# Joint Power and Antenna Selection Optimization in Large Cloud Radio Access Networks

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#### Abstract

Large multiple-input multiple-output (MIMO) networks promise high energy efficiency, i.e., much less power is required to achieve the same capacity compared to the conventional MIMO networks if perfect channel state information (CSI) is available at the transmitter. However, in such networks, huge overhead is required to obtain full CSI especially for Frequency-Division Duplex (FDD) systems. To reduce overhead, we propose a downlink antenna selection scheme, which selects S antennas from M>S transmit antennas based on the large scale fading to serve  $K\leq S$  users in large distributed MIMO networks employing regularized zero-forcing (RZF) precoding. In particular, we study the joint optimization of antenna selection, regularization factor, and power allocation to maximize the average weighted sum-rate. This is a mixed combinatorial and non-convex problem whose objective and constraints have no closed-form expressions. We apply random matrix theory to derive asymptotically accurate expressions for the objective and constraints. As such, the joint optimization problem is decomposed into subproblems, each of which is solved by an efficient algorithm. In addition, we derive structural solutions for some special cases and show that the capacity of very large distributed MIMO networks scales as  $O(K\log M)$  when  $M\to\infty$  with K,S fixed. Simulations show that the proposed scheme achieves significant performance gain over various baselines.

#### **Index Terms**

Large MIMO, Cloud Radio Access Networks, Antenna selection, Asymptotic Analysis

# I. INTRODUCTION

Large MIMO networks have been a hot research topic due to their high energy efficiency [1]. Such networks are equipped with an order of magnitude more antennas than conventional systems, i.e., a hundred antennas or more. In centralized large MIMO systems where all antennas are collocated at the base station (BS), high energy efficiency is realized by exploiting the increased spatial degrees of freedom and beamforming gain. In large distributed MIMO systems where the antennas are distributed geographically, enhanced energy efficiency is achieved from shortened distances between antennas and users as well as improved spectral efficiency per unit area. There are a number of prior works on large MIMO networks, including various topics such as information theoretical capacity [2], transceiver design [3], CSI acquisition, and pilot contamination [4]. In particular, various downlink precoding



Figure 1. Illustration of a large distributed MIMO network, which consists of M thin base stations (distributed antennas) connected to a C-RAN via high speed optical fiber.

schemes have been proposed and analyzed. Remarkably, the simple zero-forcing (ZF) precoding is shown to achieve most of the capacity of large MIMO downlink [1]. One of the main challenges towards achieving the performance predicted by the theoretical analysis is how to obtain the CSI at the transmitter (CSIT) for a large number of antennas. In most of the existing works, Time-Division Duplex (TDD) is assumed and channel reciprocity can be exploited to obtain CSIT via uplink pilot training. In [5], [6], random matrix theory is used to analyze the asymptotic performance of ZF and RZF [7] in both TDD and FDD systems, with a focus on the case when all the antennas are collocated at a BS. For FDD systems, the amount of CSI feedback required to maintain a constant per-user rate gap from the perfect CSIT case has also been analyzed in [6] under the assumption of perfect CSI estimation at the users. In practice, we need M orthogonal pilot sequences to estimate the channel corresponding to the M transmit antennas. However, the number of available orthogonal pilot sequences is limited by the channel coherent time and it may become smaller than M as M grows large.

In this paper, we consider large distributed MIMO networks operating in FDD mode in which there are M distributed antennas (thin BSs<sup>1</sup>) linked together by high speed fiber backhaul as illustrated in Fig. 1. Such networks are also called the cloud radio access networks (C-RAN) [8]. In such a scenario, only a few nearby antennas can contribute significantly to a user's communication due to path loss. To avoid expensive CSI acquisition and signal processing overheads for antennas with huge path losses to the users, a subset of S antennas is selected to serve a given set of S users using RZF precoding [7], where S0 in a two-cell system.

The existing antenna selection schemes in multi-user MIMO systems [10], [11] require global knowledge of the instantaneous CSI which is unacceptable for large M. In 3G and LTE systems, users are associated with the strongest antennas/BSs. However, this baseline algorithm is inefficient when the antennas/BSs are allowed to perform cooperative MIMO (CoMP) [12] as illustrated in the following two examples. In both examples, we assume S=2 distributed antennas are selected to serve K=2 users.

<sup>&</sup>lt;sup>1</sup>A thin base-station refers to a low cost and low power base station and this name is borrowed from the nomenclature "thin client" in cloud computing.

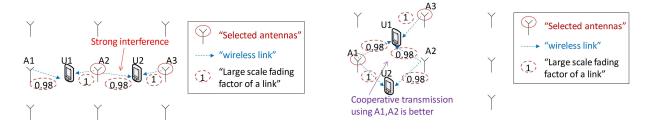


Figure 2. An example that strong cross link causes large interference

Figure 3. An example that strong cross links provide cooperative gain

**Example 1** (Strong cross link causing low SINR). Fig. 2 illustrates the path loss configuration. According to the baseline algorithm, the selected antennas will be  $\mathcal{A} = \{2,3\}$ . However, this is not a good choice because antenna 2 causes strong interference to user 2 before precoding. Although the interference can be suppressed using RZF precoding, the overall SINR is still low because the cross link from A3 to U1 is weak and the joint transmission gain is limited. A better choice would be  $\mathcal{A} = \{1,3\}$ .

**Example 2** (Strong cross link providing cooperative gain). Fig. 3 illustrates the path loss configuration. According to the baseline algorithm, the selected antennas will be  $\mathcal{A} = \{1, 3\}$ . Instead, better performance can be achieved by letting  $\mathcal{A} = \{1, 2\}$  due to cooperative transmission.

Hence, a more efficient antenna selection design is crucial for C-RAN. We study the joint optimization of antenna selection, regularization factor in RZF precoding, and power allocation, to maximize the average weighted sum-rate under per antenna power constraints. The optimization only requires the knowledge of large scale fading factors and the overhead for CSI acquisition is greatly reduced as discussed in Remark 1. The following are two first-order challenges.

- Combinatorial Optimization Problem: The antenna selection problem with CoMP processing in the C-RAN
  is combinatorial with exponential complexity w.r.t. the total number of antennas M.
- Asymptotic Performance Analysis: It is important to derive closed-form performance expressions in order to
  obtain design insights. Yet, the performance analysis is non-trivial due to the heterogeneous path loss as well
  as the lack of closed form antenna selection solution.

In this paper, we extend the results in [6] to obtain deterministic equivalent (DE) of the weighted sum-rate and the per-antenna transmit power<sup>2</sup>. By exploiting the implicit structure in the objective and constraints functions, the joint optimization problem is decomposed into simpler subproblems, each of which is solved by an efficient

<sup>2</sup>We also noticed that the downlink channel **H** in C-RAN can be modeled as the gram random matrices with a given variance profile [13]. The mutual information  $\log |\mathbf{H}\mathbf{H}^{\dagger} + \rho \mathbf{I}|$  for such channel model, where  $\rho > 0$  is a constant, has been shown in [13] to have a Gaussian limit whose parameters are identified as the dimension of **H** goes to infinity. In this paper, we focus on a different problem, i.e., the joint optimization of power and antenna selection to maximize the weighted sum-rate under RZF precoding.

algorithm. We also show that there is an asymptotic decoupling effect in very large distributed MIMO networks and the capacity grows logarithmically with the total number of antennas M even when the number of active antennas S is fixed.

The rest of the paper is organized as follows. The system model is outlined in Section II. In Section III, the antenna selection problem is formulated and its deterministic approximation is derived using random matrix theory. The solution of the problem is presented in Section IV. In Section V, we give structural solutions for some special cases. Simulations are used to verify the performance of the proposed solution in Section VI and the conclusion is given in Section VII.

#### II. SYSTEM MODEL

Consider the downlink of C-RAN with M distributed transmit antennas and K single-antenna users as illustrated in Fig. 1. The  $M\gg K$  distributed antennas are connected to a C-RAN [8] via fiber backhaul and the system operates in FDD mode. Denote  $h_{km}$  as the channel between the  $m^{\text{th}}$  transmit antenna and the  $k^{\text{th}}$  user. We consider a composite fading channel, i.e.,  $h_{km}=\sigma_{km}W_{km}, \ \forall k,m$ , where  $\sigma_{km}\geq 0$  is the large scale fading factor caused by, e.g., path loss and shadow fading, and  $W_{km}$  is the small scale fading factor.

**Assumption 1** (Channel model). The small scale fading process  $W_{km}(t) \sim \mathcal{CN}(\mathbf{0}, 1)$  is quasi-static within a time slot but i.i.d. w.r.t. time slots and the spatial indices k, m. The large scale fading process  $\sigma_{km}(t)$  is assumed to be a slow ergodic random process (i.e.,  $\sigma_{km}(t)$  remains constant for a large number of time slots) according to a general distribution.

The baseband processing is centralized at the C-RAN. To limit the signaling overheads, we consider antenna selection where a subset  $\mathcal{A}$ ,  $|\mathcal{A}| = S \geq K$  of the M antennas are selected to serve the K users. Let  $\mathcal{A}_j$  denote the  $j^{\text{th}}$  element in  $\mathcal{A}$ . Let  $\mathbf{H}(\mathcal{A}) \in \mathbb{C}^{K \times S}$  denote the composite downlink channel matrix between the selected S antennas and the K users, and define  $\mathbf{\Sigma}(\mathcal{A}) \in \mathbb{R}_+^{K \times S}$  as the corresponding large scale fading matrix, whose element at the  $k^{\text{th}}$  row and the  $j^{\text{th}}$  column is  $\sigma_{k\mathcal{A}_j}$ . For conciseness,  $\mathbf{H}(\mathcal{A})$  and  $\mathbf{\Sigma}(\mathcal{A})$  are denoted as  $\mathbf{H}$  and  $\mathbf{\Sigma}$  when there is no ambiguity.

**Assumption 2** (CSIT assumption). The C-RAN has knowledge of all the  $K \times M$  large scale fading factors  $\sigma_{km}$ 's and the  $K \times S$  instantaneous channel matrix  $\mathbf{H}(A)$  corresponding to the selected antennas in A only.

Remark 1 (CSI Acquisition). In FDD systems,  $\mathbf{H}(A)$  can be obtained via downlink channel estimation and channel feedback. The amount of training for  $\mathbf{H}(A)$  is limited by the channel coherence time, which depends on the user movement speed. Hence, for large M, the estimated CSI quality at the C-RAN will be poor if all the M antennas in the network are active. Using antenna selection and with properly chosen S, the instantaneous CSI for the S selected antennas can be estimated and fed back to the C-RAN using conventional arrangement in LTE. Hence, the problem of CSI limitation in C-RAN can be alleviated by antenna selection based on large scale fading factors. On

the other hand, the large scale fading matrix  $\Sigma$  is a long-term statistic and can be estimated at the C-RAN from the uplink reference signals [14] due to the reciprocity of large scale fading factors.

We consider the RZF precoding scheme [7]. The composite receive signal vector for the K users can be expressed as:

$$y = HFs + n,$$

where  $\mathbf{s} = [s_1, ..., s_K] \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_K)$  is the symbol vector;  $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_K)$  is the AWGN noise vector; and  $\mathbf{F} = [\mathbf{f}_1, ..., \mathbf{f}_K] \in \mathbb{C}^{S \times K}$  is the RZF precoding matrix given by

$$\mathbf{F} = (\mathbf{H}^{\dagger}\mathbf{H} + \alpha S\mathbf{I}_S)^{-1} \mathbf{H}^{\dagger} \mathbf{P}^{1/2}, \tag{1}$$

where  $\alpha$  is the regularization factor and  $\mathbf{P} = \operatorname{diag}(p_1,...,p_K)$  is a power allocation matrix. Note that the regularization factor is scaled by S to ensure that  $\alpha$  is bounded as  $S, K \to \infty$  [6]. Define power allocation vector as  $\mathbf{p} = [p_1,...,p_K]^T$ .

Define the normalized channel  $\bar{\mathbf{H}} = \mathbf{H}/\sqrt{S}$ . Let  $\mathbf{h}_k^{\dagger}$  and  $\bar{\mathbf{h}}_k^{\dagger}$  denote, respectively, the  $k^{\text{th}}$  row of  $\mathbf{H}$  and  $\bar{\mathbf{H}}$ . Define  $\bar{\mathbf{H}}_k$  as the matrix  $\bar{\mathbf{H}}$  with the  $k^{\text{th}}$  row removed, and  $\mathbf{P}_k \triangleq \text{diag}\left(p_1,...,p_{k-1},p_{k+1},...,p_K\right)$ . Assume that user k has perfect knowledge of the effective channel  $\mathbf{h}_k^{\dagger}\mathbf{f}_k$  and the interference-plus-noise power. The SINR of user k is [5]

$$\gamma_k \left( \mathcal{A}, \alpha, \mathbf{p} \right) = \frac{p_k A_k^2}{B_k + \left( 1 + A_k \right)^2},\tag{2}$$

where

$$A_k = \bar{\mathbf{h}}_k^{\dagger} \left( \bar{\mathbf{H}}_k^{\dagger} \bar{\mathbf{H}}_k + \alpha \mathbf{I}_S \right)^{-1} \bar{\mathbf{h}}_k,$$

$$B_k = \bar{\mathbf{h}}_k^{\dagger} \left( \bar{\mathbf{H}}_k^{\dagger} \bar{\mathbf{H}}_k + \alpha \mathbf{I}_S \right)^{-1} \bar{\mathbf{H}}_k^{\dagger} \mathbf{P}_k \bar{\mathbf{H}}_k \left( \bar{\mathbf{H}}_k^{\dagger} \bar{\mathbf{H}}_k + \alpha \mathbf{I}_S \right)^{-1} \bar{\mathbf{h}}_k.$$

The instantaneous transmit power of the  $j^{th}$  selected antenna is given by

$$P_{\mathcal{A}_j}\left(\mathcal{A}, \alpha, \mathbf{p}\right) = \frac{1}{S} \mathbf{1}_j^T \bar{\mathbf{F}} \bar{\mathbf{F}}^{\dagger} \mathbf{1}_j, \tag{3}$$

where  $\bar{\mathbf{F}} = \bar{\mathbf{H}}^{\dagger} \left( \bar{\mathbf{H}} \bar{\mathbf{H}}^{\dagger} + \alpha \mathbf{I}_K \right)^{-1} \mathbf{P}^{1/2}$ ; and  $\mathbf{1}_j$  is a  $K \times 1$  vector whose  $j^{\text{th}}$  element is 1 and all other elements are zeros.

#### III. OPTIMIZATION FORMULATION FOR DYNAMIC ANTENNA SELECTION

We consider long-term control policy where the active antenna set A, the regularization factor  $\alpha$ , and the power allocation  $\mathbf{p}$  are adaptive to the large scale fading  $\Sigma$  only.

**Definition 1** (Long-term control policy). A long-term antenna selection, regularization and power control policy  $\Psi = \{\Psi_{\mathcal{A}}, \Psi_{\alpha}, \Psi_{\mathbf{p}}\}$  are mappings from the large scale fading matrix  $\Sigma$  to the active antenna set  $\mathcal{A}$ , the regularization factor  $\alpha$ , and the power allocation  $\mathbf{p}$  respectively. Specifically,  $\mathcal{A}$ ,  $\alpha$ , and  $\mathbf{p}$  are given by:  $\Psi_{\mathcal{A}}(\Sigma)$ ,  $\Psi_{\alpha}(\Sigma)$  and  $\Psi_{\mathbf{p}}(\Sigma)$ .

For technical reasons, we consider a sequence of C-RAN systems indexed by  $S = \{1, 2, ...\}$ . In the S-th system, there are  $M = \lceil \overline{\beta}S \rceil$  distributed transmit antennas and  $K = \lceil \beta S \rceil$  single-antenna users, where  $\beta \in (0, 1]$  and  $\frac{1}{\overline{\beta}} \in (0, 1)$  are constant. Correspondingly, we consider a sequence of long-term control policies  $\{\Psi^{(S)}\}$  indexed by  $S = \{1, 2, ...\}$ , and apply the S-th control policy  $\Psi^{(S)}$  to the S-th system. We restrict our attention to the class of policies that satisfy the following technical assumptions.

**Definition 2** (Admissible control policy). A sequence of control policies  $\{\Psi^{(S)}\}$  is admissible if for each S,  $\Psi^{(S)}_{\mathcal{A}}$  is a mapping:  $\mathbb{R}^{K \times M}_{+} \mapsto \{\mathcal{A} : \mathcal{A} \subset \{1,...,M\} \ , \ |\mathcal{A}| = S\}, \ \Psi^{(S)}_{\alpha}$  is a mapping:  $\mathbb{R}^{K \times M}_{+} \mapsto [\alpha_{\min}, \alpha_{\max}], \ \text{and} \ \Psi^{(S)}_{\mathbf{p}}$  is a mapping:  $\mathbb{R}^{K \times M}_{+} \mapsto [0, P_{\max}]^{K}, \ \text{where the constants} \ P_{\max}, \ \alpha_{\max} \ \text{and} \ \alpha_{\min} \in (0, \infty).$ 

The objective of the S-th control policy  $\Psi^{(S)}$  is to maximize the conditional average weighted sum-rate for the S-th system. Specifically, given large scale fading matrix  $\mathbf{\Sigma}^{(S)} \in \mathbb{R}_+^{K \times M}$  and weight vector  $\mathbf{w}^{(S)} = \left[w_k^{(S)}\right]_{k=1,\dots,K} \in \mathbb{R}_+^K$  for the S-th system, the long-term control  $\Psi^{(S)}\left(\mathbf{\Sigma}^{(S)}\right)$  is given by the solution of the following joint optimization problem

$$\mathcal{P}\left(\mathbf{\Sigma}^{(S)}\right) : \max_{\Psi^{(S)}\left(\mathbf{\Sigma}^{(S)}\right)} \mathcal{I}\left(\Psi^{(S)}\left(\mathbf{\Sigma}^{(S)}\right) | \mathbf{\Sigma}^{(S)}\right)$$
s.t. 
$$\mathbf{E}\left[P_{m}\left(\Psi^{(S)}\left(\mathbf{\Sigma}^{(S)}\right)\right) \middle| \mathbf{\Sigma}^{(S)}\right] \le \frac{\rho_{m}}{S}, \ \forall m \in \Psi_{\mathcal{A}}^{(S)}\left(\mathbf{\Sigma}^{(S)}\right), \tag{4}$$

where for given  $\Sigma$ , w and  $\Psi(\Sigma) = \{A, \alpha, \mathbf{p}\}$ , the conditional average weighted sum-rate is

$$\mathcal{I}\left(\Psi\left(\mathbf{\Sigma}\right)|\mathbf{\Sigma}\right) = \mathbf{E}\left[\sum_{k=1}^{K} w_{k} \log\left(1 + \gamma_{k}\left(\Psi\left(\mathbf{\Sigma}\right)\right)\right)|\mathbf{\Sigma}\right],\tag{5}$$

 $\gamma_{k}\left(\Psi\left(\mathbf{\Sigma}\right)\right)=\gamma_{k}\left(\mathcal{A},\alpha,\mathbf{p}\right)$  is the SINR in (2),  $P_{m}\left(\Psi\left(\mathbf{\Sigma}\right)\right)=P_{m}\left(\mathcal{A},\alpha,\mathbf{p}\right)$  is the per antenna transmit power in (3) and  $\rho_{m}>0$  is a constant.

Remark 2 (Per antenna power constraint). In (4), the per antenna power constraint for the S-th system is  $\frac{\rho_m}{S}$  due to the following reason. It can be verified that  $\mathrm{E}\left[P_m\left(\Psi^{(S)}\left(\mathbf{\Sigma}^{(S)}\right)\right)\big|\mathbf{\Sigma}^{(S)}\right] = \sum_{k=1}^K R'_{m,k}p_k = O(1/S) \to 0$  as  $S \to \infty$  under an admissible control policy  $\Psi^{(S)}$ , where  $R'_{m,k} = O\left(\frac{1}{S^2}\right)$  is some coefficient independent of  $\mathbf{p}$ . Moreover, the effective channel gain per user is O(S). Hence, the per-antenna transmit power required to support a finite data rate for each user in the S-th system is O(1/S) as  $S \to \infty$  and O(1) power allocation variables  $\{p_1,...,p_K\}$  are needed to satisfy the constraint (4). Similar observation is also made in [1], [6] about in a large MIMO system with M antennas, a per-antenna power of O(1/M) is needed to support a finite SINR for each user.

There are several challenges in solving Problem  $\mathcal{P}\left(\mathbf{\Sigma}^{(S)}\right)$ . First, there is no closed-form expression for the optimization objective and constraints. Second, the problem is combinatorial w.r.t. the antenna selection and non-convex w.r.t. the regularization factor and power allocation. The first challenge is tackled in this section by using the random matrix theory in [15] to derive asymptotically accurate expressions for the optimization objective and constraints. The second challenge is tackled in Section IV.

To derive DE of the weighted sum-rate and transmit power, we require the following assumptions.

**Assumption 3** (Boundedness of  $\Sigma^{(S)}$ ,  $\mathbf{w}^{(S)}$ ).

1) Uniformly Bounded  $\Sigma^{(S)}$  w.r.t. S: Let  $\{\Sigma^{(S)} \in \mathbb{R}_+^{K \times M}\}$  be a sequence of large scale fading matrices (indexed by  $S = \{1, 2, ...\}$ ) such that

$$\limsup_{S \to \infty} \sup_{1 \le k \le K, 1 \le m \le M} \sigma_{km}^{(S)} < \infty, \tag{6}$$

$$\liminf_{S \to \infty} \inf_{1 \le k \le K, 1 \le m \le M} \sigma_{km}^{(S)} > 0, \tag{7}$$

where  $\sigma_{km}^{(S)}$  is the element at the k-th row and m-th column of  $\Sigma^{(S)}$ .

2) Uniformly Bounded  $Kw_k$  w.r.t. S: Let  $\{\mathbf{w}^{(S)} \in \mathbb{R}_+^K\}$  be a sequence of weight vectors (indexed by  $S = \{1, 2, ...\}$ ) such that

$$\limsup_{S \to \infty} \sup_{1 \le k \le K} K w_k^{(S)} < \infty.$$

The assumption in (6) ensures that the normalized channel matrix has uniformly bounded spectral norm with probability 1, which is required in the proof of Lemma 1 and 2 later.

**Proposition 1.** Let  $\{\Sigma^{(S)}\}$  be as in Assumption 3. Let  $\bar{\mathbf{H}}^{(S)}$  denote the normalized channel matrix corresponding to  $\Sigma^{(S)}$ . We have  $\limsup_{S\to\infty} \|\bar{\mathbf{H}}^{(S)\dagger}\bar{\mathbf{H}}^{(S)}\| \overset{a.s}{<} \infty$ .

Please refer to Appendix A for the proof.

The assumption in (7) ensures that  $\xi_l$  in (9) in Lemma 1 is bounded away from zero as  $S \to \infty$ . Assumption 3-2) is to ensure that  $\mathcal{I}\left(\Psi^{(S)}\left(\mathbf{\Sigma}^{(S)}\right)|\mathbf{\Sigma}^{(S)}\right)$  is bounded as  $S \to \infty$ .

**Lemma 1** (DE of SINR). Let  $\{\Sigma^{(S)}\}$  be as in Assumption 3. We have  $\gamma_k(\Psi^{(S)}(\Sigma^{(S)})) - \bar{\gamma}_k(\Psi^{(S)}(\Sigma^{(S)})) \stackrel{a.s}{\to} 0$  as  $S \to \infty$ , where for given  $\Sigma$  and  $\Psi(\Sigma) = \{A, \alpha, \mathbf{p}\}$ ,

$$\bar{\gamma}_k\left(\Psi\left(\mathbf{\Sigma}\right)\right) = \frac{p_k \xi_k^2}{\frac{1}{S} \sum_{l \neq k}^K \left[ p_l \theta_{kl} / \left(1 + \xi_l\right)^2 \right] + \left(1 + \xi_k\right)^2},\tag{8}$$

where  $\boldsymbol{\xi} = \left[\xi_1,...,\xi_K\right]^T \in \mathbb{R}_+^K$  is the unique solution of

$$\xi_{l} = \frac{1}{S} \sum_{m \in \mathcal{A}} \left[ \sigma_{lm}^{2} / f_{m} \left( \xi \right) \right], \ l = 1, ..., K,$$
 (9)

with  $f_m\left(\mathbf{\xi}\right) \triangleq \alpha + \frac{1}{S} \sum_{i=1}^K \frac{\sigma_{im}^2}{1+\xi_i}$ , and  $\boldsymbol{\theta}_k = \left[\theta_{k1}, ..., \theta_{kK}\right]^T$  is

$$\boldsymbol{\theta}_k \triangleq (\mathbf{I}_K - \mathbf{D})^{-1} \mathbf{d}_k, \ k = 1, ..., K,$$
 (10)

with  $\mathbf{D} = [D_{ln}]_{l,n=1,...,K} \in \mathbb{R}^{K \times K}$  and  $\mathbf{d}_k = [d_{kl}]_{l=1,...,K} \in \mathbb{R}^{K \times 1}$  given by

$$D_{ln} = \frac{1}{S} \sum_{m \in \mathcal{A}} \left[ \frac{1}{S} \sigma_{lm}^2 \sigma_{nm}^2 / \left( (1 + \xi_n)^2 f_m^2 \left( \boldsymbol{\xi} \right) \right) \right],$$

$$d_{kl} = \frac{1}{S} \sum_{m \in \mathcal{A}} \left[ \sigma_{km}^2 \sigma_{lm}^2 / f_m^2 \left( \xi \right) \right].$$

**Lemma 2** (DE of per-antenna transmit power). Let  $\left\{ \mathbf{\Sigma}^{(S)} \right\}$  be as in Assumption 3. We have  $SP_m\left( \Psi^{(S)}\left( \mathbf{\Sigma}^{(S)} \right) \right) - S\bar{P}_m\left( \Psi^{(S)}\left( \mathbf{\Sigma}^{(S)} \right) \right) \overset{a.s}{\to} 0, \forall m \in \Psi_{\mathcal{A}}^{(S)}\left( \mathbf{\Sigma}^{(S)} \right)$  as  $S \to \infty$ , where for given  $\mathbf{\Sigma}$  and  $\Psi\left( \mathbf{\Sigma} \right) = \left\{ \mathcal{A}, \alpha, \mathbf{p} \right\}$ 

$$\bar{P}_{m}\left(\Psi\left(\mathbf{\Sigma}\right)\right) = \frac{\alpha^{-1}}{S^{2}}\psi_{m}^{-2}\left(\mathbf{v}\right)\sum_{i=1}^{K}\sigma_{im}^{2}\left(p_{i}v_{i}-\varphi_{i}\right),\tag{11}$$

where  $\mathbf{v} = [v_1, ..., v_K]^T \in \mathbb{R}_{++}^K$  is the unique solution of

$$v_{l} = \frac{1}{\alpha + \frac{1}{S} \sum_{m \in \mathcal{A}} \left[ \sigma_{lm}^{2} / \psi_{m} \left( \mathbf{v} \right) \right]}, \ l = 1, ..., K,$$
(12)

with  $\psi_{m}\left(\mathbf{v}\right)\triangleq1+\frac{1}{S}\sum_{i=1}^{K}\sigma_{im}^{2}v_{i}$ , and  $\boldsymbol{\varphi}=\left[\varphi_{1},...,\varphi_{K}\right]^{T}$  is

$$\varphi \triangleq (\alpha \mathbf{I}_K + \mathbf{\Delta} - \mathbf{C})^{-1} \mathbf{c}, \tag{13}$$

with  $\Delta = diag(\Delta_1, ..., \Delta_K)$ ,  $\mathbf{C} = [C_{ln}]_{l,n=1,...,K} \in \mathbb{R}^{K \times K}$  and  $\mathbf{c} = [c_l]_{l=1,...,K} \in \mathbb{R}^{K \times 1}$  given by

$$\Delta_{l} = \frac{1}{S} \sum_{m \in \mathcal{A}} \left[ \sigma_{lm}^{2} / \psi_{m} \left( \mathbf{v} \right) \right], \ l = 1, ..., K,$$

$$C_{ln} = \frac{1}{S} \sum_{m \in \mathcal{A}} \left[ \frac{1}{S} \sigma_{lm}^{2} \sigma_{nm}^{2} v_{l} / \psi_{m}^{2} \left( \mathbf{v} \right) \right],$$

$$c_{l} = \frac{1}{S} \sum_{m \in \mathcal{A}} \left[ \frac{\sigma_{lm}^{2} v_{l}}{\psi_{m}^{2} \left( \mathbf{v} \right)} \left( p_{l} + \frac{1}{S} \sum_{i=1}^{K} \sigma_{im}^{2} v_{i} \left( p_{l} - p_{i} \right) \right) \right].$$

The proof of Lemma 1 is similar to that of [6, Thereom 2] and is omitted for conciseness<sup>3</sup>. The proof of Lemma 2 is given in Appendix B.

Remark 3. The equations in (9) and (12) can be solved using Proposition 1 in [6].

Then the following theorem holds.

**Theorem 1** (Asymptotic equivalence of Problem  $\mathcal{P}$ ). Let  $\left\{\mathbf{\Sigma}^{(S)}\right\}$ ,  $\left\{\mathbf{w}^{(S)}\right\}$  be as in Assumption 3 and  $\Psi^{(S)*}\left(\mathbf{\Sigma}^{(S)}\right)$  be an optimal solution of the following problem

$$\mathcal{P}_{E}\left(\mathbf{\Sigma}^{(S)}\right) : \max_{\Psi^{(S)}\left(\mathbf{\Sigma}^{(S)}\right)} \bar{\mathcal{I}}\left(\Psi^{(S)}\left(\mathbf{\Sigma}^{(S)}\right) | \mathbf{\Sigma}^{(S)}\right)$$

$$\triangleq \sum_{k=1}^{K} w_{k}^{(S)} log\left(1 + \bar{\gamma}_{k}\left(\Psi^{(S)}\left(\mathbf{\Sigma}^{(S)}\right)\right)\right)$$
s.t.  $\bar{P}_{m}\left(\Psi^{(S)}\left(\mathbf{\Sigma}^{(S)}\right)\right) \leq \frac{\rho_{m}}{S}, \ \forall m \in \Psi_{\mathcal{A}}^{(S)}\left(\mathbf{\Sigma}^{(S)}\right).$  (14)

Let  $\mathcal{I}^{(S)\circ}$  be the optimal value of Problem  $\mathcal{P}\left(\mathbf{\Sigma}^{(S)}\right)$  with weight vector  $\mathbf{w}^{(S)}$ . Then as  $S \to \infty$ , we have

$$SE\left[P_m\left(\Psi^{(S)*}\left(\mathbf{\Sigma}^{(S)}\right)\right)\middle|\mathbf{\Sigma}^{(S)}\right] - \rho_m \le 0,$$

$$\forall m \in \Psi_{\mathcal{A}}^{(S)*}\left(\mathbf{\Sigma}^{(S)}\right),$$
(15)

$$\mathcal{I}\left(\Psi^{(S)*}\left(\mathbf{\Sigma}^{(S)}\right)|\mathbf{\Sigma}^{(S)}\right) - \mathcal{I}^{(S)\circ} \to 0. \tag{16}$$

 $<sup>^3</sup>$ In [6, Thereom 2],  $\alpha$  is a positive constant. However, it can be verified that the proof of [6, Thereom 2] still holds if we replace the constant  $\alpha$  by  $\alpha^{(S)} = \Psi_{\alpha}^{(S)}\left(\mathbf{\Sigma}^{(S)}\right)$  in Lemma 1 due to the bounded constraint on  $\Psi_{\alpha}^{(S)}$ .

Please refer to Appendix C for the proof. Note that in both  $\mathcal{P}\left(\mathbf{\Sigma}^{(S)}\right)$  and  $\mathcal{P}_{E}\left(\mathbf{\Sigma}^{(S)}\right)$ ,  $\Psi^{(S)}$  must be admissible. Remark 4. In Appendix C, we prove that for a given sequence of admissible control policies  $\left\{\Psi^{(S)}\right\}$  and  $\left\{\mathbf{\Sigma}^{(S)}\right\}$ ,  $\left\{\mathbf{w}^{(S)}\right\}$  that satisfy Assumption 3, the conditional average weighted sum-rate  $\mathcal{I}\left(\Psi^{(S)}\left(\mathbf{\Sigma}^{(S)}\right)|\mathbf{\Sigma}^{(S)}\right)$  (conditioned on the large scale fading matrix  $\mathbf{\Sigma}^{(S)}$ ) converges to the DE  $\bar{\mathcal{I}}\left(\Psi^{(S)}\left(\mathbf{\Sigma}^{(S)}\right)|\mathbf{\Sigma}^{(S)}\right)$  as  $S \to \infty$ . In the proposed long-term control policy, the antenna selection  $\mathcal{A}$  is adaptive to the large scale fading matrix  $\mathbf{\Sigma}$  only. Hence, for a given sequence of  $\mathbf{\Sigma}^{(S)}$ , the antenna set  $\Psi^{(S)}_{\mathcal{A}}\left(\mathbf{\Sigma}^{(S)}\right)$  is deterministic for each S and as a result, the DE convergence holds true. On the other hand, if  $\mathcal{A}$  were adaptive to the short term CSI  $\mathbf{H}$ , then conditioned on  $\mathbf{\Sigma}^{(S)}$ ,  $\mathcal{A}$  would still be random and the DE convergence would fail. Similar conclusion has also been made in [16] that the DE of the data rate in massive MIMO system with user selection is valid as long as the user selection is independent of

Remark 5. For centralized large MIMO downlink, a DE of the SINR has been provided in Theorem 2 of [6] under per-user channel transmit correlation with correlation matrices  $\Theta_k \triangleq \operatorname{E}\left[\mathbf{h}_k\mathbf{h}_k^{\dagger}\right]$ ,  $\forall k$  and imperfect CSIT with CSIT errors  $\tau_k$ 's. Our channel model is a special case of that in [6] with  $\Theta_k = \sigma_k^2 \triangleq \operatorname{diag}\left(\sigma_{kA_1}^2,...,\sigma_{kA_S}^2\right)$ ,  $\forall k$  and  $\tau_k = 0$ ,  $\forall k$  in [6]. However, this paper and [6] focus on different network topologies (distributed versus centralized). As a result, there are some new technical challenges:

the instantaneous CSI H.

- Due to the distributed topology, we have to consider per-antenna power constraint, which is more complicated than the sum power constraint considered in [6]. For example, in the DE of the SINR in Theorem 2 of [6], the RZF precoding matrix is scaled to satisfy the sum power constraint. However, the per-antenna power constraint cannot be satisfied by simply scaling the precoding matrix **F**, and we need to derive the DE for the per-antenna transmit power as in Lemma 2. Moreover, the per-antenna power constraint has to be explicitly handled by the optimization algorithm, which makes both the algorithm design and performance analysis more difficult.
- The assumptions for the theoretical results are different between the distributed and the centralized topologies. Theorem 2 of [6] requires the following assumptions: A1)  $\bar{\mathbf{H}}^{\dagger}\bar{\mathbf{H}}$  has, almost surely, uniformly bounded spectral norm on M. The optimization problems in [6] focus on the case whereby  $\Theta_k = \Theta$ ,  $\forall k$ , under which A1 is true. However, it is not clear if A1 holds for the distributed topology. In this paper, we replace A1 with Assumption 3-1), which is a more mild assumption in practical systems.
- In Theorem 1, we formally proved the asymptotic equivalence between Problem  $\mathcal{P}$  and its deterministic approximation  $\mathcal{P}_E$ . The proof of Theorem 1 is non-trivial. However, in [6], the problem formulation is directly based on the deterministic approximation and there is no proof of the asymptotic equivalence between the "original problem" and its deterministic approximation.
- Compared to the optimization problems in [6], problem  $\mathcal{P}_E$  is much more difficult to solve due to the heterogeneous path loss and the combinatorial nature of the antenna selection problem.

<sup>&</sup>lt;sup>4</sup>This assumption is reasonable since the correlations between the geographically distributed antennas are indeed negligible.

# IV. OPTIMIZATION SOLUTION FOR $\mathcal{P}_E$

In Theorem 1, we let  $S \to \infty$  to establish the asymptotic equivalence between Problem  $\mathcal{P}$  and  $\mathcal{P}_E$ . In this section, we focus on solving  $\mathcal{P}_E$  for large but finite M, K, S, which is the case for a practical large C-RAN. By Theorem 1, the optimal value  $\mathcal{I}^*$  of  $\mathcal{P}_E(\Sigma)$  and the optimal value  $\mathcal{I}^\circ$  of  $\mathcal{P}(\Sigma)$  satisfy  $\mathcal{I}^* = \mathcal{I}^\circ + o(1)$  for fixed M, K, S, where  $o(1) \to 0$  as  $S \to \infty$ . This implies that the solution of Problem  $\mathcal{P}(\Sigma)$  can still be well approximated by the solution of  $\mathcal{P}_E(\Sigma)$  for large but finite M, K, S. Since we focus on solving  $\mathcal{P}_E(\Sigma)$  for given  $\Sigma$  in the rest of the paper, we will omit the argument  $\Sigma$  in  $\mathcal{P}_E$  and explicitly express  $\bar{\mathcal{I}}(\Psi(\Sigma)|\Sigma)$  as  $\bar{\mathcal{I}}(\Psi(\Sigma)) = \bar{\mathcal{I}}(\mathcal{A}, \alpha, \mathbf{p})$ , where  $\{\mathcal{A}, \alpha, \mathbf{p}\} = \Psi(\Sigma)$ .

#### A. Problem Decomposition

Under an admissible control policy, the power allocation vector is bounded as  $\max_{1 \le k \le K} p_k \le P_{\text{max}}$ . We first show that this bounded power constraint can be relaxed in Problem  $\mathcal{P}_E$ .

**Proposition 2.** For fixed M, K, S, let  $\mathcal{P}_E'$  denote a relaxed problem of  $\mathcal{P}_E$  obtained by removing the bounded power constraint  $\max_{1 \leq k \leq K} p_k \leq P_{max}$  in  $\mathcal{P}_E$ . For sufficiently large  $P_{max}$ , the optimal power allocation  $\mathbf{p}^* = [p_1^*, ..., p_K^*]$  of  $\mathcal{P}_E'$  satisfies  $\max_{1 \leq k \leq K} p_k^* \leq P_{max}$  and thus  $\mathcal{P}_E$  and  $\mathcal{P}_E'$  are equivalent.

Please refer to Appendix D for the proof.

Using primal decomposition [17] and Proposition 2, for sufficiently large  $P_{\text{max}}$ ,  $\mathcal{P}_E$  can be decomposed into the following two subproblems:

**Subproblem 1** (Optimization of **p** and  $\alpha$  under fixed A):

$$\mathcal{P}_{1}\left(\mathcal{A}\right): \max_{\alpha \in [\alpha_{\min}, \alpha_{\max}], \mathbf{p} \geq \mathbf{0}} \bar{\mathcal{I}}\left(\mathcal{A}, \alpha, \mathbf{p}\right), \text{ s.t. } (14) \text{ is satisfied.}$$

$$(17)$$

**Subproblem 2** (Optimization of A):

$$\mathcal{P}_{2}: \max_{\mathcal{A}} \quad \bar{\mathcal{I}}\left(\mathcal{A}, \alpha^{*}\left(\mathcal{A}\right), \mathbf{p}^{*}\left(\mathcal{A}\right)\right),$$
s.t.  $\mathcal{A} \subseteq \{1, ..., M\}$ , and  $|\mathcal{A}| = S$ ,

where  $\alpha^*(A)$ ,  $\mathbf{p}^*(A)$  is the optimal solution of  $\mathcal{P}_1(A)$ .

Subproblem 1 is non-convex. Although the gradient projection (GP) method [18] is usually used to find a stationary point for the constrained non-convex problem, it cannot be applied here because the power constraint functions in (14) are very complicated w.r.t.  $\alpha$  and it is very difficult to calculate the projection of  $\alpha$  and  $\mathbf{p}$  on the feasible set of  $\mathcal{P}_1(\mathcal{A})$ . In Section IV-B, we combine the weighted MMSE (WMMSE) approach in [19] and the bisection method to find a stationary point for  $\mathcal{P}_1(\mathcal{A})$ . In Section IV-C, we propose an efficient algorithm for Subproblem 2. For some special cases discussed in Section V, the proposed algorithms are asymptotically optimal.

#### B. Solution of Subproblem 1

We first propose an efficient algorithm to solve Subproblem 1 with fixed  $\alpha$ , which can be expressed as follows:

$$\mathcal{P}_{1a}\left(\mathcal{A},\alpha\right): \max_{\mathbf{p}>0} \bar{\mathcal{I}}\left(\mathcal{A},\alpha,\mathbf{p}\right), \text{ s.t. } (14) \text{ is satisfied.} \tag{19}$$

Then we give the overall solution of Subproblem 1.

1) Algorithm S1a for Solving  $\mathcal{P}_{1a}(\mathcal{A}, \alpha)$ :  $\mathcal{P}_{1a}(\mathcal{A}, \alpha)$  can be rewritten as a weighted sum-rate maximization problem (WSRMP) under the linear constraints for K-user interference channel as follows. First, rewrite the objective  $\bar{\mathcal{I}}(\mathcal{A}, \alpha, \mathbf{p})$  as

$$\bar{\mathcal{I}}\left(\mathcal{A}, \alpha, \mathbf{p}\right) = \sum_{k=1}^{K} w_k \log \left(1 + g_{kk} p_k / \left(1 + \sum_{l \neq k}^{K} g_{kl} p_l\right)\right),$$

$$g_{kk} \triangleq \frac{\xi_k^2}{\left(1 + \xi_k\right)^2}, \ \forall k, \ g_{kl} \triangleq \frac{\theta_{kl}}{S\left(1 + \xi_l\right)^2 \left(1 + \xi_k\right)^2}, \ \forall k \neq l.$$

Define a  $K \times S$  matrix  $\hat{\mathbf{R}}$  with the elements given by

$$\hat{R}_{kj} = \frac{1}{S} \alpha^{-1} \sigma_{k\mathcal{A}_j}^2 \psi_{\mathcal{A}_j}^{-2} (\mathbf{v}), \, \forall k, j,$$

and define  $\bar{\mathbf{R}}$  as a  $K \times K$  matrix with each element given by

$$\bar{R}_{kk} = \frac{1}{S} \sum_{m \in \mathcal{A}} \left[ \left( 1 + \frac{1}{S} \sum_{i \neq k}^{K} \sigma_{im}^{2} v_{i} \right) \sigma_{km}^{2} v_{k} / \psi_{m}^{2} \left( \mathbf{v} \right) \right], \, \forall k,$$

$$\bar{R}_{kl} = -\frac{1}{S} \sum_{m \in \mathcal{A}} \left[ \frac{1}{S} \sigma_{lm}^{2} v_{l} \sigma_{km}^{2} v_{k} / \psi_{m}^{2} \left( \mathbf{v} \right) \right], \, \forall k \neq l.$$

Let  $\mathbf{V} = \operatorname{diag}(v_1,...,v_K)$ . Then the per antenna power constraint in (14) can be rewritten as  $\mathbf{R}\mathbf{p} \leq \boldsymbol{\rho}$ , where  $\mathbf{R} \triangleq \hat{\mathbf{R}}^T \left[ \mathbf{V} - (\alpha \mathbf{I}_K + \boldsymbol{\Delta} - \mathbf{C})^{-1} \bar{\mathbf{R}} \right] \in \mathbb{R}^{S \times K}$ , and  $\boldsymbol{\rho} = \left[ \rho_{\mathcal{A}_1},...,\rho_{\mathcal{A}_S} \right]^T$ .

In [19], a WMMSE algorithm was proposed to find a stationary point for the WSRMP in MIMO interfering broadcast channels under per-BS power constraints. In the following, the WMMSE algorithm is tailored and generalized to solve  $\mathcal{P}_{1a}(\mathcal{A}, \alpha)$  under multiple linear constraints.

Following a similar proof as that of [19, Theorem 1], it can be shown that  $\mathbf{p}^*(\mathcal{A}, \alpha) = \mathbf{q}^* \circ \mathbf{q}^*$  is the optimal solution of  $\mathcal{P}_{1a}(\mathcal{A}, \alpha)$ , where the notation  $\circ$  denotes the Hadamard product; and  $\mathbf{q}^*$  is the optimal solution of

$$\min_{\mathbf{q}, \boldsymbol{v}, \boldsymbol{\omega}} \sum_{k=1}^{K} w_k \left( \omega_k e_k - \log \omega_k \right), \text{ s.t. } \mathbf{R} \left( \mathbf{q} \circ \mathbf{q} \right) \le \boldsymbol{\rho}, \tag{20}$$

where  $\mathbf{q}, \boldsymbol{v}$  and  $\boldsymbol{\omega} \geq \mathbf{0}$  are vectors in  $\mathbb{R}^K$ ; and  $e_k = \left(1 - v_k \sqrt{g_{kk}} q_k\right)^2 + \sum_{l \neq k} v_k^2 g_{kl} q_l^2 + v_k^2$ . Hence we only need to solve Problem (20), which is convex in each of the optimization variables  $\mathbf{q}, \boldsymbol{v}, \boldsymbol{\omega}$ . We can use the block coordinate decent method to solve (20). First, for fixed  $\mathbf{q}, \boldsymbol{v}$ , the optimal  $\boldsymbol{\omega}$  is given by  $\omega_k^* = e_k^{-1}, \forall k$ . Second, for fixed  $\mathbf{q}, \boldsymbol{\omega}$ , the optimal  $\boldsymbol{v}$  is given by  $v_k^* = \left(\sum_{l=1}^K g_{kl}q_l^2 + 1\right)^{-1} \sqrt{g_{kk}}q_k$ ,  $\forall k$ . Finally, for fixed  $\boldsymbol{v}, \boldsymbol{\omega}$ , the optimal  $\boldsymbol{q}$  is given by the solution of the following optimization problem:

$$\min_{\mathbf{q}} \sum_{k=1}^{K} \left( w_k \omega_k \left( 1 - v_k \sqrt{g_{kk}} q_k \right)^2 + \sum_{l \neq k} w_l \omega_l v_l^2 g_{lk} q_k^2 \right) 
\text{s.t. } \mathbf{R} \left( \mathbf{q} \circ \mathbf{q} \right) \leq \boldsymbol{\rho}.$$
(21)

Problem (21) is a convex quadratic optimization problem which can be solved using the Lagrange dual method. Specifically, the Lagrange function of Problem (21) is given by

$$L(\boldsymbol{\lambda}, \mathbf{q}) = \sum_{k=1}^{K} \left( w_k \omega_k \left( 1 - v_k \sqrt{g_{kk}} q_k \right)^2 + \sum_{l \neq k} w_l \omega_l v_l^2 g_{lk} q_k^2 \right) + \boldsymbol{\lambda}^T \left( \mathbf{R} \left( \mathbf{q} \circ \mathbf{q} \right) - \boldsymbol{\rho} \right),$$

where  $\lambda \in \mathbb{R}^{S}_{+}$  is the Lagrange multiplier vector. The dual function of Problem (21) is

$$J(\lambda) = \min_{\mathbf{q}} L(\lambda, \mathbf{q}). \tag{22}$$

The minimization problem in (22) can be decomposed into K independent problems as

$$\min_{q_k} \left\{ w_k \omega_k \left( 1 - \upsilon_k \sqrt{g_{kk}} q_k \right)^2 + \left( \sum_{l \neq k} w_l \omega_l \upsilon_l^2 g_{lk} + \boldsymbol{\lambda}^T \mathbf{r}_k \right) q_k^2 \right\}, \tag{23}$$

where  $\mathbf{r}_k$  is the  $k^{\text{th}}$  column of  $\mathbf{R}$ . For fixed  $\lambda$ , Problem (23) has a closed-form solution given by

$$q_k^*(\boldsymbol{\lambda}) = \left(\sum_{l=1}^K w_l \omega_l v_l^2 g_{lk} + \boldsymbol{\lambda}^T \mathbf{r}_k\right)^{-1} w_k \sqrt{g_{kk}} v_k \omega_k.$$
 (24)

Since (21) is a convex quadratic optimization problem, the optimal solution is given by  $\mathbf{q}_k^*\left(\tilde{\boldsymbol{\lambda}}\right) = \left[q_k^*\left(\tilde{\boldsymbol{\lambda}}\right)\right]_{k=1,\dots,K}$ , where  $\tilde{\lambda}$  is the optimal solution of the dual problem

$$\max_{\lambda} J(\lambda), \text{ s.t. } \lambda \ge 0. \tag{25}$$

The dual function  $J(\lambda)$  is concave and it can be verified that  $\mathbf{R}(\mathbf{q}^*(\lambda) \circ \mathbf{q}^*(\lambda)) - \rho$  is a subgradient of  $J(\lambda)$ . Hence, the standard subgradient based methods such as the subgradient algorithm in [20] or the Ellipsoid method in [21] can be used to solve Problem (25).

The overall algorithm for solving Problem (20) is summarized as follows.

Algorithm S1a (for solving Problem (20)):

**Initialization**: Let  $\mathbf{q} = c\mathbf{1}$ , where  $\mathbf{1}$  denotes a vector of all ones; and c is chosen such that  $\mathbf{R} (\mathbf{q} \circ \mathbf{q}) \leq \boldsymbol{\rho}$ . **Step 1** Let  $\upsilon_k = \left(\sum_{l=1}^K g_{kl}q_l^2 + 1\right)^{-1} \sqrt{g_{kk}}q_k$ ,  $\forall k$  **Step 2** Let  $\omega_k = \left(1 - \upsilon_k \sqrt{g_{kk}}q_k\right)^{-1}$ ,  $\forall k$ 

**Step 1** Let 
$$v_k = \left(\sum_{l=1}^K g_{kl}q_l^2 + 1\right)^{-1} \sqrt{g_{kk}}q_k$$
,  $\forall k$ 

Step 2 Let 
$$\omega_k = (1 - \upsilon_k \sqrt{g_{kk}} q_k)^{-1}$$
,  $\forall k$ 

**Step 3** Let  $q_k = q_k^*(\tilde{\lambda})$ ,  $\forall k$ ; where  $\tilde{\lambda}$  is the optimal solution of (25) which can be solved using, e.g., the subgradient algorithm in [20] or the Ellipsoid method in [21] with the subgradient of  $J(\lambda)$  given by  $\mathbf{R}(\mathbf{q}^*(\lambda)) \circ \mathbf{q}^*(\lambda)) - \rho$ ; and  $\mathbf{q}^*(\lambda)$ is given in (24).

#### Return to Step 1 until convergence.

The following theorem shows that Algorithm S1a converges to a stationary point of  $\mathcal{P}_{1a}(\mathcal{A},\alpha)$ .

**Theorem 2** (Convergence of Alg. S1a). For any limit point  $(\tilde{\mathbf{q}}, \tilde{\boldsymbol{v}}, \tilde{\boldsymbol{\omega}}, \tilde{\boldsymbol{\lambda}})$  of the iterates generated by Algorithm S1a, the corresponding  $\tilde{\mathbf{p}}(\mathcal{A}, \alpha) = \tilde{\mathbf{q}} \circ \tilde{\mathbf{q}}$  and  $\tilde{\boldsymbol{\lambda}}$  satisfies the KKT conditions of  $\mathcal{P}_{1a}(\mathcal{A}, \alpha)$ , which can be expressed

$$\nabla_{\mathbf{p}} \bar{\mathcal{I}} (\mathcal{A}, \alpha, \tilde{\mathbf{p}} (\mathcal{A}, \alpha)) - \mathbf{R}^{T} \tilde{\boldsymbol{\lambda}} + \tilde{\boldsymbol{\nu}} = 0;$$

$$\operatorname{diag} (\boldsymbol{\rho}) \tilde{\boldsymbol{\lambda}} - \operatorname{diag} \left( \tilde{\boldsymbol{\lambda}} \right) \mathbf{R} \tilde{\mathbf{p}} (\mathcal{A}, \alpha) = 0;$$

$$\operatorname{diag} (\tilde{\boldsymbol{\nu}}) \tilde{\mathbf{p}} (\mathcal{A}, \alpha) = 0;$$

$$(26)$$

where  $\tilde{\boldsymbol{\lambda}}$  and  $\tilde{\boldsymbol{\nu}} = \mathbf{R}^T \tilde{\boldsymbol{\lambda}} - \nabla_{\mathbf{p}} \bar{\mathcal{I}} \left( \mathcal{A}, \alpha, \tilde{\mathbf{p}} \left( \mathcal{A}, \alpha \right) \right)$  are the Lagrange multipliers associated with the constraints  $\mathbf{R}\mathbf{p} \leq \boldsymbol{\rho}$  and  $\mathbf{p} \geq 0$  respectively; and  $\nabla_{\mathbf{p}} \bar{\mathcal{I}} \left( \mathcal{A}, \alpha, \tilde{\mathbf{p}} \left( \mathcal{A}, \alpha \right) \right) = \left[ \frac{\partial \bar{\mathcal{I}}}{\partial p_1}, ..., \frac{\partial \bar{\mathcal{I}}}{\partial p_K} \right]$  with

$$\frac{\partial \bar{\mathcal{I}}}{\partial p_{k}} = \frac{w_{k}g_{kk}}{\left(\tilde{\Omega}_{k} + g_{kk}\tilde{p}_{k}\left(\mathcal{A},\alpha\right)\right)} - \sum_{l \neq k}^{K} \frac{w_{l}g_{lk}g_{ll}\tilde{p}_{l}\left(\mathcal{A},\alpha\right)}{\tilde{\Omega}_{l}\left(\tilde{\Omega}_{l} + g_{ll}\tilde{p}_{l}\left(\mathcal{A},\alpha\right)\right)}.$$

where

$$\tilde{\Omega}_{k} = 1 + \sum_{l \neq k}^{K} g_{kl} \tilde{p}_{l} \left( \mathcal{A}, \alpha \right), \, \forall k.$$
(27)

*Proof:* Following a similar proof as that of [19, Theorem 3], we can show that Algorithm S1a converges to a stationary point  $\tilde{\mathbf{q}}$ ,  $\tilde{v}$ ,  $\tilde{\omega}$  of Problem (20). At the stationary point, the corresponding  $\tilde{\mathbf{q}}$ ,  $\tilde{v}$ ,  $\tilde{\omega}$ ,  $\tilde{\lambda}$  must satisfy diag  $(\rho)$   $\tilde{\lambda}$  – diag  $(\tilde{\lambda})$   $\mathbf{R}$   $(\tilde{\mathbf{q}} \circ \tilde{\mathbf{q}}) = 0$  and (24) with  $\tilde{v}_k = \left(\sum_{l=1}^K g_{kl}\tilde{q}_l^2 + 1\right)^{-1}\sqrt{g_{kk}}\tilde{q}_k$ , and  $\tilde{\omega}_k = \left(1 - \tilde{v}_k\sqrt{g_{kk}}\tilde{q}_k\right)^{-1}$ . Using the above fact, it can be verified by a direct calculation that  $\tilde{\mathbf{p}}$   $(\mathcal{A}, \alpha)$  and  $\tilde{\lambda}$  satisfies the KKT conditions in (26).

2) Overall Solution for Subproblem 1: The following theorem characterizes the overall solution of subproblem 1.

**Theorem 3** (Stationary point of  $\mathcal{P}_1(A)$ ). Let  $\tilde{\mathbf{p}}(A, \alpha)$  denote the stationary point of  $\mathcal{P}_{1a}(A, \alpha)$  found by Algorithm S1a. Assume that  $\tilde{\mathbf{p}}(A, \alpha)$  is differentiable over  $\alpha$  and define a function

$$\hat{\mathcal{I}}(\mathcal{A}, \alpha) \triangleq \bar{\mathcal{I}}(\mathcal{A}, \alpha, \tilde{\mathbf{p}}(\mathcal{A}, \alpha)). \tag{28}$$

Then the following are true:

- 1) If  $\frac{\partial \hat{\mathcal{I}}(\mathcal{A}, \alpha_{min})}{\partial \alpha} < 0$ , then  $\alpha_{min}, \tilde{\mathbf{p}}(\mathcal{A}, \alpha_{min})$  is a stationary point of  $\mathcal{P}_1(\mathcal{A})$ .
- 2) If  $\frac{\partial \hat{\mathcal{I}}(\mathcal{A}, \alpha_{max})}{\partial \alpha} > 0$ , then  $\alpha_{max}, \tilde{\mathbf{p}}(\mathcal{A}, \alpha_{max})$  is a stationary point of  $\mathcal{P}_1(\mathcal{A})$ .
- 3) If  $\frac{\partial \hat{\mathcal{I}}(\mathcal{A}, \alpha_{min})}{\partial \alpha} > 0$  and  $\frac{\partial \hat{\mathcal{I}}(\mathcal{A}, \alpha_{max})}{\partial \alpha} < 0$ , let  $\tilde{\alpha}(\mathcal{A})$  be a solution of the following equation

$$\frac{\partial \hat{\mathcal{I}}(\mathcal{A}, \alpha)}{\partial \alpha} = 0, \ \alpha \in [\alpha_{min}, \alpha_{max}], \tag{29}$$

i.e.,  $\tilde{\alpha}(A)$  is a stationary point of  $\hat{\mathcal{I}}(A, \alpha)$ . Then  $\tilde{\alpha}(A)$ ,  $\tilde{\mathbf{p}}(A, \tilde{\alpha}(A))$  is a stationary point of  $\mathcal{P}_1(A)$ .

*Proof:* It is easy to see that if  $\frac{\partial \hat{\mathcal{I}}(\mathcal{A}, \alpha_{\min})}{\partial \alpha} < 0$  ( $\frac{\partial \hat{\mathcal{I}}(\mathcal{A}, \alpha_{\max})}{\partial \alpha} > 0$ ),  $\alpha_{\min}$  ( $\alpha_{\max}$ ) is a local maximum (and thus stationary point) of the problem  $\max_{\alpha \in [\alpha_{\min}, \alpha_{\max}]} \hat{\mathcal{I}}(\mathcal{A}, \alpha)$ . Then Theorem 3 can be proved using the facts that  $\tilde{\mathbf{p}}(\mathcal{A}, \alpha)$  ( $\alpha = \alpha_{\min}, \alpha_{\max}$  or  $\tilde{\alpha}(\mathcal{A})$  depending on different cases) is a stationary point of  $\mathcal{P}_{1a}(\mathcal{A}, \alpha)$  and  $\alpha$  is a stationary point of  $\max_{\alpha \in [\alpha_{\min}, \alpha_{\max}]} \hat{\mathcal{I}}(\mathcal{A}, \alpha)$ . The details are omitted for conciseness.

Motivated by Theorem 3, we propose the following bisection algorithm to solve  $\mathcal{P}_1\left(\mathcal{A}\right)$  .

Algorithm S1b (Bisection search for solving  $\mathcal{P}_1(\mathcal{A})$ ):

**Initialization**: If  $\frac{\partial \hat{\mathcal{I}}(\mathcal{A}, \alpha_{\min})}{\partial \alpha} < 0$ , terminate the algorithm and output  $\alpha_{\min}$ ,  $\tilde{\mathbf{p}}(\mathcal{A}, \alpha_{\min})$ . If  $\frac{\partial \hat{\mathcal{I}}(\mathcal{A}, \alpha_{\max})}{\partial \alpha} > 0$ , terminate the algorithm and output  $\alpha_{\max}$ ,  $\tilde{\mathbf{p}}(\mathcal{A}, \alpha_{\max})$ . Otherwise, choose proper  $\alpha_a$ ,  $\alpha_b$  such that  $0 < \alpha_a < \alpha_b$  and  $\frac{\partial \hat{\mathcal{I}}(\mathcal{A}, \alpha_a)}{\partial \alpha} > 0$ ,  $\frac{\partial \hat{\mathcal{I}}(\mathcal{A}, \alpha_b)}{\partial \alpha} < 0$ .

Step 1: Let  $\alpha = (\alpha_a + \alpha_b)/2$ . If  $\frac{\partial \hat{\mathcal{I}}(\mathcal{A}, \alpha)}{\partial \alpha} \leq 0$ , let  $\alpha_b = \alpha$ . Otherwise, let  $\alpha_a = \alpha$ .

**Step 2**: If  $\alpha_b - \alpha_a$  is small enough, terminate the algorithm and output  $\alpha, \tilde{\mathbf{p}}(\mathcal{A}, \alpha)$ . Otherwise, return to Step 1.

Remark 6. It is observed in the simulations that one can always choose sufficiently large  $\alpha_{\text{max}}$  and sufficiently small  $\alpha_{\text{min}} > 0$  such that  $\frac{\partial \hat{\mathcal{I}}(\mathcal{A}, \alpha_{\text{min}})}{\partial \alpha} > 0$  and  $\frac{\partial \hat{\mathcal{I}}(\mathcal{A}, \alpha_{\text{max}})}{\partial \alpha} < 0$ . Then the constraint  $\alpha_{\text{min}} \leq \alpha \leq \alpha_{\text{max}}$  is never active at the solution found by Algorithm S1b.

The calculation of  $\frac{\partial \hat{\mathcal{I}}(\mathcal{A},\alpha)}{\partial \alpha}$  in Algorithm S1b is non-trivial due to the lack of analytical expression for  $\hat{\mathcal{I}}(\mathcal{A},\alpha)$ . In the following, we show how to calculate  $\frac{\partial \hat{\mathcal{I}}(\mathcal{A},\alpha)}{\partial \alpha}$  from the output of Algorithm S1a:  $\tilde{\mathbf{p}}(\mathcal{A},\alpha)$  and  $\tilde{\boldsymbol{\lambda}}$ . Assuming that  $\frac{\partial \tilde{\mathbf{p}}(\mathcal{A},\alpha)}{\partial \alpha}$ ,  $\frac{\partial \tilde{\boldsymbol{\lambda}}}{\partial \alpha}$  and  $\frac{\partial \tilde{\boldsymbol{\nu}}}{\partial \alpha}$  exist and taking partial derivative of the equations in (26) with respect to  $\alpha$ , we obtain a linear equation with  $\frac{\partial \tilde{\mathbf{p}}(\mathcal{A},\alpha)}{\partial \alpha}$ ,  $\frac{\partial \tilde{\boldsymbol{\lambda}}}{\partial \alpha}$  and  $\frac{\partial \tilde{\boldsymbol{\nu}}}{\partial \alpha}$  as the variables. Then we can calculate  $\frac{\partial \tilde{\mathbf{p}}(\mathcal{A},\alpha)}{\partial \alpha}$  by solving this linear equation. Finally, the derivative  $\frac{\partial \hat{\mathcal{I}}(\mathcal{A},\alpha)}{\partial \alpha}$  can be calculated as

$$\frac{\partial \hat{\mathcal{I}}(\mathcal{A}, \alpha)}{\partial \alpha} = \sum_{k=1}^{K} w_k \left( \frac{\sum_{l=1}^{K} \left( \tilde{p}_l \left( \mathcal{A}, \alpha \right) \frac{\partial g_{kl}}{\partial \alpha} + g_{kl} \frac{\partial \tilde{p}_l \left( \mathcal{A}, \alpha \right)}{\partial \alpha} \right)}{g_{kk} \tilde{p}_k \left( \mathcal{A}, \alpha \right) + \tilde{\Omega}_k} - \frac{\sum_{l \neq k}^{K} \left( \tilde{p}_l \left( \mathcal{A}, \alpha \right) \frac{\partial g_{kl}}{\partial \alpha} + g_{kl} \frac{\partial \tilde{p}_l \left( \mathcal{A}, \alpha \right)}{\partial \alpha} \right)}{\tilde{\Omega}_k} \right).$$
(30)

The detailed calculations for  $\frac{\partial \tilde{\mathbf{p}}(\mathcal{A},\alpha)}{\partial \alpha}$ ,  $\frac{\partial g_{kl}}{\partial \alpha}$ 's and  $\frac{\partial \hat{\mathcal{I}}(\mathcal{A},\alpha)}{\partial \alpha}$  can be found in Appendix E.

#### C. Algorithm S2 for Solving Subproblem 2

Subproblem 2 is a combinatorial problem and the optimal solution requires exhaustive search. We shall propose a low complexity algorithm which is asymptotically optimal for large M as will be shown in Corollary 1.

Based on the insight obtained in Example 1 and 2, we propose an efficient algorithm S2 for  $\mathcal{P}_2$ . In step 1, the algorithm selects antennas that have a direct link with a single user and do not cause strong interference to others<sup>5</sup>. In step 2, the algorithm selects antennas that have strong links with several users. These antennas have the potential to provide large cooperative gain. Note that "bad" antennas that cause strong interference may also be selected; however, they will be deleted in step 4. In step 3, the algorithm selects antennas that have a strong cross link with a single user and do not cause strong interference to others. In step 4, a greedy search is performed to replace the "bad" antennas with "good" ones from a candidate antenna set  $\Gamma_j$ , which is carefully chosen to reduce the number of weighted sum-rate calculations in step 4 as well as to maintain a good performance.

We first define some notations and then give the detailed steps of Algorithm S2. Let  $\tilde{m}_k = \operatorname{argmax}_m \sigma_{km}^2$ , k = 1, ..., K. Define  $\bar{g}_k^d = \sigma_{km\bar{k}}^2$ . For k = 1, ..., K, and m = 1, ..., M, let  $G_{km} = 1$ , if  $\sigma_{km}^2 \ge \kappa \bar{g}_k^d$ , and otherwise, let

<sup>&</sup>lt;sup>5</sup>The phrase "an antenna causes interference to a user" refers to the case when an antenna causes strong interference to a user before precoding and joint transmission using RZF does not provide much gain due to some other weak cross links as shown in Example 1.

 $G_{km}=0$ , where  $\kappa\in(0,1)$ . Roughly speaking,  $G_{km}$  is an indication of whether antenna m contributes significantly to the communication of user k. Simulations show that the performance of Algorithm S2 is not sensitive to the choice of  $\kappa$  for  $\kappa\in\left[\frac{1}{4},\frac{1}{2}\right]$ . Define  $\mathcal{K}_m=\left\{k:\,G_{km}=1\right\},\,m=1,...,M$ . Let  $\tilde{\mathcal{I}}_{\mathcal{A}}\triangleq\bar{\mathcal{I}}\left(\mathcal{A},\tilde{\rho}\left(\mathcal{A}\right),\tilde{\mathbf{p}}\left(\mathcal{A},\tilde{\rho}\left(\mathcal{A}\right)\right)\right)$  denote the optimized weighted sum-rate under  $\mathcal{A}$ . For any set of antennas  $\mathcal{A}\subseteq\{1,...,M\}$ , let  $\bar{\mathcal{A}}$  denote the relative complement of  $\mathcal{A}$ .

```
Algorithm S2 (for solving Subproblem 2):
Initialization: Let A = \Phi, where \Phi denotes the void set.
Step 1 (Select antennas with a direct link and no cross link):
  For k=1 to K, if |\mathcal{K}_{\tilde{m}_k}|=1, let \mathcal{A}=\mathcal{A}\cup\tilde{m}_k. If |\mathcal{A}|=S, go to step 4.
Step 2 (Select antennas with multiple strong links):
  Let \bar{G}_m = |\mathcal{K}_m| B + \sum_{k=1}^K \sigma_{km}^2, where B can be any constant larger than \max_{1 \le m \le M} \sum_{k=1}^K \sigma_{km}^2.
  Let m^* = \operatorname{argmax} \bar{G}_m
  While |\mathcal{K}_{m^*}| \geq 2 and |\mathcal{A}| < S
    Let A = A \cup m^* and m^* = \operatorname{argmax} \bar{G}_m.
  If |\mathcal{A}| = S, go to step 4.
Step 3 (Select antennas with a single strong link):
  Let \tilde{k}_m = \underset{i}{\operatorname{argmax}} \sigma_{km}^2 and I_m = w_{\tilde{k}_m} \log \left( 1 + \sigma_{\tilde{k}_m m}^2 \right).
  While |\mathcal{A}| < \overset{\kappa}{S}
    Let m^* = \operatorname{argmax} I_m and \mathcal{A} = \mathcal{A} \cup m^*.
  End
Step 4 (Greedy search for replacing "bad" antennas with "good" ones):
  For j = 1 to S
    Let \mathcal{A}_{-j}=\mathcal{A}/\mathcal{A}_j. Let n^*=\mathop{\mathrm{argmax}}_{m\in \bar{\mathcal{A}}\cap\{m:|\mathcal{K}_m|=1\}}I_m and \Gamma_j^a=\bar{\mathcal{A}}\cap\{m:|\mathcal{K}_m|\geq 2\}. If I_{n^*}\geq I_{\mathcal{A}_j} or \left|\mathcal{K}_{\mathcal{A}_j}\right|>1, let \Gamma_j=\Gamma_j^a\cup n^*; otherwise, let \Gamma_j=\Gamma_j^a.
    Let m^* = \operatorname{argmax} \bar{\mathcal{I}}_{\mathcal{A}_{-j} \cup m}.
    If \tilde{\mathcal{I}}_{\mathcal{A}_{-j}\cup m^*} > \tilde{\mathcal{I}}_{\mathcal{A}}, let \mathcal{A}_j = m^*.
  End
```

Finally, we elaborate the choice of the candidate antenna set  $\Gamma_j$  in step 4. The  $I_m$  defined in step 3 reflects the contribution of antenna m to the weighted rate of a single user. Then  $n^*$  in step 4 is the unselected antenna which is likely to contribute the most to the weighted rate of a single user without causing strong interference to other users. If  $I_{n^*} \geq I_{\mathcal{A}_j}$ ,  $n^*$  is added in  $\Gamma_j$ . Even if  $I_{n^*} < I_{\mathcal{A}_j}$ , we still add  $n^*$  in  $\Gamma_j$  if  $\mathcal{A}_j$  has the potential to cause large interference, i.e.,  $|\mathcal{K}_{\mathcal{A}_j}| > 1$ .  $\Gamma_j^a$  are the set of unselected antennas which have the potential to provide cooperative gain, and are also added in  $\Gamma_j$ .

# V. STRUCTURAL SOLUTION FOR SOME SPECIAL CASES

# A. Large MIMO Network with Collocated Antennas

We first study the case where the antennas are collocated at the base station. Specifically, this corresponds to the case where all antennas experience the same large scale fading:  $\sigma_{km}^2 = \sigma_{k1}^2$ , k = 1, ..., K, m = 1, ..., M. In this

case, any subset A of S antennas is optimal for the antenna selection problem since all antennas are statistically equivalent. We focus on deriving the structural properties of  $\mathbf{p}^*$  and  $\alpha^*$ .

We first obtain simpler expressions for asymptotic SINR and transmit power for  $\mathcal{P}_1(\mathcal{A})$ .

**Theorem 4** (DE for collocated antennas). Let  $\Sigma$ ,  $\alpha$  be as in Assumption 3. The following statements are true for the special case of collocated antennas ( $\sigma_{km}^2 = \sigma_{k1}^2$ , k = 1, ..., K, m = 1, ..., M):

1) The system of equation:

$$u = \frac{1}{\alpha + \frac{1}{S} \sum_{i=1}^{K} \frac{\sigma_{i1}^2}{1 + \sigma_{i}^2, u}},$$
(31)

admits a unique solution u in  $\mathbb{R}_{++}$ .

2) Define

$$F_{12} = \frac{1}{S} \sum_{i=1}^{K} \frac{\sigma_{i1}^{2}}{\left(1 + \sigma_{i1}^{2} u\right)^{2}}, \ \bar{F}_{12}\left(\mathbf{p}\right) = \frac{1}{S} \sum_{i=1}^{K} \frac{p_{i} \sigma_{i1}^{2}}{\left(1 + \sigma_{i1}^{2} u\right)^{2}}.$$

As  $S \xrightarrow{K=S\beta} \infty$ ,  $\gamma_k(A, \alpha, \mathbf{p})$  in (2) and  $P_m(A, \alpha, \mathbf{p})$  in (3) converge, respectively, to the following deterministic values almost surely

$$\bar{\gamma}_{k}\left(\mathcal{A}, \alpha, \mathbf{P}\right) = \frac{p_{k}\sigma_{k1}^{4}u^{2}\left(\alpha + F_{12}\right)}{\bar{F}_{12}\left(\mathbf{p}\right)\sigma_{k1}^{2}u + \left(1 + \sigma_{k1}^{2}u\right)^{2}\left(\alpha + F_{12}\right)},$$

$$\bar{P}_{m}\left(\mathcal{A}, \alpha, \mathbf{p}\right) = \frac{\bar{F}_{12}\left(\mathbf{p}\right)u}{S\left(\alpha + F_{12}\right)}, \forall m \in \mathcal{A}.$$
(32)

The proof is similar to the proof in Appendix B.

Using Theorem 4,  $\mathcal{P}_{1a}(\mathcal{A}, \alpha)$  can be reformulated into a simpler form as follows. First, according to (32), all antennas always have the same transmit power. Furthermore, it can be verified that the per antenna power constraint must be achieved with equality at the optimal solution. Combining these facts and the asymptotic expressions in Theorem 4,  $\mathcal{P}_{1a}(\mathcal{A}, \alpha)$  is equivalent to the following optimization problem

$$\max_{\mathbf{p} \ge 0} \sum_{k=1}^{K} w_k \log \left( 1 + \frac{p_k \sigma_{k1}^4 u^2}{\sigma_{k1}^2 P_T + \left( 1 + \sigma_{k1}^2 u \right)^2} \right), \text{ s.t. } \frac{\bar{F}_{12} \left( \mathbf{p} \right) u}{\left( \alpha + F_{12} \right)} \le P_T, \tag{33}$$

where  $P_T = \min_{m \in \mathcal{A}} \rho_m$ .

1) Water-filling Structure of the Optimal Power Allocation: For fixed  $\mathcal{A}, \alpha$ , the optimal power allocation  $\mathbf{p}^* (\mathcal{A}, \alpha) = [p_1^*, ..., p_K^*]$  is given by:

$$p_k^* = \left(\frac{w_k S \left(1 + \sigma_{k1}^2 u\right)^2 (\alpha + F_{12})}{\lambda \sigma_{k1}^2 u} - \frac{\sigma_{k1}^2 P_T + \left(1 + \sigma_{k1}^2 u\right)^2}{\sigma_{k1}^4 u^2}\right)^+,\tag{34}$$

where  $\lambda$  is chosen such that  $\bar{F}_{12}\left(\mathbf{p}^{*}\left(\mathcal{A},\alpha\right)\right)u/\left(\alpha+F_{12}\right)=P_{T}.$ 

2) Properties of the optimal  $\alpha$  in High SNR Regime: The following theorem summarizes the structural properties of the optimal solution  $\alpha^*$  for  $\mathcal{P}_1$  ( $\mathcal{A}$ ).

**Theorem 5** (Properties of  $\alpha^*$  at high SNR). For fixed K, S and sufficiently small  $\alpha_{min}$ , the following are true:

1) 
$$\alpha^* = O\left(\frac{1}{P_T}\right)$$
 for large  $P_T$ .

2) There exists a small enough constant  $\alpha_t > \alpha_{min}$  such that  $\bar{\mathcal{I}}(\mathcal{A}, \alpha, \mathbf{p}^*(\mathcal{A}, \alpha))$ , is a concave function of  $\alpha$  for all  $\alpha \in [\alpha_{min}, \alpha_t)$ .

The proof is given in Appendix F. Theorem 5 implies that with sufficiently small  $\alpha_{\min}$  and initial  $\alpha_b > \alpha_a \ge \alpha_{\min}$ , Algorithm S1b will converge to the optimal  $\alpha^*$  at high enough SNR.

Remark 7 (Large MIMO with Collocated Users). When all users are collocated in the large MIMO network (i.e., the users are very close geographically), they experience the same large scale fading:  $\sigma_{km}^2 = \sigma_{1m}^2$ , k = 1, ..., K, m = 1, ..., M. As a result, the optimal active antenna set  $\mathcal{A}^*$  contains the antennas that have the largest S large scale fading factors with the users. Similarly, it can be shown that the optimal power allocation for  $\mathcal{P}_{1a}(\mathcal{A}, \alpha)$  is also a water-filling solution. The details are omitted due to limited space.

#### B. Very Large Distributed MIMO Network

In this section, we derive the asymptotic performance for very large distributed MIMO networks. Note that the analysis in this section does not rely on Assumption 3. The results in this section are derived only under the large distributed MIMO setup defined below.

**Definition 3** (Very Large Distributed MIMO Network). The coverage area is a square with side length  $R_c$ . There are  $M=N^2$  antennas evenly distributed in the square grid for some integer N. The locations of the K users are randomly generated from a uniform distribution within the square. The path loss model is given by  $\sigma_{km}^2 = G_0 r_{km}^{-\zeta}$ , where  $G_0 > 0$  is a constant,  $r_{km}$  is the distance between the  $m^{th}$  antenna and  $k^{th}$  user,  $\zeta$  is the path loss factor.

Let  $\tilde{m}_k = \operatorname*{argmax}_{m} \sigma_{km}^2$ . Define  $\bar{g}_k^d = \sigma_{k\tilde{m}_k}^2$  as the direct-link gain, and  $\bar{g}_k^c = \max_{l \neq k} \sigma_{k\tilde{m}_l}^2$  as the maximum cross-link gain. Define  $\eta = \min_{k} \bar{g}_k^d / \bar{g}_k^c$ .

**Theorem 6** (Asymptotic Decoupling and Capacity Scaling). For any  $\eta_0 > 0$ , we have

$$Pr(\eta > \eta_0) \ge 1 - K \left[ 1 - \left( 1 - \pi \left( \eta_0^{1/\zeta} + 1 \right)^2 / (2M) \right)^{K-1} \right].$$
 (35)

Furthermore, for any  $\epsilon > 0$ , the maximum achievable sum-rate  $C_s$  almost surely satisfies

$$O\left(K\left(\frac{\zeta}{2} - \epsilon\right)logM\right) \le C_s \le O\left(K\left(\frac{\zeta}{2} + \epsilon\right)logM\right),$$

as  $M \to \infty$  with K, S fixed.

Please refer to Appendix G for the proof.

Remark 8 (Asymptotic Decoupling). For large M/K, there is a high probability that  $\eta$  is large, i.e., there is a high chance that each user can find a set of nearby transmit antennas which are far from other users. Due to this decoupling effect, simplified physical layer processing (such as Matched-Filter precoder [1]) can also achieve good performance.

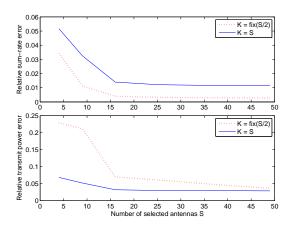


Figure 4. Relative sum-rate/transmit power error versus S

**Corollary 1** (Asymptotic Optimality of Algorithm S2). Algorithm S2 is asymptotically optimal, i.e., for any  $\epsilon > 0$ , the achieved sum-rate  $\mathcal{I}_{\mathcal{A}}$  satisfies  $\mathcal{I}_{\mathcal{A}} \overset{a.s}{\geq} O\left(K\left(\frac{\zeta}{2} - \epsilon\right) log M\right)$  as  $M \to \infty$  with K, S fixed.

The proof is given in Appendix H.

#### VI. NUMERICAL RESULTS

Consider a C-RAN serving K users lying inside a square with an area of  $2\text{km} \times 2\text{km}$ . The simulation setup is the same as that in Definition 3 with path loss factor  $\zeta = 2.5$ .

# A. Accuracy of the Asymptotic Expressions

We verify the accuracy of the DE of sum-rate (i.e.,  $w_k = 1$ ,  $\forall k$ )  $\bar{\mathcal{I}}(\mathcal{A}, \alpha, \mathbf{p})$  and the DE of per antenna transmit power  $\bar{P}_m \left( \mathcal{A}, \alpha, \mathbf{p} \right)$ ,  $\forall m \in \mathcal{A}$  by comparing them to those obtained by Monte-Carlo simulations:  $\mathcal{I}^{\text{sim}}$  and  $P_m^{\text{sim}}$ ,  $\forall m \in \mathcal{A}$ . We set  $\mathcal{A} = \{1, ..., M\}$ , i.e., S = M. Assume the per antenna power constraint is given by  $\rho_m = 10$ dB,  $\forall m$  and equal power allocation is adopted, i.e.,  $\mathbf{P} = c\mathbf{I}$  in (1), where c is chosen such that  $\bar{P}_m \left( \mathcal{A}, \alpha, \mathbf{p} \right) = \frac{10}{S}$ . The regularization factor is fixed as  $\alpha = \beta/10$ . In Fig. 4, we plot the relative sum-rate error  $\left| \bar{\mathcal{I}} \left( \mathcal{A}, \alpha, \mathbf{p} \right) - \mathcal{I}^{\text{sim}} \right| / \mathcal{I}^{\text{sim}}$  and the relative transmit power error  $\left( \sum_{m \in \mathcal{A}} \left| \bar{P}_m \left( \mathcal{A}, \alpha, \mathbf{p} \right) - P_m^{\text{sim}} \right|^2 \right)^{1/2} / \left( \sum_{m \in \mathcal{A}} \left| P_m^{\text{sim}} \right|^2 \right)^{1/2}$ , versus S. Both cases with K = S/2 (i.e.,  $\beta = 1/2$ ) and K = S (i.e.,  $\beta = 1$ ) are simulated. The large scale fading matrix  $\mathbf{\Sigma}$  is generated according to a random realization of user locations. It can be seen that the asymptotic approximation is quite accurate.

In the rest of the simulations (i.e., in Fig. 5 and 6), the per antenna power constraint is set as  $\rho_m = P_0, \forall m$ . The number of users K is fixed as 8. From user 1 to user 8, the weights  $w_k$ 's increase linearly from 0.5 to 1.5.

# B. Performance Gain of the Proposed Scheme w.r.t. Baseline

In Fig. 5, we verify the performance gain of the proposed scheme w.r.t. the traditional antenna selection baseline algorithm where each user is associated with the strongest antennas. There are a total number of M=25 antennas.

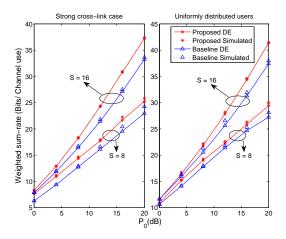


Figure 5. Comparison of proposed antenna selection scheme and baseline

We plot the weighted sum-rates averaged over different realizations of user locations versus  $P_0$  for S=8 and 16 respectively. In the subplot on the left-hand side of Fig. 5, we consider the strong cross link case, where the user locations are randomly generated but with the restriction that the distance between each user and the nearest antenna must be larger than a threshold. In this case, it can be seen that the proposed scheme achieves significant performance gain compared with the baseline. In the subplot on the right-hand side of Fig. 5, we consider the normal case where the users are uniformly distributed. Similar results can be observed, although the performance gain is smaller<sup>6</sup>.

# C. Advantages of the Proposed Scheme over the Cases without Antenna Selection

In Fig. 6, we compare the proposed antenna selection scheme with various cases without antenna selection. There are a total number of M=49 antennas and S=16 of them are selected for transmission. The performance of the following cases are compared. Case 1: All the 49 distributed antennas are used for transmission. Case 2: All the 49 antennas are collocated at the BS and are used for transmission. Case 3: There is a total of 16 antennas evenly distributed in the square and all the 16 antennas are used for transmission. We plot the weighted sum-rates averaged over different realizations of user locations versus the sum transmit power. The following advantages of the proposed antenna selection scheme can be observed. 1) Under the same sum transmit power, it achieves a weighted sum-rate that is close to Case 1, and is better than Case 2, while the pilot training overhead is lower. 2) The performance is much better than Case 3 due to large antenna gain.

<sup>6</sup>This is because the unfavored scenarios for the baseline algorithm as illustrated in Example 1 and 2 occur less frequently when the users are uniformly distributed.

<sup>7</sup>In Fig. 6, the horizontal axis is chosen to be sum transmit power to make a fair comparison of the performance for the cases with different number of active transmit antennas and different antenna deployments (i.e., distributed versus collocated).

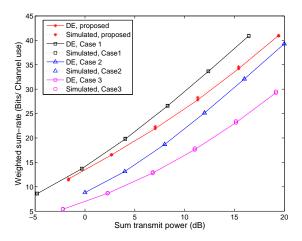


Figure 6. Comparison of weighted sum-rates for different choices of transmit antennas

#### VII. CONCLUSION

In this paper, we have considered a downlink antenna selection in a large distributed MIMO network with  $M\gg 1$  distributed antennas serving K users using RZF precoding. The objective is to maximize the average weighted sum-rate under per antenna and sum power constraints based on large scale fading. This mixed combinatorial and non-convex problem is decomposed into simpler subproblems, each of which is then solved by an efficient algorithm. We also show that the capacity of a very large distributed MIMO network scales according to  $O\left(K\frac{\zeta}{2}\log M\right)$ , where  $\zeta$  is the path loss factor.

# APPENDIX

# A. Proof of Proposition 1

First, we prove the following lemma.

**Lemma 3.** Define  $\sigma_{max} \triangleq \max_{1 \leq k \leq K, 1 \leq m \leq M} \sigma_{km}$ . Then for every  $t \geq 0$ , with probability at least  $1 - e^{-t^2/\sigma_{max}^2}$  one has

$$\|\mathbf{H}\| \le C\left(\sigma_{max}\right)\left(\sqrt{K} + \sqrt{M}\right) + t,$$
 (36)

where  $C(\sigma_{max})$  is a constant only depends on  $\sigma_{max}$ .

*Proof:* Recall that  $\mathbf{H} = \mathbf{W} \circ \mathbf{\Sigma}$ , where the notation  $\circ$  denotes the Hadamard product; and  $\mathbf{W}$  is the small scale fading matrix with i.i.d.  $\sim \mathcal{CN}(\mathbf{0},1)$  entries. For any matrix  $\mathbf{X} \in \mathbb{C}^{K \times M}$ , let  $\mathbf{x} = \mathrm{Vec}(\mathbf{X})$  represent the vector obtained by stacking the column of  $\mathbf{X}$ , and let  $\mathrm{Vec}^{-1}(\mathbf{x})$  denote the inverse mapping. Define a function  $f(\mathbf{x}) \triangleq \|\mathrm{Vec}^{-1}(\mathbf{x}) \circ \mathbf{\Sigma}\|$ . Then we have

$$|f(\mathbf{x}) - f(\mathbf{y})| \leq \|\mathbf{x} \circ \operatorname{Vec}(\mathbf{\Sigma}) - \mathbf{y} \circ \operatorname{Vec}(\mathbf{\Sigma})\|$$

$$\leq \sigma_{\max} \|\mathbf{x} - \mathbf{y}\|,$$
(37)

where (37) follows from the fact that  $\|\mathbf{x} \circ \text{Vec}(\boldsymbol{\Sigma})\|$  is a 1-Lipschitz function of  $\mathbf{x} \circ \text{Vec}(\boldsymbol{\Sigma})$ . It can be deduced from [22, Theorem 2] that

$$\mathbb{E}\|\mathbf{H}\| \le C\left(\sigma_{\max}\right)\left(\sqrt{K} + \sqrt{M}\right). \tag{38}$$

Note that  $\|\mathbf{H}\| = f(\mathbf{w})$ , where  $\mathbf{w} = \text{Vec}(\mathbf{W})$  has i.i.d.~  $\mathcal{CN}(\mathbf{0}, 1)$  entries. One has

$$\Pr\left\{f\left(\mathbf{w}\right) - C\left(\sigma_{\max}\right)\left(\sqrt{K} + \sqrt{M}\right) > t\right\}$$

$$\leq \Pr\left\{f\left(\mathbf{w}\right) - \mathrm{E}f\left(\mathbf{w}\right) > t\right\} \leq e^{-t^2/\sigma_{\max}^2},\tag{39}$$

where the last inequality is obtained by applying the Gaussian concentration in [23, Prop. 5.34] on the function  $\hat{f}(\hat{\mathbf{w}}) \triangleq f(\mathbf{w})$ , where  $\hat{\mathbf{w}} \triangleq \begin{bmatrix} \operatorname{Re}(\sqrt{2}\mathbf{w}) & \operatorname{Im}(\sqrt{2}\mathbf{w}) \end{bmatrix}^T$ . This completes the proof for Lemma 3.

Noting that  $\bar{\mathbf{H}} = \mathbf{H}/\sqrt{S}$ , Proposition 1 follows immediately from Lemma 3 by setting  $t = \sqrt{S}$  and letting  $S \to \infty$ .

#### B. Proof of Lemma 2

For conciseness, the  $\Sigma^{(S)}$ ,  $\Psi^{(S)}$  ( $\Sigma^{(S)}$ ) =  $\{A^{(S)}, \alpha^{(S)}, \mathbf{p}^{(S)}\}$  are denoted as  $\Sigma$ ,  $\Psi(\Sigma) = \{A, \alpha, \mathbf{p}\}$  and we use " $\overset{a.s}{\to}$  0" as a simplified notation for " $\overset{a.s}{\to}$  0, as  $S \to \infty$ ". Recall that  $A_j$  is the  $j^{\text{th}}$  antenna in A. By denoting  $\tilde{\mathbf{Q}}_j = \bar{\mathbf{H}}_j^{\text{c}} \bar{\mathbf{H}}_j^{\text{c}\dagger} + \alpha \mathbf{I}_K$ , where  $\bar{\mathbf{H}}_j^{\text{c}}$  is the matrix of  $\bar{\mathbf{H}}$  where the  $j^{\text{th}}$  column is removed, and applying matrix inverse lemma to (3), we have

$$SP_{\mathcal{A}_{j}}\left(\Psi\left(\mathbf{\Sigma}\right)\right) = A_{j}^{c} \left(1 + \bar{\mathbf{h}}_{j}^{c\dagger} \tilde{\mathbf{Q}}_{j}^{-1} \bar{\mathbf{h}}_{j}^{c}\right)^{-2},\tag{40}$$

where  $A^c_j=ar{\mathbf{h}}^{c\dagger}_j \tilde{\mathbf{Q}}^{-1}_j \mathbf{P} \tilde{\mathbf{Q}}^{-1}_j ar{\mathbf{h}}^c_j$ , and  $ar{\mathbf{h}}^c_j=ar{\mathbf{H}} \mathbf{1}_j$  is the j-th column of  $ar{\mathbf{H}}$ . Note that

$$A_j^c = \tilde{\mathbf{h}}_j^{c\dagger} \boldsymbol{\sigma}_j^c \tilde{\mathbf{Q}}_j^{-1} \mathbf{P} \tilde{\mathbf{Q}}_j^{-1} \boldsymbol{\sigma}_j^c \tilde{\mathbf{h}}_j^c, \tag{41}$$

where  $\boldsymbol{\sigma}_{j}^{c} = \operatorname{diag}\left(\sigma_{1j},...,\sigma_{Kj}\right)$ , and the elements of  $\tilde{\mathbf{h}}_{j}^{c} \triangleq \left(\boldsymbol{\sigma}_{j}^{c}\right)^{-1}\bar{\mathbf{h}}_{j}^{c}$  are i.i.d. complex random variables with zero mean, variance 1/S. For any  $\mathbf{A} \in \mathbb{C}^{K \times K}$ , define  $\Xi_{\boldsymbol{\sigma}_{j}^{c},\mathbf{A}} \triangleq \frac{1}{K}\operatorname{Tr}\left(\left(\boldsymbol{\sigma}_{j}^{c}\right)^{2}(\mathbf{A} + \alpha\mathbf{I}_{K})^{-1}\right)$ . Using [24, Corrolary 1], (41) and the equality  $\frac{1}{K}\operatorname{Tr}\left(\left(\boldsymbol{\sigma}_{j}^{c}\right)^{2}\tilde{\mathbf{Q}}_{j}^{-1}\mathbf{P}\tilde{\mathbf{Q}}_{j}^{-1}\right) = -\frac{\partial}{\partial z}\Xi_{\boldsymbol{\sigma}_{j}^{c},\bar{\mathbf{H}}_{j}^{c}\bar{\mathbf{H}}_{j}^{c}+z\mathbf{P}}\Big|_{z=0}$ , we have

$$A_j^c + \frac{K}{S} \left. \frac{\partial}{\partial z} \Xi_{\sigma_j^c, \bar{\mathbf{H}}_j^c \bar{\mathbf{H}}_j^{c\dagger} + z\mathbf{P}} \right|_{z=0} \stackrel{a.s}{\to} 0. \tag{42}$$

By [25, Lemma 2.1], we have

$$\frac{\partial}{\partial z} \Xi_{\boldsymbol{\sigma}_{j}^{c}, \bar{\mathbf{H}}_{j}^{c} \bar{\mathbf{H}}_{j}^{c\dagger} + z\mathbf{P}} \bigg|_{z=0} - \left. \frac{\partial}{\partial z} \Xi_{\boldsymbol{\sigma}_{j}^{c}, \bar{\mathbf{H}} \bar{\mathbf{H}}^{\dagger} + z\mathbf{P}} \right|_{z=0} \stackrel{a.s}{\to} 0.$$
(43)

Applying [6, Theorem 1] to  $\Xi_{\sigma_i^c, \bar{\mathbf{H}}\bar{\mathbf{H}}^{\dagger}+z\mathbf{P}}$ , we obtain<sup>8</sup>

$$\frac{K}{S} \left. \frac{\partial}{\partial z} \Xi_{\boldsymbol{\sigma}_{j}^{c}, \bar{\mathbf{H}}\bar{\mathbf{H}}^{\dagger} + z\mathbf{P}} \right|_{z=0} + \frac{\alpha^{-1}}{S} \sum_{i=1}^{K} \sigma_{i\mathcal{A}_{j}}^{2} \left( p_{i}v_{i} - \varphi_{i} \right) \stackrel{a.s}{\to} 0, \tag{44}$$

<sup>&</sup>lt;sup>8</sup>In [6, Thereom 1],  $\alpha$  is a positive constant. However, it can be verified that the proof of [6, Thereom 1] still holds if we replace the constant  $\alpha$  by  $\alpha^{(S)} = \Psi_{\alpha}^{(S)}\left(\mathbf{\Sigma}^{(S)}\right)$  due to the bounded constraint on  $\Psi_{\alpha}^{(S)}$ .

where  $v_i$  is defined in (12) and  $\varphi_i$  is defined in (13). Combining (42-44), we have

$$A_j^c - \frac{\alpha^{-1}}{S} \sum_{i=1}^K \sigma_{i\mathcal{A}_j}^2 \left( p_i v_i - \varphi_i \right) \stackrel{a.s}{\to} 0. \tag{45}$$

Note that  $(\alpha \mathbf{I}_K + \mathbf{\Delta} - \mathbf{C})^{-1}$  in (13) is invertible because it is diagonally dominant. Applying [24, Corrolary 1], [25, Lemma 2.1] and [6, Theorem 1] one by one, we have  $\bar{\mathbf{h}}_j^{c\dagger} \tilde{\mathbf{Q}}_j^{-1} \bar{\mathbf{h}}_j^c - \frac{K}{S} \frac{1}{K} \mathrm{Tr} \left( \left( \boldsymbol{\sigma}_j^c \right)^2 \tilde{\mathbf{Q}}_j^{-1} \right) \overset{a.s}{\to} 0$ ,  $\frac{1}{K} \mathrm{Tr} \left( \left( \boldsymbol{\sigma}_j^c \right)^2 \tilde{\mathbf{Q}}_j^{-1} \right) - \frac{1}{K} \mathrm{Tr} \left( \left( \boldsymbol{\sigma}_j^c \right)^2 \mathbf{Q}^{-1} \right) \overset{a.s}{\to} 0$  and  $\frac{1}{K} \mathrm{Tr} \left( \left( \boldsymbol{\sigma}_j^c \right)^2 \mathbf{Q}^{-1} \right) - \frac{1}{K} \sum_{i=1}^K \sigma_{iA_j}^2 v_i \overset{a.s}{\to} 0$ , where  $\tilde{\mathbf{Q}} = \bar{\mathbf{H}} \bar{\mathbf{H}}^\dagger + \alpha \mathbf{I}_K$ . Hence,

$$\bar{\mathbf{h}}_{j}^{c\dagger} \tilde{\mathbf{Q}}_{j}^{-1} \bar{\mathbf{h}}_{j}^{c} - \frac{1}{S} \sum_{i=1}^{K} \sigma_{i\mathcal{A}_{j}}^{2} v_{i} \stackrel{a.s}{\to} 0.$$

$$(46)$$

Combining (40), (45) and (46), we have  $SP_m(\Psi(\Sigma)) - S\bar{P}_m(\Psi(\Sigma)) \stackrel{a.s}{\to} 0, \forall m \in \mathcal{A}$ .

# C. Proof of Theorem 1

Let  $\Psi^{(S)\circ}\left(\Sigma^{(S)}\right)$  be the optimal solution of Problem  $\mathcal{P}\left(\Sigma^{(S)}\right)$ . For an admissible policy  $\Psi^{(S)}$ , let  $\left\{\mathcal{A}^{(S)}, \alpha^{(S)}, \mathbf{p}^{(S)}\right\} = \Psi^{(S)}\left(\Sigma^{(S)}\right)$  and define  $Y_m^{(S)} \triangleq SP_m\left(\Psi^{(S)}\left(\Sigma^{(S)}\right)\right)$  for any  $m \in \mathcal{A}^{(S)}$ . Recall that  $K = \lceil \beta S \rceil$  and  $M = \lceil \overline{\beta}S \rceil$ . Then according to the definition of admissible policy and Assumption 3, we have  $\sup_{1 \leq k \leq K} \rho_k^{(S)} \leq P_{\max}, \ \alpha^{(S)} \geq \alpha_{\min}$  and  $\sup_{1 \leq k \leq K, 1 \leq m \leq M} \sigma_{km}^{(S)} \leq \sigma_{\max} \text{ for some } \sigma_{\max} > 0, \text{ uniformly on } S. \text{ Using the expression of the per antenna transmit power in (3), it can be shown that <math>Y_m^{(S)} \leq \frac{(\beta+1)P_{\max}\sigma_{\max}^2}{\sigma_{\min}^2}\overline{Y}_m^{(S)}, \text{ where } \overline{Y}_m^{(S)} \triangleq \frac{\sum_{k=1}^K |W_{km}|^2}{K} \text{ and } W_{km} \sim \mathcal{CN}\left(0,1\right)$  is the element at the k-th row and m-th column of the small scale fading matrix W. Note that  $2K\overline{Y}_m^{(S)} \sim \chi^2\left(2K\right)$  is a chi-square random variable with 2K degrees of freedom. Using the Chernoff bounds on the upper tails of the CDF of  $\chi^2\left(2K\right)$ , we have  $F_{\overline{Y}}^{(S)}\left(t\right) \triangleq \Pr\left[\overline{Y}_m^{(S)} \geq t\right] \leq \left(te^{1-t}\right)^K \leq te^{1-t}$  for any  $t \geq 1$ . Using the relationship between expectation and CDF for a positive random variable, it can be shown that  $E\left[\overline{Y}_m^{(S)}I_{\overline{Y}_m^{(S)}\geq Y}\right] = \int_{\gamma}^{\infty} F_{\overline{Y}}^{(S)}\left(t\right) dt + YF_{\overline{Y}}^{(S)}\left(Y\right) \leq \left(1+Y+Y^2\right)e^{1-Y}$  for any  $Y \geq 1$ , where  $I_{\overline{Y}_m^{(S)}\geq Y}=1$  if  $\overline{Y}_m^{(S)}\geq Y$  and  $I_{\overline{Y}_m^{(S)}\geq Y}=0$  otherwise. It follows that  $\left\{\overline{Y}_m^{(S)}\right\}$  is uniformly integrable [26]. Hence,  $\left\{Y_m^{(S)}\right\}$  is also uniformly integrable. Combine the above and Lemma 2, it follows that

$$SE[P_m(\Psi^*(\Sigma))|\Sigma] - S\bar{P}_m(\Psi^*(\Sigma)) \to 0.$$
 (47)

Note that for conciseness,  $\Sigma^{(S)}$ ,  $\Psi^{(S)\circ}\left(\Sigma^{(S)}\right)$ ,  $\Psi^{(S)*}\left(\Sigma^{(S)}\right)$  are denoted as  $\Sigma$ ,  $\Psi^{\circ}\left(\Sigma\right)$ ,  $\Psi^{*}\left(\Sigma\right)$  when there is no ambiguity and we use " $\to$  0" as a simplified notation for " $\to$  0 as  $S\to\infty$ ". By the definition of  $\bar{P}_{m}\left(\Psi^{*}\left(\Sigma\right)\right)$ , we have

$$S\bar{P}_m\left(\Psi^*\left(\Sigma\right)\right) - \rho_m \leq 0. \tag{48}$$

Then it follows from (47) and (48) that, as  $S \to \infty$ ,

$$SE\left[P_m\left(\Psi^*\left(\Sigma\right)\right)|\Sigma\right] - \rho_m \le 0, \forall m \in \Psi_A^*\left(\Sigma\right). \tag{49}$$

Similarly, as  $S \to \infty$ , it can be shown that

$$S\bar{P}_m\left(\Psi^{\circ}\left(\Sigma\right)\right) - \rho_m \le 0, \ \forall m \in \Psi^{\circ}_{\Delta}\left(\Sigma\right).$$
 (50)

Similarly, define  $Z_k^{(S)} \triangleq \log\left(1+\gamma_k\left(\Psi^{(S)}\left(\mathbf{\Sigma}^{(S)}\right)\right)\right)$  and  $\varepsilon_k^{(S)} \triangleq \log\left(1+\bar{\gamma}_k\left(\Psi^{(S)}\left(\mathbf{\Sigma}^{(S)}\right)\right)\right) - \mathrm{E}\left[Z_k^{(S)}\left|\mathbf{\Sigma}^{(S)}\right|\right]$ . Since  $Kw_k^{(S)}$  is uniformly bounded on S according to Assumption 3, there exists a constant C>0 such that  $\sup_{1\leq k\leq K} w_k^{(S)} \leq \frac{C}{K}$ , uniformly on S. Using the expression of the SINR in (2), it can be shown that  $Z_k^{(S)} \leq \log\left(1+P_{\max}\right)$ . Hence,  $\left\{Z_k^{(S)}\right\}$  is uniformly integrable [26]. Together with Lemma 1, it follows that  $\varepsilon_k^{(S)} \to 0$ , i.e.,  $\forall \delta>0$ ,  $\exists S_0>0$  such that  $\forall S\geq S_0$ ,  $\left|\varepsilon_k^{(S)}\right|<\frac{\delta}{C}, \forall k$ . Then,  $\forall \delta>0$ ,  $\exists S_0>0$  such that  $\forall S\geq S_0$ ,  $\left|\varepsilon_k^{(S)}\right|<\frac{\delta}{C}, \forall k$ . Then,  $\forall \delta>0$ ,  $\exists S_0>0$  such that  $\forall S\geq S_0$ ,  $\left|\varepsilon_k^{(S)}\right|<\frac{\delta}{C}, \forall k$ . Then,  $\left|\varepsilon_k^{(S)}\right|<\frac{\delta}{C}, \forall k$ . Note that  $\left|\varepsilon_k^{(S)}\right|<\frac{\delta}{C}$  and  $\left|\varepsilon_k^{(S)}\right|<\frac{\delta}{C}$  i.e.,  $\left|\varepsilon_k^{(S)}\right|<\frac{\delta}{C}$  of  $\left|\varepsilon_k^{(S)}\right|<\frac{\delta}{C}$ . Note that  $\left|\varepsilon_k^{(S)}\right|<\frac{\delta}{C}$  in  $\left|\varepsilon_k^{(S)}\right|<$ 

$$\mathcal{I}\left(\Psi^{\circ}\left(\Sigma\right)|\Sigma\right) - \bar{\mathcal{I}}\left(\Psi^{\circ}\left(\Sigma\right)|\Sigma\right) \rightarrow 0,$$

$$\bar{\mathcal{I}}\left(\Psi^{*}\left(\Sigma\right)|\Sigma\right) - \mathcal{I}\left(\Psi^{*}\left(\Sigma\right)|\Sigma\right) \rightarrow 0.$$
(51)

From (49), (50) and the definition of  $\Psi^*(\Sigma)$  and  $\Psi^\circ(\Sigma)$ , we have

$$\mathcal{I}\left(\Psi^{\circ}\left(\Sigma\right)|\Sigma\right) - \mathcal{I}\left(\Psi^{*}\left(\Sigma\right)|\Sigma\right) \geq 0,$$

$$\bar{\mathcal{I}}\left(\Psi^{\circ}\left(\Sigma\right)|\Sigma\right) - \bar{\mathcal{I}}\left(\Psi^{*}\left(\Sigma\right)|\Sigma\right) \leq 0.$$
(52)

It follows from (51) and (52) that

$$\mathcal{I}\left(\Psi^{\circ}\left(\mathbf{\Sigma}\right)|\mathbf{\Sigma}\right) - \mathcal{I}\left(\Psi^{*}\left(\mathbf{\Sigma}\right)|\mathbf{\Sigma}\right) \to 0.$$

This completes the proof for Theorem 1.

#### D. Proof of Proposition 2

Using the notations defined in Section IV-B, the constraint in (14) can be rewritten as  $\mathbf{Rp} \leq \rho$ . It can be verified that  $R_{j,l} > 0, \forall j, l$ , where  $R_{j,l}$  is the element at the j-th row and l-th column of  $\mathbf{R}$ . Suppose that there exists k such that  $p_k > P_{\text{max}}$ . We must have  $\sum_{l=1}^K R_{j,l} p_l > R_{j,k} P_{\text{max}} > \rho_{\mathcal{A}_j}$  for sufficiently large  $P_{\text{max}}$ . Hence, for sufficiently large  $P_{\text{max}}$ , we must have  $\max_{1 \leq k \leq K} p_k \leq P_{\text{max}}$  in order to satisfy the per antenna power constraint in (14). This completes the proof.

# E. Calculation of the Derivative $\frac{\partial \hat{I}(A,\alpha)}{\partial \alpha}$

For convenience, define two (S + K)-dimensional vectors

$$oldsymbol{
ho}_{ ext{ext}} = \left[ egin{array}{c} 
ho \ 0 \end{array} 
ight], \ ilde{oldsymbol{\lambda}}_{ ext{ext}} = \left[ egin{array}{c} ilde{oldsymbol{\lambda}} \ - ilde{oldsymbol{
u}} \end{array} 
ight].$$

Define a  $(S+K) \times K$  matrix  $\mathbf{R}_{\mathrm{ext}} \triangleq \begin{bmatrix} \mathbf{R}^T, -\mathbf{I}_K \end{bmatrix}^T$ . Define a vector  $\mathbf{e} \in \mathbb{R}^K$  whose  $k^{\mathrm{th}}$  element is

$$e_{k} = \sum_{l=1}^{K} \left( \frac{w_{l}g_{lk} \sum_{i=1}^{K} \tilde{p}_{i} (\mathcal{A}, \alpha) \frac{\partial g_{li}}{\partial \alpha}}{\left(g_{ll}\tilde{p}_{l} (\mathcal{A}, \alpha) + \tilde{\Omega}_{l}\right)^{2}} - \frac{w_{l} \frac{\partial g_{lk}}{\partial \alpha}}{g_{ll}\tilde{p}_{l} (\mathcal{A}, \alpha) + \tilde{\Omega}_{l}} \right) + \sum_{l \neq k}^{K} \left( \frac{w_{l} \frac{\partial g_{lk}}{\partial \alpha}}{\tilde{\Omega}_{l}} - \frac{w_{l}g_{lk} \sum_{i \neq l}^{K} \tilde{p}_{i} (\mathcal{A}, \alpha) \frac{\partial g_{li}}{\partial \alpha}}{\tilde{\Omega}_{l}^{2}} \right).$$

Define a  $K \times K$  matrix  $\Upsilon$  whose element at the  $k^{\text{th}}$  row and  $l^{\text{th}}$  column is

$$\Upsilon_{kl} = \sum_{l=1}^{K} \frac{-w_l g_{lk} g_{ln}}{\left(1 + \sum_{i=1}^{K} g_{li} \tilde{p}_i \left(\mathcal{A}, \alpha\right)\right)^2} + \sum_{l \neq k, n} \frac{w_l g_{lk} g_{ln}}{\tilde{\Omega}_l^2}.$$

Finally, define a  $(2K + S) \times (2K + S)$  matrix

$$\mathbf{\Upsilon}_{\mathrm{ext}} = \left[ egin{array}{cc} \mathbf{\Upsilon}; & -\mathbf{R}_{\mathrm{ext}}^T \\ \mathrm{diag}\left( ilde{oldsymbol{\lambda}}_{\mathrm{ext}} 
ight) \mathbf{R}_{\mathrm{ext}}; & \mathrm{diag}\left( \mathbf{R}_{\mathrm{ext}} ilde{\mathbf{p}}\left( \mathcal{A}, lpha 
ight) - oldsymbol{
ho}_{\mathrm{ext}} 
ight) \end{array} 
ight].$$

Taking partial derivative of the equations in (26) with respect to  $\alpha$ , we obtain the following linear equations

$$\mathbf{\Upsilon}_{\mathrm{ext}} \left[ \begin{array}{c} \frac{\partial \tilde{\mathbf{p}}(\mathcal{A}, \alpha)}{\partial \alpha} \\ \frac{\partial \tilde{\boldsymbol{\lambda}}_{\mathrm{ext}}}{\partial \alpha} \end{array} \right] = \left[ \begin{array}{c} \left(\frac{\partial \mathbf{R}}{\partial \alpha}\right)^T \tilde{\boldsymbol{\lambda}} + \mathbf{e} \\ -\mathrm{diag}\left(\tilde{\boldsymbol{\lambda}}_{\mathrm{ext}}\right) \left(\frac{\partial \mathbf{R}_{\mathrm{ext}}}{\partial \alpha}\right)^T \tilde{\mathbf{p}}\left(\mathcal{A}, \alpha\right) \end{array} \right].$$

Then we can obtain  $\frac{\partial \tilde{\mathbf{p}}(\mathcal{A},\alpha)}{\partial \alpha}$  by solving the above linear equations.

Define  $\mathcal{J}=\left\{j:\ \bar{P}_{\mathcal{A}_j}\left(\mathcal{A},\alpha,\tilde{\mathbf{p}}\left(\mathcal{A},\alpha\right)\right)<\rho_{\mathcal{A}_j}\right\}$  and  $\mathcal{K}=\left\{k:\ \tilde{p}_k\left(\mathcal{A},\alpha\right)>0\right\}$ . Note that we have  $\tilde{\lambda}_j=0, \forall j\in\mathcal{J}$  and  $\tilde{\nu}_k=0, \forall k\in\mathcal{K}$  according to the KKT conditions. It can be verified that  $\frac{\partial \tilde{\lambda}_j}{\partial \alpha}=0, \forall j\in\mathcal{J}$  and  $\frac{\partial \tilde{\nu}_k}{\partial \alpha}=0, \forall k\in\mathcal{K}$ . Therefore, we can delete these  $|\mathcal{J}|+|\mathcal{K}|$  variables and the corresponding linear equations whose index i satisfies  $i-K\in\mathcal{J}$  or  $i-S-K\in\mathcal{K}$ . The remaining  $2K+S-|\mathcal{J}|-|\mathcal{K}|$  variables can be determined by the remaining linear equations. After obtaining  $\frac{\partial \tilde{\mathbf{p}}(\mathcal{A},\alpha)}{\partial \alpha}$ , the derivative  $\frac{\partial \hat{\mathcal{I}}(\mathcal{A},\alpha)}{\partial \alpha}$  can be calculated using (30).

To complete the calculation of  $\frac{\partial \hat{\mathcal{I}}(\mathcal{A},\alpha)}{\partial \alpha}$ , we still need to obtain  $\frac{\partial g_{kl}}{\partial \alpha}$ ,  $\forall k,l$ , and  $\frac{\partial \mathbf{R}}{\partial \alpha}$ . The following Lemma are useful and can be proved by a direct calculation.

**Lemma 4** (Derivatives of the intermediate variables). For the intermediate variables  $\theta_k$ ,  $\mathbf{v}$ ,  $\Delta$  and  $\mathbf{C}$  defined in Lemma 1 and Lemma 2, the partial derivatives of them with respective to  $\alpha$  are given below.

$$\frac{\partial \boldsymbol{\theta}_k}{\partial \alpha} = \left(\mathbf{I}_K - \mathbf{D}\right)^{-1} \left(\frac{\partial \mathbf{d}_k}{\partial \alpha} + \frac{\partial \mathbf{D}}{\partial \alpha} \boldsymbol{\theta}_k\right), \, \forall k, \tag{53}$$

where  $\frac{\partial \mathbf{d}_k}{\partial \alpha}$  is given by

$$\frac{\partial d_{kl}}{\partial \alpha} = -\frac{2}{S} \sum_{m \in A} \left[ \sigma_{km}^2 \sigma_{lm}^2 \left( 1 - \frac{1}{S} \sum_{i=1}^K \frac{\sigma_{im}^2 \varphi_i}{\left( 1 + \xi_i \right)^2} \right) / f_m^3 \left( \boldsymbol{\xi} \right) \right], \ \forall l,$$

and  $\frac{\partial \mathbf{D}}{\partial \alpha}$  is given by

$$\frac{\partial D_{ln}}{\partial \alpha} = \frac{1}{S^2} \sum_{m \in \mathcal{A}} \left[ \frac{2\sigma_{lm}^2 \sigma_{nm}^2}{\left(1 + \xi_n\right)^2} \left[ \frac{1}{S} \sum_{i=1}^K \frac{\sigma_{im}^2}{1 + \xi_i} \left( \frac{\varphi_i}{1 + \xi_i} - \frac{\varphi_n}{1 + \xi_n} \right) - 1 - \frac{\alpha \varphi_n}{1 + \xi_n} \right] / f_m^3\left(\boldsymbol{\xi}\right) \right].$$

$$\frac{\partial C_{ln}}{\partial \alpha} = \frac{1}{S} \sum_{m \in \mathcal{A}} \left[ \frac{1}{S} \sigma_{lm}^2 \sigma_{nm}^2 \frac{\partial v_l}{\partial \alpha} / \psi_m^2 \left( \mathbf{v} \right) - \frac{2}{S} \sigma_{lm}^2 \sigma_{nm}^2 v_l \frac{1}{S} \sum_{i=1}^K \sigma_{im}^2 \frac{\partial v_i}{\partial \alpha} / \psi_m^3 \left( \mathbf{v} \right) \right], \ \forall l, n,$$
 (54)

$$\frac{\partial \triangle_{l}}{\partial \alpha} = -\frac{1}{S} \sum_{m \in \mathcal{A}} \left[ \sigma_{lm}^{2} \frac{1}{S} \sum_{i=1}^{K} \sigma_{im}^{2} \frac{\partial v_{i}}{\partial \alpha} / \psi_{m}^{2} \left( \mathbf{v} \right) \right], \, \forall l,$$
 (55)

$$\frac{\partial \mathbf{v}}{\partial \alpha} = -\left(\alpha \mathbf{I}_K + \mathbf{\Delta} - \mathbf{C}\right)^{-1} \mathbf{v},\tag{56}$$

Using Lemma 4,  $\frac{\partial g_{kl}}{\partial \alpha}$ ,  $\forall k, l$ , and  $\frac{\partial \mathbf{R}}{\partial \alpha}$  can be obtained by a direct calculation as follows.

$$\frac{\partial g_{kk}}{\partial \alpha} = \frac{2\xi_k \frac{\partial \xi_k}{\partial \alpha}}{(1+\xi_k)^3}, \forall k,$$

$$\frac{\partial g_{kl}}{\partial \alpha} = \frac{\frac{\partial \theta_{kl}}{\partial \alpha}}{S(1+\xi_l)^2(1+\xi_k)^2} - \frac{2\theta_{kl} \left[\frac{\partial \xi_l}{\partial \alpha} (1+\xi_k) + \frac{\partial \xi_k}{\partial \alpha} (1+\xi_l)\right]}{S(1+\xi_l)^3(1+\xi_k)^3}, \forall k \neq l,$$

where  $\frac{\partial \xi_k}{\partial \alpha} = \varphi_k$ ,  $\forall k$  is defined in (13),  $\boldsymbol{\theta}_k = [\theta_{k1}, ..., \theta_{kK}]^T$ ,  $\forall k$  is defined in (10) and  $\frac{\partial \boldsymbol{\theta}_k}{\partial \alpha}$  is given in (53). To calculate  $\frac{\partial \mathbf{R}}{\partial \alpha}$ , we first obtain  $\frac{\partial \hat{\mathbf{R}}}{\partial \alpha}$  as

$$\frac{\partial \hat{R}_{kj}}{\partial \alpha} = -\frac{\sigma_{kA_j}^2 \alpha^{-1}}{S^2} \left[ \alpha^{-1} / \psi_{A_j}^2 \left( \mathbf{v} \right) + \frac{2}{S} \sum_{i=1}^K \sigma_{iA_j}^2 \frac{\partial v_i}{\partial \alpha} / \psi_{A_j}^3 \left( \mathbf{v} \right) \right], \ \forall k, j.$$

where  $\frac{\partial v_i}{\partial \alpha}$ ,  $\forall i$  is given in (56). Then we obtain  $\frac{\partial \bar{\mathbf{R}}}{\partial \alpha}$  as

$$\begin{split} \frac{\partial \bar{R}_{kk}}{\partial \alpha} &= \frac{1}{S} \sum_{m \in \mathcal{A}} \left[ \left( \sigma_{km}^2 \frac{\partial v_k}{\partial \alpha} + \sigma_{km}^2 \frac{1}{S} \sum_{i \neq k}^K \sigma_{im}^2 \left( v_k \frac{\partial v_i}{\partial \alpha} + v_i \frac{\partial v_k}{\partial \alpha} \right) \right) / \psi_m^2 \left( \mathbf{v} \right) \\ &- \left( 2 \sigma_{km}^2 v_k \left( 1 + \frac{1}{S} \sum_{i \neq k}^K \sigma_{im}^2 v_i \right) \frac{1}{S} \sum_{i = 1}^K \sigma_{im}^2 \frac{\partial v_i}{\partial \alpha} \right) / \psi_m^3 \left( \mathbf{v} \right) \right], \forall k, \\ \frac{\partial \bar{R}_{kl}}{\partial \alpha} &= -\frac{1}{S} \sum_{m \in \mathcal{A}} \left[ \sigma_{km}^2 \sigma_{lm}^2 \frac{1}{S} \left( v_l \frac{\partial v_k}{\partial \alpha} + v_k \frac{\partial v_l}{\partial \alpha} \right) / \psi_m^2 \left( \mathbf{v} \right) - \frac{2}{S} \sigma_{km}^2 \sigma_{lm}^2 v_k v_l \frac{1}{S} \sum_{i = 1}^K \sigma_{im}^2 \frac{\partial v_i}{\partial \alpha} / \psi_m^3 \left( \mathbf{v} \right) \right], \forall k \neq l. \\ \text{Finally, } \frac{\partial \mathbf{R}}{\partial \alpha} &= \left[ \mathbf{I}_S, \quad \mathbf{1} \right]^T \frac{\partial \bar{\mathbf{R}}}{\partial \alpha} \text{ and } \frac{\partial \bar{\mathbf{R}}}{\partial \alpha} \text{ is given by} \\ \frac{\partial \tilde{\mathbf{R}}}{\partial \alpha} &= \left( \frac{\partial \hat{\mathbf{R}}}{\partial \alpha} \right)^T \left( \mathbf{V} - (\alpha \mathbf{I}_K + \mathbf{\Delta} - \mathbf{C})^{-1} \bar{\mathbf{R}} \right) + \hat{\mathbf{R}}^T \left( \frac{\partial \mathbf{V}}{\partial \alpha} - (\alpha \mathbf{I}_K + \mathbf{\Delta} - \mathbf{C})^{-1} \frac{\partial \bar{\mathbf{R}}}{\partial \alpha} \right) \\ &+ \hat{\mathbf{R}}^T \left( \mathbf{I}_K + \frac{\partial \mathbf{\Delta}}{\partial \alpha} - \frac{\partial \mathbf{C}}{\partial \alpha} \right) (\alpha \mathbf{I}_K + \mathbf{\Delta} - \mathbf{C})^{-2} \bar{\mathbf{R}}, \end{split}$$

where  $\frac{\partial \mathbf{C}}{\partial \alpha}$  and  $\frac{\partial \mathbf{\Delta}}{\partial \alpha}$  are given in (54) and (55) respectively, and  $\frac{\partial \mathbf{V}}{\partial \alpha} = \operatorname{diag}\left(\frac{\partial v_1}{\partial \alpha}, ..., \frac{\partial v_K}{\partial \alpha}\right)$ .

# F. Proof of Theorem 5

When  $P_T$  is large enough, all users will be allocated with non-zero power. In this case, the SINR of user k under power allocation in (34) is given by

$$\hat{\gamma}_{k} (\alpha, \mathbf{p}^{*} (\mathcal{A}, \alpha))$$

$$= \frac{Sw_{k}\sigma_{k1}^{2} \left(2\alpha u P_{T} + (\beta - 1) P_{T} + \frac{1}{S} \sum_{l=1}^{K} \frac{1}{\sigma_{l1}^{2}}\right)}{\left(\sum_{l=1}^{K} w_{l}\right) \left(\frac{\sigma_{k1}^{2} P_{T}}{\left(1 + \sigma_{k1}^{2} u\right)^{2}} + 1\right)} - 1,$$
(57)

and  $\bar{\mathcal{I}}\left(\mathcal{A},\alpha,\mathbf{p}^*\left(\mathcal{A},\alpha\right)\right) = \sum_{k=1}^K w_k \log\left(1+\hat{\gamma}_k\left(\alpha,\mathbf{p}^*\left(\mathcal{A},\alpha\right)\right)\right)$ . For any k, it can be shown that the solution  $\tilde{\alpha}_k$  of  $\frac{\partial}{\partial\alpha}\hat{\gamma}_k\left(\alpha,\mathbf{p}^*\left(\mathcal{A},\alpha\right)\right) = 0$  must satisfy  $\tilde{\alpha}_k = O\left(\frac{1}{P_T}\right)$ . Since the optimal regularization factor  $\alpha^*$  must satisfy  $\min_k \tilde{\alpha}_k \leq \alpha^* \leq \max_k \tilde{\alpha}_k$ , we have  $\alpha^* = O\left(\frac{1}{P_T}\right)$ . To prove the second result, it can be verified that  $\frac{\partial^2}{\partial^2\alpha}\hat{\gamma}_k\left(\alpha,\mathbf{p}^*\left(\mathcal{A},\alpha\right)\right) < 0$  and thus  $\hat{\gamma}_k\left(\alpha,\mathbf{p}^*\left(\mathcal{A},\alpha\right)\right)$  is concave when  $\alpha$  is small enough. Since  $\bar{\mathcal{I}}\left(\mathcal{A},\alpha,\mathbf{p}^*\left(\mathcal{A},\alpha\right)\right)$  is a concave increasing function of  $\hat{\gamma}_k\left(\alpha,\mathbf{p}^*\left(\mathcal{A},\alpha\right)\right)$ ,  $\bar{\mathcal{I}}\left(\mathcal{A},\alpha,\mathbf{p}^*\left(\mathcal{A},\alpha\right)\right)$  must be a concave function of  $\alpha$  [21].

# G. Proof of Theorem 6

We first derive a lower bound for the probability that the minimum distance  $r_{\min}$  between any two users is larger than a certain value  $r_0$ : Pr  $(r_{\min} \ge r_0)$ . Let  $d^u_{kl}$  denote the distance between user k and user l. We have

$$\begin{split} \Pr\left(r_{\min} \geq r_{0}\right) &= 1 - \Pr\left(\min_{l \neq k} d_{kl}^{u} \leq r_{0}, \; \exists k \in \{1, ..., K\}\right) \\ &\geq 1 - \sum_{k=1}^{K} \Pr\left(\min_{l \neq k} d_{kl}^{u} \leq r_{0}\right) \\ &= 1 - K \Pr\left(\min_{l \neq 1} d_{1l}^{u} \leq r_{0}\right) \\ &= 1 - K \left[1 - \left(\Pr\left(d_{12}^{u} \geq r_{0}\right)\right)^{K-1}\right] \\ &\geq 1 - K \left[1 - \left(1 - \frac{\pi r_{0}^{2}}{R_{c}^{2}}\right)^{K-1}\right], \end{split}$$

where the second inequality follows from the union bound and the last inequality holds because  $\Pr(d_{12}^u \ge r_0) \ge 1 - \frac{\pi r_0^2}{R_2^2}$ .

Then we use the path loss model to transfer the probability  $\Pr\left(r_{\min} \geq r_0\right)$  to the probability  $\Pr\left(\eta > \eta_0\right)$  in (35). Note that for any k, we have  $\min_{m} r_{km} \leq \frac{\sqrt{2}R_c}{2\sqrt{M}}$ , and  $\max_{l} r_{k\tilde{m}_l} \geq r_{\min} - \frac{\sqrt{2}R_c}{2\sqrt{M}}$ . Hence  $\eta \geq \left(r_{\min} / \left(\frac{\sqrt{2}R_c}{2\sqrt{M}}\right) - 1\right)^{\zeta}$  and

$$\Pr(\eta > \eta_0) \geq \Pr\left(\left(r_{\min} / \left(\frac{\sqrt{2}R_c}{2\sqrt{M}}\right) - 1\right)^{\zeta} > \eta_0\right)$$

$$= \Pr\left(r_{\min} > \frac{\sqrt{2}R_c}{2\sqrt{M}} \left(\eta_0^{1/\zeta} + 1\right)\right)$$

$$\geq 1 - K\left[1 - \left(1 - \frac{\pi\left(\eta_0^{1/\zeta} + 1\right)^2}{2M}\right)^{K-1}\right].$$

Finally, we prove the capacity scaling by deriving an upper and a lower bound for the sum-rate. The following lemma follows directly from Definition 3.

**Lemma 5.** For any  $\epsilon > 0$ , as  $M \to \infty$  with K, S fixed, we have

$$Pr\left(\min_{k,m} r_{km} \le M^{-\frac{1}{2}-\epsilon}\right) = \frac{\pi M^{-2\epsilon}}{R_c^2} \to 0,$$

and thus

$$Pr\left(\bar{g}_k^d > G_0 M^{\frac{\zeta}{2} + \epsilon}\right) \to 0.$$

Let  $P_T^U = \max_{m \in \mathcal{A}} \rho_m$  and let  $X_S$  be a random variable with  $\chi^2(2S)$  distribution. Assuming that each user is served by S antennas without mutual interference, we obtain an upper bound for average sum-rate as follows:

$$C_s \leq K \operatorname{E} \left[ \log \left( 1 + P_T^U \bar{g}_k^d X_S / 2 \right) \right]$$
  
$$\leq K \log \left( 1 + P_T^U \bar{g}_k^d \operatorname{E} \left[ X_S \right] / 2 \right). \tag{58}$$

Combining (58) with Lemma 5, we prove that  $C_s \stackrel{a.s}{\leq} O\left(K\left(\frac{\zeta}{2} + \epsilon\right)\log M\right)$  as  $M \to \infty$  with K, S fixed. Furthermore, it follows from the lower bound provided in Appendix H that  $C_s \geq O\left(K\left(\frac{\zeta}{2} - \epsilon\right)\log M\right)$ . This

completes the proof of Theorem 6.

## H. Proof of Corollary 1

Due to (35) in Theorem 6, the step 1 in Algorithm S2 will almost surely select a set of antennas  $\mathcal{A}, |\mathcal{A}| = K$  such that each user has strong direct-link with one of the selected K antennas and weak cross-links with other selected antennas for large M/K. Assume that each selected antenna only serves the nearest user, and assume equal power allocation for each user, i.e., the transmit power for each user is  $P^L = \min_{m \in \mathcal{A}} \frac{\rho_m}{S}$ . Let  $X_m$  denote a random variable with  $\chi^2(2m)$  distribution. Let  $\eta_0 = M^{\frac{\zeta}{2} - \epsilon_1}$  in (35). Then using (35) and the fact that  $\bar{g}_k^d \geq G_0\left(\frac{\sqrt{2}R_c}{2\sqrt{M}}\right)^{-\zeta}$ , we can show that as  $M \to \infty$  with K, S fixed, the average sum-rate  $\mathcal{I}_{\mathcal{A}}$  is almost surely lower bounded by

$$\mathcal{I}_{\mathcal{A}} \stackrel{a.s}{\geq} KE \left[ \log \left( 1 + \frac{P^L G_0 \left( \frac{\sqrt{2}R_c}{2\sqrt{M}} \right)^{-\zeta} X_1/2}{1 + P^L M^{-\frac{\zeta}{2} + \epsilon_1} G_0 \left( \frac{\sqrt{2}R_c}{2\sqrt{M}} \right)^{-\zeta} \frac{X_{K-1}}{2}} \right) \right], \tag{59}$$

where  $X_1$  and  $X_{K-1}$  are independent. Choose  $B_1, B_2 > 0$  such that  $\Pr(X_1 \ge B_1) \Pr(X_{K-1} \le B_2) \ge 1 - \epsilon_2$ . Then as  $M \to \infty$  with K, S fixed, it follows from (59) that

$$\begin{split} \mathcal{I}_{\mathcal{A}} &\overset{a.s}{\geq} \quad K\left(1-\epsilon_{2}\right) \log \left(1+\frac{P^{L}G_{0}\left(\frac{\sqrt{2}R_{c}}{2\sqrt{M}}\right)^{-\zeta}B_{1}/2}{1+P^{L}M^{-\frac{\zeta}{2}+\epsilon_{1}}G_{0}\left(\frac{\sqrt{2}R_{c}}{2\sqrt{M}}\right)^{-\zeta}\frac{B_{2}}{2}}\right) \\ &= \quad O\left(K\left(1-\epsilon_{2}\right)\left(\frac{\zeta}{2}-\epsilon_{1}\right) \log M\right). \end{split}$$

Choose  $\epsilon_1, \epsilon_2$  such that  $\epsilon_1 + \frac{\zeta}{2}\epsilon_2 - \epsilon_1\epsilon_2 = \epsilon$ . Then we have  $\mathcal{I}_{\mathcal{A}} \stackrel{a.s}{\geq} O\left(K\left(\frac{\zeta}{2} - \epsilon\right)\log M\right)$  as  $M \to \infty$  with K, S fixed. The rest of the steps in S2 only increase the sum-rate by a constant. This completes the proof.

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