# Blind Spectrum Sensing by Information Theoretic Criteria for Cognitive Radios

Rui Wang and Meixia Tao

#### Abstract

Spectrum sensing is a fundamental and critical issue for opportunistic spectrum access in cognitive radio networks. Among the many spectrum sensing methods, the information theoretic criteria (ITC) based method is a promising blind method which can reliably detect the primary users while requiring little prior information. In this paper, we provide an intensive treatment on the ITC sensing method. To this end, we first introduce a new over-determined channel model constructed by applying multiple antennas or over sampling at the secondary user in order to make the ITC applicable. Then, a simplified ITC sensing algorithm is introduced, which needs to compute and compare only two decision values. Compared with the original ITC sensing algorithm, the simplified algorithm significantly reduces the computational complexity without losing any performance. Applying the recent advances in random matrix theory, we then derive closed-form expressions to tightly approximate both the probability of false alarm and probability of detection. Based on the insight derived from the analytical study, we further present a generalized ITC sensing algorithm which can provide flexible tradeoff between the probability of detection and probability of false alarm. Finally, comprehensive simulations are carried out to evaluate the performance of the proposed ITC sensing algorithms. Results show that they considerably outperform other blind spectrum sensing methods in certain cases.

#### **Index Terms**

Cognitive radio networks, spectrum sensing, information theoretic criteria, random matrix theory.

#### I. INTRODUCTION

Due to the increasing popularity of wireless devices in recent years, the radio spectrum has been an extremely scarce resource. By contrast, 90 percent of the existing licensed spectrum

The authors are with the Department of Electronic Engineering at Shanghai Jiao Tong University, Shanghai, 200240, P. R. China. Emails:{liouxingrui, mxtao}@sjtu.edu.cn

remains idle and the usage varies geographically and temporally as reported by the Federal Communication Commission (FCC) [1]. This indicates that the fixed frequency regulation policy conflicts drastically with the high demand for frequency resource. Cognitive radio (CR) is one of the most promising technologies to deal with such irrational frequency regulation policy [2], [3] and has received lots of attention. In cognitive radio networks, secondary (unlicensed) users first reliably sense the primary (licensed) channel and then opportunistically access it without causing harmful interference to primary users [4]. By doing this, the spectrum utilization of existing wireless communication networks can be tremendously improved. FCC has issued a Notice of proposed Rule Making to allow the unlicensed CR devices to operate in the unused channel [5]. The IEEE has also formed the 802.22 working group to develop the standard for wireless regional area networks (WRAN) which will operate on unused VHR/UHF TV bands based on cognitive radio technology. Both of these activities will significantly change the current wireless communication situation.

As mentioned above, the secondary users need to opportunistically access the unused licensed channel while causing negligible interference to the primary users. As a result, the detection of presence of primary users is a fundamental and critical task in the cognitive radio networks. Although the detection of presence of signals is known as a classical problem in signal processing, however, sensing the presence of primary users in a complicated communication environment, especially a CR-based network, is still a challenging problem from the practice perspective. This is mainly due to the following two limiting factors: Firstly, it is very difficult, if not possible, for the secondary user to obtain the necessary prior information about the signal characteristics of the primary user for most of the traditional detection techniques to apply. Secondly, the CR devices should be capable of sensing the very weak signals transmitted by primary users. For instance, the standard released by FCC has required that spectrum sensing algorithms need to reliably detect the transmitted TV signals at a very low signal-to-noise ratio (SNR) of at least -18dB [4].

Thus far, there are mainly four types of spectrum sensing methods: energy detection [6], [7], matched filtering (coherent detection) [8], feature detection [9] and eigenvalue-based detection [10]–[12]. Among them, energy detection is optimal if the secondary user only knows the local noise power [13]. The matched-filtering based coherent detection is optimal for maximizing the detection probability but it requires the explicit knowledge of the transmitted signal pattern (e.g.,

pilot, training sequence etc.) of the primary user. The feature detection, often referred to as the cyclostationary detection, exploits the periodicity in the modulation scheme which, however, is difficult to determine in certain scenarios. By constructing the decision variables based on eigenvalues of the sampled covariance matrix to detect the presence of the primary user, the eigenvalue-based sensing methods presented in [10]–[12] do not need to estimate the power of the noise and hence are more practical in most CR networks. Recently, several new spectrum sensing schemes by incorporating system-level design parameters have been introduced, such as throughput maximization [14]–[16] and cooperative sensing using multiple nodes [17]–[20]. Nevertheless, the aforementioned four types of sensing techniques are still treated as a basic component in these new schemes.

In this paper, we study a blind spectrum sensing method based on information theoretic criteria (ITC), an approach originally for model selection introduced by Akaike [21], [22] and by Schwartz [23] and Rissanen [24]. Applying information theoretic criteria for spectrum sensing was firstly introduced in [25]–[28]. This work provides a more intensive study on the ITC sensing algorithm and its performance. The main contributions of this paper are as follows:

- First of all, to make the information theoretic criteria applicable, a new over-determined channel model is constructed by introducing multiple antennas or over sampling at the secondary user.
- Then, a simplified information theory criteria (SITC) sensing algorithm which only involves the computation of two decision values is presented. Compared to the original information theory criteria (OITC) sensing algorithm in [25], SITC is much less complex and yet almost has no performance loss. Simulation results also demonstrate that the proposed SITC based spectrum sensing outperforms the eigenvalue based sensing algorithm in [10] and almost obtains the similar performance with [11]. The proposed sensing algorithm also enables a more tractable analytical study on the detection performance.
- Applying the recent advances in random matrix theory, we then derive closed-form expressions for both the probability of false alarm and probability of detection. which can approximate the actual results in simulation very well.
- Finally, based on the insight derived from the analytical study, we further present a generalized information theory criteria (GITC) sensing algorithm. By involving an adjustable threshold, the proposed GITC can provide flexible tradeoff between the probability of

detection and probability of false alarm in order to supply different system requirements.

The rest of paper is organized as follows. In Section II, the preliminary on the information theoretic criteria is provided. The proposed over-determined system model is presented in Section III. Section IV gives the proposed SITC sensing algorithm and the theoretical analysis of its detection performance, followed by the GITC sensing algorithm in Section V. Extensive simulation results are illustrated in Section VI. Finally, Section VII offers some concluding remarks.

*Notations*:  $\mathcal{E}[\cdot]$  denotes expectation over the random variables within the brackets. Tr(A) stands for the trace of matrix A. Superscripts  $(\cdot)^T$  and  $(\cdot)^\dagger$  denote transpose and conjugate transpose.

# II. PRELIMINARY ON THE INFORMATION THEORETIC CRITERIA

Information theoretic criteria are an approach originally for model selection introduced by Akaike [21], [22] and by Schwartz [23] and Rissanen [24]. There are two well-known criteria that have been widely used: Akaike information criterion (AIC) and minimum description length (MDL) criterion. One of the most important applications of information theoretic criteria is to estimate the number of source signals in array signal processing [29]. Consider a system model described as

$$\boldsymbol{x} = \mathbf{A}\boldsymbol{s} + \boldsymbol{\mu},\tag{1}$$

where x is the  $p \times 1$  complex observation vector, A is a  $p \times q$  (p > q) complex system matrix, s denotes the  $q \times 1$  complex source modulated signals and  $\mu$  is the additive complex white Gaussian noise vector. It is noted that the definite parameters q, A and  $\sigma^2$  are all unknown. The resulting cost functions of AIC and MDL have the following form [29]:

$$AIC(k) = -2\log\left(\frac{\prod_{i=k+1}^{p} l_i^{1/(p-k)}}{\frac{1}{p-k} \sum_{i=k+1}^{p} l_i}\right)^{N(p-k)} + 2k(2p-k) + 2,$$
(2)

$$MDL(k) = -\log\left(\frac{\prod_{i=k+1}^{p} l_{i}^{1/(p-k)}}{\frac{1}{p-k} \sum_{i=k+1}^{p} l_{i}}\right)^{N(p-k)} + \left(\frac{1}{2}k(2p-k) + \frac{1}{2}\right)\log N,$$
(3)

where N signifies the observation times and  $l_i$  denotes the *i*-th decreasing ordered eigenvalue of the sampled covariance matrix. The estimated number of source signals is determined by choosing the minimum (2) or (3). That is,

$$\hat{k}_{\text{AIC}} = \arg\min_{j=0,1,\dots,p-1} \text{AIC}(j), \tag{4}$$

$$\hat{k}_{\text{MDL}} = \arg\min_{j=0,1,\dots,p-1} \text{MDL}(j).$$
(5)

# **III. SYSTEM MODEL**

We consider a multipath fading channel model and assuming that there is only one primary user in the cogitative radio network. Let x(t) be a continuous-time baseband received signal at the secondary user's receiver. Spectrum sensing can be formulated as a binary hypothesis test between the following two hypotheses

$$\mathcal{H}_0: \quad x(t) = \mu(t), \tag{6}$$

$$\mathcal{H}_{1}: \quad x(t) = \int_{0}^{T} h(\ell) s(t-\ell) d\ell + \mu(t),$$
(7)

where s(t) denotes the signal transmitted by the primary user, h(t) is the continuous channel response between the primary transmitter and the secondary receiver,  $\mu(t)$  denotes the additive white noise, the parameter T signifies the duration of the channel. The channel response is also assumed to remain invariant during each observation. To obtain the discrete representation, we assume that the received signal is sampled at rate  $f_s$  which is equal to the reciprocal of the baseband symbol duration  $T_0$ . For notation simplicity, we define  $x(n) = x(nT_0)$ ,  $s(n) = s(nT_0)$ and  $\mu(n) = \mu(nT_0)$ . Hence, the corresponding received signal samples under the two hypotheses are described as:

$$\mathcal{H}_0: \quad x(n) = \mu(n), \tag{8}$$

$$\mathcal{H}_1: \quad x(n) = \sum_{i=0}^{L-1} h(i)s(n-i) + \mu(n), \tag{9}$$

where h(i)  $(0 \le i \le L - 1)$  denotes the discrete channel response of h(t) and L denotes the order of the discrete channel (L taps). Let each observation consist of M received signal samples. Then (8) and (9) can be rewritten in matrix form as:

$$\mathcal{H}_0: \quad \boldsymbol{x}_i = \boldsymbol{\mu}_i, \tag{10}$$

$$\mathcal{H}_1: \quad \boldsymbol{x}_i = \mathbf{H}\boldsymbol{s}_i + \boldsymbol{\mu}_i, \tag{11}$$

where H is an  $M \times (L + M - 1)$  circular channel matrix defined as

$$\mathbf{H} = \begin{bmatrix} h(L-1) \ h(L-2) & \dots & h(0) \\ h(L-1) \ h(L-2) & \dots & h(0) \\ & \ddots & \ddots & \\ & & & h(L-1) \ h(L-2) \ \dots & h(0) \end{bmatrix},$$

 $x_i$ ,  $s_i$ , and  $\mu_i$  are the  $M \times 1$  observation vector,  $(L+M-1) \times 1$  source signal vector and  $M \times 1$  noise vector, respectively, and are defined as

$$\boldsymbol{x}_{i} = [x(iM - M + 1), x(iM - M + 2), \dots, x(iM)]^{T},$$
(12)

$$\boldsymbol{s}_{i} = [s(iM - M - L + 2), s(iM - M - L + 3), \dots, s(iM)]^{T},$$
(13)

$$\boldsymbol{\mu}_{i} = [\mu(iM - M + 1), \mu(iM - M + 2), \dots, \mu(iM)]^{T}.$$
(14)

Now, comparing (11) with the array signal processing model (1), we find that a major difference is that the **H** in our considered system model is an under-determined matrix, i.e., the order of column is larger than the order of row. Therefore, the information theoretic criteria are not directly applicable here [29].

To construct an over-determined channel matrix  $\mathbf{H}$  as in (1), one needs to enlarge the observation space. Obviously, simply increasing the observation window M does not work. Here we propose to expand the observation space using one of the following two methods. One is to increase the spatial dimensionality by employing multiple receive antennas at the secondary user and the other is to increase the time dimensionality by over-sampling the received signals. It turns out that the two methods are similar to each other. Hence we shall focus on the multipleantenna approach hereafter. The difference for over-sampling method will be discussed in the end of this section. In specific, suppose that the detector at the secondary user is equipped with K antennas. Redefine (12) and (14) as

$$\boldsymbol{x}_{i} = [x_{1}^{i}(1), x_{2}^{i}(1), \dots, x_{K}^{i}(1), x_{1}^{i}(2), \dots, x_{K}^{i}(2), \dots, x_{1}^{i}(M), \dots, x_{K}^{i}(M)]^{T},$$
(15)

$$\boldsymbol{\mu}_{i} = [\mu_{1}^{i}(1), \mu_{2}^{i}(1), \dots, \mu_{K}^{i}(1), \mu_{1}^{i}(2), \dots, \mu_{K}^{i}(2), \dots, \mu_{1}^{i}(M), \dots, \mu_{K}^{i}(M)]^{T},$$
(16)

where  $\boldsymbol{x}_{k}^{i} = [x_{k}^{i}(1), x_{k}^{i}(2), \dots, x_{k}^{i}(M)]^{T}$  represents the  $M \times 1$  observation vector at the kth antenna at the *i*-th observation as in (12), and  $\boldsymbol{\mu}_{k}^{i} = [\boldsymbol{\mu}_{k}^{i}(1), \boldsymbol{\mu}_{k}^{i}(2), \dots, \boldsymbol{\mu}_{k}^{i}(M)]^{T}$  is the corresponding noise vector at the k-th antenna at the *i*-th observation as in (14). Then, the new channel matrix **H** becomes an  $MK \times (M + L - 1)$  matrix:

$$\mathbf{H} = \begin{bmatrix} h_{1}(L-1) & h_{1}(L-2) & \dots & h_{1}(0) \\ \vdots & \vdots & \vdots \\ h_{K}(L-1) & h_{K}(L-2) & \dots & h_{K}(0) \\ & h_{1}(L-1) & h_{1}(L-2) & \dots & h_{1}(0) \\ & \vdots & \vdots \\ & h_{K}(L-1) & h_{K}(L-2) & \dots & h_{K}(0) \\ & & \ddots & \ddots \\ & & & h_{1}(L-1) & h_{1}(L-2) & \dots & h_{1}(0) \\ & & \vdots & \vdots \\ & & h_{K}(L-1) & h_{K}(L-2) & \dots & h_{K}(0) \end{bmatrix}.$$
(17)

Here,  $h_k(i)$ , for i = 0, ..., L - 1, denotes the *i*-th channel tap observed at *k*-th antenna. To ensure that **H** is now an over-determined matrix (the order of row is larger than the order of column), we need to have

$$K > \frac{L+M-1}{M},\tag{18}$$

or, alternatively,

$$M > \frac{L-1}{K-1}.$$
(19)

Furthermore, we assume the noise samples come form different antennas are independent with zero mean and  $\mathcal{E}(\boldsymbol{\mu}_i \boldsymbol{\mu}_i^H) = \sigma^2 \mathbf{I}_{MK}$ . Then we can exactly ensure that the system mode under multiple antennas satisfies the over-determined condition specified in [29]. For ease of presentation, we define p = MK and q = L + M - 1 in (11).

As mentioned earlier, the second approach to construct the over-determined channel model is for the secondary user to over-sample the received signals. Suppose that the over-sampling factor is given by K. That is, the received baseband signal is sampled K times in one symbol. Then a similar system model as in (15), (16) and (17) can be obtained, except that  $x_i$  and  $\mu_i$ should be replaced with

$$\boldsymbol{x}_{i} = [x(iMK - MK + 1), x(iMK - MK + 2), \dots, x(iMK)]^{T},$$
(20)

$$\boldsymbol{\mu}_{i} = [\mu(iMK - MK + 1), \mu(iMK - MK + 2), \dots, \mu(iMK)]^{T},$$
(21)

and  $h_k(i)$ , for i = 0, ..., L - 1, becomes the k-th over-sampling point of the *i*-th channel tap. It can be verified that  $h_k(i)$ 's are different for different k [30]. The major difference between the over-sampling approach and the multiple-antenna approach is that the over-sampled noise samples in (21) are correlated, which contradicts the primary assumption of independent noise samples. Nevertheless, the pre-whiting technique can be used to whiten the correlated noises based on the known correlation matrix. The details can be referred to Appendix A.

Before leaving this section. it is noted that, though the proposed over-determined model is based on the assumption that there is only one primary user in the cognitive network, it is also applicable the scenario where there exist multiple primary users. An alternative approach to construct the over-determined model in the presence of multiple primary users is to use the cooperative sensing technique as in [28] by using multiple detectors.

# IV. SIMPLIFIED INFORMATION THEORETIC CRITERIA SENSING ALGORITHM AND PERFORMANCE ANALYSIS

Since the binary hypothesis test in the spectrum sensing is equivalent to the special case of source number estimation problem, the information theoretic criteria method can be directly applied to conduct spectrum sensing as firstly proposed in [25]–[28]. The basic idea is when the primary user is absent, the received signal  $x_i$  is only the white noise samples. Therefore, the estimated number of source signals via information theoretic criteria (AIC or MDL) should be zero. Otherwise, when the primary user is present, the number of source signals must be larger than zero. Hence, by comparing the estimated number of source signals with zero, the presence of the primary user can be detected. It is noted that the estimation of the number of source by using (4) and (5) needs very little prior information about the primary user. In particular, it does not require the knowledge of channel state information, synchronization, nor pilot design and modulation strategy. Moreover it does not need the estimation of noise power. Hence we argue that information theoretic criteria method is a blind spectrum sensing similar to [10]–[12], and it is robust and suitable for the practical applications.

However, it is known that signal detection is much easier than signal estimation. Therefore, using the estimation method to conduct the detection as in [25]–[28] may lead to unnecessary computational complexity overhead. In the mean time, it makes it difficult to carry out analytical study on the detection performance. In this section, we propose a simplified ITC algorithm to

conduct the spectrum sensing. It can significantly reduce the computational complexity while having almost no performance loss as will be illustrated in Section V. It also enables a more tractable analytical study on the detection performance.

# A. Simplified ITC sensing algorithm

Before presenting the simplified ITC sensing algorithm in detail, we have the following lemma.

**Lemma 1**: If there is one value  $\hat{k}(>0)$  which minimizes the AIC metric in (2) (MDL metric in (3)), then AIC(0) > AIC(1) (MDL(0) > MDL(1)) with high probability.

*Proof:* Please refer to Appendix B

The outline of the proposed simplified sensing algorithm is as follows.

# Algorithm 1: SITC sensing algorithm

Step 1. Compute the sampled covariance matrix of received signals, i.e.,  $\mathbf{R}_x = \frac{1}{N} \sum_{i=1}^{N} \mathbf{x}_i \mathbf{x}_i^{\dagger}$ , where  $\mathbf{x}_i$ 's are received vectors as described in (12) or (20) and N denotes the number of the observations.

Step 2. Obtain the eigenvalues of  $\mathbf{R}_x$  through eigenvalue decomposition technique, and denote them as  $\{l_1, l_2, \ldots, l_p\}$  with  $l_1 \ge l_2, \ldots, \ge l_p$ .

Step 3. Calculate the decision values AIC(0) and AIC(1) (MDL(0) and MDL(1)) according to (2)((3)). Then the detection decision metric is

$$\mathcal{T}_{\text{SITC}-\text{AIC}}(\mathbf{L}_x) : \text{AIC}(0) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrsim}} \text{AIC}(1).$$
(22)

if AIC is adopted, or

$$\mathcal{T}_{\text{SITC-MDL}}(\mathbf{L}_x) : \text{MDL}(0) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \text{MDL}(1).$$
(23)

if MDL is adopted, where  $L_x$  denote the set of eigenvalues  $\{l_i, i = 1, 2, ..., p\}$ 

Note that in the OITC sensing algorithm [25], one needs to find the exact value of  $\hat{k}$  from 0 to p - 1 to minimize the AIC in (2) or MDL in (3). In the proposed SITC algorithm, only two decision values at k = 0 and 1 should be computed and compared. Thus, the computational complexity is significantly reduced. In the next subsection, based on the proposed SITC algorithm, we present the analytical results on the detection performance. Since from the Lemma 1, the SITC algorithm almost obtains the same performance as OITC algorithm, we claim that our analytical results are also applicable for evaluating the performance of OITC algorithm.

# B. Performance Analysis

Since spectrum sensing is actually a binary hypothesis test, the performance we focus on is the probability of detection  $P_d$  (the probability for identifying the signal when the primary user is present) and the probability of false alarm  $P_f$  (the probability for identifying the signal when the primary user is absent). As no threshold value is involved in the ITC sensing algorithm,  $P_d$  is not directly related with  $P_f$ . The two probabilities are presented separately. For ease of presentation, we shall take the AIC criterion for example to illustrate the analysis throughout this section. The extension to MDL criterion is straightforward if not mentioned otherwise.

1) Probability of false alarm: According to the sensing steps in Algorithm 1, the false alarm occurs when AIC(0) is larger than AIC(1) at hypothesis  $\mathcal{H}_0$ . The probability of false alarm can be expressed as

$$P_{f-AIC} = \Pr(\operatorname{AIC}(0) > \operatorname{AIC}(1) | \mathcal{H}_0).$$
(24)

Since the primary user is absent, the received signal  $x_i$  only contains the noises. The sampled covariance matrix  $\mathbf{R}_x$  in Algorithm 1 thus turns to  $\mathbf{R}_\mu$  defined as

$$\mathbf{R}_{\mu} = \frac{1}{N} \sum_{i=1}^{N} \boldsymbol{\mu}_{i} \boldsymbol{\mu}_{i}^{\dagger}.$$
 (25)

Hence, the eigenvalues in (2) become the eigenvalues of the sampled noise covariance matrix  $\mathbf{R}_{\mu}$  in (25), which is a Wishart random matrix [31]. By applying the recent advances on the eigenvalue distribution for Wishart matrices, a closed-form expression for the probability of false alarm can be obtained.

**Proposition 1**: The probability of false alarm of the proposed spectrum sensing algorithm can be approximated as:

$$P_{f} \approx F_{2} \left( \frac{pN - (\sqrt{N} + \sqrt{p})^{2}}{(\sqrt{N} + \sqrt{p})(\frac{1}{\sqrt{N}} + \frac{1}{\sqrt{p}})^{\frac{1}{3}}} \right) - F_{2} \left( \frac{(p - \alpha_{1})N - (\sqrt{N} + \sqrt{p})^{2}}{(\sqrt{N} + \sqrt{p})(\frac{1}{\sqrt{N}} + \frac{1}{\sqrt{p}})^{\frac{1}{3}}} \right) + F_{2} \left( \frac{(p - \alpha_{2})N - (\sqrt{N} + \sqrt{p})^{2}}{(\sqrt{N} + \sqrt{p})(\frac{1}{\sqrt{N}} + \frac{1}{\sqrt{p}})^{\frac{1}{3}}} \right) - F_{2} \left( \frac{-(\sqrt{N} + \sqrt{p})^{2}}{(\sqrt{N} + \sqrt{p})(\frac{1}{\sqrt{N}} + \frac{1}{\sqrt{p}})^{\frac{1}{3}}} \right),$$
(26)

where  $F_2(\cdot)$  is the cumulative distribution function (CDF) of Tracy-Widom distribution of order two [31],  $\alpha_1$  and  $\alpha_2$  with  $\alpha_1 < \alpha_2$  are the two real roots of the function in (32) if AIC is applied, or (37) if MDL is applied. *Proof:* Recall the definition in (24), to compute the probability of false alarm is to compute the probability

$$P_{f-AIC} = \Pr(\operatorname{AIC}(0) - \operatorname{AIC}(1) > 0 | \mathcal{H}_0).$$
(27)

According to the cost function of AIC defined in (2), we have

$$AIC(0) - AIC(1) = -2\log\left[\frac{\prod_{i=1}^{p} l_i^{1/p}}{\frac{1}{p} \sum_{i=1}^{p} l_i}\right]^{pN} + 2\log\left[\frac{\prod_{i=2}^{p} l_i^{1/p-1}}{\frac{1}{p-1} \sum_{i=2}^{p} l_i}\right]^{(p-1)N} - (4p-2).$$

Then we can rewrite (27) as

$$P_{f-AIC} = \Pr\left(\log\left[\frac{(\frac{1}{p}\sum_{i=1}^{p}l_{i})^{p}}{(\frac{1}{p-1}\sum_{i=2}^{p}l_{i})^{p-1}l_{1}}\right] > \frac{4p-2}{2N} \middle| \mathcal{H}_{0}\right).$$
(28)

Note here, the sum of eigenvalues of sampled covariance matrix,  $\frac{1}{p} \sum_{i=1}^{p} l_i$ , is equivalent to  $\frac{1}{pN} \operatorname{Tr} \left( \sum_{i=1}^{N} \boldsymbol{x}_i \boldsymbol{x}_i^{\dagger} \right)$ . At hypothesis  $H_0$ , where the received vector involves only the noise samples,  $\frac{1}{pN} \operatorname{Tr} \left( \sum_{i=1}^{N} \boldsymbol{x}_i \boldsymbol{x}_i^{\dagger} \right)$  is the un-biased estimation of the covariance of the white noise. Therefore, when N is sufficiently large, we have

$$\frac{1}{p}\sum_{i=1}^{p}l_{i}\approx\sigma^{2}.$$
(29)

Substituting (29) into (28) yields:

$$P_{f-AIC} \approx \Pr\left[\frac{(\sigma^2)^p}{(\frac{p}{p-1}\sigma^2 - \frac{l_1}{p-1})^{p-1}l_1} > \exp\left(\frac{2p-1}{N}\right) \middle| \mathcal{H}_0\right].$$
(30)

From (30), it is seen that the probability of false alarm is only dependent on the largest eigenvalue of the noise sampled covariance matrix  $\mathbf{R}_{\mu}$ . Since  $\mathbf{R}_{\mu}$  is actually a Wishart random matrix , its the largest eigenvalue  $l_1$  satisfies the Tracy-Widom distribution of order two [31]. To apply this result, we rewrite (30) as

$$P_{f-AIC} \approx \Pr\left[\frac{l_1}{\sigma^2} \left(p - \frac{l_1}{\sigma^2}\right)^{p-1} < \frac{(p-1)^{p-1}}{\exp\left(\frac{2p-1}{N}\right)} \middle| \mathcal{H}_0\right]$$
$$= \Pr\left[x^p - px^{p-1} + \frac{(p-1)^{p-1}}{\exp\left(\frac{2p-1}{N}\right)} > 0 \middle| \mathcal{H}_0\right],\tag{31}$$

where  $x \triangleq p - \frac{l_1}{\sigma^2}$ .

Define a function

$$f(x) \triangleq x^{p} - px^{p-1} + \frac{(p-1)^{p-1}}{\exp\left(\frac{2p-1}{N}\right)}.$$
(32)

We next find the real roots of this function.

Taking the differentiation of f(x) and equating it to zero, we obtain

$$\frac{df(x)}{dx} = px^{p-1} - p(p-1)x^{p-2} = px^{p-2}[x - (p-1)] = 0.$$

Clearly, f(x) has two stationary points, which are x = p - 1 and x = 0. In the following, two scenarios with p being even or odd are considered respectively. When p is even, it is easily found that the function f(x) monotonically decreases over  $(-\infty, p - 1)$  and monotonically increases over  $(p - 1, \infty)$ . Simultaneously, we can verify that

$$f(p-1) = (p-1)^p - p(p-1)^{p-1} + \frac{(p-1)^{p-1}}{\exp\left(\frac{2p-1}{N}\right)} < 0.$$
(33)

and

$$f(0) = f(p) = \frac{(p-1)^{p-1}}{\exp\left(\frac{2p-1}{N}\right)} > 0.$$
(34)

So there must be two real real roots within (0, p) and around p - 1 for function f(x). Let  $\alpha_1$  and  $\alpha_2$ , with  $\alpha_1 < \alpha_2$ , denote the two real roots, then (31) is converted into an equivalent form:

$$P_{f-AIC} \approx \Pr\left[x < \alpha_1 | H_0\right] + \Pr\left[\alpha_2 < x | \mathcal{H}_0\right].$$
(35)

When p is odd, we can find f(x) decreases monotonically over (0, p-1), while it is the monotonic increasing function over both  $(-\infty, 0)$  and  $(p-1, \infty)$ . According to the fact that  $f(-\infty) < 0$ , f(0) > 0, f(p-1) < 0 and f(p) > 0, we conclude that f(x) have three real roots, which are denoted as  $\alpha_0$ ,  $\alpha_1$  and  $\alpha_2$ , with  $\alpha_0 < 0$  and  $0 < \alpha_1 < \alpha_2$ , respectively. Then (31) can be rewritten as:

$$P_{f-AIC} \approx \Pr\left[\alpha_0 < x < \alpha_1 | H_0\right] + \Pr\left[\alpha_2 < x | \mathcal{H}_0\right].$$
(36)

However, it is noted that as N is large enough, the largest eigenvalue of the sampled noise covariance matrix,  $l_1$ , is just slightly larger than the true covariance of noise  $\sigma^2$ . Hence, from the definition, x can be reasonably limited in (0, p). Therefore, both the probability of (35) and (36) can be summarized as the following form

$$P_{f-AIC} \approx \Pr\left[0 < x < \alpha_1 | H_0\right] + \Pr\left[\alpha_2 < x < p | \mathcal{H}_0\right].$$

In other words,

$$P_{f-AIC} \approx \Pr\left[p - \alpha_1 < \frac{l_1}{\sigma^2} < p \middle| \mathcal{H}_0\right] + \left[0 < \frac{l_1}{\sigma^2} < p - \alpha_2 \middle| \mathcal{H}_0\right].$$

Applying the distribution of the largest eigenvalue for Wishart matrix in random matrix theory [31], the variable  $N \frac{l_1}{\sigma^2}$  satisfies the distribution of Tracy-widom of order two, i.e.,

$$\frac{N\frac{l_1}{\sigma^2} - (\sqrt{N} + \sqrt{p})^2}{(\sqrt{N} + \sqrt{p})\left(\frac{1}{\sqrt{N}} + \frac{1}{\sqrt{p}}\right)^{\frac{1}{3}}} \sim W_2 \sim F_2.$$

Here,  $W_2$  and  $F_2$  denote the probability density function (PDF) and cumulative density function (CDF) for distribution of Tracy-widom of order two respectively. Therefore, the probability of false alarm of AIC can be concluded as (26).

Similar with the above derivation, when the MDL criterion is applied, we only need to modify the step in (31) as

$$P_{f-MDL} \approx \Pr\left[x^p - px^{p-1} + \frac{(p-1)^{p-1}}{\exp\left(\frac{(p-0.5)\log N}{N}\right)} > 0 \middle| \mathcal{H}_0\right].$$

and redefine the function f(x) in (32) as

$$f(x) \triangleq x^{p} - px^{p-1} + \frac{(p-1)^{p-1}}{\exp\left(\frac{(p-0.5)\log N}{N}\right)}.$$
(37)

From Proposition 1, it is found that the probability of false alarm is independent with noise covariances  $\sigma^2$ . Therefore, the proposed SITC sensing algorithm is robust in practical applications. It is also noted that  $P_f$  depends on the product of M and K, i.e., p = MK, rather than the individual values of M and K.

2) Probability of detection: When the primary user is present, the event of detection also occurs when AIC(0) > AIC(1). The probability of detection is thus described as

$$P_{d-AIC} = \Pr(\operatorname{AIC}(0) > \operatorname{AIC}(1) | \mathcal{H}_1).$$
(38)

Since at  $\mathcal{H}_1$ , the received vector  $x_i$  involves the signals transmitted by the primary user, the sampled covariance matrix  $\mathbf{R}_x$  can be written as

$$\mathbf{R}_{x} = \frac{1}{N} \sum_{i=1}^{N} (\mathbf{H}\boldsymbol{s}_{i} + \boldsymbol{\mu}_{i}) (\mathbf{H}\boldsymbol{s}_{i} + \boldsymbol{\mu}_{i})^{\dagger}.$$
(39)

Note that  $\mathbf{R}_x$  is no longer a Wishart matrix. The exact distribution of its eigenvalues is unknown and difficult to find, and hence so is the  $P_d$ . In the following, we resort to deriving a closed-form expression for the conditional probability of detection given the channel matrix **H**. The average probability of detection can then be obtained using a hybrid analytical-simulation approach. **Proposition 2**: Let  $\mathbf{R}_s$  denote the covariance matrix of  $s_i$  given in (13) and  $\{\delta_1, \delta_2, \ldots, \delta_p\}$ be the eigenvalues of matrix  $\mathbf{H}\mathbf{R}_s\mathbf{H}^{\dagger}$  (with  $\delta_1 \ge \delta_2 \ge \ldots \ge \delta_p$ ). Then there exists a value of  $\rho$ , for  $\delta_p \le \rho \le \delta_1$ , such that the probability of detection given  $\mathbf{H}$  can be approximated as  $P_{d|H} \approx Q(\rho)$ , where the function  $Q(\cdot)$  is

$$Q(\delta) = F_2 \left( \frac{pN - (\sqrt{N} + \sqrt{p})^2}{(\sqrt{N} + \sqrt{p})(\frac{1}{\sqrt{N}} + \frac{1}{\sqrt{p}})^{\frac{1}{3}}} \right) - F_2 \left( \frac{(\frac{(p-\pi_1)\epsilon - \delta}{\sigma_2})N - (\sqrt{N} + \sqrt{p})^2}{(\sqrt{N} + \sqrt{p})(\frac{1}{\sqrt{N}} + \frac{1}{\sqrt{p}})^{\frac{1}{3}}} \right) + F_2 \left( \frac{(\frac{(p-\pi_2)\epsilon - \delta}{\sigma_2})N - (\sqrt{N} + \sqrt{p})^2}{(\sqrt{N} + \sqrt{p})(\frac{1}{\sqrt{N}} + \frac{1}{\sqrt{p}})^{\frac{1}{3}}} \right) - F_2 \left( \frac{-(\sqrt{N} + \sqrt{p})^2}{(\sqrt{N} + \sqrt{p})(\frac{1}{\sqrt{N}} + \frac{1}{\sqrt{p}})^{\frac{1}{3}}} \right), \quad (40)$$

where  $\epsilon = \frac{1}{p} \text{Tr}(\mathbf{H}\mathbf{R}_{\mathbf{s}}\mathbf{H}^{\dagger}) + \sigma^{2}$  and  $\pi_{1}, \pi_{2}$  (with  $\pi_{1} < \pi_{2}$ ) denote the two roots of the function (46) for AIC or (49) for MDL. Furthermore, upper and lower bounds can be obtained as  $Q(\delta_{p}) \leq P_{d|H} \leq Q(\delta_{1})$ .

Proof: Please refer to Appendix C.

From Proposition 2, we find that  $P_d$  is not only related to N and p, but also depends on  $\frac{\epsilon}{\sigma^2}$ , which is the ratio of the signal strength of primary user to the noise variance. The exact value of  $\rho \in [\delta_p, \delta_1]$  in Proposition 2 is difficult to determine in an analytical way, since it is related to both the channel response **H** and the covariance matrix of source signal  $\mathbf{R}_s$ . In practice, we can simply choose  $\rho_{AIC} = \frac{1}{2}(\delta_p + \delta_1)$  and  $\rho_{MDL} = \frac{3}{4}(\delta_p + \delta_1)$ . It will be demonstrated later in Section VI that the analytical  $P_{d|H}$  based on this choice of  $\rho$  can approximate the Monte Carlo results very well in most of cases.

# V. GENERALIZED INFORMATION THEORETIC CRITERIA SENSING ALGORITHM

As mentioned in the previous section, the probability of detection of and probability of false alarm of the proposed simplified ITC sensing algorithm are not directly related to each other as the algorithm does not involve any threshold (same for the original ITC algorithm in [25]). According to the analytical results given in (26) and (40), to satisfy different system requirements, a proper set of values for the parameters M, K and N in model (11) should be chosen, which is inconvenient for practical application. In this section, based on the analytical discussion in Section IV, we propose a generalized information theoretic criteria sensing algorithm which can provide a flexible tradeoff between  $P_d$  and  $P_f$  according to different system design requirements.

From the expression given in (28) and (44), we found that the sensing decision for SITC algorithm is actually based on the decision variable  $\log \left[\frac{(\frac{1}{p}\sum_{i=1}^{p}l_i)^p}{(\frac{1}{p-1}\sum_{i=2}^{p}l_i)^{p-1}l_1}\right]$ . Thus, we generalize the decision rule as

$$\mathcal{T}_{\text{GITC}}(\mathbf{L}_x) : \frac{\left(\frac{1}{p} \sum_{i=1}^p l_i\right)^p}{\left(\frac{1}{p-1} \sum_{i=2}^p l_i\right)^{p-1} l_1} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \gamma,$$
(41)

where  $\gamma$  is a pre-set threshold. It is seen that if we set  $\gamma = \exp(\frac{2p-1}{N})$ , the decision rule given in (41) turns into the AIC based SITC sensing algorithm presented in Algorithm 1. If we fix  $\gamma = \exp(\frac{(p-0.5)\log N}{N})$ , the algorithm becomes the MDL-based SITC sensing algorithm. Furthermore, it is easy to find that the analytical results obtained in Section IV are applicable for the GITC sensing algorithm. The only change that needs to be made is to replace  $\alpha_1$  and  $\alpha_2$  in (26) (or  $\pi_1$  and  $\pi_2$  in (40)) by two real roots generated by the following equation.

$$f(x) \triangleq x^p - px^{p-1} + \frac{(p-1)^{p-1}}{\gamma}.$$
 (42)

Thus, the outline of the proposed GITC sensing algorithm can be summarized as follows.

# Algorithm 2: GITC sensing algorithm

Step 1 and Step 2: the same as Algorithm 1 in Section IV.

Step 3: According to the system requirement on  $P_f$ , choose a proper threshold  $\gamma$  based on (26) and (42).

Step 4: Conduct the decision based on (41).

According to the decision variable presented in (41), we find that the proposed GITC sensing algorithm is actually also an eigenvalue-based method similar to [10]–[12]. The advantage of the proposed GITC over the algorithms in [11], [12] is that it is able to analytically obtain the explicit decision threshold  $\gamma$  according to the system requirement on  $P_f$  before the actual deployment.

#### VI. SIMULATION RESULTS AND DISCUSSIONS

In this section, we present some numerical examples to demonstrate the effectiveness of the proposed sensing schemes and to confirm the theoretical analysis.

#### A. Comparison between simulation and analytical results for both SITC and OITC

In our first set of examples, we compare the simulation results with analytical results given in Proposition 1 and Proposition 2. The comparison between SITC and OITC are also presented. In the simulation, the channel taps are random numbers with zero-mean complex Gaussian distribution. All the results are averaged over 1000 Monte Carlo realizations. For each realization, random channel, random noise and random BPSK modulated inputs are generated. We define the SNR as the ratio of the average received signal power to the average noise power

$$SNR = \frac{\mathcal{E}[\|\boldsymbol{x}_i - \boldsymbol{\mu}_i\|^2]}{\mathcal{E}[\|\boldsymbol{\mu}_i\|^2]}.$$
(43)

The comparison of simulation and analytical results for  $P_f$  is demonstrated in TABLE I and TABLE II. According to Proposition 1,  $P_f$  is independent with the noise variance, thus remains constant over different SNR. Hence we average multiple values over different SNR as the simulated  $P_f$  and compare it with the analytical  $P_f$ . From TABLE I, we first observe that SITC and OITC perform almost the same. It is also seen that, for AIC, the analytical results are slightly larger than the simulation results especially when p = MK is small. Nevertheless, the analytical approximation is accurate enough to evaluate the performance of the proposed sensing scheme. It is also found that  $P_{f-AIC}$  gradually decreases as p = MK increases while the  $P_{f-MDL}$  remains zero in both simulation and analytical results. We conclude that the MDL method has excellent false alarm performance. From TABLE II, we find that the probability of false alarm increases very slowly as N increases. In fact, our simulation shows that  $P_{f-AIC}$  is still below 0.1 even when  $N = 10^{15}$  at M = 5 and K = 4.

Figs. 1-4 show  $P_d$  at different system parameters. In Fig. 1, we first compare the detection performance obtained by simulation between SITC and OITC. It is seen that the proposed SITC sensing algorithm do not lead to any performance loss compared to OITC algorithm. Then, comparing the semi-analytical results obtained from Proposition 2 with the simulation results, one can observe a very good match between them, especially for MDL method. Thus, Proposition 2 is validated. Fig. 2, Fig. 3 and Fig. 4 present the simulation results of  $P_d$  for variable K (at M = 5, N = 10000), M (at K = 4, N = 10000) and N (at K = 4, M = 5), respectively. It is found that the performance is improved as any of these parameters increases.

Thus far, a few efficient sensing algorithms have been proposed in the literature, with each requiring distinct prior information. In this subsection, for fair comparison, we only choose the eigenvalue-based methods proposed in [10]-[12] and the energy detection method since they both need little prior information. It should be mentioned that the proposed SITC-AIC and SITC-MDL algorithms are equivalent to the GITC algorithm provided in Section V via setting  $\gamma_{AIC} = \exp\left(\frac{(2p-1)}{N}\right)$  and  $\gamma_{MDL} = \exp\left(\frac{(p-0.5)\log N}{N}\right)$ . Therefore, we omit the performance comparison with the GITC. In the simulation, we fix the order of channel L = 10 as in [10] and choose N = 10000, K = 4 and M = 5. Fig. 5 shows the comparison with the energy detection (ED) method (with perfect estimation of noise covariance) and the four eigenvaluebased methods, namely, the maximum minimum eigenvalue detection (EV-MME) and energy with minimum eigenvalue detection (EV-EME) [10], the blindly combined energy detection (EV-BCED) [11], and the arithmetic to geometric mean (EV-AGM) [12]. We see that, under almost the same  $P_f$ , energy detection performs the best, followed by the EV-AGM method, the proposed SITC-AIC method and EV-BCED method, and then EV-MME and EV-EME methods. Among the proposed SITC-AIC and four eigenvalue-based methods, the SITC-AIC almost obtains the same performance with the EV-BCED method and they both outperform EV-MME and EV-EME while being slightly inferior to the EV-AGM method. Though the proposed SITC-MDL

method performs the worst in  $P_d$ , it is the best among all the considered schemes in terms of  $P_f$  performance.

The comparison with energy detection with noise uncertainty is presented in Fig. 6, where "ED-x dB" means that the noise uncertainty in energy detection is x-dB as defined in [10]. It is observed that, although the proposed method performs worse than the energy detection method with accurate noise covariance estimation, it significantly outperforms in both  $P_d$  and  $P_f$  when there exists some noise uncertainty. This clearly demonstrates the robustness of information theoretic criteria based blind sensing algorithm.

# C. Performance of the GITC algorithm

Results for the GITC sensing algorithm at different threshold values are demonstrated in Fig. 7. It is assumed that we should choose proper thresholds to make  $P_f = 0.1$ ,  $P_f = 0.05$  and  $P_f = 0.01$ . According to the Proposition 1 and the discussion in Section V, we choose three

thresholds  $\gamma = 1.0372$ ,  $\gamma = 1.0393$  and  $\gamma = 1.0429$  (note that since the analytical results are slightly larger than the simulation results, the thresholds we choose should make theoretic  $P_f$ larger than required  $P_f$  by about 0.02). From the plots, it is found that the  $P_f$  requirements are satisfied very well. One can also see that the probability of false alarm is very sensitive to the threshold. Hence, the GITC sensing algorithm is flexible for system design with different requirements.

# VII. CONCLUSIONS

In this paper we have provided an intensive study on the information theoretic criteria based blind spectrum sensing method. Based on the prior work on the related study, we first proposed the simplified ITC sensing algorithm. This algorithm significantly reduces the computational complexity without losing any detection performance compared with the existing ITC based sensing algorithm. Moreover, it enables a more trackable analytical study on the detection performance. Thereafter, applying the recent advances in random matrix theory, we derive closed-form expressions for both the probability of false alarm and probability of detection which can tightly approximate the actual results in simulation. We further generalized the SITC sensing algorithm to an eigenvalue based sensing algorithm which strike the balance between the probabilities of detection and false alarm by involving an adjustable threshold. Simulation results demonstrate that the proposed blind sensing algorithm outperforms the existing eigenvalue-based sensing algorithms in certain scenarios.

#### APPENDIX A

# WHITENING THE OVER-SAMPLED NOISES

At the secondary receiver, the received continuous signal is usually filtered by a low-pass filter. Therefore, the noise  $\mu(t)$  in (6) and (7) should be correlated. We assume that the white noise before the filter is  $\hat{\mu}(t)$  and the system function of the low-pass filter is g(t) which is known at the secondary receiver. In the following, we only consider the real value case, since in the communication system, the complex value signal is just the combination of two orthogonal real value signals. As we have known,  $\mu(t)$  can be described by  $\hat{\mu}(t)$  and g(t) as

$$\mu(t) = g(t) \otimes \hat{\mu}(t) = \int_0^{t_{max}} g(\ell) \hat{\mu}(t-\ell) d\ell,$$

where  $(0, t_{max})$  represents the time span of g(t) and  $\otimes$  denotes the convolution operator. Thus, the auto-correlative function of  $\mu(t)$  denoted by  $\phi_{\mu}(\tau)$  can be expressed as

$$\phi_{\mu}(\tau) = \phi_g(\tau) \otimes \phi_{\hat{\mu}}(\tau),$$

where  $\phi_g(\tau)$  and  $\phi_{\hat{\mu}}(\tau)$  are the auto-correlative functions of g(t) and  $\hat{\mu}(t)$ , respectively. Note that  $\phi_{\hat{\mu}}(\tau)$  should be equal to  $\sigma^2 \delta(\tau)$  since  $\hat{\mu}(t)$  is white (here the covariance of  $\hat{\mu}(t)$  is assumed to be  $\sigma^2$ ). Therefore, we derive that

$$\phi_{\mu}(\tau) = \sigma^2 \phi_g(\tau) = \sigma^2 \int_0^{t_{max}} g(\ell) g(\tau - \ell) d\ell, \quad 0 \le \tau \le 2t_{max}$$

Thus, if the received signal is over-sampled at rate  $Kf_s$  where  $f_s$  is the reciprocal of the baseband symbol duration  $T_0$  and K is the over-sampling factor, the covariance matrix of the noise vector  $\mu_i$  given in (21) becomes

$$\mathbf{R}_{\mu} = \sigma^2 \mathbf{Q},$$

with  $\mathbf{Q}$  having entries  $q_{i,j} = \phi_g(|i-j|\frac{T_0}{K})$ . Note that  $\mathbf{Q}$  is a positive definite symmetric matrix. It can be decomposed into  $\mathbf{Q} = \tilde{\mathbf{Q}}^2$ , where  $\tilde{\mathbf{Q}}$  is also a positive definite symmetric matrix. Hence, to obtain the independent noise samples in the over-sampling scheme, we can pre-whiten the over-sampled noise samples  $\mu_i$  as

$$ilde{oldsymbol{\mu}}_i = ilde{\mathbf{Q}}^{-1} oldsymbol{\mu}_i.$$

Then, the covariance matrix of  $\tilde{\mu}_i$  transforms into

$$\mathbf{R}_{\tilde{\mu}_i} = \tilde{\mathbf{Q}}^{-1} \mathbf{R}_{\mu} \tilde{\mathbf{Q}}^{-1} = \sigma^2 \mathbf{I}_p.$$

Now, noise samples  $\mu_i$  are whitened. It is noted that  $\tilde{\mathbf{Q}}$  is only related to the low-pass filter and over-sampling factor K and is independent to the signal and noise. Therefore, the pre-whitening process can be used blindly.

#### APPENDIX B

# **PROOF OF LEMMA 1**

We prove the lemma from two aspects. Firstly, it has been shown in [32], [33] that most of the estimation errors of AIC and MDL occur tightly around the true numbers. According to this finding, at hypothesis  $\mathcal{H}_0$  (the true number of source signal is zero), if there exists  $\hat{k} > 0$ minimizing (2) or (3), then we have  $\hat{k} = 1$  with high probability. Hence, Lemma 1 holds for the case of false alarm. Next, we prove that Lemma 1 succeeds at hypothesis  $\mathcal{H}_1$ . Since the primary user is present, the eigenvalues  $l_i$  of the sampled covariance matrix are distinct at least for  $i = 1, 2, \ldots, q$  (here q is the true source number). For  $i = q, q + 1, \ldots, p$ , the eigenvalues are actually the estimation of noise variance  $\sigma^2$ . They may be equal to each other when N is enough large. According to the expression of AIC and MDL, it is found that the second terms in (2) and (3) are monotonically increasing functions of k. To make the cost function in (2) or (3) minimum at  $\hat{k} \in [1, p-1]$ , we must have that the first terms in (2) and (3) are monotonically decreasing for  $k = 0, 1, \ldots, \hat{k}$ . We next prove this statement.

We focus on the AIC criterion and the extension to MDL is straightforward. Supposing  $k' \in [2, \hat{k}]$  and

$$f_{\rm AIC}(k) = -2 \log \left( \frac{\prod_{i=k+1}^{p} l_i^{1/(p-k)}}{\frac{1}{p-k} \sum_{i=k+1}^{p} l_i} \right)^{(p-k)N},$$

we have

$$f_{\text{AIC}}(k'-1) - f_{\text{AIC}}(k') = 2N \log \frac{\left(\frac{1}{p-k'+1} \sum_{i=k'}^{p} l_i\right)^{p-k'+1}}{\left(\frac{1}{p-k'} \sum_{i=k'+1}^{p} l_i\right)^{p-k'} l'_k}$$

Since

$$\left(\frac{1}{p-k'+1}\sum_{i=k'}^{p}l_{i}\right)^{p-k'+1}$$

$$=\left(\frac{1}{p-k'}\frac{p-k'}{p-k'+1}\sum_{i=k'+1}^{p}l_{i}+\frac{1}{p-k'+1}l_{k'}\right)^{p-k'+1}$$

$$\geq \left[\left(\frac{1}{p-k'}\sum_{i=k'+1}^{p}l_{i}\right)^{\frac{p-k'}{p-k'+1}}l_{k'}^{\frac{1}{p-k'+1}}\right]^{p-k'+1}$$

$$=\left(\frac{1}{p-k'}\sum_{i=k'+1}^{p}l_{i}\right)^{p-k'}l_{k'}^{p-k'}$$

(here, the arithmetic-mean geometric-mean inequality  $x_1^{a_1} + x_2^{a_2} \ge x_1^{a_1} x_2^{a_2}$  with  $a_1 + a_2 = 1$  is applied), we conclude that

$$\frac{\left(\frac{1}{p-k'+1}\sum_{i=k'}^{p}l_{i}\right)^{p-k'+1}}{\left(\frac{1}{p-k'}\sum_{i=k'+1}^{p}l_{i}\right)^{p-k'}l_{k}'} \ge 1.$$

It further means

$$f_{\rm AIC}(k'-1) - f_{\rm AIC}(k') > 0,$$

i.e.,  $f_{AIC}(k)$  is a monotonic decreasing function. Hence, we have

$$\lim_{N \to \infty} \frac{\text{AIC}(0) - \text{AIC}(1)}{N} = 2 \log \frac{\left(\frac{1}{p} \sum_{i=1}^{p} l_i\right)^p}{\left(\frac{1}{p-1} \sum_{i=2}^{p} l_i\right)^{p-1} l_1} + \lim_{N \to \infty} \frac{-4p+2}{N} > 0$$

If N is finite but larger enough, we claim that Lemma 1 holds with high probability. The high probability is also contributed by the fact that, due to the property of SVD decomposition technique, the first eigenvalue  $l_1$  is always much larger than other eigenvalues. Therefore,  $2N \log \frac{\left(\frac{1}{p} \sum_{i=1}^{p} l_i\right)^p}{\left(\frac{1}{p-1} \sum_{i=2}^{p} l_i\right)^{p-1} l_1}$  is larger enough to make Lemma 1 succeed at hypothesis  $\mathcal{H}_1$ . Thus, we complete the proof of Lemma 1.

# APPENDIX C

#### **PROOF OF PROPOSITION 2**

We firstly derive the derivation of the probability of misdetection  $P_m$  (the probability for misdetecting the presence of primary user at hypothesis  $H_1$ ), then obtain the probability of detection  $P_d$  through  $1 - P_m$ . Without loss of generality, the following derivation is also based on AIC. According to (38), we have

$$P_{m-AIC|H} = \Pr[AIC(0) - AIC(1) < 0|\mathcal{H}_1].$$

Similar to the process described in the proof of Proposition 1, we can rewritten  $P_{m-AIC|H}$  as

$$P_{m-AIC|H} = \Pr\left(\log\left[\frac{(\frac{1}{p}\sum_{i=1}^{p}l_{i})^{p}}{(\frac{1}{p-1}\sum_{i=2}^{p}l_{i})^{p-1}l_{1}}\right] < \frac{4p-2}{2N}\Big|\mathcal{H}_{1}\right).$$
(44)

Where  $\{l_1, l_2, ..., l_p\}$  are the decreasing ordered eigenvalues of the sampled covariance matrix  $\mathbf{R}_x$  in (39). When the number of observation N is larger enough, we obtain the approximation

$$\frac{1}{N}\sum_{i=1}^{N}\boldsymbol{x}_{i}\boldsymbol{x}_{i}^{\dagger}\approx\mathcal{E}\left(\boldsymbol{x}_{i}\boldsymbol{x}_{i}^{\dagger}\right)=\mathbf{H}\mathbf{R}_{s}\mathbf{H}^{\dagger}+\sigma^{2}\mathbf{I}_{p}$$

Thus

$$\frac{1}{p}\sum_{i=1}^{p}l_{i}\approx\frac{1}{p}\mathrm{Tr}\left(\mathbf{H}\mathbf{R}_{s}\mathbf{H}^{\dagger}\right)+\sigma^{2}$$

Hence, (44) turns to

$$P_{m-AIC|H} \approx \Pr\left[\frac{l_1}{\epsilon} \left(p - \frac{l_1}{\epsilon}\right)^{p-1} > \frac{(p-1)^{p-1}}{\exp\left(\frac{2p-1}{N}\right)} \middle| \mathcal{H}_1\right]$$
$$= \Pr\left[y^p - py^{p-1} + \frac{(p-1)^{p-1}}{\exp\left(\frac{2p-1}{N}\right)} < 0 \middle| \mathcal{H}_1\right], \tag{45}$$

where  $\epsilon = \frac{1}{p} \operatorname{Tr} (\mathbf{H} \mathbf{R}_s \mathbf{H}) + \sigma^2$  and  $y \triangleq p - \frac{l_1}{\epsilon}$ .

Assuming  $\pi_1$  and  $\pi_2$  (with  $\pi_1 < \pi_2$ ) are two real roots within (0, p) of the following function

$$g(y) = y^{p} - py^{p-1} + \frac{(p-1)^{p-1}}{\exp\left(\frac{2p-1}{N}\right)}.$$
(46)

As described in the proof of proposition 1, the probability of misdetection is concluded as

$$P_{m-AIC|H} \approx \Pr[\pi_1 < y < \pi_2 | \mathcal{H}_1],$$

i.e.,

$$P_{m-AIC|H} \approx \Pr[(p - \pi_2)\epsilon < l_1 < (p - \pi_1)\epsilon | \mathcal{H}_1].$$
(47)

Note that  $l_1$  is the largest eigenvalue of the sampled variance matrix  $\mathbf{R}_x$ . Given the channel matrix,  $\mathbf{R}_x$  can be approximated as

$$\mathbf{R}_{x} \approx \frac{1}{N} \left[ \mathbf{H} \sum_{i=1}^{N} \boldsymbol{s}_{i} \boldsymbol{s}_{i}^{\dagger} \mathbf{H}^{\dagger} \right] + \frac{1}{N} \sum_{i=1}^{N} \boldsymbol{\mu}_{i} \boldsymbol{\mu}_{i}^{\dagger} \approx \mathbf{H} \mathbf{R}_{s} \mathbf{H}^{\dagger} + \frac{1}{N} \sum_{i=1}^{N} \boldsymbol{\mu}_{i} \boldsymbol{\mu}_{i}^{\dagger},$$

when N is larger enough.

Let  $\{\delta_1, \delta_2, \dots, \delta_p\}$  and  $\{\chi_1, \chi_2, \dots, \chi_p\}$  be the decreasing ordered eigenvalues of  $\mathbf{HR}_s\mathbf{H}^{\dagger}$ and  $\frac{1}{N}\sum_{i=1}^{N} \boldsymbol{\mu}_i \boldsymbol{\mu}_i^{\dagger}$  respectively. Apply Weyl's inequality theorem in [34], the largest eigenvalue of  $\mathbf{R}_x$ ,  $l_1$ , satisfies

$$\chi_1 + \delta_p \leqslant l_1 \leqslant \chi_1 + \delta_1$$

Equivalently  $\chi_1$  satisfies

$$l_1 - \delta_1 \leqslant \chi_1 \leqslant l_1 - \delta_p. \tag{48}$$

Therefore, there must exist a constant  $\rho$  satisfying  $\delta_p \leq \rho \leq \delta_1$  which makes  $l_1 - \rho$  equal to  $\chi_1$ . Then (47) is rewritten as

$$P_{m-AIC|H} \approx \Pr[(p - \pi_2)\epsilon - \rho < \chi_1 < (p - \pi_1)\epsilon - \rho|\mathcal{H}_1],$$

i.e.,

$$P_{d-AIC|H} \approx \Pr\left[\frac{(p-\pi_1)\epsilon - \rho}{\sigma^2} < \frac{\chi_1}{\sigma^2} < p|H_1\right] + \Pr\left[0 < \frac{\chi_1}{\sigma^2} < \frac{(p-\pi_2)\epsilon - \rho}{\sigma^2}|\mathcal{H}_1\right],$$

where we use the similar constraint for  $\frac{\chi_1}{\sigma^2}$  as in the proof of Proposition 1. Since  $\chi_1$  converges to the Tracy-Widom distribution of order two, we conclude

$$P_{d-AIC|H} \approx Q(\rho),$$

where  $Q(\cdot)$  is defined in Proposition 2. Simultaneously, based on (47), the upper and lower bounds for  $P_{m-AIC|H}$  is

$$1 - Q(\delta_1) \leqslant P_{m-AIC|H} \leqslant 1 - Q(\delta_p).$$

Therefore, the upper and lower bound of  $P_{d-AIC|H}$  can be obtain straightforwardly as

$$Q(\delta_p) \leqslant P_{d-AIC|H} \leqslant Q(\delta_1).$$

The proof for MDL criterion is the same, except that the function g(y) in (46) is redefined as

$$g(y) = y^{p} - py^{p-1} + \frac{(p-1)^{p-1}}{\exp\left(\frac{(p-0.5)\log N}{N}\right)}.$$
(49)

Proposition 2 is thus proved.

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	$6 = 2 \times 3$	$12 = 3 \times 4$	$20 = 5 \times 4$	$24 = 4 \times 6$	$35 = 5 \times 7$
Simulation results for SITC-AIC	0.0948	0.0770	0.0594	0.0541	0.0460
Simulation results for OITC-AIC	0.0972	0.0773	0.0597	0.0541	0.0470
Analytical results for SITC-AIC	0.1360	0.1036	0.0791	0.0711	0.0550
Simulation results for SITC-MDL	0	0	0	0	0
Simulation results for OITC-MDL	0	0	0	0	0
Analytical results for SITC-MDL	0	0	0	0	0

 $\label{eq:TABLE I} {\mbox{Probability of false alarm with different } p = M \times K \mbox{ at } N = 10000$ 

TABLE II							
Probability of false alarm with different $N$ at $p=MK=20$							
	N = 1000	N = 5000	N = 10000				
Simulation results for SITC-AIC	0.0421	0.0558	0.0594				
Simulation results for OITC-AIC	0.0421	0.0561	0.0597				
Analytical results for SITC-AIC	0.0581	0.0744	0.0791				
Simulation results for SITC-MDL	0	0	0				
Simulation results for OITC-MDL	0	0	0				
Analytical results for SITC-MDL	0	0	0				



Fig. 1. Simulation and theoretic results about probability of detection at different (M, K, N) for both SITC and OITC.



Fig. 2. Probability of detection for different K at M = 5 and N = 10000.



Fig. 3. Probability of detection for different M at K = 4 and N = 10000.



Fig. 4. Probability of detection for different N at M = 5 and K = 4.



Fig. 5. Comparison with the eigenvalue-based methods and the energy detection method at M = 5, K = 4 and N = 10000.



Fig. 6. Comparison with energy detection with noise uncertainty at M = 5, K = 4 and N = 10000.



Fig. 7. Simulation results for GITC algorithm for different  $P_f$  at M = 5, K = 4 and N = 1000.