# WSR Maximized Resource Allocation in Multiple DF Relays Aided OFDMA Downlink Transmission

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Abstract—This paper considers the weighted sum rate (WSR) maximized resource allocation (RA) constrained by a system sum power in an orthogonal frequency division multiple access (OFDMA) downlink transmission system assisted by multiple decode-and-forward (DF) relays. In particular, multiple relays may cooperate with the source for every relay-aided transmission. A two-step algorithm is proposed to find the globally optimum RA. In the first step, the optimum source/relay power and assisting relays that maximize the rate is found for every combination of subcarrier and destination, assuming a sum power is allocated to the transmission at that subcarrier to that destination in the relay-aided transmission mode and the direct mode, respectively. In the second step, a convex-optimization based algorithm is designed to find the globally optimum assignment of destination, transmission mode, and sum power for each subcarrier to maximize the WSR. Combining the RAs found in the two steps, the globally optimum RA can be found. In addition, we show that the optimum RA in the second step can readily be derived when the system sum power is very high. The effectiveness of the proposed algorithm is illustrated by numerical experiments.

*Index Terms*—Orthogonal frequency division multiple access, resource allocation, cooperative wireless transmission, relaying, decode and forward, convex optimization.

# I. INTRODUCTION

Relay-aided cooperative wireless transmission has been attracting intensive research interest lately, motivated by the consideration that by cooperative relaying, distributed multiple input and multiple output (MIMO) links can be created for performance improvement, which is suitable especially for applications incapable of installing multiple antennas at the same radio device [1]. Two protocols, namely amplify and forward (AF) as well as decode and forward (DF), have been proposed and become the focus of recent research works [2]. In this paper, we consider half-duplex radio devices adopting the DF protocol, which carries out a relay-aided transmission in two time slots, referred to as the broadcasting slot and the relaying slot, respectively. In the broadcasting slot, the source emits a symbol, which is received by both the destination and the relays. In the relaying slot, some relays first recover the source symbol from received signals, then transmit that recovered symbol to the destination. Finally, the destination

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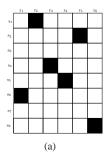
The authors would like to thank the Walloon Region for funding the projects MIMOCOM, the ARC SCOOP and the FP7 Network NEWCOM++.

combines the signals received in the two slots, then decode for the source symbol. In particular, it has been show in [2] that selection relaying DF, which uses either the direct transmission mode without any relay assisting, or the relay-aided mode depending on channel state information (CSI), achieves full diversity.

This paper addresses the resource allocation (RA) for an orthogonal frequency division multiple access (OFDMA) downlink transmission system aided by multiple relays adopting selection relaying DF. The motivation behind considering OFDMA is that it is a widely recognized multiuser transmission technique for current and future wireless systems, thanks to its flexibility to incorporate dynamic RA for performance improvement [3]. Compared to the conventional OFDMA transmission, the relay-aided OFDMA transmission raises more complicated RA problems, since it introduces extra tasks such as deciding the transmission mode of each subcarrier, determining assisting relays and the power allocation to them for each relay-aided subcarrier, besides assigning destination and source power at each subcarrier. Therefore, novel efficient RA algorithms are solicited for relay-aided OFDMA transmission systems.

For the point to point OFDM transmission aided by DF relays, some RA algorithms have been proposed lately. To name a few, RA algorithms have been proposed in [4] to minimize the sum power under rate constraints, and in [5], [6] to maximize the sum rate subject to power constraints, when only one relay exists and assists with selection relaying DF. However, at every subcarrier assigned to the direct transmission mode, the source does not transmit any symbol in the relaying slot, which wastes bandwidth resource. To address this issue, rate-optimized RA algorithms which permit source transmission in the relaying slot at each subcarrier assigned to the direct mode, have been proposed in [7]–[12].

For the OFDMA downlink transmission aided by DF relays, some RA algorithms have been proposed as well. For example, power constrained RA algorithms proposed in [13]–[15] and [16] consider respectively the maximization of the sum rate and the weighted sum goodput, whereas the one in [17] aims at maximizing a metric depending on the rates and queue lengths of the source and relays. Using those algorithms, each destination may decode the source transmitted signals, each at a distinct subcarrier in the broadcasting and relaying (except for [16]) slots, as well as the relays transmitted signals, each at a distinct subcarrier from a single relay in the relaying slot. Note that when the source transmits signals to relays in the broadcasting slot, every destination discards the received signal replicas and thus spatial diversity is not exploited. In



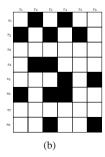


Fig. 1. Example patterns of selecting (a) one relay or (b) multiple relays assisting at each relay-aided subcarrier, where a black block at the i-th row and j-th column indicates the j-th relay  $\mathbf{r}_j$  assists the transmission at the i-th subcarrier  $\mathbf{s}_i$ .

[18], a RA algorithm is proposed to maximize the sum utility of multiple uplink/downlink data streams aided by a single destination adopting selection relaying AF or DF.

So far, the majority of the proposed RA algorithms, as the aforementioned ones, restrict that at most one relay can assist the source for every relay-aided transmission. In fact, when there are multiple relays available, allowing not just one but *each* of them to be eligible for assisting at every subcarrier, can better exploit the degrees of freedom in the system for performance improvement. For illustration purposes, example patterns of selecting single or multiple assisting relays are shown in Figure 1. In fact, these benefits have been demonstrated in [10] for the OFDM transmission to a single destination, and in [19], [20] for the OFDMA downlink transmission with each destination using only one fixed subcarrier.

Compared to the above existing works, this paper contains the following contributions:

- we consider the weighted sum rate (WSR) maximized RA problem constrained by a system sum power in an OFDMA downlink transmission system aided by multiple DF relays. In particular, multiple relays may cooperate with the source for every relay-aided transmission.
- we propose a two-step algorithm to find the globally optimum RA based on a divide-and-conquer strategy. In particular, we show that the optimum RA in the second step can be easily derived when the system sum power is very high.

The remainder of this paper is organized as follows. In Section II, we describe the considered system. We propose the two-step RA algorithm in Section III, then derive the optimum RA for the second step when the system sum power is very high in Section IV. In Section V, the effectiveness of the proposed algorithm is illustrated by numerical experiments. Finally, some conclusions complete this paper in Section VI.

### II. SYSTEM DESCRIPTION AND RA PROBLEM

We consider an OFDMA downlink transmission system from a source to U destinations aided by N DF relays collected in the set  $\Psi = \{\mathbf{r}_i | i=1,\cdots,N\}$ . All links are assumed to be frequency selective, and OFDM with sufficiently long cyclic prefix is used to transform every link into K parallel channels, each at a different subcarrier facing flat fading. At

each subcarrier, the transmission of a symbol is in either the direct mode, or the relay-aided mode spanning across two equal-duration time slots, namely the broadcasting slot and the relaying slot. Due to the OFDMA, each subcarrier is allocated to one destination exclusively. As will be described later, in the broadcasting slot only the source transmits symbols at all subcarriers. In the relaying slot, the source transmits at the subcarriers assigned to the direct mode, whereas some relays transmit simultaneously at the other subcarriers assigned to the relay-aided mode.

We make the following assumptions for the considered system. First, we assume the radio frequency (RF) nonlinearity at the source and the relays is negligible, and the carrier frequency and symbol timing of the source are perfectly synchronized with those of the relays, e.g. with the techniques in [21]. Second, every channel in the system remains unchanged within a sufficiently long duration, over which the RA algorithm can be implemented at a central controller knowing precisely the CSI of the system. Furthermore, the RA information can be reliably disseminated to the source, every relay, and every destination. To date, the related works, as introduced in Section I, are based on these assumptions as well. As an interesting topic, the RA for the system with subcarrier nonorthogonality and non-ideal CSI will be studied in our future work.

Let's consider the transmission of a unit-variance symbol s at a subcarrier to a destination, say at subcarrier k to destination u. The coefficient of the channel between any two of the source,  $\mathbf{r}_i$ , and destination u, are notated according to Table I. We first describe the transmission in the relay-aided mode. Suppose a set of relays, collected in the set  $\mathcal{R}_k(u)$ , are selected by the RA algorithm to assist relaying. Using the transmit power  $P_{\mathbf{s},u}(k)$ , the source first emits the symbol  $\sqrt{P_{\mathbf{s},u}(k)}s$ , while each relay does not transmit anything in the broadcasting slot, as illustrated in Figure 2.a. At the end of this slot, both destination u and every relay receive the source signal. The signal samples at destination u and  $\mathbf{r}_i$  can be expressed by

$$y_{u,b}(k) = \sqrt{P_{s,u}(k)} \lambda_{s,u}(k) s + n_{u,b}(k)$$
 (1)

and

$$y_{\mathbf{r}_{i},\mathbf{b}}(k) = \sqrt{P_{\mathbf{s},u}(k)}\lambda_{\mathbf{s},\mathbf{r}_{i}}(k)s + n_{\mathbf{r}_{i},\mathbf{b}}(k), \tag{2}$$

respectively, where  $n_{u,b}(k)$  and  $n_{r_i,b}(k)$  represent the corruption of the additive white Gaussian noise (AWGN) at destination u and  $r_i$ , respectively.

After decoding and then reencoding s, all relays in  $\mathcal{R}_k(u)$  transmit simultaneously to destination u in the relaying slot, which in effect establishes a distributed multiple input and single output (MISO) transmission link as illustrated in Figure 2.b. Specifically,  $\mathbf{r}_i \in \mathcal{R}_k(u)$  transmits  $\alpha_{\mathbf{r}_i} s$ , where  $\alpha_{\mathbf{r}_i}$  is

TABLE I CHANNEL COEFFICIENTS AT SUBCARRIER k.

source to destination $u$	source to $r_i$	$\mathbf{r}_i$ to destination $u$
$\lambda_{\mathrm{s},u}(k)$	$\lambda_{\mathrm{s,r}_i}(k)$	$\lambda_{\mathbf{r}_i,u}(k)$

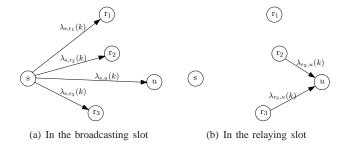


Fig. 2. Illustration of a relay-aided transmission at subcarrier k, where  $\mathcal{R}_k(u) = \{r_2, r_3\}$ .

the complex weight for transmit beamforming and satisfies  $|\alpha_{\mathbf{r}_i}|^2 = P_{\mathbf{r}_i,u}(k)$ , with  $P_{\mathbf{r}_i,u}(k)$  denoting the transmit power allocated to  $\mathbf{r}_i$ . To have the relays' signals add coherently when received at destination  $u,\ \alpha_{\mathbf{r}_i} = \sqrt{P_{\mathbf{r}_i,u}(k)}e^{-j\arg{(\lambda_{\mathbf{r}_i,u}(k))}}$  is used, where  $\arg{(\lambda_{\mathbf{r}_i,u}(k))}$  stands for the phase of  $\lambda_{\mathbf{r}_i,u}(k)$ . Note that the above described relay-aided transmission enables a flexible use of all relays opportunistically through a general form of adaptive transmit beamforming, in that  $\mathcal{R}_k(u)$  and  $\{P_{\mathbf{r}_i,u}(k)|\mathbf{r}_i\in\mathcal{R}_k(u)\}$  are determined dynamically by the RA algorithm based on the CSI, as will be developed later. At the end of the relaying slot, the signal sample at destination u is denoted by

$$y_{u,r}(k) = \sum_{\mathbf{r}_i \in \mathcal{R}_k(u)} \sqrt{P_{\mathbf{r}_i,u}(k)} |\lambda_{\mathbf{r}_i,u}(k)| s + n_{u,r}(k), \quad (3)$$

where  $n_{u,r}(k)$  represents the corruption of AWGN at destination u.

Finally, destination u combines  $y_{u,\mathbf{b}}(k)$  and  $y_{u,\mathbf{r}}(k)$  based on the maximum ratio combining (MRC), and the output is denoted by

$$y_{u,c}(k) = \sqrt{P_{s,u}(k)} (\lambda_{s,u}(k))^* y_{u,b}(k) + \left( \sum_{r_i \in \mathcal{R}_{k}(u)} \sqrt{P_{r_i,u}(k)} |\lambda_{r_i,u}(k)| \right)^* y_{u,r}(k).$$
 (4)

We assume  $\{n_{\mathbf{r}_i,\mathbf{b}}(k)|\mathbf{r}_i\in\mathcal{R}_k(u)\}$ ,  $n_{u,\mathbf{b}}(k)$ , and  $n_{u,\mathbf{r}}(k)$  are independent zero-mean circular Gaussian random variables with the same variance  $\sigma^2$ . By some mathematical arrangements, the signal to noise ratio (SNR) associated with decoding s from  $y_{\mathbf{r}_i,\mathbf{b}}(k)$  at  $\mathbf{r}_i\in\mathcal{R}_k(u)$  in the broadcasting slot is

$$\gamma_{\mathbf{r}_i}(k) = P_{\mathbf{s},u}(k)G_{\mathbf{s},\mathbf{r}_i}(k),\tag{5}$$

and the SNR associated with decoding s from  $y_{u,\mathbf{c}}(k)$  at destination u is

$$\gamma_{u,c}(k) = P_{s,u}(k)G_{s,u}(k) + \left(\sum_{\mathbf{r}_i \in \mathcal{R}_k(u)} \sqrt{P_{\mathbf{r}_i,u}(k)G_{\mathbf{r}_i,u}(k)}\right)^2, \tag{6}$$

where  $G_{\mathbf{s},\mathbf{r}_i}(k) = \frac{|\lambda_{\mathbf{s},\mathbf{r}_i}(k)|^2}{\sigma^2}$ ,  $G_{\mathbf{s},u}(k) = \frac{|\lambda_{\mathbf{s},u}(k)|^2}{\sigma^2}$ , and  $G_{\mathbf{r}_i,u}(k) = \frac{|\lambda_{\mathbf{r}_i,u}(k)|^2}{\sigma^2}$  represent the noise power normalized channel gains from the source to  $\mathbf{r}_i$ , from the source to destination u, and from  $\mathbf{r}_i$  to destination u, respectively.

To guarantee reliable decoding at destination u and every relay in  $\mathcal{R}_k(u)$ , the maximum achievable rate at subcarrier k is

$$R_{u,k,1} = \min\{\min_{\mathbf{r}_i \in \mathcal{R}_k(u)} \{\ln(1 + \gamma_{\mathbf{r}_i}(k))\}, \ln(1 + \gamma_{u,c}(k))\}$$
  
= \ln(1 + \gamma\_u(k)) (7)

in nats/two-slots, where

$$\gamma_u(k) = \min\{\gamma_{u,c}(k), \gamma_r(k)\} \tag{8}$$

and

$$\gamma_{\mathbf{r}}(k) = P_{\mathbf{s},u}(k) \cdot \min_{\mathbf{r}_i \in \mathcal{R}_k(u)} G_{\mathbf{s},\mathbf{r}_i}(k). \tag{9}$$

As for the direct transmission, we consider a protocol with a better use of system bandwidth compared to requiring the source to transmit only in the broadcasting slot considered in [5], [6]. Specifically, the source emits two independent symbols with transmit powers  $P_{\mathrm{s1},u}(k)$  and  $P_{\mathrm{s2},u}(k)$  in the two slots, respectively, and only destination u decodes the transmitted symbols from the two received signal samples. We assume the AWGN corruption associated with the two received samples are independent zero-mean circular Gaussian distributed with variance  $\sigma^2$ . By simple mathematical arrangements, the sum achievable rate to destination u in nats/two-slots can be evaluated as

$$R_{u,k,2} = \ln(1 + P_{s_1,u}(k)G_{s,u}(k)) + \ln(1 + P_{s_2,u}(k)G_{s,u}(k)).$$
(10)

We consider the RA problem of finding the optimum assignment of destination, transmission mode, and source power for each subcarrier, as well as the optimum assisting relays and their transmit powers for every relay-aided subcarrier, to maximize the WSR of the destinations when the considered system consumes a sum power no greater than  $P_t$ . Note that for the system under consideration the same subcarrier is used by the source and the relays for transmitting a symbol in the relay-aided mode. This constraint could however be relaxed, and optimized subcarrier pairing could also be implemented. This would further increase the degrees of freedom for optimization. However, it would be more involved to gain insights on the RA algorithm design, and that is why in the current work, the RA algorithm is designed under this constraint of nonoptimized subcarrier pairing. We will nevertheless use the insights gained here to guide future work, which will also consider subcarrier pairing.

## III. THE TWO-STEP RA ALGORITHM

We propose a two-step algorithm to solve the RA problem based on a divide-and-conquer strategy. We will first give an overview, then develop this algorithm in the following subsections.

## A. An overview of the two-step algorithm

In the first step, the optimum  $\mathcal{R}_k(u)$ ,  $P_{s,u}(k)$ , and  $\{P_{r_i,u}(k)|\forall\, r_i\in\mathcal{R}_k(u)\}$  that maximize  $R_{u,k,1}$ , and the optimum  $P_{s_1,u}(k)$  and  $P_{s_2,u}(k)$  that maximize  $R_{u,k,2}$ , are found for every combination of subcarrier k and destination u,

assuming that a sum power P is used by the transmission at subcarrier k to destination u in the relay-aided mode and the direct mode, respectively. Such maximized  $R_{u,k,1}$  and  $R_{u,k,2}$  are denoted by  $R_{u,k,1}(P)$  and  $R_{u,k,2}(P)$ , respectively. The associated algorithm will be developed in Section III.B.

To simplify the RA in the second step, two user sets, namely  $\mathcal{U}_{\mathrm{D}}(k) = \{u | \forall P \in [0, P_{\mathrm{t}}], R_{u,k,2}(P) \geq R_{u,k,1}(P)\}$  and  $\mathcal{U}_{\mathrm{R}}(k) = \{u | \forall P \in [0, P_{\mathrm{t}}], R_{u,k,1}(P) \geq R_{u,k,2}(P)\}$ , are found for every subcarrier k. Once a destination  $u \in \mathcal{U}_{\mathrm{D}}(k)$  (resp.  $u \in \mathcal{U}_{\mathrm{R}}(k)$ ) is allocated with subcarrier k, the direct mode (resp. the relay-aided mode) should always be used to maximize the WSR, since it results in a rate no smaller than the relay-aided mode (resp. the direct mode), independently of the sum power allocated to this subcarrier.

To formulate the maximum WSR with the  $R_{u,k,1}(P)$  and  $R_{u,k,2}(P)$  derived in the first step, we define for every destination u in  $\mathcal{U}_{\mathbf{R}}(k)$  (resp.  $\mathcal{U}_{\mathbf{D}}(k)$ ) a binary variable  $t_{u,k,1}$  (resp.  $t_{u,k,2}$ ) and a nonnegative variable  $P_{u,k,1}$  (resp.  $P_{u,k,2}$ ), where  $t_{u,k,1}=1$  (resp.  $t_{u,k,2}=1$ ) indicates that subcarrier k in the relay-aided (resp. direct) mode is allocated to destination u, and  $P_{u,k,1}$  (resp.  $P_{u,k,2}$ ) represents the corresponding sum power allocated to subcarrier k. For every destination  $u \notin \mathcal{U}_{\mathbf{D}}(k) \cup \mathcal{U}_{\mathbf{R}}(k)$ ,  $t_{u,k,1}$ ,  $t_{u,k,2}$ ,  $P_{u,k,1}$ , and  $P_{u,k,2}$  are defined with the same interpretations as explained above. Note that  $t_{u,k,1}$  and  $P_{u,k,1}$  are defined for every destination  $u \notin \mathcal{U}_{\mathbf{D}}(k)$ , and  $t_{u,k,2}$  and  $P_{u,k,2}$  are defined for every destination  $u \notin \mathcal{U}_{\mathbf{D}}(k)$ , and  $t_{u,k,2}$  and  $t_{u,k,2}$  are defined for every destination  $t_{u,k,2}$  and  $t_{u,k,3}$  are defined for every destination  $t_{u,k,3}$  and  $t_{u,k,4}$  and  $t_{u,k,4}$  are defined for every destination  $t_{u,k,4}$  and  $t_{u,k,4}$  and  $t_{u,k,4}$  are defined for every destination  $t_{u,k,4}$  and  $t_{u,k,4}$  and  $t_{u,k,4}$  and  $t_{u,k,4}$  are defined for every destination  $t_{u,k,4}$  and  $t_{u,k,4}$  and  $t_{u,k,4}$  and  $t_{u,k,4}$  are defined for every destination  $t_{u,k,4}$  and  $t_{u,k,4}$  and  $t_{u,k,4}$  and  $t_{u,k,4}$  are defined for every destination  $t_{u,k,4}$  and  $t_{u,k,4}$  and  $t_{u,k,4}$  and  $t_{u,k,4}$  are defined for every destination  $t_{u,k,4}$  and  $t_{u,k,4}$  and  $t_{u,k,4}$  and  $t_{u,k,4}$  are defined for every destination  $t_{u,k,4}$  and  $t_{u,k,4}$  and  $t_{u,k,4}$  and  $t_{u,k,4}$  and  $t_{u,k,4}$  are defined for every destination  $t_{u,k,4}$  and  $t_{u,k,4}$  and  $t_{u,k,4}$  and  $t_{u,k,4}$  are defined for every destination  $t_{u,k,4}$  and  $t_{u,k,4}$  and  $t_{u,k,4}$  and  $t_{u,k,4}$  and  $t_{u,k,4}$  are defined for every destination  $t_{u,k,4}$  and  $t_{u,k,4}$  and  $t_{u,k,4}$  and  $t_{u,k,4}$  and  $t_{u,k,4}$  and

$$f = \sum_{k=1}^{K} \left( \sum_{u \notin \mathcal{U}_{D}(k)} w_{u} t_{u,k,1} R_{u,k,1}(P_{u,k,1}) + \sum_{u \notin \mathcal{U}_{R}(k)} w_{u} t_{u,k,2} R_{u,k,2}(P_{u,k,2}) \right)$$
(11)

where  $w_u > 0$  satisfying  $\sum_{u=1}^{U} w_u = 1$  is the weight assigned by system designers to the rate of destination u. In particular, increasing  $w_u$  leads to a higher priority given to destination u.

In the second step, for every subcarrier k the optimum  $\{t_{u,k,1}, P_{u,k,1} | \forall u \notin \mathcal{U}_D(k)\}$  and  $\{t_{u,k,2}, P_{u,k,2} | \forall u \notin \mathcal{U}_R(k)\}$  that maximize f are found subject to the system sum power constraint

$$\sum_{k=1}^{K} \left( \sum_{u \notin \mathcal{U}_{D}(k)} t_{u,k,1} P_{u,k,1} + \sum_{u \notin \mathcal{U}_{R}(k)} t_{u,k,2} P_{u,k,2} \right) \le P_{t}, \quad (12)$$

as well as the constraint

$$\sum_{u \notin \mathcal{U}_{\mathcal{D}}(k)} t_{u,k,1} + \sum_{u \notin \mathcal{U}_{\mathcal{R}}(k)} t_{u,k,2} \le 1, \forall k, \tag{13}$$

due to the OFDMA. It is important to note that there exist default constraints that  $\forall k, u, \mathcal{R}_k(u)$  should be a subset of  $\Psi$ , and every power (resp. indicator) variable should be non-negative (resp. binary). This problem, consisting of both binary and continuous optimization variables, is not convex since the feasible set for the binary variables is not convex. In Section III.C, we will develop an algorithm to find the globally optimum solution. Combining the RAs found in the

two steps, the globally optimum RA for maximizing the WSR can be found.

### B. RA algorithm in the first step

First, we consider the maximization of  $R_{u,k,2}$  under the constraint  $P_{\mathbf{s}_1,u}(k)+P_{\mathbf{s}_2,u}(k)=P.$  According to the Jensen's inequality, the maximum  $R_{u,k,2}$  in this case can be easily found as

$$R_{u,k,2}(P) = 2\ln\left(1 + G_{s,u}(k)\frac{P}{2}\right),$$
 (14)

and the optimum RA is to allocate the sum power P equally to  $P_{\mathbf{s}_1,u}(k)$  and  $P_{\mathbf{s}_2,u}(k)$ .

Next, we consider the maximization of  $R_{u,k,1}$  with respect to  $\mathcal{R}_k(u)$ ,  $P_{s,u}(k)$ , and  $\{P_{r_i,u}(k)|\forall r_i \in \mathcal{R}_k(u)\}$  constrained by

$$P_{s,u}(k) + \sum_{r_i \in \mathcal{R}_k(u)} P_{r_i,u}(k) = P.$$
 (15)

To facilitate derivation, we define  $G_{\mathcal{R}_k(u),u}(k) = \sum_{\mathbf{r}_i \in \mathcal{R}_k(u)} G_{\mathbf{r}_i,u}(k)$ , which represents the sum of the channel gains from all assisting relays to destination u.  $\Psi_k$  is defined as the set incorporating all relays sorted in the increasing order of  $G_{\mathbf{s},\mathbf{r}_i}(k)$ , and the i-th relay in  $\Psi_k$  is denoted by  $\mathbf{r}_k(i)$ . In particular,  $G_{\mathbf{s},\mathbf{r}_k(N)}(k) = \max_{\mathbf{r}_i \in \Psi} G_{\mathbf{s},\mathbf{r}_i}(k)$ . We define  $\Psi_k(i)$  as the set containing all relays in  $\Psi_k$  with indices from i up to N. When  $G_{\mathbf{s},u}(k) < G_{\mathbf{s},\mathbf{r}_k(N)}(k)$ ,  $x_{u,k}$  is defined as the smallest i satisfying  $G_{\mathbf{s},u}(k) < G_{\mathbf{s},\mathbf{r}_k(i)}(k)$ .

By means of intuitive figure illustrations, the procedure of maximizing  $R_{u,k,1}$  is derived and put in Appendix A for clarity. In particular, we find that

$$R_{u,k,1}(P) = \ln(1 + G_{u,1}(k)P),$$
 (16)

where  $G_{u,1}(k)$  can be evaluated by one of the following formulas:

- 1) when  $G_{s,u}(k) \geq G_{s,r_k(N)}(k)$ ,  $G_{u,1}(k) = G_{s,r_k(N)}(k)$ . In this case,  $\forall \ P \in [0,P_{\rm t}], \ R_{u,k,2}(P) \geq R_{u,k,1}(P)$  holds.
- 2) when  $G_{\Psi_k(x_{u,k}),u}(k) \leq G_{s,u}(k) < G_{s,r_k(N)}(k),$   $G_{u,1}(k) = G_{s,u}(k).$  In this case,  $\forall P \in [0, P_t],$   $R_{u,k,2}(P) \geq R_{u,k,1}(P)$  holds.
- $\begin{array}{ll} R_{u,k,2}(P) \geq R_{u,k,1}(P) \text{ holds.} \\ \text{3) when } G_{\mathbf{s},u}(k) < G_{s,\mathbf{r}_k(N)}(k) \quad \text{and} \quad G_{\mathbf{s},u}(k) < G_{\Psi_k(x_{u,k}),u}(k), \end{array}$

$$G_{u,1}(k) = \max_{x_{u,k} \le b \le y_{u,k}} \frac{G_{s,r_k(b)}(k)G_{\Psi_k(b),u}(k)}{G_{\Psi_k(b),u}(k) + G_{s,r_k(b)}(k) - G_{s,u}(k)}$$
(17)

where  $y_{u,k}$  is the greatest i satisfying  $G_{\Psi_k(i),u}(k) > G_{s,u}(k)$ . In this case,  $G_{u,1}(k) > G_{s,u}(k)$  always holds as shown in Appendix A. Suppose  $b = z_{u,k}$  is the maximizer for the right-hand side of (17), the optimum  $\mathcal{R}_k(u)$  is  $\Psi_k(z_{u,k})$ . The optimum  $P_{s,u}(k)$  is computed by (46) with  $b = z_{u,k}$ , and the optimum  $P_{r_i,u}(k)$ ,  $\forall r_i \in \Psi_k(z_{u,k})$  is

$$P_{r_i,u}(k) = \frac{(P - P_{s,u}(k))G_{r_i,u}(k)}{G_{\Psi_k(z_{u,k}),u}(k)}.$$
 (18)

The above analysis reveals that, the relay-aided mode with the optimum RA in effect transforms all channels at subcarrier k into a source to destination u channel with normalized gain  $G_{u,1}(k)$  and half of the system bandwidth. As a matter of fact, only one symbol can be sent during the two slots, while the system bandwidth can actually support sending two independent symbols per two slots. When  $G_{u,1}(k) \leq G_{s,u}(k)$ , the direct mode should always be used. When  $G_{u,1}(k) > G_{s,u}(k)$ , using the relay-aided mode leads to an increased channel gain but sacrificing half of the bandwidth, compared to using the direct mode. In this case,  $R_{u,k,1}(P) \geq R_{u,k,2}(P)$  holds if  $P \leq \frac{4(G_{u,1}(k) - G_{s,u}(k))}{(G_{s,u}(k))^2}$ , otherwise  $R_{u,k,2}(P) > R_{u,k,1}(P)$ , which means that the relay-aided and direct modes should be used for the low and high power regimes, respectively. The interpretation is that, in the low power regime, it is more beneficial to increase the received power, while in the high power regime, it is better to increase the number of channel uses per time unit [22]. Based on the above analysis,  $\mathcal{U}_{D}(k)$ and  $\mathcal{U}_{R}(k)$  for every subcarrier k can be expressed as

$$U_{D}(k) = \{ u | G_{u,1}(k) \le G_{s,u}(k) \}$$
(19)

and

$$\mathcal{U}_{R}(k) = \{u | G_{u,1}(k) > G_{s,u}(k), P_{t} \le \frac{4(G_{u,1}(k) - G_{s,u}(k))}{(G_{s,u}(k))^{2}}\}.$$
(20)

### C. RA algorithm in the second step

We propose an algorithm to solve the RA problem in the second step based on the following strategy. First, for every subcarrier k,  $t_{u,k,1}$  and  $t_{u,k,2}$  are relaxed to be real variables between 0 and 1. Now,  $t_{u,k,1}$  (resp.  $t_{u,k,2}$ ) can be interpreted as the fraction of the whole duration allocated to transmitting at subcarrier k to destination u in the relayaided (resp. direct) mode, and  $\eta_{u,k,1} = t_{u,k,1} R_{u,k,1} (P_{u,k,1})$ (resp.  $\eta_{u,k,2} = t_{u,k,2}R_{u,k,2}(P_{u,k,2})$ ) can be interpreted as the rate averaged over the whole duration for transmitting to destination u at subcarrier k in the relay-aided (resp. direct) mode. Then, the relaxed RA problem can be transformed with change of variables into a convex optimization problem, and its globally optimum RA can be found as will be shown later. Most interestingly, we will show that every optimum  $t_{u,k,1}$ and  $t_{u,k,2}$  for the relaxed problem are still equal to either 0 or 1, which means that the globally optimum RA to the relaxed problem is also globally optimum to the original RA problem.

To make the change of variables,  $P_{u,k,1}$  is substituted with  $\frac{\rho_{u,k,1}}{t_{u,k,1}}$  for every  $u \notin \mathcal{U}_{\mathrm{D}}(k)$ , and  $P_{u,k,2}$  with  $\frac{\rho_{u,k,2}}{t_{u,k,2}}$  for every  $u \notin \mathcal{U}_{\mathrm{R}}(k)$ . Now, the sum power constraint,  $\eta_{u,k,1}$ , and  $\eta_{u,k,2}$  are respectively expressed as

$$\sum_{k=1}^{K} \left( \sum_{u \notin \mathcal{U}_{\mathcal{D}}(k)} \rho_{u,k,1} + \sum_{u \notin \mathcal{U}_{\mathcal{R}}(k)} \rho_{u,k,2} \right) \le P_{\mathbf{t}}, \tag{21}$$

$$\eta_{u,k,1} = t_{u,k,1} \ln \left( 1 + G_{u,1}(k) \frac{\rho_{u,k,1}}{t_{u,k,1}} \right),$$
(22)

$$\eta_{u,k,2} = 2 t_{u,k,2} \ln \left( 1 + \frac{G_{s,u}(k)}{2} \frac{\rho_{u,k,2}}{t_{u,k,2}} \right).$$
(23)

Note that  $\eta_{u,k,1}$  (resp.  $\eta_{u,k,2}$ ) is a concave function of  $t_{u,k,1}>0$  and  $\rho_{u,k,1}$  (resp.  $t_{u,k,2}>0$  and  $\rho_{u,k,2}$ ), since it is a perspective of the concave function  $R_{u,k,1}(P_{u,k,1})$  (resp.  $R_{u,k,2}(P_{u,k,2})$ ) (refer to page 89 in [23] for more details). One delicate issue is that after the change of variables,  $\eta_{u,k,1}$  (resp.  $\eta_{u,k,2}$ ) can not be evaluated at  $t_{u,k,1}=0$  (resp.  $t_{u,k,2}=0$ ). To address this issue, we expand the domain of  $\eta_{u,k,1}$  to incorporate  $t_{u,k,1}=0$ , and define  $\eta_{u,k,1}=0$  when  $t_{u,k,1}=0$ . After the expansion and definition,  $\eta_{u,k,1}$  is still a concave and continuous function of  $t_{u,k,1}\geq 0$ , because  $\lim_{t_{u,k,1}\to 0}\eta_{u,k,1}=0$ . Motivated by the same consideration, we expand the domain of  $\eta_{u,k,2}$  to incorporate  $t_{u,k,2}=0$ , and define  $\eta_{u,k,2}=0$  when  $t_{u,k,2}=0$ , so as to ensure  $\eta_{u,k,2}$  is a concave and continuous function of  $t_{u,k,2}\geq 0$ .

Based on the above analysis, the relaxed RA problem with new variables, i.e., maximizing f subject to the constraints (13) and (21), is a convex optimization problem. Obviously, this problem satisfies the Slater constraints qualification, i.e. there exists at least one feasible solution satisfying all inequality constraints strictly. This justifies the zero duality gap for the problem [24]. Therefore, the dual method can be used to solve this problem for the globally optimum RA [25]. Specifically, dual variables are defined for certain constraints, then an iterative algorithm consisting of an inner loop and an outer loop is implemented. In the inner loop, the Lagrangian maximization problem (LMP), which is to find the optimum RA variables that maximize the Lagrangian, is solved with the dual variables fixed. In the outer loop, the optimum dual variables are found by iteratively updating the dual variables with the subgradient method, until the Karush-Kuhn-Tucker (KKT) conditions are satisfied. The optimum RA variables for the LMP given the optimum dual variables, are the globally optimum solution for the relaxed RA problem.

To use the dual method, a nonnegative dual variable  $\mu$  is introduced for the system sum power constraint (21), and the associated LMP is

$$\max L(\mu) = f + \mu \left( P_{t} - P_{x} \right)$$
s.t.(13),
$$t_{u,k,1} \geq 0, \rho_{u,k,1} \geq 0, \forall k, \forall u \notin \mathcal{U}_{D}(k),$$

$$t_{u,k,2} \geq 0, \rho_{u,k,2} \geq 0, \forall k, \forall u \notin \mathcal{U}_{R}(k),$$
(24)

where  $L(\mu)$  represents the Lagrangian given  $\mu$ , and  $P_x$  is the left-hand side of (21). The optimum dual variable denoted by  $\mu^*$ , must satisfy the KKT conditions consisting of (a)  $\mu^* \geq 0$ , (b)  $\mu^*(P_{\rm t} - P_x(\mu^*)) = 0$ , and (c)  $P_x(\mu^*) \leq P_{\rm t}$ , where  $P_x(\mu)$  represents the  $P_x$  evaluated with the optimum power variables to (24) given  $\mu$  [23], [24]. It is important to note that  $\mu^*$  must be strictly positive, which can be justified as follows. Suppose  $\mu^* = 0$ . In this case, the maximum  $L(\mu^*)$  subject to the constraints of (24) is infinity, since f, as a function of  $\eta_{u,k,1}$  and  $\eta_{u,k,2}$ , is monotonically increasing with  $\rho_{u,k,1}$  and  $\rho_{u,k,2}$ , which can be increased unboundedly while remaining feasible to (24). This means that  $P_x(\mu^*)$  is infinity, which contradicts (c) of the KKT conditions. Therefore,  $\mu^* \neq 0$ , and the KKT conditions are reduced to  $\mu > 0$  and  $P_{\rm t} = P_x(\mu^*)$ .

Let's now find the optimum RA variables to (24) given  $\mu > 0$ . To this end, we find for every subcarrier k,  $\forall u \notin \mathcal{U}_D(k)$ , the

optimum  $\rho_{u,k,1}$  given  $t_{u,k,1}$ , and  $\forall u \notin \mathcal{U}_{\mathbf{R}}(k)$ , the optimum  $\rho_{u,k,2}$  given  $t_{u,k,2}$ , to maximize  $L(\mu)$ . After mathematical arrangements, we can show these optimum values are

$$\rho_{u,k,i} = \begin{cases} t_{u,k,1} \left[ \frac{w_u}{\mu} - \frac{1}{G_{u,1}(k)} \right]^+ & \text{if } i = 1, \\ 2t_{u,k,2} \left[ \frac{w_u}{\mu} - \frac{1}{G_{s,u}(k)} \right]^+ & \text{if } i = 2, \end{cases}$$
(25)

where  $[x]^+ = \max\{x, 0\}$ . Using (25),  $L(\mu)$  is reduced to

$$L(\mu) = \sum_{k=1}^{K} L_k(\mu) + \mu P_{t},$$
 (26)

where

$$L_k(\mu) = \sum_{u \notin \mathcal{U}_D(k)} t_{u,k,1} X_k(u) + \sum_{u \notin \mathcal{U}_R(k)} t_{u,k,2} Y_k(u), \quad (27)$$

and

$$X_{k}(u) = \ln\left(1 + G_{u,1}(k) \left[\frac{w_{u}}{\mu} - \frac{1}{G_{u,1}(k)}\right]^{+}\right)$$

$$-\mu\left[\frac{w_{u}}{\mu} - \frac{1}{G_{u,1}(k)}\right]^{+}, \qquad (28)$$

$$Y_{k}(u) = 2\ln\left(1 + G_{s,u}(k) \left[\frac{w_{u}}{\mu} - \frac{1}{G_{s,u}(k)}\right]^{+}\right)$$

$$-\mu\left[\frac{2w_{u}}{\mu} - \frac{2}{G_{s,u}(k)}\right]^{+}. \qquad (29)$$

Now, the problem of finding the optimum  $\{t_{u,k,1}|\forall u\notin \mathcal{U}_{\mathbb{D}}(k), \forall k\}$  and  $\{t_{u,k,2}|\forall u\notin \mathcal{U}_{\mathbb{R}}(k), \forall k\}$  for (24) can be decomposed into K subproblems, the k-th of which is to find the optimum  $\{t_{u,k,1}|\forall u\notin \mathcal{U}_{\mathbb{D}}(k)\}$  and  $\{t_{u,k,2}|\forall u\notin \mathcal{U}_{\mathbb{R}}(k)\}$  for maximizing  $L_k(\mu)$  subject to the constraints in (24). It can be readily shown that the optimum  $\{t_{u,k,1}|\forall u\notin \mathcal{U}_{\mathbb{D}}(k)\}\cup \{t_{u,k,2}|\forall u\notin \mathcal{U}_{\mathbb{R}}(k)\}$  has all entries equal to 0, except for one entry  $t_{u_k,k,i_k}$  equal to 1. Obviously,  $(u_k,i_k)$  is the (u,i) with the maximum metric  $m_{u,i}(k)$ , expressed as

$$m_{u,i}(k) = \begin{cases} X_k(u) & \text{if } i = 1, \\ Y_k(u) & \text{if } i = 2. \end{cases}$$
 (30)

Note that when computing  $m_{u,i}(k)$ , i can only be 1 (resp. 2) if  $u \in \mathcal{U}_{\mathbf{R}}(k)$  (resp.  $u \in \mathcal{U}_{\mathbf{D}}(k)$ ), while i may be either 1 or 2 if  $u \notin \mathcal{U}_{\mathbf{R}}(k) \cup \mathcal{U}_{\mathbf{D}}(k)$ . There might exist multiple combinations of (u,i) corresponding to the maximum  $m_{u,i}(k)$ . In this case, any combination of them can be chosen to be  $(u_k,i_k)$ . According to (25), the optimum power variables related to subcarrier k are all equal to zero, except for  $P_{u_k,k,i_k}$  computed with

$$P_{u_k,k,i_k} = \begin{cases} \left[ \frac{w_{u_k}}{\mu} - \frac{1}{G_{u_k,1}(k)} \right]^+ & \text{if } i_k = 1, \\ 2 \left[ \frac{w_{u_k}}{\mu} - \frac{1}{G_{s,u_k}(k)} \right]^+ & \text{if } i_k = 2. \end{cases}$$
(31)

The procedure of finding the optimum RA variables to (24) is summarized in **Algorithm** 1. Next, we address the problem of finding  $\mu^*$  in the outer loop iteratively. To speed up the

### Algorithm 1 Procedure to solve (24)

for k = 1 to K do

 $\forall u$  and the associated i, compute  $m_{u,i}(k)$  defined in (30);

 $\{u_k, i_k\} = \arg\max_{u,i} m_{u,i}(k);$  $t_{u_k,k,i_k} = 1$  and compute  $P_{u_k,k,i_k}$  by (31);

 $P_{u,k,i} = 0$  and  $t_{u,k,i} = 0$  if either  $u \neq u_k$  or  $i \neq i_k$ .

end for

### Algorithm 2 The RA algorithm in the second step

Find  $\mu_{\rm U}$  satisfying  $P_{\rm U}(\mu_{\rm U}) = P_{\rm t}$ ;

Find  $\mu_L$  satisfying  $P_L(\mu_L) = P_t$ ;

For each  $\mu$  in  $S = \{\mu_L + \frac{n}{N_s}(\mu_U - \mu_L), n = 0, 1, \dots, N_s - 1\}$ , use **Algorithm** 1 to find the optimum RA variables to (24), and compute  $P_x(\mu)$ ;

Initialize  $\mu$  with the  $\mu \in S$  satisfying  $P_x(\mu) \leq P_t$  and having the minimum  $P_t - P_x(\mu)$ ;

#### repeat

Update  $\mu$  with  $[\mu - \delta(P_t - P_x(\mu))]^+$ ;

Use **Algorithm** 1 to find the optimum solution for (24); Compute  $P_x(\mu)$ ;

**until**  $\mu > 0$  and  $0 \le P_{\rm t} - P_x(\mu) < \epsilon$ 

The optimum solution for (24) found in the last iteration corresponds to the optimum sum power and mode for each subcarrier in the second step.

convergence rate,  $\mu$  is first initialized by a value close to  $\mu^*$ . To this end, we define

$$P_{\rm L}(\mu) = \sum_{k=1}^{K} \left[ \frac{\min_{u} \{ w_u \}}{\mu} - M_{\rm U}(k) \right]^{+}$$
 (32)

$$P_{\rm U}(\mu) = \sum_{k=1}^{K} \left[ \frac{2 \max_{u} \{w_u\}}{\mu} - M_{\rm L}(k) \right]^+, \quad (33)$$

where

$$M_{\mathrm{U}}(k) = \max \left\{ \max_{u \notin \mathcal{U}_{\mathrm{D}}(k)} \frac{1}{G_{u,1}(k)}, \max_{u \notin \mathcal{U}_{\mathrm{R}}(k)} \frac{2}{G_{\mathrm{s},u}(k)} \right\}$$
(34)

$$M_{\rm L}(k) = \min \left\{ \min_{u \notin \mathcal{U}_{\rm D}(k)} \frac{1}{G_{u,1}(k)}, \min_{u \notin \mathcal{U}_{\rm R}(k)} \frac{2}{G_{\rm S,u}(k)} \right\}. \tag{35}$$

It can be readily shown that  $P_{\rm L}(\mu^\star) \leq P_x(\mu^\star) \leq P_{\rm U}(\mu^\star)$  always holds. Suppose  $\mu_{\rm U}$  and  $\mu_{\rm L}$  satisfy  $P_{\rm U}(\mu_{\rm U}) = P_{\rm t}$  and  $P_{\rm L}(\mu_{\rm L}) = P_{\rm t}$ , respectively. Since  $P_{\rm U}(\mu_{\rm U}) = P_{\rm t} = P_x(\mu^\star) \leq P_{\rm U}(\mu^\star)$  and  $P_{\rm U}(\mu)$  is a non-increasing function of  $\mu$ ,  $\mu_{\rm U} \geq \mu^\star$ . Similarly, we can show  $\mu^\star \geq \mu_{\rm L}$ , thus  $\mu^\star$  is confined in the interval  $[\mu_{\rm L}, \mu_{\rm U}]$ . To initialize  $\mu$  with a value close to  $\mu^\star$ , this interval is sampled uniformly at  $N_s$  points to build the set  $S = \{\mu_{\rm L} + \frac{n}{N_s}(\mu_{\rm U} - \mu_{\rm L}), n = 0, 1, \cdots, N_s - 1\}$ . Given each  $\mu$  in S, (24) is solved and  $P_x(\mu)$  computed, then  $\mu$  is initialized with the  $\mu \in S$  satisfying  $P_x(\mu) \leq P_{\rm t}$  and having the minimum  $P_{\rm t} - P_x(\mu)$ .

After initialization,  $\mu$  is iteratively updated based on the subgradient method, i.e.,  $\mu$  is updated with  $[\mu - \delta(P_{\rm t} - P_x(\mu))]^+$  where  $\delta$  is a sufficiently small value to guarantee convergence, until  $\mu^{\star}$  is found when  $P_{\rm t} = P_x(\mu)$  and  $\mu > 0$ . The RA algorithm in the second step is summarized in **Algorithm 2**. Due to numerical issues, we regard the KKT conditions to be satisfied when  $\mu > 0$  and  $0 \le P_{\rm t} - P_x(\mu) \le \epsilon$ , where  $\epsilon$  is a very small positive value.

# IV. THE OPTIMUM RA IN THE SECOND STEP FOR A SPECIAL CASE

We will show the optimum RA in the second step can be easily derived when  $P_{\rm t}$  is very high so that the associated  $\mu_{\rm U}$  satisfies

$$\forall k, \frac{\min_{u} w_{u}}{\mu_{U}} \gg \max_{u} \{ \max\{\frac{1}{G_{u,1}(k)}, \frac{1}{G_{s,u}(k)} \} \},$$
 (36)

$$\forall k, \frac{\min_{u} w_{u}}{\mu_{U}} \gg \max_{u} \{\max\{G_{u,1}(k), G_{s,u}(k)\}\}.$$
 (37)

Since (36) holds and  $\mu^* \leq \mu_U$ , the optimum sum power  $P_k$  for subcarrier k satisfies

$$\forall k, P_k \approx \begin{cases} \frac{w_{u_k}}{\mu^*} & \text{if } i_k = 1\\ \frac{2w_{u_k}}{\mu^*} & \text{if } i_k = 2, \end{cases}$$
(38)

according to (31), where  $(u_k, i_k)$  corresponds to the optimum destination and mode for subcarrier k found by the RA algorithm in the second step. From (36), (37), and (38), the conditions

$$\forall k, u, P_k G_{s,u}(k) \gg 1, P_k \gg G_{s,u}(k), \tag{39}$$

$$\forall k, u, P_k G_{u,1}(k) \gg 1, P_k \gg G_{u,1}(k),$$
 (40)

hold. This means that in the high power regime, even though the optimum sum power for each subcarrier is not precisely known, it is for sure that  $\forall \ k, u$ , the following approximations hold:

$$w_u R_{u,k,2}(P_k) \approx 2w_u \left( \ln(P_k) + \ln \frac{G_{s,u}(k)}{2} \right) \approx 2w_u \ln(P_k),$$
(41)

$$w_u R_{u,k,1}(P_k) \approx w_u \left( \ln(P_k) + \ln G_{u,1}(k) \right) \approx w_u \ln(P_k). \tag{42}$$

By comparing (41) and (42), it can easily be seen that the direct mode for each subcarrier k is the optimum for maximizing the WSR, independently of the destination assigned. All this confirms the former interpretation that, for the high power regime, increasing the number of channel uses per time unit is more beneficial than increasing the received power. If equal weights are used for all destinations, the optimum destination for subcarrier k is the one with the maximum  $G_{s,u}(k)$ . Otherwise, the optimum destination for all subcarriers is the same and given by the one with the maximum weight  $w_u$ . In either case, the optimum sum power to subcarrier k is  $P_k \approx \frac{P_t}{K}$  in order to satisfy the KKT conditions according to (38).

# V. NUMERICAL EXPERIMENTS

In numerical experiments, we consider a multiple DF relays aided OFDMA downlink transmission system with N=4 relays. The source is allocated at the origin, and the relays  $\mathbf{r}_1$ ,  $\mathbf{r}_2$ ,  $\mathbf{r}_3$ , and  $\mathbf{r}_4$  are located at coordinates (-15, -5), (-5, -5), (5, -5), and (15, -5), respectively. Note that all the aforementioned coordinate-related values have the unit of meter. The random channel between every two of the source, the relays, and the destinations is generated based on a 6-tap delay line model, with the i-th tap's amplitude as a circular Gaussian random variable of zero mean and variance

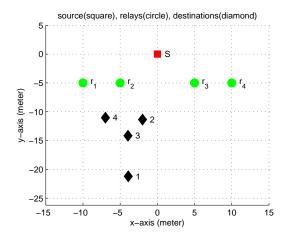


Fig. 3. The OFDMA system considered in numerical experiments.

as  $\sigma_i^2 = \sigma_0^2 e^{-3i}$   $(i=0,1,\cdots,5)$ . Moreover, the average received power attenuation is equal to  $\alpha d^{-3}$  at a distance of d. Note that  $10\log_{10}(\alpha)$ , representing the log-normal shadowing effect, is Gaussian distributed with zero mean and variance equal to 0 dB. This means that the transmit power is attenuated by 30 dB in average at a distance of 10 m.

To illustrate the effectiveness of the optimum RA found for the proposed protocol, we compare it to the optimum RA for a reference protocol. Both protocols are the same except that with the reference protocol, at every subcarrier in the direct mode the source only transmits in the broadcasting slot (and remains idle during the relaying slot). Interestingly, when the destinations use equal weights, the optimum RA for the reference protocol can easily be derived on a per subcarrier basis. For subcarrier k, the optimum destination  $u_k$  is the one with the maximum  $G_u(k)$  where  $G_u(k) =$  $\max\{G_{u,1}(k),G_{s,u}(k)\}$ , i.e., the relay-aided mode is optimum only if  $G_{u_k,1}(k) > G_{s,u_k}(k)$ . The optimum per subcarrier sum power can be calculated according to water filling of the system sum power  $P_{\rm t}$  to K parallel channels with normalized gain  $G_{u_k}(k)$  for the k-th channel. It is important to note that for the reference protocol, the above RA might *not* be optimum when destinations use *unequal* weights. In this case, the RA needs to be optimized jointly over all subcarriers with the same two-step optimization strategy as proposed in Section III, which is not developed here due to space limitation. On the contrary, the RA for the proposed protocol always needs to be jointly optimized over all subcarriers.

### A. Results for a random realization of channels

For clarity of presentation, we first assume K=32 subcarriers are used,  $\sigma^2=-30$  dBW, U=4 destinations exist with equal weights,  $N_s=100$ ,  $\delta=10^{-3}(\mu_{\rm U}-\mu_{\rm L})$ , and  $\epsilon=0.1$ . For the randomly generated destination coordinates shown in Figure 3, we have produced a random realization of all channels (not shown here due to space limitation), over which the optimum RAs for the proposed and reference protocols are evaluated. For every destination u, the  $G_{u,1}(k)$  computed by the proposed RA algorithm in the first step, and  $G_{s,u}(k)$  are shown in Figure 4 for every subcarrier k and

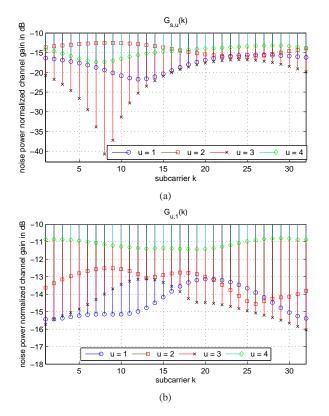


Fig. 4. (a)  $G_{s,u}(k)$  and (b)  $G_{u,1}(k)$  computed by the proposed RA algorithm in the first step, at each subcarrier k for every destination.

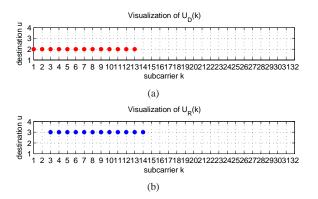


Fig. 5. When  $P_{\rm t}=35$  dBW, (a) the visualization of  $\mathcal{U}_{\rm D}(k)$ ,  $\forall$  k, where a dot marked at the coordinate (k,u) indicates destination  $u\in\mathcal{U}_{\rm D}(k)$ , otherwise  $u\notin\mathcal{U}_{\rm D}(k)$ . (b) the visualization of  $\mathcal{U}_{\rm R}(k)$ ,  $\forall$  k, where a dot marked at the coordinate (k,u) indicates destination  $u\in\mathcal{U}_{\rm R}(k)$ , otherwise  $u\notin\mathcal{U}_{\rm R}(k)$ .

## destination.

First, we set  $P_{\rm t}=35$  dBW which corresponds to the low power regime, since if  $P_{\rm t}$  is uniformly allocated to the subcarriers of the source, the average SNR is  $2.17,\,5.80,\,-2.38,\,{\rm and}\,5.23$  dB at four destinations, respectively. The corresponding  $\mathcal{U}_{\rm D}(k)$  and  $\mathcal{U}_{\rm R}(k)$  are found and visualized in Figure 5 for every subcarrier. The optimum RA and the corresponding WSR have been evaluated for the proposed protocol with the RA algorithm developed in Section III, and for the reference protocol with the aforementioned method. The results are shown in Figure 6. We can see that for the reference protocol, the optimum destination and mode at every subcarrier k are  $u_k=4$  with the maximum  $G_u(k)=\max\{G_{u,1}(k),G_{s,u}(k)\}$ 

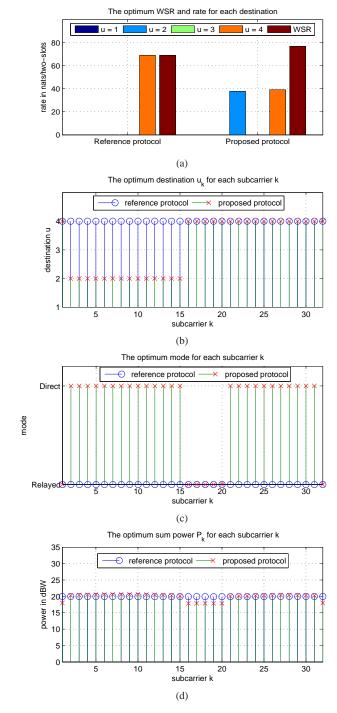
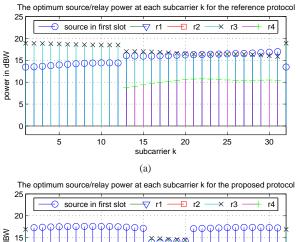


Fig. 6. For the reference and proposed protocols, the optimum RA found in the second step for each subcarrier k when  $P_{\rm t}=35$  dBW.

and the relay-aided mode, respectively. The uniform sum power allocation to all subcarriers is optimum. The reason is that  $G_{u,1}(k)$  for destination 4 varies slightly over all subcarriers. It is very interesting to note that, compared to the reference protocol, assigning a few subcarriers to destination 2 or 4 in the direct mode with a higher per subcarrier sum power, leads to a higher optimum WSR for the proposed protocol, even though at those subcarriers  $G_{s,u}(k)$  for destination 2 or 4 is smaller than  $G_{u,1}(k)$  for destination 4. For both protocols, the optimum  $P_{s_1,u}(k)$  and  $P_{r_i,u}(k) \mid \forall r_i \mid c$  computed by the



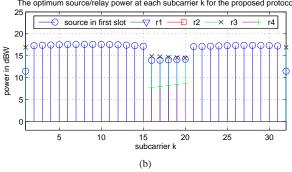


Fig. 7. The optimum source and relay power in the broadcasting slot for the two protocols when  $P_{\rm t}=35$  dBW.

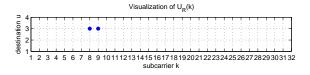


Fig. 8. The visualization of  $\mathcal{U}_{\mathrm{R}}(k)$  for each subcarrier k when  $P_{\mathrm{t}}=60$  dBW, where a dot marked at the coordinate (k,u) means that destination  $u\in\mathcal{U}_{\mathrm{R}}(k)$ , otherwise  $u\notin\mathcal{U}_{\mathrm{R}}(k)$ .

RA algorithm in the first step are shown in Figure 7. It is shown that  $r_3$  and  $r_4$  assist relaying simultaneously at a few subcarriers for both protocols.

Next, we set  $P_{\rm t}=60$  dBW, which satisfies the conditions (36) and (37). For each subcarrier k, the  $\mathcal{U}_D(k)$  is the same as when  $P_{\rm t}=35$  dBW, since it is determined independently of  $P_{\rm t}$  according to the conditions in (19). The  $\mathcal{U}_{\rm R}(k)$  is visualized in Figure 8 for every subcarrier. Now,  $\mathcal{U}_{R}(k)$  contains none of the destinations at most of the subcarriers, since  $P_t$  is so high that the conditions in (20) are not satisfied by any destination at many subcarriers. The optimum RAs and WSRs have been evaluated for the two protocols and shown in Figure 9. It is shown that the optimum RA for the proposed protocol leads to a much higher WSR compared to that for the reference protocol. For the reference protocol, the optimum mode and destination at each subcarrier are the same as when  $P_{\rm t}=35$  dBW. This is because when the destinations have equal weights, the optimum mode and destination for the reference protocol only depends on the  $G_{s,u}(k)$  and  $G_{u,1}(k)$ of the destinations. The uniform sum power allocation to all subcarriers is still optimum, as explained for the case when  $P_{\rm t} = 35$  dBW. For the proposed protocol, the optimum destination and mode for every subcarrier k are the destination

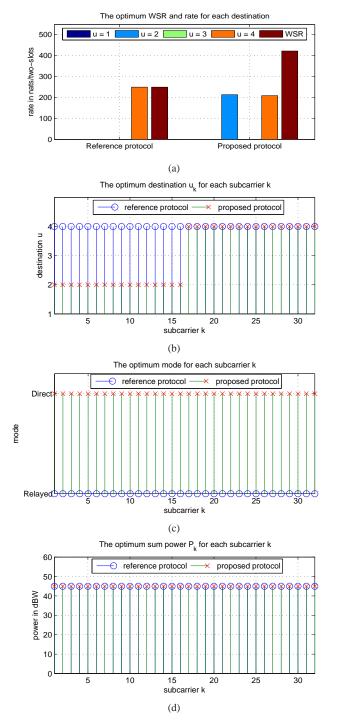


Fig. 9. For the reference and proposed protocols, the optimum RA found in the second step for each subcarrier k when  $P_{\rm t}=60$  dBW and destinations have equal weights.

with the maximum  $G_{s,u}(k)$  and the direct mode, respectively, and the uniform sum power allocation is also optimum. For the reference protocol, the optimum  $P_{s_1,u}(k)$  and  $\{P_{r_i,u}(k)|\forall\ r_i\}$  computed by the RA algorithm in the first step are shown in Figure 10. It is shown that  $r_3$  and  $r_4$  assist relaying simultaneously at a few subcarriers. Note that for the proposed protocol, the optimum sum power at subcarrier k is equally allocated to  $P_{s_1,u}(k)$  and  $P_{s_2,u}(k)$ , which is not shown here.

For the considered channel realization, the above observa-

tions illustrate the effectiveness of the proposed protocol and algorithm to adapt the RA dynamically for WSR maximization. They also support the analysis of Section IV for the high power regime. When  $P_{\rm t}=60$  dBW and the weights for destination 1, 2, 3, and 4 are 0.4, 0.2, 0.2, 0.2, respectively, the optimum RA for the proposed protocol has also been evaluated and shown in Figure 11. We can see that destination 1 with the maximum weight and the direct mode is optimum for each subcarrier. The uniform sum power allocation is optimum. These observations illustrate the effectiveness of the analysis of Section IV for unequal destination weights in the high power regime.

### B. Results for a set of random realizations of channels

For the comprehensive illustration of scenarios close to reality, we have generated 1000 random channel realizations. For each realization, the optimum RAs and the corresponding WSRs for the reference and proposed protocols are evaluated. We assume K=64 subcarriers are used,  $\sigma^2=-30$  dBW, U=8 destinations exist with equal weights,  $N_s=100$ ,  $\delta=10^{-3}(\mu_{\rm U}-\mu_{\rm L})$ , and  $\epsilon=0.1$ . To produce each channel realization, the destination coordinates are first generated in the region  $\{(x,y)|-10\leq x\leq 10, -30\leq y\leq -10\}$  uniformly and independently of each other, then the random channels are produced.

For better clarity, the optimum WSRs for only the first 100 channel realizations, and the average WSR for all channel realizations are shown in Figure 12 for both protocols when  $P_{\rm t}$  is 35 and 60 dBW, respectively. It is found that for every channel realization, the optimum WSR of the proposed protocol is equal to or greater than that for the reference protocol. The reason is that the set of feasible RAs searched for the optimum RA of the reference protocol, is a subset of that of the proposed protocol, since extra constraints are imposed on the RA optimization for the reference protocol, i.e.  $P_{s_2,u}(k)$  must be zero for every subcarrier in the direct mode. Compared to the reference protocol, the optimum RA for the proposed protocol leads to either the same or a slightly greater optimum WSR for each channel realization when  $P_{\rm t}=35$ dBW. However, a much higher optimum WSR results from the optimum RA and the proposed protocol for the majority of channel realizations when  $P_{\rm t}=60$  dBW. As a result,

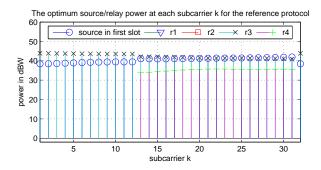


Fig. 10. The optimum source and relay power in the broadcasting slot for the reference protocol when  $P_{\rm t}=60$  dBW. Note that for the proposed protocol, the optimum sum power at each subcarrier k is equally allocated to  $P_{{\rm s}_1,u}(k)$  and  $P_{{\rm s}_2,u}(k)$ , which is not shown here.

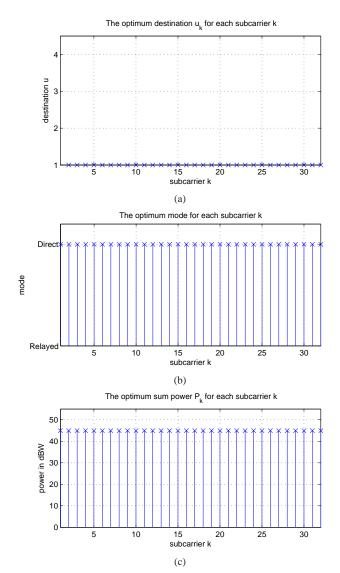


Fig. 11. For the proposed protocol, the optimum RA found in the second step for each subcarrier k when  $P_{\rm t}=60$  dBW and destination 4 has the maximum weight.

the average optimum WSRs for the two protocols are very close when  $P_{\rm t}=35$  dBW, while the proposed protocol and RA result in a much higher average optimum WSR than the reference protocol when  $P_{\rm t}=60$  dBW. As already said, this is due to the better exploitation of channel uses by the direct mode of the proposed protocol in the high power regime.

For both protocols, the rate cumulative distribution functions (CDF) and average rates of user 1 are shown in Figure 13 when  $P_{\rm t}$  is 35 and 60 dBW, respectively. Note that for the same protocol and  $P_{\rm t}$ , all other users have similar rate CDFs and average rates as user 1, since all users' coordinates are independently and identically distributed in the same region. It can be seen that both protocols lead to almost the same rate CDFs for user 1 when  $P_{\rm t}=35$  dBW. Nevertheless, using the proposed protocol makes it more likely to transmit with high rate to user 1 when  $P_{\rm t}=60$  dBW. This means that for a fixed outage probability, the proposed protocol leads to a higher outage rate for user 1 than the reference protocol. As a result,

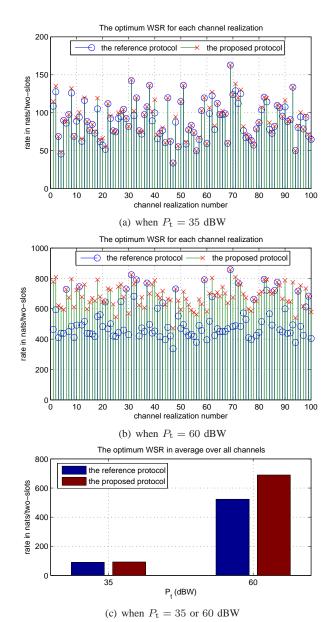


Fig. 12. The optimum WSRs for the first 100 channel realizations for the reference and proposed protocols, and the average WSR for all channel realizations.

the average rates of user 1 for both protocols are almost the same when  $P_{\rm t}=35$  dBW, while the proposed protocol leads to a much higher average rate of user 1 than the reference protocol when  $P_{\rm t}=60$  dBW. These observations illustrate the effectiveness of the proposed protocol and algorithm for the random channel realizations.

### VI. CONCLUSION

We have considered the WSR maximized RA problem constrained by a system sum power in an OFDMA downlink transmission system assisted by multiple DF relays. In particular, multiple relays may cooperate with the source for every relay-aided transmission. We have proposed a two-step algorithm to optimize the RA. Furthermore, we have shown that the optimum RA in the second step can be easily derived when the system sum power is very high. The effectiveness

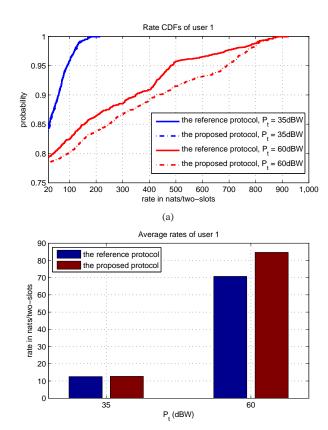


Fig. 13. User 1's rate CDFs and average rates for the reference and proposed protocols

(b)

of the proposed algorithm has been illustrated by numerical experiments.

### APPENDIX A

Maximization of  $R_{u,k,1}$  in the relay-aided mode

When  $G_{s,u}(k) \geq G_{s,r_k(N)}(k)$ ,  $\gamma_r(k) \leq \gamma_{u,c}(k)$  always holds, thus  $R_{u,k,1} = \ln(1 + P_{s,u}(k) \cdot \min_{r_i \in \mathcal{R}_k(u)} G_{s,r_i}(k))$ , which means that  $R_{u,k,1}$  is equal to the maximum rate allowed for reliable transmission from the source to all relays in  $\mathcal{R}_k(u)$ . In this case, the maximum  $R_{u,k,1}$  is  $R_{u,k,1}(P) = \ln(1 + G_{s,r_k(N)}(k)P)$ . To achieve it,  $\mathcal{R}_k(u)$  and  $P_{s,u}(k)$  should be equal to  $\{r_k(N)\}$  and P, respectively.

Now we consider the case when  $G_{s,r_k(N)}(k) > G_{s,u}(k)$ . The maximization of  $R_{u,k,1}$  is equivalent to the maximization of  $\gamma_u(k)$ , as the minimum of  $\gamma_{u,c}(k)$  and  $\gamma_r(k)$ . Note that  $\gamma_r(k)$  depends on  $P_{s,u}(k)$  and  $\mathcal{R}_k(u)$ , while besides them  $\gamma_{u,c}(k)$  depends on  $\{P_{r_i,u}(k)|r_i\in\mathcal{R}_k(u)\}$  as well. To maximize  $\gamma_u(k)$  given  $P, \gamma_{u,c}(k)$  can be first maximized with  $P_{s,u}(k)$  and  $\mathcal{R}_k(u)$  fixed, by finding the optimum allocation of the sum relay power  $P-P_{s,u}(k)$  to  $\{P_{r_i,u}(k)|r_i\in\mathcal{R}_k(u)\}$ . Denoting such maximized  $\gamma_{u,c}(k)$  by  $\gamma'_{u,c}(k)$ , the optimum  $\{P_{s,u}(k),\mathcal{R}_k(u)\}$  that maximizes  $\min\{\gamma'_{u,c}(k),\gamma_r(k)\}$  is then found. Finally, that  $\{P_{s,u}(k),\mathcal{R}_k(u)\}$ , as well as the associated optimum allocation of  $P-P_{s,u}(k)$  to  $\{P_{r_i,u}(k)|r_i\in\mathcal{R}_k(u)\}$ , constitute the optimum solution to maximizing  $\gamma_u(k)$  given P.

To facilitate derivation, we assume the relays in  $\mathcal{R}_k(u)$  are sorted in the increasing order of  $G_{s,r_i}(k)$ , and the first relay in

 $\mathcal{R}_k(u)$  is  $\mathbf{r}_k(b)$ , namely the b-th relay in  $\Psi_k$ . This means that  $\mathcal{R}_k(u)$  is a subset of  $\Psi_k(b)$ , and  $G_{\mathcal{R}_k(u),u}(k) \leq G_{\Psi_k(b),u}(k)$ . It is important to note that  $G_{s,\mathbf{r}_k(i)}(k)$  is an increasing function of i, while  $G_{\Psi_k(i),u}(k)$  is a decreasing function of i.

Now,  $\gamma_{\rm r}(k)=P_{{\rm s},u}(k)G_{s,{\rm r}_k(b)}(k)$ , and according to the Schwartz Inequality,

$$\gamma_{u,c}(k) \le P_{s,u}(k)G_{s,u}(k) + (P - P_{s,u}(k))G_{\mathcal{R}_k(u),u}(k),$$
 (43)

where the equality holds when

$$\forall \mathbf{r}_i \in \mathcal{R}_k(u), \ P_{\mathbf{r}_i,u}(k) = \frac{(P - P_{\mathbf{s},u}(k))G_{\mathbf{r}_i,u}(k)}{\sum_{\mathbf{r}_i \in \mathcal{R}_k(u)} G_{\mathbf{r}_i,u}(k)}. \tag{44}$$

Obviously, the right hand side of (43) is equal to  $\gamma'_{u,c}(k)$ . As  $P_{s,u}(k)$  increases from 0 to P,  $(P_{s,u}(k), \gamma_r(k))$  moves along the line from the origin to A, whereas  $(P_{s,u}(k), \gamma'_{u,c}(k))$  along the line from B to C in the coordinate space shown in Figure 14, where the coordinates of A, B, and C are given in Table II. Note that C is fixed, while both A and B depend on  $\mathcal{R}_k(u)$ . In particular, with  $\mathcal{R}_k(u)$  and P fixed, the maximum of  $\min\{\gamma'_{u,c}(k),\gamma_r(k)\}$  with respect to  $P_{s,u}(k)$ , denoted by  $\gamma'_u(k)$ , is achieved at A, C, and D for a given  $\mathcal{R}_k(u)$  satisfying respectively

- 1)  $b < x_{u,k}$ , where  $x_{u,k}$  is the smallest i satisfying  $G_{s,r_b(i)}(k) > G_{s,u}(k)$ .
- 2)  $b \ge x_{u,k}$  and  $G_{\mathcal{R}_k(u),u}(k) \le G_{s,u}(k)$ ,
- 3)  $b \ge x_{u,k}$  and  $G_{\mathcal{R}_k(u),u}(k) > G_{s,u}(k)$ .

We can see that only in case 3,  $\gamma_u'(k)$  is greater than  $G_{s,u}(k)P$ . Among all  $\mathcal{R}_k(u)$ 's having the same first relay as  $\mathbf{r}_k(b)$  and corresponding to case 3, the  $\mathcal{R}_k(u)$  with the maximum  $G_{\mathcal{R}_k(u),u}(k)$  corresponds to the greatest height of D, which suggests that  $\Psi_k(b)$  is the one achieving the maximum  $\gamma_u'(k)$  in that set. The corresponding  $\gamma_u'(k)$  and  $P_{s,u}(k)$  are computed respectively as

$$\gamma_u'(k) = \frac{G_{s, r_k(b)}(k)G_{\Psi_k(b), u}(k)P}{G_{\Psi_k(b), u}(k) + G_{s, r_k(b)}(k) - G_{s, u}(k)}$$
(45)

and

$$P_{s,u}(k) = \frac{G_{\Psi_k(b),u}(k)}{G_{\Psi_k(b),u}(k) + G_{s,r_k(b)}(k) - G_{s,u}(k)} P.$$
 (46)

Based on the above analysis, we can decide what the optimum  $\mathcal{R}_k(u)$ ,  $P_{s,u}(k)$ , and  $P_{r_i,u}(k)$  should be. When  $G_{\Psi_k(x_{u,k}),u}(k) \leq G_{s,u}(k)$ , every possible  $\mathcal{R}_k(u)$  corresponds to either case 1 or 2, which means that the  $\gamma_u'(k)$  maximized over  $\mathcal{R}_k(u)$  and  $P_{s,u}(k)$  is equal to  $G_{s,u}(k)P$ . In this case, the maximum  $R_{u,k,1}$  is  $R_{u,k,1}(P) = \ln(1 + G_{s,u}(k)P)$ , and to achieve it  $P_{s,u}(k)$  should be equal to P.

When  $G_{\Psi_k(x_{u,k}),u}(k) > G_{\mathrm{s},u}(k)$ , every possible  $\mathcal{R}_k(u)$  corresponds to one of cases 1, 2, and 3. Obviously, the optimum  $\mathcal{R}_k(u)$  should correspond to case 3. To have  $\mathcal{R}_k(u)$  corresponding to case 3, b must be between  $x_{u,k}$  and  $y_{u,k}$ , where  $y_{u,k}$  is the greatest i satisfying  $G_{\Psi_k(i),u}(k) > G_{\mathrm{s},u}(k)$ . As pointed out before,  $\Psi_k(b)$  is the  $\mathcal{R}_k(u)$  achieving the maximum  $\gamma_u'(k)$  among all  $\mathcal{R}_k(u)$ 's having the same first relay as  $\mathbf{r}_k(b)$ . Therefore, the optimum  $\mathcal{R}_k(u)$  should be  $\Psi_k(z_{u,k})$ , where  $z_{u,k}$  is the b between  $x_{u,k}$  and  $y_{u,k}$  and maximizing  $\gamma_u'(k)$ , namely the maximizer of the right-hand side of (17). In this case, the maximum  $R_{u,k,1}$  and the optimum RA can

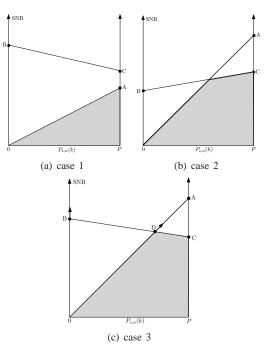


Fig. 14. Illustrations of  $\gamma'_{u,c}(k)$ ,  $\gamma_{\rm r}(k)$ , and  $\gamma'_u(k)$  versus  $P_{\rm s,u}(k)$ .  $\gamma'_u(k)$  is highlighted by painting the area it bounds in light grey.

TABLE II COORDINATES OF A, B, AND C.

	A	В	С
$P_{s,u}(k)$	P	0	P
SNR	$G_{s,\mathbf{r}_k(b)}(k)P$	$G_{\mathcal{R}_k(u),u}(k)P$	$G_{s,u}(k)P$

be evaluated with the formulas for the third case shown in Section III.B.

### ACKNOWLEDGEMENT

The authors would like to thank Prof. Josep Vidal for coordinating the review, as well as the anonymous reviewers for their valuable comments and suggestions to improve the quality of this paper.

### REFERENCES

- R. Pabst, B. Walke, D. Schultz et al., "Relay-based deployment concepts for wireless and mobile broadband radio," *IEEE Communications Magazine*, vol. 42, no. 9, pp. 80–89, sept. 2004.
- [2] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: efficient protocols and outage behavior," *IEEE Transactions on Information Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.
- [3] T. Wang and L. Vandendorpe, "Iterative resource allocation for maximizing weighted sum min-rate in downlink cellular OFDMA systems," *IEEE Transactions on Signal Processing*, vol. 59, no. 1, pp. 223–234, Jan. 2011.
- [4] B. Gui and L. J. Climini-Jr, "Bit loading algorithms for cooperative OFDM systems," Eurasip Journal on Wireless Communications and Networking, 2008.
- [5] Y. Li, W. Wang, J. Kong et al., "Power allocation and subcarrier pairing in OFDM-based relaying networks," in *IEEE International Conference* on Communications, 2008, pp. 2602–2606.
- [6] L. Vandendorpe, R. Duran, J. Louveaux et al., "Power allocation for OFDM transmission with DF relaying," in *IEEE International Confer*ence on Communications, 2008, pp. 3795 –3800.
- [7] L. Vandendorpe, J. Louveaux, O. Oguz et al., "Improved OFDM transmission with DF relaying and power allocation for a sum power constraint," in ISWPC, 2008, pp. 665–669.

[8] ——, "Power allocation for improved DF relayed OFDM transmission: the individual power constraint case," in *IEEE International Conference* on Communications, 2009, pp. 1–6.

- [9] —, "Rate-optimized power allocation for DF-relayed OFDM transmission under sum and individual power constraints," *Eurasip Journal on Wireless Communications and Networking*, vol. 2009.
- [10] ——, "Rate-optimized power allocation for OFDM transmission with multiple DF/regenerative relays and an improved protocol," in *IEEE Wireless Communications and Networking Conference*, 2009, pp. 1–6.
- [11] —, "Rate-optimized power allocation for OFDM transmission with multiple DF relays and individual power constraints," in *European Signal Processing Conference*, 2009, pp. 145–149.
- [12] T. Wang and L. Vandendorpe, "Sum rate maximized resource allocation in multiple DF relays aided OFDM transmission,," to appear in IEEE Journal on Selected Areas in Communications, Sept. 2011.
- [13] W. Nam, W. Chang, S.-Y. Chung et al., "Transmit optimization for relay-based cellular OFDMA systems," in *IEEE International Conference on Communications*, 2007, pp. 5714 –5719.
- [14] M. Kaneko and P. Popovski, "Radio resource allocation algorithm for relay-aided cellular OFDMA system," in *IEEE International Conference* on Communications, 2007, pp. 4831 –4836.
- [15] R. Kwak and J. Cioffi, "Resource-allocation for OFDMA multi-hop relaying downlink systems," in *IEEE Global Telecommunications Con*ference, 2007, pp. 3225 –3229.
- [16] Y. Cui, V. K. N. Lau, and R. Wang, "Distributive subband allocation, power and rate control for relay-assisted OFDMA cellular system with imperfect system state knowledge," *IEEE Transactions on Wireless Communications*, vol. 8, no. 10, pp. 5096–5102, Oct. 2009.
- [17] M. Salem, A. Adinoyi, M. Rahman et al., "Fairness-aware radio resource management in downlink OFDMA cellular relay networks," *IEEE Trans*actions on Wireless Communications, vol. 9, no. 5, pp. 1628 –1639, May 2010
- [18] T. C.-Y. Ng and W. Yu, "Joint optimization of relay strategies and resource allocations in cooperative cellular networks," *IEEE Journal on Selected Areas in Communications*, vol. 25, no. 2, pp. 328 –339, Feb. 2007
- [19] S. Kadloor and R. Adve, "Relay selection and power allocation in cooperative cellular networks," *IEEE Transactions on Wireless Communications*, vol. 9, no. 5, pp. 1676–1685, May 2010.
- [20] K. Vardhe, D. Reynolds, and B. D. Woerner, "Joint power allocation and relay selection for multiuer cooperative communications," *IEEE Transactions on Wireless Communications*, vol. 9, no. 4, pp. 1255–1260, Apr. 2010.
- [21] M. Morelli, C. Kuo, and M. Pun, "Synchronization techniques for orthogonal frequency division multiple access (ofdma): a tutorial review," Proceeding of IEEE, vol. 95, no. 7, pp. 1394–1427, July 2007.
- [22] D. Tse and P. Viswanath, Fundamentals of Wireless Communication. Cambridge University Press, 2005.
- [23] S. Boyd and L. Vandenberghe, Convex optimization. Cambridge University Press, 2004.
- [24] D. P. Bertsekas, Nonlinear programming, 2nd edition. Athena Scientific, 2003.
- [25] W. Yu and R. Lui, "Dual methods for nonconvex spectrum optimization of multicarrier systems," *IEEE Transactions on Communications*, vol. 54, no. 7, pp. 1310–1322, July 2006.



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