Blind Adaptive Interference Suppression Based on Set-Membership Constrained Constant-Modulus Algorithms with Time-Varying Bounds

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Abstract—This work presents blind constrained constant modulus (CCM) adaptive algorithms based on the set-membership filtering (SMF) concept and incorporates dynamic bounds for interference suppression applications. We develop stochastic gradient and recursive least squares type algorithms based on the CCM design criterion in accordance with the specifications of the SMF concept. We also propose a blind framework that includes channel and amplitude estimators that take into account parameter estimation dependency, multiple access interference (MAI) and inter-symbol interference (ISI) to address the important issue of bound specification in multiuser communications. A convergence and tracking analysis of the proposed algorithms is carried out along with the development of analytical expressions to predict their performance. Simulations for a number of scenarios of interest with a DS-CDMA system show that the proposed algorithms outperform previously reported techniques with a smaller number of parameter updates and a reduced risk of overbounding or underbounding.

Index Terms—Interference suppression, blind adaptive estimation, set-membership estimation, spread spectrum systems.

I. INTRODUCTION

Set-membership filtering (SMF) [2], [3], [5], [6] is a class of recursive estimation algorithms that, on the basis of a pre-determined error bound, seeks a set of parameters that yield bounded filter output errors. These algorithms have been applied to a variety of applications including adaptive equalization [5] and multi-access interference suppression [6], [7]. The SMF algorithms are able to combat conflicting requirements such as fast convergence and low misadjustment by introducing a modification on the objective function. These algorithms exhibit reduced complexity due to data-selective updates, which involve two steps: a) information evaluation and b) update of parameter estimates. If the filter update does not occur frequently and the information evaluation does not involve much computational complexity, the overall complexity can be significantly reduced.

Adaptive SMF algorithms usually achieve good convergence and tracking performance due to the use of an adaptive step size or an adaptive forgetting factor for each update. This translates into reduced complexity due to the data selective updating. However, the performance of SMF techniques depends on the error-bound specification, which is very difficult

to obtain in practice due to the lack of knowledge of the environment and its dynamics. In wireless networks characterized by non-stationary environments, where users often enter and exit the system, it is very difficult to choose an error bound and the risk of overbounding (when the error bound is larger than the actual one) and underbounding (when the error bound is smaller than the actual one) is significantly increased, leading to a performance degradation. In addition, when the measured noise in the system is time-varying and the multiple access interference (MAI) and the intersymbol interference (ISI) encountered by a receiver in a wireless network are highly dynamic, the selection of an error-bound is further complicated. This is especially relevant for low-complexity estimation problems encountered in applications that include mobile units and wireless sensor networks [9], [10], where the sensors have limited signal processing capabilities and power consumption is of central importance. These problems suggest the deployment of mechanisms to automatically adjust the error bound in order to guarantee good performance and a low update rate (UR).

A. Prior and Related Work

In this context, blind methods are appealing because they can alleviate the need for training sequences or pilots, thereby increasing the throughput and efficiency of wireless networks. In particular, blind algorithms based on constrained optimization techniques are important in several areas of signal processing and communications such as beamforming and interference suppression. The constrained optimization required in these applications deals with linear constraints that correspond to prior knowledge of certain parameters such as the direction of arrival (DoA) of user signals in antenna array processing [23] and the signature sequence of the desired signal in DS-CDMA systems [24], [25]. Therefore, linear signal models and constraints can be used to describe various wireless communications systems including multi-input multi-output (MIMO) and orthogonal frequency-division multiplexing (OFDM) systems. For instance, linear constraints that incorporate the knowledge of user signatures of a DS-CDMA system can also be used to exploit the knowledge of the spatial signatures of MIMO systems. A number of blind algorithms with different trade-offs between performance and complexity have been reported in the last decades [24]-[36]. Local scattering, synchronization and estimation errors, imperfectly calibrated arrays and imprecisely

known wave field propagation conditions are typical sources of uncertainties that can lead to a performance degradation of blind algorithms. The literature indicates that the CCM-based algorithms [30]-[36] have a superior performance to algorithms based on the constrained minimum variance (CMV) criterion [24]- [27]. The CCM-based algorithms exploit prior knowledge about the constant modulus property of signals like M-PSK, which results in an improved performance over CMV-type techniques and a performance that is very close to the linear minimum mean-square error (MMSE) training-based techniques. Moreover, the CCM-type algorithms are robust against errors in the effective signature sequence required for blind parameter estimation, which prevents a severe performance degradation in the presence of uncertainties. These features make CCM-type algorithms excellent candidates for interference suppression in wireless networks. The need for the adjustment of parameters (step size, forgetting factor) and the computational complexity of CCM-based techniques calls for approaches like SMF. Prior work on SMF blind algorithms for interference suppression is very limited [7], [8]. The use of time-varying bounds is also restricted to applications where one assumes that the "true" error bound is constant [16] and to the parameter-dependent error bound recently proposed in [13], [15]. The time-varying bound techniques so far reported are not blind and do not exploit these mechanisms for channel and parameter estimation.

B. Contributions

In this work, we propose set-membership blind adaptive constrained algorithms based on the CCM criterion. Preliminary results have been reported in [22]. In particular, we derive stochastic gradient (SG) and recursive least squares (RLS)-type CCM algorithms designed in accordance with the specifications of the SMF concept. The second contribution is a low-complexity blind framework for parameter estimation, and tracking of parameter evolution, MAI and ISI that relies on simple estimation techniques and employs the proposed set-membership CCM algorithms with a time-varying bound. The third contribution of this work is an analysis of the optimization problem that gives rise to the proposed algorithms along with a mean-squared error (MSE) convergence and tracking analysis using the energy conservation approach [39] for predicting the performance of the algorithms. A simulation study considers an interference suppression application to DS-CDMA systems, which compares the performance of the proposed and existing algorithms, and discusses the main features of the algorithms.

This paper is structured as follows. Section II describes the DS-CDMA system model and briefly reviews linearly constrained receivers. Section III introduces the SM blind algorithms with time-varying bound. Section IV proposes blind parameter dependent and interference dependent bounds along with blind channel and amplitude estimation algorithms. Section V is dedicated to the analysis of the proposed algorithms. Section VI is devoted to the presentation and discussion of numerical results, while Section VII gives the conclusions.

II. DS-CDMA SYSTEM MODEL AND LINEARLY CONSTRAINED RECEIVERS

Let us consider the uplink of a symbol synchronous DS-CDMA system with K users, N chips per symbol and L_p propagation paths. A synchronous model is assumed for simplicity since it captures most of the features of more realistic asynchronous models with small to moderate delay spreads. The modulation is assumed to have constant modulus. Let us assume that the signal has been demodulated at the base station, the channel is constant during each symbol and the receiver is perfectly synchronized with the main channel path. The received signal after filtering by a chip-pulse matched filter and sampled at chip rate yields an M-dimensional received vector at time i

$$\boldsymbol{r}[i] = \sum_{k=1}^{K} A_k[i]b_k[i]\boldsymbol{C}_k\boldsymbol{h}_k[i] + \boldsymbol{\eta}_k[i] + \boldsymbol{n}[i], \qquad (1)$$

where $M = N + L_p - 1$, $\boldsymbol{n}[i] = [n_1[i] \ \dots \ n_M[i]]^T$ is the complex Gaussian noise vector with zero mean and $E[\boldsymbol{n}[i]\boldsymbol{n}^H[i]] = \sigma^2 \boldsymbol{I}$ independent and identically distributed samples, $(.)^T$ and $(.)^H$ denote transpose and Hermitian transpose, respectively, and $E[.]$ stands for expected value. The user symbols are denoted by $b_k[i]$, the amplitude of user k is $A_k[\mathrm{i}]$, and $\boldsymbol{\eta}_k[i]$ is the intersymbol interference (ISI) for user k. The signature of user k is represented by $\boldsymbol{c}_k = [c_k(1)\dots c_k(N)]^T$, the $M \times L_p$ constraint matrix $\boldsymbol{C}_k$ that contains one-chip shifted versions of the signature sequence for user k and the $L_p \times 1$ vector $\boldsymbol{h}_k[i]$ with the multipath components are described by

$$\boldsymbol{C}_k = \begin{bmatrix} c_k(1) & & \boldsymbol{0} \\ \vdots & \ddots & c_k(1) \\ c_k(N) & & \vdots \\ \boldsymbol{0} & \ddots & c_k(N) \end{bmatrix}, \boldsymbol{h}_k[i] = \begin{bmatrix} h_{k,0}[i] \\ \vdots \\ h_{k,L_p-1}[i] \end{bmatrix}.$$
$$(2)$$

The MAI comes from the non-orthogonality between the received signature sequences. The ISI originates from the multipath propagation effects of the channel, depends on the length of the channel response and how it is related to the length of the chip sequence. We define L_s as the ISI span, i.e., the number of symbols affected by the channel. For $L_p = 1$, $L_s = 1$ (no ISI), for $1 < L_p \leq N, L_s = 3$, for $N < L_p \leq 2N, L_s = 5$, and so on. At time instant i we will have ISI coming not only from the previous time instants but also from the next symbols. The linear model in (1) can be used to represent other wireless communications systems including multi-input multi-output (MIMO) and orthogonal frequency-division multiplexing (OFDM) systems. For example, the user signatures of a DS-CDMA system are equivalent to the spatial signatures of a MIMO system.

In order to describe the design of linearly constrained receivers, we consider the received vector $\boldsymbol{r}[i]$, the $M \times L_p$ constraint matrix $\boldsymbol{C}_k$ that contains one-chip shifted versions of the signature sequence for user k and the $L_p \times 1$ vector $\boldsymbol{h}_k[i] = [h_{k,0}[i] \ \dots \ h_{k,L_p-1}[i]]^T$ with the multipath components to be estimated. The CCM linear receiver design is equivalent to determining an FIR filter $\boldsymbol{w}_k[i]$ with M coefficients that

provide an estimate of the desired symbol as follows

$$z_k[i] = \boldsymbol{w}_k^H[i]\boldsymbol{r}[i], \qquad (3)$$

where the detected symbol is given by $\hat{b}_k[i] = Q(\boldsymbol{w}_k^H[i]\boldsymbol{r}[i])$, where $Q(\cdot)$ is a function that performs the detection according to the constellation employed.

The design of the receive filter $\boldsymbol{w}_k[i]$ is based on the optimization of the CM cost function

$$J_{CM}(\boldsymbol{w}_k[i]) = E\Big[(|\boldsymbol{w}_k^H[i]\boldsymbol{r}[i]|^2 - 1)^2\Big] \qquad (4)$$

subject to the constraints given by $\boldsymbol{w}_k^H[i]\boldsymbol{p}_k[i] = \nu$, where $\boldsymbol{p}_k[i] = \boldsymbol{C}_k\boldsymbol{h}_k[i]$ is the effective signature vector that corresponds to the convolution between the original signature sequence and the channel gains, and ν is a constant to ensure the convexity of the optimization problem as will be discussed later on. This approach assumes the knowledge of the channel. However, when multipath is present these parameters are unknown and time-varying, requiring channel estimation. The CCM receive filter expression that iteratively solves the constrained optimization problem in (4) is given by

$$\boldsymbol{w}_k[i+1] = \boldsymbol{R}_k^{-1}[i]\Big[\boldsymbol{d}_k[i] - \Big(\boldsymbol{p}_k^H[i]\boldsymbol{R}_k^{-1}[i]\boldsymbol{p}_k[i]\Big)^{-1}$$
$$\cdot \Big(\boldsymbol{p}_k^H[i]\boldsymbol{R}_k^{-1}[i]\boldsymbol{d}_k[i] - \nu\Big)\boldsymbol{p}_k[i]\Big], \quad i = 1, 2, \dots \qquad (5)$$

where $z_k[i] = \boldsymbol{w}_k^H[i]\boldsymbol{r}[i]$, $\boldsymbol{R}_k[i] = E[|z_k[i]|^2\boldsymbol{r}[i]\boldsymbol{r}^H[i]]$, $\boldsymbol{d}_k[i] = E[z_k^*[i]\boldsymbol{r}[i]]$. A detailed derivation of the CCM estimation approach can be found in [33], [34], [36]. It should be remarked that the expression in (5) is a function of previous values of the filter $\boldsymbol{w}_k[i]$ and therefore must be iterated in order to reach a solution. In addition to this, the iterative method in (5) assumes the knowledge of the channel parameters. Since there is a large number of applications that have to deal with unknown multipath propagation, it is also important to be able to blindly estimate the multipath components.

In order to blindly estimate the channel, a designer can adopt the blind channel estimation procedure based on the subspace approach reported in [26], [41] and which is described by

$$\hat{\boldsymbol{h}}_k[i] = \arg\min_{\boldsymbol{h}_k[i]} \boldsymbol{h}_k^H[i]\boldsymbol{C}_k^H\boldsymbol{R}^{-1}[i]\boldsymbol{C}_k\boldsymbol{h}_k[i] \qquad (6)$$

subject to $||\boldsymbol{h}_k[i]|| = 1$, where $\boldsymbol{R}[i] = E[\boldsymbol{r}[i]\boldsymbol{r}^H[i]]$. The solution is the eigenvector of the $L_p \times L_p$ matrix corresponding to the minimum eigenvalue of $\boldsymbol{C}_k^H\boldsymbol{R}^{-1}[i]\boldsymbol{C}_k$ obtained by an eigenvalue decomposition (EVD). Here, we use $\boldsymbol{R}_k[i]$ in lieu of $\boldsymbol{R}[i]$ to avoid the estimation of both $\boldsymbol{R}[i]$ and $\boldsymbol{R}_k[i]$, which shows no performance loss as reported and investigated in [33], [34], [36].

III. SET-MEMBERSHIP BLIND ADAPTIVE CONSTRAINED ALGORITHMS WITH TIME VARYING ERROR BOUNDS

In this section, we describe an adaptive filtering framework that combines the set-membership (SM) concept with blind constrained algorithms based on the CCM design. We also introduce simple time-varying error bounds to take into consideration the evolution of the receive filter and the MAI and ISI

effects. In SM filtering [2], the parameter vector $\boldsymbol{w}_k[i]$ for user k is designed to achieve a specified bound on the magnitude of an estimated quantity $z_k[i]$. As a result of this constraint, the SM blind adaptive algorithm will only perform filter updates for certain data. Let $\Theta[i]$ represent the set containing all $\boldsymbol{w}_k[i]$ that yield an estimation quantity upper bounded in magnitude by a time-varying error bound $\gamma_k[i]$. Thus, we can write

$$\Theta[i] = \bigcap_{(\boldsymbol{r}[i]) \in \mathcal{S}} \{\boldsymbol{w}_k \in \mathcal{C}^M : |z_k[i]| \le \gamma_k[i]\}, \qquad (7)$$

where $\boldsymbol{r}[i]$ is the observation vector, $\mathcal{S}$ is the set of all possible data pairs $(b_k[i], \boldsymbol{r}[i])$ and the set $\Theta[i]$ is referred to as the feasibility set, and any point in it is a valid estimate $z_k[i] = \boldsymbol{w}_k^H[i]\boldsymbol{r}[i]$. Since it is not practical to predict all data pairs, adaptive methods work with the membership set $\psi_i = \bigcap_{m=1}^{i} \mathcal{H}_m$ provided by the observations, where $\mathcal{H}_m = \{\boldsymbol{w}_k \in \mathcal{C}^M : |z_k[i]| \le \gamma_k[i]\}$. In order to devise an effective SM algorithm, the bound $\gamma_k[i]$ must be appropriately chosen. Due to the time-varying nature of many practical environments, this bound should also be adaptive and capable of estimating certain characteristics for the SM estimation technique. We devise SM-CCM algorithms equipped with variable step sizes and forgetting factors that are able to automatically tune to different situations, which is an advantage over methods operating with fixed parameters in time-varying scenarios. In what follows, we derive SM-CCM algorithms for blind parameter estimation that assume time-varying error bounds.

A. Set-Membership CCM Stochastic Gradient-Type Algorithm

Here we develop the set-membership CCM stochastic gradient-type (SM-CCM-SG) algorithm. The basic idea is to devise a gradient descent strategy to compute a parameter vector $\boldsymbol{w}_k$ for user k that minimizes the instantaneous CM cost function subject to certain constraints, which requires for adaptation that the square of the error $e_k^2[i]$ exceeds a specified error bound $\gamma_k^2[i]$. Mathematically, the proposed SM-CCM-SG algorithm solves the following optimization problem

$$\begin{aligned} \text{minimize} \quad & J_{CM}(\boldsymbol{w}_k[i]) = (|\boldsymbol{w}_k^H[i]\boldsymbol{r}[i]|^2 - 1)^2 = e_k^2[i] \\ \text{subject to} \quad & \boldsymbol{w}_k^H[i]\boldsymbol{p}_k[i] = \nu \\ & \text{whenever} \ \ e_k^2[i] \ge \gamma_k^2[i] \end{aligned} \qquad (8)$$

This problem can be solved using the method of Lagrange multipliers using the equality constraint $e_k^2[i] = \gamma_k^2[i]$ [1]:

$$\mathcal{L}(\boldsymbol{w}_k[i], \kappa_k) = (|\boldsymbol{w}_k^H[i]\boldsymbol{r}[i]|^2 - 1)^2 + [(\boldsymbol{w}_k^H[i]\boldsymbol{p}_k[i] - \nu)\kappa_k^*]$$
$$+ [\kappa_k^*(\boldsymbol{p}_k^H[i]\boldsymbol{w}_k[i] - \nu)], \qquad (9)$$

where κ_k is a Lagrange multiplier. Computing the gradient terms of (9) and equating them to zero, we obtain

$$\begin{aligned} \nabla_{\boldsymbol{w}_k^*[i]}\mathcal{L}(\boldsymbol{w}_k[i], \kappa_k) &= 2(e_k[i]\boldsymbol{r}[i]z_k^*[i]) + \boldsymbol{p}_k[i]\kappa_k = 0 \\ \nabla_{\kappa_k^*}\mathcal{L}(\boldsymbol{w}_k[i], \kappa_k) &= \boldsymbol{w}_k^H[i]\boldsymbol{p}_k[i] - \nu = 0. \end{aligned} \qquad (10)$$

We employ a gradient descent rule to solve for the above equations. Using the first equation of (10), we have

$$\boldsymbol{w}_k[i+1] = \boldsymbol{w}_k[i] - \mu_k(e_k[i]z_k^*[i]\boldsymbol{r}[i] + \boldsymbol{p}_k[i]\kappa_k) \qquad (11)$$

Using the second equation of (10), we obtain

$$\boldsymbol{w}_k[i+1] = \boldsymbol{\Pi}_k[i](\boldsymbol{w}_k[i]-\mu_k e_k[i]z_k^*[i]\boldsymbol{r}[i])+\nu\boldsymbol{p}_k[i](\boldsymbol{p}_k^H[i]\boldsymbol{p}_k[i])^{-1}$$
(12)

where μ_k is the effective step size, $e_k[i] = |z_k[i]|^2 - 1$ is the error signal for user k, and $\boldsymbol{\Pi}_k[i] = \boldsymbol{I} - \boldsymbol{p}_k[i](\boldsymbol{p}_k^H[i]\boldsymbol{p}_k[i])^{-1}\boldsymbol{p}_k^H[i]$ is a projection matrix that ensures the constraint and $\boldsymbol{I}$ is an identity matrix. By imposing the condition to update whenever $e_k^2[i] \geq \gamma_k^2[i]$ we arrive at the set of all $\boldsymbol{w}_k$ that satisfy

$$\sqrt{1-\gamma_k[i]} \leq |\boldsymbol{w}_k^H[i+1]\boldsymbol{r}[i]| \leq \sqrt{1+\gamma_k[i]} \quad (13)$$

It can be verified that the set above is non-convex and comprises two parallel hyper-strips in the parameter space. From the above conditions we consider two cases: i) $|\boldsymbol{w}_k^H[i+1]\boldsymbol{r}[i]| \leq \sqrt{1+\gamma_k[i]}$ and ii) $|\boldsymbol{w}_k^H[i+1]\boldsymbol{r}[i]| \geq \sqrt{1-\gamma_k[i]}$. By substituting the recursion obtained in (12) into $|\boldsymbol{w}_k^H[i+1]\boldsymbol{r}[i]|$, we have

$$|\boldsymbol{w}_k^H[i+1]\boldsymbol{r}[i]| = |z_k[i]-\mu_k[i]e_k[i]z_k[i]\boldsymbol{r}^H[i]\boldsymbol{\Pi}_k[i]\boldsymbol{r}[i]|. \quad (14)$$

Using the above expression we have for case i):

$$|z_k[i] - \mu_k[i]e_k[i]z_k[i]\boldsymbol{r}^H[i]\boldsymbol{\Pi}_k[i]\boldsymbol{r}[i]| = \sqrt{1+\gamma_k[i]} \quad (15)$$

which leads to

$$\mu_k[i] = \left(\frac{1-\sqrt{1+\gamma_k[i]}}{|z_k[i]|}\right)\frac{1}{e_k[i]\boldsymbol{r}^H[i]\boldsymbol{\Pi}_k[i]\boldsymbol{r}[i]} \quad (16)$$

Using (14) we have for case ii):

$$|z_k[i] - \mu_k[i]e_k[i]z_k[i]\boldsymbol{r}^H[i]\boldsymbol{\Pi}_k[i]\boldsymbol{r}[i]| = \sqrt{1-\gamma_k[i]} \quad (17)$$

which results in the following

$$\mu_k[i] = \left(\frac{1-\sqrt{1-\gamma_k[i]}}{|z_k[i]|}\right)\frac{1}{e_k[i]\boldsymbol{r}^H[i]\boldsymbol{\Pi}_k[i]\boldsymbol{r}[i]} \quad (18)$$

The resulting SM-CCM-SG algorithm is described by

$$\boldsymbol{w}_k[i+1] = \boldsymbol{\Pi}_k[i](\boldsymbol{w}_k[i]-\mu_k[i]e_k[i]z_k^*[i]\boldsymbol{r}[i])+\nu\boldsymbol{p}_k[i](\boldsymbol{p}_k^H[i]\boldsymbol{p}_k[i])^{-1}$$
(19)

where

$$\mu_k[i] = \begin{cases} \left(\frac{1-\sqrt{1+\gamma_k[i]}}{|z_k[i]|}\right)\frac{1}{e_k[i]\boldsymbol{r}^H[i]\boldsymbol{\Pi}_k[i]\boldsymbol{r}[i]} & \text{if } |z_k[i]| \geq \sqrt{(1+\gamma_k[i])} \\ \left(\frac{1-\sqrt{1-\gamma_k[i]}}{|z_k[i]|}\right)\frac{1}{e_k[i]\boldsymbol{r}^H[i]\boldsymbol{\Pi}_k[i]\boldsymbol{r}[i]} & \text{if } |z_k[i]| \leq \sqrt{(1-\gamma_k[i])} \\ 0 & \text{otherwise} \end{cases}$$
(20)

The SM-CCM-SG algorithm described in (19)-(20) requires $M + \text{UR}(5M - 1)$ additions and $M + 1 + \text{UR}(5M + 4)$ multiplications per received symbol, where UR is the update rate.

B. Set-Membership CCM RLS-Type Algorithm

In this part, we derive the set-membership CCM recursive least-squares-type (SM-CCM-RLS) algorithm. The idea is to devise a least-squares method to calculate a parameter vector $\boldsymbol{w}_k$ used at the receiver for user k that minimizes the exponentially-weighted CM cost function subject to constraints that require that the squared error of the filter exceed a specified time-varying error bound $\gamma_k^2[i]$. Mathematically, the proposed SM-CCM-RLS algorithm solves the optimization problem

$$\text{minimize} \quad J_{CM}^{LS}(\boldsymbol{w}_k[i]) = \sum_{l=1}^{i} \lambda_k^{i-l}[i](|\boldsymbol{w}_k^H[i]\boldsymbol{r}[l]|^2 - 1)^2$$
$$\text{subject to} \quad \boldsymbol{w}_k^H[i]\boldsymbol{p}_k[i] = \nu$$
$$\text{whenever} \quad e_k^2[i] \geq \gamma_k^2[i],$$
(21)

where $\lambda_k[i]$ is a time-varying forgetting factor. Similarly to the case of the SM-CCM-SG algorithm, this problem can be solved with the method of Lagrange multipliers [1] along with the use of the condition $e_k^2[i] \geq \gamma_k^2[i]$ to save computations

$$\mathcal{L}(\boldsymbol{w}_k[i], \epsilon_k) = \sum_{l=1}^{i} \lambda_k^{i-l}[i](|\boldsymbol{w}_k^H[i]\boldsymbol{r}[l]|^2 - 1)^2 + [\epsilon_k^*(\boldsymbol{p}_k^H[i]\boldsymbol{w}_k[i] - \nu)],$$
(22)

where ϵ_k is a Lagrange multiplier. Computing the gradient terms of the Lagrangian in (22) and equating them to zero, we obtain

$$\nabla_{\boldsymbol{w}_k^*[i]}\mathcal{L}(\boldsymbol{w}_k[i], \epsilon_k) = \sum_{l=1}^{i} \lambda_k^{i-l}[i](|z_k[l]|^2 - 1)(\boldsymbol{r}[i]\boldsymbol{r}^H[i]\boldsymbol{w}_k[i] + \boldsymbol{p}_k[i]\epsilon_k =$$
$$\nabla_{\epsilon_k^*}\mathcal{L}(\boldsymbol{w}_k[i], \epsilon_k) = \boldsymbol{w}_k^H[i]\boldsymbol{p}_k[i] - \nu = 0.$$
(23)

Solving for the above equations, we have

$$\boldsymbol{w}_k[i+1] = \hat{\boldsymbol{R}}_k^{-1}[i]\Big[\hat{\boldsymbol{d}}_k[i] - \left(\boldsymbol{p}_k^H[i]\hat{\boldsymbol{R}}_k^{-1}[i]\boldsymbol{p}_k[i]\right)^{-1}$$
$$\cdot \left(\boldsymbol{p}_k^H[i]\hat{\boldsymbol{R}}_k^{-1}[i]\hat{\boldsymbol{d}}_k[i] - \nu\right)\boldsymbol{p}_k[i]\Big],$$
(24)

where $z_k[i] = \boldsymbol{w}_k^H[i]\boldsymbol{r}[i]$, $\hat{\boldsymbol{R}}_k[i] = \sum_{l=1}^{i} \lambda_k^{i-l}[i]|z_k[l]|^2\boldsymbol{r}[l]\boldsymbol{r}^H[l]$, $\hat{\boldsymbol{d}}_k[i] = \sum_{l=1}^{i} \lambda_k^{i-l}[i]z_k^*[l]\boldsymbol{r}[l]$. Firstly, we need to compute $\hat{\boldsymbol{d}}_k[i]$ and this is performed by the following recursion

$$\hat{\boldsymbol{d}}_k[i] = \hat{\boldsymbol{d}}_k[i-1] + \lambda_k[i]z_k^*[i]\boldsymbol{r}[i]. \quad (25)$$

At this point, we need to compute $\hat{\boldsymbol{R}}_k^{-1}[i]$ efficiently and this is done by applying the matrix inversion lemma [1], which yields

$$\hat{\boldsymbol{R}}_k^{-1}[i] = \hat{\boldsymbol{R}}_k^{-1}[i-1] - \frac{\lambda_k[i]|z_k[i]|^2\hat{\boldsymbol{R}}_k^{-1}[i]\boldsymbol{r}[i]\boldsymbol{r}^H[i]\hat{\boldsymbol{R}}_k^{-1}[i-1]}{1 + \lambda_k[i]|z_k[i]|^2\boldsymbol{r}^H[i]\hat{\boldsymbol{R}}_k^{-1}[i-1]\boldsymbol{r}[i]}$$
(26)

The last step of the SM-CCM-RLS algorithm is the use of the condition $e_k^2[i] \geq \gamma_k^2[i]$ to save computations and to adjust the optimal $\lambda_k[i]$. In order to adjust $\lambda_k[i]$, the authors in [6] have advocated a strategy that yields bounding ellipsoids that lead to a simple innovation check with linear complexity, which considers the cost function

$$\mathcal{C}_{\lambda_k[i]} = \lambda_k[i]\left[\frac{e_k[i]}{\gamma_k^2[i]}\left(\frac{1}{1 + \lambda_k[i]\boldsymbol{r}^H[i]\hat{\boldsymbol{R}}_k^{-1}[i-1]\boldsymbol{r}[i]}\right) - 1\right].$$
(27)

The maximization of the cost function in (27) leads to the innovation check of the proposed SM-CCM-RLS algorithm:

$$\lambda_k[i] = \begin{cases} \frac{1}{\boldsymbol{r}^H[i]\hat{\boldsymbol{R}}_k^{-1}[i-1]\boldsymbol{r}[i]}\left(\frac{|e_k[i]|}{\gamma_k[i]}-1\right) & \text{if } |e_k^*[i]| > \gamma_k[i], \\ 0 & \text{otherwise.} \end{cases}$$
(28)

The SM-CCM-RLS algorithm described in (24)-(26) and (28) requires $M + \text{UR}(4M^2 + 3M)$ additions and $M + 2 + \text{UR}(4M^2 + 6M + 3)$ multiplications per received symbol. It is worth mentioning that the computational savings can be quite substantial if the algorithm operates with a low UR.

IV. BLIND PARAMETER ESTIMATION AND TIME-VARYING BOUNDS

This section presents a blind framework employed to compute time-varying error bounds $\gamma_k[i]$ based on parameter and interference dependency. The proposed blind framework is an extension of the approach in [21] that computes time-varying error bounds, and performs interference estimation and tracking. In contrast to [21] that considers training-based recursions, a blind procedure for estimating MAI and ISI power levels is presented with a set-membership blind channel estimator and a blind amplitude estimator, which are employed in the adaptive error bound for the SM adaptive algorithms.

A. Parameter Dependent Bound

Here, we describe a parameter dependent bound (PDB), that is similar to the one proposed in [13] and considers the evolution of the parameter vector $\boldsymbol{w}_k[i]$ for the desired user (user k). The PDB recursion computes a bound for SM adaptive algorithms and is described by

$$\gamma_k[i+1] = (1-\beta)\gamma_k[i] + \beta\sqrt{\alpha||\boldsymbol{w}_k[i]||^2\hat{\sigma}^2[i]}, \quad (29)$$

where β is a forgetting factor that should be adjusted to ensure an appropriate time-averaged estimate of the evolution of the power of the parameter vector $\boldsymbol{w}_k[i]$. The quantity $\alpha||\boldsymbol{w}_k[i]||^2\hat{\sigma}^2[i]$ is the variance of the inner product of $\boldsymbol{w}_k[i]$ with $\boldsymbol{n}[i]$ which provides information on the evolution of $\boldsymbol{w}_k[i]$, where α is a tuning parameter and $\hat{\sigma}^2[i]$ is an estimate of the noise power. This kind of recursion helps avoiding too high or low values of the squared norm of $\boldsymbol{w}_k[i]$ and provides a smoother evolution of its trajectory for use in the time-varying bound. The noise power at the receiver should be estimated via a time average recursion. In this work, we will assume that it is known at the receiver.

B. Parameter and Interference Dependent Bound

In this part, we develop a blind interference estimation and tracking procedure to be combined with a parameter dependent bound and incorporated into a time-varying error bound for SM recursions. The MAI and ISI power estimation scheme, outlined in Fig. 1, employs both the RAKE receiver and the linear receiver described in (3) for subtracting the desired user signal from $\boldsymbol{r}[i]$ and estimating MAI and ISI power levels. With the aid of adaptive algorithms, we design the linear

receiver, estimate the channel modeled as an FIR filter for the RAKE receiver and obtain the detected symbol $\hat{b}_k[i]$, which is combined with an amplitude estimate $\hat{A}_k[i]$ for subtracting the desired signal from the output $x_k[i]$ of the RAKE. Then, the difference $d_k[i]$ between the desired signal and $x_k[i]$ is used to estimate the MAI and ISI power.
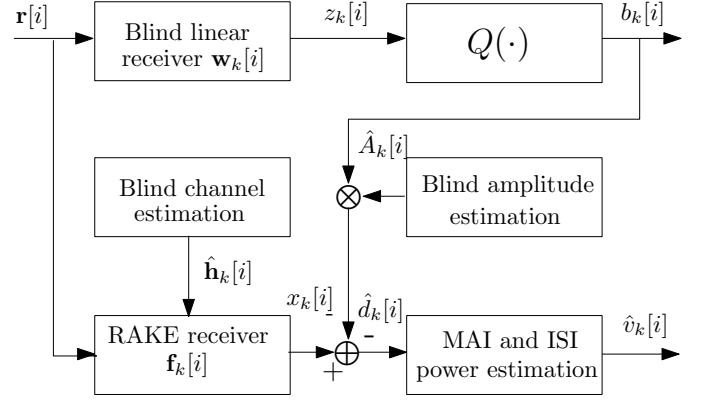

Fig. 1. Block diagram of the proposed blind scheme.

C. Blind Interference Estimation and Tracking

Let us consider the RAKE receiver with perfect channel knowledge, whose parameter vector $\boldsymbol{f}_k[i] = \boldsymbol{C}_k\boldsymbol{h}_k[i]$ for user k (desired one) corresponds to the effective signature sequence at the receiver, i.e. $\tilde{\boldsymbol{c}}_k[i] = \boldsymbol{C}_k\boldsymbol{h}_k[i]$. The output of the RAKE receiver is given by

$$x_k[i] = \boldsymbol{f}_k^H[i]\boldsymbol{r}[i] = \underbrace{A_k[i]b_k[i]\boldsymbol{f}_k^H[i]\tilde{\boldsymbol{c}}_k[i]}_{\text{desired signal}} + \underbrace{\sum_{\substack{j=2 \\ j\neq k}}^{K} A_j[i]b_j[i]\boldsymbol{f}_j^H[i]\tilde{\boldsymbol{c}}_j[i]}_{\text{MAI}} + \underbrace{\boldsymbol{f}_k^H[i]\boldsymbol{\eta}_k[i]}_{\text{ISI}} + \underbrace{\boldsymbol{f}_k^H[i]\boldsymbol{n}[i]}_{\text{noise}},$$
(30)

where $\boldsymbol{f}_k^H[i]\tilde{\boldsymbol{c}}_k[i] = \rho_k[i]$ and $\boldsymbol{f}_k^H[i]\tilde{\boldsymbol{c}}_j[i] = \rho_{1,j}[i]$ for $j \neq 1$. The symbol ρ_k represents the cross-correlation (or inner product) between the effective signature and the RAKE with perfect channel estimates. The symbol $\rho_{1,j}[i]$ represents the cross-correlation between the RAKE receiver and the effective signature of user j. The second-order statistics of the output of the RAKE in (30) are described by

$$E[|x_k[i]|^2] = A_k^2[i]\rho_k^2[i]\underbrace{E[|b_k[i]|^2]}_{\to 1}$$
$$+ \sum_{\substack{j=1 \\ j\neq k}}^{K}\sum_{\substack{l=1 \\ l\neq k}}^{K}\underbrace{A_j^2[i]E[b_j[i]b_l^*[i]]\boldsymbol{f}_j^H\tilde{\boldsymbol{c}}_j\tilde{\boldsymbol{c}}_l^H\boldsymbol{f}_j}_{\to \sum_{j=1, j\neq k}^{K}\boldsymbol{f}_j^H\tilde{\boldsymbol{s}}_j\tilde{\boldsymbol{c}}_j^H[i]\boldsymbol{f}_j}$$
$$+ \boldsymbol{f}_k^H E[\boldsymbol{\eta}_k[i]\boldsymbol{\eta}_k^H[i]]\boldsymbol{f}_k + \underbrace{\boldsymbol{f}_k^H E[\boldsymbol{n}[i]\boldsymbol{n}^H[i]]\boldsymbol{f}_k}_{\to \sigma^2\boldsymbol{f}_k^H\boldsymbol{f}_k}.$$
(31)

From the previous development, we can identify the sum of the power levels of MAI, ISI and noise terms from the second-order statistics. Our approach is to obtain instantaneous estimates of the MAI, the ISI and the noise from the output of a RAKE receiver, subtract the detected symbol in (3) from this output (using the more reliable linear multiuser receiver ($\boldsymbol{w}_k[i]$)) and to track the interference (MAI + ISI + noise) power as shown in Fig. 1. Let us define the difference between the output of the RAKE receiver and the detected symbol for user 1:

$$d_k[i] = x_k[i] - \hat{A}_k[i]\hat{b}_k[i] \approx \underbrace{\sum_{k=2}^{K} A_k[i]b_k[i]\boldsymbol{f}_k^H[i]\tilde{\boldsymbol{s}}_k[i]}_{\text{MAI}} + \underbrace{\boldsymbol{f}_k^H[i]\boldsymbol{\eta_k}[i]}_{\text{ISI}}$$
$$+ \underbrace{\boldsymbol{f}_k^H[i]\boldsymbol{n}[i]}_{\text{noise}}.$$
$$(32)$$

By taking expectations on $|d_k[i]|^2$ and taking into account the assumption that MAI, ISI and noise are uncorrelated we have:

$$E[|d_k[i]|^2] \approx \sum_{k=2}^{K} \boldsymbol{f}_k^H[i]\tilde{\boldsymbol{c}}_k[i]\tilde{\boldsymbol{c}}_k^H[i]\boldsymbol{f}_k[i]$$
$$+ \boldsymbol{f}_k^H[i]E[\boldsymbol{\eta_k}[i]\boldsymbol{\eta_k}^H[i]]\boldsymbol{f}_k[i] + \sigma^2\boldsymbol{f}_k^H[i]\boldsymbol{f}_k[i],$$
$$(33)$$

where the above equation represents the interference power. Based on time averages of the instantaneous values of the interference power, we consider the following algorithm to estimate and track $E[|d_k[i]|^2]$

$$\hat{v}[i+1] = (1-\beta)\hat{v}[i] + \beta|d_k[i]|^2, \qquad (34)$$

where β is a forgetting factor. To incorporate parameter dependency and interference power for computing a more effective bound as an alternative to replace (29), we employ the parameter and interference dependent bound (PIDB)

$$\gamma_k[i+1] = (1-\beta)\gamma_k[i] + \beta\left(\sqrt{\tau\ \hat{v}^2[i]} + \sqrt{\alpha||\boldsymbol{w}_k||^2\hat{\sigma}^2[i]}\right),$$
$$(35)$$

where $\hat{v}[i]$ is the estimated interference power in the multiuser system and τ is a weighting parameter that must be set. The equations in (34) and (35) are time-averaged recursions that are aimed at tracking the quantities $|d_k[i]|^2$ and $(\sqrt{\tau\hat{v}^2[i]} + \sqrt{\alpha||\boldsymbol{w}_k||^2\hat{\sigma}^2[i]})$, respectively. The equations in (34) and (35) also avoid undesirable too high or low instantaneous values which may lead to inappropriate time-varying bound $\gamma_k[i]$.

D. Blind Channel and Amplitude Estimation

Let us now present a set-membership blind channel estimation (SM-BCE) algorithm to design the RAKE and linear receivers. Consider the constraint matrix $\boldsymbol{C}_k$ that contains one-chip shifted versions of the signature sequence for user k defined in (2) and the assumption that the symbols $b_k[i]$ are independent and identically distributed (i.i.d), and statistically independent from the symbols of the other users. Consider the covariance matrix $\boldsymbol{R} = [\boldsymbol{r}[i]\boldsymbol{r}^H[i]]$ and the transmitted signal

$\boldsymbol{x}_k[i] = A_k[i]b_k[i]\boldsymbol{p}_k[i]$, where $\boldsymbol{p}_k[i] = \boldsymbol{C}_k\boldsymbol{h}_k[i]$. Let us now perform an eigen-decomposition on $\boldsymbol{R}$

$$\boldsymbol{R} = \sum_{k=1}^{K} E[\boldsymbol{x}_k[i]\boldsymbol{x}_k^H[i]] + E[\boldsymbol{\eta}_k[i]\boldsymbol{\eta}_k^H[i]] + \sigma^2\boldsymbol{I}$$
$$= [\boldsymbol{\phi}_s\ \boldsymbol{\phi}_n]\begin{bmatrix} \boldsymbol{\Lambda}_s + \sigma^2\boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & \sigma^2\boldsymbol{I} \end{bmatrix}[\boldsymbol{\phi}_s\ \boldsymbol{\phi}_n]^H,$$
$$(36)$$

where $\boldsymbol{\phi}_s$ and $\boldsymbol{\phi}_n$ are the signal and noise subspaces, respectively. Since $\boldsymbol{\phi}_s$ and $\boldsymbol{\phi}_n$ are orthogonal, we have the condition $\boldsymbol{\phi}_n^H\boldsymbol{x}_k[i] = \boldsymbol{\phi}_n^H A_k[i]b_k[i]\boldsymbol{p}_k[i] = \boldsymbol{\phi}_n^H A_k[i]b_k[i]\boldsymbol{C}_k\boldsymbol{h}_k[i] = \boldsymbol{0}$. Hence, we have

$$\Gamma = \boldsymbol{h}_k^H[i]\underbrace{\boldsymbol{C}_k^H b_k^*[i]A_k^*[i]\boldsymbol{\phi}_n\boldsymbol{\phi}_n^H A_k[i]b_k[i]\boldsymbol{C}_k}_{\boldsymbol{\Upsilon}_k}\boldsymbol{h}_k[i]$$
$$(37)$$

The above relation allows to blindly estimate the channel $\boldsymbol{h}_k[i]$. To this end, we need to compute the eigenvector corresponding to the smallest eigenvalue of $\boldsymbol{\Upsilon}_k$. It turns out that we can use the fact that $\lim_{p\to\infty}(\boldsymbol{R}/\sigma^2)^{-p} = \boldsymbol{\phi}_n\boldsymbol{\phi}_n^H$ [36] and, in practice, it suffices to use $p = 1$ or 2. Therefore, to blindly estimate the channel of user k in the DS-CDMA system we need to solve the optimization problem

$$\hat{\boldsymbol{h}}_k[i] = \arg\min_{\boldsymbol{h}_k[i]} \boldsymbol{h}_k^H[i]\boldsymbol{\Upsilon}_k\boldsymbol{h}_k[i], \quad \text{subject to} \quad ||\boldsymbol{h}_k[i]|| = 1,$$
$$(38)$$

In order to solve (38) efficiently, we rely on the SM estimation strategy and a variant of the power method [33] that uses a simple shift is adopted to yield the SM-BCE

$$\hat{\boldsymbol{h}}_k[i] = (\boldsymbol{I} - \tau_k[i]\hat{\boldsymbol{\Upsilon}}_k[i])\hat{\boldsymbol{h}}_k[i-1], \qquad (39)$$

where $\tau_k[i] = 1/tr[\hat{\boldsymbol{\Upsilon}}_k[i]]$ and $\hat{\boldsymbol{h}}_k[i] \leftarrow \hat{\boldsymbol{h}}_k[i]/||\hat{\boldsymbol{h}}_k[i]||$ to normalize the channel. The quantity $\hat{\boldsymbol{\Upsilon}}_k[i]$ is estimated by

$$\hat{\boldsymbol{\Upsilon}}_k[i] = \hat{\boldsymbol{\Upsilon}}_k[i-1] + \lambda_k[i]\boldsymbol{C}_k^H\hat{A}_k^*[i]\hat{\boldsymbol{P}}_k^p[i]\hat{A}_k[i]\boldsymbol{C}_k, \quad (40)$$

where $\lambda_k[i]$ is a variable forgetting factor that is obtained by (28) and $\hat{\boldsymbol{P}}_k^p[i]$ is computed according to (26). Next, we describe a procedure to estimate the amplitude.

In general, amplitude estimation is an important task at the receiver that is useful for interference cancellation and power control. The proposed blind interference estimation and tracking algorithm needs some form of amplitude estimation in order to accurately compute the interference power. To estimate the amplitudes of the associated user signals, we describe the following procedure to estimate the absolute value of the output of the RAKE receiver defined in (30) as given by

$$q_k[i+1] = (1-\beta)q_k[i] + |x_k[i]|. \qquad (41)$$

The amplitude can be estimated by removing the square-root of the interference power from the above estimate according to

$$\hat{A}_k[i+1] = (1-\beta)\hat{A}_k[i] + (|q_k[i]| - |v_k[i]|) \qquad (42)$$

The above procedure is simple and effective to estimate the amplitude for use in the interference power estimation procedure. Since one recursion depends on the other, a designer shall start the procedure with an interference power equal to zero (or equivalently $|v_k[i]| = 0$).

V. ANALYSIS OF THE ALGORITHMS

In this section, we study of the properties of the optimization problems associated with the design of the SM-CCM-based algorithms and examine the convergence and tracking performances of the proposed algorithms. In order to ensure the convergence of the SM-CCM-RLS a persistence of excitation condition on the received data $\boldsymbol{r}[i]$ must hold and the transmitted symbols $b_k[i]$ and the noise $\boldsymbol{n}[i]$ have to be uncorrelated. To this end, the variable bound must adapt such that the above conditions are met, following the same procedure established for OBE algorithms in [18]. Since the SM-CCM-RLS algorithm is expected to converge without misadjustment to the Wiener filter and certain aspects of the convergence analysis of the SM-CCM-SG algorithm are of greater interest, we will focus on the latter. The CCM-based algorithms are inherently nonlinear and deal with time-varying environments, which leads to difficulties in the study of their performance. For this reason, we will resort to an effective approach termed energy conservation principle [38] that lends itself to such analysis.

A. Analysis of the Optimization Problem

Let us consider the optimization problem that needs to be solved for the design of the blind receiver and can be solved by the algorithms proposed in Section III:

$$\text{minimize } J_{CM}(\boldsymbol{w}_k[i]) = E[(|\boldsymbol{w}_k^H[i]\boldsymbol{r}[i]|^2 - 1)^2]$$
$$= E[|z_k[i]|^4 - 2|z_k[i]|^2 + 1]$$
$$\text{subject to } \boldsymbol{w}_k^H[i]\boldsymbol{p}_k[i] = \nu$$
$$\text{whenever } (|\boldsymbol{w}_k^H[i]\boldsymbol{p}_k[i]|^2 - 1)^2 \geq \gamma_k^2[i]$$

$$(43)$$

Let us rewrite the received vector as

$$\boldsymbol{r}[i] = \sum_{k=1}^{K} \underbrace{A_k b_k[i]\boldsymbol{p}_k[i]}_{\boldsymbol{x}_k[i]} + \boldsymbol{\eta}_k[i] + \boldsymbol{n}[i], \qquad (44)$$

where $\boldsymbol{p}_k[i] = \boldsymbol{C}_k \boldsymbol{h}_k[i]$, $\boldsymbol{R} = \boldsymbol{S} + \boldsymbol{G} + \sigma^2 \boldsymbol{I}$, $\boldsymbol{S} = E[\sum_{k=1}^{K} \boldsymbol{x}_k[i]\boldsymbol{x}_k^H[i]]$, and $\boldsymbol{G} = E[\sum_{k=1}^{K} \boldsymbol{\eta}_k[i]\boldsymbol{\eta}_k^H[i]]$, $b_k[i]$ are independent and identically distributed random variables, and are statistically independent from $\boldsymbol{n}[i]$.

Consider user k as the desired user and let $\boldsymbol{w}_k[i]$ be the receive filter for this user. Our strategy to analyze the above optimization problem and its properties is to transform the variables and rewrite the problem in a convenient form that provides more insight about the nature of the problem. Let us now define the signal of the desired and the signal of all the users after applying the receive filter $\boldsymbol{w}_k[i]$:

$$t_k \triangleq A_k \boldsymbol{p}_k^H \boldsymbol{w}_k[i], \quad \text{and} \quad \boldsymbol{t} \triangleq \boldsymbol{A}\boldsymbol{P}^H \boldsymbol{w}_k = [t_1 \ldots t_K]^T, \quad (45)$$

where $\boldsymbol{P} \triangleq [\boldsymbol{p}_1 \ldots \boldsymbol{p}_K]$, $\boldsymbol{A} \triangleq \text{diag}[A_1 \ldots A_K]$ and $\boldsymbol{b} \triangleq [b_1 \ldots b_K]^T$. The relation between the receive filter, the channel and the signature sequence can be written as

$$\boldsymbol{w}_k^H[i]\boldsymbol{p}_k[i] = \boldsymbol{w}_k^H[i]\boldsymbol{C}_k \boldsymbol{h}_k[i] = \nu. \qquad (46)$$

Therefore, we have for the desired user k the following equivalences

$$t_k[i] = A_k \boldsymbol{p}_k^H[i]\boldsymbol{w}_k[i] = A_k \boldsymbol{h}_k^H[i] \underbrace{\boldsymbol{C}_k^H \boldsymbol{w}_k[i]}_{\boldsymbol{h}_k[i]} = A_k \boldsymbol{h}_k^H[i]\boldsymbol{h}_k[i].$$

$$(47)$$

By considering the noise and the ISI negligible we can write the cost function in (43) as

$$J_{CM}(\boldsymbol{t}) = E[|z_k[i]|^4 - 2|z_k[i]|^2 + 1]$$
$$= E[(\boldsymbol{t}^H \boldsymbol{b}\boldsymbol{b}^H \boldsymbol{t})^2] - 2E[(\boldsymbol{t}^H \boldsymbol{b}\boldsymbol{b}^H \boldsymbol{t}] + 1$$
$$= 8(\sum_{j=1}^{K} t_j t_j^*)^2 - 4\sum_{j=1}^{K}(t_j t_j^*)^2 - 4\sum_{j=1}^{K} t_j t_j^* + 1$$
$$= 8(D + \sum_{j=1}^{K} t_j t_j^*)^2 - 4D^2 - 4\sum_{j=2}^{K}(t_j t_j^*)^2$$
$$- 4D - 4\sum_{j=1, j\neq k}^{K}(t_j t_j^*) + 1, \qquad (48)$$

where $\boldsymbol{t} = f(\boldsymbol{w}_k[i])$ is a linear function of the receive filter $\boldsymbol{w}_k[i]$, the terms multiplying the summations in the third line of (48) are obtained by evaluating the expected values in the second line of (48) [31], and $D = t_j t_j^* = \nu^2 |A_k|^2 |\hat{\boldsymbol{h}}_k^H[i]\boldsymbol{h}_k[i]|$. The strategy we employ to enforce the convexity of the optimization problem relies on the adjustment of the parameter ν and a transformation of variables on the cost function in (48), which will be detailed in what follows.

Let us now transform the above cost function taking into account the constraint $\boldsymbol{w}_k^H[i]\boldsymbol{p}_k[i] = \nu$ and the fact that we are interested in demodulating user k and rejecting the remaining users. Therefore, we introduce another parameter vector $\bar{\boldsymbol{t}} = [t_1 \ldots t_{k-1} \; t_{k+1} \ldots t_K]^T = \bar{\boldsymbol{B}}\boldsymbol{w}_k[i]$ that excludes the user k and is responsible for the interference suppression task of the remaining users (all but user k), where $\bar{\boldsymbol{B}} = \bar{\boldsymbol{A}}\bar{\boldsymbol{P}}^H$, $\bar{\boldsymbol{P}}[i] = [\boldsymbol{p}_1[i] \ldots \; \boldsymbol{p}_{k-1}[i] \; \boldsymbol{p}_{k+1}[i] \ldots \boldsymbol{p}_K[i]]$ and $\bar{\boldsymbol{A}} = \text{diag}[A_1 \ldots A_{k-1} \; A_{k+1} \ldots A_K]$. The transformed cost function is given by

$$J_{CM}(\bar{\boldsymbol{t}}) = 8(D + \bar{\boldsymbol{t}}^H \bar{\boldsymbol{t}})^2 - 4(D^2 + \bar{\boldsymbol{t}}^H \bar{\boldsymbol{t}})^2$$
$$- 4(D + \bar{\boldsymbol{t}}^H \bar{\boldsymbol{t}}) + 1. \qquad (49)$$

At this point we need to take into account the constraint $J_{CM}(\boldsymbol{w}_k[i]) \leq \gamma_k^2[i]$. Since $\bar{\boldsymbol{t}} = \bar{\boldsymbol{B}}\boldsymbol{w}_k[i]$ is a linear mapping we have an equivalent constraint $J_{CM}(\bar{\boldsymbol{t}}) \leq \bar{\gamma}_k^2[i]$, where $\bar{\gamma}_k^2[i]$ is the bound modified by the linear mapping. It can be verified that the constraint set generated by $J_{CM}(\bar{\boldsymbol{t}}) \leq \bar{\gamma}_k^2[i]$ is not convex and leads to two disjoint parallel hyperstrips in the parameter space.

Given that the constraint is not convex, the optimization problem is clearly non convex. However, in this context non-convexity only poses a problem if local minima are present and prevent an algorithm from reaching the global minimum. It turns out that a designer can adjust the parameter ν in order to enforce the convexity for each hyperstrip. Computing the

Hessian [43] $\boldsymbol{H} = \frac{\partial}{\partial \bar{\boldsymbol{t}}^H} \frac{\partial J_{CM}(\boldsymbol{t})}{\partial \boldsymbol{t}}$ we obtain

$$
\boldsymbol{H} = 16\big[(D - 1/4)\boldsymbol{I} + \\
\underbrace{\bar{\boldsymbol{t}}^H \bar{\boldsymbol{t}} \boldsymbol{I} + \bar{\boldsymbol{t}} \bar{\boldsymbol{t}}^H - \mathrm{diag}\big(|t_1[i]|^2 \ldots |t_{k-1}|^2 \ |t_{k+1}|^2 \ldots |t_K[i]|^2\big)}_{\text{positive or positive semi-definite terms}}\big]
$$
(50)

The condition $D = \nu^2 |A_k|^2 |\hat{\boldsymbol{h}}_k^H[i]\hat{\boldsymbol{h}}_k[i]|^2 \geq 1/4$ ensures that there is no local minimum in each of the hyperstrips resulting from the optimization problem and convexity can be enforced in each of the hyperstrips. Since $\boldsymbol{t} = \bar{\boldsymbol{B}} \boldsymbol{w}_k[i]$ is a linear function of $\boldsymbol{w}_k[i]$ then $J_{CM}(\bar{\boldsymbol{t}})$ preserves the maxima and minima properties of $J_{CM}(\boldsymbol{w}_k[i])$. For sufficiently high signal-to-noise ratio (SNR) values, the extrema of the cost function can be considered a small perturbation of the noise-free case [31].

B. Stability Analysis

In this part, we discuss the stability analysis of the SM-CCM-SG algorithm described in subsection III.A. In particular, we consider the range of step-size values for convergence. Let us now rewrite the update equation of the algorithm as

$$
\boldsymbol{w}_k[i+1] = \boldsymbol{\Pi}_k[i](\boldsymbol{w}_k[i] - \mu_k[i]e_k[i]z_k^*[i]\boldsymbol{r}[i]) + \nu \boldsymbol{p}_k[i](\boldsymbol{p}_k^H[i]\boldsymbol{p}_k[i])^{-1}
$$
$$
= \boldsymbol{w}_k[i] - \mu_k[i]e_k[i]\boldsymbol{r}^H[i]\boldsymbol{w}_k[i]\Big(\boldsymbol{I} - \frac{\boldsymbol{p}_k[i]\boldsymbol{p}_k^H[i]}{\boldsymbol{p}_k^H[i]\boldsymbol{p}_k[i]}\Big)\boldsymbol{r}[i]
$$
$$
= \boldsymbol{w}_k[i] - \mu_k[i]e_k[i]\boldsymbol{v}[i]\boldsymbol{r}^H[i]\boldsymbol{w}_k[i]
$$
$$
= (\boldsymbol{I} - \mu_k[i]e_k[i]\boldsymbol{v}[i]\boldsymbol{r}^H[i])\boldsymbol{w}_k[i],
$$
(51)

where $\boldsymbol{v}_k[i] = (\boldsymbol{I} - \frac{\boldsymbol{p}_k[i]\boldsymbol{p}_k^H[i]}{\boldsymbol{p}_k^H[i]\boldsymbol{p}_k[i]}\boldsymbol{r}[i])$ and the expression in the second line of (51) is obtained by substituting $\boldsymbol{\Pi}_k[i] = \boldsymbol{I} - \boldsymbol{p}_k[i](\boldsymbol{p}_k^H[i]\boldsymbol{p}_k[i])^{-1}\boldsymbol{p}_k^H[i]$ into the first line and further manipulating the terms. Let us now define the error vector

$$
\boldsymbol{\epsilon}_{\boldsymbol{w}_k}[i+1] = \boldsymbol{w}_{k,\mathrm{opt}} - \boldsymbol{w}_k[i+1]
$$
$$
= (\boldsymbol{I} - \mu_k[i]e_k[i]\boldsymbol{v}_k[i]\boldsymbol{r}^H[i])\boldsymbol{\epsilon}_{\boldsymbol{w}_k}[i] + \mu_k[i]e_k[i]\boldsymbol{v}_k[i]\boldsymbol{r}^H[i]\boldsymbol{w}_{k,\mathrm{opt}}.
$$
(52)

In order to proceed with the analysis, we need to resort to an assumption.

Assumption 1: Let us suppose that for the algorithm in (51) when $i \to \infty$

$$
E[\mu_k[i]e_k^2[i]] = E[\mu_k[i]]E[e_k^2[i]].
$$
(53)

This assumption holds if $\mu_k[i]$ is a constant, and we claim that it is approximately true if $\mu_k[i]$ varies slowly around its mean value. By writing

$$
E[\mu_k[i]e_k^2[i]] = E[\mu_k[i]]E[e_k^2[i]] + E[(\mu_k[i] - E[\mu_k[i]])e_k^2[i]],
$$
(54)

we can notice that the second term on the right-hand side will be small compared with the first one provided that $\mu_k[i]$ varies slowly around its mean value.

By taking expectations on both sides of (52) and using the previous assumption, we have

$$
E[\boldsymbol{\epsilon}_{\boldsymbol{w}_k}[i+1]] = (\boldsymbol{I} - E[\mu_k[i]]\boldsymbol{R}_{rv})E[\boldsymbol{\epsilon}_{\boldsymbol{w}_k}[i]],
$$
(55)

where $\boldsymbol{R}_{rv} = E[e_k[i]\boldsymbol{v}_k[i]\boldsymbol{r}^H[i]]$ and $\boldsymbol{R}_{vr}\boldsymbol{w}_{\mathrm{opt}} = \boldsymbol{0}$.

From the above, it can be concluded that $\boldsymbol{w}_k[i]$ converges to $\boldsymbol{w}_{k,\mathrm{opt}}$ and (55) is stable if and only if $\prod_{i=0}^{\infty}(\boldsymbol{I} - E[\mu_k[i]]\boldsymbol{R}_{vr}) \to 0$, which is a necessary and sufficient condition for $\lim_{i \to \infty} E[\boldsymbol{\epsilon}_{\boldsymbol{w}_k}] = \boldsymbol{0}$ and $E[\boldsymbol{w}_k[i]] \to \boldsymbol{w}_{k,\mathrm{opt}}$. For stability, a sufficient condition for (55) to hold implies that

$$
0 \leq E[\mu(\infty)] < \min_k \frac{2}{|\lambda_k^{vr}|},
$$
(56)

where λ_k^{vr} is the kth eigenvalue of $\boldsymbol{R}_{vr}$ that is not necessarily real since $\boldsymbol{R}_{rv}$ is not symmetric.

C. Steady-State Analysis

In this part of the analysis we are interested in devising a formula to predict the excess MSE, which depends on the MAI, the ISI, the noise at the receiver and the parameters of the SM-CCM-SG algorithm. The excess MSE is related to the error in the filter coefficients $\boldsymbol{\epsilon}_{\boldsymbol{w}_k}[i]$ via the *a priori* estimation error, which is defined as

$$
e_a[i] \triangleq \boldsymbol{\epsilon}_{\boldsymbol{w}_k}^H[i]\boldsymbol{r}[i],
$$
(57)

where $\boldsymbol{\epsilon}_{\boldsymbol{w}_k}[i] = \boldsymbol{w}_{\mathrm{opt}} - \boldsymbol{w}_k[i]$, and $\boldsymbol{w}_{\mathrm{opt}}$ is the optimum linear MMSE receiver. Consider the MSE at time i:

$$
\mathrm{MSE}[i] = E[|b_k[i] - \boldsymbol{w}_k^H[i]\boldsymbol{r}[i]|^2]
$$
$$
= \epsilon_{\min} + E[|e_a[i]|^2] + \boldsymbol{p}_k^H[i]E[\boldsymbol{\epsilon}_{\boldsymbol{w}_k}[i]] + E[\boldsymbol{\epsilon}_{\boldsymbol{w}_k}^H[i]]\boldsymbol{p}_k[i]
$$
$$
- E[\boldsymbol{w}_{\mathrm{opt}}^H\boldsymbol{r}[i]\boldsymbol{r}^H[i]\boldsymbol{\epsilon}_{\boldsymbol{w}_k}[i]] - E[\boldsymbol{\epsilon}_{\boldsymbol{w}_k}^H[i]\boldsymbol{r}[i]\boldsymbol{r}^H[i]\boldsymbol{w}_{k,\mathrm{opt}}],
$$
(58)

where $e_a[i] \triangleq \boldsymbol{\epsilon}_{\boldsymbol{w}_k}^H[i]\boldsymbol{r}[i] = (\boldsymbol{w}_{\mathrm{opt}} - \boldsymbol{w}_k[i])^H\boldsymbol{r}[i]$ and $\epsilon_{\min} = E[|b_k[i] - \boldsymbol{w}_{\mathrm{opt}}^H\boldsymbol{r}[i]|^2]$. When $i \to \infty$, we have $\boldsymbol{w}_k[i] \to \boldsymbol{w}_{k,\mathrm{opt}}$ and $E[\boldsymbol{\epsilon}_{\boldsymbol{w}_k}[i]] \to 0$ and the steady-state MSE

$$
\lim_{i \to \infty} \mathrm{MSE}[i] = \epsilon_{\min} + \lim_{i \to \infty} E[|e_a[i]|^2]
$$
(59)

The steady-state excess MSE is then defined [1] as

$$
\xi \triangleq \lim_{i \to \infty} E[|e_a[i]|^2]
$$
(60)

Using the energy conservation principle [38], the proposed SM-CCM-SG algorithm can be written in the form

$$
\boldsymbol{w}_k[i+1] = \boldsymbol{w}_k[i] + \mu_k[i]\underbrace{(-e_k[i]z_k^*[i])}_{F_{e_k}[i]}\underbrace{\Big(\boldsymbol{I} - \frac{\boldsymbol{p}_k[i]\boldsymbol{p}_k^H[i]}{\boldsymbol{p}_k^H[i]\boldsymbol{p}_k[i]}\Big)\boldsymbol{r}[i]}_{\boldsymbol{u}_k[i]},
$$
(61)

where $F_{e_k}[i]$ is a generic scalar function determined by the adaptive algorithm. Subtracting the above recursion for $\boldsymbol{w}_k[i+1]$ from $\boldsymbol{w}_{k,\mathrm{opt}}$, we obtain

$$
\boldsymbol{\epsilon}_{\boldsymbol{w}_k}[i+1] = \boldsymbol{\epsilon}_{\boldsymbol{w}_k}[i] - \mu_k[i]\boldsymbol{u}_k[i]F_{e_k}[i]
$$
(62)

Using the *a priori* estimation error $e_a[i] \triangleq \boldsymbol{\epsilon}_{\boldsymbol{w}_k}^H[i]\boldsymbol{r}[i]$. Rewriting the previous equation, we obtain

$$
e_a[i] = e_p[i] + \mu_k[i]\boldsymbol{u}_k^H[i]\boldsymbol{r}[i]F_{e_k}^*[i]
$$
$$
= e_p[i] + \mu_k[i]\boldsymbol{r}^H[i]\Big(\boldsymbol{I} - \frac{\boldsymbol{p}_k[i]\boldsymbol{p}_k^H[i]}{\boldsymbol{p}_k^H[i]\boldsymbol{p}_k[i]}\Big)\boldsymbol{r}[i]F_{e_k}^*[i]
$$
(63)

Since $r^H[i]\left(I - \frac{p_k[i]p_k^H[i]}{p_k^H[i]p_k[i]}\right)r[i] = u_k^H[i]u_k[i]$, we have

$$e_p[i] = e_a[i] - \mu_k[i]u_k^H[i]u_k[i]F_{e_k}^*[i]. \tag{64}$$

Using the fact that $F_{e_k}^*[i] = \frac{e_a[i]-e_p[i]}{\mu_k[i]u_k^H[i]u_k[i]}$, we can rewrite $\epsilon_{w_k}[i]$ as

$$\epsilon_{w_k}[i+1] = \epsilon_{w_k}[i] - \frac{u_k[i]}{u_k^H[i]u_k[i]}[e_a^*[i] - e_p^*[i]] \tag{65}$$

Rearranging, we obtain

$$\epsilon_{w_k}[i+1]+\frac{u_k[i]}{u_k^H[i]u_k[i]}e_p^*[i] = \epsilon_{w_k}[i]+\frac{u_k[i]}{u_k^H[i]u_k[i]}e_a^*[i] \tag{66}$$

By defining $\bar{\mu}[i] = 1/(u_k^H[i]u_k[i])$, squaring the previous equation and taking expectations, we obtain the relation

$$E[||\epsilon_{w_k}[i+1]||^2]+E[\bar{\mu}_k[i]|e_p[i]|^2] = E[||\epsilon_{w_k}[i+1]||^2]+E[\bar{\mu}_k[i]|e_a[i]|^2] \tag{67}$$

In the steady state, we can write

$$E[||\epsilon_{w_k}[i]||^2] = E[||\epsilon_{w_k}[i+1]||^2], \text{ and } E[\bar{\mu}_k[i]|e_a[i]|^2] = E[\bar{\mu}_k[i]|e_p[i]|^2] \tag{68}$$

The energy preserving equation can be used to compute the excess MSE. It can be obtained by cancelling the terms $E[||\epsilon_{w_k}[i+1]||^2]$ and $E[||\epsilon_{w_k}[i]||^2]$ in (67) and by substituting (64) into (67), which yields

$$E[\bar{\mu}_k[i]|e_a[i]|^2] = E[\bar{\mu}_k[i]|e_a[i] - \frac{\mu_k[i]}{\bar{\mu}_k[i]}F_{e_k}^*[i]|^2], \tag{69}$$

where $F_{e_k}[i] = -e_k[i]z_k^*[i] = (1-|z_k[i]|^2)z_k^*[i]$ and $z_k[i] = (w_{k,opt} - \epsilon_{w_k}[i])^H r[i] = w_{k,opt}^H r[i] - e_a[i] = A_k b_k[i] + \text{MAI} + \eta_k[i] + v[i] - e_a[i]$. Substituting $F_{e_k}[i]$ and (64) into (69) and manipulating the terms, we obtain

$$\underbrace{E[\mu_k[i]]E[e_a[i]z_k[i](1-|z_k[i]|^2)]}_{C} \\ +E[\mu_k[i]]E[\mu_k[i]]E[e_a[i]z_k^*[i](1-|z_k[i]|^2)] \\ = \underbrace{E[\mu_k^2[i]]E[u_k^H[i]u_k[i]|z_k[i]|^2(1-|z_k[i]|^2)^2]}_{F}. \tag{70}$$

At this point we need to resort to another assumption to continue with our analysis.

Assumption 2: In the steady state, $u_k^H[i]u_k[i]$ and $|F_{e_k}[i]|^2$ are uncorrelated. The quantities $\{b_k[i], \text{MAI}, \eta[i], v[i], e_a[i]\}$ are zero mean random variables, and are mutually independent. We use the fact that $E[b_k^{2m}] = 1$ for any positive integer m and that the residual MAI and ISI are Gaussian random variables.

Using the previous assumption and substituting $z_k[i] =$

$w_k^H[i]r[i]$ into (70), we obtain

$$E[\mu_k^2[i]]E[u_k^H[i]u_k[i]]AE[|e_a[i]|^2] \\ + 3E[\mu_k^2[i]]E[u_k^H[i]u_k[i]]E[\text{MAI}^2]E[|e_a[i]|^4] \\ + 3E[\mu_k^2[i]]E[u_k^H[i]u_k[i]]E[v^2[i]]E[|e_a[i]|^4] \\ + E[\mu^2[i]]E[u_k^H[i]u_k[i]](3E[\mu^2[i]]+1)E[|e_a[i]|^4] \\ \underbrace{E[\mu_k^2[i]]E[u_k^H[i]u_k[i]]B + E[\mu_k^2[i]]E[u_k^H[i]u_k[i]]E[|e_a[i]|^6]}_{C} \\ = 2E[\mu_k[i]](E[\text{MAI}^2]E[|e_a[i]|^2] + E[v^2[i]]E[|e_a[i]|^2] \\ \underbrace{+E[\eta^2[i]]E[|e_a[i]|^2] + E[|e_a[i]|^4]}_{D},$$

$$\tag{71}$$

where $A = 3 + 3\sigma^4\text{MAI} + 6\sigma_{\text{MAI}}^2\sigma_v^2 + 6\sigma_{\text{MAI}}\sigma_n^2 + 3\sigma_v^4 + 6\sigma_v^2\sigma_n^2 + 3\sigma_n^4$ and $B = \sigma_\eta^6 + 3\sigma_v^2\sigma_\eta^4 + 3\sigma_{\text{MAI}}^2\sigma_\eta^4 + \sigma_v^6 + 6\sigma_v^2 + \sigma_\eta^2\sigma_{\text{MAI}}^2 + 3\sigma_{\text{MAI}}^4\sigma_n^2 + 3\sigma_{\text{MAI}}^4\sigma_v^2 + \sigma_{\text{MAI}}^6 + \sigma_{\text{MAI}}^4 + 2\sigma_v^2\sigma_n^2 + \sigma_v^4 + 2\sigma_{\text{MAI}}^2 + 2\sigma_{\text{MAI}}^2\sigma_v^2 + \sigma_{\text{MAI}}^4 + 4\sigma_v^2 + 2\sigma_\eta^2 + 2\sigma_{\text{MAI}}^2 + 2$.

Upon convergence($i \to \infty$), we can assume that $E[\text{MAI}^{2m}](E[\text{MAI}^2])^m = \sigma_{\text{MAI}}^{2m}$, $E[\eta^{2m}] = (E[\eta^2])^m = \sigma_\eta^{2m}$, and $E[v^{2m}] = (E[v^2])^m = \sigma_v^{2m}$, where σ_{MAI}, σ_η, and σ_v are the variances of the Gaussian distribution. In this situation, the high power terms $E[|e_a[i]|^4]$ and $E[|e_a[i]|^6]$ may be neglected as these values are typically very small compared with the remaining terms. The excess MSE is obtained as follows

$$\xi = E[|e_a[i]|^2] = \frac{E[\mu_k^2(\infty)]E[u_k^H[i]u_k[i]]B}{2E[\mu_k(\infty)](\sigma_{\text{MAI}}^2 + \sigma_v^2 + \sigma_\eta^2) - E[\mu_k^2(\infty)]E[u_k^H[i]u_k[}} \tag{72}$$

D. Tracking Analysis

In this subsection, we assess the proposed SM-CCM-SG algorithm in a non-stationary environment, in which the algorithm has to track the minimum point of the error-performance surface. Specifically, we derive an expression for the excess MSE of a blind adaptive linear receiver when the channel varies in time. Differently from the works in [38], [40], where expressions were derived for the constant modulus (CM) algorithm and the CCM algorithm, respectively, we consider a set-membership approach. In the time-varying scenarios of interest, the optimum receive filter coefficients are assumed to vary according to the model $w_{opt}[i+1] = w_{opt} + q[i]$, where $q[i]$ denotes a random perturbation. This is consistent with tracking analyses of adaptive filtering algorithms and requires an assumption.

Assumption 3: The sequence $q[i]$ is a stationary sequence of independent zero-mean vectors and positive definite auto-correlation matrix $Q = E[q[i]q^H[i]]$, which is mutually independent of the sequences $\{u_k[i]\}$, $\{v[i]\}$, $\{\text{MAI}[i]\}$, $\{\eta[i]\}$.

Let us consider the weight-error vector $\epsilon_{w_k}[i] = w_{k,opt} - w_k[i]$, which satisfies

$$\epsilon_{w_k}[i+1] = \epsilon_{w_k}[i] - \mu_k[i]u_k[i]F_{e_k}[i] + q[i]. \tag{73}$$

With $e_a[i] = \epsilon_{w_k}^H[i]r[i]$ and $e_p[i] = \epsilon_{w_k}^H[i+1]r[i]$, we have

$$e_a[i] = e_p[i] + \mu_k[i]||u_k[i]||^2 F_{e_k}^*[i]. \tag{74}$$

Using (73) and (74), we obtain

$$\boldsymbol{\epsilon}_{\boldsymbol{w}_k}[i+1] + \bar{\mu}_k[i]\boldsymbol{u}_k[i]e_a^*[i] = \boldsymbol{\epsilon}_{\boldsymbol{w}_k}[i] + \boldsymbol{q}[i] + \bar{\mu}_k[i]\boldsymbol{u}_k[i]e_p^*[i]. \tag{75}$$

Squaring (75) and taking the expected value on both sides, we obtain

$$E[||\boldsymbol{\epsilon}_{\boldsymbol{w}_k}[i+1]||^2] + E[\bar{\mu}_k[i]|e_a[i]|^2] = E[||\boldsymbol{\epsilon}_{\boldsymbol{w}_k}[i] + \boldsymbol{q}[i]||^2] \\ + E[\bar{\mu}_k[i]|e_p[i]|^2], \tag{76}$$

where

$$E[||\boldsymbol{\epsilon}_{\boldsymbol{w}_k}[i] + \boldsymbol{q}[i]||^2] = E[||\boldsymbol{\epsilon}_{\boldsymbol{w}_k}[i]||^2] + E[\boldsymbol{\epsilon}_{\boldsymbol{w}_k}[i] + \boldsymbol{q}[i]] \\ + E[\boldsymbol{q}^H[i]\boldsymbol{\epsilon}_{\boldsymbol{w}_k}[i]] + E[\boldsymbol{q}^H[i]\boldsymbol{q}[i]]. \tag{77}$$

Using *Assumption 3*, we have $E[\boldsymbol{\epsilon}_{\boldsymbol{w}_k}^H[i]\boldsymbol{q}[i]] = E[\boldsymbol{q}^H[i]\boldsymbol{\epsilon}_{\boldsymbol{w}_k}[i]] = 0$. When $i \rightarrow \infty$ $E[||\boldsymbol{\epsilon}_{\boldsymbol{w}_k}[i+1]||^2] = E[||\boldsymbol{\epsilon}_{\boldsymbol{w}_k}[i]||^2]$, the energy preserving equation describing the tracking performance is given by

$$E[\bar{\mu}_k[i]|e_a[i]|^2] = \text{Tr}(\boldsymbol{Q}) + E[\bar{\mu}_k[i]|e_a[i] - \frac{\mu_k[i]}{\bar{\mu}_k[i]}F_{e_k}^*[i]|^2 \tag{78}$$

Expanding the equation above, it can be simplified to

$$C = \text{Tr}(\boldsymbol{Q}) + D, \tag{79}$$

where C and D are defined in (71). The excess MSE is then obtained as

$$\xi = E[|e_a[i]|^2] \\ = \frac{E[\mu_k^2(\infty)]E[||\boldsymbol{u}_k[i]||^2]B + \text{Tr}(\boldsymbol{Q})}{2E[\mu_k(\infty)(\sigma_{\text{MAI}}^2 + \sigma_v^2 + \sigma_\eta^2) - E[\mu_k^2(\infty)]E[|\boldsymbol{u}_k[i]|^2]A} \tag{80}$$

E. Computation of Moments

In order to compute the expressions obtained in (72) and (80), we need to obtain the first and second-order moments of $\mu_k[\infty]$. To this end, we resort to the expression for the variable step size in (20) and the methodology employed in [3], which after some algebraic manipulations yields

$$E[\mu_k[\infty]] = E[\gamma_k[i]]P_{\text{up}} + \frac{(1 - P_{\text{up}})}{E[\gamma_k[i]]}\frac{E[A_k[i]]}{E[|\boldsymbol{u}_k[i]|^2]}, \tag{81}$$

$$E[\mu_k^2[\infty]] = E[\gamma_k[i]]P_{\text{up}} + \frac{(1 - P_{\text{up}})}{E[\gamma_k[i]]}\frac{E[A_k^2[i]]}{E[|\boldsymbol{u}_k[i]|^4]}, \tag{82}$$

where the probability of update P_{up} is given by

$$P_{\text{up}} = Pr\big[|e_k[i]| > E[\gamma_k[i]]\big] = Pr\big[|e_k[i]|^2 > E[|\gamma_k[i]|^2]\big] \\ = 2Q\left(\frac{E[\gamma_k[i]]}{\sigma_e}\right), \tag{83}$$

where $Pr[\cdot]$ denotes the probability $Q(x)$ is the complementary Gaussian cumulative distribution function [44] given by

$$Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}}e^{-t^2/2}dt. \tag{84}$$

VI. SIMULATION RESULTS

In this section we assess the performance of the proposed and analyzed adaptive algorithms in terms of mean-square error (MSE) and bit error rate (BER). In particular, we consider the proposed SM-CCM-SG and SM-CCM-RLS adaptive algorithms and the existing CMV-SG and CMV-RLS reported in [26], the CCM-SG [32] and the CCM-RLS [33] algorithms, the SM-CMV-SG and SM-CMV-RLS reported in [7] with and without the proposed blind PDB and PIDB time-varying bounds described in Section IV. The DS-CDMA network employs Gold sequences of length $N = 31$, the users are randomly distributed and communicate over multipath channels. The channels experienced by different users are independent and different since we focus on an uplink scenario. The DS-CDMA system under consideration employs quadrature phase-shift keying (QPSK) modulation. The channel coefficients are given by $h_{k,l}[i] = p_{k,l}\alpha_{k,l}[i]$, where $\alpha_{k,l}[i]$ $(l = 0, 1, \ldots, L_p - 1, k = 1, 2, \ldots, K)$ are obtained with Clarke's model [44] and $p_{k,l}$ represent the powers of each channel path. We show the results in terms of the normalized Doppler frequency f_dT (cycles/symbol), where f_d is the maximum Doppler shift and T is the symbol interval. We use three-path channels with relative powers given by 0, -3 and -6 dB,where in each run the spacing between paths is obtained from a discrete uniform random variable between 1 and 2 chips. The proposed blind channel estimator described in Section IV and that of [41] model the channel as a finite impulse response (FIR) filter and we employ a filter with 6 taps as an upper bound for the experiments. The phase ambiguity derived from the blind channel estimation method in [41] is addressed in our simulations by using the phase of $\hat{\mathbf{h}}_k(0)$ as a reference to remove the ambiguity and for time-varying channels we assume ideal phase tracking. Alternatively, differential modulation can be used to account for the phase rotations. The tuning parameter α, the forgetting factor β and the weighting parameter τ required by the time-varying bounds described in Section IV have been obtained by experimentation and chosen such that the performance of the algorithms is optimized. The update rate (UR) has been computed by counting, for each simulation trial t, the number of updates $(N_{u,t})$ performed and then dividing it by the number of received symbols $(N_{s,t})$. Then, the UR is given by $UR = \frac{1}{T}\sum_t^T \frac{N_{u,t}}{N_{s,t}}$, where $T = 1000$ is the total number of trials.

A. MSE Analytical Results

The aim of this part is to verify the validity of the analytical results obtained in Section V. Specifically, we shall evaluate the analytical MSE obtained with the analytical formulas in (72) and (80), and compare them to the results obtained by simulations. We consider first a scenario with fixed channels $(f_d = 0)$ and assess the MSE curves using (72) against the number of received symbols and also versus the signal-to-noise ratio (SNR) defined by E_b/N_0, as shown in Fig. 2. The results show that the analytical curves match very well those obtained via simulations, showing the validity of our analysis and assumptions.
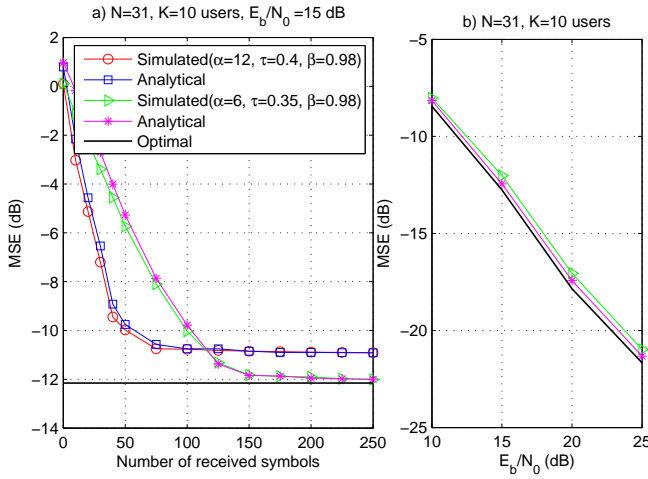
Fig. 2. MSE performance against a) the number of received symbols at $E_b/N_0 = 15dB$ and b) E_b/N_0 (dB).

Fig. 4. Performance of the a) interference power estimation and tracking and b) amplitude estimation algorithms at $E_b/N_0 = 12dB$.

The tracking analysis of the proposed SM-CCM-SG algorithm in a time-varying multipath fading channel is discussed next. We consider the same scenario as before except for the fact that the channel is now time-varying and the analytical results of the tracking analysis in (80) are employed. We consider a Jakes' model with a typical normalized fading rate f_dT. We compute $\mathrm{Tr}(\boldsymbol{Q})$ with the aid of $J_0(2\pi f_dT)$ [44], which is the autocorrelation function of the Jakes' model and where J_0 is the zero-order Bessel function of the first kind. A comparison of the curves obtained by simulations and by the analytical formulas is shown in Fig. 3. Similarly to the case of fixed channels, the analytical and simulated curves agree and show the validity of the proposed formulas. Nevertheless, the curves indicate a higher misadjustment as compared to the curves in Fig. 2 due to the time-varying process and extra effort of the adaptive filters to track the channel variations.
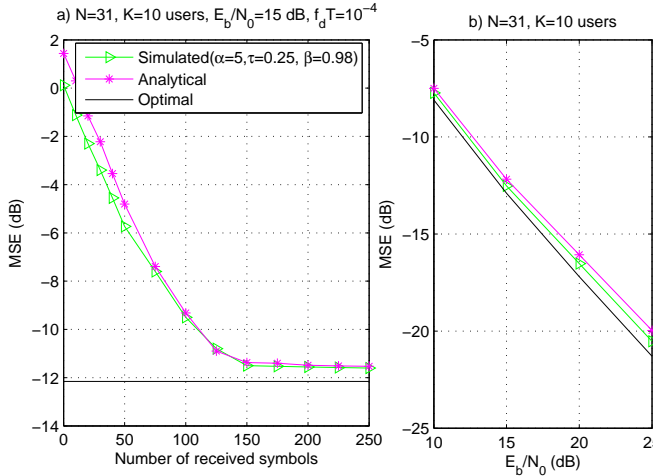

Fig. 3. MSE performance against a) the number of received symbols at $E_b/N_0 = 15dB$ and b) E_b/N_0 (dB).

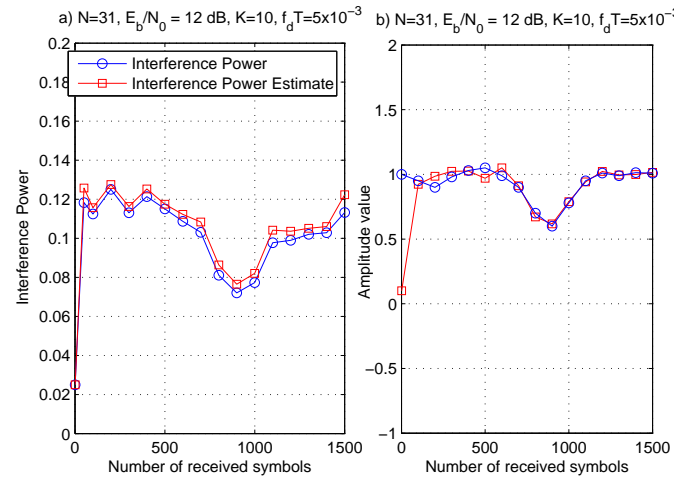### B. Interference Power and Channel Estimation

At this point, we wish to evaluate the effectiveness of the proposed algorithms for estimating and tracking the interference power and the amplitude. To this end, we have carried out an experiment, depicted in Fig. 4, where the proposed algorithm estimates of the MAI and ISI powers and the amplitude of the desired user are compared to the actual interference power and amplitude. The results show that the proposed algorithms are very effective for estimating and tracking the interference power in dynamic environments, as depicted in Fig. 4.

We assess the proposed channel estimation (CE) algorithm, called SM-CCM-CE, with the time-varying bounds PDB and PIDB, and also with a fixed bound, and compare them to the CMV-based method (CMV-CE) reported in [41] and the subspace algorithm of [42] in terms of the MSE between the actual and the estimated channels using the following dynamic scenario. The system has initially 10 users, the power distribution among the interferers follows a log-normal distribution with associated standard deviation of 3 dB. After 1500 symbols, 6 additional users enter the system and the power distribution among interferers is loosen to 6 dB. The results, shown in Fig 5, reveal that the proposed SM-CCM-CE algorithm outperforms the CMV-CE technique reported in [41] because SM-CCM-CE uses a variable forgetting factor. The proposed SM-CCM is also computationally simpler than CMV-CE due to the sparse updates, whereas it is substantially less complex than the subspace method of [42]. This is because the SM-CCM-CE needs to compute the principal eigenvector of the $L_p \times L_p$ matrix $\boldsymbol{C}_k \hat{\boldsymbol{R}}_k^{-1}[i]\boldsymbol{C}_k$, while the subspace technique in [42] requires the principal eigenvector of the $M \times M$ matrix $\hat{\boldsymbol{R}}^{-1}[i]$. Since $L_p \ll M$ in most practical scenarios SM-CCM-CE is typically considerably simpler than the subspace method.

C. BER Performance

The SM-CCM algorithms are assessed in a non-stationary environment where users enter and exit the system, as depicted
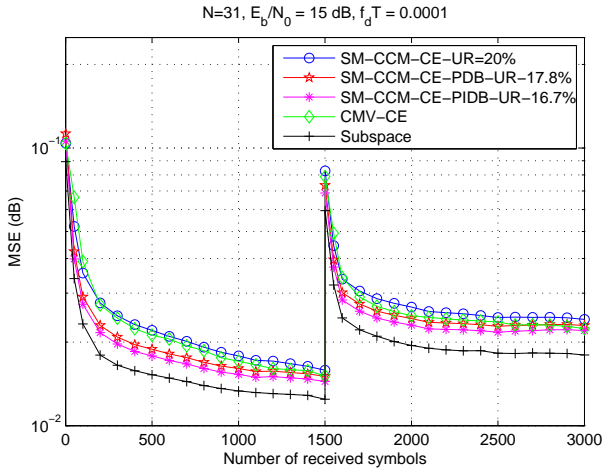
Fig. 5. MSE performance of channel estimation versus number of received symbols in a dynamic scenario where receivers operate at $SNR = E_b/N_0 = 15$ dB for the desired user.

Fig. 6. BER performance of the algorithms versus number of symbols for a non-stationary scenario. The parameters are $\gamma = 1.3$ for the SM-CMV, $\gamma = 0.65$ for the SM-CCM.

in Figs. 6 and 7. The system starts with 2 interferers with 7 dB above the desired user's power level and 5 interferers with the same power level of the desired one, which corresponds to the signal-to-noise ratio $E_b/N_0 = 15$ dB. At 1000 symbols, 2 interferers with 10 dB above the desired signal power level and 1 with the same power level enter the system, whereas 1 interferer with 7 dB above the desired signal power level leaves it. At 2000 symbols, 1 interferer with 10 dB above, and 5 interferers with the same power level of the desired signal exit the system, while 1 interferer with 15 dB above the desired user enters the system. The results for 100 runs show that the proposed SM-CCM-RLS algorithm achieves the best performance, followed by the proposed SM-CMV-RLS recursion, the CCM-SG and the CMV-SG methods. In summary the SM-CCM algorithms outperform the SM-CMV techniques in all scenarios and the SM-CCM-RLS algorithm is the best among the analyzed algorithms.

In a near-far scenario the eigenvalue spread of the covariance matrix of the received vector $r[i]$ is large, affecting the convergence performance of the SG algorithms with fixed step size and making it very difficult to compute a pre-determined value [1] for the step size. In this case, the SM-CCM algorithms are able to deal with near-far situations since they adopt variable step size or variable forgetting factor mechanisms, ensuring good tracking performance in dynamic scenarios and an improved performance over the existing CCM-type algorithms. In addition, due to their data-selective update feature the SM algorithms can save significant computational resources as they only require parameter updates for about 20% of the time for the SG-type recursions and around 12% for the RLS-based techniques.

The same scenario illustrated in Fig. 6 is considered for the SM-CCM algorithms with time-varying error bounds, as shown in Figs. 8 and 9. The results indicate that the proposed time-varying bounds are capable of improving the performance of SM algorithms, while further reducing the number of updates. The RLS-type algorithms outperform the SG-based
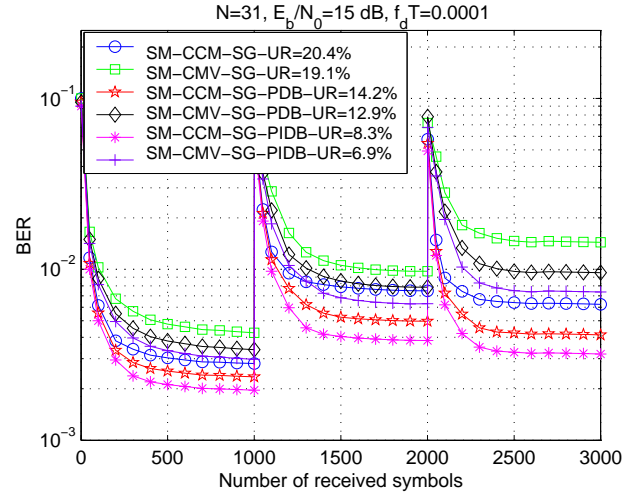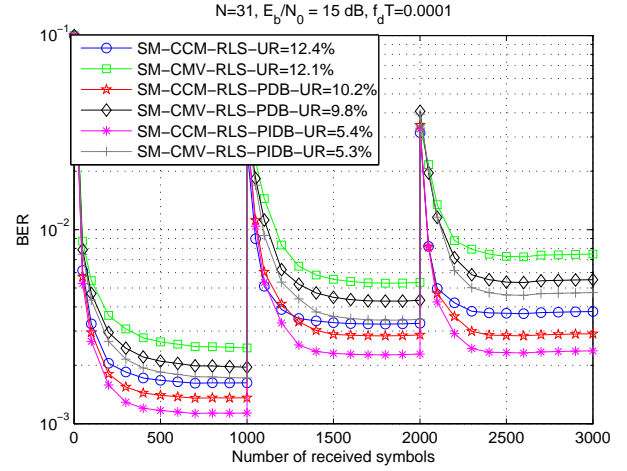

Fig. 7. BER performance of the algorithms versus number of symbols for a non-stationary scenario. The parameters are $\gamma = 1.3$ for the SM-CMV, $\gamma = 0.65$ for the SM-CCM.

techniques as expected and have lower UR. Moreover, the algorithms with the PIDB approach achieve the best performance, followed by the algorithms with the PDB method and the SM recursions with fixed bounds. With respect to the UR, the PIDB technique results in the smallest number of updates, followed by the PDB approach and the fixed bound.

The BER performance versus E_b/N_0 and number of users is shown in Fig. 10. We consider data packets of $P = 1500$ symbols and compare the proposed SM-CCM-RLS algorithm with a fixed bound, with the PIDB mechanism, the training-based BEACON algorithm with the PIDB mechanism [21] and the linear MMSE receiver that assumes perfect knowledge of the channels and the noise variance. We have measured the BER after 200 independent transmissions and the BEACON algorithm [6],[21] is trained with 200 symbols and is then switched to decision-directed mode. The curves illustrate that the proposed blind SM-CCM-RLS algorithm can approach the
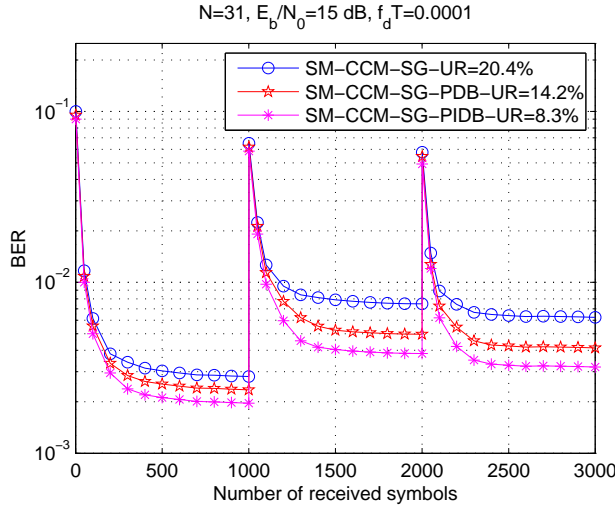
Fig. 8. BER performance of the SG algorithms versus number of symbols for a non-stationary scenario with time-varying bounds. The parameters are $\alpha = 8$, $\tau = 0.35$ and $\beta = 0.95$ for the time-varying bounds.
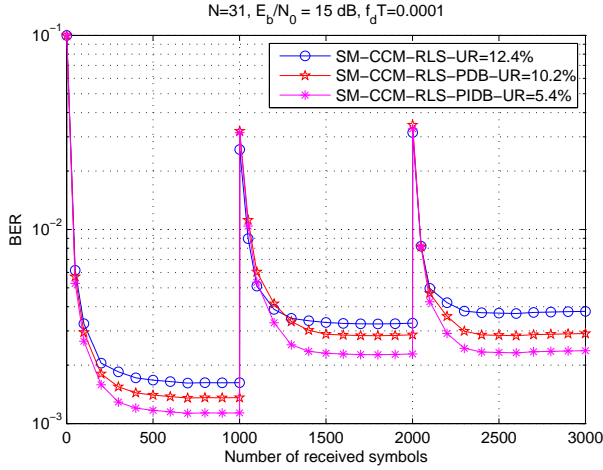

Fig. 10. BER performance versus (a) E_b/N_0 with $K = 10$ users and (b) number of users (K) at $E_b/N_0 = 15$ dB .

outperform previously reported blind techniques for several scenarios.

Fig. 9. BER performance of the RLS algorithms versus number of symbols for a non-stationary scenario with time-varying bounds. The parameters are $\alpha = 8$, $\tau = 0.35$ and $\beta = 0.95$ for the time-varying bounds.

performance of the supervised BEACON algorithm and that of the linear MMSE receiver.

VII. CONCLUSIONS

We have proposed SM-CCM adaptive algorithms for interference suppression in DS-CDMA systems. We have analyzed the optimization problem that gives rise to the SM-CCM algorithms and devised analytical expressions to predict their MSE performance with a good accuracy. The proposed SM-CCM algorithms are equipped with variable step sizes and forgetting factors and are only updated for a small fraction of time without incurring any performance degradation. A blind framework for SM estimation that takes into account parameter estimation dependency and MAI and ISI for multiuser communications has also been introduced to provide a time-varying bound that is robust against channel and SNR variations. Simulations have shown that the proposed SM-CCM algorithms
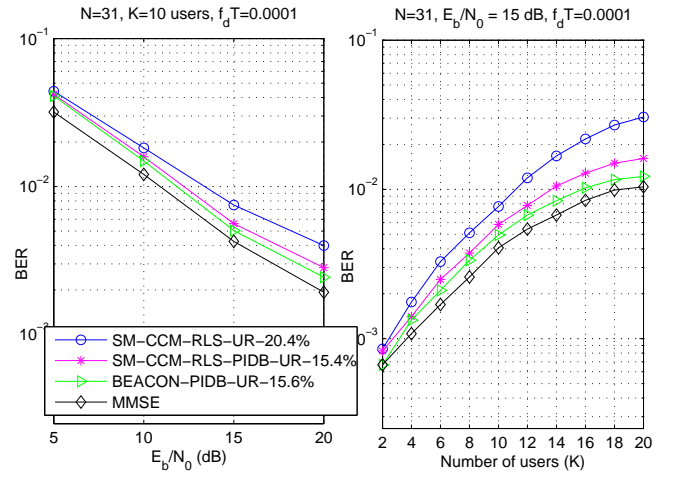
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