robust precoding with imperfect channel state information is address in Section III. The proposed precoding schemes are extended to multi-user scenario in Section IV. The effectiveness of the proposed schemes are demonstrated in Section V with computer simulations, and conclusions are drawn in Section VI. The performance of the conventional and improved schemes are compared analytically in the appendix.

Throughout this paper,  $(\cdot)^T$  denotes matrix transpose,  $(\cdot)^H$  matrix conjugate transpose,  $(\cdot)^*$  matrix conjugate,  $E[\cdot]$  expectation,  $\|\cdot\|$  Euclidian norm,  $\|\cdot\|_F$  Frobenius norm,  $\text{Tr}(\cdot)$  trace operation,  $\det(\cdot)$  determinant of a matrix, and  $\mathbf{I}_N$  an  $N \times N$  identity matrix. We define a complex derivative as [23]

$$\frac{\partial J}{\partial A} = \frac{1}{2} \left( \frac{\partial J}{\partial A_R} - j \frac{\partial J}{\partial A_I} \right),$$

where the complex variable  $A = A_R + jA_I$ .

## II. IMPROVED PRECODING SCHEMES FOR SU-MIMO SYSTEMS WITH PERFECT CSI

In this section, we consider the downlink single-user (SU)-MIMO system depicted in Fig. 1. It is assumed that the transmitter has perfect CSI. The proposed design will be extended to imperfect CSI and multi-user scenario in Section III and IV, respectively. Denote  $N_t$ ,  $N_r$ ,  $B$  as the number of transmit, receive antennas and the number of information symbols, respectively. The transmitted symbol vector  $\mathbf{s} \in \mathbb{C}^{B \times 1}$  is precoded using the precoding matrix  $\mathbf{P} \in \mathbb{C}^{N_t \times B}$ , i.e.,  $\mathbf{x} = \mathbf{P}\mathbf{s} \in \mathbb{C}^{N_t \times 1}$ , where the symbol vector  $\mathbf{s} = [s_1 \ \dots \ s_B]^T$  comprises the transmit symbol of  $B$  parallel data streams. It is assumed that  $B$  does not exceed  $\min(N_t, N_r)$  [12]. These streams can be due to a parallel (layered) encoding of a high-rate data signal, or they may belong to different and independent users. The data symbols are assumed to be uncorrelated and to have zero mean and identical energy  $\sigma_s^2$ , i.e.,  $E[\mathbf{s}\mathbf{s}^H] = \sigma_s^2 \mathbf{I}_B$ . The signal after the precoder satisfies the total power constraint such that

$$E[\|\mathbf{x}\|^2] = E[\|\mathbf{P}\mathbf{s}\|^2] = \text{Tr}(\mathbf{P}\mathbf{P}^H \sigma_s^2) = N_t \sigma_s^2. \quad (1)$$

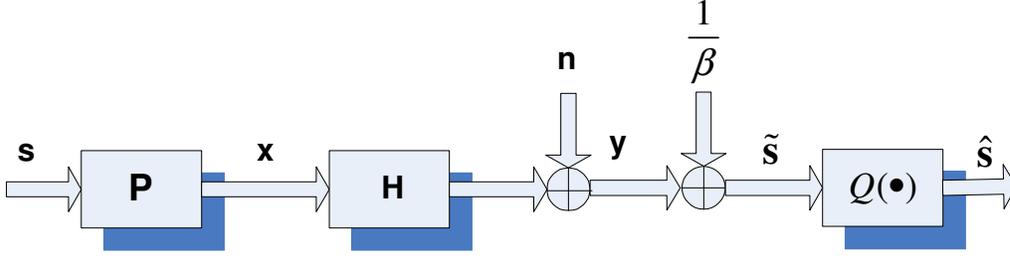


Fig. 1. Block diagram of the precoding design for SU-MIMO systems.

We consider an uncoded MIMO system using improper modulation (for which  $E[ss^T] \neq \mathbf{0}$ ), such as  $M$ -ary amplitude shift-keying (ASK), offset quadrature phase shift keying (OQPSK) [24], etc.. After transmitting  $\mathbf{x}$  over the channel, the received signal is perturbed by noise. The soft decision statistic for the symbol vector received at the  $N_r$  receive antenna array can be expressed as

$$\tilde{\mathbf{s}} = \beta^{-1}(\mathbf{H}\mathbf{P}\mathbf{s} + \mathbf{n}) \in \mathbb{C}^{N_r \times 1}, \quad (2)$$

where  $\mathbf{n} \in \mathbb{C}^{N_r \times 1}$  denotes the complex additive white Gaussian noise vector with zero mean and covariance matrix  $\sigma_n^2 \mathbf{I}_{N_r}$ , i.e.,  $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \sigma_n^2 \mathbf{I}_{N_r})$ . The scalar  $\beta$  is an automatic gain control factor, chosen to meet the transmit power constraint for the precoder. The channel matrix  $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$  contains the complex channel gains between every transmit and receive antenna pair. No special structure is required for the channel matrix  $\mathbf{H}$ . The algorithms developed in this paper are applicable to systems in either flat fading or frequency selective fading channels ( $\mathbf{H}$  is a block Toeplitz matrix in this case).

The conventional precoder  $\mathbf{P}$  is derived by minimizing the minimum square error (MSE)

$$\epsilon = E[\|\tilde{\mathbf{s}} - \mathbf{s}\|^2] = E[\|\beta^{-1}(\mathbf{H}\mathbf{P}\mathbf{s} + \mathbf{n}) - \mathbf{s}\|^2], \quad (3)$$

with transmit power constraint specified by (1). This design criterion is optimum for systems with proper modulations, such as  $M$ -QAM and  $M$ -PSK (for which  $E[ss^T] = \mathbf{0}$ ). However, for the improper modulation schemes considered in this paper, such as  $M$ -ary ASK, OQPSK (for which  $E[ss^T] = \sigma_s^2 \mathbf{I}_B$ ), the design criterion expressed by (3) is suboptimum. We propose a new precoding scheme based on an error criterion defined by

$$\mathbf{e} = \beta^{-1} \text{Re}\{\mathbf{H}\mathbf{P}\mathbf{s} + \mathbf{n}\} - \mathbf{s}, \quad (4)$$

and the new precoder results from the following optimization

$$\begin{aligned} \mathbf{P} &= \arg \min_{\beta, \mathbf{P}} E[\|\mathbf{e}\|^2] \\ &= \arg \min_{\beta, \mathbf{P}} E[\|\beta^{-1} \text{Re}\{\mathbf{H}\mathbf{P}\mathbf{s} + \mathbf{n}\} - \mathbf{s}\|^2], \end{aligned} \quad (5)$$

with the transmit power constraint

$$E[\|\mathbf{P}\mathbf{s}\|^2] = N_t \sigma_s^2. \quad (6)$$

The reason for the above modification is that the conventional optimization approach expressed by (3) yields a complex valued filter output. However, only the real part of this output is relevant for the decision in a system with an improper constellation. Thus, minimization of the modified cost function in (4) will result in a better estimator. This is similar to the derivation of widely linear receivers<sup>1</sup>. The modified MSE function can be written as follows

$$\begin{aligned} E[\|\mathbf{e}\|^2] &= \text{Tr}\{0.25\beta^{-2}\sigma_s^2(\mathbf{H}\mathbf{P}\mathbf{P}^H\mathbf{H}^H + \mathbf{H}\mathbf{P}\mathbf{P}^T\mathbf{H}^T \\ &+ \mathbf{H}^*\mathbf{P}^*\mathbf{P}^H\mathbf{H}^H + \mathbf{H}^*\mathbf{P}^*\mathbf{P}^T\mathbf{H}^T) - 0.5\beta^{-1}\sigma_s^2(\mathbf{H}\mathbf{P} \\ &+ \mathbf{H}^*\mathbf{P}^* + \mathbf{P}^H\mathbf{H}^H + \mathbf{P}^T\mathbf{H}^T) + 0.5(\beta^{-2}\sigma_n^2 + \sigma_s^2)\mathbf{I}\}. \end{aligned} \quad (7)$$

For the solution to (5), a Lagrange multiplier  $\lambda$  is introduced in order to take into account the power constraint expressed by (6). The improved MMSE solution is derived by solving the minimum of the following function with respect to  $\mathbf{P}$  and  $\beta$

$$\begin{aligned} \eta &= \text{Tr}\{0.25\beta^{-2}\sigma_s^2(\mathbf{H}\mathbf{P}\mathbf{P}^H\mathbf{H}^H + \mathbf{H}\mathbf{P}\mathbf{P}^T\mathbf{H}^T \\ &+ \mathbf{H}^*\mathbf{P}^*\mathbf{P}^H\mathbf{H}^H + \mathbf{H}^*\mathbf{P}^*\mathbf{P}^T\mathbf{H}^T) \\ &- 0.5\beta^{-1}\sigma_s^2(\mathbf{H}\mathbf{P} + \mathbf{H}^*\mathbf{P}^* + \mathbf{P}^H\mathbf{H}^H + \mathbf{P}^T\mathbf{H}^T) \\ &+ (0.5\beta^{-2}\sigma_n^2 + \sigma_s^2)\mathbf{I}\} + \lambda[\sigma_s^2 \text{Tr}(\mathbf{P}\mathbf{P}^H) - N_t \sigma_s^2]. \end{aligned} \quad (8)$$

Using the cyclic property of the trace, we obtain the

<sup>1</sup>It was shown in [25] that widely linear receivers achieve better error performance, less sensitivity to channel estimation errors and more robust to the fading unbalance problem than the conventional receivers. Moreover, they enable MIMO systems to operate with a number of transmit antennas larger (up to a factor of two) than the number of receive antennas. It was reported in [26] that for heavily loaded DS-CDMA systems, widely linear reception of real constellations achieves a higher spectral efficiency than linear reception of complex constellations. It was also shown in [27] that joint transceiver design for real-valued sequences over MIMO channels using WL filters yields significant performance advantages compared to the conventional linear transceiver.

following:

$$\begin{aligned}
\text{Tr}(\mathbf{HPP}^H \mathbf{H}^H) &= \text{Tr}(\mathbf{P}^H \mathbf{H}^H \mathbf{HP}); \\
\frac{\partial \text{Tr}(\mathbf{P}^H \mathbf{H}^H \mathbf{HP})}{\partial \mathbf{P}} &= (\mathbf{H}^H \mathbf{HP})^*; \\
\text{Tr}(\mathbf{HPP}^T \mathbf{H}^T) &= \text{Tr}(\mathbf{P}^T \mathbf{H}^T \mathbf{HP}); \\
\frac{\partial \text{Tr}(\mathbf{P}^T \mathbf{H}^T \mathbf{HP})}{\partial \mathbf{P}} &= 2\mathbf{H}^T \mathbf{HP}; \\
\text{Tr}(\mathbf{H}^* \mathbf{P}^* \mathbf{P}^H \mathbf{H}^H) &= \text{Tr}(\mathbf{P}^H \mathbf{H}^H \mathbf{H}^* \mathbf{P}^*); \\
\frac{\partial \text{Tr}(\mathbf{P}^H \mathbf{H}^H \mathbf{H}^* \mathbf{P}^*)}{\partial \mathbf{P}} &= \mathbf{0}; \\
\text{Tr}(\mathbf{H}^* \mathbf{P}^* \mathbf{P}^T \mathbf{H}^T) &= \text{Tr}(\mathbf{P}^T \mathbf{H}^T \mathbf{H}^* \mathbf{P}^*); \\
\frac{\partial \text{Tr}(\mathbf{P}^T \mathbf{H}^T \mathbf{H}^* \mathbf{P}^*)}{\partial \mathbf{P}} &= \mathbf{H}^T \mathbf{H}^* \mathbf{P}^*. \tag{9}
\end{aligned}$$

Furthermore, we also obtain the following partial derivatives

$$\begin{aligned}
\frac{\partial \text{Tr}(\mathbf{PP}^H)}{\partial \mathbf{P}} &= \mathbf{P}^*; & \frac{\partial \text{Tr}(\mathbf{HP})}{\partial \mathbf{P}} &= \mathbf{H}^T; \\
\frac{\partial \text{Tr}(\mathbf{P}^T \mathbf{H}^T)}{\partial \mathbf{P}} &= \mathbf{H}^T; \\
\frac{\partial \text{Tr}(\mathbf{H}^* \mathbf{P}^*)}{\partial \mathbf{P}} &= \mathbf{0}; & \frac{\partial \text{Tr}(\mathbf{P}^H \mathbf{H}^H)}{\partial \mathbf{P}} &= \mathbf{0}. \tag{10}
\end{aligned}$$

1. Setting  $\frac{\partial \eta}{\partial \mathbf{P}} = \mathbf{0}$ , and using (9), (10) yields

$$\begin{aligned}
0.25\beta^{-2}\sigma_s^2[(\mathbf{H}^H \mathbf{HP})^* + 2\mathbf{H}^T \mathbf{HP} + \mathbf{H}^T \mathbf{H}^* \mathbf{P}^*] \\
- 0.5\beta^{-1}\sigma_s^2(2\mathbf{H}^T) + \lambda\sigma_s^2 \mathbf{P}^* = \mathbf{0}. \tag{11}
\end{aligned}$$

Subsequently, we obtain

$$\mathbf{H}^H \mathbf{HP} + \mathbf{H}^H \mathbf{H}^* \mathbf{P}^* + 2\lambda\beta^2 \mathbf{P} = 2\beta \mathbf{H}^H. \tag{12}$$

2. Setting  $\frac{\partial \eta}{\lambda} = 0$  yields the power constraint

$$\text{Tr}(\mathbf{PP}^H) = N_t. \tag{13}$$

3. Minimizing (8) with respect to  $\beta$ , i.e., setting  $\frac{\partial \eta}{\partial \beta} = 0$  yields the solution for the optimum value of  $\beta$  as follows

$$\begin{aligned}
\text{Tr}\{-0.5\beta^{-3}\sigma_s^2(\mathbf{HPP}^H \mathbf{H}^H + \mathbf{HPP}^T \mathbf{H}^T \\
+ \mathbf{H}^* \mathbf{P}^* \mathbf{P}^H \mathbf{H}^H + \mathbf{H}^* \mathbf{P}^* \mathbf{P}^T \mathbf{H}^T) + 0.5\beta^{-2}\sigma_s^2(\mathbf{HP} \\
+ \mathbf{H}^* \mathbf{P}^* + \mathbf{P}^H \mathbf{H}^H + \mathbf{P}^T \mathbf{H}^T) - \beta^{-3}\sigma_n^2 \mathbf{I}\} = 0, \tag{14}
\end{aligned}$$

which can be simplified as

$$\begin{aligned}
\text{Tr}\{\mathbf{H}^H \mathbf{HPP}^H + \mathbf{H}^T \mathbf{HPP}^T + \mathbf{H}^H \mathbf{H}^* \mathbf{P}^* \mathbf{P}^H \\
+ \mathbf{H}^T \mathbf{H}^* \mathbf{P}^* \mathbf{P}^T - \beta(\mathbf{HP} + \mathbf{H}^* \mathbf{P}^* + \mathbf{P}^H \mathbf{H}^H \\
+ \mathbf{P}^T \mathbf{H}^T) + 2\frac{\sigma_n^2}{\sigma_s^2} \mathbf{I}\} = 0. \tag{15}
\end{aligned}$$

By multiplying each side of equation (12) with  $\mathbf{P}^H$  and  $\mathbf{P}^T$ , then taking conjugate transpose and applying

the cyclic property of the trace, we obtain

$$\begin{aligned}
\beta \text{Tr}(\mathbf{P}^H \mathbf{H}^H) &= \frac{1}{2} \text{Tr}\{\mathbf{H}^H \mathbf{HPP}^H + \mathbf{H}^H \mathbf{H}^* \mathbf{P}^* \mathbf{P}^H \\
&\quad + 2\lambda\beta^2 \mathbf{P}^H \mathbf{P}\}; \\
\beta \text{Tr}(\mathbf{P}^T \mathbf{H}^T) &= \frac{1}{2} \text{Tr}\{\mathbf{H}^T \mathbf{H}^* \mathbf{P}^* \mathbf{P}^T + \mathbf{H}^T \mathbf{HPP}^T \\
&\quad + 2\lambda\beta^2 \mathbf{P}^T \mathbf{P}^*\}; \\
\beta \text{Tr}(\mathbf{HP}) &= \frac{1}{2} \text{Tr}\{\mathbf{H}^H \mathbf{HPP}^H + \mathbf{H}^T \mathbf{HPP}^T \\
&\quad + 2\lambda\beta^2 \mathbf{P}^H \mathbf{P}\}; \\
\beta \text{Tr}(\mathbf{H}^* \mathbf{P}^*) &= \frac{1}{2} \text{Tr}\{\mathbf{H}^T \mathbf{H}^* \mathbf{P}^* \mathbf{P}^T + \mathbf{H}^H \mathbf{H}^* \mathbf{P}^* \mathbf{P}^H \\
&\quad + 2\lambda\beta^2 \mathbf{P}^T \mathbf{P}^*\}. \tag{16}
\end{aligned}$$

Substituting (16) into (15), and using the fact that  $\text{Tr}(\mathbf{PP}^H) = \text{Tr}(\mathbf{P}^* \mathbf{P}^T) = \text{Tr}(\mathbf{I}_{N_t}) = N_t$ , we obtain

$$\text{Tr}\left\{\left(\frac{\sigma_n^2}{\sigma_s^2} - 2\lambda\beta^2\right) \mathbf{PP}^H\right\} = 0. \tag{17}$$

Let us define  $\xi = 2\lambda\beta^2$ . From (17), we can derive

$$\xi = 2\lambda\beta^2 = \frac{\sigma_n^2}{\sigma_s^2}. \tag{18}$$

In order to facilitate the derivation of the proposed precoder, we express Equation (12) as

$$\mathbf{H}^H \mathbf{H}\tilde{\mathbf{P}} + \mathbf{H}^H \mathbf{H}^* \tilde{\mathbf{P}}^* + \xi \tilde{\mathbf{P}} = 2\mathbf{H}^H, \tag{19}$$

where  $\tilde{\mathbf{P}} = \mathbf{P}/\beta$ . Let us denote  $\tilde{\mathbf{P}} = \tilde{\mathbf{P}}_r + j\tilde{\mathbf{P}}_i$ ,  $\mathbf{H}^H \mathbf{H} = \mathbf{X}_r + j\mathbf{X}_i$ ,  $\mathbf{H}^H \mathbf{H}^* = \mathbf{Y}_r + j\mathbf{Y}_i$ ,  $2\mathbf{H}^H = \mathbf{Z}_r + j\mathbf{Z}_i$ . Then  $\mathbf{Z}_r$  and  $\mathbf{Z}_i$  can be expressed in vector form as

$$\begin{bmatrix} \mathbf{Z}_r \\ \mathbf{Z}_i \end{bmatrix} = \begin{bmatrix} \mathbf{X}_r + \mathbf{Y}_r + \xi & \mathbf{Y}_i - \mathbf{X}_i \\ \mathbf{X}_i + \mathbf{Y}_i & \mathbf{X}_r - \mathbf{Y}_r + \xi \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{P}}_r \\ \tilde{\mathbf{P}}_i \end{bmatrix}, \tag{20}$$

which leads to the improved MMSE solution  $\mathbf{P}^m = \beta \tilde{\mathbf{P}}^m = \beta \tilde{\mathbf{P}}_r^m + j\beta \tilde{\mathbf{P}}_i^m$ , where  $\tilde{\mathbf{P}}_r^m$  and  $\tilde{\mathbf{P}}_i^m$  are derived as

$$\begin{bmatrix} \tilde{\mathbf{P}}_r^m \\ \tilde{\mathbf{P}}_i^m \end{bmatrix} = \begin{bmatrix} \mathbf{X}_r + \mathbf{Y}_r + \xi & \mathbf{Y}_i - \mathbf{X}_i \\ \mathbf{X}_i + \mathbf{Y}_i & \mathbf{X}_r - \mathbf{Y}_r + \xi \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{Z}_r \\ \mathbf{Z}_i \end{bmatrix}, \tag{21}$$

where  $\xi = \frac{\sigma_n^2}{\sigma_s^2}$ . From (13), we have  $\text{Tr}(\mathbf{P}^m \mathbf{P}^{mH}) = \text{Tr}(\beta^2 \tilde{\mathbf{P}}^m \tilde{\mathbf{P}}^{mH}) = N_t$ . Therefore,

$$\beta = \sqrt{\frac{N_t}{\text{Tr}(\tilde{\mathbf{P}}^m \tilde{\mathbf{P}}^{mH})}}. \tag{22}$$

The zero-forcing (ZF) solution is obtained by minimizing the MSE for unconstrained transmit power [12, 19] and can be formed as  $\mathbf{P}^z = \beta' \tilde{\mathbf{P}}^z = \beta' \tilde{\mathbf{P}}_r^z + j\beta' \tilde{\mathbf{P}}_i^z$ , where  $\tilde{\mathbf{P}}_r^z$  and  $\tilde{\mathbf{P}}_i^z$  are derived as

$$\begin{bmatrix} \tilde{\mathbf{P}}_r^z \\ \tilde{\mathbf{P}}_i^z \end{bmatrix} = \begin{bmatrix} \mathbf{X}_r + \mathbf{Y}_r & \mathbf{Y}_i - \mathbf{X}_i \\ \mathbf{X}_i + \mathbf{Y}_i & \mathbf{X}_r - \mathbf{Y}_r \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{Z}_r \\ \mathbf{Z}_i \end{bmatrix}, \tag{23}$$

and

$$\beta' = \sqrt{\frac{N_t}{\text{Tr}(\tilde{\mathbf{P}}^z \tilde{\mathbf{P}}^{zH})}}. \tag{24}$$

Unlike the MMSE solution, the ZF precoder does not need any knowledge of the noise as shown by (23). The conventional linear MMSE and ZF precoders [12] can be derived by optimizing the criterion expressed by (3) with and without transmit power constraint, respectively. The solutions are given below for comparison purpose

$$\begin{aligned}\mathbf{P}_c^m &= \beta_c \tilde{\mathbf{P}}_c^m = \beta_c \mathbf{H}^H \left( \mathbf{H}\mathbf{H}^H + \frac{\sigma_n^2}{\sigma_s^2} \mathbf{I} \right)^{-1}; \\ \mathbf{P}_c^z &= \beta'_c \tilde{\mathbf{P}}_c^{zf} = \beta'_c (\mathbf{H}^H \mathbf{H})^{-1} \mathbf{H}^H,\end{aligned}\quad (25)$$

where  $\beta_c$  and  $\beta'_c$  are derived using transmit power constraint as shown in (22) and (24). The computation of the improved linear precoding matrices  $\mathbf{P}^m$  and  $\mathbf{P}^z$  is slightly more complex than the computation of the conventional linear precoding matrices  $\mathbf{P}_c^m$  and  $\mathbf{P}_c^z$  due to inversion of a dimension doubled matrix (comparing  $\begin{bmatrix} \mathbf{X}_r + \mathbf{Y}_r & \mathbf{Y}_i - \mathbf{X}_i \\ \mathbf{X}_i + \mathbf{Y}_i & \mathbf{X}_r - \mathbf{Y}_r \end{bmatrix}^{-1}$  with  $(\mathbf{H}^H \mathbf{H})^{-1}$  in the ZF case). However, once the precoding matrices are derived, the improved and the conventional systems have exactly the same pre-processing complexity in the transmitter. Therefore, the complexity increase by the improved schemes is not significant compared to the conventional ones, especially in slow-fading channels for which the precoding matrices do not need to be updated frequently.

For the performance comparison, we have the following result.

*Proposition 1:* Let us denote  $P_t^c$  as the symbol error probability of the system employing the conventional ZF precoder,  $P_t$  as the symbol error probability of the system employing the improved ZF precoder, then  $P_t < P_t^c$  always holds.

*Proof:* See Appendix.

*Proposition 2:* Let us denote  $C(\mathbf{P}_c)$  as the capacity of the MIMO channel for a fixed realization of  $\mathbf{H}$  and the conventional ZF precoder  $\mathbf{P}_c$ ; denote  $C(\mathbf{P})$  as the capacity of the MIMO channel for a fixed realization of  $\mathbf{H}$  and the improved ZF precoder  $\mathbf{P}$ . Then  $C(\mathbf{P}_c) < C(\mathbf{P})$  always holds.

*Proof:* See Appendix.

### III. ROBUST PRECODING SCHEMES WITH IMPERFECT CSI

Since obtaining channel information at the transmitter can be difficult due to channel dynamics, CSI is usually not an accurate instantaneous channel information. In this section, channel is described statistically, the imperfect CSI consists of the first and second order statistics of the actual channel, i.e., the channel is modeled as a channel estimate and its estimation error covariance in the form of a non-zero channel mean  $\bar{\mathbf{H}}$  and a covariance matrix  $\mathbf{R}$ .

For the downlink MIMO system considered in this work, we assume that the channel experiences transmit correlation which is known to the transmitter and no receive correlation. This kind of model occurs when the BS antennas are separated by less than the coherence

distance, while the receive antennas are fully decorrelated since MSs are sufficiently apart from each other, and they are surrounded by a rich scattering environment. These propagation conditions correspond to a cellular communication systems typically characterized by a low angular spread at the BS. This kind of channel can be modeled as [3, 28–30]

$$\mathbf{H} = \bar{\mathbf{H}} + \mathbf{H}_w \mathbf{R}^{1/2}, \quad (26)$$

where  $\mathbf{H}_w$  is the unknown part of the fading coefficient matrix. It is assumed to be uncorrelated Rayleigh fading and  $\mathbb{E}[\mathbf{H}_w^H \mathbf{H}_w] = \alpha \mathbf{I}$ .  $\mathbf{R}^{1/2}$  is defined such that  $\mathbf{R}^{1/2} (\mathbf{R}^{1/2})^H = \mathbf{R}$ , which is called the long-term transmit correlation matrix [28]. The mean and correlation values correspond to a channel estimate and its error covariance. These statistics can be obtained by time-averaging operations on channel measurement [30]. Our goal is to use the channel model given in (26) and design precoders which are robust to imperfect CSI conditions.

Recall that the improved precoder for improper signals with perfect CSI can be derived from (19), i.e.,

$$\mathbf{H}^H \mathbf{H} \tilde{\mathbf{F}} + \mathbf{H}^H \mathbf{H}^* \tilde{\mathbf{F}}^* + \xi \tilde{\mathbf{F}} = 2\mathbf{H}^H, \quad (27)$$

where  $\xi = \frac{\sigma_n^2}{\sigma_s^2}$ , and  $\mathbf{F} = \beta \tilde{\mathbf{F}}$  is the precoding matrix to be derived. Now, we consider the case when the transmitter only knows the statistics of the channel ( $\bar{\mathbf{H}}$  and  $\mathbf{R}$ ). In contrast to the perfect CSI case where the instantaneous MSE is considered, we need to formulate the problem with the average MSE, (i.e., the expected value of MSE where the expectation is taken with respect to  $\mathbf{H}$ ), leading to the the problem formulation

$$\mathbb{E}[\mathbf{H}^H \mathbf{H}] \tilde{\mathbf{F}} + \mathbb{E}[\mathbf{H}^H \mathbf{H}^*] \tilde{\mathbf{F}}^* + \xi \tilde{\mathbf{F}} = 2\mathbb{E}[\mathbf{H}^H] = 2\bar{\mathbf{H}}^H. \quad (28)$$

Since

$$\begin{aligned}\mathbb{E}[\mathbf{H}^H \mathbf{H}] &= \mathbb{E}[(\bar{\mathbf{H}}^H + \mathbf{R}^{H/2} \mathbf{H}_w^H)(\bar{\mathbf{H}} + \mathbf{H}_w \mathbf{R}^{1/2})] \\ &= \bar{\mathbf{H}}^H \bar{\mathbf{H}} + \mathbb{E}[\mathbf{R}^{H/2} \mathbf{H}_w^H \mathbf{H}_w \mathbf{R}^{1/2}] \\ &= \bar{\mathbf{H}}^H \bar{\mathbf{H}} + \alpha \mathbf{R}^{H/2} \mathbf{R}^{1/2}; \\ \mathbb{E}[\mathbf{H}^H \mathbf{H}^*] &= \mathbb{E}[(\bar{\mathbf{H}}^H + \mathbf{R}^{H/2} \mathbf{H}_w^H)(\bar{\mathbf{H}}^* + \mathbf{H}_w^* \mathbf{R}^{*/2})] \\ &= \bar{\mathbf{H}}^H \bar{\mathbf{H}}^*,\end{aligned}\quad (29)$$

Equ. (28) can be reformed as

$$(\bar{\mathbf{H}}^H \bar{\mathbf{H}} + \alpha \mathbf{R}^{H/2} \mathbf{R}^{1/2}) \tilde{\mathbf{F}} + \bar{\mathbf{H}}^H \bar{\mathbf{H}}^* \tilde{\mathbf{F}}^* + \xi \tilde{\mathbf{F}} = 2\bar{\mathbf{H}}^H. \quad (30)$$

Denoting  $\tilde{\mathbf{F}} = \tilde{\mathbf{F}}_r + j\tilde{\mathbf{F}}_i$  and

$$\begin{aligned}(\bar{\mathbf{H}}^H \bar{\mathbf{H}} + \alpha \mathbf{R}^{H/2} \mathbf{R}^{1/2}) &= \mathbf{A}_r + j\mathbf{A}_i; \\ \bar{\mathbf{H}}^H \bar{\mathbf{H}}^* &= \mathbf{B}_r + j\mathbf{B}_i; \quad 2\bar{\mathbf{H}}^H = \mathbf{C}_r + j\mathbf{C}_i,\end{aligned}\quad (31)$$

Equ. (30) can be written in vector form as

$$\begin{bmatrix} \mathbf{C}_r \\ \mathbf{C}_i \end{bmatrix} = \begin{bmatrix} \mathbf{A}_r + \mathbf{B}_r + \xi & \mathbf{B}_i - \mathbf{A}_i \\ \mathbf{A}_i + \mathbf{B}_i & \mathbf{A}_r - \mathbf{B}_r + \xi \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{F}}_r \\ \tilde{\mathbf{F}}_i \end{bmatrix}, \quad (32)$$

which leads to the improved MMSE solution  $\mathbf{F}^m = \beta \tilde{\mathbf{F}}^m = \beta \tilde{\mathbf{F}}_r^m + j\beta \tilde{\mathbf{F}}_i^m$ , where  $\tilde{\mathbf{F}}_r^m$  and  $\tilde{\mathbf{F}}_i^m$  are derived as

$$\begin{bmatrix} \tilde{\mathbf{F}}_r^m \\ \tilde{\mathbf{F}}_i^m \end{bmatrix} = \begin{bmatrix} \mathbf{A}_r + \mathbf{B}_r + \xi & \mathbf{B}_i - \mathbf{A}_i \\ \mathbf{A}_i + \mathbf{B}_i & \mathbf{A}_r - \mathbf{B}_r + \xi \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{C}_r \\ \mathbf{C}_i \end{bmatrix}. \quad (33)$$

From the power constraint (1), we have  $\text{Tr}(\mathbf{F}^m \mathbf{F}^{mH}) = \text{Tr}(\beta^2 \tilde{\mathbf{F}}^m \tilde{\mathbf{F}}^{mH}) = N_t \sigma_s^2$ . Therefore,

$$\beta = \sqrt{\frac{N_t \sigma_s^2}{\text{Tr}(\tilde{\mathbf{F}}^m \tilde{\mathbf{F}}^{mH})}}. \quad (34)$$

The conventional precoder  $\mathbf{F}_c$  is derived by minimizing the expected value of MSE expressed by (3), the conventional precoder  $\tilde{\mathbf{F}}_c$  with imperfect CSI can be derived as

$$\mathbb{E}[\mathbf{H}^H \mathbf{H}] \tilde{\mathbf{F}}_c + \xi \tilde{\mathbf{F}}_c = \mathbb{E}[\mathbf{H}^H] = \tilde{\mathbf{H}}^H, \quad (35)$$

which can be reformulated as

$$\begin{aligned} \tilde{\mathbf{H}}^H &= (\tilde{\mathbf{H}}^H \tilde{\mathbf{H}} + \alpha \mathbf{R}^{H/2} \mathbf{R}^{1/2}) \tilde{\mathbf{F}}_c + \xi \tilde{\mathbf{F}}_c \\ &= (\tilde{\mathbf{H}}^H \tilde{\mathbf{H}} + \alpha \mathbf{R}^{H/2} \mathbf{R}^{1/2} + \xi \mathbf{I}) \tilde{\mathbf{F}}_c, \end{aligned} \quad (36)$$

leading to the conventional MMSE solution  $\mathbf{F}_c = \beta_c \tilde{\mathbf{F}}_c$ , where  $\tilde{\mathbf{F}}_c$  is derived as

$$\tilde{\mathbf{F}}_c = (\tilde{\mathbf{H}}^H \tilde{\mathbf{H}} + \alpha \mathbf{R}^{H/2} \mathbf{R}^{1/2} + \xi \mathbf{I})^{-1} \tilde{\mathbf{H}}^H, \quad (37)$$

where  $\beta_c = \sqrt{\frac{N_t \sigma_s^2}{\text{Tr}(\tilde{\mathbf{F}}_c \tilde{\mathbf{F}}_c^H)}}$ .

#### IV. PRECODING SCHEMES FOR MU-MIMO SYSTEMS

The MU-MIMO wireless communication systems have attracted considerable attention recently due to their potential to improve reliability and capacity at system level. Their major advantages over SU-MIMO systems include significant gain in multiple access capacity, better immunity to most of propagation limitations plaguing single-user MIMO communications, etc. [31]. Precoding plays a pivotal role in MU-MIMO systems in order to reap those performance benefits. In this section, we investigate the application of the proposed precoding schemes to MU-MIMO systems. Here, we consider the downlink multi-user scenario where the BS communicates with several MSs, each equipped with multiple antennas. The configuration of the MU-MIMO system is shown in Fig. 2, where  $M$  antennas are located at the BS and  $N$  antennas are located at each of the  $K$  MSs. For simplicity, we assume equal number of receive antennas at each mobile station. However, the discussed algorithms can be readily extended to the systems where different users have different number of receive antennas. At the BS, the data symbol vector for user  $k$  (denoted as  $\mathbf{s}_k \in \mathbb{C}^{L \times 1}$  where  $L$  is the number of parallel data symbols transmitted from each user) is precoded using the precoding matrix  $\mathbf{P}_k \in \mathbb{C}^{M \times L}$ , and transmitted via  $M$  transmit antennas.

The  $k$ th user's received signal vector can be written as

$$\mathbf{r}_k = \mathbf{H}_k \sum_{i=1}^K \mathbf{P}_i \mathbf{s}_i + \mathbf{n}_k \in \mathbb{C}^{N \times 1}, \quad (38)$$

where  $\mathbf{H}_k \in \mathbb{C}^{N \times M}$  is the channel matrix corresponding to the  $k$ th user, its  $(i, j)$ th entry represents the channel gain from the  $j$ th transmit antenna at BS to the  $i$ th receive antenna at the  $k$ th MS. We also assume a rich scattering environment so that entries are independently identically distributed (i.i.d) complex Gaussian random variable with zero mean and unit variance. The noise term  $\mathbf{n}_k$  is assumed to be zero mean complex Gaussian random vector with covariance matrix  $\sigma_n^2 \mathbf{I}_N$ .

MU-MIMO schemes can be categorized into linear and nonlinear approaches. In the former case, users are assigned different precoding matrices at the transmitter. The precoders are designed jointly based on CSI and all the users. The latter involves additional transmit signal processing for performance improvement. There are two representative nonlinear methods available in the literature, one is based on the vector perturbation [32], and the other is based on the spatial extension of THP [33]. For simplicity, we focus on the linear approach, and investigate multi-user transmitter pre-processing schemes to suppress multi-user interference with linear signal processing at the transmitter. In particular, we use the decomposition technique introduced in [34].

Under the condition that the number of transmit antennas is larger than the sum of the number of receive antennas of any  $K - 1$  users (which ensures the existence of a nullspace), interference from other users can be completely removed using a decomposition approach proposed in [34]. Let us denote  $\tilde{\mathbf{H}}_k = [\mathbf{H}_1^T \cdots \mathbf{H}_{k-1}^T \mathbf{H}_{k+1}^T \cdots \mathbf{H}_K^T]^T$ , the  $k$ th user is free from multi-user interference if  $\mathbf{P}_k$  lies in the nullspace of  $\tilde{\mathbf{H}}_k$ , i.e.,  $\tilde{\mathbf{H}}_k \mathbf{P}_k = \mathbf{0}$ . In this case, the  $k$ th user received signal become

$$\mathbf{r}_k = \mathbf{H}_k \sum_{i=1}^K \mathbf{P}_i \mathbf{s}_i + \mathbf{n}_k = \mathbf{H}_k \mathbf{P}_k \mathbf{s}_k + \mathbf{n}_k, \quad (39)$$

which is equivalent to a SU-MIMO system free from multi-user interference.

Let us denote  $\mathbf{V}_k \in \mathbb{C}^{M \times n_k}$  (where  $n_k = M - (K - 1)N$ ) as the nullspace of  $\tilde{\mathbf{H}}_k$ , which can be derived by singular value decomposition (SVD)

$$\tilde{\mathbf{H}}_k = [\tilde{\mathbf{U}}_k \quad \mathbf{U}_k] \begin{bmatrix} \Sigma & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{V}}_k^H \\ \mathbf{V}_k^H \end{bmatrix}, \quad (40)$$

where columns of  $\mathbf{V}_k$  form an orthonormal basis whose dimension is  $n_k$  with probability one under rich-scattering assumptions. The precoding matrix can be chosen as

$$\mathbf{P}_k = \mathbf{V}_k \mathbf{A}_k, \quad (41)$$

where  $\mathbf{A}_k$  is the precoding matrix for the equivalent single-user MIMO system (39).

Fig. 3 shows the equivalent system model for the MU-MIMO channel after being decomposed into  $K$  parallel single-user channels. The design of  $\mathbf{A}_k$  is the same as designing the transmit processing for a SU-MIMO channel for user  $k$ , where the equivalent channel matrix is  $\mathbf{H}_k \mathbf{V}_k$ . The conventional and the improved precoding

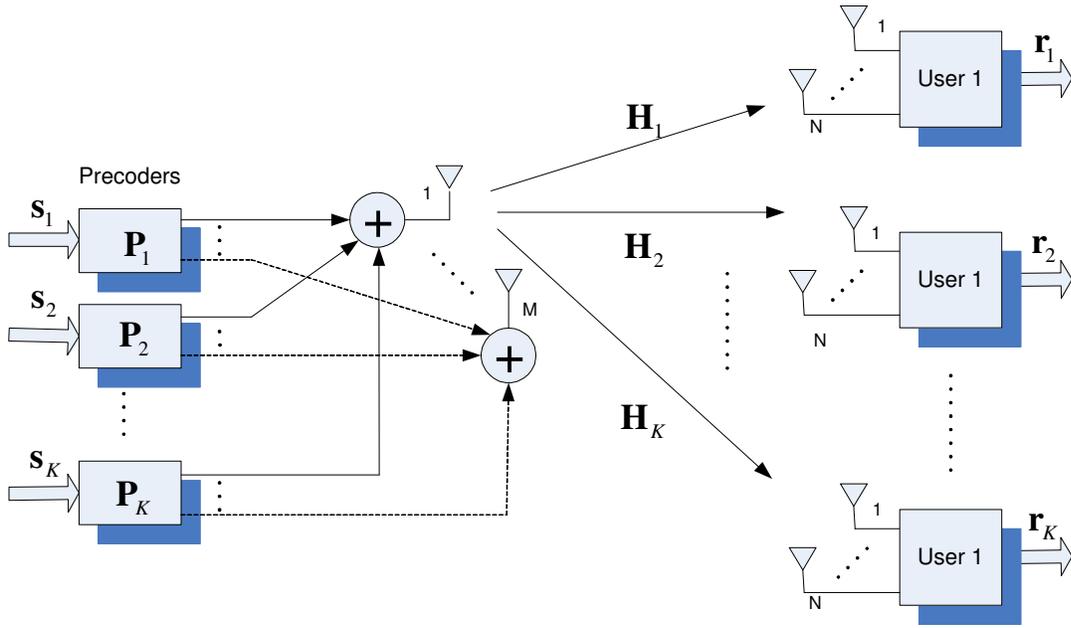


Fig. 2. Block diagram of a precoded downlink MU-MIMO system with  $M$  transmit antennas at the BS and  $K$  MSs, each has  $N$  receive antennas.

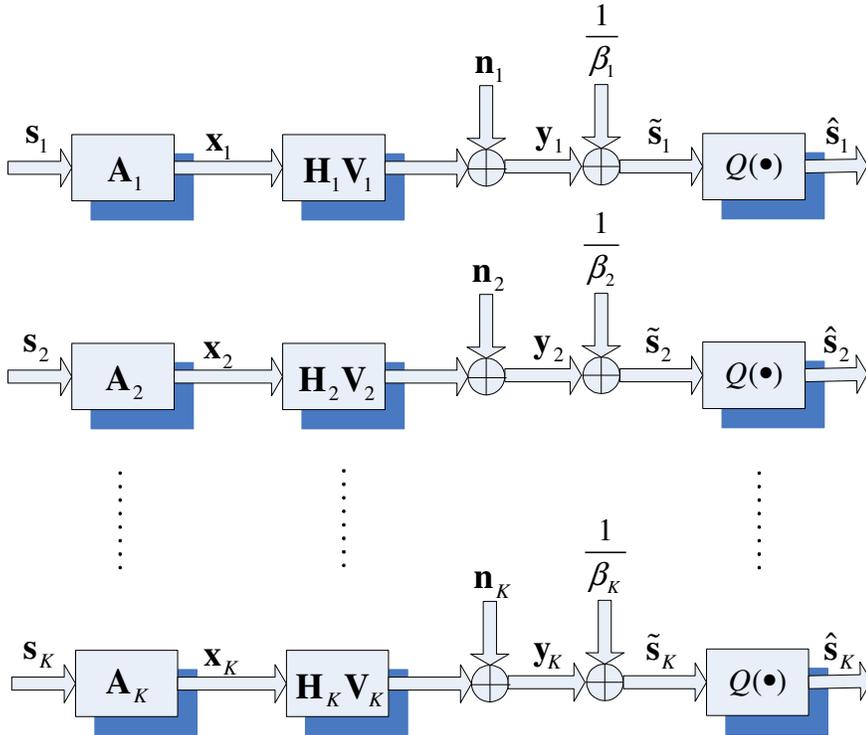


Fig. 3. Equivalent system model in which a MU-MIMO channel is decomposed into  $K$  parallel SU-MIMO channels.

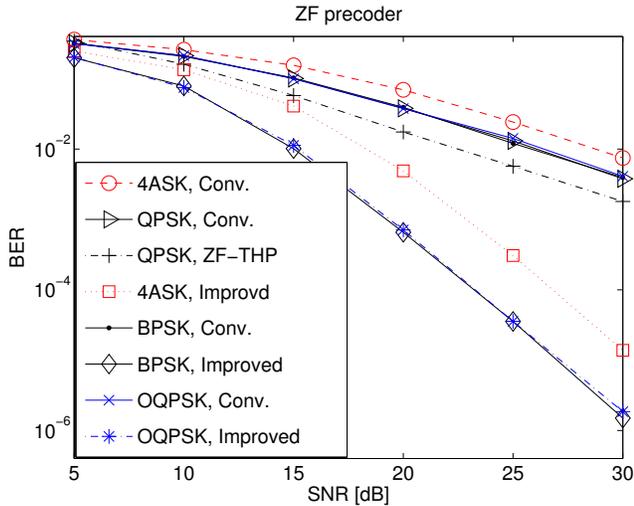


Fig. 4. Performance comparison of the ZF precoding schemes for 4ASK, QPSK, OQPSK.

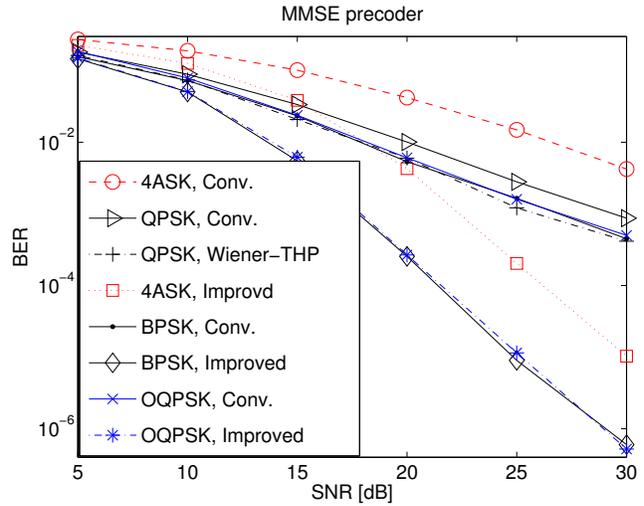


Fig. 5. Performance comparison of the MMSE precoding schemes for 4ASK, QPSK, OQPSK.

schemes introduced in Section II can be applied to derive  $\mathbf{A}_k$  for each user.

Note that this decomposition approach imposes a restriction on the system configuration in terms of the number of antennas. More specifically, the number of transmit antennas has to be larger than the sum of the number of receive antennas of any  $K - 1$  users in order to guarantee the existence of a nonzero precoding matrix. However, this problem can be easily tackled by a dynamic antenna scheduling strategy [35] to activate a subset of the receive antennas for every user that meets the signal to noise plus interference-ratio (SINR) requirement. Alternatively, we can use user scheduling [36] techniques such that multi-user diversity is exploited and the system throughput performance is optimized by scheduling transmissions simultaneously to a group of users who have the best instantaneous channel quality.

## V. NUMERICAL RESULTS

We first compare different algorithms by applying them to a SU-MIMO system with  $4 \times 4$  antennas ( $N_t = N_r = 4$ ). The transmit power is set to  $N_t \sigma_s^2 = 4$ , i.e., unit average transmit power is used for each transmitted symbol. We assume uncorrelated Rayleigh fading channel and the channel matrix is normalized such that  $E[\|\mathbf{H}\|_F^2] = 1$ . The simulation results are averaged over at least 50,000 channel realizations.

Figs. 4 and 5 show the performance comparison of the conventional and improved ZF and MMSE algorithms, respectively, for the downlink transmit processing. The employed modulation schemes are 4ASK, QPSK, OQPSK, which are chosen such that all the systems have the same data transmission rate or spectrum efficiency. One can see from the figures that with the conventional linear ZF and MMSE precoding schemes, the QPSK and OQPSK modulated systems outperform the 4ASK system. Direct implementation of the ZF scheme expressed by (23)

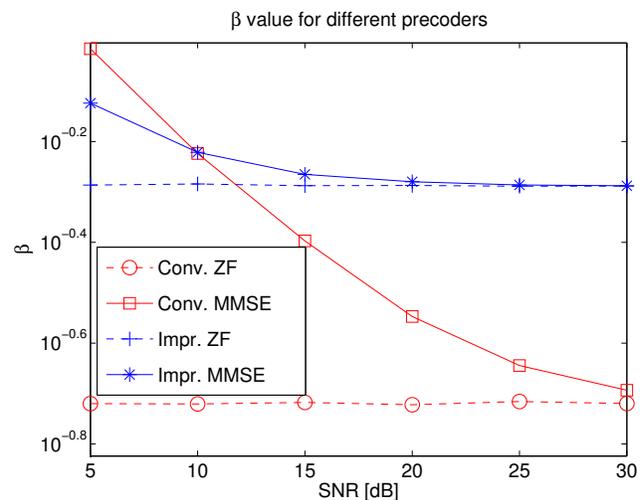


Fig. 6. Comparison of the power control value  $\beta$  between the conventional and the improved precoding schemes.

leads to the numerical problem caused by the inversion of an ill-conditioned matrix. However, the problem can be easily resolved by Tikhonov regularization, and the regularization parameter is chosen to be a very small number. We also compare the linear precoding schemes to the spatio-temporal THP, which is implemented according to [6]. Apparently, the linear precoders are outperformed by their nonlinear counterparts, such as the ZF-THP [5] shown in Fig. 4 and Wiener-THP [5] shown in Fig. 5. By comparison, the improved ZF and MMSE precoding schemes for 4ASK and OQPSK systems achieve significant performance gains compared to the conventional linear and even nonlinear THP schemes. The results for the OQPSK system are obtained by applying the improved transmit filtering to the I and Q components separately.

Also shown in Figs. 4 and 5 are the curves corre-

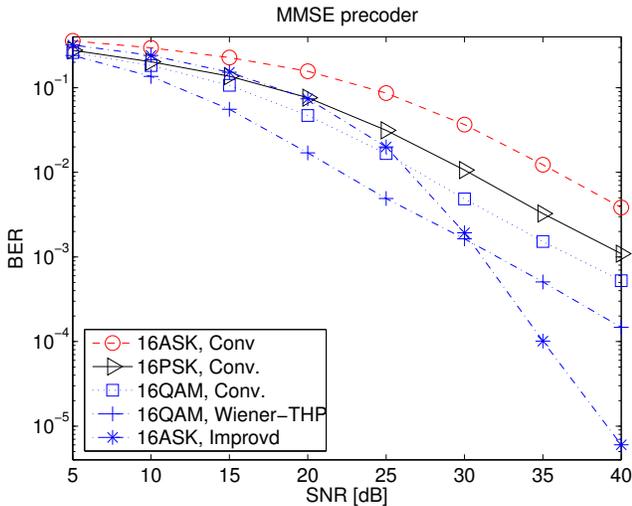


Fig. 7. Performance comparison of the MMSE precoding schemes for 16ASK, 16QAM, 16PSK.

sponding to the BPSK system. BPSK is an improper modulation, therefore, applying the proposed scheme to the BPSK system will lead to improved performance. One can see from the figures that the BPSK system with the conventional (improved) precoder yields almost the same performance as the OQPSK system with the conventional (improved) precoder. However, the use of OQPSK is preferred because of its higher spectral efficiency.

In Fig. 6, we examine the value of the automatic gain control factor  $\beta$  which plays an important role in the precoding schemes. For the MMSE precoders, the  $\beta$  values of the improved scheme are higher than that of the conventional scheme at medium to high SNRs. The ZF precoders do not take the noise into account, therefore,  $\beta$  remains constant despite the changes in the SNR value. The  $\beta$  value of the improved ZF scheme is always higher than that of the conventional ZF scheme. See Appendix for a theoretical proof of it. The  $\beta$  value of the MMSE precoder converges to that of the ZF precoder since two precoders become close to each other at sufficiently high SNRs.

In Fig. 7, different precoding schemes are compared for high order constellations with the same spectral efficiency, i.e., 16ASK, 16QAM, 16PSK. It can be observed that with conventional linear precoding, the 16ASK system has the worst performance; and the 16QAM system is more power efficient than 16PSK. The 16ASK system with improved precoder has worse (better) performance than the 16QAM system at low (high) SNRs. The crossover point is around 25 dB. The nonlinear THP performs better than that of all the linear schemes for SNRs less than 30 dB. It was shown in [5] that THP is particularly advantageous with higher-order modulation schemes. However, it is outperformed by the 16ASK system with improved linear MMSE precoder when SNR increases further, which shows advantage of applying the improved schemes at high SNRs for higher order constellations.

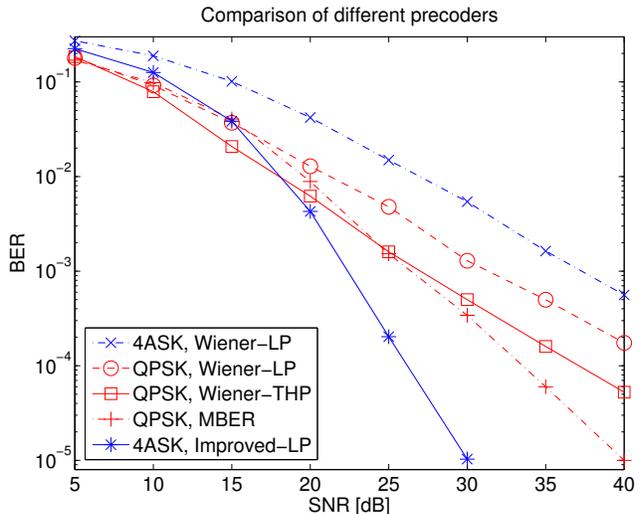


Fig. 8. Performance comparison of different precoding schemes for 4ASK and QPSK.

The conventional and the improved linear precoders are compared with the asymptotically minimum BER (MBER) precoder for QPSK and 4ASK systems in Fig. 8. The latter is implemented according to Equations (2) and (18) in [22]. The unitary matrix  $\mathbf{D}_m$  is chosen to be the normalized Hadamard matrix. The automatic gain control factor  $\beta$  is applied to the MBER precoder to fulfill the transmit power constraint expressed by (6) or (13). It can be seen that the MBER precoder outperforms the Wiener (MMSE) linear precoder and even the nonlinear Wiener-THP at medium to high SNRs. One can also see from the figure that the 4ASK system with improved precoding has the best performance when SNR is sufficiently high, e.g., greater than 20 dB.

In Figs. 9 and 10, we examine our robust precoding designs with imperfect CSI which is in the form of a mean and a covariance as shown in (26). The channel mean  $\bar{\mathbf{H}}$  is drawn from a complex Gaussian distribution and kept fixed for all the simulations. The specific value of  $\bar{\mathbf{H}}$  used in the simulations is taken from [29] and given as

$$\bar{\mathbf{H}} = \begin{bmatrix} .33 + .47i & 1.03 - 0.96i & .88 - .17i & -.94 + .82i \\ .58 + .01i & .93 - .08i & -.56 - .12i & 1.02 - .32i \\ .73 - .05i & .49 - .56i & -.36 - .67i & -.39 + .72i \\ -.62 - 1.72i & .51 + .95i & 1.00 - .88i & -.09 - .05i \end{bmatrix}. \quad (42)$$

The covariance matrix  $\mathbf{R}$  is Toeplitz, defined by the correlation coefficient  $\rho$  ( $0 < \rho < 1$ ) as  $\mathbf{R}(i, j) = \rho^{|i-j|}$  [29]. The value of  $\rho$  is set to be 0.6 in the simulations. The unknown part of the channel  $\mathbf{H}_w$  in (26) is assumed to be uncorrelated Rayleigh fading and is normalized such that  $\mathbb{E}[\|\mathbf{H}_w\|_F^2] = 1$ . In this case,  $\alpha = 1/N$  in (31) and (37). The simulation results are averaged over at least 50,000 realizations of  $\mathbf{H}_w$ .

The numerical comparisons of different precoders with perfect and imperfect CSI are shown in Fig. 9. With perfect CSI, we assume the transmitter has perfect knowledge of the actual channel realization  $\mathbf{H}$ . The precoders expressed by (21) and (25) are employed for OQPSK

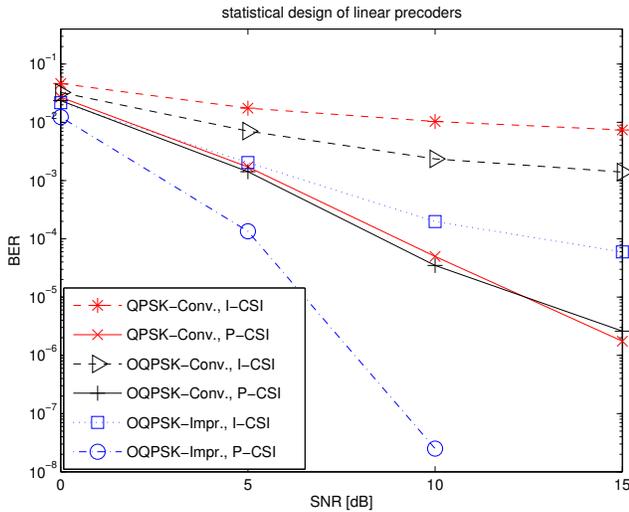


Fig. 9. Comparison of different precoding schemes. I-CSI (imperfect CSI) vs. P-CSI (perfect CSI).

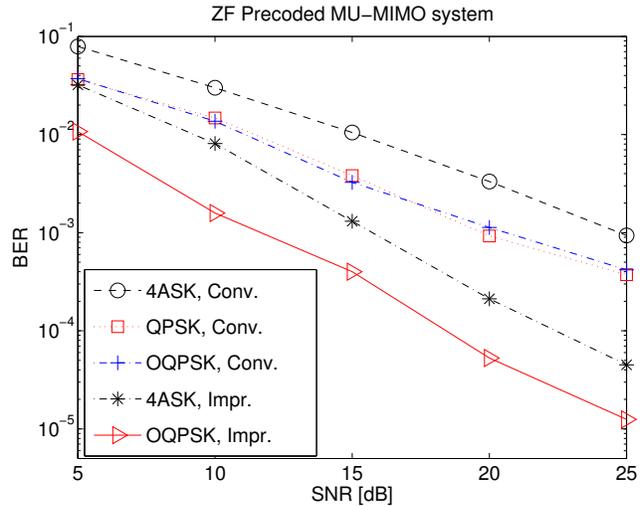


Fig. 11. Performance comparison of the ZF precoding schemes for the MU-MIMO system.

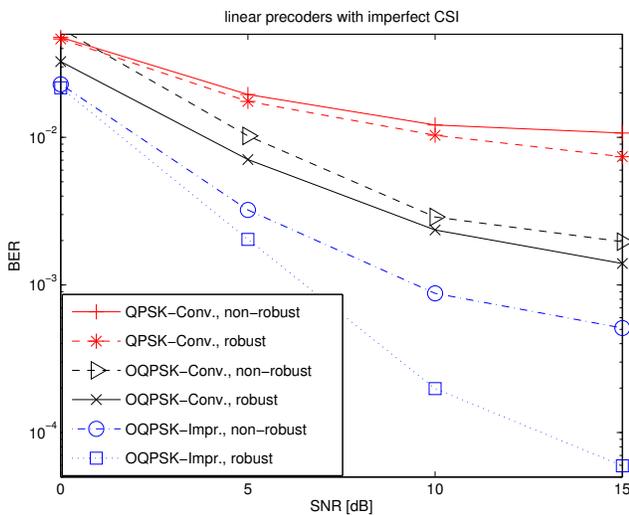


Fig. 10. Comparison of different precoding schemes. Robust vs. non-robust design.

and QPSK systems, respectively. While with imperfect CSI, we assume the transmitter only knows the statistical information about the channel, i.e.,  $\bar{\mathbf{H}}$  and  $\mathbf{R}$ , but not the actual channel realization  $\mathbf{H}$ . In this case, the statistically robust designs expressed by (33) and (37) are employed for the improved and the conventional systems, respectively. Fig. 9 shows that for both systems, imperfect CSI leads to significant performance degradation compared to the case with perfect CSI. This follows from the fact that a statistical robust design does not guarantee any performance target for a particular realization of the random channel. The problem is much more severe with the conventional QPSK and OQPSK systems, for which error floors are clearly visible. On the contrary, no error floor has occurred for the OQPSK system with the improved precoding scheme.

Statistically robust design of linear precoding is com-

pared with non-robust design in Fig. 10. The non-robust design assumes the channel estimate  $\bar{\mathbf{H}}$  to be a perfect channel estimate of the instantaneous realization of  $\mathbf{H}$ ; while the robust design also takes the channel covariance information into consideration. Apparently, the robust precoder significantly improves the performance of the OQPSK system with the improved precoder, especially at high SNR; whereas the difference is relatively small for the QPSK and OQPSK systems with the conventional precoder, robust design only slightly improves the system performance in this case.

Finally, we examine the performance of the conventional and the improved precoding schemes in an MU-MIMO system. The number of transmit antennas at BS and the number of users are set to  $M = 6$  and  $K = 3$ , respectively. Each user employs  $N = 2$  receive antennas. As shown in [34], a 3-user MU-MIMO system with 6 transmit antennas at the BS and 2 receive antennas at each MS is equivalent to 3 parallel  $2 \times 2$  single-user MIMO system, therefore, the MU-MIMO system increases the capacity by 3 folds<sup>2</sup>.

Figs. 11 and 12 show the performance of the MU-MIMO system with the conventional and improved ZF and MMSE precoders, respectively. The results similar to the SU-MIMO case are observed here, i.e., the 4ASK system has the worst (better) performance than the QPSK and OQPSK systems with the conventional (improved) linear precoders. The OQPSK system with the improved precoding schemes achieve the best performance. The results verify the advantage of employing improper signal constellation with improved precoding schemes for MU-MIMO systems.

<sup>2</sup>MU-MIMO provides much increased capacity when the number of receive antennas at each MS is limited compared to SU-MIMO. By allowing multiple users, each still with a small number of receive antennas, the overall capacity increases.

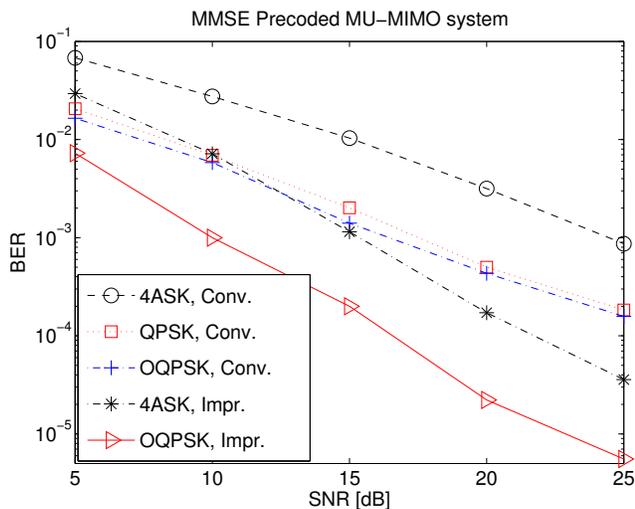


Fig. 12. Performance comparison of the MMSE precoding schemes for the MU-MIMO system.

## VI. CONCLUSIONS

In this paper, we proposed a novel linear precoding scheme for SU- and MU-MIMO systems. The proposed schemes are derived by optimization of modified cost functions for  $M$ -ary improper constellations, and are shown to outperform the conventional linear and nonlinear schemes with both  $M$ -ary improper and proper constellations. Consequently, the proposed schemes can be applied to practical systems to improve power efficiency without loss of spectral efficiency. We also addressed the issue of robust precoding design in the presence of imperfect CSI with only the first and second order statistics of the channel. Under such a condition, optimization is possible only in a statistical sense. Different precoders have been designed based on the expected value of MSEs for both improper and proper signals. The results reveal that by utilizing improperness of the signal constellation, the precoder can be designed to be more robust to imperfect CSI, while the conventional precoder relies more on the perfect knowledge of CSI.

## ACKNOWLEDGEMENT

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## REFERENCES

- [1] G. Foschini. "Layered space-time architecture for wireless communication in a fading environment when using multiple-element antennas", *Bells Labs Tech. J.*, vol. 1, pp. 41–59, 1996.
- [2] G. Foschini, M. Gans. "On limits of wireless communications in a fading environment when using multiple antennas", *Wireless Personal Commun.*, vol. 6, no. 3, pp. 311–335, March 1998.
- [3] A. Paulraj, R. Nabar, D. Gore. *Introduction to Space-Time Wireless Communications*, Cambridge, University Press, 2003.
- [4] C. Windpassinger, R. Fischer, J. Huber. "Precoding in multi-antenna and multiuser communications", *IEEE Trans. Wireless Commun.*, vol. 3, no. 4, pp. 1305–1316, July 2004.

- [5] M. Joham, J. Brehmer, W. Utschick. "MMSE approaches to multiuser spatio-temporal Tomlinson-Harashima precoding". *Proc. ITG Conf. Source and Channel Coding*, pp. 387–394, Jan. 2004.
- [6] R. Fischer, C. Windpassinger, A. Lampe, J. Huber. "Space-time transmission using Tomlinson-Harashima precoding". *Proc. ITG Conf. Source and Channel Coding*, pp. 139–147, Jan. 2002.
- [7] D. Palomar, J. Cioffi, M. Lagunas. "Joint Tx-Rx beamforming design for multicarrier MIMO channels: a unified framework for convex optimization", *IEEE Trans. Signal Proc.*, vol. 51, no. 9, pp. 2381–2401, Sept. 2003.
- [8] H. Sampath, P. Stoica, A. Paulraj. "Generalized linear precoder and decoder design for MIMO channels using the weighted MMSE criterion", *IEEE Trans. Commun.*, vol. 49, no. 12, pp. 2198–2206, Dec. 2001.
- [9] M. Shenouda and T. Davidson. "A Framework for Designing MIMO Systems with Decision Feedback Equalization or Tomlinson-Harashima Precoding," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 2, pp. 401–411, Feb. 2008.
- [10] J.-K. Zhang, A. Kavcic, and K. M. Wong. "Equal-diagonal QR decomposition and its application to precoder design for successive cancellation detection," *IEEE Trans. Inform. Theory*, pp. 154–171, Jan. 2005.
- [11] F. Xu, T. Davidson, J.-K. Zhang, and K. Wong. "Design of block transceivers with decision feedback detection," *IEEE Trans. Sig. Proc.*, vol. 54, pp. 964–978, March 2006.
- [12] M. Joham, W. Utschick, J. Nosssek. "Linear transmit processing in MIMO communications systems". *IEEE Trans. on Sig. Proc.*, vol. 53, no. 8, pp. 2700–2712, August 2005.
- [13] M. Tomlinson. "New automatic equaliser employing modulo arithmetic", *Electronic Letters*, vol. 7, no. 5, pp. 138–139, March 1971.
- [14] H. Harashima. "Matched-transmission technique for channels with intersymbol interference", *IEEE Trans. Commun.*, vol. 20, no. 4, pp. 774–780, August 1972.
- [15] M. Costa. "Writing on dirty paper", *IEEE Trans. Inform. Theory*, vol. 29, no. 3, pp. 439–441, May 1983.
- [16] U. Erez, S. Brink. "A close-to-capacity dirty paper coding scheme", *IEEE Trans. Inform. Theory*, vol. 51, no. 10, pp. 3417–3432, Oct. 2005.
- [17] W. Yu, D. Varodayan, M. Cioffi. "Trellis and convolutional precoding for transmitter-based interference presubtraction", *IEEE Trans. Commun.*, vol. 53, no. 7, pp. 1220–1230, July 2005.
- [18] U. Erez, S. Shamai, R. Zamir. "Capacity and lattice strategies for canceling known interference", *IEEE Trans. Inform. Theory*, vol. 51, no. 11, pp. 3820–3833, Nov. 2005.
- [19] B. Vojcic, W. Jang. "Transmitter precoding in synchronous multiuser communication", *IEEE Trans. Commun.*, vol. 46, no. 10, pp. 1346–1355, Oct. 1998.
- [20] R. Choi, R. Murch. "New transmit schemes and simplified receivers for MIMO wireless communication systems", *IEEE Trans. Wireless Commun.*, vol. 2, no. 6, pp. 1217–1230, Nov. 2003.
- [21] Y. Ding, T. Davidson, Z.-Q. Luo and K. Wong. "Minimum BER block precoders for zero-forcing equalization." *IEEE Trans. on Sig. Proc.*, vol. 51, no. 9, pp. 2410–2423, Sept. 2003.
- [22] S. Chan, T. Davidson, and K. Wong. "Asymptotically minimum BER linear block precoders for MMSE equalization," *IEE Proc. Commun.*, vol. 151, pp. 297–304, Aug. 2004.
- [23] S. Kay. *Fundamentals of statistical signal processing*, Prentice Hall, NJ, 1998.
- [24] J. Proakis. *Digital Communications*, 4th edition, McGraw-Hill, New York, 2000.
- [25] S. Buzzi, M. Lops, S. Sardellitti. "Widely linear reception strategies for layered space-time wireless communications". *IEEE Trans. on Sig. Proc.*, vol. 54, no. 6, pp. 2252–2262, June 2006.
- [26] A. Lampe, M. Breiling. "Asymptotic analysis of widely linear MMSE multiuser detection – complex vs. real modulation". *Proc. IEEE Inf. Theory Workshop*, pp. 55–57, Sept. 2001.
- [27] D. Mattered, L. Paura, F. Sterle. "Widely linear MMSE transceiver for real-valued sequences over MIMO channel". *Proc. ISPA*, pp. 1176–1180, Sept. 2003.
- [28] M. Kiessling and J. Speidel, "Statistical prefilter design for MIMO ZF and MMSE receivers based on majorization theory," *In Proc. Int. Conf. Acoust., Speech, Signal Process.*, May 2004.
- [29] X. Zhang, D. Palomar, B. Ottersten. "Statistically robust design of linear MIMO transceiver". *IEEE Trans. on Signal Processing*, vol. 56, no. 8, pp. 3678–3689, August 2008.

- [30] M. Vu, A. Paulraj. "Optimal linear precoders for MIMO wireless correlated channels with nonzero mean in space-time coded systems". *IEEE Trans. on Signal Processing*, vol. 54, no. 6, pp. 2328–2332, June 2006.
- [31] D. Gesbert, M. Kountouris, R. Heath, C. Chae, T. Sizer. "From single user to multiuser communications: shifting the MIMO paradigm". *IEEE Sig. Proc. Magazine*, vol. 24, no. 5, pp. 36–46, Oct. 2007.
- [32] B. Hoochwald, C. Peel, A. Swindlehurst. "A vector-perturbation technique for near capacity multi-antenna multiuser communication – part II: perturbation". *IEEE Trans. Commun.*, vol. 53, no. 3, pp. 537–544, March 2005.
- [33] D. Wang, E. Jorswieck, A. Sezgin, E. Costa. "Joint Tomlinson-Harashima precoding with diversity techniques for multiuser MIMO systems". *Proc. IEEE VTC'05 Spring*, vol. 2, pp. 1017–1021, June 2005.
- [34] L. Choi, R. Murch. "A transmit preprocessing technique for multiuser MIMO system using a decomposition approach". *IEEE Trans. on Wireless Commun.*, vol. 3, no. 1, pp. 20–24, January 2004.
- [35] M. Sadek, A. Tarighat, A. Sayed. "Active antenna selection in multiuser MIMO communications". *IEEE Trans. on Sig. Proc.*, vol. 55, no. 4, pp. 1498–1510, April 2007.
- [36] X. Zhang, J. Lee, H. Liu. "Low complexity multiuser MIMO scheduling with channel decomposition". *Proc. IEEE WCNC'07*, vol. 3, no. 1, pp. 2454–2458, March 2007.
- [37] U. Madhow, M. Honing. "MMSE interference suppression for direct-sequence spread-spectrum CDMA". *IEEE Trans. Commun.*, vol. 42, no. 12, pp. 3178–3187, Dec. 1994.
- [38] M. Simon, M. Alouini. *Digital Communication over Fading Channels – A Unified Approach to Performance Analysis*, John Wiley & Sons, 2000.

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## APPENDIX

### A. Performance comparison between improved and conventional precoders (Proof of Propositions 1 and 2)

Using the ZF precoders as an example, we conduct a theoretical analysis for the conventional and the improved precoding schemes. Their performance will be compared based on the analytical results. For simplicity, we assume  $N_t = N_r = N$  in our analysis. For the conventional system, the received signal vector can be expressed as

$$\tilde{\mathbf{s}} = \beta'^{-1}(\mathbf{H}\mathbf{P}_c\mathbf{s} + \mathbf{n}) = \mathbf{H}\tilde{\mathbf{P}}_c\mathbf{s} + \mathbf{n}/\beta'. \quad (43)$$

We can reform  $\mathbf{H}\tilde{\mathbf{P}}_c\mathbf{s}$  as

$$\mathbf{H}\tilde{\mathbf{P}}_c\mathbf{s} = \begin{bmatrix} \bar{\mathbf{h}}_1 \\ \vdots \\ \bar{\mathbf{h}}_N \end{bmatrix} \sum_{i=1}^N \mathbf{p}_i s_i = \begin{bmatrix} \bar{\mathbf{h}}_1 \sum_{i=1}^N \mathbf{p}_i s_i \\ \vdots \\ \bar{\mathbf{h}}_N \sum_{i=1}^N \mathbf{p}_i s_i \end{bmatrix} \quad (44)$$

where  $\bar{\mathbf{h}}_i$  is the  $i$ th row of  $\mathbf{H}$ , and  $\mathbf{p}_i$  is the  $i$ th column of  $\tilde{\mathbf{P}}_c$ . The vector  $\bar{\mathbf{h}}_i$  can be expressed as [37]

$$\bar{\mathbf{h}}_i = \bar{\mathbf{h}}_i^\epsilon + \bar{\mathbf{h}}_i^\perp, \quad (45)$$

where  $\bar{\mathbf{h}}_i^\epsilon$  denotes the projection of  $\bar{\mathbf{h}}_i$  onto the subspace spanned by the interference; and  $\bar{\mathbf{h}}_i^\perp$  denotes the projection of  $\bar{\mathbf{h}}_i$  onto the orthogonal complement of the subspace spanned by the interference. According to [37], the ZF solution for  $s_i$  is

$$\mathbf{p}_i = \frac{\bar{\mathbf{h}}_i^{\perp H}}{\|\bar{\mathbf{h}}_i^\perp\|^2}. \quad (46)$$

Since  $\bar{\mathbf{h}}_i^\perp \bar{\mathbf{h}}_j^H = 0$  for  $j \neq i$ , we have

$$\mathbf{H}\tilde{\mathbf{P}}_c\mathbf{s} = \begin{bmatrix} \bar{\mathbf{h}}_1 \sum_{i=1}^N \mathbf{p}_i s_i \\ \vdots \\ \bar{\mathbf{h}}_N \sum_{i=1}^N \mathbf{p}_i s_i \end{bmatrix} = \begin{bmatrix} \bar{\mathbf{h}}_1 \sum_{i=1}^N \frac{\bar{\mathbf{h}}_i^{\perp H}}{\|\bar{\mathbf{h}}_i^\perp\|^2} s_i \\ \vdots \\ \bar{\mathbf{h}}_N \sum_{i=1}^N \frac{\bar{\mathbf{h}}_i^{\perp H}}{\|\bar{\mathbf{h}}_i^\perp\|^2} s_i \end{bmatrix} = \begin{bmatrix} s_1 \\ \vdots \\ s_N \end{bmatrix}. \quad (47)$$

The soft decision for the signal vector becomes

$$\tilde{\mathbf{s}} = \mathbf{H}\tilde{\mathbf{P}}_c\mathbf{s} + \mathbf{n}/\beta' = \mathbf{s} + \mathbf{n}/\beta'. \quad (48)$$

According to the power constraint  $\mathbb{E}[\|\mathbf{P}\mathbf{s}\|^2] = \beta'^2 \mathbb{E}[\|\tilde{\mathbf{P}}\mathbf{s}\|^2] = N\sigma_s^2$

$$\beta'^2 \sum_{i=1}^N \|\mathbf{p}_i\|^2 = \beta'^2 \sum_{i=1}^N \frac{1}{\|\bar{\mathbf{h}}_i^\perp\|^2} = N\sigma_s^2, \quad (49)$$

therefore,

$$\beta'^2 = \frac{N\sigma_s^2}{\sum_{i=1}^N 1/\|\bar{\mathbf{h}}_i^\perp\|^2} = \frac{N\sigma_s^2}{\sum_{i=1}^N \frac{1}{\delta_i^\epsilon \|\bar{\mathbf{h}}_i\|^2}}, \quad (50)$$

where  $\delta_i^\epsilon$  is called fading unbalanced resistance (FUR) [25], which measures the system's ability to cope with situations in which the transmitted symbols are received with large power disparities due to the hostile channel conditions. It is defined as

$$\delta_i^\epsilon = \frac{\|\bar{\mathbf{h}}_i^\perp\|^2}{\|\bar{\mathbf{h}}_i\|^2} = \frac{1}{(\mathbf{H}^H \mathbf{H})_{i,i}^{-1}}. \quad (51)$$

The hard decision on the transmitted symbol vector is made as  $\hat{\mathbf{s}} = Q(\text{Re}\{\tilde{\mathbf{s}}\})$ , where  $Q(\cdot)$  represents the quantizer (decision device) which maps the input onto the signal constellation to generate the symbol estimates. From (48) and (50), we can derive the signal-to-noise ratio (SNR) for decision statistic  $\text{Re}\{\tilde{s}_i\}$  at the input of the quantizer as

$$\gamma_t^c = \frac{\beta'^2 \sigma_s^2}{\sigma_n^2/2} = \frac{2N\sigma_s^2}{\sigma_n^2 \sum_{i=1}^N \frac{1}{\delta_i^c \|\mathbf{h}_i\|^2}}. \quad (52)$$

When the transmitter precodes with the conventional precoder before transmission, we can view the system as experiencing an effective channel of  $\mathbf{H}\tilde{\mathbf{P}}_c$ . Therefore, the capacity of the MIMO channel for a fixed realization of  $\mathbf{H}$  and a fixed  $\mathbf{P}_c$  is given by [3]

$$C(\mathbf{P}_c) = \log_2 \det \left( \mathbf{I}_N + \frac{\sigma_s^2 \beta'^2}{\sigma_n^2} (\mathbf{H}\tilde{\mathbf{P}}_c)^H (\mathbf{H}\tilde{\mathbf{P}}_c) \right). \quad (53)$$

As indicated by (47),  $\tilde{\mathbf{H}}\tilde{\mathbf{P}}_c = \mathbf{I}_N$  for the conventional ZF precoder. Equ. (53) can be simplified to

$$C(\mathbf{P}_c) = \log_2 \det \left( \frac{\sigma_s^2 \beta'^2 + \sigma_n^2}{\sigma_n^2} \mathbf{I}_N \right) = N \log_2 \left( \frac{\sigma_s^2 \beta'^2 + \sigma_n^2}{\sigma_n^2} \right). \quad (54)$$

For the improved ZF precoding, the received signal vector can be expressed as

$$\tilde{\mathbf{s}}' = \beta^{-1} \text{Re}(\mathbf{H}\tilde{\mathbf{P}}\mathbf{s} + \mathbf{n}) = \text{Re}(\tilde{\mathbf{H}}\tilde{\mathbf{P}}\mathbf{s} + \mathbf{n}/\beta). \quad (55)$$

Since

$$\begin{aligned} \text{Re}(\tilde{\mathbf{H}}\tilde{\mathbf{P}}\mathbf{s}) &= \frac{1}{2} \begin{bmatrix} \bar{\mathbf{h}}_1 \sum_{i=1}^N \mathbf{p}_i s_i \\ \vdots \\ \bar{\mathbf{h}}_N \sum_{i=1}^N \mathbf{p}_i s_i \end{bmatrix} + \frac{1}{2} \begin{bmatrix} \bar{\mathbf{h}}_1^* \sum_{i=1}^N \mathbf{p}_i^* s_i \\ \vdots \\ \bar{\mathbf{h}}_N^* \sum_{i=1}^N \mathbf{p}_i^* s_i \end{bmatrix} \\ &= \begin{bmatrix} \bar{\mathbf{h}}_1 & \bar{\mathbf{h}}_1^* \\ \vdots & \vdots \\ \bar{\mathbf{h}}_N & \bar{\mathbf{h}}_N^* \end{bmatrix} \sum_{i=1}^N \frac{1}{2} \begin{bmatrix} \mathbf{p}_i \\ \mathbf{p}_i^* \end{bmatrix} s_i = \begin{bmatrix} \bar{\mathbf{h}}_{1a} \\ \vdots \\ \bar{\mathbf{h}}_{Na} \end{bmatrix} \sum_{i=1}^N \mathbf{p}_{ia} s_i, \end{aligned} \quad (56)$$

where the augmented vectors are defined as

$$\bar{\mathbf{h}}_{ia} = [\bar{\mathbf{h}}_i \quad \bar{\mathbf{h}}_i^*]; \quad \mathbf{p}_{ia} = \frac{1}{2} \begin{bmatrix} \mathbf{p}_i \\ \mathbf{p}_i^* \end{bmatrix}. \quad (57)$$

Obviously,  $\|\bar{\mathbf{h}}_{ia}\|^2 = 2\|\bar{\mathbf{h}}_i\|^2$ . Decomposing  $\bar{\mathbf{h}}_{ia} = \bar{\mathbf{h}}_{ia}^\xi + \bar{\mathbf{h}}_{ia}^\perp$ , the improved ZF solution for decoding  $s_i$  is

$$\mathbf{p}_{ia} = \frac{\bar{\mathbf{h}}_{ia}^{\perp H}}{\|\bar{\mathbf{h}}_{ia}^\perp\|^2}. \quad (58)$$

Since  $\bar{\mathbf{h}}_{ia}^\perp \bar{\mathbf{h}}_{ja}^H = 0$  for  $j \neq i$ , we have

$$\begin{aligned} \text{Re}(\tilde{\mathbf{H}}\tilde{\mathbf{P}}\mathbf{s}) &= \begin{bmatrix} \bar{\mathbf{h}}_{1a} \\ \vdots \\ \bar{\mathbf{h}}_{Na} \end{bmatrix} \sum_{i=1}^N \mathbf{p}_{ia} s_i = \begin{bmatrix} \bar{\mathbf{h}}_{1a} \sum_{i=1}^N \frac{\bar{\mathbf{h}}_{ia}^{\perp H}}{\|\bar{\mathbf{h}}_{ia}^\perp\|^2} s_i \\ \vdots \\ \bar{\mathbf{h}}_{Na} \sum_{i=1}^N \frac{\bar{\mathbf{h}}_{ia}^{\perp H}}{\|\bar{\mathbf{h}}_{ia}^\perp\|^2} s_i \end{bmatrix} \\ &= \begin{bmatrix} s_1 \\ \vdots \\ s_N \end{bmatrix}, \end{aligned} \quad (59)$$

therefore

$$\tilde{\mathbf{s}} = \text{Re}(\tilde{\mathbf{H}}\tilde{\mathbf{P}}\mathbf{s} + \mathbf{n}/\beta) = \mathbf{s} + \text{Re}(\mathbf{n})/\beta. \quad (60)$$

According to the power constraint

$$\beta^2 \sum_{i=1}^N \|\mathbf{p}_{ia}\|^2 = \beta^2 \sum_{i=1}^N \frac{1}{\|\bar{\mathbf{h}}_{ia}^\perp\|^2} = N\sigma_s^2, \quad (61)$$

therefore,

$$\beta^2 = \frac{N\sigma_s^2}{\sum_{i=1}^N 1/\|\bar{\mathbf{h}}_{ia}^\perp\|^2} = \frac{N\sigma_s^2}{\sum_{i=1}^N \frac{1}{\delta_i \|\mathbf{h}_i\|^2}} = \frac{N\sigma_s^2}{\sum_{i=1}^N \frac{1}{2\delta_i \|\mathbf{h}_i\|^2}}, \quad (62)$$

where the FUR value  $\delta_i$  for the improved scheme is calculated according to [25] as

$$\delta_i = \frac{\|\bar{\mathbf{h}}_{ia}^\perp\|^2}{\|\bar{\mathbf{h}}_{ia}\|^2} = \frac{1}{(\text{Re}\{\mathbf{H}^H \mathbf{H}\})_{i,i}^{-1}}. \quad (63)$$

Based on (60) and (62), the SNR for the real-valued decision static  $\tilde{s}'_i$  is

$$\gamma_t = \frac{\beta^2 \sigma_s^2}{\sigma_n^2/2} = \frac{2N\sigma_s^2}{\sigma_n^2 \sum_{i=1}^N \frac{1}{\|\bar{\mathbf{h}}_{ia}^\perp\|^2}} = \frac{2N\sigma_s^2}{\sigma_n^2 \sum_{i=1}^N \frac{1}{2\delta_i \|\mathbf{h}_i\|^2}}. \quad (64)$$

According to [25],  $\delta_i \geq \delta_i^c$ , therefore

$$\frac{1}{2\delta_i \|\mathbf{h}_i\|^2} < \frac{1}{\delta_i^c \|\mathbf{h}_i\|^2}. \quad (65)$$

Comparing  $\gamma_t^c$  in (52) and  $\gamma_t$  in (64), and using (65), we can come to the conclusion that  $\gamma_t > \gamma_t^c$ . In what follows, we use the M-ASK modulation as an example to show that the improved scheme achieves a better error performance. The symbol error probabilities for M-ASK are given by [38]

$$\begin{aligned} P_t &= \frac{2(M-1)}{M} Q(\sqrt{2g_{\text{ASK}}\gamma_t}); \\ P_t^c &= \frac{2(M-1)}{M} Q(\sqrt{2g_{\text{ASK}}\gamma_t^c}), \end{aligned} \quad (66)$$

where  $g_{\text{ASK}} = 3/(M^2 - 1)$ . Since  $\gamma_t > \gamma_t^c$ , based on (66), we can conclude that the improved precoder results in a better performance than the conventional precoder, i.e.,  $P_t < P_t^c$ .

When the transmitter precodes with the improved precoder before transmission, we can view the system as experiencing an effective channel of  $\text{Re}\{\tilde{\mathbf{H}}\tilde{\mathbf{P}}\}$ . Therefore, the capacity of the MIMO channel for a fixed realization of  $\mathbf{H}$  and a fixed  $\mathbf{P}$  is given by

$$C(\mathbf{P}) = \log_2 \det \left( \mathbf{I}_N + \frac{2\sigma_s^2 \beta^2}{\sigma_n^2} \text{Re}(\tilde{\mathbf{H}}\tilde{\mathbf{P}})^H \text{Re}(\tilde{\mathbf{H}}\tilde{\mathbf{P}}) \right). \quad (67)$$

As indicated by (59),  $\text{Re}\{\tilde{\mathbf{H}}\tilde{\mathbf{P}}\} = \mathbf{I}$  for the improved ZF precoder. Equ. (67) can be simplified to

$$C(\mathbf{P}) = \log_2 \det \left( \frac{2\sigma_s^2 \beta^2 + \sigma_n^2}{\sigma_n^2} \mathbf{I} \right) = N \log_2 \left( \frac{2\sigma_s^2 \beta^2 + \sigma_n^2}{\sigma_n^2} \right). \quad (68)$$

From (50), (62) and (65), it can be easily shown that

$$\beta^2 > \beta'^2. \quad (69)$$

Comparing (68) with (54), and using (69), we can come to the conclusion that  $C(\mathbf{P}) > C(\mathbf{P}_c)$ . Note that (69) coincides with the simulation results shown in Fig. 6.