Distributed Diffusion-Based LMS for Node-Specific Adaptive Parameter Estimation

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Abstract—A distributed adaptive algorithm is proposed to solve a node-specific parameter estimation problem where nodes are interested in estimating parameters of local interest, parameters of common interest to a subset of nodes and parameters of global interest to the whole network. To address the different nodespecific parameter estimation problems, this novel algorithm relies on a diffusion-based implementation of different Least Mean Squares (LMS) algorithms, each associated with the estimation of a specific set of local, common or global parameters. Coupled with the estimation of the different sets of parameters, the implementation of each LMS algorithm is only undertaken by the nodes of the network interested in a specific set of local, common or global parameters. The study of convergence in the mean sense reveals that the proposed algorithm is asymptotically unbiased. Moreover, a spatial-temporal energy conservation relation is provided to evaluate the steady-state performance at each node in the mean-square sense. Finally, the theoretical results and the effectiveness of the proposed technique are validated through computer simulations in the context of cooperative spectrum sensing in Cognitive Radio networks.

Index Terms—Adaptive distributed networks, diffusion algorithm, cooperation, node-specific parameter estimation.

I. INTRODUCTION

Two major groups of energy aware and low-complex distributed strategies for estimation over networks have been studied in the literature, i.e., consensus strategies and the algorithms based on incremental or diffusion mode of cooperation. Motivated by the procedure obtained in [1] and [2], the most recent implementations of the consensus strategy (e.g., [3]-[4]) allow the cooperating nodes to reach an agreement regarding a vector of parameters of interest in a single timescale. The second group, which is in the focus of this paper, consists of a single time-scale distributed algorithms that obtain a linear estimator of a vector of parameters by distributing a specific stochastic gradient method under an incremental or a diffusion mode of cooperation. In the incremental mode (e.g.,[5]-[6]), each node communicates with only one neighbor

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and the data are processed in a cyclic manner throughout the network. This strategy achieves the centralized-like solution. However, the determination of a cyclic path that covers all nodes of the network is an NP hard problem [7] and, in addition, cyclic trajectories are more sensitive to node failures and to link failures. Alternatively, better reliability can be achieved at the expense of increased energy consumption in the so-called diffusion mode considered, for instance, in [8]-[10]. Under this strategy, each node interacts with a subset of neighboring nodes. As a result, unlike incremental-based strategies, the cooperation is undertaken in a fully ad-hoc fashion.

In many of the distributed estimation problems, it is considered that the nodes have the same interest. This scenario can be viewed as a special case of a more general problem where the nodes of the network have overlapping but different estimation interests. Some examples of this kind of networks can be found in the context of power system state estimation in smart grids, speech enhancement and active noise control in wireless acoustic networks and cooperative spectrum sensing in Cognitive Radio (CR) networks. Perhaps some of the first works explicitly considering a network with nodespecific estimation interests are [11]-[12]. In these works, for networks with a fully connected and tree topology, Bertrand *et al.* proposed distributed algorithms that allow to estimate node-specific desired signals sharing a common latent signal subspace.

In this paper, we consider the estimation scenarios which can be formulated as Node-Specific Parameter Estimation (NSPE) problems. Within this category, most of the existing works are based on consensus implementations. For instance, the consensus approach presented in [13] is based on optimization techniques that force different nodes to reach an agreement when estimating parameters of common interest. At the same time, the consensus-based technique in [13] allows each node to estimate a vector of parameters that is only of its own interest. In the case of schemes based on a distributed implementation of adaptive filtering techniques, NSPE problems are recently receiving an increasing attention. In [14], a diffusion-based scheme is used to solve an NSPE problem where the node-specific estimation interests are expressed as the multiplication of a node-specific matrix of basis functions with a vector of global parameters. Since the matrix of basis functions is known by each node, the problem finally reduces to the estimation of a vector of global interest. In [15], the authors use diffusion adaption and scalarization techniques to obtain a Pareto-optimal solution for the multi-objective cost function that appears in a distributed estimation problem where each node has a different interest. For a network formed by non-overlapping clusters of nodes, each with a different estimation interest, a diffusion-based strategy with an adaptive combination rule is proposed in [16]. However, in the proposed strategy the cooperation is finally limited to nodes that have exactly the same objectives. For the same network, Chen et al. have recently derived a diffusion-based algorithm with spatial regularization that simultaneously provides biased estimates of the multiple vectors of parameters [17]. Unlike previous works, the proposed algorithm allows cooperation among neighboring nodes as long as they have numerically similar parameter estimation interests. Additionally, in [18] the authors analyze the performance of the diffusion-based LMS algorithm derived in [9] when it is run in the NSPE setting considered in [17].

As far as the authors are concerned, there are no diffusionbased strategies that provide unbiased solutions of a NSPE problem where the nodes can have overlapping and arbitrarily different estimation interests at the same time. Only in [19]-[21], the aforementioned NSPE problem is solved by employing incremental implementations of the Least Mean Squares (LMS) and Recursive Least Squares (RLS) algorithms. Motivated by this fact, we build on our preliminary work [22] in order to design a diffusion-based algorithm that solves a NSPE problem in a network where the nodes can simultaneously be interested in estimating parameters of local, common and/or global interest. In particular, we adopt two peer-to-peer diffusion protocols, Combine-then-Adapt (CTA) and Adapt-then-Combine (ATC), to allow each node to estimate its node-specific vector of parameters in real time under the LMS criterion. Under both CTA and ATC schemes, each node undertakes a local adaptive filtering task where its local observations are fused with an estimate of its parameters of local interest as well as estimates of the parameters of global and common interest, which have been exchanged with its neighbors. As a result, the network is able to adapt in real time to variations of the data statistics related to parameters of local, common and global interest in the network. Moreover, as a detailed performance analysis of the resulting adaptive network shows, the proposed NSPE techniques are asymptotically unbiased in the mean sense.

The paper is organized as follows. Section II mathematically describes the considered NSPE problem. In Section III we derive an ATC and CTA diffusion-based techniques to solve the NSPE problem of Section II by employing the LMS algorithm. Next, Section IV is devoted to the theoretical performance analysis of the proposed techniques. Initially, the convergence in the mean sense is analyzed to show that the proposed techniques are asymptotically unbiased. Afterwards, we provide closed-form expressions for the Mean Square Error (MSE) and Mean Square Deviation (MSD) achieved by each node with respect to the estimation of its parameters of local, common and global interest in the steady state. In Section V, the theoretical analysis is first verified via generic simulations, and also simulation results are provided in the context of cooperative spectrum sensing in CR networks. Finally, Section VI summarizes our work and gives a description of the future research lines.

The following notation is used throughout the paper. We use boldface letters for random variables and normal fonts for deterministic quantities. Capital letters refer to matrices and small letters refer to both vectors and scalars. The notation $(\cdot)^H$ and $E\{\cdot\}$ stand for the Hermitian transposition and the expectation operator, respectively. For a set, e.g., \mathcal{X} , the operator $|\cdot|$ stands for the cardinality. If the set \mathcal{X} is ordered, then $\mathcal{X}(j)$ equals the j-th element of \mathcal{X} . We use the weighted norm notation $\|x\|_{\Sigma}^2 \triangleq x^H \Sigma x$ with a vector x and a Hermitian positive semi-definite matrix $\Sigma \geq 0$. Moreover, $R_{\mathbf{A}} = E\{\mathbf{A}^H \mathbf{A}\}, R_{\mathbf{A},\mathbf{B}} = E\{\mathbf{A}^H \mathbf{B}\}$ and $r_{\mathbf{A},\mathbf{b}} = E\{\mathbf{A}^H \mathbf{b}\}$ for any random matrices \mathbf{A} , \mathbf{B} and any random vector \mathbf{b} . The notation blockdiag $\{\cdot\}$ denotes a block-diagonal matrix. Finally, $0_{L\times M}$ denotes a $L\times M$ zero matrix, while 1_L stands for a $(L\times 1)$ vector of ones.

II. PROBLEM STATEMENT

Let us consider a connected network consisting of N nodes that are randomly deployed over some geographical region. Nodes that are able to share information with each other are said to be neighbors. The neighborhood of any particular node k, including also node k, is denoted as \mathcal{N}_k . Since the network is connected, as shown in Fig. 1, the neighborhoods are set so that there is a path between any pair of the nodes in the network.

At discrete time i, each node k has access to data $\{d_{k,i}, U_{k,i}\}$, corresponding to time realizations of zero-mean random processes $\{\mathbf{d}_{k,i}, \mathbf{U}_{k,i}\}$, with dimensions $L_k \times 1$ and $L_k \times M_k$, respectively. These data are related to events that take place in the monitored area through the subsequent model

$$\mathbf{d}_{k,i} = \mathbf{U}_{k,i} w_k^o + \mathbf{v}_{k,i} \tag{1}$$

where, for each time instant i,

- w_k^o equals the deterministic but unknown vector of dimension M_k that gathers all parameters of interest for node k
- $\mathbf{v}_{k,i}$ denotes the random noise vector with zero mean and covariance matrix $R_{v_k,i}$ of dimensions $L_k \times L_k$,
- $\mathbf{d}_{k,i}$ and $\mathbf{U}_{k,i}$ are zero-mean random variables with dimensions $L_k \times 1$ and $L_k \times M_k$, respectively.

Given the previous observation model, by processing data set $\{d_{k,i}, U_{k,i}\}$ the objective of the network consists in estimating the node-specific vector of parameters $\{w_k\}_{k=1}^N$ that minimize the subsequent cost function

$$J_{\text{glob}}(\{w_k\}_{k=1}^N) = \sum_{k=1}^N E\{\|\mathbf{d}_{k,i} - \mathbf{U}_{k,i} w_k\|^2\}.$$
 (2)

The vast majority of works dealing with distributed estimation algorithms in the context of adaptive filtering (e.g., [5]-[9]) considered the case where the nodes' interests are the same, i.e. $w_k^o = w^o$ for all $k \in \{1, 2, \dots, N\}$. However, similarly to [19]-[21], the formulation of this paper goes beyond by considering that the node-specific interests are different but overlapping.

As depicted in Fig. 1, each node-specific vector w_k^o might consist of a sub-vector w^o of parameters of global interest to

the whole network, sub-vectors $\{\varsigma_i^o\}$ of parameters of common interest to subsets of nodes including node k, and a sub-vector ξ_k^o of local parameters for node k. In particular, the global parameters w^o $(M_g \times 1)$ might be related to a phenomenon that can be monitored by all the nodes. In contrast, a set of parameters of common interest $\varsigma_i^o (M_{j_c} \times 1)$ might be related to a phenomenon that can be observed by a subset of nodes in the network. The ordered set of indices k associated with the connected subset of nodes interested in ς_i^o is denoted as C_j . For instance, in Fig. 1, $C_1 = \{1, 2, 3\}$. Depending on the areas of influence associated with the events of common interest, note that a node might be interested in more than one set of common parameters. As a result, subsets of nodes C_i and $C_{j'}$, with $j \neq j'$, might be partially or fully overlapped. For instance, Figure 1 indicates that node k is interested in estimating both vectors of common parameters ς_i^o and ς_{i-1}^o , i.e. $C_{j-1} \cap C_j = \{k\}$. Finally, each vector of local parameters ξ_k^o $(M_{l_k} \times 1)$ may represent the influence of some local phenomena that only affects the area monitored by node k. In this way, considering a scenario where there are J different subsets of common parameters (see Fig. 1), the observation model provided in (1) can be rewritten as

$$\mathbf{d}_{k,i} = \mathbf{U}_{k_g,i} w^o + \sum_{j \in \mathcal{I}_k} \mathbf{U}_{k_{jc},i} \varsigma_j^o + \mathbf{U}_{k_l,i} \xi_k^o + \mathbf{v}_{k,i}$$
(3)

where, for $k \in \{1, 2, ..., N\}$, $j \in \{1, 2, ..., J\}$ and $i \ge 1$,

- \mathcal{I}_k equals an ordered set of indices j associated with the vectors ς_i^o that are of interest for sensor k,
- $\mathbf{U}_{k_g,i}$, $\mathbf{U}_{k_{jc},i}$ and $\mathbf{U}_{k_l,i}$ are matrices of dimensions $L_k \times M_g$, $L_k \times M_{jc}$ and $L_k \times M_{k_l}$ that might be correlated, and consist of the columns of $\mathbf{U}_{k,i}$ associated with w^o , ς_i^o and ξ_k^o , respectively.

Thus, according to (2) and (3), our NSPE problem can be restated as minimizing the following cost

$$\sum_{k=1}^{N} E \left\{ \| \mathbf{d}_{k,i} - \mathbf{U}_{k_g,i} w - \sum_{j \in \mathcal{I}_k} \mathbf{U}_{k_{jc},i} \varsigma_j - \mathbf{U}_{k_l,i} \xi_k \|^2 \right\}$$
(4)

with respect to variables $w, \{\varsigma_j\}_{j=1}^J$ and $\{\xi_k\}_{k=1}^N$.

III. A SOLUTION OF THE NEW NSPE PROBLEM

In this section, acting as a starting point for the derivation of the distributed algorithms and allowing us to introduce some useful notation, we briefly describe the centralized solution provided in [21] to the NSPE problem stated in the previous section. Later, via diffusion-based approach we focus on the derivation of distributed algorithms that approximate the centralized solution. For the sake of simplicity and without losing generality, we assume that $M_{l_k} = M_l$, $M_{j_c} = M_c$ and $L_k = L$ for all $k \in \{1, 2, \ldots, N\}$ and $j \in \{1, 2, \ldots, J\}$.

A. Centralized solution

An inspection of (4) reveals that the solution of the considered NSPE problem entails the optimization of a scalar real-valued cost function w.r.t. multiple vector variables, i.e.,

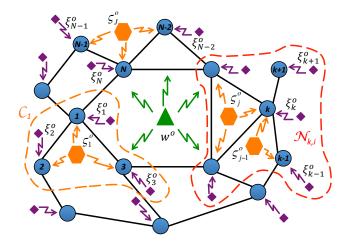


Fig. 1. A network of N nodes with node-specific parameter estimation interests.

 $\{w,\{\varsigma_j\}_{j=1}^J,\{\xi_k\}_{k=1}^N\}$. If we gather these variables into the following augmented vector

$$\bar{w} = \begin{bmatrix} w^T & \varsigma_1^T & \varsigma_2^T & \cdots & \varsigma_J^T & \xi_1^T & \xi_2^T & \cdots & \xi_N^T \end{bmatrix}^T \quad (\bar{M} \times 1) \quad (5)$$

where $\bar{M} = M_g + J \cdot M_c + N \cdot M_l$, from [21] we know that our optimization problem can be cast as

$$\widehat{\overline{w}} = \underset{\widetilde{w}}{\operatorname{argmin}} \left\{ J_{\text{glob}}(\overline{w}) \right\}$$

$$= \underset{\overline{w}}{\operatorname{argmin}} \left\{ \sum_{k=1}^{N} E\left\{ \|\mathbf{d}_{k,i} - \bar{\mathbf{U}}_{k,i} \overline{w} \|^{2} \right\} \right\}$$
(6)

where $\bar{\mathbf{U}}_{k,i}$ is defined in (7) at the top of the following page with $M_a = (k-1)M_l$ and $M_b = (N-k)M_l$ and

$$\mathbb{1}_{\{\mathcal{X}\in\mathcal{A}\}} = \begin{cases} 1 & \text{if } \mathcal{X} \subseteq \mathcal{A}, \\ 0 & \text{otherwise.} \end{cases}$$
 (8)

From [23], we know that the resulting solutions \widehat{w} are optimal if the random processes are $\{\mathbf{d}_{k,i}, \mathbf{U}_{k,i}\}$ are jointly wide-sense stationary are given by the normal equations

$$\left(\sum_{k=1}^{N} R_{\bar{U}_k}\right) \cdot \hat{\overline{w}} = \sum_{k=1}^{N} r_{\bar{U}_k d_k}.$$
 (9)

Notice that the solution of the previous system of equations requires the transmission of all sensor observations to the fusion center and the inversion of a square matrix whose dimension is proportional to the network size. As a result, for large networks, the centralized solution in (9) is not scalable with respect to both computational power and communication resources, which motivates the derivation of distributed solutions

B. Diffusion-based NSPE solutions

By relying on in-network processing of the data $\{d_{k,i}, U_{k,i}\}$, the incremental-based algorithms proposed in [19] and [21] converge to the centralized solution in the mean sense with an increase of the energy efficiency and an improved scalability. Attaining more robustness to link or node failures than the incremental strategies, other alternative mode of cooperation

$$\bar{\mathbf{U}}_{k,i} = \begin{bmatrix} \mathbf{U}_{k_g,i} & \mathbb{1}_{\{1 \in \mathcal{I}_k\}} \mathbf{U}_{k_{1c},i} & \mathbb{1}_{\{2 \in \mathcal{I}_k\}} \mathbf{U}_{k_{2c},i} & \cdots & \mathbb{1}_{\{J \in \mathcal{I}_k\}} \mathbf{U}_{k_{Jc},i} & 0_{L \times M_a} & \mathbf{U}_{k_l,i} & 0_{L \times M_b} \end{bmatrix}$$
(7)

to process the data $\{d_{k,i}, U_{k,i}\}$ in a distributed fashion is based on diffusion strategies, e.g., Combine-then-Adapt (CTA) and Adapt-then-Combine (ATC). In the case where the nodes are interested in estimating the same vector of global parameters, the aforementioned strategies are known to well approximate the centralized solution when all nodes want to estimate the same vector of parameters [9]. In this work, we extend them so as to be applicable in the NSPE described in Section II.

First, let us define $\bar{\psi}_k^{(i)}$ as the local estimate of \bar{w}^o at time instant i and node k. Note that $\bar{\psi}_k^{(i)}$ is generally a noisy version of the optimal augmented vector \bar{w}^o . By using a diffusion mode of cooperation, at each time instant i-1, each node k has access to the set of local estimates of its neighbors, i.e., \mathcal{N}_k . Thus, node k can fuse its local estimate with the local estimates of its neighbors as follows

$$\bar{\phi}_k^{(i-1)} = f_k \left(\{ \bar{\psi}_\ell^{(i-1)} \}_{\ell \in \mathcal{N}_k} \right) \tag{10}$$

where f_k is a local combiner function. In this work, we will focus on linear combiners of the form

$$\bar{\phi}_k^{(i-1)} = \sum_{\ell \in \mathcal{N}_k} \bar{C}_{k,\ell} \, \bar{\psi}_\ell^{(i-1)} \tag{11}$$

where

$$\bar{C}_{k,\ell} = \operatorname{diag}\{c_{k,\ell}^{w} I_{M_g}, c_{k,\ell}^{\varsigma_1} I_{M_c}, \dots, c_{k,\ell}^{\varsigma_J} I_{M_c} \\ c_{k,\ell}^{\varsigma_1} I_{M_I}, \dots, c_{k,\ell}^{\varsigma_N} I_{M_I}\}.$$
(12)

In (12), $c_{k,\ell}^w$ equals the weight coefficient used by node k when combining the local estimate of the global vector w^o from node ℓ . Similarly, for $\ell, m \in \{1, \dots, N\}$, $c_{k,\ell}^{\varsigma_j}$ and $c_{k,\ell}^{\xi_m}$ denote the combination coefficients employed by node k when fusing the local estimates of ς_j^o and ξ_m^o , from node ℓ with its corresponding local estimates, respectively. Since the contribution of each node to the different estimation tasks might be different depending on the statistics of its observations as well as its own estimation interests, note that we allow each node to have different coefficients when combining the local estimates of each vector of global, common or local parameters performed by a neighbor node ℓ .

To determine the combination coefficients at each node k, we can interpret (11) as a weighted least squares estimate of the augmented vector of parameters \bar{w}^o given its local estimate as well as the local estimates at the neighbour nodes [8]. This way, by collecting the local estimates of the augmented vector \bar{w}^o at the neighbour nodes

$$\bar{\psi}_{\mathcal{N}_k} = \operatorname{col}\left\{\{\bar{\psi}_{\ell}^{(i-1)}\}_{\ell \in \mathcal{N}_k}\right\} \tag{13}$$

and defining

$$B = \operatorname{col}\{I_{\bar{M}}, I_{\bar{M}}, \dots, I_{\bar{M}}\} \qquad (n_k \cdot \bar{M} \times \bar{M}) \tag{14}$$

and

$$\bar{C}_k = \text{diag}\{\bar{C}_{k,1}, \bar{C}_{k,2}, \dots, \bar{C}_{k,n_k}\}$$
 (15)

with $n_k = |\mathcal{N}_k|$, we can formulate the subsequent local weighted least-squares problem

$$\underset{\bar{\phi}_k}{\operatorname{argmin}} \left\{ \|\bar{\psi}_{\mathcal{N}_k} - B\bar{\phi}_k\|_{\bar{C}_k}^2 \right\},\tag{16}$$

whose solution is given by

$$\bar{\phi}_{k}^{(i-1)} = \left[B^{H} \bar{C}_{k} B \right]^{-1} B^{H} \bar{C}_{k} \bar{\psi}_{\mathcal{N}_{k}}. \tag{17}$$

In more detail, focusing on the different sub-vectors that form $\bar{\phi}_k^{(i-1)}$, for $k,m\in\{1,2,\ldots,N\}$ and $j\in\{1,2,\ldots,J\}$ the solution provided in (17) can be rewritten as

$$\phi_{k,w}^{(i-1)} = \sum_{\ell \in \mathcal{N}_k} \frac{c_{k,\ell}^w}{\sum_{k' \in \mathcal{N}_k} c_{k,k'}^w} \psi_{\ell,w}^{(i-1)}$$
(18)

$$\phi_{k,\varsigma_{j}}^{(i-1)} = \sum_{\ell \in \mathcal{N}_{c}} \frac{c_{k,\ell}^{\varsigma_{j}}}{\sum_{k' \in \mathcal{N}_{c}} c_{k,k'}^{\varsigma_{j}}} \psi_{\ell,\varsigma_{j}}^{(i-1)}$$
(19)

and

$$\phi_{k,\xi_m}^{(i-1)} = \sum_{\ell \in \mathcal{N}_k} \frac{c_{k,\ell}^{\xi_m}}{\sum_{k' \in \mathcal{N}_k} c_{k,k'}^{\xi_m}} \psi_{\ell,\xi_m}^{(i-1)} \tag{20}$$

where $\phi_{k,w}^{(i-1)}$, $\phi_{k,\varsigma_j}^{(i-1)}$ and $\phi_{k,\xi_m}^{(i-1)}$ denote the subvectors of combiner $\bar{\phi}_k^{i-1}$ associated with the local estimation of w^o , ς_j^o and ξ_m at node k and time instant i-1, respectively. Analogously, $\psi_{k,w}^{(i-1)}$, $\psi_{k,\varsigma_j}^{(i-1)}$ and $\psi_{k,\xi_m}^{(i-1)}$ denote the sub-vectors of the local estimate $\bar{\psi}_k^{i-1}$ associated with the local estimation of w^o , ς_i^o and ξ_m at node j and time instant i-1, respectively.

At this point, after a suitable redefinition of the combination coefficients that appear in (18), (19) and (20), we can now verify that the combination coefficients in (11) and (12) have to satisfy

$$c_{k,\ell}^w = 0 \text{ if } \ell \notin \mathcal{N}_k; \quad \sum_{\ell \in \mathcal{N}_k} c_{k,\ell}^w = 1$$
 (21)

$$c_{k,\ell}^{\varsigma_j} = 0 \text{ if } \ell \notin \mathcal{N}_k; \quad \sum_{\ell \in \mathcal{N}_k} c_{k,\ell}^{\varsigma_j} = 1$$
 (22)

and

$$c_{k,\ell}^{\xi_m} = 0 \text{ if } \ell \notin \mathcal{N}_k; \quad \sum_{\ell \in \mathcal{N}_k} c_{k,\ell}^{\xi_m} = 1$$
 (23)

for $j \in \{1, 2, \dots, J\}$ and $k, m \in \{1, 2, \dots, N\}$.

Next, in order to estimate \bar{w}^o at each node k in an adaptive fashion, the corresponding local aggregate estimate $\bar{\phi}_k^{(i-1)}$ is fed into the local LMS-type adaptive algorithm that minimizes the cost associated with node k in (6). This way, the resulting

diffusion based strategy can be described as

$$\begin{cases} & \text{Combination step:} \\ & \bar{\phi}_{k}^{(i-1)} = \sum_{\ell \in \mathcal{N}_{k}} \bar{C}_{k,\ell} \, \bar{\psi}_{\ell}^{(i-1)} \\ & \text{Adaptation step:} \\ & \bar{\psi}_{k}^{(i)} = \bar{\phi}_{k}^{(i-1)} - \mu_{k} \bar{\mathbf{U}}_{k,i}^{H} \left[\mathbf{d}_{k,i} - \bar{\mathbf{U}}_{k,i} \, \bar{\phi}_{k}^{(i-1)} \right] \end{cases}$$

with $i \geq 1$, $\{\bar{\psi}_{\ell}^{(0)}\}_{\ell \in \mathcal{N}_k}$ equal to some initial guess, $\bar{C}_{k,\ell}$ defined in (12) and $\mu_k > 0$ equal to a suitably chosen positive step-size parameter.

Due to the structure of the augmented regressors $\bar{\mathbf{U}}_{k.i}$ defined in (7), only $2 + |\mathcal{I}_k|$ sub-vectors of $\bar{\psi}_k^{(i)}$ are updated when a specific node k performs the adaptation step at each time instant i (see (24)). According to (5) and (7), based on $\{\mathbf{d}_{k,i},\mathbf{U}_{k,i}\}$ and the aggregate estimates $\phi_{k,w}^{(i-1)},\{\phi_{k,\varsigma_j}^{(i-1)}\}_{j\in\mathcal{I}_k}$ and $\phi_{k,\xi_k}^{(i-1)}$, the updated sub-vectors correspond with the local estimates of w^o , $\{\varsigma_{k,j}^o\}_{j\in\mathcal{I}_k}$ and ξ_k^o at node k and time i, respectively. Therefore, note that each node only updates the sub-vectors that are within its interest, which will be now denoted as $\psi_k^{(i)}$, $\{\varsigma_{k,j}^{(i)}\}_{j\in\mathcal{I}_k}$ and $\xi_k^{(i)}$ for the sake of simplicity. The previous fact allows to set the subsequent equalities in the combination coefficients

$$\begin{cases}
c_{k,\ell}^{\xi_m} = 0 & \text{if } k \neq \ell \text{ or } k \neq m \\
c_{k,\ell}^{\varsigma_j} = 0 & \text{if } j \notin \mathcal{I}_k \text{ or } j \notin \mathcal{I}_\ell
\end{cases}$$
(25)

First set of equalities together with (23) show that $c_{k,k}^{\xi_k} = 1$ for each node k. Hence, the vector of local parameters ξ_k^o is only estimated by node k, which is the only node of the network performing measurements where ξ_k^o is involved. Continuing the analysis of (25), from the second set of equalities we can verify that node k only cooperate to estimate the vectors of common parameters that are within its interests, i.e. $\{\varsigma_i^o\}_{j\in\mathcal{I}_k}$. Then, taking into account (22) we can easily show that

$$c_{k,\ell}^{\varsigma_j} = 0 \text{ if } \ell \notin \mathcal{N}_k \cap \mathcal{C}_j; \quad \sum_{\ell \in \mathcal{N}_k \cap \mathcal{C}_j} c_{k,\ell}^{\varsigma_j} = 1$$
 (26)

As a result, when a node k estimates a specific vector of common parameters that is within its interest, i.e. ς_i^o with $j \in \mathcal{I}_k$, it will only cooperate with the subset of neighbour nodes $\mathcal{N}_k \cap \mathcal{C}_j$, which is composed of the neighbour nodes whose measurements are dependent on ς_i^o .

At this point, from (24) together with (21)-(23) and (25), we can obtain the Combine-then-Adapt (CTA) diffusion-based LMS algorithm summarized below.

CTA Diffusion-based LMS for NSPE (CTA D-NSPE)

- Start with some initial guesses $\psi_k^{(0)}$, $\{\varsigma_j^{(0)}\}_{j\in\mathcal{I}_k}$ and $\xi_k^{(0)}$ at each node $k\in\{1,2,\ldots,N\}$.
- For the estimation of w^o and any ς_i^o , choose $N \times N$ combination matrices C^w and C^{ς_j} whose elements in each row k, i.e., $\{c_{k,\ell}^w\}_{\ell=1}^N$ and $\{c_{k,\ell}^{\varsigma_j}\}_{\ell=1}^N$, satisfy (21) and (26), respectively.
- At each time i, for each $k \in \{1, 2, ..., N\}$, execute

- Combination step:

$$\phi_{k,w}^{(i-1)} = \sum_{\ell \in \mathcal{N}_k} c_{k,\ell}^w \, \psi_{\ell}^{(i-1)} \tag{27}$$

and

$$\phi_{k,\varsigma_j}^{(i-1)} = \sum_{\ell \in \mathcal{N}_k \cap \mathcal{C}_i} c_{k,\ell}^{\varsigma_j} \varsigma_{\ell,j}^{(i-1)} \tag{28}$$

for each $j \in \mathcal{I}_k$.

- Adaptation step:

$$\begin{bmatrix} \psi_{k}^{(i)} \\ \boldsymbol{\varsigma}_{k}^{(i)} \\ \boldsymbol{\xi}_{k}^{(i)} \end{bmatrix} = \begin{bmatrix} \phi_{k,w}^{(i-1)} \\ \phi_{k,s}^{(i-1)} \\ \boldsymbol{\xi}_{k}^{(i-1)} \end{bmatrix} + \mu_{k} U_{k,i}^{H} \begin{bmatrix} d_{k,i} - U_{k,i} & \phi_{k,w}^{(i-1)} \\ \phi_{k,s}^{(i-1)} \\ \boldsymbol{\xi}_{k}^{(i-1)} \end{bmatrix} \end{bmatrix}$$
(29)

$$\begin{array}{lll} \text{with} & \varsigma_k^{(i)} &=& \operatorname{col}\Big\{\big\{\varsigma_{k,j}^{(i)}\big\}_{j\in\mathcal{I}_k}\Big\} & \text{and} & \phi_{k,\varsigma}^{(i)} &=& \operatorname{col}\Big\{\big\{\phi_{k,\varsigma_j}^{(i)}\big\}_{j\in\mathcal{I}_k}\Big\}. \end{array}$$

Now, let us consider that each node k firstly performs the adaptation step and afterwards, it solves its local weighted least squares problem given in (16). Then, by following a derivation that is analogous to the one undertaken for the CTA D-NSPE scheme and that has been omitted for the sake of brevity, we can obtain the Adapt-then-Combine (ATC) diffusion-based LMS algorithm. Basically, as it is summarized in the table shown below, the new NSPE algorithm consists in reversing the order under which the adaptation and combination steps are performed for each node k according to the CTA D-NSPE strategy.

ATC Diffusion-based LMS for NSPE (ATC D-NSPE)

- Start with some initial guesses $\phi_{k,w}^{(0)}, \{\phi_{k,\varsigma_j}^{(0)}\}_{j\in\mathcal{I}_k}$ and $\xi_k^{(0)}$ at each node $k \in \{1, 2, \dots, N\}$.
- For the estimation of w^o and any ς_j^o , choose $N \times N$ combination matrices C^w and C^{c_j} whose elements in each row k, i.e., $\{c_{k,\ell}^w\}_{\ell=1}^N$ and $\{c_{k,\ell}^{s_j}\}_{\ell=1}^N$, satisfy (21) and (26), respectively.
- At each time i, for each $k \in \{1, 2, ..., N\}$, execute - Adaptation step:

$$\begin{bmatrix} \psi_k^{(i)} \\ \varsigma_k^{(i)} \\ \xi_k^{(i)} \end{bmatrix} = \begin{bmatrix} \phi_{k,w}^{(i-1)} \\ \phi_{k,\varsigma}^{(i-1)} \\ \xi_k^{(i-1)} \end{bmatrix} + \mu_k \, U_{k,i}^H \begin{bmatrix} d_{k,i} - U_{k,i} & \phi_{k,s}^{(i-1)} \\ \phi_{k,\varsigma}^{(i-1)} \\ \xi_k^{(i-1)} \end{bmatrix} \end{bmatrix}$$

with
$$\varsigma_k^{(i)} = \operatorname{col}\left\{\left\{\varsigma_{k,j}^{(i)}\right\}_{j\in\mathcal{I}_k}\right\}$$
 and $\phi_{k,\varsigma}^{(i)} = \operatorname{col}\left\{\left\{\phi_{k,\varsigma_j}^{(i)}\right\}_{j\in\mathcal{I}_k}\right\}$. - Combination step:

$$\phi_{k,w}^{(i)} = \sum_{\ell \in \mathcal{N}_k} c_{k,\ell}^w \, \psi_{\ell}^{(i)} \tag{31}$$

and

$$\phi_{k,\varsigma_{j}}^{(i)} = \sum_{\ell \in \mathcal{N}_{k} \cap \mathcal{C}_{i}} c_{k,\ell}^{\varsigma_{j}} \, \varsigma_{\ell,j}^{(i)}. \tag{32}$$

for each $j \in \mathcal{I}_k$.

Although the algorithms have been designed for the case where parameters of local, common and global interest coexist, note that the derived algorithms can be simplified straightforwardly to any other NSPE setting. For instance, the derived algorithms can be easily simplified to a setting where there not parameters of global interest or where some of the nodes do not have parameters of local interest. Nevertheless, independently of the considered NSPE setting, we can check that both diffusion-based NSPE algorithms are scalable in terms of computational burden and energy resources. On the one hand, regarding the computational complexity, at each time instant, each node k only needs to update a maximum of $1+2(1+|\mathcal{I}_k|)$ vectors whose dimensions are independent of the number of nodes. On the other hand, at each time instant i, in both algorithms each node k is required to transmit a maximum of $1 + |\mathcal{I}_k|$ vectors, whose dimensions are again independent of the number of nodes.

IV. PERFORMANCE ANALYSIS

This section is devoted to the performance analysis of CTA D-NSPE and ATC D-NSPE algorithms proposed in Section III. We start by considering a general recursion that includes both algorithms and that captures the behavior of individual nodes across the network. We then study the convergence in the mean of the general model. Finally, we characterize its mean-square performance in the steady-state in terms of Mean-Square Deviation (MSD) and Excess Mean-Square Error (EMSE).

A. Network-wide recursion

In this subsection, we derive a general algorithmic form that includes CTA D-NSPE and ATC D-NSPE as special cases. In particular, let us write the first combination step as

$$\phi_{k,w}^{(i-1)} = \sum_{\ell \in \mathcal{N}_k} c_{k,\ell}^w \, q_{\ell,w}^{(i-1)} \tag{33}$$

and

$$\phi_{k,\varsigma_{j}}^{(i-1)} = \sum_{\ell \in \mathcal{N}_{\ell} \cap \mathcal{C}_{\ell}} c_{k,\ell}^{\varsigma_{j}} \, q_{\ell,\varsigma_{j}}^{(i-1)} \tag{34}$$

for each j belonging to \mathcal{I}_k . Moreover, the adaptation step is expressed in the following

$$\begin{bmatrix} \psi_k^{(i)} \\ \zeta_k^{(i)} \\ \xi_k^{(i)} \end{bmatrix} = \begin{bmatrix} \phi_{k,w}^{(i-1)} \\ \phi_{k,s}^{(i-1)} \\ \xi_k^{(i-1)} \end{bmatrix} + \mu_k U_{k,i}^H \begin{bmatrix} d_{k,i} - U_{k,i} & \phi_{k,w}^{(i-1)} \\ \phi_{k,s}^{(i-1)} \\ \xi_k^{(i-1)} \end{bmatrix}$$
(35)

where, with a slight abuse of notation, $\varsigma_k^{(i)} = \operatorname{col}\left\{\left\{\varsigma_{k,j}^{(i)}\right\}_{j\in\mathcal{I}_k}\right\}$ and $\phi_{k,\varsigma}^{(i-1)} = \operatorname{col}\left\{\left\{\phi_{k,\varsigma_j}^{(i-1)}\right\}_{j\in\mathcal{I}_k}\right\}$. The last step of each iteration of the general algorithmic form is the second combination

step described as

$$q_{k,w}^{(i)} = \sum_{\ell \in \mathcal{N}_{t}} a_{k,\ell}^{w} \, \psi_{\ell}^{(i)} \tag{36}$$

and

$$q_{k,\varsigma_j}^{(i)} = \sum_{\ell \in \mathcal{N}_k \cap \mathcal{C}_j} a_{k,\ell}^{\varsigma_j} \varsigma_{\ell,j}^{(i)}.$$
 (37)

for each j belonging to \mathcal{I}_k . In (33), the non-negative real coefficient $c_{k,\ell}^w$ corresponds to the (k,ℓ) -th entries of the $(N\times N)$ combination matrix C^w , which satisfies $C^w1_N=1_N$. Moreover, in (34), the non-negative real coefficient $c_{k,\ell}^{\varsigma_j}$ corresponds to the $(|\mathcal{C}_{j,k}|,|\mathcal{C}_{j,\ell}|)$ entry of a $(|\mathcal{C}_j|\times|\mathcal{C}_j|)$ combination matrix C^{ς_j} , which satisfies $C^{\varsigma_j}1_{|\mathcal{C}_j|}=1_{|\mathcal{C}_j|}$ with

$$\mathcal{C}_{i,k} = \{ k' \in \mathcal{C}_i : k' \le k \}. \tag{38}$$

and $k, \ell \in \mathcal{C}_j$ for any $j \in \{1, 2, \dots, J\}$. Similarly, in (36)-(37) the non-negative real coefficients $\{a_{k,\ell}^w\}$ and $\{a_{k,\ell}^{\varsigma_j}\}$ correspond to the (k,ℓ) -th and the $(|\mathcal{C}_{j,k}|, |\mathcal{C}_{j,\ell}|)$ -th entries of the $(N \times N)$ and $(|\mathcal{C}_j| \times |\mathcal{C}_j|)$ combination matrices A^w and A^{ς_j} , respectively, which satisfy

$$A^{w}1_{N} = 1_{N}, \qquad A^{\varsigma_{j}}1_{|\mathcal{C}_{j}|} = 1_{|\mathcal{C}_{j}|}$$

for any $j \in \{1, 2, ..., J\}$.

Also, note that if we set $C^w = I_N$, $C^{\varsigma_j} = I_{|\mathcal{C}_j|}$ for $j \in \{1, 2, \ldots, J\}$, equations (33)-(37) represent ATC D-NSPE. On the other hand, its CTA counterpart corresponds to selecting $A^w = I_N$, $A^{\varsigma_j} = I_{|\mathcal{C}_j|}$ for $j \in \{1, 2, \ldots, J\}$.

Now, let us interpret data as random variables. Associated with the quantities in the general form in (33)-(37), we define the weight-error vectors, for $k = \{1, \ldots, N\}$ and $j = \{1, \ldots, J\}$, as follows

$$\tilde{\phi}_{k,w}^{(i)} = w^{o} - \phi_{k,w}^{(i)}, \, \tilde{p}_{k,w}^{(i)} = w^{o} - \psi_{k}^{(i)}, \, \tilde{q}_{k,w}^{(i)} = w^{o} - q_{k,w}^{(i)}
\tilde{\phi}_{k,\varsigma_{j}}^{(i)} = \varsigma_{j}^{o} - \phi_{k,\varsigma_{j}}^{(i)}, \, \tilde{p}_{k,\varsigma_{j}}^{(i)} = \varsigma_{j}^{o} - \varsigma_{k,j}^{(i)}, \, \tilde{q}_{k,\varsigma_{j}}^{(i)} = \varsigma_{j}^{o} - q_{k,\varsigma_{j}}^{(i)}
\tilde{\phi}_{k,\xi_{k}}^{(i)} = \xi_{k}^{o} - \xi_{k}^{(i)}, \, \tilde{p}_{k,\xi_{k}}^{(i)} = \xi_{k}^{o} - \xi_{k}^{(i)}, \, \tilde{q}_{k,\xi_{k}}^{(i)} = \xi_{k}^{o} - \xi_{k}^{(i)}.$$
(39)

Next, we collect these quantities across all agents into the corresponding $(\sum_{k=1}^N M_k \times 1)$ block vectors, i.e., network weight-error vectors,

$$\tilde{\phi}_{i} = \operatorname{col}\left\{\left\{\tilde{\phi}_{k,w}^{(i)}, \{\tilde{\phi}_{k,\varsigma_{j}}^{(i)}\}_{j \in \mathcal{I}_{k}}, \tilde{\phi}_{k,\xi_{k}}^{(i)},\right\}_{k=1}^{N}\right\}$$
(40)

In the same vein, the network vectors $\tilde{\boldsymbol{p}}_i$ and $\tilde{\boldsymbol{q}}_i$ are formed, by stacking the corresponding weight-error vectors. For notational convenience, hereafter we use

$$\check{M} = \sum_{k=1}^{N} M_k \,.$$

To proceed, let us introduce the diagonal matrix

$$\mathcal{M} = \operatorname{diag}\{\mu_1 I_{M_1}, \dots, \mu_N I_{M_N}\} \qquad (\breve{M} \times \breve{M}), \tag{41}$$

the block-diagonal matrix

$$\mathcal{D}_i = \operatorname{diag}\{\mathbf{U}_{1,i}^H \mathbf{U}_{1,i}, \dots, \mathbf{U}_{N,i}^H \mathbf{U}_{N,i}\} \qquad (\check{M} \times \check{M}), \quad (42)$$

and the vector

$$\mathbf{\mathcal{V}}_i = \operatorname{col}\{\mathbf{U}_{1\ i}^H \mathbf{v}_{1,i}, \dots, \mathbf{U}_{N\ i}^H \mathbf{v}_{N,i}\} \qquad (\breve{M} \times 1). \tag{43}$$

Finally, the network-wide behavior can be characterized by these relations for the block quantities:

$$\tilde{\boldsymbol{\phi}}_{i-1} = \tilde{\boldsymbol{\mathcal{C}}} \, \tilde{\boldsymbol{q}}_{i-1} \tag{44}$$

$$\tilde{\boldsymbol{p}}_i = (I - \mathcal{M}\boldsymbol{\mathcal{D}}_i)\tilde{\boldsymbol{\phi}}_{i-1} - \mathcal{M}\boldsymbol{\mathcal{V}}_i \tag{45}$$

$$\tilde{\boldsymbol{q}}_i = \breve{\mathcal{A}} \, \tilde{\boldsymbol{p}}_i \tag{46}$$

where the structure of the extended weighting matrices \check{A} and \check{B} is explained in the following subsection.

Note that equations (44)-(46) can be summarized in the following equivalent form,

$$\tilde{\boldsymbol{q}}_{i} = \breve{\mathcal{A}} \left(I - \mathcal{M} \boldsymbol{\mathcal{D}}_{i} \right) \breve{\mathcal{C}} \, \tilde{\boldsymbol{q}}_{i-1} - \breve{\mathcal{A}} \, \mathcal{M} \boldsymbol{\mathcal{V}}_{i}, \tag{47}$$

which will be used in the Subsections IV-D and IV-E to perform the mean and the mean-square steady-state analysis, respectively.

B. Structure of the extended weighting matrices

The extended weighting matrices \check{C} and \check{A} have the same form, only the weights are different. Therefore, in order to define them, let us consider, for instance, the \check{A} matrix,

$$\breve{\mathcal{A}} = \operatorname{col}\left\{A_1^w, \{A_1^{\varsigma_j}\}_{j \in \mathcal{I}_1}, A_1^{\xi_1}, \dots, A_N^w, \{A_N^{\varsigma_j}\}_{j \in \mathcal{I}_N}, A_N^{\xi_N}\right\},$$
(48)

where the blocks being stacked are defined in (49)-(52) on the top of the following page, with \mathcal{I}_{jk} in (51) defined as $\mathcal{I}_{jk} = \{j' \in \mathcal{I}_k : j' < j\}$.

An alternative way to define $\check{\mathcal{A}}$ is the following relation

$$\breve{\mathcal{A}} = \mathcal{P}\,\breve{\mathcal{A}}^{\text{blkd}}\,\mathcal{P}^T \tag{53}$$

where the block-diagonal matrix is given by

$$\check{\mathcal{A}}^{\text{blkd}} = \text{blockdiag} \left\{ A^w \otimes I_{M_g}, \left\{ A^{\varsigma_j} \otimes I_{M_c} \right\}_{j=1}^J, I_N \otimes I_{M_l} \right\}$$
(54)

while \otimes stands for the Kronecker product, and \mathcal{P} is the $M \times M$ permutation matrix that stacks appropriately chosen $1 \times M$ row basis vectors. In particular, a basis vector e_k has the unity at the k^{th} position and zeros elsewhere. For more details how the \mathcal{P} matrix is specified, see Appendix A.

C. Data assumptions

To proceed, we state the following independence assumptions on the data:

- A1) $\mathbf{v}_{k,i}$ is temporally and spatially white noise whose covariance matrix is $R_{v_k,i}$ and which is independent of $\mathbf{U}_{k',i'}$ for all k' and i', with $k,k' \in \{1,2,\ldots,N\}$ and i,i'>0;
- A2) $U_{k,i}$ is independent of $U_{k,i'}$, with i,i' > 0 and $i \neq i'$ (temporal independence).
- A3) $U_{k,i}$ is independent of $U_{k',i}$, with $k, k' \in \{1, 2, ..., N\}$ and $k \neq k'$ (spatial independence),

A4) $\mathbf{U}_{k_g,i}$, $\mathbf{U}_{k_{jc},i}$ and $\mathbf{U}_{k_l,i}$ are independent for all $k \in \{1,2,\ldots,N\}$ and $j \in \{1,2,\ldots,J\}$;

In order to evaluate the fourth-order moment of the matrix-valued regression data in Subsection IV-E, we further assume:

A5) $\mathbf{U}_{k,i} \ (L_k \times M_k)$ has a real matrix variate normal distribution specified by mean $0_{L_k \times M_k}$ and positive-semidefinite matrices $\Psi_k \ (M_k \times M_k)$ and $\Omega_k \ (L_k \times L_k)$ (see [24, Chapter 2]). Equivalently, using standard notation for multivariate normal distribution, the distribution of $\mathbf{U}_{k,i}$ can be defined as $\mathrm{vec}(\mathbf{U}_{k,i}) \sim \mathcal{N}_{M_k L_k} \ (\mathrm{vec}(0_{L_k \times M_k}), \Psi_k \otimes \Omega_k)$.

Remark 1: Note that even for the vector-valued regression data, in order to evaluate the fourth-order moment, the Guassian assumption is required (e.g. see [8] and [9]). The results of the fourth-order moment of the matrix-valued regression data appear to be quite a bit more challenging than those on its vector counterpart, due to the extra dimension involved. Therefore, the assumption A5) seems well-justified.

D. Mean stability

By taking the expectation of (47) and using assumptions (A1-A3), we obtain

$$E\tilde{\boldsymbol{q}}_{i} = \breve{\mathcal{A}}\left(I - \mathcal{M}\mathcal{R}_{U}\right) \breve{\mathcal{C}} E\tilde{\boldsymbol{q}}_{i-1}, \tag{55}$$

where

$$\mathcal{R}_U = E \mathcal{D}_i = \text{blockdiag}\{R_{U_1}, R_{U_2}, \dots, R_{U_N}\}, \quad (56)$$

and

$$R_{U_k} = E \mathbf{U}_{k,i}^H \mathbf{U}_{k,i} = \text{blockdiag}\{R_{U_{k_a}}, R_{U_{k_{ic}}}, R_{U_{k_l}}\}.$$
 (57)

The algorithm in (47) is asymptotically unbiased, i.e, $E\tilde{q}_i \to 0_{\check{M}\times 1}$ as $i\to\infty$, if the matrix $\check{\mathcal{A}}(I-\mathcal{M}\mathcal{R}_U)\check{\mathcal{C}}$ is stable. In order to prove its stability, we will build on the approaches taken in [25] and [26], by selecting a convenient matrix norm $\|\cdot\|$ and exploit its submultiplicativity property, i.e., $\|AB\| \le \|A\| \|B\|$.

Here, we use the induced block maximum matrix norm [25], [26], however, defined over a block matrix with different block sizes. In particular, let x be a $\check{M} \times 1$ vector consisting of \check{N} blocks, where $\check{N} = N + \sum_{k=1}^{n} |\mathcal{I}_k| + N$, given as

$$x = \operatorname{col}\left\{\left\{x_k^{(w)}, \{x_k^{(\varsigma_j)}\}_{j \in \mathcal{I}_k}, x_k^{(\xi)}, \right\}_{k=1}^N\right\}.$$

The block maximum norm is defined by

$$\|x\|_{b,\infty} = \max_{1 \le k \le N} \left\{ \{\|x_k^{(w)}\|, \left\{\|x_k^{(\varsigma_j)}\|\right\}_{j \in \mathcal{I}_k}, \|x_k^{(\xi_k)}\| \right\}$$

where $\|\cdot\|$ denotes the Euclidean norm of its argument. Next, we define the matrix norm induced from the block maximum norm, i.e.,

$$||A||_{b,\infty} = \max_{||x||_{b,\infty}=1} ||Ax||_{b,\infty}$$

where A is $\check{M} \times \check{M}$ matrix. As in [25], it can be straightforwardly shown that the block maximum norm has the unitary invariance property of the Euclidean norm under properly defined block-wise transformation.

$$A_k^w = \begin{bmatrix} a_{k,1}^w I_{M_g} & 0_{M_g \times (M_c|\mathcal{I}_1| + M_l)} & a_{k,2}^w I_{M_g} & 0_{M_g \times (M_c|\mathcal{I}_2| + M_l)} & \dots & a_{k,N}^w I_{M_g} & 0_{M_g \times (M_c|\mathcal{I}_N| + M_l)} \end{bmatrix}$$
(49)

$$A_k^{\varsigma_j} = \begin{bmatrix} A_{k1}^{\varsigma_j} & A_{k2}^{\varsigma_j} & \dots & A_{kN}^{\varsigma_j} \end{bmatrix}$$
 (50)

$$A_{k\ell}^{\varsigma_j} = \begin{cases} \begin{bmatrix} 0_{M_c \times (M_g + M_c \mid \mathcal{I}_{j\ell}|)} & a_{k,\ell}^{\varsigma_j} I_{M_c} & 0_{M_c \times (M_c \mid \mathcal{I}_{\ell}| - \mid \mathcal{I}_{j\ell}| - 1] + M_l)} \end{bmatrix} & \text{if } \ell \in \mathcal{C}_j, \\ 0_{M_c \times M_\ell} & \text{if } \ell \notin \mathcal{C}_j \end{cases}$$

$$(51)$$

$$A_k^{\xi_k} = \begin{bmatrix} 0_{M_l \times (\sum_{\ell=1}^{k-1} M_\ell + M_g + M_c | \mathcal{I}_k |)} & I_{M_l} & 0_{M_l \times (\sum_{\ell=k+1}^{N} M_\ell)} \end{bmatrix}$$
 (52)

Next, by evaluating the block maximum norm of (55) and by applying its submultiplicativity property, we obtain the following relation

$$\|E\tilde{\boldsymbol{q}}_{i}\|_{b,\infty} \leq \|\check{\boldsymbol{\mathcal{A}}}\|_{b,\infty} \|I - \mathcal{M}\mathcal{R}_{U}\|_{b,\infty} \|\check{\boldsymbol{\mathcal{C}}}\|_{b,\infty} \|E\tilde{\boldsymbol{q}}_{i-1}\|_{b,\infty}. \tag{58}$$

Let us now evaluate the block maximum norms of the extended combination matrices $\check{\mathcal{A}}$ and $\check{\mathcal{C}}$ given in (53), e.g., $\|\check{\mathcal{A}}\|_{b,\infty}$, while the same holds for $\|\check{\mathcal{C}}\|_{\infty}$. Since $\check{\mathcal{A}}$ is a right stochastic matrix, we can bound $\|\check{\mathcal{A}}\|_{\infty}$ as shown in (59) at the top of the next page. Thus, $\|\check{\mathcal{A}}\|_{b,\infty} < 1$, given that A^w , A^{ς_j} are row-stochastic, i.e., $A^w 1_N = 1_N$ and $A^{\varsigma_j} 1_{|\mathcal{C}_j|} = 1_{|\mathcal{C}_j|}$, for $j = \{1, \ldots, J\}$.

At this point, we only need to find the conditions that secure

$$||I - \mathcal{MR}_U||_{b,\infty} < 1.$$

Under assumption A4, due to the unitary invariance of the block maximum these conditions correspond to the mean stability conditions of stand-alone LMS filters and can be easily realized to be

$$\mu_k < \frac{2}{\lambda_{\max}(R_{U_{k_a}}, R_{U_{k_{ic}}}, R_{U_{k_l}})} \qquad \text{for each } k,$$

where $k = \{1, ..., N\}$ and $\lambda_{\max}(X, Y, Z)$ denotes the maximum of the maximum eigenvalues of the Hermitian matrix arguments X, Y and Z.

The above discussion is summarized in the subsequent theorem.

Theorem 1. For any initial conditions, under the assumptions A1-A4 made in Subsection IV-C, if the positive step-size of each node satisfies $\mu_k < 2/\lambda_{\max}(R_{U_{k_g}}, R_{U_{k_{j_c}}}, R_{U_{k_l}})$, then the estimates generated by ATC (or CTA) D-NSPE algorithm converge in the mean, i.e.,

$$\lim_{i \to \infty} E\tilde{\boldsymbol{q}}_i = 0_{\check{M} \times 1},\tag{60}$$

if the combination matrices related to the estimates of global and common parameters are row-stochastic.

E. Steady-state performance

At this point, we aim to evaluate the mean-square performance of the general diffusion model in (47). In particular, we will examine the performance in the stady-state in terms of MSD and EMSE.

To this end, we use the energy conservation arguments [23], [26]. Specifically, after equating the weighted norm of (47) and taking the expectation under Assumptions A1-A3, we obtain the subsequent variance relation

$$E\|\tilde{\boldsymbol{q}}_i\|_{\Sigma}^2 = E\|\tilde{\boldsymbol{q}}_{i-1}\|_{\Sigma'}^2 + E\left\{\boldsymbol{\mathcal{V}}_i^H \mathcal{M} \breve{\mathcal{A}}^T \Sigma \breve{\mathcal{A}} \mathcal{M} \boldsymbol{\mathcal{V}}_i\right\}$$
(61)

where Σ is an arbitrary $(\check{M}\times\check{M})$ Hermitian nonnegative-definite matrix that we are free to choose, and

$$\Sigma' = E\left\{ \check{\mathcal{C}}^T \left(I - \mathcal{M} \mathcal{D}_i \right)^H \check{\mathcal{A}}^T \Sigma \check{\mathcal{A}} \left(I - \mathcal{M} \mathcal{D}_i \right) \check{\mathcal{C}} \right\}. \quad (62)$$

To proceed, we have to extract Σ from r.h.s. of (62) and from the second term on r.h.s. in (61). To do so, we will use vectorization operator and exploit some useful properties of the trace operator and Kronecker product, i.e.,

$$\operatorname{vec}(ABC) = (C^T \otimes A)\operatorname{vec}(B) \tag{63}$$

and

$$Tr(AB) = vec(A^T)^T vec(B).$$
 (64)

Furthermore, in addition to Assumptions A1-A3, here we also use Assumption A5, stated in Subsection IV-C.

Thus, after defining $V = E \mathcal{V}_i \mathcal{V}_i^H = \text{blockdiag} \left(\left\{ (\operatorname{Tr} R_{v,k} \Omega_k) \Psi_k \right\}_{k=1}^N \right)$ [27], we get

$$E\|\tilde{\boldsymbol{q}}_i\|_{\Sigma}^2 = E\|\tilde{\boldsymbol{q}}_{i-1}\|_{\Sigma'}^2 + \operatorname{Tr}\left(\breve{\boldsymbol{\mathcal{A}}}\,\boldsymbol{\mathcal{M}}\,\boldsymbol{V}\,\boldsymbol{\mathcal{M}}\breve{\boldsymbol{\mathcal{A}}}^T\,\boldsymbol{\Sigma}\right). \tag{65}$$

Next, we introduce $\sigma = \text{vec}(\Sigma)$. In order to extract Σ from Σ' , we take the following steps

$$\sigma' = \text{vec}(\Sigma') = F\sigma \tag{66}$$

where F is a matrix, of dimensions $\check{M}^2 \times \check{M}^2$, given by

$$F = E\left\{ \left(\check{\mathcal{A}} \left(I - \mathcal{M} \mathcal{D}_{i} \right) \check{\mathcal{C}} \right)^{T} \otimes \left(\check{\mathcal{C}}^{T} \left(I - \mathcal{M} \mathcal{D}_{i} \right)^{T} \check{\mathcal{A}}^{T} \right) \right\}$$

$$= \left(\check{\mathcal{C}}^{T} \otimes \check{\mathcal{C}}^{T} \right) E\left\{ \left(I - \mathcal{M} \mathcal{D}_{i} \right)^{T} \check{\mathcal{A}}^{T} \otimes \left(\left(I - \mathcal{M} \mathcal{D}_{i} \right)^{T} \check{\mathcal{A}}^{T} \right) \right\}$$

$$= \left(\check{\mathcal{C}}^{T} \otimes \check{\mathcal{C}}^{T} \right) G\left(\check{\mathcal{A}}^{T} \otimes \check{\mathcal{A}}^{T} \right)$$
(67)

with

$$G = E\left\{ (I - \mathcal{M}\mathcal{D}_i)^T \otimes (I - \mathcal{M}\mathcal{D}_i)^T \right\}$$

= $I \otimes I - \mathcal{R}_U \mathcal{M} \otimes I - I \otimes \mathcal{R}_U \mathcal{M} + E\{\mathcal{D}_i^T \mathcal{M} \otimes \mathcal{D}_i^T \mathcal{M}\}$
(68)

and $\mathcal{R}_U = \text{blockdiag}(\{\text{Tr}(\Omega_k)\Psi_k\})$ (see (56)).

$$\|\breve{\mathcal{A}}x\|_{b,\infty} = \max_{1 \le k \le N} \left\{ \left\| \sum_{\ell \in \mathcal{N}_k} a_{k,\ell}^w x_{\ell}^{(w)} \right\|, \left\{ \left\| \sum_{\ell \in \mathcal{N}_k \cap \mathcal{C}_j} a_{k,\ell}^{\varsigma_j} x_{\ell}^{(\varsigma_j)} \right\| \right\}_{j \in \mathcal{I}_k}, \left\| x_k^{(\xi_k)} \right\| \right\}$$

$$\leq \max_{1 \le k \le N} \left\{ \sum_{\ell \in \mathcal{N}_k} |a_{k,\ell}^w|, \max_{j \in \mathcal{I}_k} \sum_{\ell \in \mathcal{N}_k \cap \mathcal{C}_j} |a_{k,\ell}^{\varsigma_j}|, 1 \right\} \|x\|_{b,\infty} = \|x\|_{b,\infty}.$$

$$(59)$$

For sufficiently small step sizes, the forth-order moment of regressors, i.e., the rightmost term in (68), can be discarded. However, this term can be evaluated as follows

$$E\{\mathcal{D}_i^T \mathcal{M} \otimes \mathcal{D}_i^T \mathcal{M}\} = S \cdot (\mathcal{M} \otimes \mathcal{M}) \tag{69}$$

where

$$S = E\{\mathcal{D}_{i}^{T} \otimes \mathcal{D}_{i}^{T}\}$$

$$= \operatorname{blockdiag}\left(\left\{E\{\mathbf{U}_{k,i}^{H}\mathbf{U}_{k,i} \otimes \mathcal{D}_{i}\}\right\}_{k=1}^{N}\right)$$
(70)

with

$$E\{\mathbf{U}_{k,i}^{H}\mathbf{U}_{k,i}\otimes\boldsymbol{\mathcal{D}}_{i}\}=K_{(M_{k},\check{M})}E\{\boldsymbol{\mathcal{D}}_{i}\otimes\mathbf{U}_{k,i}^{H}\mathbf{U}_{k,i}\}K_{(\check{M},M_{k})}$$
(71)

and $K_{(m,n)}$ denoting the $mn \times mn$ commutation matrix that satisfies

$$K_{(m,n)}\operatorname{vec}(A) = \operatorname{vec}(A^T)$$

for any $m \times n$ matrix A [28]. In (71), it can be shown that

$$E\{\mathcal{D}_{i} \otimes \mathbf{U}_{k,i}^{H} \mathbf{U}_{k,i}\}$$

$$= \operatorname{blockdiag}\left(\left\{E\{\mathbf{U}_{\ell,i}^{H} \mathbf{U}_{\ell,i} \otimes \mathbf{U}_{k,i}^{H} \mathbf{U}_{k,i}\}\right\}_{\ell=1}^{N}\right).$$
(72)

Moreover, from [27] and [29], we can obtain closed-form expressions for the the expectations that appear in (71). In particular, we can check that

$$E\{\mathbf{U}_{k,i}^{H}\mathbf{U}_{k,i} \otimes \mathbf{U}_{k,i}^{H}\mathbf{U}_{k,i}\}$$

$$= \operatorname{Tr}(\Omega_{k})\operatorname{Tr}(\Omega_{k})\Psi_{k} \otimes \Psi_{k}$$

$$+ \operatorname{Tr}(\Omega_{k}\Omega_{k})\operatorname{vec}(\Psi)\operatorname{vec}(\Psi)^{T}$$

$$+ \operatorname{Tr}(\Omega_{k}\Omega_{k})K_{(M_{k},M_{k})}(\Psi_{k} \otimes \Psi_{k})$$

$$(73)$$

and that

$$E\{\mathbf{U}_{\ell,i}^H\mathbf{U}_{\ell,i}\otimes\mathbf{U}_{k,i}^H\mathbf{U}_{k,i}\} = [\operatorname{Tr}(\Omega_\ell)\Psi_\ell]\otimes[\operatorname{Tr}(\Omega_k)\Psi_k]$$
 (74) for any $k,\ell\in\{1,2,\ldots,N\}$ with $k\neq\ell$.

To evaluate the performance measures in the steady state, i.e., $i \to \infty$, by using (64), we first rewrite (65) as

$$E\|\tilde{\boldsymbol{q}}_{\infty}\|_{\sigma}^{2} = E\|\tilde{\boldsymbol{q}}_{\infty}\|_{F\sigma}^{2} + \left[\operatorname{vec}(\check{\mathcal{A}}\,\mathcal{M}\,V^{T}\,\mathcal{M}\check{\mathcal{A}}^{T})\right]^{T}\sigma. \tag{75}$$

After rearranging, we obtain the following relation

$$E\|\tilde{\boldsymbol{q}}_{\infty}\|_{(I-F)\sigma}^{2} = \left[\operatorname{vec}(\check{\mathcal{A}}\,\mathcal{M}\,V^{T}\,\mathcal{M}\check{\mathcal{A}}^{T})\right]^{T} \cdot \sigma. \tag{76}$$

Now, we can evaluate MSD averaged across the whole network

$$MSD^{net} = \frac{1}{N} E \|\tilde{\boldsymbol{q}}_{\infty}\|_{I}^{2}$$
 (77)

by selecting $\sigma = (I - F)^{-1} \frac{1}{N} \text{vec}(I_{\check{M}})$, we obtain

$$MSD^{net} = \frac{1}{N} \left[vec(\breve{\mathcal{A}} \mathcal{M} V^T \mathcal{M} \breve{\mathcal{A}}^T) \right]^T (I - F)^{-1} vec(I_{\breve{M}}).$$
(78)

Now, in order to evaluate MSD at each node k, let us first define the *Khatri-Rao* matrix product.

Definition 2: Consider matrices A and B of dimensions $m \times n$ and $p \times q$, respectively. Let $A = (A_{ij})$ be partitioned with A_{ij} of dimensions $m_i \times n_j$ as the (i,j)-th block submatrix and let $B = (B_{ij})$ be partitioned with B_{ij} as the (i,j)-th block submatrix of dimensions $p_i \times q_j$ ($\sum m_i = m$, $\sum n_j = n$, $\sum p_i = p$ and $\sum q_j = q$). The Khatri-Rao matrix product is defined as

$$A \odot B = (A_{ij} \otimes B_{ij})_{ij}$$

where $A_{ij} \otimes B_{ij}$ is of dimensions $m_i p_i \times n_j q_j$, while $A \odot B$ is of dimensions $(\sum m_i p_i) \otimes \sum n_j q_j$, (see [30]).

Based on the previous definition, MSD at node k is

$$MSD_k = \left[\text{vec}(\breve{\mathcal{A}} \mathcal{M} V^T \mathcal{M} \breve{\mathcal{A}}^T) \right]^T (I - F)^{-1} m_k, \quad (79)$$

where

$$m_k = \text{vec}\left(\text{diag}(e_k) \odot Y\right)$$
 (80)

with $N \times N$ partitioned matrices $Y = \operatorname{blockdiag}(I_{M_1}, \ldots, I_{M_N})$ and block-diagonal matrix made of the elements of a $1 \times N$ vector e_k with the unity at the k^{th} position and zeros elsewhere.

On the other hand, MSD related to the estimation of the global, some specific common or the local vector of parameter at node k can be evaluated by redefining Y as a $(2N+\sum_{k=1}^N |\mathcal{I}_k|) \times (2N+\sum_{k=1}^N |\mathcal{I}_k|)$ partitioned matrix, i.e.

$$Y = \operatorname{blockdiag}(I_{M_g}, I_{|\mathcal{I}_1|M_c}, I_{M_l}, \dots, I_{M_g}, I_{|\mathcal{I}_N|M_c}, I_{M_l}),$$
(81)

and by taking $1 \times 2N + \sum_{k=1}^{N} |\mathcal{I}_k|$ vector e_k with the unity at the appropriate position and zeros elsewhere.

Similarly.

$$EMSE^{net} = \frac{\left[vec(\check{\mathcal{A}} \mathcal{M} V^T \mathcal{M} \check{\mathcal{A}}^T)\right]^T (I - F)^{-1} vec(\mathcal{R}_U)}{N}.$$
(82)

Additionally, EMSE at each node k is

$$EMSE_k = \left[\text{vec}(\check{\mathcal{A}} \mathcal{M} V^T \mathcal{M} \check{\mathcal{A}}^T) \right]^T (I - F)^{-1} p_k, \quad (83)$$

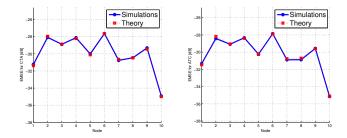


Fig. 2. Steady-state EMSE per node for CTA D-NSPE and ATC D-NSPE.

where we select a node k by

$$p_k = \operatorname{vec}\left(\operatorname{diag}(e_k) \odot \mathcal{R}_U\right)$$
 (84)

where \mathcal{R}_U is defined as $N \times N$ partitioned matrix as in (56). Under the independence of $\mathbf{U}_{k_g,i}$, $\mathbf{U}_{k_{jc},i}$ and $\mathbf{U}_{k_l,i}$, we can evaluate EMSE performance measure related to the global, specific common or local parameter at some node k. To do so, we need to properly redefine the partitions of R_U and the size of vector e_k .

V. SIMULATION RESULTS

In this section, we initially discuss some generic simulations that verify mean-square theoretical results (see Section IV-E). Afterwards, the effectiveness of the proposed algorithms are illustrated in the context of cooperative spectrum sensing in CR networks.

A. Validation of mean-square theoretical results

We assume a network with N=10 nodes where the measurements follow the observation model provided in (3) with $L_k=2$ for all k. In the considered setting, two different vectors of common parameters coexist, i.e., ς_1^o and ς_2^o . The vector ς_1^o is composed of 3 parameters, while ς_2^o consists of 2 parameters. Moreover, we consider that the area of influence of ς_1^o and ς_2^o is formed by $\mathcal{C}_1 \in \{2,3,4,5,6\}$ and $\mathcal{C}_2 \in \{5,6,7,8\}$, respectively. As a result, there are nodes that are interested in estimating zero, one or two different vector of common parameters. In addition, each node k is interested in estimating a vector of global parameters and a vector of local parameters, each one of length equal to $M_g=2$ and $M_{l_k}=3$, respectively.

The data observed by each node, i.e., $\{d_{k,i}, U_{k,i}\}$, have been generated under the assumption of a background noise $\mathbf{v}_{k,i}$ with covariance $\sigma_{v_k}^2 I_2$, where $\sigma_{v_k}^2 = \sigma_v^2 = 10^{-3}$ across the network. Furthermore, each one of the L_k rows of the regressor

$$U_{k,i} = \text{col}\left\{U_{k_g,i}^T \{U_{k_j_c,i}^T\}_{j \in \mathcal{I}_k} U_{k_l,i}^T\right\}^T$$

have been independently drawn from a time-correlated spatially independent Gaussian distribution. In particular, the c-th row of $U_{k,i}$ is generated according to a first-order autoregressive (AR) model with correlation function $r_{k,c}(i)=\sigma_{u_k}^2\alpha_k^{|i|}$ where the pair of parameters $\{\sigma_{u_k},\alpha_k\}$ are randomly chosen in (0,1) so that the the Signal-to-Noise-Ratio (SNR) at each node ranges from 10 dB to 20 dB. Hence, $\mathbf{U}_{k,i}$ follows a

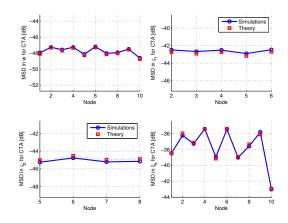


Fig. 3. Steady-state MSD per node for CTA D-NSPE LMS.

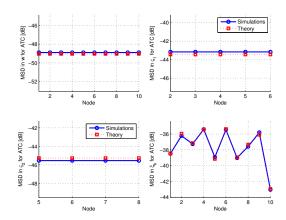


Fig. 4. Steady-state MSD per node for ATC D-NSPE LMS.

real matrix variate normal distribution specified by the mean matrix $0_{2\times M_k}$ and the positive-semidefinite matrices $\Omega_k=I_2$ and $\Psi_k=\text{toeplitz}\{[\sigma_{u_k}^2\,\sigma_{u_k}^2\,\alpha_k\,\ldots\,\sigma_{u_k}^2\,\alpha_k^{M_k-1}]^T\}$.

When implementing both CTA D-NSPE and ATC D-NSPE, static uniform combination weights have been assumed, i.e., $a_{k,\ell}^w = 1/|\mathcal{N}_k|$ for all $k,\ell \in \{1,2,\ldots,10\}$, and $a_{k,\ell}^{\varsigma_j} = 1/|\mathcal{N}_k\cap\mathcal{C}_j|$ for all $k,\ell\in\mathcal{C}_j$ and $j\in\{1,2\}$. The neighborhood of each node k has been set so that the network graph as well as the subsets \mathcal{C}_1 and \mathcal{C}_2 are connected. Moreover, in order to validate the theoretical expressions for non-fully connected networks and non-fully connected subsets \mathcal{C}_j , we have assumed that $\max_{1\leq k\leq N}\{|\mathcal{N}_k|\}\leq 4$ and that $\max_{1\leq k\leq N}\{|\mathcal{N}_k\cap\mathcal{C}_j|\}\leq 2$.

The experimental values in Figs. 2-4 result from averaging the mean-square measures over 100 independent experiments where both CTA D-NSPE LMS and ATC D-NSPE LMS are run for 10 000 iterations. Despite the temporal correlation of the regressors as well as the correlation among $\mathbf{U}_{k_g,i}$, $\mathbf{U}_{k_{jc},i}$ and $\mathbf{U}_{k_l,i}$, which was not assumed for the derivation of the theoretical results, all figures show a good match between the simulated curves and the theoretical expressions for the MSD and EMSE at each node k.

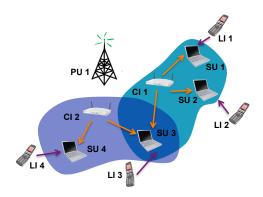


Fig. 5. An illustrative CR scenario. Each one of the 4 secondary users (SU) aims at estimating the aggregated spectrum transmitted by the Primary User (PU) and its interferer(s). Apart from the PU and its Local Interferer (LI), each SU is influenced by one or two Common Interferers (CI). SU 3 is influenced by CI 1 and CI 2, while SUs 1, 2 and 4 are only influenced by one CI.

B. Illustrative application

In the following, we will also demonstrate the performance of the proposed algorithm when used for cooperative spectrum sensing in CR networks (see [26, Section 2.4] and [31]-[33]). In brief, there are Q primary users (PU) transmitting and N secondary users (SU) sensing the power spectrum. In addition to PUs, for each SU we also assume two types of low-power interference sources, i.e., local interferer (LI) and common interferers (CI). The former is affecting only one SU, while the latter are influencing several SUs. Therefore, the aim for each SU is to estimate the aggregated spectrum transmitted by all the PUs as well as the spectrum of its own LI and CI. An example of such a scenario is given in Fig. 5.

Next, the power spectral density (PSD) of the signal transmitted by the q-th PU, denoted by $\Phi_q^t(f)$, can be approximated by using the subsequent model of X basis functions

$$\Phi_q^t(f) = \sum_{x=1}^X b_x(f) \check{w}_{qx}^o = b_0^T(f) \check{w}_q^o$$
 (85)

where $b_0(f) = [b_1(f), \dots, b_X(f)]^T \in \mathbb{R}^X$ is a vector of basis functions evaluated at frequency f and $\check{w}_q^o = [\check{w}_{q1}^o, \dots, \check{w}_{qX}^o]^T \in \mathbb{R}^X$ is a vector of weighting coefficients representing the power transmitted by the q-th PU over each basis

Let $p_{tk,i}(f) = |H_{tk}(f,i)|^2$ be the frequency-dependent attenuation coefficient, where $H_{tk}(f,i)$ is the channel frequency response between the t-th transmitter and k-th receiver [33]. For each time i and frequency f, we define

- $p_{qk,i}(f)$ denoting the attenuation coefficient between the q-th PU and the k-th SU,
- $p_{Ik,i}(f)$ referring to the attenuation coefficient between the local interferer and the k-th SU,
- $p_{jk,i}(f)$ being the attenuation coefficient between the j-th common interferer and the k-th SU, where $j \in \mathcal{I}_k$.

Then, under the assumption of spatial uncorrelation among the channels, the signal received by the k-th SU at time instant i can be expressed as

$$\Phi_{k,i}^{r}(f) = b_{k,i}^{T}(f)w_k^o + z_{k,i}, \tag{86}$$

where $w_k^o = \operatorname{col} \left\{ \check{w}_1^o, \ldots, \check{w}_Q^o, \varsigma_{\mathcal{I}_k(1)}^o, \ldots, \varsigma_{\mathcal{I}_k(|\mathcal{I}_k|)}^o, \xi_k^o \right\} \in \mathbb{R}^{(Q+|\mathcal{I}_k|+1)X}$ with ξ_k^o and ς_j^o equal to the vectors of weighting coefficients representing the power transmitted by the LI and j-th CI associated with the k-th SU, respectively. Also, $b_{k,i}(f) = p_{k,i}(f) \otimes b_0(f) \in \mathbb{R}^{(Q+|\mathcal{I}_k|+1)X}$, and

$$p_{k,i}(f) = [p_{1k,i}, \dots, p_{qk,i}, p_{\mathcal{I}_k(1),i}, \dots, p_{\mathcal{I}_k(|\mathcal{I}_k|),i}, p_{Ik,i}]^T,$$
(87)

while $z_{k,i}$ is the measurement and/or model noise. In the above expression, we dropped the frequency index for compactness of notation. Also note that, in practice, the attenuation factors $p_{tk,i}$ cannot be estimated accurately, so we assume access only to noisy estimates $\hat{p}_{tk,i}$ hereafter.

Considering that, at discrete time i, each node k observes the received PSD in (86) over L frequency samples $\{f_m\}_{m=1}^L$, the subsequent vector linear model is obtained

$$\mathbf{d}_{k,i} = \mathbf{U}_{k,i} w_k^o + \mathbf{v}_{k,i} \tag{88}$$

where $\mathbf{v}_{k,i}$ denotes noise with zero mean and covariance matrix R_{v_k} of dimension $L \times L$ and $\mathbf{U}_{k,i} = [b_{k,i}(f_1) \dots b_{k,i}(f_L)]^T$ is of dimension $L \times (Q + |\mathcal{I}_k| + 1)X$ with $L > (Q + |\mathcal{I}_k| + 1)X$.

For the computer simulations presented here, we consider a scenario where there is only one common interferer whose PSD can be sensed by nodes in $C_1 = \{2, 4, 7, 9\}$. Furthermore, we analyze the ATC D-NSPE LMS scheme for several different combining strategies and degrees of connectivity. In particular, we consider the ATC D-NSPE LMS algorithms with

- a) the same neighborhood size at all the nodes, i.e., $|\mathcal{N}_k| = 5$, while $|\mathcal{N}_k \cap \mathcal{C}_1| = 3$ for all $k \in C_1$. In this scenario, we employ the static uniform combination weights, i.e., $a_{k,\ell}^w = 1/5$ and $a_{k,\ell}^{\varsigma_1} = 1/3$.
- b) the clique topology, i.e., $|\mathcal{N}_k| = N$ and $|\mathcal{N}_k \cap \mathcal{C}_1| = |\mathcal{C}_1|$ for all $k \in C_1$, with corresponding static uniform combination weights,
- c) the topology set as in a), while the combination weights are adaptive. Specifically, the weights corresponding to both global and common parameter estimation processes are being adapted according to the adaptive combination mechanism proposed in [16]. For instance, the weights $a_{k,\ell}^w$, for $\ell \in \mathcal{N}_k$, evolve as

$$a_{k,\ell}^{w}(i) = \frac{\gamma_{k,\ell}^{-2}(i)}{\sum_{m \in \mathcal{N}_k} \gamma_{k,m}^{-2}(i)}$$

with

$$\gamma_{k,\ell}^2(i) = (1-\nu)\gamma_{k,\ell}^2(i-1) + \nu||\psi_{\ell}^{(i)} - \phi_{k,w}^{(i-1)}||^2.$$

We also compare these schemes with an LMS-based non-cooperative strategy as well as with the incremental-based NSPE LMS (I-NSPE LMS), developed in [21], that is used as a benchmark.

The step-size of the LMS adaptation at each node is set equal to $\mu_k=0.04$ for all the algorithms, expect for the incremental NSPE where μ_k is the step-size for estimating the local parameters only. In the I-NSPE LMS, the step-sizes for estimating global and common parameters are set to $\mu_w^{I-NSPE}=\mu_k/N$ and $\mu_{\varsigma_j}^{I-NSPE}=\mu_k/|\mathcal{C}_j|$, respectively.

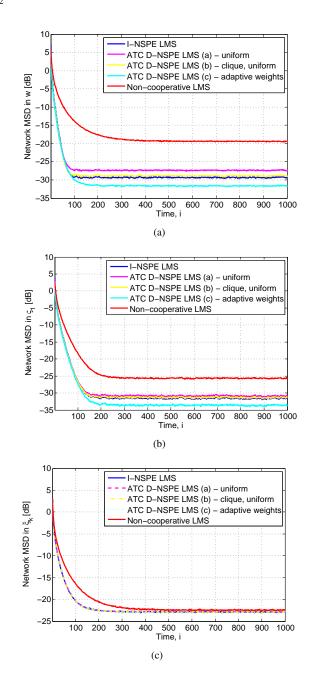


Fig. 6. Learning behavior of network MSD with respect to the parameters of global interest (a), common interest (b) and for the parameters of local interest (c).

Accordingly, we obtain a fair comparison among the strategies.

Figure 6 depicts the learning behavior of the two schemes in terms of the network MSD associated with the estimation of w^o , ς_1^o and ξ_k^o . Each network MSD is the result of averaging the local MSDs associated with the estimation of w^o and ξ_k^o at each node, except for the network MSD associated with the estimation of ς_1^o , which is averaged over the nodes belonging to the set \mathcal{C}_1 . To generate each plot, we have averaged the results over 100 independent experiments where we assumed Q=2 PUs, N=10 SUs and X=16 Gaussian basis functions, of amplitude normalized to one and standard deviation $\sigma_b=0.05$. Furthermore, we have considered that each SU scans L=80 channels over the normalized frequency

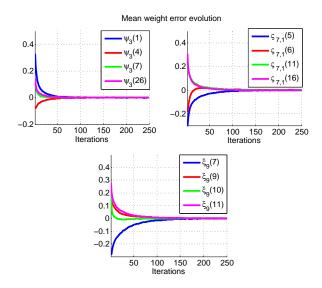


Fig. 7. The mean error trajectories of some vector coefficients related to the global (upper left), common (upper right) and local parameters (bottom) at randomly selected nodes.

axis between 0 and 1, whereas the noise $z_{k,i}$ in (86) is zero-mean Gaussian with standard deviation varying between 0.04 and 0.16 for different k.

Each attenuation coefficient follows $\hat{p}_{tk,i}(f) = p_{tk,i}(f) + n_{tk}$, where n_{tk} denotes a zero-mean Gaussian variable with standard deviation in the range between 0.3 and 1.25, while $p_{tk,i}(f)$ is related to the frequency response of the channel modeled as a static 3-tap FIR filter. Each tap is assumed to be a zero-mean complex Gaussian random variable with variance $\sigma_h^2 = 0.25$. Under this setting, we observe that all the proposed D-NSPE schemes outperform the non-cooperative one, especially when estimating w^o and ς_1^o . Note that D-NSPE a) and b) well-approximate the centralized-like performance of the incremental strategy. Finally, due to the fact that the adaptive combiners integrate some additional knowledge regarding the quality of the estimates at the different nodes, D-NSPE c) outperforms all other schemes including the incremental.

Finally, to illustrate the asymptotic unbiasedness of the proposed technique, in Fig. 7 we plot its mean weight behavior under the previously described setting. The figure indicates the mean weight evolution of some vector coefficients related to the global, common and local parameters at randomly selected nodes, whereas the optimal values from (9) are indicated by the black lines. As expected by Theorem 1, D-NSPE LMS has estimated the optimum weight vectors without bias.

VI. CONCLUSIONS AND FUTURE WORK

We have addressed a novel NSPE problem where the estimation interests of the nodes consist of a set of local parameters, network-wide global parameters as well as common parameters to a subset of nodes. To do so, we have proposed two distributed adaptive schemes where a local LMS is run at each node in order to estimate each set of local parameters. Coupled among themselves and with all these local estimation processes, the parameters of global and common interests are estimated by LMS-based schemes

implemented under a diffusion mode of cooperation. After obtaining conditions under which the proposed strategies are asymptotically unbiased, the mean-square steady-state performance has been evaluated. All the theoretical results have been validated through generic computer simulations. Moreover, the performance of the proposed algorithms have been illustrated in the context of cooperative spectrum sensing in Cognitive Radio networks.

APPENDIX A

Here, we aim to specify the structure of the permutation matrix \mathcal{P} in (53). To this end, first note that there are N blocks, each corresponding to a specific node, i.e., $\mathcal{P} = \text{col}\{E_1,\ldots,E_N\}$, where the k^{th} block E_k , of dimensions $M_k \times M$, takes the following form

$$E_{k} = \operatorname{col}\{e_{f_{g}(k,1)}, \dots, e_{f_{g}(k,M_{g})}, e_{f_{c}(k,\mathcal{I}_{k}(1),1)}, \dots, e_{f_{c}(k,\mathcal{I}_{k}(1),M_{c})}, \dots, e_{f_{c}(k,\mathcal{I}_{k}(1),M_{c})}, \dots, e_{f_{c}(k,\mathcal{I}_{k}(|\mathcal{I}_{k}|),1)}, \dots, e_{f_{c}(k,\mathcal{I}_{k}(|\mathcal{I}_{k}|),M_{c})}, e_{f_{l}(k,1)}, \dots, e_{f_{l}(k,M_{l})}\}$$
(89)

with the three counter functions, specifying the position of the unity in the basis vectors $e_{(\cdot)}$, defined by

$$f_g(k,c) = (k-1) \cdot M_g + c,$$
 (90)

$$f_c(k, j, c) = N \cdot M_g + \sum_{j'=1}^{j-1} |\mathcal{C}_{j'}| \cdot M_c + (|\mathcal{C}_{j,k}| - 1) \cdot M_c + c$$
(91)

and

$$f_l(k,c) = N \cdot M_g + \sum_{j=1}^{J} |\mathcal{C}_j| \cdot M_c + (k-1) \cdot M_l + c$$
 (92)

with $C_{j,k}$ given in (38).

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