On the Feasibility of Wireless Shadowing Correlation Models

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***Abstract*—There is emerging interest in more detailed models for wireless shadowing, which may include nonconstant shadowing variance, non-lognormal shadowing, and, most importantly, correlation between paths; we focus on this last aspect. This paper offers a structured synthesis of the existing literature on autocorrelation and cross-correlation in wireless shadowing and attempts to fill existing gaps in the analysis of correlation models. We make a survey of these models and argue, as has previously been observed, that certain models are not mathematically feasible, which may lead to problems in simulations or analysis. We then state some theorems that test whether the models are positive semidefinite, which is the central necessary condition for feasibility, and evaluate the existing models accordingly. Additionally, we evaluate the models according to their physical plausibility, which leads us to choose one model among many as arguably the best one in existence so far. This paper should be useful as a guide on how to implement shadowing correlation in one's work, how to choose an appropriate correlation model, and how to modify existing models or create new models so that they fulfill mathematical feasibility.**

***Index Terms*—Correlation, wireless channel modeling, wireless shadowing.**

I. Introduction

CORRELATION in wireless shadowing is a significant step in obtaining more realistic channel propagation models. This paper proposes to discuss in detail the existing literature on shadowing correlation and the existing correlation models, focusing on evaluating the mathematical feasibility of these models while also making comments on their physical plausibility. The purpose of this paper is to facilitate the choice and implementation of correlation models in works that involve shadowing.

A. Motivation

Recent work on various aspects of wireless communications has indicated a wide gap between results obtained assuming independent shadowing paths and between those that introduce correlation in shadowing propagation models [1]–[5]. Older works show that shadowing correlation significantly affects handover behavior [6]–[9], interference power [10]–[14] (and consequently system performance) [15]–[17], and the performance of macrodiversity schemes [10], [18]–[23]. Furthermore, shadowing in decibels has been measured [18], [24]–[39] to have significant correlation in various scenarios. Because of these two facts, we believe that the general wireless community is becoming convinced of the importance of modeling correlation in shadowing [2]–[5], [12], [23], [40]–[49]. This is part of a more general trend suggesting that most propagation models used in simulation (and analysis) work today are sometimes too simplistic and may lead to misleading results [50]. Shadowing correlation can also be positively exploited in some algorithms or protocols, e.g., for wireless positioning [2], [5], [51], cognitive radio and spectrum sensing [49], [52]–[54], or neighbor discovery [48] applications.

Meanwhile, there already exist several models for shadowing correlation. The questions of which model to use and how to simulate channel realisations accordingly are essential for every researcher wanting to implement correlation. However, because correlation models are based on estimating a complex phenomenon from a small data sample, they may lose some properties with respect to reality; in particular, some estimated models may not be mathematically feasible, and thus, it would be impossible, in certain cases, to generate Monte Carlo samples from them.

In [45], it is deplored that this problem is rarely taken into account; yet, it has received some attention in [6], [7], [45], [46], and [55]–[57]. It has previously been addressed [43], [57], [58] by slightly modifying particular correlation matrices so that they become feasible (specifically, positive semidefinite (*psd*) for lognormal shadowing). However, we prefer to address the problem at its root: let us use only such correlation models that will *always* produce *psd* correlation matrices, as suggested in [6], [45], and [56]. This has the following advantages:

1) a faster implementation time, as no provision for correcting non-*psd* matrices needs to be made;
2) probably a faster execution time for every realisation, as no decisions or corrections need to be made during the Monte Carlo simulation;
3) mathematical consistency and elegance: if a model is feasible, then it is safe for mathematical analysis [9]–[11], [13], [21], [22], [44], [52], [59]–[64].

If we begin with a set of independent random variables and construct correlated shadowing variables by combining them in some way, then we can always calculate their correlation

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structure. However, the inverse operation cannot always be performed: a construction to obtain a desired correlation structure does not always exist. Those that cannot be constructed are termed not feasible.

B. Mathematical Tools

Some mathematical definitions and properties will be necessary in this paper. All algebraic operations are taken over the $\mathbb{R}$ field.

Definition 1: A square matrix $\mathbf{A}_{N\times N}$ is positive definite if for every column vector $\mathbf{x} \in \mathbb{R}^N \setminus \mathbf{0}$, $\mathbf{x}^{\mathrm{T}}\mathbf{A}\mathbf{x} > 0$ [65, p. 82, Def. 3.1.1.xvii].

Definition 2: A square matrix $\mathbf{A}_{N\times N}$ is *psd* if for every column vector $\mathbf{x} \in \mathbb{R}^N$, $\mathbf{x}^{\mathrm{T}}\mathbf{A}\mathbf{x} \geq 0$ [65, p. 82, Def. 3.1.1.xv].

Definition 3: The Hadamard (also called Schur) product of two $M \times N$ matrices ($\mathbf{A}$ with entries $a_{i,j}$ and $\mathbf{B}$ with entries $b_{i,j}$) is a $M \times N$ matrix $\mathbf{C} = \mathbf{A} \circ \mathbf{B}$ with entries $c_{i,j}$ such that the matrix entries are multiplied elementwise: $c_{i,j} = a_{i,j}b_{i,j}$ [65, p. 252, eq. (7.3.1)].

Property 1 (Schur Product Theorem): If two $N \times N$ matrices $\mathbf{A}$ and $\mathbf{B}$ are both *psd*, then their Hadamard product $\mathbf{C} = \mathbf{A} \circ \mathbf{B}$ is also *psd* [65, p. 335, Fact 8.16.8].

Definition 4: The Kronecker (also called Zehfuss) product of a $M \times N$ matrix $\mathbf{A}$ with entries $a_{i,j}$ and a $K \times L$ matrix $\mathbf{B}$ is a $MK \times NL$ matrix $\mathbf{C} = \mathbf{A} \otimes \mathbf{B}$ with $M \times N$ block entries $a_{i,j}\mathbf{B}$ [65, p. 248, Def. 7.1.2].

Property 2: If $\mathbf{A}$ and $\mathbf{B}$ are both *psd*, then so is $\mathbf{C} = \mathbf{A} \otimes \mathbf{B}$ [65, p. 254, Fact 7.4.15].

Definition 5: An even function $f(t)$ is said to be *positive definite* if and only if for any N, for any $t_1, \ldots, t_N$ on $\mathbb{R}^+$, the matrix with entries $a_{i,j} = f(|t_i - t_j|)$ is *psd*. This can directly be derived from [66, p. 58, eq. (1.29)] and Definition 2.

Property 3 (Bochner's Theorem): A continuous even function $f(\tau)$ is positive definite if and only if there exists a nondecreasing bounded function $F(\omega)$, $\omega \in \mathbb{R}^+$ such that [66, p. 92, eq. (2.53)]

$$f(\tau) = \int_0^\infty \cos \omega\tau dF(\omega). \tag{1}$$

Property 4: Every covariance matrix is necessarily *psd*[1] and symmetric [66, p. 15, eq. (0.36)].

Property 5 (Pólya's Theorem): A bounded even function $f(\tau)$ with $f(\infty) = 0$ that is concave (up) on $(0, \infty)$ is necessarily positive definite [66, p. 136]. (The converse is not true.)

C. Structure of This Paper

In Section II, we discuss the different types of physical situations that lead us to discuss correlation in shadowing before focusing on autocorrelation and cross-correlation. In Section III, we define correlation in general mathematical terms. In Section IV, we define positive semidefiniteness, which is the primary property to determine feasibility, and give some indications on how this property can be proved. In Section V, we discuss why models derived from real-world measurements may nevertheless prove not feasible. In Section VI, we make a comprehensive study of the correlation models in the literature and summarised these models' properties in a table. In Section VII, we impose further conditions on correlation models by talking about their physical plausibility. These physical properties are also summarised in the table, which allows us to formulate a best model that meets both mathematical and physical criteria. In Section VIII, we discuss the relationship between feasibility and positive semidefiniteness in the cases of both lognormal and non-lognormal shadowing and discuss how to generate correlated jointly lognormal shadowing. We conclude in Section IX with practical guidelines for using shadowing correlation models in one's work.

This paper represents a synthesis of existing knowledge and some new contributions. As such, all non-original statements are given appropriate references.

II. Types of Correlation

In wireless communications, shadowing is a phenomenon that corresponds to a random variability of the power gain over a (directed) propagation path. A transmitting node radiates a radio signal toward a receiver node. The receiver may receive a desired signal or an interfering signal. Shadowing represents the variability of the logarithm of the received power around its expected value: we denote this quantity S, with $\mathbb{V}\mathrm{AR}\{S\} < \infty$. The small-scale effect of fading has been removed in the spatial dimension by averaging over a local region [45] of about 50 wavelengths [56], [62], that is, approximately 8–34 m for classic cellular channels [18], [24], [25], [27], [29], [37], [38].

Consider two directed paths $\overrightarrow{X_1Y_1}$ and $\overrightarrow{X_2Y_2}$ with shadowing values S_1 and S_2, respectively. Their correlation coefficient is defined as

$$\rho_{1,2} = \frac{\mathbb{E}\{S_1S_2\}}{\sqrt{\mathbb{V}\mathrm{AR}\{S_1\}\mathbb{V}\mathrm{AR}\{S_2\}}}. \tag{2}$$

In general, correlation may exist for any set of four points $X_1 \neq Y_1, X_2 \neq Y_2$. However, we usually examine correlation under more specific scenarios.

A. Scenarios

Shadowing correlation can occur in various particular scenarios, specifically in peer-to-peer links [2], [67], [68], in indoor, cross-floor, and indoor–outdoor links [27], [30], and in satellite–ground communications [31], [69]–[71]. It should be noted however that most of the literature on correlation in shadowing is driven by cellular (usually urban) scenarios, where the base station/mobile and uplink/downlink dualities apply. In cellular communications, the distinction between autocorrelation and cross-correlation applies [45]. Cross-sector correlation has also been considered [72].

An important consideration for satellite and indoor scenarios is that propagation is usually considered in three, not just two, Cartesian dimensions. In this paper, we will analyze 2-D

[1]Reference [56] says "positive definite," but this is too restrictive, whereas [66] also says "positive definite," but this is a difference in nomenclature, and clearly, *psd* is meant.

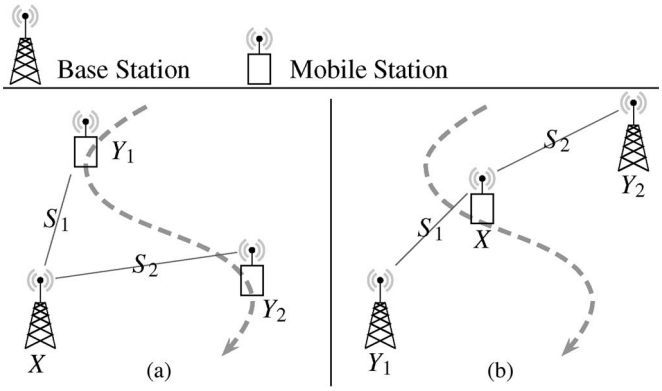

Fig. 1. (a) Shadowing autocorrelation for a mobile $Y(t)$. (b) Shadowing cross-correlation for a mobile X.

channel models only, although the analysis usually translates easily to three dimensions.

B. Autocorrelation

Called serial correlation [73], this model considers a transmitting base station X received by the same moving mobile Y at two moments in time t_1 and t_2 and at distinct locations $Y_1 = Y(t_1)$ and $Y_2 = Y(t_2)$. Alternatively, the signal may be received by two distinct mobiles Y_1 and Y_2 at the same moment in time. These scenarios may also be reversed to consider the uplink: either the same mobile transmits to the same base station X at moments t_1 and t_2 at locations Y_1 and Y_2, respectively, or the same base station X simultaneously receives from two distinct mobiles at locations Y_1 and Y_2. All four cases can come under the category of autocorrelation and are illustrated in Fig. 1(a).

C. Cross-Correlation

This is the central object of our study. Cross-correlation, or site-to-site correlation [73], considers two transmitting base stations Y_1 and Y_2 that transmit to a common mobile receiver X. Alternatively, cross-correlation can consider a mobile transmitter X whose signal is picked up by two base stations Y_1 and Y_2. These are illustrated in Fig. 1(b).

Now, observing the previous section on autocorrelation and the systematic nomenclature we gave to nodes (X a common node, and Y_1 and Y_2 nodes on two separate links), we may abstract from the nature of these nodes and the link direction and conclude that autocorrelation and cross-correlation are very similar in the mathematical sense and can be studied in the same manner. Furthermore, we see in [46] and [74] how an autocorrelation model can also be used to model cross-correlation. We therefore assume that X is the common node to all paths, which we locate at the origin for simplicity, and all links are between X and the points Y_i.

D. Generalised Correlation

If we allow total freedom for the positions of the two paths, then the correlation model becomes more complex; in fact, it becomes a function of up to eight free variables (four free positions on a 2-D plane).

We will not study this type of correlation for two reasons: first, because of its increased complexity; and second, because the methods given in the literature begin with an explicit construction of the shadowing realisations, and hence, it is not necessary to study their feasibility. They are always feasible. Contrast this with the correlation models studied here. In Section VIII-A, it is necessary to factorise a matrix, which is not always possible, and thus, the method is implicit, and feasibility must be studied.

Such methods rely on generating shadowing maps [42] in some way. Three algorithms for doing so are the sum-of-sinusoids (SoS) algorithm [67], the network shadowing (NeSh) method [2], [3], [44], [68], and the over-obstacle multiple-edge diffraction model [75].

E. Time Correlation

Time correlation is different in that it only considers one path $\overrightarrow{XY}$ but looks at the correlation between shadowing at various moments in time. Shadowing can thus be represented as a random process $S(t)$ [8], [64]. The feasibility of the model is a simple problem here because the problem simply evolves in one dimension, i.e., time. The correlation model is then specified by the autocorrelation function of $S(t)$, and the model will be *psd* if and only if the autocorrelation function is *psd*. We are not aware of any measurements of correlation in time only, and it is not evident if shadowing significantly changes over a fixed path.

It should be understood that shadowing evolution in time should (like in space) have fast fading removed through time averaging of some duration.

F. Uplink–Downlink Correlation

Consider the path $\overrightarrow{XY}$ and then its return path $\overrightarrow{YX}$. By channel symmetry, one might conclude that the shadowing experienced in both directions is identical, which would correspond to a correlation coefficient of $\rho_{\mathrm{ud}} = 1$. In practice, measurements indicate a high degree of correlation: $\rho_{\mathrm{ud}} \geq 0.66$ [35]. From these measurements, Kim and Han [76] assumed $\rho_{\mathrm{ud}} = 0.7$. Furthermore, Kotz *et al.* [50] demonstrated asymmetry but positive correlation in link connectivity, which can be interpreted as unequal but correlated shadowing in each direction.

III. Shadowing Model Description

Autocorrelation and cross-correlation models can generically be described as follows: consider N points $Y_1, \ldots, Y_N$ located on a plane at positions $\vec{r}_1, \ldots \vec{r}_N$ with $\vec{r}_i \in \mathbb{R}^2 \setminus \{0\}$. We will write $r_i = \|\vec{r}_i\|$. We assume, without loss of generality, that the common point X is located at the origin, and thus, $\vec{r}_i = \overrightarrow{XY_i}$. Consider S_i as the logarithm of the power attenuation due to shadowing on each path $\vec{r}_i$ (see Fig. 2). We have $\mathbb{E}\{S_i\} = 0$. We will, for the moment, not commit to any particular shadowing distribution. We only require the condition that $\mathbb{V}\mathrm{AR}\{S_i\} < \infty$.

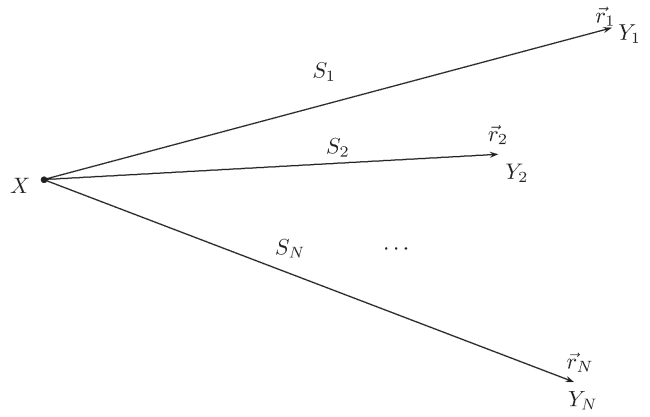

Fig. 2. Generic geometry of shadowing autocorrelation and cross-correlation.

The shadowing log-variance can most generally be characterised as

$$\sigma_i^2 = \mathbb{V}\mathrm{AR}\{S_i\} = \sigma_{\mathrm{s}}^2(\vec{r}_i) \tag{3}$$

although in an isotropic medium, it will simply be a function of distance: $\mathbb{V}\mathrm{AR}\{S_i\} = \sigma_{\mathrm{s}}^2(r_i)$. Usually, shadowing log-variance is considered as constant after some distance [1].

The pairwise correlation coefficients necessarily exist (by the Cauchy–Schwarz inequality) and can be defined as

$$\begin{aligned}\rho_{i,j} &= \mathbb{E}\{S_i S_j\}/\sigma_i\sigma_j = h(\vec{r}_i, \vec{r}_j), \quad i \neq j \\ \rho_{i,i} &= 1.\end{aligned} \tag{4}$$

We then have the following properties.

1) $-1 \leq h(\vec{r}_i, \vec{r}_j) \leq 1$.
2) $h(\vec{r}_i, \vec{r}_j) = h(\vec{r}_j, \vec{r}_i)$.

A shadowing model that includes pairwise correlation can most generally be defined by the pair of functions $(\sigma_{\mathrm{s}}^2(\vec{r}), h(\vec{r}_1, \vec{r}_2))$.

Now, the correlation matrix of $\mathbf{S} = [S_1, \ldots, S_N]$ is

$$\mathbf{K} = \begin{bmatrix} \sigma_1^2 & \sigma_1\sigma_2\rho_{1,2} & \cdots & \sigma_1\sigma_N\rho_{1,N} \\ \sigma_1\sigma_2\rho_{1,2} & \sigma_2^2 & \cdots & \sigma_2\sigma_N\rho_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_1\sigma_N\rho_{1,N} & \sigma_2\sigma_N\rho_{2,N} & \cdots & \sigma_N^2 \end{bmatrix}. \tag{5}$$

IV. Positive Semidefiniteness of Correlation Models

From Property 4, for $\mathbf{K}$ to be a valid covariance matrix of $\mathbf{S}$, it is necessary (but not always sufficient, as seen in Section VIII) that $\mathbf{K}$ be *psd* ($\mathbf{K}$ is already symmetric by construction).

Definition 6: We say that a shadowing model $(\sigma_{\mathrm{s}}^2(\vec{r}), h(\vec{r}_1, \vec{r}_2))$ is *psd* if $\forall N$, $\forall [\vec{r}_1, \ldots, \vec{r}_N] \in (\mathbb{R}^2 \setminus \{0\})^N$, the correlation matrix $\mathbf{K}$ is always *psd*.

A. Models With Variable Shadowing Variance

Theorem 1: If a model $(1, h(\vec{r}_1, \vec{r}_2))$ with constant log-variance 1 is *psd*, then for any $\sigma_{\mathrm{s}}^2(\vec{r})$, the model $(\sigma_{\mathrm{s}}^2(\vec{r}), h(\vec{r}_1, \vec{r}_2))$ is also *psd*. Conversely, if the model $(\sigma_{\mathrm{s}}^2(\vec{r}), h(\vec{r}_1, \vec{r}_2))$ is *psd* and $\sigma_{\mathrm{s}}^2(\vec{r}) > 0\ \forall \vec{r} \in \mathbb{R}^2 \setminus \{0\}$, then the model $(1, h(\vec{r}_1, \vec{r}_2))$ with constant log-variance 1 is *psd*.

Proof: Consider N shadowing paths $\vec{r}_i \in \mathbb{R}^2 \setminus \{0\}$ and a shadowing model $(\sigma_{\mathrm{s}}^2(\vec{r}), h(\vec{r}_1, \vec{r}_2))$, where $(1, h(\vec{r}_1, \vec{r}_2))$ is a *psd* shadowing model. We call $\mathbf{H}$ the $N \times N$ matrix with entries $h(\vec{r}_i, \vec{r}_j)$ and $\mathbf{s}$ the column vector with entries $\sigma_{\mathrm{s}}(\vec{r}_i)$. Then, the correlation matrix of $\mathbf{S}$ can be written as

$$\mathbf{K} = (\mathbf{s}\mathbf{s}^{\mathrm{T}}) \circ \mathbf{H}. \tag{6}$$

Now, the matrix $\mathbf{s}\mathbf{s}^{\mathrm{T}}$ is *psd*, as can be seen from Definition 2: $\mathbf{x}^{\mathrm{T}}(\mathbf{s}\mathbf{s}^{\mathrm{T}})\mathbf{x} = (\mathbf{x}^{\mathrm{T}}\mathbf{s})^2 \geq 0\ \forall \mathbf{x} \in \mathbb{R}^N$. In addition, since $(1, h(\vec{r}_1, \vec{r}_2))$ is *psd*, it follows that $\mathbf{H}$ is *psd*. Applying the Schur product theorem, we find that $\mathbf{K}$ is always *psd*, which implies that the model $(\sigma_{\mathrm{s}}^2(\vec{r}), h(\vec{r}_1, \vec{r}_2))$ is *psd*.

To prove the converse, we write

$$\mathbf{H} = (\mathbf{z}\mathbf{z}^{\mathrm{T}}) \circ \mathbf{K} \tag{7}$$

where $\mathbf{z}$ is the column vector with entries $\sigma_{\mathrm{s}}^{-1}(\vec{r}_i)$, and the proof is analogous. Of course, here, the additional requirement that $\sigma_{\mathrm{s}}(\vec{r}) \neq 0\ \forall \vec{r} \in \mathbb{R}^2 \setminus \{0\}$ is required. ■

Thus, to study the positive semidefiniteness of a shadowing model, it is sufficient to study the correlation function $h(\vec{r}_1, \vec{r}_2)$ in isolation, which simplifies the problem. We will therefore say that h is *psd* if $(1, h)$ is *psd*.

B. Methods for Proving Positive Semidefiniteness

While we do not have a general criterion for proving that some given h is *psd* or not, the following approaches will nevertheless help us analyze most particular cases.

1) If there exists an explicit constructive algorithm for generating data according to h, then h is necessarily *psd*, since the resulting covariance matrix will always be *psd* (see Property 4).
2) If there exists at least one choice of $[\vec{r}_1, \ldots, \vec{r}_N]$ for which $\mathbf{K}$ is not *psd*, then h is not *psd*. For this test, we need $N \geq 3$, since every correlation matrix of size 2 is *psd*, as seen from this decomposition:

$$\begin{aligned}\begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} &= \begin{bmatrix} u & v \\ v & u \end{bmatrix}^2, \quad -1 \leq \rho \leq 1 \\ u &= \frac{1}{2}\sqrt{1+\rho} + \frac{1}{2}\sqrt{1-\rho} \\ v &= \frac{1}{2}\sqrt{1+\rho} - \frac{1}{2}\sqrt{1-\rho}.\end{aligned} \tag{8}$$

However, $N = 3$ may not be enough [57].

3) Several theorems can also be used to prove that some h is *psd*, as seen in the next section.

C. One-Parameter and Separable Correlation

Proving that isotropic models (functions of $d = \|\vec{r}_1 - \vec{r}_2\|$ only, with $\vec{r}_i \in \mathbb{R}^2$) are *psd* is not trivial. Indeed, while for all *psd* models $h(d)$ the function $h(x)$ on $\mathbb{R}^+$ is positive definite, the converse is not true, and tighter conditions are needed [66, p. 361]. The following two theorems are examples of tests that can be applied to verify that $h(d)$ is *psd*.

Theorem 2: A one-parameter correlation model $h(d)$ with $d = \|\vec{r}_1 - \vec{r}_2\|$ with $\vec{r}_i \in \mathbb{R}^2$ is *psd* if[2] the integral

$$f(\omega) = \int_0^\infty h(x) J_0(\omega x) x dx \tag{9}$$

exists and is nonnegative $\forall \omega \geq 0$ [66, p. 357]. $J_0(x)$ is the Bessel function of the first kind of order 0.

Theorem 3: A one-parameter correlation model $h(d)$ with $d = \|\vec{r}_1 - \vec{r}_2\|$ with $\vec{r}_i \in \mathbb{R}^2$ is *psd* if we have the following[2]:

1) the function $h(x)$ with $x \in \mathbb{R}^+$ is positive definite;
2) the Fourier transform $f(\omega)$ of $h(|x|)$ is nonincreasing on $\omega \in (0, \infty)$.

The converse is not true [66, p. 361].

Many other useful properties of isotropic correlation models are found in [66, Ch. 22].

Theorem 4: A one-parameter correlation model $h(\theta)$ with $\theta = |\angle \vec{r}_1 - \angle \vec{r}_2 + 2k\pi|$, $k \in \mathbb{Z}$, such that $\theta \in [0^\circ, 180^\circ]$ is *psd* if it may be written as

$$h(\theta) = \sum_{n=0}^{\infty} a_n \cos(n\theta) \tag{10}$$

for some nonnegative and bounded sequence $a_0, a_1, \ldots$ with $0 < \sum_{n=0}^{\infty} a_n < \infty$.

Proofs are given in [55] and [56].

Theorem 5: A one-parameter correlation model $h(|x|)$ with $|x| = |g(\vec{r}_1) - g(\vec{r}_2)|$ for some function $g : \mathbb{R}^2 \setminus \{0\} \mapsto \mathbb{R}$ is *psd* if it may be written as

$$h(x) = \int_0^\infty \cos(2\pi x \omega) f(\omega)\, d\omega \tag{11}$$

for some nonnegative and finite-area $f(\omega)$ on $0 < \omega < \infty$.

Proof: From Bochner's theorem, $h(x)$ in (11) is a *psd* function. From Definition 5, it follows that for any N the matrix $\mathbf{H}_{N\times N}$ with entries $\rho_{i,j} = h(|t_i - t_j|)$, $t_i = g(\vec{r}_i)$ is *psd*. ■

Theorem 6: If a correlation model h may be written as

$$h(\vec{r}_1, \vec{r}_2) = h_1(\vec{r}_1, \vec{r}_2) h_2(\vec{r}_1, \vec{r}_2) \tag{12}$$

with h_1 and h_2 both *psd*, then h is also *psd*.

Proof: Let $\mathbf{H}$, $\mathbf{H_1}$, and $\mathbf{H_2}$ be the $N \times N$ matrices with entries $h(\vec{r}_i, \vec{r}_j)$, $h_1(\vec{r}_i, \vec{r}_j)$, and $h_2(\vec{r}_i, \vec{r}_j)$, respectively. We may then write $\mathbf{H} = \mathbf{H_1} \circ \mathbf{H_2}$. Now, since h_1 and h_2 are *psd*, it follows that $\mathbf{H_1}$ and $\mathbf{H_2}$ are both *psd*. Applying Schur's product theorem, we have that $\mathbf{H}$ is also *psd*, which implies that h is *psd*. ■

[2]These two theorems apply for shadowing on $\mathbb{R}^2$ and take a more restricted form on $\mathbb{R}^3$ [66].

A similar argument was given in [56] but for positive definite instead of *psd* matrices.

D. Incorporating Time and Uplink–Downlink Correlation

As shown in Section II-E, shadowing may evolve in time in a correlated manner. In addition, as seen in Section II-F, shadowing may be different on the same propagation path in both directions, although these are usually highly correlated. We will show that, given a positive definite temporal autocorrelation $R_S(\tau)$ and a *psd* spatial correlation model h, and assuming correlation separability between the spatial, time, and uplink–downlink dimensions, the resulting combined correlation matrix is always *psd*. Combining cross-correlation with temporal autocorrelation was described in [8].

Consider a common node X and N nodes $Y_1, \ldots, Y_N$. Let $S_i(t)$ be the shadowing on path $\overrightarrow{XY_i}$ at time instant t with variance $\sigma_i^2 = \sigma_{\mathrm{s}}^2(\vec{r}_i)$ and $\tilde{S}_i$ the shadowing on the return path $\overrightarrow{Y_iX}$ with variance $\tilde{\sigma}_i^2 = \tilde{\sigma}_{\mathrm{s}}^2(\vec{r}_i)$.

Consider the spatial correlation function h such that $\rho_{i,j} = h(\vec{r}_i, \vec{r}_j)$, $i \neq j$, and $\rho_{i,i} = 1$, i.e.,

$$\begin{aligned} \mathbb{E}\left\{S_i(t) S_j(t)\right\} &= \sigma_i \sigma_j \rho_{i,j} \quad \forall t \\ \mathbb{E}\left\{\tilde{S}_i(t) \tilde{S}_j(t)\right\} &= \tilde{\sigma}_i \tilde{\sigma}_j \rho_{i,j} \quad \forall t. \end{aligned} \tag{13}$$

We will assume that the correlation between the two directions of the same path is constant, as in [76], i.e.,

$$\mathbb{E}\left\{S_i(t) \tilde{S}_i(t)\right\} / \sigma_i \tilde{\sigma}_i = \rho_{\mathrm{ud}} \quad \forall i\, \forall t. \tag{14}$$

Consider also the normalised temporal autocorrelation function $\zeta(\tau)$, i.e.,

$$\frac{\mathbb{E}\left\{S_i(t) S_i(t+\tau)\right\}}{\sigma_i^2} = \frac{\mathbb{E}\left\{\tilde{S}_i(t) \tilde{S}_i(t+\tau)\right\}}{\tilde{\sigma}_i^2} = \zeta(\tau) \quad \forall i\, \forall t \tag{15}$$

where $\zeta(\tau)$ is positive definite, and $\zeta(0) = 1$.

We will further assume separability of cross-correlation and uplink–downlink correlation so that the correlation terms can be found as follows:

$$\begin{aligned} \frac{\mathbb{E}\left\{S_i(t_1) S_j(t_2)\right\}}{\sigma_i \sigma_j} &= \frac{\mathbb{E}\left\{\tilde{S}_i(t_1) \tilde{S}_j(t_2)\right\}}{\tilde{\sigma}_i \tilde{\sigma}_j} = \rho_{i,j} \zeta(t_2 - t_1) \\ \mathbb{E}\left\{S_i(t_1) \tilde{S}_j(t_2)\right\} &= \sigma_i \tilde{\sigma}_j \rho_{\mathrm{ud}} \rho_{i,j} \zeta(t_2 - t_1). \end{aligned} \tag{16}$$

From this equation, we see that separability implies that the uplink and downlink cross-correlation models must be the same, i.e., $h = \tilde{h}$.

Consider M time instances $t_1, \ldots, t_M$. Then, the correlation matrix of $S_1(t_1), \tilde{S}_1(t_1), \ldots, S_N(t_1), \tilde{S}_N(t_1), \ldots, S_1(t_M), \tilde{S}_1(t_M), \ldots, S_N(t_M), \tilde{S}_N(t_M)$ is

$$\begin{aligned} \bar{\mathbf{K}}_{2NM\times 2NM} = &\left[\zeta(t_j - t_i)\right]_{M\times M} \\ &\otimes \left((\breve{\mathbf{s}} \breve{\mathbf{s}}^{\mathrm{T}}) \circ \left([\rho_{i,j}]_{N\times N} \otimes \begin{bmatrix} 1 & \rho_{\mathrm{ud}} \\ \rho_{\mathrm{ud}} & 1 \end{bmatrix} \right) \right) \end{aligned} \tag{17}$$

where $\breve{\mathbf{s}}$ is the column vector with entries $\sigma_1, \tilde{\sigma}_1, \ldots, \sigma_N, \tilde{\sigma}_N$.

Theorem 7: Given a *psd* correlation function h, uplink and downlink shadowing variances $\sigma_{\rm s}^2(\vec{r})$ and $\tilde{\sigma}_{\rm s}^2(\vec{r})$, an uplink–downlink correlation coefficient $-1 \leq \rho_{\rm ud} \leq 1$, and a normalised shadowing positive definite time autocorrelation function $\zeta(\tau)$ with $\zeta(0) = 1$, and assuming that the correlation is separable, as in (16), we can conclude that the composite correlation model is *psd*.

Proof: Consider (17). The matrix $\breve{\mathbf{s}}\breve{\mathbf{s}}^{\rm T}$ is always *psd*, as seen in the proof of Theorem 1. The matrix $[\zeta(t_j - t_i)]_{M\times M}$ is *psd* because $\zeta(\tau)$ is positive definite, and the matrix $[\rho_{i,j}]_{N\times N}$ is *psd* because h is *psd*. Finally, all 2×2 correlation matrices are *psd*, as seen in (8). Both the Hadamard and the Kronecker products are *psd* if both their factors are *psd*, as seen in Properties 1 and 2. It follows that $\bar{\mathbf{K}}$ is *psd*. ■

Simply speaking, one can incorporate (separable) directional and time correlation without upsetting the positive semidefiniteness of the correlation model. Other dimensions, notably frequency [49], may also be similarly incorporated.

One can note that the Kronecker product has similarly been used in [77] in a multiantenna context and in [45] for correlation between shadowing, delay spread, and angle spread, all under the assumptions of separability, which is natural when no further information is available [45].

V. Estimating the Correlation Function From Measured Data

To better understand why the estimated models may be *psd* or not, it is worthwhile to examine some aspects of how these models are constructed from measured data.

A. Autocorrelation as Mixed Time–Space Measurements

Autocorrelation is measured [24]–[26], [28], [32], [36]–[39] by taking a single moving mobile Y with velocity v in uniform rectilinear motion, measuring the power loss between Y and a base station X, correcting for path loss and small-scale fading, and considering the shadowing S along the path $\overrightarrow{XY}$ (or $\overrightarrow{YX}$) as a process of time $S(t)$ (equivalently, of space $S(d)$, with $d = vt$ [78], [79]). The underlying assumption is that shadowing over a fixed link does not vary with time (or perhaps varies little or very slowly compared with variation when in motion). More specifically, consider the shadowing $S(\vec{r}, t)$ as would be experienced by a virtual mobile $\tilde{Y}$ at time t and location $\vec{r}$. Now, let the real mobile Y begin at location $\vec{r}_0$ at time $t = 0$ and move in uniform rectilinear motion with velocity $\vec{v}$. Thus, its location at time t is $\vec{r} = \vec{r}_0 + t\vec{v}$. The observed shadowing process at Y is thus $S(t) = S(\vec{r}_0 + t\vec{v}, t)$, or equivalently, $S(d) = S(\vec{r}_0 + d\vec{v}/v, d/v)$. Clearly, the observed process varies in both time and space, and assuming that $S(\vec{r}, t)$ varies in t when $\vec{r}$ is fixed (see Section II-E), these measurements do not directly translate into a correlation model. For example, a different model $S(d)$ might be extracted if the velocity v is changed. This is generally true for other motion trajectories.

An additional issue about autocorrelation models is the level-crossing rate, which is only properly defined for some models. This topic is addressed in [61], [62], and [79]–[81], and we do not further explore it here.

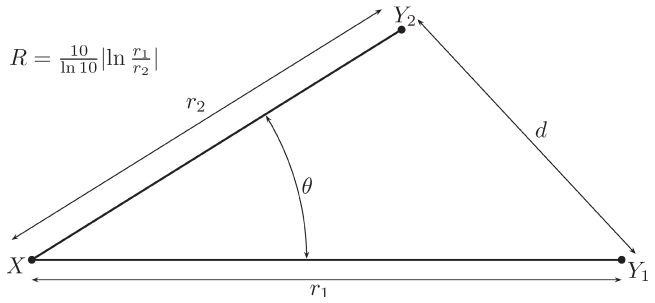

Fig. 3. Pair of autocorrelated or cross-correlated paths with the most relevant dimensional variables: d, θ, R.

B. Cross-Correlation as Spatial-Only Correlation

Several field measurements have been performed [18], [24], [28], [29], [33], [34], [36]–[38] to gather data about the true physical correlation model, which we call $\bar{h}$. Field measurements produce a set of data points, from which an estimate of the cross-correlation function, which we call $\hat{h}$, is obtained by fitting the data with plausible-looking simple mathematical functions. It must be understood that $\bar{h}$ is always a *psd* model by its very mathematical definition in (4). It was argued in [57] that a particularly convenient *psd* model $\hat{h}$ may not accurately reflect the true correlation. This is true. However, because $\bar{h}$ must be *psd*, it is likely that there exists some *psd* model $\hat{h}$ that is close to $\bar{h}$. Therefore, we argue that it is always best and possible to choose a *psd* model $\hat{h}$ that is also close to $\bar{h}$.

In general, $\bar{h}(\vec{r}_1, \vec{r}_2)$ is a function of four variables. However, estimating a four-variable function accurately requires many more points than can realistically be obtained from costly and cumbersome field measurements. Additionally, every point of the function $\bar{h}$ is itself estimated as an expectation (4) and requires several data points. Two consequences follow.

1) The measurement data are usually grouped by collapsing them from four variables into a single variable.
2) On that one variable, the curve fitting is still relatively crude [26], [28], [31], [32], [34], [39].

These two approximation steps further distance $\hat{h}$ from $\bar{h}$ and may cause the model $\hat{h}$ to be non-*psd*. Nevertheless, for practical reasons, it is simplest to measure correlation along one dimension only [66, pp. 358–359].

C. Collapsing Correlation Onto One Dimension

The measurements can be grouped along one variable in several ways. In such cases, $\hat{h}$ is expressed as a function of a single free variable. The most common forms are the following:

1) absolute distance (between Y_1 and Y_2) [24]–[26], [28], [31], [32], [36]–[39]: $d = \|\vec{r}_1 - \vec{r}_2\|$;
2) angle (not oriented) of arrival separation [24], [28], [33], [34], [36]–[38], [56]: $\theta = |\angle\vec{r}_1 - \angle\vec{r}_2| \in [0°, 180°]$;
3) arrival distance ratio (in decibels) [33], [37], [56], [72]: $R = |10\log_{10} r_1/r_2| = (10/\ln 10)|\ln r_1 - \ln r_2|$.

These quantities are illustrated in Fig. 3.

For correlation functions expressed in d, one may also estimate and fit the power spectral density (which is the Fourier transform of the autocorrelation function in differences of d) of the measured data to that of a particular model [28], [32].

TABLE I
SUMMARY OF EXISTING SHADOWING CORRELATION MODELS AND THEIR PROPERTIES

Model Name	Equation	Tunable Parameters	References	Dimensions	*psd*	Physical Properties[b] Violated
Constant	(18)	$0 < \rho < 1$	many	none	Yes	$1, 5, 7$
Exp(d)	(19)	$d_0 > 0$	[25], [26], ...	d	Yes	7
Exp+Exp(d)	(20)	$0 \leq a \leq 1$, $0 < d_1, 0 < d_2$	[28], [32], [90]	d	Yes	7
Gaussian(d)	(21)	$\bar{d} > 0$	[79]	d	Yes	7
Exp$*$Gaussian(d)	(22)	$d_0 > 0$, $\bar{d} > 0$	[81]	d	Yes	7
Exp(d^ν)	(23)	$d_0 > 0$, $0 < \nu \leq 2$	[91]	d	Yes	7
		$d_0 > 0$, $\nu > 2$			No	7
Cos(θ)	(24)	$A \geq 0, B \geq 0$, $A + B \leq 1$	[15], [55], [84], [92]	θ	Yes	$(1)^c, (2)^c, 4, (5)^c$
Piecewise(θ)	(25)	none	[34]	θ	No	$1, 6$
Stepwise(θ)	(26)	none	[33, Table 1]	θ	No	$1, 5, 6$
Triangular(θ)	(27)	$0 \leq b < a \leq 1$, $0^\circ < \theta_0 < 180^\circ$	[72]	θ	Yes	$(1)^c, (2)^c, (5)^c$
Exp(θ)	(28)	$\alpha > 0$	[4], [41], [56], [93]	θ	Yes	$2, (5)^c$
"1.0/0.0 RX"	(29), (30), (31)	$R_0 > 0$	[72]	θ, R	No	6
Triangular(θ) $\times$ Triangular(R)	(29), (27), (31)	$0 \leq b < a \leq 1$, $0^\circ < \theta_0 < 180^\circ$, $R_0 > 0$	modified [72]	θ, R	Yes	$(1)^c, (5)^c$
		$b = 0$, $a = 1$			Yes	none
Exp(d)$\times(\theta \leq 90^\circ)$	(32)	$d_0 > 0$	[46]	d, θ	No	6
Exp(d)$\times \cos^+(\theta)$	(33)	$d_0 > 0$	[94]	d, θ	No	none
Exp(d)$\times \cos(\theta)$	(34)	$d_0 > 0$	modified [94]	d, θ	Yes	4
Saunders'	(35)	$d_0 > 0$, $\gamma > 0$	[73], [96]	$(\theta, r_1, r_2)^a$, $r_i > d_0/2$	No	$2, (5)^c$
Modified Saunders'	(36)	$d_0 > 0$, $\gamma > 0$	[1]	$(\theta, r_1, r_2)^a$, $r_2, \ldots, r_N > d_0/2$	No	$2, 3, (5)^c, 6$
Stepwise(θ, R)	(37)	none	[33, Table 2]	$(\theta, R)^a$	No	$1, 5, 6$
Piecewise(θ)–Triangular$^\alpha$(R)	(38)	$R_0 > 0, \alpha \geq 0, a > 0, b > 0, a+b \leq 0.22$	[22]	$(\theta, R)^a$	No	$(1)^c, (5)^c, 6$
"1.0/0.4 RX"	(39)	$R_0 > 0$	[72]	$(\theta, R)^a$	No	$5, 6$

[a] Non–separable
[b] As described in Section VII
[c] Physical property met only for some parameter choices

VI. SPECIFIC CORRELATION MODELS AND THEIR POSITIVE SEMIDEFINITENESS

We have made a wide (if not exhaustive) investigation of the correlation models h used in the literature. All existing models imply jointly lognormal shadowing (which is by no means a requirement of our analysis). Most models assume a constant shadowing variance $\sigma_s^2(\vec{r}) = \sigma_s^2$ but not all [1], [39]. As we have shown in Theorem 1, the positive semidefiniteness of a shadowing model is separate from the shadowing spread function. As such, we may safely study the existing correlation models while looking forward to more complex point-to-point shadowing spread models in the future.

The various existing models and their properties are summarised in Table I. In particular, we show which models are *psd*. While [45] stated that "most" models are non-*psd*, we find that actually a slight majority of the models are in fact *psd*.

It should be understood that these models are not necessarily mutually exclusive when they are expressed in different (d, R, θ) domains, since this implies a different reduction (integration) of the original 4-D model $\bar{h}$ onto one or two dimensions, possibly representing different projections of the same $\bar{h}$ onto those dimensions.

A. Constant Model

The simplistic model that assigns $\rho_{i,j} = \rho$, $i \neq j$

$$h(\vec{r}_1, \vec{r}_2) = \rho \tag{18}$$

with $0 < \rho < 1$, is sometimes used [5], [9]–[11], [20], [21], [29], [42], [45], [47], [53], [61], [63], [76], [82], [83] ($\rho = 0.5$ [7], [23], [59], [60]) when more information is lacking. However, [7] argued that this may be a too-simplistic model, comparing simulations that use constant versus nonconstant models. On the other hand, we have shown [13] that for a high number of highly correlated lognormal-shadowed interferers, the total interference power may be well approximated with

the knowledge of the average correlation coefficient only. (It is important to actually estimate this average. This was not done in [7].)

The model is claimed *psd* for $\rho = 0.5$ in [45]. It does, in fact, give a *psd* $\mathbf{H}$ for $\rho \geq -1/(N-1)$ [83], and therefore, the model is *psd* according to Definition 6 for $0 < \rho < 1$ (as well as $\rho = 0, 1$) but would be non-*psd* for $-1 \leq \rho < 0$. It can quickly be simulated [42], [83] in the case of jointly lognormal correlated shadowing.

B. Absolute Distance-Only Models

All the autocorrelation models are expressed as a function of d (equivalently, of time t, given a constant velocity v).

1) Exponential Model: The most common autocorrelation model is a decaying exponential of distance [6], [8], [25], [26], [31], [36], [38]–[40], [42], [45], [48], [49], [51], [52], [56], [62], [70], [73], [74], [78], [80], [84]–[88]. This model is often attributed to Gudmundson [26], although the less-cited work of Marsan *et al.* [25] also proposes this model, and measurements by Graziano [24, Figs. 5 and 7] suggest it. This model has also been interpreted as a cross-correlation model [74]. It may be expressed as

$$h(d) = e^{-d/d_0} \tag{19}$$

where $d_0 > 0$ is the tunable parameter called the decorrelation distance (sometimes defined for 50% correlation rather than $1/e$). Literature surveys have been made [85], [86] of the values that d_0 might take in field measurements. While this model can be understood as being based on a first-order autoregressive (AR(1)), i.e., first-order Markov process [45], its positive semidefiniteness is not thereby evident, since, on a nonlinear trajectory, nonsuccessive points may have different correlation coefficients than simply those constructed by an AR(1) filter. The fact that e^{-x/d_0} is a *psd* function on $\mathbb{R}^+$ is a necessary but not sufficient condition [66, p. 361] (the same applies for any model depending on d). Nevertheless, this model is proved to be *psd* using Theorem 2 in [66, p. 362] and similarly in [56].

For equally spaced ordered points $Y_1, \ldots, Y_N$ on a straight line with separation d_{sep}, we have the correlation coefficients equal to $\rho_{i,j} = \rho^{|i-j|}$, where $\rho = e^{-d_{\text{sep}}/d_0}$. We then have a correlation matrix as in [89], which is necessarily *psd*, because the model (19) is *psd*.

2) More Complex Models: The exponential model has inspired some more complex models, which may be interpreted as autocorrelation or cross-correlation models.

In [28], [32], and [90], the sum of two independent exponential processes is used, which leads to the following correlation model:

$$h(d) = ae^{-d/d_1} + (1-a)e^{-d/d_2} \tag{20}$$

where $0 \leq a \leq 1$, $0 < d_1$, and $0 < d_2$ are tunable parameters. This model is always *psd*.

Proof: Applying (20) in (9), we may separate the resulting integral by linearity into two integrals, both of which are nonnegative, as seen for the model (19), and weighed by the nonnegative coefficients a and $1-a$. The resulting integral is thus nonnegative. It follows from Theorem 2 that this model is *psd*. ■

In [61] and [79] (and implicitly in [81]), a Gaussian correlation model is used, i.e.,

$$h(d) = e^{-(d/\bar{d})^2} \tag{21}$$

where $\bar{d} > 0$ is the tunable parameter. This model is proved to be *psd* using Theorem 2 in [66, p. 364].

In [81], a convolution of an exponential and a Gaussian function is proposed, i.e.,

$$\begin{aligned} h(d) &= Ke^{-|d|/d_0} * e^{-(d/\bar{d})^2} \\ &= K \int_{-\infty}^{\infty} e^{-|d-t|/d_0 - t^2/\bar{d}^2}\, dt \end{aligned} \tag{22}$$

where $d_0 > 0$ and $\bar{d} > 0$ are the tunable parameters, and $K = \exp(-\bar{d}^2/4d_0^2)/\sqrt{\pi}\bar{d}$. This model is *psd*.

Proof: The Fourier transform of (22) with $x = d$ is

$$f(\omega) = K \frac{2/d_0}{\omega^2 + d_0^{-2}} \bar{d}\sqrt{\pi} e^{-\frac{1}{4}(\bar{d}\omega)^2}$$

which meets the conditions of Theorem 3. ■

In [91], a model is proposed that can be written as

$$h(d) = e^{-(d/d_0)^\nu} \tag{23}$$

with $d_0 > 0$ and $\nu > 0$ as tunable parameters. The positive semidefiniteness of this model is dependent on ν: for $0 < \nu \leq 2$, it is proved to be *psd* with the aid of Theorem 3 in [66, p. 364]. This of course includes models (19) and (21), as well as the choice of $\nu = 0.9682$ in [91]. For $\nu > 2$, however, the model is in non-*psd* [66, p. 137]. As a counterexample, consider three aligned points with equal consecutive spacings of $0.2d_0$, for which we find that the correlation matrix is not *psd* for $\nu = 2.1, 2.2, 2.5, 3, 5, 50$.

C. Angle-Only Models

Some of the first shadowing correlation measurements along θ were reported in [18] and [24], but no analytical model was extracted. A similar measurement campaign [37] reported a much lower angular correlation, suggesting the need for a more complex model. However, it has been argued [45] that θ is the most significant parameter in cross-correlation, which justifies using these models as a first approximation.

In [55] and [84], and later in [15] and [92], a cosine model was proposed, i.e.,

$$h(\theta) = A\cos\theta + B \tag{24}$$

with two tunable parameters $A \geq 0$ and $B \geq 0$, $A + B \leq 1$. The model was used and assumed *psd* in [45]. Typical parameter choices have been $A = 0.3$, $B = 0.5$ [17], [19], [21], [62] and $A = 0.3$, $B = 0.699(9)$ [15], [92].

The model was proved to be *psd* in [6], [55], and [56] using Theorem 4 by setting $a_0 = A$, $a_1 = B$, and all other $a_n = 0$.

In [34], a piecewise-linear model was proposed, i.e.,

$$h(\theta) = \begin{cases} 0.78 - 7\theta/1250^\circ, & 0^\circ \le \theta < 15^\circ \\ 0.48 - 7\theta/1250^\circ, & 15^\circ \le \theta < 60^\circ \\ 0, & 60^\circ \le \theta \le 180^\circ. \end{cases} \tag{25}$$

It was shown in [57] that this model is non-*psd*. Here is an example: $N = 7$ with $\angle\vec{r}_i = 0, 5, \ldots, 30^\circ$.

In [33, Tab. I], a lookup table for intervals of θ, which is effectively a piecewise-constant model, was given, i.e.,

$$h(\theta) = \begin{cases} 0.6, & 0^\circ \le \theta < 30^\circ \\ 0.25, & 30^\circ < \theta < 60^\circ \\ \alpha \ge 0.2, & \theta \ge 90^\circ. \end{cases} \tag{26}$$

This model is undefined for $\theta \in \{30^\circ\} \cup [60^\circ, 90^\circ]$. We understand that a value for α may be chosen on [0.2, 0.25]. Regardless of the missing information, the model is non-*psd*, as shown in the following example: $N = 11$ with $\angle\vec{r}_i = 0, 4, \ldots, 40^\circ$.

A triangular model in θ was proposed in [72], i.e.,

$$h(\theta) = \begin{cases} a - (a-b)\theta/\theta_0, & \theta \le \theta_0 \\ b, & \theta > \theta_0 \end{cases} \tag{27}$$

with $0 \le b < a \le 1$, and $0^\circ < \theta_0 \le 180^\circ$: three tunable parameters. In [72], the model $a = 0.8$, $b = 0.4$, and $\theta_0 = 60^\circ$ was used to fit the measurements in [24]. The same choice of parameters was proposed for cross-correlation modeling for the 802.16-m standard [87]. In addition, $a = 0.8$, $b = 0$, $\theta_0 = 60^\circ$ was proposed to fit the measurements in [34], which were previously fitted with (25). In [28], the same model with $a = 0.9$, $b = 0$, and $\theta_0 = 180^\circ$ was used. This model is always *psd* for any choice of parameters.

Proof: Consider

$$a_0 = b + (a-b)\frac{\theta_0}{2\pi}$$

$$a_n = 2(a-b)\frac{1 - \cos\theta_0 n}{\pi\theta_0 n^2}.$$

Using Theorem 4 yields (27). ■

A decaying exponential angle model has also been proposed [4], [41], [56], [93], i.e.,

$$h(\theta) = e^{-\alpha\theta} \tag{28}$$

with $\alpha > 0$ as a tunable parameter. This model can be seen as inspired from the exponential d model (19), which it approximates for small θ. This model is *psd* [56].

Proof: Consider

$$a_0 = \frac{1 - e^{-\alpha\pi}}{\alpha\pi}$$

$$a_n = \frac{2}{\pi}\frac{\alpha}{n^2 + \alpha^2}\left(1 - e^{-\alpha\pi}(-1)^n\right).$$

Using Theorem 4 yields (28). ■

D. Separable Models

Separable models are easily constructed from the multiplication of 1-D models. Theorem 6 shows that if the component 1-D models are *psd*, then so is the composite separable model.

1) Angle–Distance Ratio: Separable θ–R models may always be written as

$$h(\theta, R) = h_\Theta(\theta) h_\mathrm{R}(R). \tag{29}$$

The use of the θ and R dimensions for shadowing correlation models has been argued in [37], [38], [45], [56], [62], and [72], although separability may be a simplistic assumption [56].

In [72], the model "1.0/0.0 RX" is given as

$$h_\Theta(\theta) = \begin{cases} 1 - \theta/75^\circ, & \theta \le 60^\circ \\ 0, & \theta \le 60^\circ \end{cases} \tag{30}$$

$$h_\mathrm{R}(R) = \max(0, 1 - R/R_0) \tag{31}$$

with $R_0 > 0$, with R_0 usually in [6 dB, 20 dB]. This model is not *psd*, as shown in the following example: let $N = 7$ and all r_i equal, and let $\vec{r}_i = 0, 15, \ldots, 90^\circ$.

The problem with the preceding model is its discontinuity at $\theta = 60^\circ$. This is simple to resolve: if we use the general form (27) as $h_\Theta(\theta)$ and (31) as $h_\mathrm{R}(R)$, then we have a very flexible model that is always *psd*.

Proof: $h_\mathrm{R}(R)$ is shown to be *psd* by choosing

$$f(\omega) = \frac{2\sin^2(\pi R_0 \omega)}{\pi^2 R_0 \omega^2}$$

with $g(\vec{r}) = (10/\ln 10)\ln\|\vec{r}\|$ in Theorem 5. Equivalently, we may use Pólya's theorem, as in [56].

The model (27) was already shown to be *psd*.

Now, h is expressed as a product of two *psd* models, and it follows from Theorem 6 that $h(\theta, R)$ is *psd*. ■

2) Angle-Absolute Distance: In [46], a separable model depending on d and θ is used and can be rewritten as

$$h_\mathrm{d}(d) = e^{-d/d_0}$$

$$h_\Theta(\theta) = \begin{cases} 1, & \theta \le 90^\circ \\ 0, & \theta > 90^\circ \end{cases}$$

$$h(d, \theta) = h_\mathrm{d}(d) h_\Theta(\theta). \tag{32}$$

(There is a typo in the original equation [46, eq. (18)]: the term $\sigma^2_{\tilde{\psi}_p}$ should be removed, as confirmed in a private communication with D. Kaltakis).

The authors say that this model is *psd*; however, this claim is not substantiated, although we understand that their simulations always gave *psd* correlation matrices. We already recognise the model $h_\mathrm{d}(d)$ from (19), which we know to be *psd*. However, it is simple to show that h_Θ is non-*psd*; therefore, the product of the two might not be *psd* either. In fact, the following counterexample shows it is not *psd*: $N = 14$ with $r_i = d_0$, and $\angle\vec{r}_i = 0, 180/7, \ldots, 2340/7^\circ$.

The authors of [46] claim that this model is *psd* based on their simulations. This is to be expected: because the model can be approximated by (19) for $r_i \gg d_0$, as might be the case in the cellular context, it can often appear *psd* in simulations.

However, strictly speaking, it is non-*psd*, and our example shows that it can fail, particularly in cases of small r_i.

In [94], the same author proposes

$$\begin{aligned} h_{\mathrm{d}}(d) &= e^{-d/d_0} \\ h_{\Theta}(\theta) &= \max(0, \cos(\theta)) \\ h(d,\theta) &= h_{\mathrm{d}}(d) h_{\Theta}(\theta). \end{aligned} \tag{33}$$

Again, while the model appears *psd* [94], in fact, it is not, as attested to by the following counterexample: $N = 8$ with $r_i = 0.4 d_0$ and $\angle \vec{r}_i = 0, 45, \ldots, 325^\circ$. However, this is a case with very small r_i. In practice, for $r_i \gg d_0$, this model is well approximated by

$$h(d,\theta) = e^{-d/d_0} \cos(\theta) \tag{34}$$

which is *psd*.

Proof: Model (34) is the product of models (19) and (24) with $A = 1$ and $B = 0$, both of which were shown to be *psd*. Then, by Theorem 6, (34) is *psd*. ■

E. More Elaborate (Nonseparable) Models

It has been suggested [56] that separable models might not be sufficient to accurately model shadowing correlation.

Saunders' model [73] has been used in [7], [43], [45], [57], and [95] (but watch for various transcription errors), i.e.,

$$\begin{aligned} h_1 &= 10^{-0.05R} \\ h_2 &= \begin{cases} 1, & \theta < \theta_{\mathrm{T}} \\ (\theta_T/\theta)^{\gamma}, & \theta \geq \theta_{\mathrm{T}} \end{cases} \\ \theta_{\mathrm{T}} &= 2 \arcsin \frac{d_0}{2 \min\{r_1, r_2\}} \\ h(\theta, r_1, r_2) &= h_1 h_2 \end{aligned} \tag{35}$$

where d_0 is the same decorrelation distance as measured for model (19), and $\gamma > 0$ is the other tunable parameter (a typical value is $\gamma = 0.3$). The model is not separable in θ and R because θ_{T} depends on r_1 and r_2. This model is non-*psd*, as seen in [45], [46], and [57] and as is apparent from this example: $N = 3$ with $r_i = r \geq d_0/\sqrt{2}$, and $\angle \vec{r}_i = 0, \theta_T, 2\theta_T$.

In addition, we see that this model is undefined for $r_i < d_0/2$, since the domain of $\arcsin$ is $[-1, 1]$. To rectify this problem, Saunders' model has been modified [1] (also [88], but note some transcription errors) by augmenting its domain, i.e.,

$$h(\theta, r_1, r_2) = \begin{cases} \text{same as (35)}, & r_1, r_2 \geq d_0/2 \\ \sqrt{d_0/2 \max\{r_1, r_2\}}, & \text{otherwise.} \end{cases} \tag{36}$$

Notice, however, that if more than one r_i is less than $d_0/2$, then we will again have $h > 1$, which is not a valid correlation value.

In addition, extending the model's domain cannot make it *psd*: the same counterexamples used to show that (35) is non-*psd* can be used to show that (36) is non-*psd*.

Another model was given in [33, Tab. II] and used in [45]. It is piecewise constant on rectangles in the $\theta - R$ domain, i.e.,

$$h(\theta, R) = \begin{cases} R < 2, & 2 < R < 4, & R \geq 4 & \\ 0.8, & 0.6, & 0.4, & \theta < 30^\circ \\ 0.5, & 0.4, & 0.2, & 30^\circ < \theta < 60^\circ \\ 0.4, & 0.4, & 0.2, & 60^\circ < \theta < 90^\circ \\ 0.2, & 0.2, & 0.2, & \theta \geq 90^\circ. \end{cases} \tag{37}$$

The entries of this table cannot be obtained from an outer product, and thus the model is nonseparable. The model is non-*psd*, as observed in [45, 46, 57]; for example: $N = 6$ with $r_i = r \; \forall i$ and $\angle \vec{r}_i = 0, 7, 14, 21, 28, 35^\circ$.

Another model is given in [22], i.e.,

$$\begin{aligned} h_{\Theta}(\theta) &\quad \text{as in (25)} \\ h_{\mathrm{R}}(R) &= \max(0, (1 - R/R_0)^{\alpha}) \\ h(\theta, R) &= h_{\mathrm{R}}(R) \left(h_{\Theta}(\theta) + a\right) + b \end{aligned} \tag{38}$$

with $a > 0$, $b > 0$, $a + b \leq 0.22$, $\alpha \geq 0$, and $R_0 > 0$. When $b \neq 0$, this model is not quite separable. It is not *psd*, as seen from the same counterexample used for model (25), with $a = b = 0$ and $r_i = r \; \forall i$.

Finally, the model "1.0/0.4 RX" in [72] takes the following form:

$$h(\theta, R) = \begin{cases} \max\left(0, 1 - \frac{R}{R_0}\right) \left(0.6 - \frac{\theta}{150^\circ}\right) + 0.4, & \theta \leq 60^\circ \\ 0.4, & \theta > 60^\circ \end{cases} \tag{39}$$

where R_0 is chosen between 6 and 20 dB. This model is not *psd*, as attested to by the following example: with $N = 4$, regardless of the value of R_0, choose r_i all equal, and $\angle \vec{r}_i = 0, 30, 60, 90^\circ$.

VII. Some Thoughts on Physical Realism

While this paper focuses on the mathematical feasibility of correlation models, we would like to give a few thoughts about physical realism as well: can the correlation model h be considered realistic based on what we know about wireless propagation? Does it make sense intuitively? It is important to keep in mind that, while mathematical feasibility is an objective criterion, physical plausibility is relative to the understanding of wireless shadowing, and therefore, the criteria presented in this section are merely tentative. We hope that these initial ideas will encourage thought and discussion on what a shadowing correlation model should realistically look like. Nevertheless, these initial criteria can give a first approximation in choosing a good correlation model.

Proposition 1: Shadowing is a large-scale phenomenon, averaged over small displacements in space (and time), and should not by nature vary quickly in these dimensions [26], [45], [56], [62].

Proposition 2: As argued in [7], [18], [37], [59], [60], [72], [73], [86], [95], and [96], correlation in shadowing may be explained by a partial overlap of the large-scale propagation medium, as seen in Fig. 4. The nonoverlapping propagation areas are considered independent. When a propagation front

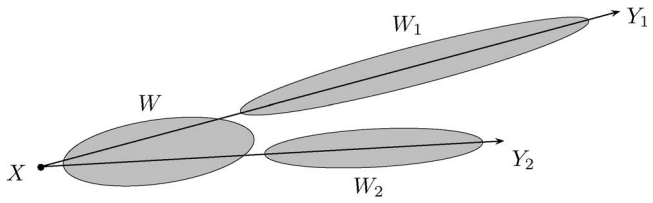

Fig. 4. Physical argument of common propagation area for correlation in shadowing.

passes through successive areas, the gains are multiplicative and thus add in the logarithmic domain. We may then write the following:

Let W, W_1, and W_2 be independent random variables with zero mean and variances a^2, b^2, and c^2, respectively. We have

$$\begin{aligned} S_1 &= W + W_1 \\ S_2 &= W + W_2. \end{aligned} \tag{40}$$

It follows that

$$\begin{aligned} \mathbb{VAR}\{S_1\} &= a^2 + b^2 = \sigma_{\mathrm{s}}^2(\vec{r}_1) \\ \mathbb{VAR}\{S_2\} &= a^2 + c^2 = \sigma_{\mathrm{s}}^2(\vec{r}_2) \\ \mathbb{E}\{S_1 S_2\} &= a^2 \end{aligned} \tag{41}$$

and therefore

$$h(\vec{r}_1, \vec{r}_2) = \frac{a^2}{\sqrt{(a^2+b^2)(a^2+c^2)}}. \tag{42}$$

This applies to any pair of shadowing terms.

This interpretation leads us to formulate the following criteria.

1) $h(\vec{r}_i, \vec{r}_j) \approx 1$ for $\vec{r}_i \approx \vec{r}_j$.
2) $h(\vec{r}_i, \vec{r}_j) \ll 1$ for $\|\vec{r}_i - \vec{r}_j\| \gg 0$.
3) h should be a nonincreasing function in θ [24], [33], [45], [56] R [33] and d.
4) h should be nonnegative, according to (41). However, this is contradicted by some measurement campaigns [24], [26], [27], [30], [34], [36]–[39], which reported some significantly negative estimated correlation coefficients in some cases. It would be interesting to study whether these observations are statistically significant. In addition, we suspect that negative correlations in measurements may appear in a particular obstacle scenario. However, averaged over very many obstacle realisations, the correlations are perhaps less likely to be negative.
5) h should be small for large θ and approach zero for $\theta \approx 180^\circ$, and r_1 and r_2 large, as the propagation regions are then essentially nonoverlapping.
6) Continuity: a small change in $\vec{r}_i$ should result in small changes in $h(\vec{r}_i, \vec{r}_j)$, at least when r_i is large.
7) Correlation should not depend on d only, as the distance between $\vec{r}_1$ and $\vec{r}_2$ tells us little about the overlap between the corresponding propagation areas.

Furthermore, it was argued [37], [38], [45], [56], [62], [72] that cross-correlation should depend on θ and R.

We have checked each model against these physical constraints and summarised the results (without proof) in Table I. The only model that is both *psd* and fulfills all these physical criteria is the model we proposed based on [72], as given by (27), (29), and (31), with $a = 1$, and $b = 0$.

VIII. Feasibility According to Marginal Distribution

Until now, we have only verified whether a given model h gives *psd* covariance matrices $\mathbf{K}$. This is a necessary but not sufficient condition to ensure that a random vector $\mathbf{S}$ can be constructed with covariance $\mathbf{K}$. To further study feasibility, it is necessary to look at the marginal distribution of S_i.

A. Jointly Lognormal Shadowing

In the most common case of lognormal shadowing (i.e., each S_i being Gaussian), a natural and effective way to jointly model shadowing paths is by making $S_1, \ldots, S_N$ jointly Gaussian. We then say that shadowing is jointly lognormal. In this case, every symmetric *psd* correlation matrix is feasible by the following explicit construction.

We begin by solving for $\mathbf{C}_{N\times N}$ in

$$\mathbf{K} = \mathbf{C}\mathbf{C}^{\mathrm{T}}. \tag{43}$$

In general, there are many solutions to this equation if and only if $\mathbf{K}$ is *psd*. Then, we generate an independent column vector of standard Gaussian random variables $\mathbf{Z} = [Z_1, \ldots, Z_N]$ and obtain the shadowing terms with

$$\mathbf{S} = \mathbf{C}\mathbf{Z}. \tag{44}$$

There exist various algorithms for calculating $\mathbf{C}$. In the case when $\mathbf{K}$ is positive definite, Cholesky factorization is a stable algorithm [57], [97] that gives a triangular solution for $\mathbf{C}$. However, $\mathbf{K}$ is allowed to be *psd* in general.[3] In addition, numerical rounding can make $\mathbf{K}$ slightly non-*psd*. To fix this double problem, there are at least two procedures. One is to slightly modify $\mathbf{K}$ so that it becomes positive definite [57]. However, $\mathbf{K}$ need not be positive definite (as suggested in [43], [56], and [57]) but may be *psd* in general. In this case, Cholesky factorization is not applicable, and matrix diagonalization (eigenvalue decomposition) [1], [7], [58], [95] should be used. In addition, should some of the resulting eigenvalues of $\mathbf{K}$ be slightly negative due to rounding, they can be set to zero [58]. Then, if there are eigenvalues equal to zero, $\mathbf{K}$ is not full rank and therefore not positive definite, but $\mathbf{C}$ can still be found. Additionally, if $\mathbf{K}$ is highly correlated, then a fast approximation exists [98].

We may conclude that, for lognormal shadowing, it is only necessary for a model h to be *psd* for it to be possible to explicitly construct any number of correlated shadowing paths.

[3]The case when $\mathbf{K}$ is *psd* but not positive definite corresponds to a singular matrix. In practice, for randomly generated positions $\vec{r}_i$ and using double-precision arithmetic, we find this event to be extremely rare [13], [14], and thus, Cholesky factorization is *almost* safe.

B. Non-Lognormal Shadowing

While lognormal is a well-established [8], [27], [73], [78], [86] and by far the most common model for the marginal distribution of shadowing, it is not the only model available. Other models have been proposed, either from physical plausibility arguments or for easier mathematical tractability. The two other models that we have encountered for the distribution of e^{S_i} are truncated lognormal [86], [99] and Gamma [100]–[102]. See [86, Sec. V-B4] for a survey of shadowing distribution measurements.

We have seen in Section III that the condition for the finite variance of S_i is required for an adequate definition of correlation. For the truncated lognormal model, which corresponds to a truncated Gaussian S_i, it is easy to see that truncation reduces the variance; hence, it is finite. For Gamma shadowing, it is easy to verify that all the moments of the logarithm of a Gamma random variable are finite, since they can be expressed as the integral in [103, Integral 4.352.1].

It is, however, not evident that the vector $\mathbf{S}$ can be generated according to non-Gaussian marginal distributions and a given *psd* correlation matrix $\mathbf{K}$. One constructive method for obtaining such jointly distributed vectors is Normal-to-Anything (NORTA), as described in [104]. This method requires some more stringent conditions on the marginal distributions to be feasible. These conditions might merit further study should non-lognormal shadowing models gain popularity.

IX. Conclusion

We propose the criteria developed in this paper as a basis for evaluating, designing, and correcting existing future correlation models. Naturally, there is more to be said both on testing more complex models for being *psd* and on their physical plausibility.

A. For Those Wanting to Incorporate Shadowing Correlation

For lognormal shadowing, only certain models that have the *psd* property (see Table I) guarantee to always give feasible joint shadowing distributions. For non-lognormal shadowing, some additional conditions might be required.

Based on both mathematical feasibility and physical arguments, we conclude that a subset of the family of models inspired by [72] is the best existing candidate for modeling correlation in shadowing. This model is given by (29), (31), and (27) with $a = 1$ and $b = 0$. With two tunable parameters θ_0 and R_0, it may be tuned to approximate many other correlation models that might have less-desirable properties.

B. For Those Using a Model That Is Not Feasible

Models that are not *psd* may still be used in particular application scenarios. For example, all models give *psd* correlation matrices for $N = 2$. In addition, we have argued that a correlation model that exactly corresponds to reality must per force be *psd*. It follows that for any model that approximates reality well (whether from extensive measurement campaigns or from theoretical arguments), there exists a model that is close to it (according to some reasonable metric) while also being *psd*. For example, the authors in [72] fit the *psd* model (27) to the data of [24] to replace the non-*psd* model (25).

Our suggestion is to take non-*psd* models and slightly correct them to make them *psd*, as we have done for (33).

C. For Those Designing New Correlation Models

We suggest that all new correlation models be designed as *psd* for the reasons described in Section I-A. Theorem 6 can be used to construct more detailed separable models.

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