OMP Based Joint Sparsity Pattern Recovery Under Communication Constraints

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Abstract-We address the problem of joint sparsity pattern recovery based on low dimensional multiple measurement vectors (MMVs) in resource constrained distributed networks. We assume that distributed nodes observe sparse signals which share the same sparsity pattern and each node obtains measurements via a low dimensional linear operator. When the measurements are collected at distributed nodes in a communication network, it is often required that joint sparse recovery be performed under inherent resource constraints such as communication bandwidth and transmit/processing power. We present two approaches to take the communication constraints into account while performing common sparsity pattern recovery. First, we explore the use of a shared multiple access channel (MAC) in forwarding observations residing at each node to a fusion center. With MAC, while the bandwidth requirement does not depend on the number of nodes, the fusion center has access to only a linear combination of the observations. We discuss the conditions under which the common sparsity pattern can be estimated reliably. Second, we develop two collaborative algorithms based on Orthogonal Matching Pursuit (OMP), to jointly estimate the common sparsity pattern in a decentralized manner with a low communication overhead. In the proposed algorithms, each node exploits collaboration among neighboring nodes by sharing a small amount of information for fusion at different stages in estimating the indices of the true support in a greedy manner. Efficiency and effectiveness of the proposed algorithms are demonstrated via simulations along with a comparison with the most related existing algorithms considering the trade-off between the performance gain and the communication overhead.

Index Terms—Multiple measurement vectors, Sparsity pattern recovery, Compressive Sensing, Orthogonal matching pursuit (OMP), Decentralized algorithms

I. INTRODUCTION

The problem of joint recovery of sparse signals based on reduced dimensional multiple measurement vectors (MMVs) is very important with a variety of applications including sub-Nyquist sampling of multiband signals [1], multivariate regression [2], sparse signal recovery with multiple sensors [3], spectrum sensing in cognitive radio networks with multiple cognitive radios [4]–[6], neuromagnetic imaging [7], [8], and medical imaging [9]. While MMV problems have been traditionally addressed using sensor array signal processing techniques, they have attracted considerable recent attention in the context of *compressive sensing* (CS). When all the low dimensional multiple measurements are available at a central processing unit, recent research efforts have considered extending algorithms developed for sparse recovery with a single measurement vector (SMV) in the context of CS to joint recovery with MMVs and discuss performance guarantees [8], [10]–[16].

Application of random matrices to obtain low dimensional (compressed) measurements at distributed nodes in sensor and cognitive radio networks was explored in [3]–[6]. When low dimensional measurements are collected at multiple nodes in a distributed network, to perform simultaneous recovery of sparse signals in the optimal way, it is required that each node transmit measurements and the information regarding the low dimensional sampling operators to a central processing unit. However, this centralized processing requires large communication burden in terms of communication bandwidth and transmit power. In sensor and cognitive radio networks, power and communication bandwidth are limited and long range communication may be prohibitive due to the requirement of higher transmit power. Thus, there is a need for the development of sparse solutions when all the measurements are not available at a central fusion center. In this paper, our goal is to investigate the problem of joint sparsity pattern recovery with multiple measurement vectors utilizing minimal amount of transmit power and communication bandwidth.

A. Motivation and our contributions

We consider a distributed network in which multiple nodes observe sparse signals that share the same sparsity pattern. Each node is assumed to make measurements by employing a low dimensional linear projection operator. The goal is to recover the common sparsity pattern jointly under communication constraints. We present two approaches in this paper. The first approach that takes communication constraints into account in providing a centralized solution for sparsity pattern recovery with MMV is to transmit only a summary or a function of the observations to a fusion center instead of transmitting raw observations. Use of shared multiple access channels (MAC) in forwarding observations to a fusion center is attractive in certain distributed networks whose bandwidth does not depend on the number of transmitting nodes [17], [18]. With the MAC output, the fusion center does not have access to individual observation vectors in contrast to employing a bank of parallel channels. However, due to the gain in communication bandwidth of MAC over parallel communication, MAC is attractive in applications where communication

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²This material is based upon work supported by the National Science Foundation (NSF) under Grant No. 1307775

³A part of this work was presented at ICASSP 2013, in Vancouver, Canada with the title 'Cooperative Sparsity Pattern Recovery in Distributed Networks Via Distributed-OMP'

bandwidth is limited [17], [18]. We discuss the conditions under which the common sparsity pattern can be estimated reliably. Further, we show that the performance is comparable under certain conditions to the case where all the observation vectors are available at the fusion center separately.

In the second approach, we develop two decentralized algorithms with low communication overhead to estimate the common sparsity pattern by appropriate collaboration and fusion among nodes. In addition to the low communication cost, decentralized algorithms are attractive since they are robust to node and link failures. In contrast to existing decentralized solutions, we consider a greedy approach, namely, orthogonal matching pursuit (OMP) for joint sparsity pattern recovery. Standard OMP with a SMV was introduced in [19]. Simultaneous OMP (S-OMP) for MMVs when all the measurements with a common measurement matrix are available at a central processing unit was discussed in [20] while in [3] authors extended it to the case where multiple measurements are collected via different measurement matrices.

The two proposed algorithms, called DC-OMP 1 and DC-OMP 2, perform sparsity pattern recovery in a greedy manner with collaboration and fusion at each iteration. In the first algorithm DC-OMP 1, single-stage fusion is performed at each iteration via only one hop communication in estimating the sparsity pattern. DC-OMP 2, is shown to have performance that is close to S-OMP performed in a centralized manner when a reasonable neighborhood size is employed. Although DC-OMP 1 lacks in performance, the communication overhead required by DC-OMP 1 is much less compared to DC-OMP 2. However, in terms of performance, DC-OMP 1 still provides a significant gain compared to the case where each node performs standard OMP independently to get an estimate of the complete support set and then the results are fused to get a global estimate. We further show that, in both DC-OMP 1 and DC-OMP 2, each node has to perform less number of iterations compared to the sparsity index of the sparse signal to reliably estimate the sparsity pattern. (in both OMP and S-OMP, at least k iterations are required where k is the sparsity index of the sparse signals).

This work is based on our preliminary work presented in [21]. In [21], we provided the algorithm development of DC-OMP 1 for common sparsity pattern recovery. In this paper, we significantly extend the work by (i). proposing a second decentralized algorithm for joint sparsity pattern recovery named DC-OMP 2 which provides a larger performance gain compared to DC-OMP 1 at the price of additional communication cost, (ii). providing theoretical analysis to show the performance gain achieved by DC-OMP 1 and DC-OMP 2 via collaboration and fusion compared to that without any collaboration among nodes, and (iii). presenting a new joint sparsity pattern recovery approach based on the MAC output.

B. Related work

While most of the work related to joint sparsity pattern recovery with multiple measurement vectors assumes that the measurements are available at a central fusion center [8], [10]–[12], [14], [22], there are several recent research

efforts in [4]-[6], [21], [23]-[25], that address the problem of joint sparse recovery in a decentralized manner where nodes communicate with only their neighboring nodes. The problem of spectrum sensing in multiple cognitive radio networks is cast as a sparse recovery problem in [4]-[6] and the authors propose decentralized solutions based on optimization techniques. However, performing optimization at sensor nodes may be computationally prohibitive. Further, in these approaches, each node needs to transmit the whole estimated vector to its neighbors at each iteration. In [25], a distributed version of iterative hard thresholding (IHT) is developed for sparse signal recovery in sensor networks in which computational complexity per node is less compared to optimization based techniques. In a recent work [26], the authors have proposed a decentralized version of a subspace pursuit algorithm for distributed CS. While this paper was under review, another paper appeared that also considers a decentralized version of OMP, called DiOMP, for sparsity pattern recovery [27] which is different from DC-OMP 1 and DC-OMP 2 proposed in this paper. In DC-OMP 1 and DC-OMP 2, the goal in performing collaboration is to improve the accuracy via fusion for common support recovery compared to the standard OMP, whereas the main idea in DiOMP is to exploit the collaboration to identify the common support when the sparse signals have common plus independent supports among multiple sparse signals.

The rest of the paper is organized as follows. In Section II, the observation model, and background on sparsity pattern recovery with a central fusion center are presented. In Section III, the problem of joint sparsity pattern recovery with multiple access communication is discussed. In Section IV, two decentralized greedy algorithms based on OMP are proposed for joint sparsity pattern recovery. Numerical results are presented in Section V and concluding remarks are given in Section VI.

II. PROBLEM FORMULATION

We consider a scenario where L nodes in a distributed network observe sparse signals. More specifically, each node makes measurements of a length-N signal denoted by \mathbf{x}_l which is assumed to be sparse in the basis Φ . Further, we assume that all signals \mathbf{x}_l 's for $l = 0, 1, \dots, L-1$ are sparse with a common sparsity pattern in the same basis. Let $\mathbf{x}_l = \Phi \mathbf{s}_l$ where \mathbf{s}_l contains only $k \ll N$ significant coefficients and the locations of the significant coefficients are the same for all \mathbf{s}_l for $l = 0, 1, \dots, L-1$. A practical situation well modeled by this joint sparsity pattern in distributed networks is where multiple sensors in a sensor network acquire the same Fourier sparse signal but with phase shifts and attenuations caused by signal propagation [3]. Another application is estimation of the sparse signal spectrum in a cognitive radio network with multiple cognitive radios [4], [5].

We assume that each node obtains a compressed version of the sparse signal observed via the following linear system,

$$\mathbf{y}_l = \mathbf{A}_l \mathbf{x}_l + \mathbf{v}_l; \tag{1}$$

for $l = 0, 1, \dots, L - 1$ where \mathbf{A}_l is the $M \times N$ projection matrix, M is the number of compressive measurements with

M < N, and \mathbf{v}_l is the measurement noise vector at the *l*-th node. The noise vector \mathbf{v}_l is assumed to be iid Gaussian with zero mean vector and the covariance matrix $\sigma_v^2 \mathbf{I}_M$ where \mathbf{I}_M is the $M \times M$ identity matrix.

A. Sparsity pattern recovery

Our goal is to jointly estimate the common sparsity pattern of \mathbf{x}_l for $l = 0, 1, \dots, L-1$ based on the underdetermined linear system (1). Once the sparse support is known, the problem of estimating the coefficients can be cast as a linear estimation problem and standard techniques such as the least squares method can be employed to estimate the amplitudes of the non zero coefficients. Further, in certain applications including spectrum sensing in cognitive radio networks, it is sufficient only to identify the sparse support.

Let $\mathbf{B}_l = \mathbf{A}_l \mathbf{\Phi}$ so that \mathbf{y}_l can be expressed as

$$\mathbf{y}_l = \mathbf{B}_l \mathbf{s}_l + \mathbf{v}_l \tag{2}$$

for $l = 0, 1, \dots, L - 1$. Define the support set of s_l , U, to be the set which contains the indices of locations of non zero coefficients in s_l at *l*-th node:

$$\mathcal{U} := \{ i \in \{0, 1, \cdots, N-1\} \mid \mathbf{s}_l(i) \neq 0 \}$$
(3)

where $\mathbf{s}_l(i)$ is the *i*-th element of \mathbf{s}_l for $i = 0, 1, \dots, N-1$ and $l = 0, 1, \dots, L-1$. Then we have $k = |\mathcal{U}|$ where $|\mathcal{U}|$ denotes the cardinality of the set \mathcal{U} . It is noted that the sparse support is denoted by the same notation \mathcal{U} at each node due to the assumption of common sparsity pattern. Further, let $\boldsymbol{\zeta}$ be a length-N vector which contains binary elements: i.e.

$$\boldsymbol{\zeta}(i) = \begin{cases} 1 & if \ \mathbf{s}_l(i) \neq 0\\ 0 & \text{otherwise} \end{cases}$$
(4)

for any l and $i = 0, 1, \dots, N - 1$. In other words, elements in ζ are 1's corresponding to indices in \mathcal{U} while all other elements are zeros. The goal of sparsity pattern recovery is to estimate the set \mathcal{U} (or the vector ζ).

B. Sparsity pattern recovery via OMP in a centralized setting

To make all the observation vectors \mathbf{y}_l for $l = 0, 1, \dots, L-1$ available at a fusion center, a bank of dedicated parallel access channels (PAC) which are independent across nodes has to be used. With PAC, the observation matrix at the fusion center can be written in the form,

$$\mathbf{Y} = \tilde{\mathbf{S}} + \mathbf{V} \tag{5}$$

where $\mathbf{Y} = [\mathbf{y}_0, \dots, \mathbf{y}_{L-1}], \, \tilde{\mathbf{S}} = [\mathbf{B}_0 \mathbf{s}_0, \dots, \mathbf{B}_{L-1} \mathbf{s}_{L-1}]$ and $\mathbf{V} = [\mathbf{v}_0, \dots, \mathbf{v}_{L-1}]$. In the special case where $\mathbf{A}_l = \mathbf{A}$ for $l = 0, 1, \dots, L-1$, the PAC output can be expressed as,

$$\mathbf{Y} = \mathbf{B}\mathbf{S} + \mathbf{V} \tag{6}$$

where $\mathbf{S} = [\mathbf{s}_0, \cdots, \mathbf{s}_{L-1}]$ and $\mathbf{B} = \mathbf{A}\boldsymbol{\Phi}$. It is noted that the model (6) is the widely used form for simultaneous sparse approximation with MMV which has been studied quite extensively [8], [10]–[12], [22].

While optimization techniques for sparse signal recovery such as l_1 norm minimization provide more promising results

Algorithm 1 Standard OMP with SMV

Inputs: $\mathbf{y}, \mathbf{B}, k$

- 1) Initialize t = 1, $\hat{\mathcal{U}}(0) = \emptyset$, residual vector $\mathbf{r}_0 = \mathbf{y}$
- 2) Find the index $\lambda(t)$ such that $\lambda(t) = \underset{\omega=0,\dots,N-1}{\arg \max} |\langle \mathbf{r}_{t-1}, \mathbf{b}_{\omega} \rangle|$
- 3) Set $\hat{\mathcal{U}}(t) = \hat{\mathcal{U}}(t-1) \cup \{\lambda(t)\}$
- 4) Compute the projection operator $\mathbf{P}(t) = \mathbf{B}(\hat{\mathcal{U}}(t)) \left(\mathbf{B}(\hat{\mathcal{U}}(t))^T \mathbf{B}(\hat{\mathcal{U}}(t)) \right)^{-1} \mathbf{B}(\hat{\mathcal{U}}(t))^T$. Update the residual vector: $\mathbf{r}_t = (\mathbf{I} \mathbf{P}(t))\mathbf{y}$ (note: $\mathbf{B}(\hat{\mathcal{U}}(t))$) denotes the submatrix of \mathbf{B} in which columns are taken from \mathbf{B} corresponding to the indices in $\hat{\mathcal{U}}(t)$)
- 5) Increment t = t+1 and go to step 2 if $t \le k$, otherwise, stop and set $\hat{\mathcal{U}} = \hat{\mathcal{U}}(t-1)$

in terms of accuracy, their computational complexity is higher than that of greedy techniques such as OMP. With a single measurement vector, the computational complexity with the best known convex optimization based algorithm is in the order of $\mathcal{O}(M^2 N^{3/2})$ while the complexity with OMP is in the order of $\mathcal{O}(MNk)$ [28]. Further, it was shown in [29] that for sparsity pattern recovery with large Gaussian measurement matrices in high signal-to-noise ratio (SNR) environments, l_1 norm based Lasso and OMP have almost identical performance. The standard OMP as presented in Algorithm 1 is developed in [19] for the SMV case. We omit the subscripts of the vectors and matrices whenever we consider the SMV case. In OMP, at each iteration, the location of one non zero coefficient of the sparse signal (or the index of a column of **B** that participates in the measurement vector y) is identified. Then the selected column's contribution is subtracted from y and iterations on the residual are carried out.

Algorithm 2 S-OMP	with	different	projection	matrices
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Inputs: Inputs: $\{\mathbf{y}_l, \mathbf{B}_l\}_{l=0}^{L-1}, k$

1) Initialize t = 1, $\hat{\mathcal{U}}(0) = \emptyset$, residual vector $\mathbf{r}_{l,0} = \mathbf{y}_l$

- 2) Find the index $\lambda(t)$ such that $\lambda(t) = \arg \max_{\omega=0,\dots,N-1} \sum_{l=0}^{L-1} |\langle \mathbf{r}_{l,t-1}, \mathbf{b}_{l,\omega} \rangle|$
- 3) Set $\hat{\mathcal{U}}(t) = \hat{\mathcal{U}}(t-1) \cup \{\lambda(t)\}$
- 4) Compute the orthogonal projection operator: $\mathbf{P}_{l}(t) = \mathbf{B}_{l}(\hat{\mathcal{U}}(t)) \left(\mathbf{B}_{l}(\hat{\mathcal{U}}(t))^{T} \mathbf{B}_{l}(\hat{\mathcal{U}}(t)) \right)^{-1} \mathbf{B}_{l}(\hat{\mathcal{U}}(t))^{T}$ Update the residual: $\mathbf{r}_{l,t} = (\mathbf{I} - \mathbf{P}_{l}(t))\mathbf{y}_{l}$
- 5) Increment t = t+1 and go to step 2 if $t \le k$, otherwise, stop and set $\hat{\mathcal{U}} = \hat{\mathcal{U}}(t-1)$

With MMV when all the measurement vectors are sampled via the same projection matrix as in (6), the support of the sparse signal can be estimated, using simultaneous OMP (S-OMP) algorithm as presented in [20]. An extension of S-OMP to estimate the common sparsity pattern with MMV considering different projection matrices as in (5) is presented in [3] which is summarized in Algorithm 2.

To perform S-OMP with the MMV model in (5) or (6),

a high communication burden to make all the information available at a central processing unit is required. In the following sections, we investigate several schemes to perform sparsity pattern recovery with MMV taking the communication constraints into account.

III. SPARSITY PATTERN RECOVERY WITH MULTIPLE ACCESS CHANNELS (MACS)

The use of a shared MAC for sending information to the fusion center is attractive in many bandwidth constrained communication networks [17], [18]. In this section, we explore the applicability of the MAC transmission scheme for common sparsity pattern recovery. With MAC, the received signal vector at the fusion center after M independent transmissions is given by,

$$\mathbf{z} = \sum_{l=0}^{L-1} \mathbf{y}_l = \sum_{l=0}^{L-1} \mathbf{B}_l \mathbf{s}_l + \mathbf{w}$$
(7)

where $\mathbf{w} = \sum_{l=0}^{L-1} \mathbf{v}_l$. It is further noted that we consider noise free observations at the fusion center to make the analysis simpler. With the MAC output (7), the fusion center does not have access to individual observation vectors acquired at each node. Recovering the set of sparse vectors \mathbf{s}_l 's for $l - 0, \dots, L - 1$ from (7) becomes a data separation problem [30] which can be equivalently represented by the following underdetermined linear system:

$$\mathbf{z} = \begin{bmatrix} \mathbf{B}_0 | \mathbf{B}_1 | \cdots | \mathbf{B}_{L-1} \end{bmatrix} \begin{bmatrix} \mathbf{s}_0 \\ \vdots \\ \mathbf{s}_{L-1} \end{bmatrix} + \mathbf{w}$$
(8)

where the projection matrix $[\mathbf{B}_0|\mathbf{B}_1|\cdots|\mathbf{B}_{L-1}]$ in (8) is $M \times NL$. We can cast the problem of sparse signal recovery based on (8) as a problem of block sparse signal recovery when all the signals \mathbf{s}_l 's for $l = 0, \dots, L-1$ share the same support as discussed below. Let \mathbf{b}_{li} be the *i*-th column vector of the matrix \mathbf{B}_l for $i = 0, 1, \dots, N-1$ and $l = 0, 1, \dots, L-1$. Then \mathbf{B}_l can be expressed by concatenating the column vectors as $\mathbf{B}_l = [\mathbf{b}_{l0} \cdots \mathbf{b}_{l(N-1)}]$. Further, let $\mathbf{s}_l(i)$ denote the *i*-th element of the vector \mathbf{s}_l for $i = 0, 1, \dots, N-1$ and $l = 0, 1, \dots, L-1$. Then, we can express $\sum_{l=0}^{L-1} \mathbf{B}_l \mathbf{s}_l$ as

$$\sum_{l=0}^{L-1} \mathbf{B}_l \mathbf{s}_l = (\mathbf{b}_{00} \cdots \mathbf{b}_{(L-1)0} | \mathbf{b}_{01} \cdots \mathbf{b}_{(L-1)1} |$$
$$\cdots | \mathbf{b}_{0(N-1)} \cdots \mathbf{b}_{(L-1)(N-1)} \rangle_{M \times LN}$$
$$[\mathbf{s}_0(0) \cdots \mathbf{s}_{L-1}(0) | \mathbf{s}_0(1) \cdots \mathbf{s}_{L-1}(1) |$$
$$\cdots | \mathbf{s}_0(N-1) \cdots \mathbf{s}_{L-1}(N-1)]_{LN \times 1}^T$$
$$= \mathbf{D} \mathbf{c} \qquad (9)$$

where $\mathbf{D} = (\mathbf{d}_0 | \mathbf{d}_1 | \cdots | \mathbf{d}_{N-1})$ is a $M \times LN$ matrix where $\mathbf{d}_j = (\mathbf{b}_{0j} \cdots \mathbf{b}_{(L-1)j})$ for $j = 0, 1, \cdots, N-1$ and $\mathbf{c} = [\mathbf{r}_0 | \mathbf{r}_1 | \cdots | \mathbf{r}_{N-1}]^T$ is a $LN \times 1$ vector where $\mathbf{r}_j = [\mathbf{s}_0(j) \cdots \mathbf{s}_{L-1}(j)]$ for $j = 0, 1, \cdots, N-1$. Since the sparse vectors \mathbf{s}_l 's for $l = 0, 1, \cdots, L-1$, share the common support, \mathbf{c} can be treated as a block sparse vector with Nblocks each of length of L in which only k blocks are non zero. Then the MAC output in (8) can be expressed as,

$$z = Dc + w \tag{10}$$

where \mathbf{c} is a block sparse vector with N blocks of size L each. The capability of recovering \mathbf{c} from (10) depends on the properties of the matrix \mathbf{D} .

In the case where the projection matrix A_l at the *l*-th node contains elements drawn from a Gaussian ensemble which are independent over $l = 0, 1, \dots, L - 1$, the elements of **D** become independent realizations of a Gaussian ensemble. When **D** contains Gaussian entries, it has been shown in [31] that a block sparse vector of the form (10) can be recovered reliably if the following condition is satisfied when the noise power is negligible.

Definition 1 ([31]). Block RIP: The matrix **D** satisfies block RIP with parameter $0 < \delta_k < 1$ if for every $\mathbf{c} \in \mathbb{R}^N$ that is block k-sparse we have that,

$$(1 - \delta_k) ||\mathbf{c}||_2^2 \le ||\mathbf{D}\mathbf{c}||_2^2 \le (1 + \delta_k) ||\mathbf{c}||_2^2.$$
(11)

Proposition 1 ([31]). When the matrix **D** satisfies block RIP conditions, the block sparse signal can be reliably recovered with high probability when

$$M \ge \frac{36}{7\delta_0} \left(\ln \left(2 \binom{N}{k} \right) + kL \ln \left(\frac{12}{\delta_0} \right) + t \right)$$
(12)

for $0 < \delta_0 < 1$ and t > 0.

It is seen that, the number of compressive measurements required per node, M, for reliable sparse signal recovery based on the MAC output is proportional to L. Thus, as the network size L increases, M should be proportionally increased to recover the set of sparse signals. However, when the goal is not to recover the complete sparse signals, but only the common sparsity pattern, in the following we show that in the case where $A_l = A$ for $l = 0, \dots, L - 1$, the sparsity pattern can be recovered using the MAC output without increasing M. Further, under certain conditions as discussed below, the sparsity pattern recovery with the MAC output can be performed with performance that is comparable to the case when all the observations are available at the fusion center (via PAC).

When $\mathbf{A}_l = \mathbf{A}$ for $l = 0, 1, \dots, L - 1$, (7) reduces to

$$\mathbf{z} = \mathbf{B}\bar{\mathbf{s}} + \mathbf{w} \tag{13}$$

where $\bar{\mathbf{s}} = \sum_{l=0}^{L-1} \mathbf{s}_l$. Since all the signals share a common sparsity pattern, \bar{s} is also a sparse vector with the same sparsity pattern. Thus, the problem of joint sparsity pattern recovery reduces to finding the sparsity pattern of \bar{s} based on (13) which is the standard model as considered in CS. Even though, when individual vectors are sparse with significant non zero coefficients, coefficients corresponding to non zero locations of $\bar{\mathbf{s}}$ can be negligible under certain cases. For example, assume that the elements of non zero coefficients of s_l are independent realizations of a random variable with mean zero. Then, from the central limit theorem, coefficients at non zero locations of $\bar{\mathbf{s}}$ may reach zero as L becomes large resulting in the vector \bar{s} with all zeros. However, when the amplitudes of all $\mathbf{s}_l(k)$'s (for $l = 0, \dots, L-1$) for a given k have the same sign, \bar{s} becomes a sparse vector with significant non zero coefficients. For example, consider the case where multiple nodes observing the same Fourier sparse signal $\tilde{\mathbf{x}} = \boldsymbol{\Phi}\boldsymbol{\theta}$

with different attenuation factors so that we may express, $\mathbf{s}_l = \mathbf{H}_l \boldsymbol{\theta}$ for $l = 0, 1, \dots, L - 1$ where \mathbf{H}_l is a diagonal matrix with positive diagonal elements that correspond to attenuation factors. Since, elements in \mathbf{H}_l are assumed to be positive, each non zero element in \mathbf{s}_l has the same sign as the corresponding non zero element in $\boldsymbol{\theta}$. Thus, in such cases, $\bar{\mathbf{s}}$ becomes a sparse vector with significant nonzero coefficients. In the following, we examine the conditions under which (13) provides performance that is comparable to the case where all the observations are available at a fusion center (6).

A. Necessary conditions for support recovery based on MAC output with any classification rule

To compare the performance with MAC and PAC outputs, we find a lower bound on the probability of error in support recovery irrespective of the recovery scheme used for support identification. Since we assume that there are k nonzero elements in each signal s_l for $l = 0, \dots, L - 1$, there are $\Pi = {N \choose k}$ possible support sets. Selecting the correct support set can be formulated as a multiple hypothesis testing problem with Π hypotheses. Based on Fano's inequality, the probability of error of a multiple hypothesis testing problem with any decision rule is lower bounded by [32]

$$P_e \ge 1 - \frac{\Xi + \log 2}{\log \Pi} \tag{14}$$

where Ξ denotes the average Kullback Leibler (KL) distance between any pair of densities. Let $\mathcal{D}_M(p_m(\mathbf{z})||p_n(\mathbf{z}))$ denote the KL distance between pdfs of the MAC output (13) when the sparse supports are \mathcal{U}_m and \mathcal{U}_n respectively, where m, n = $0, \dots, \Pi$. Thus, $\Xi_{MAC} = \frac{1}{\Pi^2} \sum_{m,n} \mathcal{D}_M(p_m(\mathbf{z})||p_n(\mathbf{z}))$.

When the projection matrix \mathbf{A} is given, we have $\mathbf{z}|\mathbf{B}\bar{\mathbf{s}} \sim \mathcal{N}(\mathbf{B}\bar{\mathbf{s}}, \sigma_v^2 L \mathbf{I}_M)$ where $\mathbf{x} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ denotes that the random vector \mathbf{x} has a joint Gaussian distribution with mean $\boldsymbol{\mu}$ and the covariance matrix $\boldsymbol{\Sigma}$. Then we have,

$$\mathcal{D}_M(p_m(\mathbf{z})||p_n(\mathbf{z})) = \frac{1}{2\sigma_v^2 L} \left\| \sum_{l=0}^{L-1} (\mathbf{B}_{\mathcal{U}_n} \mathbf{s}_{l,\mathcal{U}_n} - \mathbf{B}_{\mathcal{U}_m} \mathbf{s}_{l,\mathcal{U}_m}) \right\|_2^2$$

for $m, n = 0, 1, \dots, \Pi$ where $||.||_p$ denotes the l_p norm, $\mathbf{B}_{\mathcal{U}_n}$ is a $M \times k$ submatrix of \mathbf{B} so that $\mathbf{B}_{\mathcal{U}_n}$ contains the columns of \mathbf{B} corresponding to the column indices in the support set \mathcal{U}_n for $n = 0, 1, \dots, T - 1$. We denote $\mathbf{s}_{l,\mathcal{U}_n}$ to be the $k \times 1$ non sparse vector corresponding to the support \mathcal{U}_n .

Similarly, let $\mathcal{D}_P(p_m(\mathbf{Y})||p_n(\mathbf{Y}))$ be the KL distance between any two pdfs with the PAC output (6). Then, we have $\Xi_{PAC} = \frac{1}{\Pi^2} \sum_{m,n} \mathcal{D}_P(p_m(\mathbf{Y})||p_n(\mathbf{Y}))$. With the PAC output, the observation matrix in (6) has a matrix variate normal distribution conditioned on **BS** with mean **BS** and covariance matrix $\sigma_v^2 \mathbf{I}_M \otimes \mathbf{I}_L$, denoted by, $\mathbf{Y}|\mathbf{BS} \sim \mathcal{MN}_{M,L}(\mathbf{BS}, \sigma_v^2 \mathbf{I}_M \otimes \mathbf{I}_L)$ where \otimes denotes the Kronecker product. The corresponding KL distance between pdfs when the support sets are \mathcal{U}_m and \mathcal{U}_n , respectively, with the PAC output is given by,

$$\mathcal{D}_P(p_m(\mathbf{Y})||p_n(\mathbf{Y})) = \frac{1}{2\sigma_v^2} \sum_{l=0}^{L-1} \left\| \left(\mathbf{B}_{\mathcal{U}_n} \mathbf{s}_{l,\mathcal{U}_n} - \mathbf{B}_{\mathcal{U}_m} \mathbf{s}_{l,\mathcal{U}_m} \right) \right\|_2^2$$

for $m, n = 0, 1, \dots, \Pi$.

Lemma 1. The average KL distance between any pair of densities with MAC and PAC outputs have the following relationship

$$\Xi_{MAC} \le \Xi_{PAC} \tag{15}$$

with the equality only if \mathbf{s}_l 's are the same for all $l = 0, \dots, L-1$.

Proof: The proof follows from Proposition 2 given below.

Proposition 2. For given U_m and U_n , we have,

$$\frac{1}{L} \left\| \sum_{l=0}^{L-1} (\mathbf{B}_{\mathcal{U}_{n}} \mathbf{s}_{l,\mathcal{U}_{n}} - \mathbf{B}_{\mathcal{U}_{m}} \mathbf{s}_{l,\mathcal{U}_{m}}) \right\|_{2}^{2} \leq \sum_{l=0}^{L-1} \left\| (\mathbf{B}_{\mathcal{U}_{n}} \mathbf{s}_{l,\mathcal{U}_{n}} - \mathbf{B}_{\mathcal{U}_{m}} \mathbf{s}_{l,\mathcal{U}_{m}}) \right\|_{2}^{2} \qquad (16)$$

with equality only when all \mathbf{s}_l 's for $l = 0, \dots, L-1$ are equal.

Proof: When $\mathbf{s}_l = \mathbf{s}$ for $l = 0, \dots, L-1$, it is obvious that the right and left hand sides of (16) are equal. To prove the result when \mathbf{s}_l 's for $l = 0, 1, \dots, L-1$ are different, let $\beta_{m,n}(l) = \mathbf{B}_{\mathcal{U}_n}\mathbf{s}_{l,\mathcal{U}_n} - \mathbf{B}_{\mathcal{U}_m}\mathbf{s}_{l,\mathcal{U}_m}$ be a length-M vector for given \mathcal{U}_m and \mathcal{U}_n and for $l = 0, \dots, L-1$. Then we can write,

$$\Delta = \frac{1}{L} \left\| \sum_{l=0}^{L-1} (\mathbf{B}_{\mathcal{U}_n} \mathbf{s}_{l,\mathcal{U}_n} - \mathbf{B}_{\mathcal{U}_m} \mathbf{s}_{l,\mathcal{U}_m}) \right\|_2^2$$
$$- \sum_{l=0}^{L-1} \| (\mathbf{B}_{\mathcal{U}_n} \mathbf{s}_{l,\mathcal{U}_n} - \mathbf{B}_{\mathcal{U}_m} \mathbf{s}_{l,\mathcal{U}_m}) \|_2^2$$
$$= \frac{1}{L} \left(\left\| \sum_{l=0}^{L-1} \beta_{m,n}(l) \right\|_2^2 - L \sum_{l=0}^{L-1} \| \beta_{m,n}(l) \|_2^2 \right). (17)$$

From Cauchy Schwarz inequality, we have,

$$\left\|\sum_{l=0}^{L-1} \beta_{m,n}(l)\right\|_{2}^{2} \leq \left(\sum_{l=0}^{L-1} \|\beta_{m,n}(l)\|_{2}\right)^{2}$$
$$= \sum_{l=0}^{L-1} \|\beta_{m,n}(l)\|_{2}^{2}$$
$$+ \sum_{k \neq l} \|\beta_{m,n}(l)\|_{2} \|\beta_{m,n}(k)\|_{2}.$$
(18)

Thus, we can write Δ in (17) as,

$$\Delta \leq \frac{-1}{L} \left((L-1) \sum_{l=0}^{L-1} \|\beta_{m,n}(l)\|_{2}^{2} + \sum_{k \neq l} \|\beta_{m,n}(l)\|_{2} \|\beta_{m,n}(k)\|_{2} \right)$$
$$= \frac{-1}{L} \left(\sum_{l \neq k, l < k} (\|\beta_{m,n}(l)\|_{2} - \|\beta_{m,n}(k)\|_{2})^{2} \right) \leq 0$$

which completes the proof.

From Lemma 1, it can be seen that, when each node samples via the same projection matrix and the sparse signals s_l 's are not significantly different from each other, we may approximate $\Xi_{MAC} \approx \Xi_{PAC}$. Then, the lower bound on the probability of error (14) in sparsity pattern recovery with the MAC output becomes very close to that with access to all observation vectors. From numerical results, we observe a comparable performance as long as $s_l(k)$'s for all $l = 0, \dots, L-1$ for given k have the same sign even though they are significantly different from each other in amplitude.

1) Minimum number of measurements with Gaussian matrices: When the entries of the measurement matrix **A** are drawn from a Gaussian ensemble with mean zero and variance 1, the author in [33] derived the conditions under which the probability of error in (14) is bounded away from zero with any recovery technique with a single measurement vector. The main difference in the sparsity pattern recovery with the MAC output (13), and that with the SMV appears in terms of the SNR. Based on the results in [33], (13) is capable of recovering the sparsity pattern with any recovery technique if,

$$M > \max\left\{\frac{\log\binom{N}{k}}{8k\frac{\bar{s}_{\min}^2}{L\sigma_v^2}}, \frac{\log(N-k)}{4\frac{\bar{s}_{\min}^2}{L\sigma_v^2}}\right\}$$
(19)

where $\bar{s}_{\min} = \min_{\substack{j=0,1,\cdots,N-1 \\ j=0,1,\cdots,N-1}} \bar{\mathbf{s}}(j)$. With the assumption that $\mathbf{s}_l(k)$'s for all $l = 0, \cdots, L-1$ for a given k have the same sign, we will get $\bar{s}_{\min} = L \min_{\substack{l,j \\ l,j}} \{\mathbf{s}_l(j)\}$ where the minimization is over $l = 0, \cdots, L-1$ and $j = 0, \cdots, N-1$. Define the minimum component SNR to be $\gamma_{c,\min} = \frac{\left(\min_{\substack{l,j \\ \sigma_v^2}} \mathbf{s}_l(j)\right)^2}{\sigma_v^2}$. Then, the lower bound on M in (19) can be written as,

$$M > \max\left\{\frac{\log\binom{N}{k}}{8kL\gamma_{c,\min}}, \frac{\log(N-k)}{4L\gamma_{c,\min}}\right\}.$$
 (20)

Thus, (20) gives the necessary conditions for the sparsity pattern recovery based on the MAC output (13).

2) Sparsity pattern recovery based on (13) via OMP: Based on the MAC output in (13), the standard OMP as in Algorithm 1 can be used to estimate the sparse support by replacing y by z.

Although the MAC transmission scheme saves communication bandwidth compared to PAC, and provides comparable performance in terms of common sparsity pattern recovery under certain conditions, its use is still restrictive due to several reasons. (i). It still requires the knowledge of the measurement matrices at the fusion center which involves a certain communication overhead. (ii). Since the fusion center does not have access to individual observation vectors but only to their linear combination, the capability of recovering the common sparsity pattern is limited by the nature of the sparse signals. (iv). Extension to estimate the amplitudes of individual observations is not straight forward since individual measurements are not accessible.

In certain communication networks, it is desirable for each node to have an estimate of the sparsity pattern of the signal with less communication overhead in contrast to centralized solutions. Thus, in the following, we consider decentralized algorithms for sparse support recovery where there is no central fusion center to make the final estimation.

IV. DECENTRALIZED SPARSITY PATTERN RECOVERY VIA OMP

In a naive approach to implement S-OMP in a distributed manner, each node needs to have the knowledge of the measurement vectors and the projection matrices at every other node, which requires high communication burden and usage of a large memory at distributed nodes. A simple approach to minimize the communication overhead compared to employing S-OMP at each node is to estimate the common sparsity pattern independently at each node based on only its own measurement vector using standard OMP and exchange the estimated sparsity pattern among the nodes to get a fused estimate. Although this scheme (we call this scheme as D-OMP in subsequent analysis) requires each node to transmit only its estimated support set for fusion, it lacks accuracy especially when the number of measurements collected at each node is small. The two proposed decentralized algorithms in this paper can be considered as an intermediate approach between these two extreme cases.

When the standard OMP algorithm is performed at a given node to estimate the support of sparse signals with only k non zero elements, at each iteration, there are N-k possible incorrect indices that can be selected. The probability of selecting an incorrect index at each iteration increases as the number of measurements at each node, M, decreases. In the following, we modify the standard OMP to exploit collaboration and fusion among distributed nodes in a decentralized manner to reduce the probability of selecting an incorrect index at each iteration compared to that with the standard OMP.

We introduce the following additional notations. Let $\mathcal{I} = (\mathcal{G}, \Upsilon)$ be an undirected connected graph with node set $\mathcal{G} = \{0, 1, \dots, L-1\}$ and edge set Υ , where each edge $(i, j) \in \Upsilon$ is an undirected pair of distinct nodes within the one-hop communication range. Define $\mathcal{G}_l = \{k | (l, k) \in \Upsilon\}$ to be the set containing the indices of neighboring nodes of the *l*-th node. As defined in Section II, let \mathcal{U} be the support set which contains the indices of non zero coefficients of the sparse signals and $\hat{\mathcal{U}}_l$ be the estimated support set at the *l*-th node for $l = 0, 1, \dots, L - 1$. B(\mathcal{A}) denotes the submatrix of B which has columns of B corresponding to the elements in the set \mathcal{A} for $\mathcal{A} \subset \{0, 1, \dots, N-1\}$. We use |x| to denote the absolute value of a scalar x (it is noted that we use the same notation to denote the cardinality of a set and it should be clear by the context).

A. Algorithm development and strategies: DC-OMP 1

In the proposed distributed and collaborative OMP algorithm 1 (DC-OMP 1) stated in Algorithm 3, the goal is to improve the performance by inserting an index fusion stage to the standard OMP algorithm at each iteration. More specifically, the following two phases are performed during each iteration t.

1) **Phase 1 :** In Phase I, each node estimates an index based on only its own observations similar to index selection stage in standard OMP (step 2 in Algorithm 1).

2) **Phase II:** Once an index $\lambda_l(t)$ is estimated in Phase I, it is exchanged among the neighborhood \mathcal{G}_l . Subsequently, the *l*-th node will have estimated indices at all the nodes in its neighborhood \mathcal{G}_l . Via fusion, each node selects a set of indices (from $|\mathcal{G}_l \cup l|$ number of indices) such that most of the nodes agree upon on the set of indices as detailed next.

Algorithm 3 DC-OMP 1 for joint sparsity pattern recovery: Executed at *l*-th node, $\hat{\mathcal{U}}_l$ contains the estimated indices of the support

Inputs: $\mathbf{y}_l, \mathbf{B}_l, \mathcal{G}_l, k$

- 1) Initialize t = 1, $\hat{\mathcal{U}}_l(0) = \emptyset$, residual vector $\mathbf{r}_{l,0} = \mathbf{y}_l$ **Phase I**
- 2) Find the index $\lambda_l(t)$ such that

$$\lambda_l(t) = \underset{\omega=0,\cdots,N-1}{\arg} \max_{\substack{|\langle \mathbf{r}_{l,t-1}, \mathbf{b}_{l,\omega} \rangle|}} |\langle \mathbf{1} \rangle$$
(21)

Phase II

- 3) Local communication
 - a) Transmit $\lambda_l(t)$ to \mathcal{G}_l
 - b) Receive $\lambda_i(t), i \in \mathcal{G}_l$
- 4) Update the index set $\lambda_l^*(t)$, as discussed in subsection IV-A3
- 5) Set $\hat{\mathcal{U}}_l(t) = \hat{\mathcal{U}}_l(t-1) \cup \{\lambda_l^*(t)\}$ and $l_t = |\hat{\mathcal{U}}_l(t)|$
- 6) Compute the projection operator $\mathbf{P}_{l}(t) = \mathbf{B}_{l}(\hat{\mathcal{U}}_{l}(t)) \left(\mathbf{B}_{l}(\hat{\mathcal{U}}_{l}(t))^{T} \mathbf{B}_{l}(\hat{\mathcal{U}}_{l}(t))\right)^{-1} \mathbf{B}_{l}(\hat{\mathcal{U}}_{l}(t))^{T}.$ Update the residual vector: $\mathbf{r}_{l,t} = (\mathbf{I} - \mathbf{P}_{l}(t))\mathbf{y}_{l}$
- 7) Increment t = t+1 and go to step 2 if $l_t \le k$, otherwise, stop and set $\hat{\mathcal{U}}_l = \hat{\mathcal{U}}_l(t-1)$

3) Performing step 4 in Algorithm 3: For small networks (e.g. cognitive radio networks with few nodes), it is reasonable to assume that $\mathcal{G}_l \cup l = \mathcal{G}$. When $\mathcal{G}_l \cup l = \mathcal{G}$, each node has the estimated indices at step 2 in Algorithm 3. Let $\alpha(t) = \{\lambda_l(t)\}_{l=0}^{L-1}$. If there are any two indices such that $\lambda_l(t) = \lambda_m(t)$ for $l \neq m$, then it is more likely that the corresponding index belongs to the true support. This is because, the probability that two nodes estimate the same index which does not belong to the true support by performing step 2 is negligible especially when N-k becomes large. The updated set of indices $\lambda_l^*(t)$ at the *t*-th iteration is computed as below;

- If there are indices in $\alpha(t)$ with more occurrences, such than one indices are put in the set $\lambda_{I}^{*}(t)$ (such that $\lambda_i^*(t)$ = {set of indices in $\alpha(t)$ which occur more than once}.
- If there is no index obtained from step 2 that agrees with one or more nodes so that all L indices in $\alpha(t)$ are distinct, then select one index from $\alpha(t)$ randomly and put in $\lambda_l^*(t)$. In this case, to avoid the same index being selected at subsequent iterations, we force all nodes to use the same index.

Next, consider the case where $\mathcal{G}_l \cup l \subset \mathcal{G}$. Then, at the *l*-th node, we have $\alpha_l(t) = \{\lambda_l(t), \{\lambda_i(t)\}_{i \in \mathcal{G}_l}\}$. If there are indices in $\alpha_l(t)$ which have more than one occurrences, those indices are put in $\alpha_l^*(t)$. Otherwise, if all indices in $\alpha_l(t)$ are distinct, we set $\alpha_l^*(t) = \lambda_l(t)$. When $\mathcal{G}_l \cup l \subset \mathcal{G}$, since

the *l*-th node does not receive the estimated indices from the whole network at a given iteration, different nodes may agree upon different sets of indices at a given iteration. When two neighboring nodes agree upon two different sets of indices during the t-th iteration, there is a possibility that one node selects the same index at a later iteration beyond t. To avoid the *l*-th node selecting the same index twice, we perform an additional step in updating $\lambda_{l}^{*}(t)$ compared to the case where $\mathcal{G}_l \cup l = \mathcal{G}$; i.e., check whether at least one index in $\alpha_l^*(t)$ belongs to $\hat{\mathcal{U}}_l(t-1)$. More specifically, if $\alpha_l^*(t) \cup \hat{\mathcal{U}}_l(t-1) =$ $\mathcal{U}_l(t-1)$, then set $\lambda_l^*(t) = \lambda_l(t)$, otherwise update $\lambda_l^*(t) =$ $\alpha_l^*(t)$. It is noted that, DC-OMP 1 with $\mathcal{G}_l \cup l \subset \mathcal{G}$ in the worst case (i.e. all the indices in $\alpha_l(t)$ are distinct for all t) coincides with the standard OMP. Moreover, when $|\mathcal{G}_l \cup l| > k$, it is more likely that the vector $\alpha_l(t)$ has at least one set of two (or more) indices with the same value, thus, $\lambda_l^*(t)$ is updated appropriately most of the time at each iteration.

B. Algorithm development and strategies: DC-OMP 2

The proposed distributed and collaborative OMP algorithm 2 (DC-OMP 2) is presented in Algorithm 4. Compared to DC-OMP 1, DC-OMP 2 has a measurement fusion stage in Phase I in addition to the index fusion stage in Phase II.

In the case where all the observation vectors and projection matrices are available at each node, all the nodes in the network can perform S-OMP as presented in Algorithm 2. To perform S-OMP at each node, the quantity $f_{\omega} = \sum_{l=0}^{L-1} |\langle \mathbf{r}_{l,t-1}, \mathbf{b}_{l,\omega} \rangle|$ for each $\omega = 0, \dots, N-1$ needs to be computed at each iteration where $\mathbf{r}_{l,t-1}$ and $\mathbf{b}_{l,\omega}$ are as defined in Algorithm 2. In the first phase of DC-OMP 2, an approximation to this quantity is computed at a given node via only one-hop *local communication*.

1) **Phase 1 :** At the *t*-th iteration, *l*-th node computes

$$f_{l,\omega}(t) = |\langle \mathbf{r}_{l,t-1}, \mathbf{b}_{l,\omega} \rangle|$$
(22)

for $\omega = 0, 1, \dots, N - 1$ and exchanges it with the onehop neighborhood \mathcal{G}_l . Similarly, every node receives such information from its one-hop neighbors so that the *l*-th node computes the quantity

$$g_{l,\omega}(t) = f_{l,\omega}(t) + \sum_{j \in \mathcal{G}_l} f_{j,\omega}(t)$$
(23)

for $\omega = 0, 1, \dots, N-1$ and $l = 0, 1, \dots, L-1$. Then an estimate for the index in the support set, $\lambda_l(t)$, at the *l*-th node is computed as $\lambda_l(t) = \underset{\omega=0,\dots,N-1}{\arg \max} g_{l,\omega}(t)$ as given in Step 3 of Algorithm 4. This *'initial estimate'*, $\lambda_l(t)$ at the *t*-th iteration will be used to get an updated estimate in the next phase.

2) **Phase II:** In Phase II, as in DC-OMP 1, an updated index (or a set of indices) for the true support with higher accuracy compared to the one that is computed in Phase I, is obtained by performing collaboration and fusion via global communication. More specifically, each node transmits its estimated index to the whole network so that every node in the network receives all the estimated indices denoted by, as before, $\alpha(t) = {\lambda_l(t)}_{l=0}^{L-1}$ from step 3 of Algorithm 4.

Algorithm 4 DC-OMP 2 for joint sparsity pattern recovery; Executed at *l*-th node, $\hat{\mathcal{U}}_l$ contains the estimated indices of the support

Inputs: $\mathbf{y}_l, \mathbf{B}_l, \mathcal{G}_l, \mathcal{G}, k, L$

- 1) Initialize t = 1, $\hat{\mathcal{U}}_l(0) = \emptyset$, residual vector $\mathbf{r}_{l,0} = \mathbf{y}_l$ Phase I
- 2) Local Communication:
 - a) Compute $f_{l,\omega}(t) = |\langle \mathbf{r}_{l,t-1}, \mathbf{b}_{l,\omega} \rangle|$ for $\omega =$ $0, 1, \cdots, N-1$ and transmit to neighborhood \mathcal{G}_l b) Receive $f_{j,\omega}(t)$ from \mathcal{G}_l to com $g_{l,\omega}(t) = f_{l,\omega}(t) + \sum_{j \in \mathcal{G}_l} f_{j,\omega}(t)$ compute for $\omega = 0, 1, \cdots, N-1$

3) Find
$$\lambda_l(t) = \underset{\omega=0,\cdots,N-1}{\arg \max} g_{l,\omega}(t)$$

Phase II

- 4) Global Communication:
 - a) Transmit $\lambda_l(t)$ to \mathcal{G}
 - b) Receive $\lambda_i(t), i \in \mathcal{G} \setminus l$
- 5) Find the set $\lambda_{l}^{*}(t)$ as in Subsection IV-B3
- 6) Update the set of estimated indices: $\hat{\mathcal{U}}_l(t) = \hat{\mathcal{U}}_l(t-1) \cup$ $\{\lambda_l^*(t)\}, \text{ set } l_t = |\mathcal{U}_l(t)|$
- 7) Compute the orthogonal projection operator: $\mathbf{P}_l(t) =$ $\mathbf{B}_{l}(\hat{\mathcal{U}}_{l}(t)) \left(\mathbf{B}_{l}(\hat{\mathcal{U}}_{l}(t))^{T} \mathbf{B}_{l}(\hat{\mathcal{U}}_{l}(t)) \right)^{-1} \mathbf{B}_{l}(\hat{\mathcal{U}}_{l}(t))^{T}$ 8) Update the residual: $\mathbf{r}_{l,t} = (\mathbf{I} - \mathbf{P}_{l}(t))\mathbf{y}_{l}$
- 9) Increment t = t+1 and go to step 2 if $l_t \leq k$, otherwise, stop and set $\hat{\mathcal{U}}_l = \hat{\mathcal{U}}_l(t-1)$

3) Index fusion in Phase II : The index fusion stage is performed as stated in Subsection IV-A3 for $\mathcal{G}_l \cup l = \mathcal{G}$. It is noted that when L > k, it is more likely that the vector $\alpha(t)$ has at least one set of two (or more) indices with the same value, thus, $\lambda_l^*(t)$ is updated appropriately most of the time at each iteration. Since each node has the indices received from all the other nodes in the network, every node has the same estimate for \mathcal{U}_l when the algorithm terminates. Further, since more than one index can be selected for the set $\lambda_{l}^{*}(t)$ at a given iteration, the algorithm can be terminated in less than kiterations. In the index fusion stage, the reason for imposing global communication is that after performing index fusion via global communication, all nodes in the network have the same estimated index set. Thus, the residual computed at step 8 at a given node corresponds to the same remaining column indices of the dictionary at every node.

C. Performance analysis

1) DC-OMP 1: The main difference between DC-OMP 1 and the standard OMP is the additional index fusion stage in Phase II in Algorithm 3. In Phase II in Algorithm 3, two things can happen at a given iteration t. (i). $\lambda_l^*(t) = \lambda_l(t)$ if there are no indices that occur more than once in $\alpha_l(t)$ or the agreed upon indices are already in $\hat{\mathcal{U}}_l(t-1)$. Thus, in this case $Pr(\lambda_l^{\star}(t) \in \mathcal{U}) = Pr(\lambda_l(t) \in \mathcal{U})$. In the worst case where $\lambda_l^*(t) = \lambda_l(t)$ for all t, DC-OMP 1 in Algorithm 3 coincides

with the standard OMP, and the convergence properties are the same as the standard OMP. (ii). There can be two or more indices with the same value in $\alpha_l(t)$ and in general there can be multiple such occurrences. In particular, when the neighborhood size at each node exceeds the sparsity index k, $\alpha_l(t)$ contains at least two indices with the same value as discussed below.

Let $M_1(\delta)$ be the number of measurements required by the standard OMP to estimate the sparsity pattern with probability exceeding $1 - \delta$ for $0 < \delta < 1$ with a given projection matrix. In particular, when the projection matrices contain realizations of iid Gaussian random variables, $M_1(\delta) = ck \log(N/\delta)$ for $\delta \in (0, 0.36)$ and constant c [19]. It is noted that we do not restrict our analysis to only the Gaussian case; it is applicable more generally. Also, in the following analysis, we assume $\mathcal{G}_l \cup l = \mathcal{G}$ in DC-OMP 1.

Lemma 2. Assume L > k. Let $M_1(\delta)$ be the number of measurements required by the standard OMP to recover the sparsity pattern correctly with probability exceeding $1 - \delta$. Then, when $M \ge M_1(\delta)$, DC-OMP 1 at a given node recovers the sparsity pattern with probability exceeding $1 - \frac{1}{(N-2k)}\delta^2$.

Proof: When the index fusion stage is ignored, DC-OMP 1 coincides with the standard OMP. Thus, when $M \ge M_1(\delta)$ we have that

$$Pr(\lambda_l(t) \in \mathcal{U}) \ge 1 - \delta.$$
 (24)

for $0 < \delta < 1$ where $\lambda_l(t)$ is estimated in step 2 in DC-OMP 1. Let u_0, u_1, \dots, u_{k-1} denote the k indices in the true support \mathcal{U} and u_k, \cdots, u_{N-1} denote the N-k indices in \mathcal{U}^c . Thus, the *l*-th node selects one index from the set $\{u_0, \dots, u_{k-1}\}$ at the t-th iteration with probability exceeding $1-\delta$. Since each node in \mathcal{G} selects an index from $\{u_0, \dots, u_{k-1}\}$ with probability exceeding $1 - \delta$ by performing step 2 in Algorithm 3 when $M \geq M_1(\delta)$, there should be at least two nodes which estimate the same index at a given iteration t when L > k. When there are at least two indices with the same value in $\alpha_l(t)$, we set $\lambda_l^*(t) = \alpha_l^*(t)$ where $\alpha_l^*(t)$ contains all the indices which have more than one occurrence in $\alpha_l(t)$.

Let $p_{l,t} = Pr(\lambda_l(t) \in \mathcal{U})$ which is determined by the statistical properties of the projection matrix and the noise at the *l*-th node for $l = 0, \dots, L-1$. Since no index is selected twice in Algorithm 3, at the *t*-th iteration, we have,

$$Pr(\lambda_{l}(t) \in \mathcal{U}^{c})$$

$$= \sum_{u_{m} \in \mathcal{U}^{c} \setminus \hat{\mathcal{U}}_{l}(t-1)} Pr(\lambda_{l}(t) = u_{m})$$

$$\geq |\mathcal{U}^{c} \setminus \hat{\mathcal{U}}_{l}(t-1)| \min_{u_{m} \in \mathcal{U}^{c} \setminus \hat{\mathcal{U}}_{l}(t-1)} Prob(\lambda_{l}(t) = u_{m})$$

$$\geq (N-2k) \min_{u_{m} \in \mathcal{U}^{c} \setminus \hat{\mathcal{U}}_{l}(t-1)} Prob(\lambda_{l}(t) = u_{m}) \quad (25)$$

where the last inequality is because $|\mathcal{U}^c \setminus \hat{\mathcal{U}}_l(t-1)| \ge (N-2k)$. Since $p_{l,t} = Pr(\lambda_l(t) \in \mathcal{U})$ so that $Pr(\lambda_l(t) \in \mathcal{U}^c) = 1 - p_{l,t}$ and (25), we have,

$$\min_{u_m \in \mathcal{U}^c \setminus \hat{\mathcal{U}}_l(t-1)} \operatorname{Prob}(\lambda_l(t) = u_m) \le \frac{1}{N - 2k} (1 - p_{l,t}) \quad (26)$$

It is noted that when the absolute values of non significant coefficients of the sparse signals are almost zero, we can approximate $\min_{\substack{u_m \in \mathcal{U}^c \setminus \hat{\mathcal{U}}_l(t-1) \\ u_m \in \mathcal{U}^c \setminus \hat{\mathcal{U}}_l(t-1)}} Prob(\lambda_l(t) = u_m) \approx (1 - \nu_l) \max_{\substack{u_m \in \mathcal{U}^c \setminus \hat{\mathcal{U}}_l(t-1) \\ \text{Then we have,}}} Prob(\lambda_l(t) = u_m) \text{ for some } 0 \leq \nu_l \ll 1.$

$$\max_{u_m \in \mathcal{U}^c \setminus \hat{\mathcal{U}}_l(t-1)} \operatorname{Prob}(\lambda_l(t) = u_m) \le \frac{1}{k(1+\nu_l)} p_{l,t}$$
(27)

resulting in

$$Pr(\lambda_{l}(t) = \tilde{\lambda}(t))|_{\tilde{\lambda}(t) \in \mathcal{U}^{c}} \leq \max_{u_{m} \in \mathcal{U}^{c} \setminus \hat{\mathcal{U}}_{l}(t-1)} Prob(\lambda_{l}(t) = u_{m})$$
$$\leq \frac{1}{(N-2k)(1-\nu_{l})}(1-p_{l,t}) \quad (2)$$

for $0 \leq \nu_l \ll 1$ and $\tilde{\lambda}(t)$ can take any value from u_0, \cdots, u_{N-1} . Similarly, for any other node in \mathcal{G} , we have,

$$\begin{aligned} \Pr(\lambda_{j}(t) = \tilde{\lambda}(t))|_{\tilde{\lambda}(t) \in \mathcal{U}^{c}} &\leq \max_{u_{m} \in \mathcal{U}^{c} \setminus \hat{\mathcal{U}}_{j}(t-1)} \operatorname{Prob}(\lambda_{j}(t) = u) \\ &\leq \frac{1}{(N-2k)(1-\nu_{j})}(1-p_{j,t}) \end{aligned}$$

for $0 \leq \nu_j \ll 1$ and $j \in \mathcal{G}_l$.

For a given node l, when $M \ge M_1(\delta)$ we have $p_{l,t} \ge 1-\delta$. With the assumption that $\nu_j \ll 1$, we may approximate (29) by,

$$Pr(\lambda_j(t) = \tilde{\lambda}(t))|_{\tilde{\lambda}(t) \in \mathcal{U}^c} \le \frac{\delta}{(N - 2k)}$$
(30)

when $M \ge M_1(\delta)$. Thus, whenever there are $n_l(\lambda(t))$ number of nodes in $\alpha_l(t)$ that estimate the same index $\tilde{\lambda}(t)$, the probability that the corresponding index does not belong to the true support is upper bound by

$$Pr(\lambda_{l}^{*}(t) = \lambda(t))|_{\tilde{\lambda}(t) \in \mathcal{U}^{c}}$$

$$= \prod_{j=1}^{n_{l}(\tilde{\lambda}(t))} Pr(\lambda_{j}(t) = \tilde{\lambda}(t))|_{\tilde{\lambda}(t) \in \mathcal{U}^{c}} \leq \left(\frac{\delta}{(N-2k)}\right)^{n_{l}(\tilde{\lambda}(t))} (31)$$

When L > k, we have $|n_l(\tilde{\lambda}(t))| \ge 2$ when $M \ge M_1(\delta)$. Thus, we have,

$$Pr(\lambda_l^*(t) = \tilde{\lambda}(t))|_{\tilde{\lambda}(t) \in \mathcal{U}^c} \leq \left(\frac{\delta}{(N-2k)}\right)^2.$$
(32)

Thus, taking the union bound we get,

$$Pr(\lambda_l^*(t) \in \mathcal{U}^c) \le \delta^2 \left(\frac{(N-k)}{(N-2k)^2}\right).$$
(33)

Further, when $k \ll N$, we may approximate $\frac{k}{(N-2k)^2} \rightarrow 0$. Thus, we have,

$$Pr(\lambda_l^*(t) \in \mathcal{U}^c) \le \frac{\delta^2}{(N-2k)}$$
 (34)

completing the proof.

Thus, with the same number of measurements per node, the index fusion stage in DC-OMP 1 improves the performance of sparsity pattern recovery significantly compared to performing only the standard OMP. The performance gain is illustrated in the numerical results section.

2) DC-OMP 2: In contrast to DC-OMP 1, in DC-OMP 2, the initial estimate in Phase I is obtained via one-hop communication and the index fusion in Phase II is performed via global communication. Thus, the estimates obtained during both phases are more accurate in DC-OMP 2 than those in DC-OMP 1. Further, due to global communication during Phase II, each node performing DC-OMP 2 has the same estimate for the sparsity pattern when the algorithm terminates. When the one-hop neighborhood in Phase I in DC-OMP 2 becomes the whole network, the performance of DC-OMP 2 coincides with S-OMP being performed at each node. However, due to ^{n_{t}} the index fusion stage in Phase II, DC-OMP 2 terminates with less number of iterations compared to S-OMP. Further, with ⁸ reasonable neighborhood size for local communication in Phase I, DC-OMP 2 provides performance that is comparable to S-OMP as observed from Simulation results. It is noted that, S-OMP requires the global knowledge of the observations and the measurement matrices at each node, while in global *communication* in Phase II in DC-OMP 2, only one index is transmitted at each iteration. 29)

D. Communication complexity

To analyze the communication complexity, we concentrate on the amount of information to be transmitted by each node and whether that information is transmitted to only one hop neighbors or to the whole network. Communication complexity of the two proposed decentralized algorithms are compared with two extreme cases: performing S-OMP at each node and performing standard OMP at each node independently and then fusing the estimated supports to get a global estimate (D-OMP).

1) S-OMP: When S-OMP as stated in Algorithm 2 is performed at each node, each node is required to transmit kN messages to the whole network after k iterations. Thus, the communication burden in terms of the total amount of information to be exchanged in the network is L(L-1)kN.

2) *D-OMP*: In D-OMP, the standard OMP is performed at each node based on its own measurement vector to estimate the support set, and the estimated supports sets are fused via the majority rule to get the final estimate. Each node is required to transmit k indices, thus the communication complexity is in the order of O(k(L-1)L).

3) DC-OMP 1: In DC-OMP 1, each node has to communicate only one index to the neighborhood at a given iteration, thus, communication cost per node after $T_1 \leq k$ iterations is in the order of $\mathcal{O}(T_1)$. The total number of messages to be transmitted by all the sensors assuming that each node talks to the neighboring nodes one by one, is $\sum_{l=0}^{L-1} |\mathcal{G}_l| T_1$. It is also noted that for sufficiently small networks (such as cognitive radios with 10-20 radios) the assumption that $\mathcal{G}_l \cup l = \mathcal{G}$ for all l is also reasonable in performing DC-OMP 1 since it is required to transmit only one index to the whole network at each iteration with a maximum of $T_1 < k$ iterations. Then the total number of messages transmitted by all nodes is $(L-1)LT_1$.

4) DC-OMP 2: In DC-OMP 2, each node has to transmit N values to its one-hop neighbors during each iteration during

TABLE I COMPARISON OF COMMUNICATION COMPLEXITY OF THE PROPOSED ALGORITHMS ($T_1, T_2 \le k < M < N$):

Algorithm	global commun.	local commun.
S-OMP	L(L-1)kN	—
D-OMP (with no collab.)	k(L-1)L	_
DC-OMP 1	_	$\sum_{l=0}^{L-1} \mathcal{G}_l T_1$
DC-OMP 2	$L(L-1)T_2$	$\sum_{l=0}^{L-1} \mathcal{G}_l T_2 N$

Phase I. Thus, after $T_2 \leq k$ iterations, each node requires $\mathcal{O}(T_2N)$ transmissions in the neighborhood. In this phase, it is desirable to have as small a number of neighbors as possible since N messages are to be transmitted per iteration. During Phase II, each node exchanges one index with the whole network, thus after $T_2 \leq k$ iterations, each requires T_2 transmissions to the whole network. Thus, the total number of messages transmitted by all nodes is $\sum_{l=0}^{L-1} |\mathcal{G}_l| NT_2 + L(L-1)T_2$. Compared to performing S-OMP at each node, in DC-OMP 2, communication of length-N messages is restricted to the one-hop neighbors at each node.

It is also noted that the communication complexity above is computed assuming that each node communicates with the other nodes one by one. The communication complexity can be further reduced if an efficient broadcast mechanism is used.

Communication complexities in terms of the amount of information to be transmitted are summarized in Table IV-E. As defined before, $T_1, T_2 (\leq k)$ denotes the number of iterations required for the algorithm to be terminated with DC-OMP 1 and DC-OMP 2, respectively.

E. Comparison with optimization based decentralized approach [5]

We further compare the communication complexity with the most related decentralized algorithm for common sparse recovery as considered in [5]. As observed in Algorithm 1 in [5], to implement the decentralized sparsity pattern recovery, each node is required to iteratively solve a quadratic optimization problem in an iterative manner to get an estimate of the sparsity pattern. In contrast, in the proposed decentralized algorithms in this paper, the computational complexity is dominated by the greedy selection stage (which requires less computational complexity compared to performing quadratic optimization). Further, in [5], at a given iteration, each node is required to communicate a length-N vector to its one hop neighbors, thus the communication complexity in terms of the total number messages to be transmitted by all nodes is $\sum_{l=0}^{L-1} |\mathcal{G}_l| NT_0$ where T_0 is the number of iterations which is different from DC-OMP 1 and DC-OMP 2. It is noted that while DC-OMP 2 has similar communication complexity, DC-OMP 1 requires much less communication overhead compared to this scheme. Thus, in addition to the computational gain at each node, the communication complexity is less (in DC-OMP 1) or in the same order (in DC-OMP 2) in the proposed algorithms compared to the optimization based approach presented in [5].

V. SIMULATION RESULTS

For simulation purposes, we consider that each sampling matrix \mathbf{A}_l is a random orthoprojector so that $\mathbf{A}_l \mathbf{A}_l^T = \mathbf{I}_M$ for $l = 0, 1, \dots, L - 1$. We illustrate the performance using the probability of exact support recovery, P_d , which is defined as

$$P_d = Pr(\boldsymbol{\zeta} = \boldsymbol{\zeta}) \tag{35}$$

where ζ is defined in (4) and ζ contains binary elements $\{0, 1\}$ in which 1's corresponding to the locations in $\hat{\mathcal{U}}$.



Fig. 1. Performance of sparsity pattern recovery via OMP with MAC output (13) and PAC output (6); $N=256,\,k=5,\,L=10$

A. Sparsity pattern recovery with the MAC output

First, we illustrate the capability of the MAC transmission scheme (13) in sparsity pattern recovery. In Fig. 1, we consider the case where $A_l = A$ and different choices for the sparse signals s_l for $l = 0, \dots, L-1$. With (13), the sparsity pattern recovery is performed via the standard OMP. We compare the results with the sparsity pattern recovery via S-OMP when all the observations are available at the fusion center (i.e. as in (6)). In Fig. 1, we plot the probability of complete support recovery vs the number of measurements per node M when the nonzero elements are drawn from a uniform distribution in the range $[a_1, b_1]$ with different values for a_1 and b_1 . The variance of the measurement noise $\sigma_v^2 = 0.01$. The average SNR is defined as $\bar{\gamma} = \frac{1}{L} \sum_{l=0}^{L-1} \frac{||\mathbf{s}_l||^2}{N\sigma_v^2}$. In Fig. 1, the dimension of the sparse signals is taken as N = 256, the sparsity index k = 5 and the number of nodes L = 10. It is observed that when a_1 and b_1 are such that s_l 's have the same sign, the performance gap between the MAC and the PAC outputs in common sparsity pattern recovery is not significant even though their amplitudes can differ significantly. In other words, when the fusion center does not have separate observations vectors but has only their linear combination, the sparsity pattern recovery can still be performed reliably (with almost the same performance as the case where all observations are available) when the coefficients of the sparse signals have the same sign. However, as claimed in Section III, the performance of the sparsity pattern recovery based on the MAC output (13)

is not comparable with the performance with the PAC output (6) when the nonzero elements of s_l 's have zero mean (i.e. $a_1 = -25$ and $b_1 = 25$ in Fig. 1).



Fig. 2. Performance of the decentralized sparsity pattern recovery with Algorithms 4 and 3; N = 256, k = 10, L = 10, $\bar{\gamma} = \frac{1}{L} \sum_{l=0}^{L-1} \frac{||\mathbf{s}_l||^2}{N\sigma_*^2} = 28dB$



Fig. 3. Fraction of the support correctly recovered form Algorithms 4 and 3; N = 256, k = 10, L = 10, $\bar{\gamma} = \frac{1}{L} \sum_{l=0}^{L-1} \frac{||s_l||^2}{N\sigma_v^2} = 28dB$

B. Performance of DC-OMP 1 and DC-OMP 2

To compare the performance of the proposed Algorithms 3 and 4 with other comparable approaches, we consider two existing benchmark cases. (i). Distributed OMP with no collaboration (D-OMP), in this case, each node performs standard OMP in Algorithm 1 independently to obtain the support set estimate $\hat{\mathcal{U}}_l$. To fuse the estimated support sets, $\hat{\mathcal{U}}_l$'s, at individual nodes, each node transmits indices in $\hat{\mathcal{U}}_l$ to the rest of the nodes, and employs a majority rule based fusion scheme to obtain a final estimate $\hat{\mathcal{U}}$. (ii). S-OMP [20]: S-OMP algorithm as stated in Algorithm 2 is carried out at each node where each node has the access to all the information residing at at every other node.

In Figures 2 and 3, we let N = 256, k = 10 and L = 10. Non zero coefficients of sparse vectors are generated as

realizations of a uniform random variable with the support [10, 15] and the variance of the measurement noise $\sigma_v^2 = 0.01$. The average SNR is taken as $\bar{\gamma} = 28dB$. In Figures 2-??, for DC-OMP 1, we consider the case where $\mathcal{G}_l \cup l = \mathcal{G}$ for $l = 0, 1, \dots, L-1$. Thus, it shows the maximum performance achievable via DC-OMP 1 and also each node has the same estimated support set based on Algorithm 3. For DC-OMP 2 in Algorithm 4, we consider different neighborhood sizes for local communication phase at each node as $|\mathcal{G}_l| = n_0 = 3, 5, 7$ for $l = 0, 1, \dots, L-1$. Since in DC-OMP 2, index fusion stage is performed via global communication, each node has the same estimate after T_2 iterations.

We plot the probability of correctly recovering the full support set, P_d , and the fraction of the support set that is estimated correctly vs M in Figures 2 and 3, respectively. Results are obtained by performing 10^4 runs and averaging over 50 trials. It can be seen from Figures 2 and 3 that the two proposed decentralized algorithms outperform D-OMP with no collaboration. The performance of DC-OMP 2 gets closer to S-OMP as the neighborhood size for local communication phase increases. While a considerable performance gain over D-OMP with no collaboration is achieved, DC-OMP 1 still has a certain performance gap compared to S-OMP and DC-OMP 2. It is noted that, DC-OMP 2 performs measurement fusion within a neighborhood prior to index fusion based on the estimated indices at all nodes in the network. On the other hand, DC-OMP 1 estimates indices based on each node's own observations and only the estimated indices are fused. Thus, as discussed in Table IV-E, DC-OMP 1 incurs smaller communication overhead compared to DC-OMP 2. As Mincreases, P_d converges to 1 in all algorithms. This is because when the number of compressive measurements at each node increases, OMP (with or without collaboration) works better and recovers the sparsity pattern almost exactly with even a single measurement vector. In resource constrained distributed networks, especially in sensor networks, it is desirable to perform the desired task by employing less measurement data (i.e. with small M) at each node distributing the computational complexity among nodes to save the overall node power, and the proposed algorithms are promising in this case compared to performing OMP at each node independently (based on Algorithm 1) followed by fusion.

To further illustrate the efficiency of the proposed algorithms, in Fig. 4, we plot the average number of iterations required by DC-OMP 1 and DC-OMP 2 to terminate at each node. It is observed from Fig. 4 that, DC-OMP 2 conducts a smaller number of iterations per node compared to sparsity index ($\approx k/2$) before algorithm terminates for all values of M. As M increases, the number of iterations per node required for Algorithm DC-OMP 1 to terminate also reduces considerably compared to the sparsity index. Although, as M increases, D-OMP also correctly recovers the sparsity pattern, DC-OMP 1 does it with very few iterations per node. D-OMP with no collaboration requires k iterations at each node irrespective of the value of M.

Next, we demonstrate the performance of the proposed algorithms as the number of nodes in the network, L, increases while keeping the number of measurements per node, M, at a



Fig. 4. Number of iterations required for algorithm to be terminated at each node for Algorithms 4 and 3; N = 256, k = 10, L = 10, $\bar{\gamma} = \frac{1}{L} \sum_{l=0}^{L-1} \frac{||\mathbf{s}_l||^2}{N\sigma_v^2} = 28 dB$

fixed value. In Figures 5-6, we plot, the probability of correctly recovering the common sparsity pattern, and the fraction of the support correctly recovered, respectively, vs L. For the local communication phase of DC-OMP 2 we assume that the neighborhood size for each node as L/2. We set M = 30, N = 256, k = 10 and $\bar{\gamma} = \frac{1}{L} \sum_{l=0}^{L-1} \frac{||\mathbf{s}_l||^2}{N\sigma_v^2} = 28 dB$.



Fig. 5. Probability of exact sparsity pattern recovery vs L; N = 256, k = 10, M = 30, $\bar{\gamma} = \frac{1}{L} \sum_{l=0}^{L-1} \frac{||\mathbf{s}_l||^2}{N\sigma_v^2} = 28dB$

From Fig 5 and 6, it can be seen that the performance of both proposed algorithms DC-OMP 1 and DC-OMP 2 improves as L increases. In particular, DC-OMP 2 correctly recovers the sparsity pattern and yields similar performance as S-OMP with relatively very small L for a given M. The performance of DC-OMP 1, which performs only index fusion, is improved significantly as L increases for a given small value of M (slightly greater than k). The performance of D-OMP with no collaboration does not improve considerably as Lincreases. It is noted that in D-OMP (with no collaboration), individual nodes estimate the support by running standard OMP at each node independently, and then the estimated



Fig. 6. Fraction of the support correctly recovered vs L; N = 256, k = 10, M = 30, $\bar{\gamma} = \frac{1}{L} \sum_{l=0}^{L-1} \frac{||\mathbf{s}_l||^2}{N\sigma_n^2} = 28dB$



Fig. 7. Performance of DC-OMP 1 as the size of one hop neighborhood varies; N = 256, k = 2, L = 10, $\bar{\gamma} = \frac{1}{L} \sum_{l=0}^{L-1} \frac{||\mathbf{s}_l||^2}{N\sigma_{\star}^2} = 21 dB$

support sets are fused to get a global estimate. Since the fusion is performed after estimating the support sets, the performance of D-OMP is ultimately limited by the number of compressive measurements per node, M. However, in DC-OMP 1, estimated indices are fused at each iteration and when the number of nodes increases, more accurate indices for the true support can be estimated by step 4 in DC-OMP 1 in Algorithm 3 at each iteration.

In Figures 2-6, we assumed that $|\mathcal{G}_l \cup l| = L$ for DC-OMP 1 so that when the algorithm terminates each node has the same estimated support set. In Fig. 7, we compare the performance of DC-OMP 1 when $|\mathcal{G}_l \cup l| < L$ and the one hop neighborhood size varies. In Fig. 7, we plot the average probability of sparsity pattern recovery as the neighborhood size varies where the average is taken over the nodes. In Fig. 7, we let k = 2, L = 10 and average SNR $\bar{\gamma} = 21 dB$. We assume that each node has the same number of one-hop neighbors. We also, plot the errorbars which represent the minimum and the maximum deviations of the probability of sparsity pattern recovery over individual nodes. It can be observed that, as the number of one hop neighbors increases, the length of the error bars decreases, thus, each node has almost the same performance in sparsity pattern recovery. Further, it is observed that Algorithm 3 when $|\mathcal{G}_l| \geq k$ converges with almost the same number of measurements per node as required when $\mathcal{G}_l = \mathcal{G}$.

VI. CONCLUSION

In this paper, we considered the problem of recovering a common sparsity pattern in a distributed network under communication constraints in a centralized as well as decentralized manner. In a centralized setting, the communication constraint is taken into account by employing a shared multiple access channel to forward observations at each node to a central fusion center. Then, we showed that under certain conditions, the sparsity pattern can be reliably recovered based on the MAC output with performance that is comparable to the case when the fusion center has access to all the observations separately.

We further proposed two decentralized OMP based algorithms for joint sparsity pattern recovery without depending on a central fusion center. These algorithms use minimal communication and computational burden at each node. More specifically, in the proposed algorithms, collaboration and fusion are exploited at different stages of the standard OMP algorithm to have a more accurate estimate for the common sparsity pattern with small number of compressive measurements per node as well as less communication burden.

In future work, we will develop decentralized algorithms for simultaneous approximation of sparse signals with structured and more complex joint sparsity models. We will further investigate the impact of channel effects on the joint sparse recovery in distributed networks.

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