Partial Interference Alignment for *K*-user MIMO Interference Channels

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Abstract

In this paper, we consider a Partial Interference Alignment and Interference Detection (PIAID) design for K-user quasi-static MIMO interference channels with discrete constellation inputs. Each transmitter has M antennas and transmits L independent data streams to the desired receiver with N receive antennas. We focus on the case where not all K-1 interfering transmitters can be aligned at every receiver. As a result, there will be residual interference at each receiver that cannot be aligned. Each receiver detects and cancels the residual interference based on the constellation map. However, there is a window of unfavorable interference profile at the receiver for Interference Detection (ID). In this paper, we propose a low complexity Partial Interference Alignment scheme in which we dynamically select the user set for IA so as to create a favorable interference profile for ID at each receiver. We first derive the average symbol error rate (SER) by taking into account of the non-Guassian residual interference due to discrete constellation. Using graph theory, we then devise a low complexity user set selection algorithm for the PIAID scheme, which minimizes the asymptotically tight bound for the average end-to-end SER performance. Moreover, we substantially simplify interference detection at the receiver using Semi-Definite Relaxation (SDR) techniques. It is shown that the SER performance of the proposed PIAID scheme has significant gain compared with various conventional baseline solutions.

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I. INTRODUCTION

Interference has been a very difficult problem in wireless communications.. For instance, the capacity region of two-user Gaussian interference channels has been an open problem for over 30 years [1]. Recently, there are some progress made in understanding the interference [2], [3] and extensive studies have been done regarding the Interference Alignment (IA) [3]. For instance, IA is a signal processing approach that attempts to simultaneously align the interference on a lower dimension subspace at each receiver so that the desired signals can be transmitted on the interference-free dimensions. In [4], the authors show that IA (using infinite dimension symbol extension in time or frequency selective fading channels) is optimal in Degrees-of-Freedom (DoF) sense. In [5] the authors propose a variation of the IA scheme, the ergodic alignment scheme, for K-user time or frequencyselective interference channels. In practice, since it is not possible to realize infinite dimension symbol extensions, there are a number of works [6]-[8] and the reference therein that consider IA in the spatial domain, without symbol extensions, in the K-user quasi-static MIMO interference channels. Furthermore, the authors in [9] investigate the asymptotic performance of these different IA solutions. However, for quasi-static (or constant) channels, conventional IA might be infeasible depending on the system parameters. For example, it is conjectured in [10] that conventional IA on quasi-static MIMO (M transmit and N receive antennas) interference channels is not feasible to achieve a per user DoF greater than $\frac{M+N}{K+1}$. As a result, we cannot rely on IA to eliminate all interference in quasistatic MIMO interference channels especially when K is large. There are other works that consider IA over signal scale in [11]–[14]. In [11] and [12], signal scale alignment schemes are introduced for the many-to-one interference channel and fully connected interference networks, respectively. In [13], [14], the authors propose a lattice alignment scheme for K-user MIMO interference channels. However, the scheme requires infinite SNR and serves only as a proof of concept. It is not clear whether this approach can be applied at finite SNR.

Due to the fact that not all the interferers can be aligned at each receiver, there will be residual interference at the receiver. In this paper, we assume the receiver has interference detection (ID) capability. Specifically, the receiver detects and cancels the residual interference based on the constellation map derived from the discrete constellation inputs. However, there is a window of *unfavorable interference profile* for ID at the receiver. For instance, ID at the receiver is more effective when the interference is stronger than the desired signal [14], [15]. In [15], [16], the authors propose an ID scheme for quasi-static interference channels based on lattices. However the proposed scheme can only work under idealized assumptions such as the symmetric SISO interference channels (where all cross links have the same fading coefficients) or a specialized class of 3-user SISO interference channels (where the product of fading coefficients is assumed to be rational). In fact, IA and ID are complementary approaches to deal with interference in quasi-static MIMO interference channels. The IA approach¹ can be used to first eliminate some interference and the ID approach can be used to deal with the residual interference at each receiver. While we can potentially benefit from the concepts of IA and ID approaches in dealing with interference, there are still some key technical challenges to be addressed.

- Feasibility Issue of IA and Path Loss Effects: Sometimes brute-force IA in quasi-static MIMO interference channels (without symbol extensions) might be infeasible depending on system parameters. Furthermore, existing literature has completely ignored the effects of path loss, which may also be exploited when dealing with interference. For instance, nodes with large path loss may not need to be interference-aligned and hence, it is important to jointly consider the feasibility issue and the path loss effects.
- Coupling between IA and ID: While both IA and ID are effective means to mitigate interference, their designs are coupled together in an intricate manner. For instance, the performance of ID at the receiver depends heavily on the interference profile. Hence, IA can potentially contribute to creating a more desirable interference profile for ID by careful selecting a subset of users for IA. However, the problem of user selection in IA to optimize the symbol error rate (SER) performance is very complicated. First, the optimization space is combinatorial and brute force exhaustive search is not viable. Second, even if we can afford the search complexity, obtaining the search metric is highly non-trivial because it is also very challenging to analyze the closed form average SER under non-Gaussian interferences.

In this paper, we propose a low complexity *Partial Interference Alignment* and *Interference Detection* (PIAID) scheme for *K*-user quasi-static MIMO interference channels with discrete constellation inputs. We consider QPSK constellations² at the inputs of the *K* transmitters and each transmit-receive pair may have different path losses. The proposed PIAID scheme dynamically selects the *interference alignment set* at each receiver based on the path loss information to create a *favorable interference profile* at each receiver for ID processing as illustrated in Fig. 2. Interference alignment is applied only to the members of the alignment set. We derive the average SER by taking into account the non-Guassian residual interference due to discrete constellation. Using graph theory, we transform the combinatorial problem into a linear programming (LP) problem and obtain a low complexity user set selection algorithm for the PIAID scheme, which minimizes the asymptotically tight bound for the average end-to-end SER performance. Furthermore, using Semi-Definite Relaxation (SDR) technique

¹ In the remaining papers, the mentioned IA approach specifically refers to the signal space alignment approach.

² QPSK constellations are easier to analyze. However, the proposed framework can be extended to QAM constellations easily.

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[17]–[20], we propose a low complexity ID algorithm at the receiver. The SER performance of the proposed PIAID scheme is shown to have significant gain compared with various conventional baseline solutions.

Outline: The rest of this paper is organized as follows. In Section II, we outline the system model and the proposed PIAID scheme. In Section III, we discuss the optimization of the IA user selection. In Section IV, we derive the average SER by taking into account the non-Guassian residual interference and propose a low complexity ID algorithm at the receiver that uses SDR technique. The numerical simulation results are illustrated in Section V. Finally we conclude with a brief summary in Section VI.

II. SYSTEM MODEL

A. K-user Quasi-Static MIMO Interference Channels

We consider K-user quasi-static MIMO Gaussian interference channels as illustrated in Fig. 1. Specifically, each M-antenna transmitter, tries to communicate to its corresponding N-antenna receiver. The channel output at the k-th receive node is described as follows:

$$\mathbf{y}_{k} = \sum_{i \in \mathcal{K}} \sqrt{P_{i} L_{ki}} \mathbf{H}_{ki} \mathbf{\overline{x}}_{i} + \mathbf{z}_{k}, \tag{1}$$

where $\mathcal{K} = \{1, \dots, K\}$, $\mathbf{H}_{ki} \in \mathbb{C}^{N \times M}$ is the MIMO complex fading coefficients from the *i*-th transmitter to the *k*-th receiver, L_{ki} is the long term path gain from the *i*-th transmitter to the *k*-th receiver, and P_i is the average transmit power of the *i*-th transmitter. In (1), $\overline{\mathbf{x}}_i \in \mathbb{C}^{M \times 1}$ is the complex signal vector transmitted by transmit node *i*, and $\mathbf{z}_k \in \mathbb{C}^{N \times 1}$ is the circularly symmetric Additive White Gaussian Noise (AWGN) vector at receive node *k*. We assume all noise terms are i.i.d zero mean complex Gaussian with $\mathbb{E}[\mathbf{z}_k(\mathbf{z}_k)^H] = 2\mathbf{I}_N$. Furthermore, the assumption on channel model is given as follows:

Assumption 1 (Assumption on Channel Model [21]): We assume that the long term path gain is given by $L_{ki} = \omega d_{ki}^{-\gamma}$, where d_{ki} is the distance between transmit node *i* and receive node *k*, ω is the Log-normal shadow fading with a standard deviation σ_{ω} , and γ is the path loss exponent. Furthermore, we assume that the entries of \mathbf{H}_{ki} for all *k*, *i* are i.i.d. complex Gaussian random variables given by $[\mathbf{H}_{ki}]_{(n,m)} \sim \mathcal{CN}(0,1)$ for all *k*, *i*, *n*, *m*, where $[\mathbf{H}_{ki}]_{(n,m)}$ denotes the $(n,m)^{\text{th}}$ element of \mathbf{H}_{ki} .

In this paper, we assume that the *i*-th transmit node transmits $D \leq \min(M, N)$ independent QPSK data streams $\{x_i^1, \dots, x_i^D\}$ to the *i*-th receive node where $x_i^d \in S = \{\frac{\sqrt{2}}{2}(1+j), \frac{\sqrt{2}}{2}(1-j), \frac{\sqrt{2}}{2}(-1+j), \frac{\sqrt{2}}{2}(-1-j)\}, \forall d \in \{1, \dots, D\}$. Let \mathbf{v}_i^d ($||\mathbf{v}_i^d|| = 1$, where $|| \cdot ||$ denotes the Frobenius norm.) denote the precoder for the x_i^d symbol. Hence, the transmitted vector at the *i*-th transmitter is given by $\overline{\mathbf{x}}_i = \sum_d \mathbf{v}_i^d x_i^d$.

B. Partial Interference Alignment and Interference Detection (PIAID) Scheme

The proposed PIAID scheme consists of two major components, namely the *Partial Interference Alignment* (PIA) at the transmitters and the *Interference Detection* (ID) at the receivers as illustrated in Fig. 2.

1) Overview of PIA: PIA is motivated by the feasibility issue³ of the MIMO interference alignment without symbol extension [6], [10]. For instance, it is conjectured in [10] that only when $K \leq \frac{M+N}{D} - 1$, the K - 1 interfering transmitters can be aligned at every receiver node. As a result, not all the K - 1 interfering transmitters can be aligned at every receiver node for large K. Furthermore, existing IA schemes do not consider or exploit the effects of different path losses between transmit and receive pairs. When the path loss effects are taken into consideration, not all transmitters will contribute the same effect at the receiver and hence, there should be different priority in determining which nodes should be aligned given the feasibility constraint. Fig. 3 illustrates an example of 4-user interference channels where M = N = 2 and D = 1. Using the feasibility condition of MIMO IA in [10], only two transmitters can be aligned at each receiver. Combining with the path costs (which depend on the path gains and transmit power), it is obvious that transmitters 2 and 3 should be aligned at receiver 1 as indicated in the Fig. 3.

Motivated by the above example, the proposed PIAID scheme dynamically selects α transmitters to be aligned at each receiver node based on the path costs. The index of the aligned transmitters at each receiver is given by a PIA set with cardinality α . Specifically, the PIA set is defined below.

Definition 1 (PIA Set): A PIA set is defined as $\mathcal{A} = \{\mathcal{A}_k, \forall k\}$, where $\mathcal{A}_k = \{k_1, k_2, \cdots, k_\alpha \neq k\}$ denotes the index of aligned transmitters at the receiver k for some constant α .

Only the transmit nodes that belong to A_k will have to align their transmit signals by choosing the precoders and equalizers according to the traditional IA requirement⁴ [10], [22]

$$(\mathbf{U}_{k})^{\dagger} \mathbf{H}_{ki} \mathbf{V}_{i} = \mathbf{0}, \forall i \in \mathcal{A}_{k}$$
$$(\mathbf{U}_{k})^{\dagger} \mathbf{H}_{kk} \mathbf{V}_{k} = \operatorname{diag}(\lambda_{1}, \cdots, \lambda_{D}), \forall k$$
(2)

where $(\cdot)^{\dagger}$ denotes the Hermitian transpose, diag $(\lambda_1, \dots, \lambda_D)$ denotes a diagonal matrix with diagonal entries $\lambda_1, \dots, \lambda_D$, $\mathbf{U}_k = [\mathbf{u}_k^1, \dots, \mathbf{u}_k^D]$ are the $N \times D$ decorrelators at receiver k with $||\mathbf{u}_k^l|| = 1, \forall l$, and $\mathbf{V}_i = [\mathbf{v}_i^1, \dots, \mathbf{v}_i^D]$ are the $M \times D$ precoders at transmitter i with $||\mathbf{v}_i^l|| = 1, \forall l$. Based on the

³ For constant MIMO interference channels, it is not always possible to completely align all the K - 1 interference at each receiver.

⁴ Note that the requirement is equivalent to that used in [10], [22], i.e., choosing the precoders and equalizers satisfying rank $((\mathbf{U}_k)^{\dagger}\mathbf{H}_{kk}\mathbf{V}_k) = D, \forall k.$

conjecture in [10], a sufficient condition⁵ for a feasible PIA set A is given by [10]:

$$\alpha(\mathcal{A}) \le \min\left(\frac{M+N}{D} - 2, K - 1\right) \text{ and } \sum_{k} \mathbf{1}(i \in \mathcal{A}_k) = \alpha(\mathcal{A}), \forall i$$
 (3)

where $\mathbf{1}(\cdot)$ is the indicator function, $\alpha(\mathcal{A}) = |\mathcal{A}_k|$ is the cardinality of \mathcal{A}_k (i.e., the number of aligned interferers at each receiver). The requirement as per (3) means that each transmitter should be selected by $\alpha(\mathcal{A})$ receivers. The IA set \mathcal{A} is a design parameter in the proposed PIA scheme, and how to choose the IA set is presented in Section III.

Remark 1 (Feasibility Condition): Note that the feasibility conditions for (2) are still open in the literature. Since the feasibility condition and computation of $\{\mathbf{U}_k, \mathbf{V}_k, \forall k\}$ are not the focus of the paper, we have adopted the results in [10] to derive a sufficient condition (3). While this condition restricts the choice of the feasible set \mathcal{A} , it introduces graph structure for the optimization w.r.t. \mathcal{A} . Furthermore, as shown in Fig. 6, the proposed PIA algorithm with condition (3) has similar performance as the solution obtained by brute-force exhaustive search (without (3)).

2) Overview of ID: In this paper, we focus on the case where not all the K - 1 interference can be aligned at each receivers. As a result, there will be residual interference at every receiver. The ID processing at the receiver first estimates the aggregate interference signal by using the constellation maps derived from the QPSK inputs. The desired signal is detected after subtracting the estimated aggregate interference. For instance, the normalized received signal at the k-th receiver is given by:

$$\mathbf{y}_{k} = \sqrt{P_{k}L_{kk}}\mathbf{H}_{kk}\mathbf{v}_{k}^{l}x_{k}^{l} + \underbrace{\sum_{d\neq l}\sqrt{P_{k}L_{kk}}\mathbf{H}_{kk}\mathbf{v}_{k}^{d}x_{k}^{d}}_{\text{inter-stream interference}} + \underbrace{\sum_{i\notin\mathcal{A}_{k};d}\sqrt{P_{i}L_{ki}}\mathbf{H}_{ki}\mathbf{v}_{i}^{d}x_{i}^{d}}_{\text{non-aligned interferens}} + \underbrace{\sum_{i\in\mathcal{A}_{k};d}\sqrt{P_{i}L_{ki}}\mathbf{H}_{ki}\mathbf{v}_{i}^{d}x_{i}^{d}}_{\text{aligned interferens}} + \mathbf{z}_{k}.$$
(4)

We adopt linear processing at the receiver and the detection process for l-th data stream at the k-th receiver is divided into two stages, namely the *aggregate interference detection stage* (stage I) and the *desired signal detection stage* (stage II). The two stages are elaborated below:

• Stage I Processing: Using the *l*-th column of \mathbf{U}_k in (2), \mathbf{u}_k^l , as the decorrelator, the post-processed signal of the *l*-th stream is given by:

$$y_{k}^{l} = (\mathbf{u}_{k}^{l})^{\dagger} \mathbf{y}_{k} = \sqrt{P_{k} L_{kk}} H_{kk}^{ll} x_{k}^{l} + \underbrace{\sum_{i \notin \mathcal{A}_{k}; d} \sqrt{P_{i} L_{ki}} H_{ki}^{ld} x_{i}^{d}}_{\text{non-aligned interferers}} + (\mathbf{u}_{k}^{l})^{\dagger} \mathbf{z}_{k}, \tag{5}$$

⁵ From [10], we know that the total number of equations for the IA requirement $\{(\mathbf{U}_k)^{\dagger}\mathbf{H}_{ki}\mathbf{V}_i = \mathbf{0}, \forall i \in \mathcal{A}_k, \forall k\}$ is $N_e = \sum_k |\mathcal{A}_k|D^2 = \alpha KD^2$, and the total number of variables is $N_v = \sum_k D(M + N - 2D) = KD(M + N - 2D)$. When each transmitter is selected by α receivers, the feasibility condition is simply given by $N_e \leq N_v$ [10], i.e., $\alpha(\mathcal{A}) \leq \min\left(\frac{M+N}{D} - 2, K - 1\right)$. where $H_{ki}^{ld} = (\mathbf{u}_k^l)^{\dagger} \mathbf{H}_{ki} \mathbf{v}_i^d$ is the equivalent channel gain for the *d*-th data stream of transmitter *i* at receiver *k*. Note that the inter-stream interference and the interference contributed by users in the IA set \mathcal{A}_k are completely eliminated due to the PIA requirement in (2). Let $\mathcal{Q}_k = \{i : P_i L_{ki} \ge P_k L_{kk}, \forall i \notin \mathcal{A}_k, i \neq k\} \subseteq \{1, \dots, K\}$ denotes the set of *strong residual interference*. Since ID at the receiver is more effective when the interference is stronger than the desired signal [14], [15], the first stage processing estimates the *aggregate strong interference* $I_k^l = \sum_{i \in \mathcal{Q}_k; d} \sqrt{P_i L_{ki}} H_{ki}^{ld} x_i^d$ using the following nearest neighbor detection rule.

Algorithm 1 (Stage I Interference Detection Algorithm): Based on the decorrelator output, y_k^l , the detected aggregate strong interference \hat{I}_k^l is given by:

$$(\hat{I}_k^l)^* = \underset{c \in \mathcal{I}_k^l}{\operatorname{arg\,min}} |y_k^l - c|,\tag{6}$$

where $\mathcal{I}_k^l = \left\{ \sum_{i \in \mathcal{Q}_k; d} \sqrt{P_i L_{ki}} H_{ki}^{ld} s_i^d : s_i^d \in \mathcal{S} \right\}$ is the set of possible values the strong interference from \mathcal{Q}_k can take.

Note that when $Q_k = \emptyset$, there will be no stage I decoding for the desired data stream x_k^l . In this case, the proposed PIAID scheme reduces to the conventional receiver with one-stage decoding.

• Stage II Processing: The estimated aggregate strong interference $(\hat{I}_k^l)^*$ is first subtracted from the decorrelator output y_k^l as illustrated:

$$\widetilde{y}_{k}^{l} = y_{k}^{l} - (\widehat{I}_{k}^{l})^{*} = \sqrt{P_{k}L_{kk}}H_{kk}^{ll}x_{k}^{l} + I_{k}^{l} - (\widehat{I}_{k}^{l})^{*} + \sum_{i \in \mathcal{O}_{k}; d} \sqrt{P_{i}L_{ki}}H_{ki}^{ld}x_{i}^{d} + (\mathbf{u}_{k}^{l})^{\dagger}\mathbf{z}_{k}, \quad (7)$$
residual interference

where $\mathcal{O}_k = \{i : P_i L_{ki} < P_k L_{kk}, \forall i \notin \mathcal{A}_k, i \neq k\} \subseteq \{1, \dots, K\}$ denotes the set of *weak* residual interference, and obviously we have $\mathcal{O}_k \bigcup \mathcal{Q}_k \bigcup \mathcal{A}_k \bigcup \{k\} = \mathcal{K}$. In turn, the desired signal for receiver k is detected based on \tilde{y}_k^l using the following algorithm.

Algorithm 2 (Stage II Signal Detection Algorithm): The *l*-th data symbol at the *k*-th receiver x_k^l is detected based on \tilde{y}_k^l according to the minimum-distance rule given by:

$$\Re\{(\hat{x}_{k}^{l})^{*}\} = \begin{cases} \frac{\sqrt{2}}{2} & \text{if } \Re\left\{\frac{\tilde{y}_{k}^{l}}{\sqrt{P_{k}L_{kk}}H_{kk}^{ll}}\right\} \ge 0\\ -\frac{\sqrt{2}}{2} & \text{else} \end{cases} \qquad \Im\{(\hat{x}_{k}^{l})^{*}\} = \begin{cases} \frac{\sqrt{2}}{2} & \text{if } \Im\left\{\frac{\tilde{y}_{k}^{l}}{\sqrt{P_{k}L_{kk}}H_{kk}^{ll}}\right\} \ge 0\\ -\frac{\sqrt{2}}{2} & \text{else} \end{cases}$$
(8)

Note that the performance of the ID processing depends heavily on the *interference profile*, which contains the relative power of the residual interference at the receiver. Fig. 4 illustrates the average end-to-end SER performance of the ID detector versus the interference power. Observe that there is a window of unfavorable interference power for which the performance of the ID is quite poor. As such, the user selection of the PIA stage can contribute significantly to the end-to-end SER performance of the PIA stage can contribute significantly to the end-to-end SER performance of the PIAID scheme. Intuitively, the user selection \mathcal{A} of PIAID should not aim at

removing the *strongest interference*. On the contrary, it should remove the *unfavorable interference* characterized by the ID stage requirement (similar to Fig. 4). As a result, the PIA and ID processing are complementary approaches to combat interference and their designs are tightly coupled together.

III. DYNAMIC IA SET SELECTION IN PARTIAL INTERFERENCE ALIGNMENT

In this section, we shall formulate the dynamic selection of PIA set as a combinatorial optimization problem, and derive a low complexity optimal solution using graph theory and linear programming.

A. Problem Formulation

Due to heterogeneous path losses and transmit powers, interference links have different contributions to the average end-to-end SER of the PIAID scheme. We shall first formulate the dynamic PIA set selection problem using a general cost metric $C = \{c_{ki}, \forall k, i\}$. In the next section, we shall obtain a specialized cost metric related to the end-to-end SER of the PIAID. The PIA set optimization problem is summarized below.

Problem 1 (MaxPIA Problem): Given a general cost matrix of the interference links $C = \{c_{ki}, \forall k, i\}$, the MaxPIA problem is given by:

MaxPIA:
$$\mathcal{A}^* = \underset{\mathcal{A} \in \mathbb{A}}{\operatorname{arg\,max}} \sum_{k,i} c_{ki} \mathbf{1}(i \in \mathcal{A}_k),$$
 (9)

where \mathcal{A}^* is the solution to the MaxPIA problem, and \mathbb{A} denotes the collection of all the PIA sets that satisfies the IA feasibility condition (3).

B. Optimal Solution of the Dynamic PIA Set Selection

Optimization problem (9) is a constrained combinatorial optimization problem, which is difficult in general. Solving problem (9) using brute force exhaustive search has a high complexity of $O(\exp(K))$ and is not viable in practice. In this section, we shall exploit specific problem structure and visualize the optimization problem in (9) using graph theory.

We first review some preliminaries on graph theory from [23], [24] and the reference therein. A graph \mathcal{G} is defined by a pair $\mathcal{G} = (\mathcal{W}, \mathcal{E})$, where \mathcal{W} is a finite set of nodes and \mathcal{E} is a finite set of edges. Specifically, the nodes in \mathcal{W} are denoted as $w_1, w_2, \dots, w_{|\mathcal{W}|}$, and an edge in \mathcal{E} connecting nodes w_i and w_k is denoted as $[w_i, w_k]$. If an edge $e = [w_1, w_2] \in \mathcal{E}$, then we say that e is incident upon w_1 (and w_2). The degree of a node w of \mathcal{W} is the number of edges incident upon w. A bipartite graph is a graph $\mathcal{B} = (\mathcal{W}, \mathcal{E})$ such that \mathcal{W} can be partitioned into two sets, \mathcal{W}_1 and \mathcal{W}_2 , and each edge in \mathcal{E} has one node in \mathcal{W}_1 and one node in \mathcal{W}_2 . The bipartite graph is usually denoted by $\mathcal{B} = (\mathcal{W}_1, \mathcal{W}_2, \mathcal{E})$. An example of a bipartite graph is illustrated in Fig. 3.

In fact, interference networks can be represented by a bipartite graph $\mathcal{B} = (\mathcal{R}, \mathcal{T}, \mathcal{E})$, where $\mathcal{R} = \{r_1, \dots, r_K\}$ is the set of the receive nodes, $\mathcal{T} = \{t_1, \dots, t_K\}$ is the set of the transmit nodes, and $\mathcal{E} = \{[r_k, t_i], \forall k, i \text{ and } k \neq i\}$ is the set of the edges. A feasible PIA set \mathcal{A} is equivalent to a subset of the edges \mathcal{E}_s with the property that the degree of each receive and transmit node of $\mathcal{G}_s = (\mathcal{T}, \mathcal{R}, \mathcal{E}_s)$ is $\alpha(\mathcal{A})$, and \mathcal{G}_s is called a α -factor of graph \mathcal{G} .

Example 1 (Graph Illustration of the PIA set): Suppose the PIA set is given by $\mathcal{A} = \{\mathcal{A}_1 = \{2,3\}, \mathcal{A}_2 = \{3,4\}, \mathcal{A}_3 = \{4,1\}, \mathcal{A}_4 = \{1,2\}\}$, the corresponding subset of edges \mathcal{E}_s is given by $\mathcal{E}_s = \{[r_1, t_2], [r_1, t_3], [r_2, t_3], [r_2, t_4], [r_3, t_4], [r_3, t_1], [r_4, t_1], [r_4, t_2]\}$ as illustrated in Fig. 3.

Let c_{ki} denote the cost of edge $[r_k, t_i] \in \mathcal{E}$. Problem 1 is equivalent to finding a α -factor of \mathcal{G} with the largest sum of costs. Hence, the MaxPIA problem in (9) is similar to a matching problem (finding a "best" 1-factor of graph \mathcal{G}) on a bipartite graph, and exploiting this equivalence we shall derive a low complexity optimal solution. Let $\mathbf{e} = \{e_{ki}, \forall k, i\}$ be a set of variables. If the edge $[r_k, t_i]$ is included in the α -factor (i.e. the transmit node *i* is chosen as one of the aligned interferers at receive node *k*) then $e_{ki} = 1$, otherwise $e_{ki} = 0$. As a result, the problem 1 is equivalent to

$$\{e_{ki}^*\} = \begin{cases} \arg \max_{e_{ki}} & \sum_{k,i} c_{ki} e_{ki} \\ \text{s.t} & \sum_i e_{ki} = \alpha, \forall k \\ & \sum_k e_{ki} = \alpha, \forall i \\ & & e_{ki} \in \{0,1\}, \forall k, i \end{cases}$$
(10)

where $\alpha = \min\left(\frac{M+N}{D} - 2, K - 1\right)$.

The above problem is a non-convex problem due to the non-convex constraint $e_{ki} \in \{0, 1\}$. To get a low complexity solution, we first relax the constraints $e_{ki} \in \{0, 1\}$ to $0 \le e_{ki} \le 1$. As a result, (10) becomes a standard LP problem, which can be solved efficiently by the well known simplex algorithm [23], [24]. The following Lemma summarizes the optimality of this relaxation.

Lemma 1 (Optimality of the LP Relaxation): The optimal solution of the LP relaxation problem is also the optimal solution of (10), i.e. $\tilde{e}_{ki}^* \in \{0, 1\}$ where \tilde{e}_{ki}^* is the optimal solution of the LP relaxed problem.

Proof: Please refer to Appendix A.

IV. SER ANALYSIS AND LOW COMPLEXITY ID PROCESSING

In this section, we first derive the average SER of the PIAID scheme for a given PIA set A and the path gains $\{L_{ki}P_i, \forall k, i\}$. Based on the SER results, we obtain an equivalent cost metric for the PIA set selection optimization problem in (10), which is order-optimal w.r.t. the average end-to-end SER. Finally, we propose a low complexity ID algorithm using SDR technique.

A. SER Analysis of ID with Non-Gaussian Residual Interference

Unlike standard SER analysis in existing literature [25], [26], a key challenge of SER analysis in the PIAID scheme is that the residual interference in the stage I and stage II ID processing are non-Gaussian due to the discrete constellation inputs. As a result, we focus on deriving an asymptotically tight SER expression for the interference dominated regime. Theorem 1 summarizes our main results.

Theorem 1 (Average SER of the Two Stage PIAID Processing): For a given PIA set A, the SER of the *l*-th data stream at the *k*-th receiver in the interference limited regime of the PIAID scheme is given by

$$\overline{\operatorname{SER}}_{k}^{l}(\mathcal{A}) \triangleq \mathbb{E}_{\mathcal{H}}\left[\operatorname{SER}_{k}^{l}(\mathcal{A},\mathcal{H})\right] \triangleq \mathbb{E}_{\mathcal{H}}\left[\sum_{x_{k}^{l}} \operatorname{Pr}\left\{x_{k}^{l}\right\} \operatorname{Pr}\left\{\hat{x}_{k}^{l} \neq x_{k}^{l} | x_{k}^{l}, \mathcal{A}, \mathcal{H}\right\}\right] \\
= \Theta\left(\sum_{i \in \mathcal{Q}_{k}} \frac{P_{k}L_{kk}}{P_{i}L_{ki}} + \sum_{i \in \mathcal{O}_{k}} \frac{P_{i}L_{ki}}{P_{k}L_{kk}}\right),$$
(11)

where $\mathcal{H} = \{\mathbf{H}_{ki}, \forall k, i\}$. $g(x) = \Theta(f(x))$ denotes $\limsup_{x \to 0} \frac{g(x)}{f(x)} \leq C$ and $\liminf_{x \to 0} \frac{g(x)}{f(x)} \geq c$ for some constants $C \geq c \geq 0$.

Proof: Please refer to the Appendix B.

Remark 2: In this paper, the precoders and decorrelators $\{\mathbf{V}_k, \mathbf{U}_k, \forall k\}$ are determined based on the IA requirements in (2). Hence, they are only dependent on the channels in the set \mathcal{A} for which interference is aligned. Since the interference from these channels involved are nulled, the remaining interference has a random channel matrix even though it is now projected on the space \mathbf{U}_k . Furthermore, the SER in (11) is averaged over realizations of the channels and noise.

Remark 3 (Interpretation of Theorem 1): The result in (11) indicates that the SER of the PIAID scheme favors either very strong or very weak residual interference. In other words, there is always an unfavorable *window of residual interferences* as illustrated in Fig. 4. The role of PIA is to eliminate these unfavorable windows of interferences so that the ID processing is given a more favorable interference profile.

Motivated by Theorem 1, we set the interference cost metric in the PIA set optimization problem as

$$c_{ki} = \begin{cases} -C & \text{if } i = k \\ \frac{P_k L_{kk}}{P_i L_{ki}} & \text{else if } P_i L_{ki} \ge P_k L_{kk} \\ \frac{P_i L_{ki}}{P_k L_{kk}} & \text{otherwise} \end{cases}$$
(12)

where C > 0 is a large constant (a sufficiently large C can be chosen as: $C > \sum_{k,i \neq k} |c_{ki}|$). Based on these interference cost metrics, the PIA set selection solutions solved by the LP relaxation is

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order-optimal⁶ w.r.t. the following problem:

$$\mathcal{A}^* = \operatorname*{arg\,min}_{\mathcal{A} \in \mathbb{A}} \sum_{k,l} \overline{\operatorname{SER}}_k^l(\mathcal{A}).$$
(13)

B. Low Complexity ID

Note that the complexity of decoding algorithm 1 in the stage I processing is exponential w.r.t. the cardinality of the set of strong residual interferences, i.e., $|Q_k|$. Using the SDR technique [17]–[20], we shall derive a low complexity ID algorithm, which has polynomial complexity w.r.t. $|Q_k|$. The SDR technique has been widely used in multiuser detection [17], [18] and MIMO systems [19], [20] to derive low complexity suboptimal detectors. It has been shown that the SDR detector can provide better performance compared with other suboptimal detectors.

To utilize the SDR technique, we first simplify equation (5) as follows:

$$y_{k}^{l} = (\mathbf{h}_{\mathcal{Q}})^{T} \mathbf{x}_{\mathcal{Q}} + \underbrace{\sqrt{P_{k} L_{kk}} H_{kk}^{ll} x_{k}^{l} + \sum_{i \in \mathcal{O}_{k}; d} \sqrt{P_{i} L_{ki}} H_{ki}^{ld} x_{i}^{d} + (\mathbf{u}_{k}^{l})^{\dagger} \mathbf{z}_{k},}_{n_{0}}$$
(14)

where $\mathbf{x}_{Q} = [x_{i_{1}}^{1}, \cdots, x_{i_{1}}^{D}, x_{i_{2}}^{1}, \cdots, x_{i_{Q}}^{D}]^{T} \in S^{DQ \times 1}$ is DQ interference symbols, and $Q = |Q_{k}|$ is the cardinality of Q_{k} . $\mathbf{h}_{Q} = [\sqrt{P_{i_{1}}L_{ki_{1}}}H_{ki_{1}}^{l1}, \cdots, \sqrt{P_{i_{1}}L_{ki_{1}}}H_{ki_{1}}^{lD}, \sqrt{P_{i_{2}}L_{ki_{2}}}H_{ki_{2}}^{l1}, \cdots, \sqrt{P_{i_{Q}}L_{ki_{Q}}}H_{ki_{Q}}^{lD}]^{T} \in \mathbb{C}^{DQ \times 1}$ is the channel gain for the interference symbols \mathbf{x}_{Q} . Furthermore, the real valued form of (14) can be expressed as:

$$\underbrace{\begin{bmatrix} \Re\{y_k^l\}\\ \Im\{y_k^l\} \end{bmatrix}}_{\mathbf{y}_R} = \underbrace{\begin{bmatrix} \Re\{(\mathbf{h}_{\mathcal{Q}})^T\} & -\Im\{(\mathbf{h}_{\mathcal{Q}})^T\} \\ \Im\{(\mathbf{h}_{\mathcal{Q}})^T\} & \Re\{(\mathbf{h}_{\mathcal{Q}})^T\} \end{bmatrix}}_{\mathbf{H}_R} \underbrace{\begin{bmatrix} \Re\{\mathbf{x}_{\mathcal{Q}}\}\\ \Im\{\mathbf{x}_{\mathcal{Q}}\} \end{bmatrix}}_{\mathbf{x}_R} + \underbrace{\begin{bmatrix} \Re\{n_0\}\\ \Im\{n_0\} \end{bmatrix}}_{\mathbf{n}_R},$$
(15)

where $\mathbf{y}_R \in \mathbb{R}^{2 \times 1}$, $\mathbf{H}_R \in \mathbb{R}^{2 \times 2DQ}$, $\mathbf{x}_R \in \{\pm 1\}^{2DQ \times 1}$ and $\mathbf{n}_R \in \mathbb{R}^{2 \times 1}$. Decoding algorithm 1 in stage I processing is equivalent to

$$(\mathbf{x}_R)^* = \operatorname*{arg\,min}_{\mathbf{x}_R \in \{\pm 1\}^{2DQ \times 1}} ||\mathbf{y}_R - \mathbf{H}_R \mathbf{x}_R||, \tag{16}$$

such that the detected aggregate strong interference $(\hat{I}_k^l)^* = (\mathbf{h}_Q)^T (\mathbf{x}_Q)^*$, where $(\mathbf{x}_Q)^*$ is determined from $(\mathbf{x}_R)^*$ as indicated in (15). The above problem (16) can be equivalently expressed as

$$\mathbf{s}^{*} = \begin{cases} \underset{\mathbf{s}}{\operatorname{arg\,min}} & \operatorname{trace}(\mathbf{WS}) \\ \text{s.t} & \operatorname{diag}(\mathbf{S}) = \mathbf{I}_{2DQ+1} \\ & \mathbf{S} = \mathbf{ss}^{T} \end{cases}$$
(17)

⁶ It can be observed from the following fact. From Theorem 1, we have $\mathcal{A}^* = \arg \min_{\mathcal{A} \in \mathbb{A}} \Theta \left(\sum_{k,i \in \mathcal{Q}_k} \frac{P_k L_{kk}}{P_i L_{ki}} + \sum_{k,i \in \mathcal{O}_k} \frac{P_i L_{ki}}{P_k L_{kk}} \right) = \arg \min_{\mathcal{A} \in \mathbb{A}} \Theta \left(\sum_{k,i \neq k} \frac{P_k L_{kk}}{P_i L_{ki}} \mathbf{1}(P_i L_{ki} \ge P_k L_{kk}) - \sum_{k,i \in \mathcal{A}_k} \frac{P_k L_{kk}}{P_i L_{ki}} \mathbf{1}(P_i L_{ki} \ge P_k L_{kk}) + \sum_{k,i \neq k} \frac{P_i L_{ki}}{P_k L_{kk}} \mathbf{1}(P_i L_{ki} < P_k L_{kk}) - \sum_{k,i \in \mathcal{A}_k} \frac{P_i L_{ki}}{P_k L_{kk}} \mathbf{1}(P_i L_{ki} \ge P_k L_{kk}) + \sum_{k,i \neq k} \frac{P_i L_{ki}}{P_k L_{kk}} \mathbf{1}(P_i L_{ki} < P_k L_{kk}) - \sum_{k,i \in \mathcal{A}_k} \frac{P_i L_{ki}}{P_k L_{kk}} \mathbf{1}(P_i L_{ki} < P_k L_{kk}) \right) = \arg \max_{\mathcal{A} \in \mathbb{A}} \Theta \left(\sum_{k,i \in \mathcal{A}_k} \frac{P_k L_{kk}}{P_i L_{ki}} \mathbf{1}(P_i L_{ki} \ge P_k L_{kk}) + \sum_{k,i \in \mathcal{A}_k} \frac{P_i L_{ki}}{P_k L_{kk}} \mathbf{1}(P_i L_{ki} < P_k L_{kk}) \right).$ Hence, the $\left(\sum_{k,i \in \mathcal{Q}_k} \frac{P_k L_{kk}}{P_i L_{ki}} + \sum_{k,i \in \mathcal{O}_k} \frac{P_i L_{ki}}{P_k L_{kk}} \right)$ is an asymptotically tight bound for SER when $P_k L_{kk} \gg P_i L_{ki}$ or $P_k L_{kk} \ll P_i L_{ki}$ for all $\{k, i\}$, and an order-optimal solution means that it minimizes the asymptotically tight bound for the SER.

where $(\cdot)^T$ denotes the transpose,

$$\mathbf{W} = \begin{bmatrix} (\mathbf{H}_R)^T \mathbf{H}_R & -(\mathbf{H}_R)^T \mathbf{y}_R \\ -(\mathbf{y}_R)^T \mathbf{H}_R & 0 \end{bmatrix} \text{ and } \mathbf{s} = \begin{bmatrix} \mathbf{x}_R \\ 1 \end{bmatrix}.$$
(18)

By means of SDR, we relax the constraint $\mathbf{S} = \mathbf{ss}^T$ by $\mathbf{S} \succeq 0$ (i.e., $\mathbf{S} \in \mathbb{C}^{n \times n}$ is positivesemidefinite), and problem (17) degenerates into the following Semi-Definite Program (SDP) that can be solved efficiently in $O(Q^{3.5})$ time [17]–[20], e.g., using the interior-point optimization technique [27]:

$$\mathbf{S}^{*} = \begin{cases} \underset{\mathbf{S}}{\operatorname{arg\,min}} & \operatorname{trace}(\mathbf{WS}) \\ \text{s.t.} & \operatorname{diag}(\mathbf{S}) = \mathbf{I}_{2DQ+1} \\ & \mathbf{S} \succeq 0. \end{cases}$$
(19)

If the optimal value S^* of the above problem (19) is rank one, then the relaxation is tight, and the optimal solution of the $(\mathbf{x}_R)^*$ is given by [17]–[19]:

$$[(\mathbf{x}_R)^*]_{(n,1)} = f_{\mathbf{x}}(\mathbf{r}, n) = \begin{cases} 1 & \text{if } \frac{[\mathbf{r}]_{(n,1)}}{[\mathbf{r}]_{(|\mathbf{r}|,1)}} \ge 0\\ -1 & \text{else} \end{cases}, \quad \forall n \in \{1, \cdots, |\mathbf{r}| - 1\}$$
(20)

where \mathbf{r} is the eigenvector of \mathbf{S}^* associated with the only one non-zero eigenvalue.

On the other hand, if S^* is not rank one, then we shall approximate $(\mathbf{x}_R)^*$ based on S^* . Specifically, there are a few standard techniques to determine $(\mathbf{x}_R)^*$, e.g., *Randomization*, *Rank-1 approximation* and *Dominant eigenvector approximation* [17]–[19].

To further improve the quality of the approximation, we propose a SDR-SID algorithm based on the dominant eigenvector approximation as follows (and illustrated in Fig. 5).

Algorithm 3 (SDR-SID Algorithm):

• Step 0: Set active set $\Lambda = \{x_{i_1}^1, \dots, x_{i_1}^D, x_{i_2}^1, \dots, x_{i_Q}^D\}$ that contains all the decoding data streams, and the cardinality is $\lambda = |\Lambda| = DQ$.

Repeat

- Step 1: According to the active set Λ , solve optimization problem (19) to obtain S^* .
- Step 2: If S^* is rank one, determine $(x_R)^*$ from (20) and terminate.
- Step 3: Extract the λ dominant eigenvectors of S*, {r₁, · · · , r_λ}. Compute [x̂_i]_(n,1) = f_x (r_i, n) from (20) ∀i ∈ {1, · · · , λ} and ∀n ∈ {1, · · · , 2λ}.
- Step 4: Compute $\mathbf{S}_i = [\hat{\mathbf{x}}_i, 1]^T [\hat{\mathbf{x}}_i, 1], \forall i \in \{1, \dots, \lambda\}$. Choose $(\mathbf{x}_R)^* = \hat{\mathbf{x}}_{i^*}$, where $i^* = \arg\min_{i \in \{1, \dots, \lambda\}} \operatorname{trace}(W\mathbf{S}_i)$.
- Step 5: Determine $x_{i^*}^{d^*}$ from $(\mathbf{x}_R)^*$, where $\{i^*, d^*\} = \arg \max_{i,d} |\sqrt{P_i L_{ki}} H_{ki}^{ld} | \mathbf{1}(x_i^d \in \Lambda)$.
- Step 6: Set y^l_k = y^l_k √L_{ki*}H^{ld*}_{ki*}x^{d*}_{i*}, delete x^{d*}_{i*} from the active set Λ and set λ = λ − 1.
 Until the active set Λ is empty.

Example 2 (Illustration of Algorithm 3): Suppose y_k^l in (14) is given by $y_k^l = \sqrt{P_1 L_{k1}} H_{k1}^{l1} x_1^1 + \sqrt{P_2 L_{k2}} H_{k2}^{l2} x_2^1 + n_0$ with $|\sqrt{P_2 L_{k2}} H_{k2}^{l2}| > |\sqrt{P_1 L_{k1}} H_{k1}^{l1}|$. The details of the implementation of Algorithm 3 are given below.

- Step 0: Set active set $\Lambda = \{x_1^1, x_2^1\}$ and $\lambda = 2$.
- Step 1: Suppose S* is not rank one. Go to step 3.
- Step 3: Extract the 2 dominant eigenvectors of S^* , $\{r_1, r_2\}$, and obtain $\{\hat{x}_1, \hat{x}_2\}$ from (20).
- Step 4: Suppose trace $(WS_1) < trace(WS_2)$, we choose $(\mathbf{x}_R)^* = \hat{\mathbf{x}}_1$.
- Step 5: Since $|\sqrt{P_2 L_{k2}} H_{k2}^{l2}| > |\sqrt{P_1 L_{k1}} H_{k1}^{l1}|$, we determine x_2^1 from $(\mathbf{x}_R)^*$, i.e., $x_2^1 = [(\mathbf{x}_R)^*]_{(2,1)} + j[(\mathbf{x}_R)^*]_{(4,1)}$, which by definition $\mathbf{x}_Q = [x_1^1, x_2^1]$.
- Step 6: Set $y_k^l = y_k^l \sqrt{P_2 L_{k2}} H_{k2}^{l1} x_2^1 = \sqrt{P_1 L_{k1}} H_{k1}^{l1} x_1^1 + n_0$, $\Lambda = \{x_1^1\}$ and $\lambda = 1$. Repeat from step 1 to obtain x_1^1 .

Algorithm 3 is motivated from the intuition that the error probability of decoding symbol x_i^d , $\forall i \in Q_k$, $\forall d$ is small if its channel gain $\sqrt{P_i L_{ki}} H_{ki}^{ld}$ is large. Note that the complexity of Algorithm 3 is mainly determined by the complexity of solving the optimization problem (19) in the step 1. Since the complexity for step 1 is only in $O(Q^{3.5})$ time [17]–[20], the overall complexity of Algorithm 3 is $O(Q^{4.5})$. Furthermore, it can be easily generalized to other approximation techniques by simply modifying the way to determine $\hat{\mathbf{x}}_i$ in step 3. Finally, using simulation, we illustrate that the average end-to-end SER performance of the low complexity SDR-SID algorithm is similar to the performance of Algorithm 1.

V. SIMULATION RESULTS AND DISCUSSIONS

In this section, we evaluate the performance of the proposed PIAID scheme via numerical simulations. In particular, we compare the performance of the proposed schemes against various baseline schemes:

• Proposed Scheme 1: PIAID with Algorithm 1 (PIAID Alg1)

- The PIA set optimization stage tries to align the unfavorable interference links by setting the interference cost metric according to (12).
- ID processing is adopted at each of the K receivers after PIA. Specifically, Algorithm 1 and 2 are applied at stage I and stage II processing, respectively.
- Proposed Scheme 2: PIAID with SDR-SID (PIAID Alg3)
 - The PIA set optimization stage tries to align the unfavorable interference links by setting the interference cost metric according to (12).
 - ID processing is adopted at each of the K receivers after PIA. Specifically, Algorithm 3 and 2 are applied at stage I and stage II processing, respectively.

- Baseline 1: Randomized PIA (Randomized PIA)
 - The PIA set A is chosen randomly from A, i.e., the collection of all the PIA sets that satisfies the IA feasibility condition (3).
 - ID processing is adopted at each of the K receivers after PIA. Specifically, Algorithm 1 and 2 are applied at stage I and stage II processing, respectively.
- Baseline 2: Iterative interference alignment (Iterative IA) [6, Algorithm 1]
 - Alternating optimization is utilized to minimize the weighted sum leakage interference.
 - Conventional one-stage decoding is adopted at each of the K receivers by treating all the interference as noise.
- Baseline 3: Maximizing SINR (Max SINR) [6, Algorithm 2]
 - Alternating optimization is utilized to maximize the SINR at the receivers.
 - Conventional one-stage decoding is adopted at each of the K receivers by treating all the interference as noise.
- Baseline 4: Maximizing Sum-Rate (Max Sum-Rate) [7]
 - A gradient ascent approach combined with the alternating optimization is utilized to maximize the sum-rate of the receivers.
 - Conventional one-stage decoding is adopted at each of the K receivers by treating all the interference as noise.
- Baseline 5: Minimizing Mean Square Error (Min MSE) [8]
 - A joint design to minimize the sum of the MSE of the receivers.
 - Conventional one-stage decoding is adopted at each of the K receivers by treating all the interference as noise.

In the simulations, all the transmit and receive nodes are assumed randomly distributed in a $2\text{km} \times 1\text{km}$ rectangular area as shown in Fig. 1. The channel model is given by Assumption 1. Specifically, we set the log-normal shadowing standard deviation as $\sigma_{\omega} = 12$ dB and the path loss exponent as $\gamma = 6$ as per ITU-R recommendation M.1225 [21]. Each transmitter delivers a single stream (D = 1) of QPSK symbols. The transmit power of each node is assumed to be the same.

A. Performance w.r.t. Receive E_s/N_0

Fig. 6 and Fig. 7 illustrate the average end-to-end SER performance per data stream versus receive E_s/N_0 (dB) of the desired link for K = 5 and K = 6 respectively. The number of transmit and receive antennas is given by $\{M = 3, N = 2\}$. The number of aligned users for the feasible interference alignment is $\alpha = 3$. The average SER performance is evaluated with 10⁷ realizations of noise, complex fading coefficients $\{\mathbf{H}_{ki}, \forall k, i\}$ and path loss $\{L_{ki}, \forall k, i\}$. Observe that the average SER of

all the schemes decreases as the receive E_s/N_0 increases, and there is significant performance gain of the proposed schemes compared to all baselines, even for low complexity PIAID with SDR-SID (Algorithm 3). The performance gain is contributed by the user selection of the PIA stage that moves the ID processing out of the unfavorable interference profile as shown in Fig. 4. Furthermore, it can also be observed that PIAID with SDR-SID (Algorithm 3) has similar performance as PIAID (Algorithm 1). Finally, Fig. 6 shows that the proposed PIA algorithm with condition (3) has similar performance as the solution obtained by brute-force exhaustive search (without (3)).

B. Cumulative Distribution Function (CDF) of the SER

Fig. 8 and Fig. 9 illustrate the Cumulative Distribution Function (CDF) of the SER per data stream with receive $E_s/N_0 = 25$ dB for K = 5 and K = 6 respectively, where the randomness of SER is induced by { $\mathbf{H}_{ki}, \forall k, i$ } and { $L_{ki}, \forall k, i$ }. The number of transmit and receive antennas is given by {M = 3, N = 2}. The number of aligned users for the feasible interference alignment is $\alpha = 3$. The CDF performance is evaluated with 10⁷ realizations of noise, complex fading coefficients { $\mathbf{H}_{ki}, \forall k, i$ } and path loss { $L_{ki}, \forall k, i$ }. It can also be verified that the proposed scheme achieves not only a smaller average SER but also a smaller SER percentile compared with the baselines.

VI. CONCLUSIONS

In this paper, we propose a low complexity and novel Partial Interference Alignment (PIA) and Interference Detection (PIAID) scheme for *K*-user quasi-static MIMO interference channels with general irrational channel coefficients. Based on the path loss information, the proposed PIAID scheme dynamically selects the IA interferers at each receiver such that it moves the ID processing out of the unfavorable interference profiles. We derive the average SER by taking into account the non-Guassian residual interference, and obtain a low complexity user set selection algorithm for the PIAID scheme, which minimizes the asymptotically tight bound for the average end-to-end SER performance. The SER performance of the proposed PIAID scheme is shown to have significant gain compared with various conventional baseline solutions.

APPENDIX A

PROOF OF LEMMA 1

By introducing the slack variables s_{ki} , the LP relaxation problem of (10) becomes

$$\{e_{ki}^*\} = \begin{cases} \arg \max_{e_{ki}} \sum_{k,i} c_{ki} e_{ki} \\ \text{s.t} & \sum_i e_{ki} = \alpha, \forall k \\ & \sum_k e_{ki} = \alpha, \forall i \\ & e_{ki} + s_{ki} = 1, \forall k, i \\ & e_{ki} \ge 0, s_{ki} \ge 0, \forall k, i \end{cases}$$
(21)

and it is equivalent to the following matrix form

$$\{\mathbf{e}^*\} = \begin{cases} \arg \max_{\mathbf{e}} & \mathbf{c}^T \mathbf{e} \\ \mathbf{s.t} & \mathbf{A} \mathbf{e} = \mathbf{b} \\ & \mathbf{e} \succeq \mathbf{0} \end{cases}$$
(22)

where $\mathbf{e} = [e_{11}, \dots, e_{1K}, e_{21}, \dots, e_{KK}, s_{11}, \dots, s_{1K}, s_{21}, \dots, s_{KK}]^T$ is formed by the optimization variables, $\mathbf{b} = [\alpha, \dots, \alpha, \underbrace{1, \dots, 1}_{2K}]^T$, $\mathbf{c} = [c_{11}, \dots, c_{1K}, c_{21}, \dots, c_{KK}, \underbrace{0, \dots, 0}_{K^2}]^T$ is formed by the path costs, and the matrix \mathbf{A} is given by

$$\mathbf{A} = \begin{bmatrix} \mathbf{B} & \mathbf{0}_{2K \times K^2} \\ \mathbf{I}_{K^2} & \mathbf{I}_{K^2} \end{bmatrix} = \begin{bmatrix} \mathbf{1}_{1 \times K} & \mathbf{0}_{1 \times K} & \cdots & \mathbf{0}_{1 \times K} \\ \mathbf{0}_{1 \times K} & \mathbf{1}_{1 \times K} & \cdots & \mathbf{0}_{1 \times K} \\ \vdots & \vdots & \vdots & \vdots & \mathbf{0}_{2K \times K^2} \\ \mathbf{0}_{1 \times K} & \mathbf{0}_{1 \times K} & \cdots & \mathbf{1}_{1 \times K} \\ \mathbf{I}_{K} & \mathbf{I}_{K} & \cdots & \mathbf{I}_{K} \\ \hline \mathbf{I}_{K^2} & \mathbf{I}_{K^2} \end{bmatrix}.$$
(23)

where $\mathbf{0}_{n \times m}$ denotes an $n \times m$ matrix of zeros and \mathbf{I}_n denotes an $n \times n$ identity matrix.

Note that the feasible set for the LP (22) is given by the polytope $\mathcal{R}(\mathbf{A}) = {\mathbf{A}\mathbf{e} = \mathbf{b}, \mathbf{e} \succeq 0}$. If all the vertices of $\mathcal{R}(\mathbf{A})$ are integers, there exists an optimal solution such that all the optimization variables are integers, and hence the LP (22) will always lead to an integer optimum when solved by the well known simplex algorithm. Since $e_{ki} \leq 1, \forall k, i$, if e_{ki} is an integer, we have $e_{ki} \in {0, 1}$. As a result, it is equivalent to proving that all the vertices of the polytope $\mathcal{R}(\mathbf{A})$ are integers. We first define *Totally Unimodular* (TUM) as follows.

Definition 2 (Definition of Totally Unimodular): An integer matrix \mathbf{Z} is totally unimodular if the determinant of each square submatrix of \mathbf{Z} is equal to 0,1, or -1.

It has been shown in [23] that if A is TUM, then all the vertices of $\mathcal{R}(A)$ are integers. Therefore, the proof reduces to proving that A is TUM. Note that the matrix B satisfies the following conditions:

- **B** is a $\{0,1\}$ matrix with no more than two nonzero elements in each column.
- Each column contains two nonzero elements that have the same sign, where one element is in a row contained in the subset Ω₁ = {1, · · · , K} and the other element is in a row contained in the subset Ω₂ = {K + 1, · · · , 2K}.

Therefore, **B** is TUM [24]. It is easy to verify that the matrix $\mathbf{C} = [\mathbf{B}^T, \mathbf{I}]^T$ is also TUM. Let **D** be a square, nonsingular submatrix of **A**. The rows of **D** can be permuted so that it can be written as

$$\mathbf{D} = \begin{bmatrix} \mathbf{E} & \mathbf{0} \\ \hline \mathbf{F} & \mathbf{I} \end{bmatrix},\tag{24}$$

where E is a square submatrix of C, and possibly with its rows permuted. Therefore, $det(D) = det(B) = \pm 1$, which completes the proof.

APPENDIX B

PROOF OF THEOREM 1

A. Upper Bound of the Average SER Performance

In this subsection, we shall obtain an upper bound of the average SER $\overline{\text{SER}}_k^l(\mathcal{A})$. Specifically, we shall first obtain an upper bound of the SER under a given channel realization, i.e., an upper bound of $\text{SER}_k^l(\mathcal{A}, \mathcal{H})$.

1) Upper bound of $SER_k^l(\mathcal{A}, \mathcal{H})$: Let $\mathcal{H}_k^l = \{H_{ki}^{ld}, \forall i, d\}$ denotes the set of equivalent channel gains after applying the equalizer \mathbf{u}_k^l when decoding the desired signal x_k^l at the receiver k, $\delta_k^l = I_k^l - \hat{I}_k^l$ denotes the difference between the real and estimated aggregate strong interference, and $\zeta_k^l = \sum_{i \in \mathcal{O}_k, n} \sqrt{P_i L_{ki}} H_{ki}^{ld} x_i^d$ denotes the aggregate weak residual interference. Specifically, we start to consider the two stage decoding separately.

• Stage II decoding: In stage II decoding, \tilde{y}_k^l in (7) is given by:

$$\widetilde{y}_{k}^{l} = y_{k}^{l} - \widehat{I}_{k}^{l} = \sqrt{P_{k}L_{kk}}H_{kk}^{ll}x_{k}^{l} + \delta_{k}^{l} + \zeta_{k}^{l} + (\mathbf{u}_{k}^{l})^{\dagger}\mathbf{z}_{k},$$
(25)

Given \mathcal{H}_k^l and suppose $\Re\{x_k^l\} = \frac{\sqrt{2}}{2}$, the error rate of decoding $\Re\{x_k^l\}$ is given by [26]: $\Pr\left\{\Re\{\hat{x}_k^l\} \neq \frac{\sqrt{2}}{2} | \Re\{x_k^l\} = \frac{\sqrt{2}}{2}, I_k^l, \hat{I}_k^l, \mathcal{H}_k^l\right\} = \Pr\left\{\Re\{\frac{\tilde{y}_k^l}{\sqrt{P_k}I_{kk}}H_k^{ll}\} \le 0 | \Re\{x_k^l\} = \frac{\sqrt{2}}{2}, I_k^l, \hat{I}_k^l, \mathcal{H}_k^l\right\}$

$$= Q\left(\left(\frac{\sqrt{2}}{2} + \Re\left\{\frac{\delta_{k}^{l} + \zeta_{k}^{l}}{\sqrt{P_{k}L_{kk}}H_{kk}^{ll}}\right\}\right)|\sqrt{P_{k}L_{kk}}H_{kk}^{ll}|\right) = Q\left(\frac{\sqrt{2}}{2}|\sqrt{P_{k}L_{kk}}H_{kk}^{ll}| + \Re\left\{e^{-j\theta}(\delta_{k}^{l} + \zeta_{k}^{l})\right\}\right),$$
(26)

where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-x^2/2} dx$ denotes the Q-function, and $e^{-j\theta} = \frac{\sqrt{P_k L_{kk}} H_{kk}^{ll}}{|\sqrt{P_k L_{kk}} H_{kk}^{ll}|}$. Similarly, we have

$$\Pr\{\Re\{\hat{x}_{k}^{l}\}\neq-\frac{\sqrt{2}}{2}|\Re\{x_{k}^{l}\}=-\frac{\sqrt{2}}{2},I_{k}^{l},\hat{I}_{k}^{l},\mathcal{H}_{k}^{l}\}=Q\left(\frac{\sqrt{2}}{2}|\sqrt{P_{k}L_{kk}}H_{kk}^{ll}|-\Re\{e^{-j\theta}(\delta_{k}^{l}+\zeta_{k}^{l})\}\right),\\\Pr\{\Im\{\hat{x}_{k}^{l}\}\neq\frac{\sqrt{2}}{2}|\Im\{x_{k}^{l}\}=\frac{\sqrt{2}}{2},I_{k}^{l},\hat{I}_{k}^{l},\mathcal{H}_{k}^{l}\}=Q\left(\frac{\sqrt{2}}{2}|\sqrt{P_{k}L_{kk}}H_{kk}^{ll}|+\Im\{e^{-j\theta}(\delta_{k}^{l}+\zeta_{k}^{l})\}\right),\\\Pr\{\Im\{\hat{x}_{k}^{l}\}\neq-\frac{\sqrt{2}}{2}|\Im\{x_{k}^{l}\}=-\frac{\sqrt{2}}{2},I_{k}^{l},\hat{I}_{k}^{l},\mathcal{H}_{k}^{l}\}=Q\left(\frac{\sqrt{2}}{2}|\sqrt{P_{k}L_{kk}}H_{kk}^{ll}|-\Im\{e^{-j\theta}(\delta_{k}^{l}+\zeta_{k}^{l})\}\right).$$
(27)

Then the error rate of decoding x_k^l is given by:

$$\Pr\{\hat{x}_{k}^{l} \neq x_{k}^{l} | x_{k}^{l}, I_{k}^{l}, \hat{I}_{k}^{l}, \mathcal{H}_{k}^{l}\} \\
\leq \Pr\{\Re\{\hat{x}_{k}^{l}\} \neq \Re\{x_{k}^{l}\} | \Re\{x_{k}^{l}\}, I_{k}^{l}, \hat{I}_{k}^{l}, \mathcal{H}_{k}^{l}\} + \Pr\{\Im\{\hat{x}_{k}^{l}\} \neq \Im\{x_{k}^{l}\} | \Im\{x_{k}^{l}\}, I_{k}^{l}, \hat{I}_{k}^{l}, \mathcal{H}_{k}^{l}\} \quad (28) \\
\leq 2Q\left(\frac{\sqrt{2}}{2}|\sqrt{P_{k}L_{kk}}H_{kk}^{l}| - |\delta_{k}^{l}| - |\zeta_{k}^{l}|\right).$$

• Stage I decoding: In the stage I decoding, y_k^l in (5) is given by:

$$y_k^l = I_k^l + \sqrt{P_k L_{kk}} H_{kk}^{ll} x_k^l + \zeta_k^l + (\mathbf{u}_k^l)^{\dagger} \mathbf{z}_k.$$
⁽²⁹⁾

Note that when the detected aggregate strong interference $\hat{I}_k^l \neq I_k^l$, an upper bound of the probability that the estimated aggregate strong interference is \hat{I}_k^l is given by

$$\Pr\{\hat{I}_{k}^{l}|x_{k}^{l}, I_{k}^{l}, \mathcal{H}_{k}^{l}\} \leq \Pr\{\hat{I}_{k}^{l} \neq I_{k}^{l}|x_{k}^{l}, I_{k}^{l}, \mathcal{H}_{k}^{l}\} \leq 2Q\left(|\delta_{k}^{l}|/2 - |\sqrt{P_{k}L_{kk}}H_{kk}^{ll}| - |\zeta_{k}^{l}|\right), \quad (30)$$

where $\Pr\{\hat{I}_k^l \neq I_k^l | x_k^l, I_k^l, \mathcal{H}_k^l\}$ is the conditional probability that $\hat{I}_k^l \neq I_k^l$ when I_k^l is transmitted. Therefore, we can follow the same way as (26) and (28), which obtain the conditional probability that $\hat{x}_k^l \neq x_k^l$ when x_k^l is transmitted. When the detected aggregate strong interference $\hat{I}_k^l = I_k^l$, obviously we have following expression at interference limited regime

$$\Pr\{\hat{I}_k^l | x_k^l, I_k^l, \mathcal{H}_k^l\} = \Pr\{I_k^l | x_k^l, I_k^l, \mathcal{H}_k^l\} \le 2Q\left(-|\sqrt{P_k L_{kk}} H_{kk}^{ll}| - |\zeta_k^l|\right) \approx 2.$$
(31)

As a result, the probability that the estimated aggregate strong interference is \hat{I}_k^l in the stage I decoding is given by

 $\Pr\{\hat{I}_{k}^{l}|x_{k}^{l}, I_{k}^{l}, \mathcal{H}_{k}^{l}\} \leq 2Q\left(|\delta_{k}^{l}|/2 - |\sqrt{P_{k}L_{kk}}H_{kk}^{ll}| - |\zeta_{k}^{l}|\right). \tag{32}$ From the upper bound expressions (32) and (28) in the stage I and stage If decoding, respectively, an upper bound of the SER under a given channel realization $\operatorname{SER}_{k}^{l}(\mathcal{A}, \mathcal{H})$ is given by

$$\begin{aligned} \operatorname{SER}_{k}^{l}(\mathcal{A},\mathcal{H}) &\triangleq \sum_{x_{k}^{l} \in \mathcal{S}} \operatorname{Pr}\{x_{k}^{l}\} \operatorname{Pr}\{\hat{x}_{k}^{l} \neq x_{k}^{l} | x_{k}^{l}, \mathcal{H}_{k}^{l} \} \\ &= \sum_{x_{k}^{l} \in \mathcal{S}} \operatorname{Pr}\{x_{k}^{l}\} \left(\sum_{I_{k}^{l} \in \mathcal{I}_{k}^{l}} \operatorname{Pr}\{I_{k}^{l}\} \operatorname{Pr}\{\hat{x}_{k}^{l} \neq x_{k}^{l} | x_{k}^{l}, I_{k}^{l}, \mathcal{H}_{k}^{l} \} \right) \\ &= \sum_{x_{k}^{l} \in \mathcal{S}} \operatorname{Pr}\{x_{k}^{l}\} \left(\sum_{I_{k}^{l} \in \mathcal{I}_{k}^{l}} \sum_{\hat{I}_{k}^{l} \in \mathcal{I}_{k}^{l}} \operatorname{Pr}\{I_{k}^{l}\} \operatorname{Pr}\{\hat{I}_{k}^{l} | x_{k}^{l}, I_{k}^{l}, \mathcal{H}_{k}^{l} \} \operatorname{Pr}\{\hat{x}_{k}^{l} \neq x_{k}^{l} | x_{k}^{l}, I_{k}^{l}, \hat{I}_{k}^{l}, \mathcal{H}_{k}^{l} \} \right) \\ &\leq \frac{1}{|\mathcal{I}_{k}|} \sum_{x_{k}^{l} \in \mathcal{S}} \sum_{I_{k}^{l} \in \mathcal{I}_{k}^{l}} \sum_{\hat{I}_{k}^{l} \in \mathcal{I}_{k}^{l}} Q\left(|\delta_{k}^{l}|/2 - |\sqrt{P_{k}L_{kk}}H_{kk}^{ll}| - |\zeta_{k}^{l}| \right) \cdot \\ Q\left(\frac{\sqrt{2}}{2} |\sqrt{P_{k}L_{kk}}H_{kk}^{ll}| - |\delta_{k}^{l}| - |\zeta_{k}^{l}| \right). \end{aligned}$$

$$(33)$$

2) Upper bound of $SER_k^l(A)$: From (33), the average SER is given by

$$\overline{\operatorname{SER}}_{k}^{l}(\mathcal{A}) \triangleq \mathbb{E}_{\mathcal{H}} \left[\operatorname{SER}_{k}^{l}(\mathcal{A}, \mathcal{H}) \right] \\
\leq \frac{1}{|\mathcal{I}_{k}|} \mathbb{E}_{\mathcal{H}_{k}^{l}} \left[\sum_{I_{k}^{l} \in \mathcal{I}_{k}^{l}} \sum_{\hat{I}_{k}^{l} \in \mathcal{I}_{k}^{l}} \mathbf{1} \left(|\delta_{k}^{l}|/2 - |\sqrt{P_{k}L_{kk}}H_{kk}^{ll}| - |\zeta_{k}^{l}| < 0 \right) \cdot \\
\mathbf{1} \left(\frac{\sqrt{2}}{2} |\sqrt{P_{k}L_{kk}}H_{kk}^{ll}| - |\delta_{k}^{l}| - |\zeta_{k}^{l}| < 0 \right) \right] \\
= \frac{1}{|\mathcal{I}_{k}|} \sum_{I_{k}^{l} \in \mathcal{I}_{k}^{l}} \sum_{\hat{I}_{k}^{l} \in \mathcal{I}_{k}^{l}} \Pr\left\{ \frac{\sqrt{2}}{2} |\sqrt{P_{k}L_{kk}}H_{kk}^{ll}| - |\zeta_{k}^{l}| \le |\delta_{k}^{l}| \le 2(|\sqrt{P_{k}L_{kk}}H_{kk}^{ll}| - |\zeta_{k}^{l}|) \right\}.$$
(34)

Therefore, from the above equation (34), we shall discuss the following two cases given by: successful stage I decoding $\hat{I}_k^l = I_k^l$ (i.e., $\delta_k^l = 0$) and unsuccessful stage I decoding $\hat{I}_k^l \neq I_k^l$ (i.e., $\delta_k^l \neq 0$).

• Successful stage I decoding $\hat{I}_k^l = I_k^l$ ($\delta_k^l = 0$): Note that $H_{ki}^{ld} = (\mathbf{u}_k^l)^{\dagger} \mathbf{H}_{ki} \mathbf{v}_i^d$, and the aggregate weak residual interference ζ_k^l is given by

$$\begin{aligned} \zeta_k^l &= \sum_{i \in \mathcal{O}_k, d} \sqrt{P_i L_{ki}} H_{ki}^{ld} x_i^d = \sum_{i \in \mathcal{O}_k, d} \sqrt{P_i L_{ki}} (\mathbf{u}_k^l)^{\dagger} \mathbf{H}_{ki} \mathbf{v}_i^d x_i^d \\ &= \sum_{i \in \mathcal{O}_k} \sqrt{P_i L_{ki}} \sum_{m, n} [(\mathbf{u}_k^l)^{\dagger}]_{(1, n)} [\mathbf{H}_{ki}]_{(n, m)} \sum_d [\mathbf{v}_i^d x_i^d]_{(m, 1)}. \end{aligned}$$
(35)

Since $\{\mathbf{u}_k^l, \mathbf{v}_i^d, \forall i \in \mathcal{O}_k, \forall d\}$ are determined by the channel gains from the aligned links given by $\{\mathbf{H}_{ki}, \forall i \in \mathcal{A}_k, \forall k\}$ [10], [22], they are independent of the random variables $\{\mathbf{H}_{ki}, \forall i \in \mathcal{O}_k\}$. From Assumption 1, $[\mathbf{H}_{ki}]_{(n,m)} \sim \mathcal{CN}(0,1)$, and hence given $\{\mathbf{u}_k^l, \mathbf{v}_i^d x_i^d, \forall i \in \mathcal{O}_k, \forall d\}$, ζ_k^l is a complex Gaussian variable with zero mean and variance

$$\sigma^{2} = \sum_{i \in \mathcal{O}_{k}} P_{i}L_{ki} \sum_{m,n} \left| \left[(\mathbf{u}_{k}^{l})^{\dagger} \right]_{(1,n)} \right|^{2} \left(\left| \sum_{d} [\mathbf{v}_{i}^{d}x_{i}^{d}]_{(m,1)} \right| \right)^{2} \right.$$

$$= \sum_{i \in \mathcal{O}_{k}} P_{i}L_{ki} \sum_{m} \left(\left| \sum_{d} [\mathbf{v}_{i}^{d}x_{i}^{d}]_{(m,1)} \right| \right)^{2} \le D \sum_{i \in \mathcal{O}_{k}} P_{i}L_{ki} \sum_{m} \left(\sum_{d} \left| [\mathbf{v}_{i}^{d}x_{i}^{d}]_{(m,1)} \right|^{2} \right) \right.$$

$$= D^{2} \sum_{i \in \mathcal{O}_{k}} P_{i}L_{ki}, \qquad (36)$$

because of $||\mathbf{u}_k^l|| = ||\mathbf{v}_i^d x_i^d|| = 1$. From ζ_k^l is a complex Gaussian variable with zero mean and variance σ^2 , and the probability density function of the random variable $Z = |\frac{\zeta_k^l}{\sigma H_{kk}^{ll}}|^2$ is given by $f_Z(z) = \frac{1}{(z+1)^2}$. Therefore, when $\delta_k^l = 0$ the conditional probability is given by:

$$\Pr\left\{\frac{\sqrt{2}}{2}|\sqrt{P_k L_{kk}} H_{kk}^{ll}| - |\zeta_k^l| \le 0 |\mathbf{u}_k^l, \mathbf{v}_i^d x_i^d, \forall i \in \mathcal{O}_k, \forall d\right\} = \Pr\{\frac{P_k L_{kk}}{2\sigma^2} \le Z\}$$

$$= 1/(\frac{P_k L_{kk}}{2\sigma^2} + 1) \le 1/(\frac{P_k L_{kk}}{4D\sum_{i \in \mathcal{O}_k} P_i L_{ki}} + 1) = \Theta\left(\sum_{i \in \mathcal{O}_k} \frac{P_i L_{ki}}{P_k L_{kk}}\right).$$
(37)

In turn, the probability of the event $\frac{\sqrt{2}}{2} |\sqrt{P_k L_{kk}} H_{kk}^{ll}| \le |\zeta_k^l|$ is given by

$$\Pr\left\{\frac{\sqrt{2}}{2}|\sqrt{P_k L_{kk}} H_{kk}^{ll}| - |\zeta_k^l| \le 0\right\} \le \Theta\left(\sum_{i \in \mathcal{O}_k} \frac{P_i L_{ki}}{P_k L_{kk}}\right).$$
(38)

• Unsuccessful stage I decoding $\hat{I}_k^l \neq I_k^l$ ($\delta_k^l \neq 0$): Given $\hat{I}_k^l = \sum_{i \in Q_k; d} \sqrt{P_i L_{ki}} H_{ki}^{ld} \hat{x}_i^d$, where $\hat{x}_i^d \in S$, δ_k^l is given by:

$$\delta_{k}^{l} = I_{k}^{l} - \hat{I}_{k}^{l} = \sum_{i \in \mathcal{Q}_{k}; d} \sqrt{P_{i} L_{ki}} (\mathbf{u}_{k}^{l})^{\dagger} \mathbf{H}_{ki} \mathbf{v}_{i}^{d} (x_{i}^{d} - \hat{x}_{i}^{d})$$

$$= \sum_{i \in \mathcal{Q}_{k}} \sqrt{P_{i} L_{ki}} \sum_{m,n} [(\mathbf{u}_{k}^{l})^{\dagger}]_{(1,n)} [\mathbf{H}_{ki}]_{(n,m)} \sum_{d} [\mathbf{v}_{i}^{d} \psi_{i}^{d}]_{(m,1)},$$
(39)

where $\psi_i^d = x_i^d - \hat{x}_i^d \in \{0, \pm\sqrt{2} \pm j\sqrt{2}, \pm\sqrt{2}, \pm j\sqrt{2}\}$. Given $\{\mathbf{u}_k^l, \mathbf{v}_i^d \psi_i^d, \forall i \in \mathcal{Q}_k, \forall d\}$, δ_k^l is a complex Gaussian variable with zero mean and variance given by

$$\sigma^{2} = \sum_{i \in \mathcal{Q}_{k}} P_{i} L_{ki} \sum_{m,n} \left| \left[(\mathbf{u}_{k}^{l})^{\dagger} \right]_{(1,n)} \right|^{2} \left| \sum_{d} [\mathbf{v}_{i}^{d} \psi_{i}^{d}]_{(m,1)} \right|^{2} = \sum_{i \in \mathcal{Q}_{k}} P_{i} L_{ki} \sum_{m} \left| \sum_{d} [\mathbf{v}_{i}^{d} \psi_{i}^{d}]_{(m,1)} \right|^{2} \ge P_{i^{*}} L_{ki^{*}} \sum_{i \in \mathcal{Q}_{k}} \sum_{m} \left| \sum_{d} [\mathbf{v}_{i}^{d} \psi_{i}^{d}]_{(m,1)} \right|^{2},$$
(40)

where $i^* = \arg \min_{i \in \Xi} P_i L_{ki}$, and $\Xi = \{i : \sum_d |\psi_i^d| \neq 0\}$, since $||\mathbf{u}_k^l|| = 1$ and $||\mathbf{v}_i^d \psi_i^d|| = |\psi_i^d|$. Therefore, when $\delta_k^l \neq 0$, the conditional probability is given by

$$\Pr\left\{\frac{\sqrt{2}}{2}|\sqrt{P_{k}L_{kk}}H_{kk}^{ll}| - |\zeta_{k}^{l}| \leq |\delta_{k}^{l}| \leq 2(|\sqrt{P_{k}L_{kk}}H_{kk}^{ll}| - |\zeta_{k}^{l}|)|\mathbf{u}_{k}^{l}, \mathbf{v}_{i}^{d}\psi_{i}^{d}, \forall i \in \mathcal{Q}_{k}, \forall d\right\}$$

$$\leq \Pr\left\{0 \leq |\delta_{k}^{l}| \leq 2|\sqrt{P_{k}L_{kk}}H_{kk}^{ll}|\right\} = \Pr\left\{0 \leq |\frac{\delta_{k}^{l}}{\sigma H_{kk}^{ll}}|^{2} \leq \frac{4P_{k}L_{kk}}{\sigma^{2}})\right\}$$

$$= \frac{4P_{k}L_{kk}/\sigma^{2}}{4P_{k}L_{kk}/\sigma^{2}+1} < \frac{4P_{k}L_{kk}}{\sigma^{2}}.$$
(41)

In turn, the unconditional probability is given by:

$$\Pr\left\{\frac{\sqrt{2}}{2}|\sqrt{P_{k}L_{kk}}H_{kk}^{ll}| - |\zeta_{k}^{l}| \le |\delta_{k}^{l}| \le 2(|\sqrt{P_{k}L_{kk}}H_{kk}^{ll}| - |\zeta_{k}^{l}|)\right\}$$

$$\le \mathbb{E}_{\mathbf{u}_{k}^{l},\mathbf{v}_{i}^{d}\psi_{i}^{d}}\left[\frac{4P_{k}L_{kk}}{\sigma^{2}}\right] = C\frac{P_{k}L_{kk}}{P_{i^{*}}L_{ki^{*}}},$$
(42)

where $C = \mathbb{E}_{\mathbf{u}_k^l, \mathbf{v}_i^d \psi_i^d} \left[4/(\sum_{i \in \mathcal{Q}_k} \sum_m |\sum_d [\mathbf{v}_i^d \psi_i^d]_{(m,1)}^2) \right]$ is a positive constant.

Finally, substitute (38) and (42) into (34), we have

$$\overline{\operatorname{SER}}_{k}^{l}(\mathcal{A}) \leq \Theta\left(\sum_{i \in \mathcal{Q}_{k}} \frac{P_{k}L_{kk}}{P_{i}L_{ki}} + \sum_{i \in \mathcal{O}_{k}} \frac{P_{i}L_{ki}}{P_{k}L_{kk}}\right).$$
(43)

B. Lower Bound of the Average SER Performance

In this subsection, we shall obtain a lower bound of the average SER $\overline{\text{SER}}_k^l(\mathcal{A})$. Specifically, we shall prove $\overline{\text{SER}}_k^l(\mathcal{A}) \ge \Theta(\frac{P_i L_{ki}}{P_k L_{kk}}), \forall i \in \mathcal{O}_k$ and $\overline{\text{SER}}_k^l(\mathcal{A}) \ge \Theta(\frac{P_k L_{kk}}{P_i L_{ki}}), \forall i \in \mathcal{Q}_k$, respectively.

Proof of SER^l_k(A) ≥ Θ(P_{iL_{ki}}/P<sub>kL_{kk}), ∀i ∈ O_k: Suppose receiver k has perfect knowledge of all the data streams except x^d_i, ∀i ∈ O_k, ∀d. After cancelling the known data streams in stage II decoding, ỹ^l_k in (7) is given by:
</sub>

$$\widetilde{y}_{k}^{l} = \sqrt{P_{k}L_{kk}}H_{kk}^{ll}x_{k}^{l} + \sqrt{P_{i}L_{ki}}H_{ki}^{ld}x_{i}^{d} + (\mathbf{u}_{k}^{l})^{\dagger}\mathbf{z}_{k}$$

$$\tag{44}$$

Suppose $\Re\{x_k^l\} = \frac{\sqrt{2}}{2}$, the error rate of decoding $\Re\{x_k^l\}$ is given by (cf. (26)):

$$\Pr\left\{\Re\{\hat{x}_{k}^{l}\}\neq\Re\{x_{k}^{l}\}|\Re\{x_{k}^{l}\}=\frac{\sqrt{2}}{2}\right\}$$

$$=\sum_{x_{i}^{d}\in\mathcal{S}}\Pr\{x_{i}^{d}\}Q\left(\frac{\sqrt{2}}{2}|\sqrt{P_{k}L_{kk}}H_{kk}^{ll}|+\Re\{e^{-j\theta}\sqrt{P_{i}L_{ki}}H_{ki}^{ld}x_{i}^{d}\}\right).$$
(45)

where $e^{j\theta} = \frac{\sqrt{P_k L_{kk}} H_{kk}^{ll}}{|\sqrt{P_k L_{kk}} H_{kk}^{ll}|}$. Note that $\Re\{e^{-j\theta} H_{ki}^{ld} x_i^d\} \in \mathcal{N}(0, \frac{1}{2}\sigma^2)$, and hence the average error rate of decoding $\Re\{x_k^l\}$ is given by:

$$\mathbb{E}\left[\Pr\{\Re\{\hat{x}_{k}^{l}\}\neq\Re\{x_{k}^{l}\}|\Re\{x_{k}^{l}\}=\frac{\sqrt{2}}{2}\}\right] = \Pr\left\{\frac{\sqrt{2}}{2}|\sqrt{P_{k}L_{kk}}H_{kk}^{ll}|+\Re\{e^{-j\theta}\sqrt{P_{i}L_{ki}}H_{ki}^{ld}x_{i}^{d}\}<0\right\} \\
= \Pr\left\{|\sqrt{P_{k}L_{kk}}H_{kk}^{ll}|<\sqrt{P_{i}L_{ki}}\kappa\right\} = \frac{1}{2}\left(1-\sqrt{\frac{1}{1+P_{i}L_{ki}/P_{k}L_{kk}}}\right) = \frac{P_{i}L_{ki}}{4P_{k}L_{kk}} + O(\frac{P_{i}^{2}L_{ki}^{2}}{P_{k}^{2}L_{kk}^{2}}) = \Theta(\frac{P_{i}L_{ki}}{P_{k}L_{kk}}) \\$$
(46)

where $\kappa \in \mathcal{N}(0, \sigma^2)$ and $|H_{kk}^{ll}| \in \operatorname{Rayleigh}(\sigma^2)$. Similarly, we have

$$\mathbb{E}\left[\Pr\{\Re\{\hat{x}_{k}^{l}\}\neq\Re\{x_{k}^{l}\}|\Re\{x_{k}^{l}\}=-\frac{\sqrt{2}}{2}\}\right] \\
= \Pr\{\frac{\sqrt{2}}{2}|\sqrt{P_{k}L_{kk}}H_{kk}^{ll}|-\Re\{e^{-j\theta}\sqrt{P_{i}L_{ki}}H_{ki}^{ld}x_{i}^{d}\}<0\} = \Theta(\frac{P_{i}L_{ki}}{P_{k}L_{kk}}), \\
\mathbb{E}\left[\Pr\{\Im\{\hat{x}_{k}^{l}\}\neq\Im\{x_{k}^{l}\}|\Im\{x_{k}^{l}\}=\frac{\sqrt{2}}{2}\}\right] \\
= \Pr\{\frac{\sqrt{2}}{2}|\sqrt{P_{k}L_{kk}}H_{kk}^{ll}|+\Im\{e^{-j\theta}\sqrt{P_{i}L_{ki}}H_{ki}^{ld}x_{i}^{d}\}<0\} = \Theta(\frac{P_{i}L_{ki}}{P_{k}L_{kk}}), \\
\mathbb{E}\left[\Pr\{\Im\{\hat{x}_{k}^{l}\}\neq\Im\{x_{k}^{l}\}|\Im\{x_{k}^{l}\}=-\frac{\sqrt{2}}{2}\}\right] \\
= \Pr\{\frac{\sqrt{2}}{2}|\sqrt{P_{k}L_{kk}}H_{kk}^{ll}|-\Im\{e^{-j\theta}\sqrt{P_{i}L_{ki}}H_{ki}^{ld}x_{i}^{d}\}<0\} = \Theta(\frac{P_{i}L_{ki}}{P_{k}L_{kk}}).$$
(47)

Therefore, given that receiver k has perfect knowledge of all the data streams except $x_i^d, \forall i \in O_k, \forall d$, the average SER of decoding x_k^l is given by

$$\overline{\operatorname{SER}}_{i}^{d} \triangleq \mathbb{E}\left[\sum_{x_{k}^{l} \in \mathcal{S}} \operatorname{Pr}\{x_{k}^{l}\} \operatorname{Pr}\{\hat{x}_{k}^{l} \neq x_{k}^{l} | x_{k}^{l}\}\right] = \Theta\left(\frac{P_{i}L_{ki}}{P_{k}L_{kk}}\right),\tag{48}$$

and obviously we have $\overline{\operatorname{SER}}_{k}^{l}(\mathcal{A}) \geq \overline{\operatorname{SER}}_{i}^{d} = \Theta\left(\frac{P_{i}L_{ki}}{P_{k}L_{kk}}\right), \forall i \in \mathcal{O}_{k}.$

Proof of SER^l_k(A) ≥ Θ(P_{kLkk}/P_{iLki}), ∀i ∈ Q_k: Using similar arguments above and suppose receiver k has perfect knowledge of all the data streams except x^d_i, ∀i ∈ Q_k, ∀d. After cancelling the known data streams in the stage I decoding, y^l_k in (5) is given by:

$$y_k^l = \sqrt{P_k L_{kk}} H_{kk}^{ll} x_k^l + \sqrt{P_i L_{ki}} H_{ki}^{ld} x_i^d + (\mathbf{u}_k^l)^{\dagger} \mathbf{z}_k.$$
(49)

Given $\Re\{x_k^l\} = \frac{\sqrt{2}}{2}, \Re\{x_i^d\} = \frac{\sqrt{2}}{2}$, the error rate of decoding $\Re\{x_k^l\}$ is given by

$$\Pr\left\{\Re\{\hat{x}_{k}^{l}\}\neq\Re\{x_{k}^{l}\}|\Re\{x_{k}^{l}\},\Re\{x_{i}^{d}\}\right\}$$

$$\geq \Pr\left\{\Re\{\hat{x}_{i}^{d}\}\neq\Re\{x_{i}^{d}\}|\Re\{x_{k}^{l}\},\Re\{x_{i}^{d}\}\right\}\Pr\left\{\Re\{\hat{x}_{k}^{l}\}\neq\Re\{x_{k}^{l}\}|\Re\{x_{k}^{l}\},\Re\{x_{i}^{d}\},\Re\{\hat{x}_{i}^{d}\}\right\}$$

$$= Q\left(\frac{\sqrt{2}}{2}|\sqrt{P_{i}L_{ki}}H_{ki}^{ld}|+\Re\{e^{-j\psi}\sqrt{P_{k}L_{kk}}H_{kk}^{ll}\sqrt{2}\}\right)\cdot\left(Q\left(\frac{\sqrt{2}}{2}|\sqrt{P_{k}L_{kk}}H_{kk}^{ll}|+\Re\{e^{-j\theta}\sqrt{P_{i}L_{ki}}H_{ki}^{ld}\sqrt{2}\}\right)\right).$$
(50)

where $e^{j\theta} = \frac{\sqrt{P_k L_{kk}} H_{kk}^{lk}}{|\sqrt{P_k L_{kk}} H_{kk}^{ll}|}, e^{j\psi} = \frac{\sqrt{P_i L_{ki}} H_{ki}^{ld}}{|\sqrt{P_i L_{ki}} H_{ki}^{ld}|}, \vartheta = \theta - \psi$. The probability density function of $Z = |\frac{H_{kk}^{ll}}{H_{ki}^{ld}}|^2$ is given by $f_Z(z) = \frac{1}{(z+1)^2}$, and ϑ follows the uniform distribution between 0 and 2π . As a result, the average error rate of decoding $\Re\{x_k^l\}$ is given by

$$\mathbb{E}\left[\Pr\left\{\Re\{\hat{x}_{k}^{l}\}\neq\Re\{x_{k}^{l}\}|\Re\{x_{k}^{l}\},\Re\{x_{i}^{d}\}\right\}\right] \\
\geq \Pr\left\{\frac{1}{2\cos(\vartheta)}|\sqrt{P_{i}L_{ki}}H_{ki}^{ld}| < |\sqrt{P_{k}L_{kk}}H_{kk}^{ll}| < \cos(\vartheta)|\sqrt{P_{i}L_{ki}}H_{ki}^{ld}|\right\} \\
= \int_{-\pi/4}^{\pi/4} \frac{(\cos^{2}(\vartheta)-1/(4\cos^{2}(\vartheta)))P_{k}L_{kk}/P_{i}L_{ki}}{(1+P_{k}L_{kk}/(4P_{i}L_{ki}\cos^{2}(\vartheta)))(1+\cos^{2}(\vartheta)P_{k}L_{kk}/P_{i}L_{ki})}\mathrm{d}\vartheta = \Theta\left(\frac{P_{k}L_{kk}}{P_{i}L_{ki}}\right).$$
(51)

Furthermore, given that receiver k has perfect knowledge of all the data streams except $x_i^d, \forall i \in Q_k, \forall d$, the average error rate of decoding x_k^l is given by

$$\overline{\operatorname{SER}}_{i}^{d} \triangleq \mathbb{E}\left[\operatorname{Pr}\left\{\hat{x}_{k}^{l} \neq x_{k}^{l} | x_{k}^{l}, x_{i}^{d}\right\}\right] \geq \Theta(\frac{P_{k}L_{kk}}{P_{i}L_{ki}}).$$
(52)

Obviously we have $\overline{\text{SER}}_k^l(\mathcal{A}) \geq \overline{\text{SER}}_i^d \geq \Theta(\frac{P_k L_{kk}}{P_i L_{ki}}), \forall i \in \mathcal{Q}_k.$

Finally, from the results of upper and lower bound, we can conclude that

$$\overline{\operatorname{SER}}_{k}^{l}(\mathcal{A}) \triangleq \mathbb{E}_{\mathcal{H}}\left[\operatorname{SER}_{k}^{l}(\mathcal{A},\mathcal{H})\right] \triangleq \Theta\left(\sum_{i \in \mathcal{Q}_{k}} \frac{P_{k}L_{kk}}{P_{i}L_{ki}} + \sum_{i \in \mathcal{O}_{k}} \frac{P_{i}L_{ki}}{P_{k}L_{kk}}\right).$$
(53)

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Fig. 1. Quasi-static K-user MIMO complex Gaussian interference channels. Each M-antenna transmitter tries to transmit D independent data streams to its corresponding N-antenna receiver. All the nodes are randomly distributed in the $a \times b$ rectangular area.

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Fig. 2. System level block diagram of the proposed PIAID Scheme.



Fig. 3. An example of 4-user interference channels where M = N = 2 and D = 1. Using the feasibility condition of MIMO IA in [10], only two transmitters can be aligned at each receiver. Specifically, the 4-user interference channels are represented by a bipartite graph $\mathcal{B} = (\mathcal{R}, \mathcal{T}, \mathcal{E})$, where $\mathcal{R} = \{r_1, \dots, r_4\}$ is the set of the receive nodes, $\mathcal{T} = \{t_1, \dots, t_4\}$ is the set of the transmit nodes, and $\mathcal{E} = \{[r_k, t_i], \forall k, i \text{ and } k \neq i\}$ is the set of the edges. Furthermore, the PIA set is given by $\mathcal{A} = \{\mathcal{A}_1 = \{2, 3\}, \mathcal{A}_2 = \{3, 4\}, \mathcal{A}_3 = \{4, 1\}, \mathcal{A}_4 = \{1, 2\}\}$ and the corresponding subset of edges is given by $\mathcal{E}_s = \{[r_1, t_2], [r_1, t_3], [r_2, t_3], [r_2, t_4], [r_3, t_4], [r_3, t_1], [r_4, t_1], [r_4, t_2]\}.$



Fig. 4. Average end-to-end SER performance of the ID detector versus the interference power at the interference limited regime. Specifically, the input to the ID detector is given by $y = H_1x_1 + \sqrt{P_2}H_2x_2$, where $x_1 \in S$ is the desired signal, $x_2 \in S$ is the interference, $H_1, H_2 \sim C\mathcal{N}(0, 1)$, and the interference power is P_2 . The average end-to-end SER is given by $\overline{\text{SER}}(P_2) \triangleq \sum_{x_1} \Pr{\{\hat{x}_1 \neq x_1 | x_1, P_2\}}$.

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Fig. 5. Illustration of Algorithm 3 (SDR-SID Algorithm).



Fig. 6. Average end-to-end SER performance versus the receive E_s/N_0 (dB). The setup is given by K = 5 (number of users), $\{M = 3, N = 2\}$ (number of transmit and receive antennas), D = 1 (number of data stream), and $\alpha = 3$ (number of aligned users for feasible interference alignment).



Fig. 7. Average end-to-end SER performance versus the receive E_s/N_0 (dB). The setup is given by K = 6 (number of users), $\{M = 3, N = 2\}$ (number of transmit and receive antennas), D = 1 (number of data stream), and $\alpha = 3$ (number of aligned users for feasible interference alignment).



Fig. 8. Cumulative Distribution Function (CDF) of the SER per data stream with receive $E_s/N_0 = 25$ dB. The setup is given by K = 5 (number of users), $\{M = 3, N = 2\}$ (number of transmit and receive antennas), D = 1 (number of data stream), and $\alpha = 3$ (number of aligned users for feasible interference alignment).



Fig. 9. Cumulative Distribution Function (CDF) of the SER per data stream with receive $E_s/N_0 = 25$ dB. The setup is given by K = 6 (number of users), $\{M = 3, N = 2\}$ (number of transmit and receive antennas), D = 1 (number of data stream), and $\alpha = 3$ (number of aligned users for feasible interference alignment).