Outage Analysis of Superposition Modulation Aided Network Coded Cooperation in the Presence of Network Coding Noise

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***Abstract*—We consider a network, where multiple source-destination pairs communicate with the aid of a half-duplex relay node (RN), which adopts decode-forward (DF) relaying and superposition-modulation (SPM) for combining the signals transmitted by the source nodes (SNs) and then forwards the composite signal to all the destination nodes (DNs). Each DN extracts the signals transmitted by its own SN from the composite signal by subtracting the signals overheard from the unwanted SNs. We derive tight lower-bounds for the outage probability for transmission over Rayleigh fading channels and invoke diversity combining at the DNs, which is validated by simulation for both the symmetric and the asymmetric network configurations. For the high signal-to-noise ratio regime, we derive both an upper-bound as well as a lower-bound for the outage performance and analyse the achievable diversity gain. It is revealed that a diversity order of 2 is achieved, regardless of the number of SN-DN pairs in the network. We also highlight the fact that the outage performance is dominated by the quality of the worst overheated link, because it contributes most substantially to the network coding noise. Finally, we use the lower bound for designing a relay selection scheme for the proposed SPM based network coded cooperative communication (SPM-NC-CC) system.**

***Index Terms*—Outage Analysis, Superpostion Modulation, Network Coding, Decode-Forward, Cooperative Communications.**

I. INTRODUCTION

Cooperative communication (CC) is capable of providing spatial diversity for single-antenna aided wireless devices [1]. In order to mitigate the multiplexing loss due to half-duplex relaying, a beneficial redundancy-reduction leading to a multiplexing gain may be attained with the aid of network coding (NC) [2], which was originally conceived for wired networks and was later extended to wireless networks. The family of NC schemes may be categorised into digital network coding (DNC) [3] and analog network coding (ANC) [4], where the DNC adopts the decode-and-forward relaying strategy, while the ANC adopts the amplify-and-forward scheme. The ANC has been widely studied as a benefit of its appealing simplicity, where the RN amplifies and forwards the analog signals received from all SNs. On the other hand, DNC is capable of correcting the source-relay (SR) link's errors at the RNs by flawlessly regenerating and retransmitting the SNs' signals and therefore eliminating the noise-amplification-induced performance limitation of the ANC scheme. Both ANC and DNC are capable of compressing the potentially redundant source sequences, hence reducing either the required number of time-slots in time-division multiple access (TDMA) or the required bandwidth [5–7]. The performance of ANC has been widely studied in the literature, with an emphasis on bidirectional transmissions between a pair of SNs. The outage performance of ANC for bidirectional transmissions is studied from different aspects in the literature, including the optimal power allocation [8, 9], adaptive AF/DF scheme [10], relay selection [11] and multi-hop relaying [12].

The ANC proposed in [4] is a multiple-access broadcast (MABC) scheme, which allows two SNs to access the channel simultaneously, therefore requires two time slots to exchange the information of two SNs, hence having a spectral efficiency of one channel use per SN. However, the MABC scheme cannot be readily generalised to multiple SN based scenarios, since the SNs cannot overhear each other. Furthermore, due to the half-duplex constraint, the MABC scheme cannot exploit the direct links, which leads to a reduced diversity gain. On the other hand, the time-division broadcast (TDBC) scheme of [3] is capable of exploiting the overheard links in order to regenerate the SNs' signals at the DNs. For M SN-DN pairs, a spectral efficiency of $(1 + 1/M)$ channel use per source is achieved, which approaches the spectral efficiency of the MABC scheme, as M increases. Furthermore, the TDBC scheme may exploit the diversity of the direct links, which is not the case for the MABC scheme due to the half-duplex constraint. In this paper, we adopt the TDBC based NC-CC scheme, where multiple SN-DN pairs share a single RN.

Superposition modulation (SPM) [13] constitutes a promising technique of realizing TDBC-based NC-CC mechanisms, where the signals received by the RN are superimposed on a symbol-by-symbol basis. The outage performance of SPM-NC-CC has been studied in [14] for the scenario of two-user cooperation. The authors of [15] analysed the outage performance of a superposition-modulated CC (SPM-NC-CC) system supporting multiple SN-DN pairs sharing a common RN. However, it was assumed in [15] that the destination node (DN) cancels the effects of the unwanted SNs' signals by perfectly recovering them. However, the authors of [15] consider amplify-and-forward (AF) relaying, while we considered decode-and-forward (DF) relaying, which is capable of mitigating the relay-induced error-propagation.

Sharma *et al.* point out in [16] that the employment of NC-CC systems may not always be beneficial, because the detrimental interference of the undesired SNs, which is often referred to as *network coding noise (NC noise)*, may outweigh the benefits of NC, as the number of sessions[1] involved in NC increases. Sharma *et al.* [17] have addressed the joint 'session-

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[1] We use the terminology of *session* as defined in [16], where a session represents the transmissions from a source to its destination.

grouping and relay-selection' problem in the context of dual-hop NC-CC systems, while adopting the optimisation criterion of requiring the *highest sum-rate of all sessions*. On the other hand, the authors of [18] proposed a joint-rate and power-control scheme for maximising the data rates of a NC-CC network, where a single RN assists two S-D pairs, while the authors of [19] jointly optimise the power allocation and group assignment. The main objective of [17–19] is to maximise the total information rate of all the SN-DN pairs in the network.

The maximised achievable rates obtained in [17–19] rely on instantaneous rate adaptation, albeit the packet-outage probability was not considered. When the knowledge of the instantaneous channel state information (CSI) is unavailable at the transmitters for rate adaptation, the packet-outage probability becomes non-negligible [16]. Sharma *et al.* also analysed in [16] the effects of NC noise on the achievable rate, but the effects of NC noise on the outage performance were evaluated only by simulations, rather than analytically.

On the other hand, the cell-edge coverage of direct transmission is of low quality, hence relaying is invoked. However, relaying substantially reduces the achievable throughput in order to enhance the diversity order, especially for multi-SN scenarios, where multiple time slots are required for the relaying phase. The next logical step is to invoke network coding, since this is the most realistic short-term solution for the industry, which justifies our system scenario. Using a single relay might be the worst-case but it is the most realistic practical scenario, which also achieves the lowest co-channel interference, since multiple RNs may not be available and if they are, they may imposes excessive co-channel interference on the entire network.

Against this background, *we consider the SPM-NC-CC scheme, where each DN extracts the information of the desired SN from the SPM signal forwarded by the RN and as our main contribution, we present the outage analysis of this system assuming a DF relay node (RN) and consider the effects of NC noise. More explicitly, the analytical outage probability expressions are derived for an arbitrary number of SN-DN pairs with the aid of tight lower-bounds. Simulation results are also provided for validating the analytical results. Furthermore, the lower-bounds and upper-bounds of the outage probability are analysed and the diversity order is shown to be* 2 *for an arbitrary number of SN-DN pairs. It is also revealed that the quality of the worst overheated source is the dominant factor imposed on the outage performance. Finally, we use the closed-form outage probability lower-bound for designing a relay selection scheme for SPM-NC-CC systems.*

The paper is organized as follows. Our system model is introduced in Section II. Then we derive a tight lower-bound approximation of the outage probability and determine the diversity order in Section III. In Section IV, we use the outage expressions derived for designing the relay selection scheme of our SPM-NC-CC and then in Section V we compare the simulation as well as analytical results and characterize the performance of the proposed relay selection scheme. Our conclusions are presented in Section VI.

II. System Model

We consider a network comprised of M SN-DN pairs $\{(S_1, D_1), \ldots, (S_M, D_M)\}$ and a single RN, as shown in Fig. 1, where each node is equipped with a single antenna and operates in a half-duplex mode. We denote the set of the SNs as $S_t = \{S_1, S_2, ..., S_M\}$. A SPM-NC-CC scheme employing $(M + 1)$ time slots (TSs) for the transmission of M packets from the SNs to the DNs using TDMA is considered in this paper. During the k-th TS of the first M TSs, the SN S_k broadcasts its packet to all the DNs and the RN, while all other SNs remain silent. We refer to the first M TSs as the *broadcast phase*, where the signals received at the DN D_k and the RN may be expressed as: $y_{s_i d_k} = \sqrt{P_s} h_{s_i d_k} x_i + n_k^i$ and $y_{s_i r} = \sqrt{P_s} h_{s_i r} x_i + n_r^i$, where P_s is the transmit power of the SNs, $h_{s_i d_k}$ and $h_{s_i r}$ are the channel coefficients capturing the effects of both the fading as well as the pathloss between the SN S_i and the DN D_k as well as between the SN S_i and the RN, respectively. Furthermore, x_i is the signal transmitted by S_i and its power is normalized to 1, while n_k and n_r denote the additive white Gaussian noise (AWGN) at the DNs and the RN, respectively.

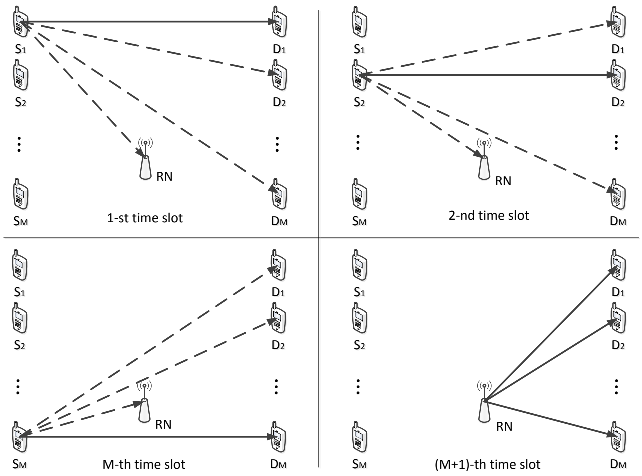

Figure 1: Schematic of the SPM-NC-CC scheme.

The RN overhears the M signals transmitted by the SNs and tries to decode them. In order to avoid error propagation, the RN superimposes the signals, which have been successfully decoded as well as re-encoded and then forwards the composite signal in the $(M + 1)$-st TS termed as the *relay phase*. We assume that a perfect error detection code is employed to identify whether the signal is correctly decoded. We define the set of SNs, whose signals have been successfully decoded by the RN as the *forwarding set*, which is denoted as F_s and its size is denoted as $|F_s|$. Therefore, the forwarding set F_s satisfies $F_s \subseteq S_t$ and $|F_s| \leq M$.

We assume that an equal amount of transmit power is allocated to each SN in the forwarding set. The composite signal transmitted by the RN may be expressed as $x_r = \frac{1}{|F_s|} \sum_{S_i \in F_s} x_i$, while the signal received by D_k may be formulated as $y_{rd_k} = \sqrt{P_r} h_{rd_k} x_r + n_k^r$, where P_r represents the transmit power of the RN.

If we have $S_k \in F_s$, the DN D_k is capable of extracting x_k from y_{rd_k} by subtracting the overheard signals after weighting them using the appropriate gain factors. In our analysis we rely on perfect channel estimation. We assume that the information is forwarded by the RN to D_k error-freely. The noise-contaminated signal extracted by D_k is expressed as

$$\hat{y}_{rd_k} = y_{rd_k} - \frac{\sqrt{P_r}}{|F_s|} \sum_{S_i \in F_s, i \neq k} \frac{h_{rd_k}}{h_{s_i d_k}} y_{s_i d_k}$$
$$= \frac{\sqrt{P_r}}{|F_s|} h_{rd_k} \left(x_k - \sum_{S_i \in F_s, i \neq k} \frac{1}{h_{s_i d_k}} n_k^i \right) + n_k^r. \quad (1)$$

Therefore, the signal of S_k at D_k may be contaminated by multiple noise terms, as shown in Eq. (1), which results in an increased noise power and was referred to as *NC noise* in [16].

III. Outage Analysis

In this section, we aim for analyzing the outage probability of the network described in Section II. We assume a narrow-band Rayleigh block fading channel model, where the fading coefficients remain constant for the duration of a packet and then they are faded independently from one packet to another both in time and space. The receivers are assumed to rely on perfect channel estimation. The additive noise at the receivers is modeled by independent zero-mean circularly symmetric complex Gaussian random variables.

Firstly, we formulate the received SNRs at the DNs and then the outage probability of the network. According to Eq. (1), the received SNRs at the DN D_k during the broadcast and relay phases are expressed as follows:

$$\gamma_k^1 = \gamma_{s_k d_k}$$
$$\gamma_k^2 = \frac{1}{\frac{1}{\gamma_{rd_k/|F_s|}} + \sum_{S_i \in F_s, i \neq k} \frac{1}{\gamma_{s_i d_k}}}, \quad (2)$$

where γ_{ab} is the instantaneous received SNR experienced at node b for the signal transmitted by node a. We denote the instantaneous channel coefficient between node a and b by h_{ab} and the average received SNR by $\bar{\gamma}_{ab}$. Then the instantaneous received SNR may be expressed as $\gamma_{ab} = |h_{ab}|^2 \bar{\gamma}_{ab}$, since we are considering block fading channels. It can be observed in Eq. (2) that γ_k^2 decreases, as the number of SNs in the forwarding set increases.

A. Formulation of Outage Probability

Assuming that the k-th SN S_k transmits to D_k at a data rate R_k, an outage event of the k-th SN-DN pair is defined as the event when the maximum achievable rate is below the target rate [6]. If diversity combining (DC) is adopted at the DN, the outage probability may be formulated as follows:

$$P_{o,k} = \Pr\left\{\log_2\left(1+\gamma_k^1\right) < R_k\right\} \Pr\left\{S_k \notin F_s\right\}$$
$$+ \Pr\left\{\log_2\left(1+\gamma_k^1+\gamma_k^2\right) < R_k\right\} \Pr\left\{S_k \in F_s\right\}, \quad (3)$$

where the first term represents the outage probability, when RN fails to decode the S_k's signal. Therefore D_k relies only on the direct transmission from S_k to recover the message, while the second term represents the outage probability when S_k is successfully decoded by the RN and therefore forwarded to D_k via the SPM composite signal.

For brevity, we define the first term in Eq. (3) as $P_{o,k}^{DT}$, which may be formulated as:

$$P_{o,k}^{DT} = \Pr\left\{\log_2\left(1+\gamma_k^1\right) < R_k\right\} \Pr\left\{S_k \notin F_s\right\}$$
$$= \Pr\left\{\log_2\left(1+\gamma_{s_k d_k}\right) < R_k\right\} \Pr\left\{\log_2\left(1+\gamma_{s_k r}\right) < R_k\right\}$$
$$= \Pr\left\{\gamma_{s_k d_k} < 2^{R_k} - 1\right\} \Pr\left\{\gamma_{s_k r} < 2^{R_k} - 1\right\}$$
$$= \left(1 - e^{-\frac{2^{R_k}-1}{\bar{\gamma}_{s_k d_k}}}\right)\left(1 - e^{-\frac{2^{R_k}-1}{\bar{\gamma}_{s_k r}}}\right). \quad (4)$$

The second term in Eq. (3) indicates the outage probability when S_k is in the forwarding set F_s, which may be expressed as follows:

$$\Pr\left\{\log_2\left(1+\gamma_k^1+\gamma_k^2\right) < R_k\right\} \Pr\left\{S_k \in F_s\right\}$$
$$= \sum_{F_s | S_k \in F_s} \Pr\left\{\log_2\left(1+\gamma_k^1+\gamma_k^2\right) < R_k | F_s\right\} \Pr\left\{F_s\right\}$$
$$\triangleq \sum_{F_s | S_k \in F_s} P_e\left(F_s\right) \Pr\left\{F_s\right\}, \quad (5)$$

where $P_e\left(F_s\right) = \Pr\left\{\log_2\left(1+\gamma_k^1+\gamma_k^2\right) < R_k | F_s\right\}$ is the error probability conditioned on the forwarding set F_s, while $\Pr\left\{F_s\right\}$ is the probability that a forwarding set F_s is adopted at the RN, which may be expressed as:

$$\Pr\left\{F_s\right\} = \prod_{S_i \in F_s} \Pr\left\{\log_2\left(1+\gamma_{s_i r}\right) \geq R_i\right\}$$
$$\times \prod_{S_j \notin F_s} \Pr\left\{\log_2\left(1+\gamma_{s_j r}\right) < R_j\right\}$$
$$= \prod_{S_i \in F_s} e^{-\frac{2^{R_i}-1}{\bar{\gamma}_{s_i r}}} \prod_{S_j \notin F_s} \left(1 - e^{-\frac{2^{R_j}-1}{\bar{\gamma}_{s_j r}}}\right). \quad (6)$$

Then, using the definitions given above, we may rewrite the outage probability in Eq. (3) in a simpler form as:

$$P_{o,k} = P_{o,k}^{DT} + \sum_{F_s | S_k \in F_s} P_e\left(F_s\right) \Pr\left\{F_s\right\}$$

B. Lower Bounds for Outage Probability

In order to derive $P_e\left(F_s\right)$ in Eq. (5), we may have to formulate the probability density function (PDF) of the second-hop received SNR γ_k^2 in Eq. (2). However, the PDF of a reciprocal of the sum of exponential random variables is challenging to formulate for the outage performance analysis. Therefore, instead of pursuing the exact PDF of γ_k^2, we investigate the following inequalities:

$$\gamma_k^2 = \frac{1}{\frac{1}{\gamma_{rd_k/|F_s|}} + \sum_{S_i \in F_s, i \neq k} \frac{1}{\gamma_{s_i d_k}}}$$
$$\leq \frac{1}{max_{S_i \in F_s, i \neq k}\left(\frac{1}{\gamma_{rd_k/|F_s|}}, \frac{1}{\gamma_{s_i d_k}}\right)}$$
$$= min_{S_i \in F_s, i \neq k}\left(\gamma_{rd_k/|F_s|}, \gamma_{s_i d_k}\right) \triangleq \gamma_{min}. \quad (7)$$

Hence, γ_k^2 is upper-bounded by γ_{min}. The cumulative distribution function (CDF) of γ_{min} may be deduced as follows:

$$F_{\gamma_{min}}(\gamma) = \Pr\left\{min_{S_i \in F_s}\left(\gamma_{rd_k/|F_s|}, \gamma_{s_i d_k}\right) \leq \gamma\right\}$$
$$= 1 - \Pr\left\{\gamma_{rd_k/|F_s|} > \gamma\right\} \prod_{S_i \in F_s, i \neq k} \Pr\left\{\gamma_{s_i d_k} > \gamma\right\}$$
$$= 1 - \exp\left\{-\gamma\left(\frac{1}{\bar{\gamma}_{rd_k/|F_s|}} + \sum_{S_i \in F_s, i \neq k} \frac{1}{\bar{\gamma}_{s_i d_k}}\right)\right\}. \quad (8)$$

For brevity, we define

$$\bar{\gamma}_{min} = \left(\frac{1}{\bar{\gamma}_{rd_k/|F_s|}} + \sum_{S_i \in F_s, i \neq k} \frac{1}{\bar{\gamma}_{s_i d_k}} \right)^{-1}.$$

Based on the CDF in Eq. (8), we may arrive at the PDF of γ_{min} as:

$$f_{\gamma_{min}}(\gamma) = \frac{dF_{\gamma_{min}}(\gamma)}{d\gamma} = \frac{1}{\bar{\gamma}_{min}} \exp\left(-\frac{\gamma}{\bar{\gamma}_{min}} \right). \quad (9)$$

If γ_k^2 is replaced by its upper-bound γ_{min} in Eq. (7), the ***lower-bound*** $P_e^{low}(F_s)$ ***of*** $P_e(F_s)$ may be attained as follows:

$$\begin{aligned} P_e(F_s) =& \Pr\left\{ \log_2\left(1 + \gamma_k^1 + \gamma_k^2\right) < R_k | F_s \right\} \\ \geq& \Pr\left\{ \log_2\left(1 + \gamma_k^1 + \gamma_{min}\right) < R_k | F_s \right\} \triangleq P_e^{low}(F_s), \\ & P_e^{low}(F_s) \\ =& \Pr\left\{ \gamma_k^1 + \gamma_{min} < 2^{R_k} - 1 \right\} \\ =& \int_0^1 \Pr\left\{ \frac{\gamma_{s_k d_k}}{2^{R_k} - 1} < u \right\} \Pr\left\{ \frac{\gamma_{min}}{2^{R_k} - 1} < (1-u) \right\} du \\ =& 1 - e^{-\alpha} - \alpha e^{-\beta} \frac{e^{\beta - \alpha} - 1}{\beta - \alpha}, \end{aligned} \quad (10)$$

where $\alpha = \frac{2^{R_k}-1}{\bar{\gamma}_{s_k d_k}}$ and $\beta = \frac{2^{R_k}-1}{\bar{\gamma}_{min}}$. Using the relation of $P_e(F_s) \geq P_e^{low}(F_s)$ shown in Eq. (10), we may formulate the ***lower-bound of the outage probability*** as follows:

$$\begin{aligned} P_{o,k} =& P_{o,k}^{DT} + \sum_{F_s | S_k \in F_s} P_e(F_s) \Pr\{F_s\}, \\ P_{o,k} \geq& P_{o,k}^{low} = P_{o,k}^{DT} + \sum_{F_s | S_k \in F_s} P_e^{low}(F_s) \Pr\{F_s\}, \end{aligned} \quad (11)$$

where $P_{o,k}^{DT}$, $\Pr\{F_s\}$ and $P_e^{low}(F_s)$ are derived in closed-form as in Eq. (4), (6) and (10), respectively.

C. Diversity Order Analysis

1) Diversity Order of the Outage Probability Lower-bound: The lower-bound derived in Eq. (11) enables us to approximate the exact outage probability in Eq. (3). However, the diversity performance is not explicit from Eq. (11). Let us now embark on deriving the asymptotic outage probability expression based on Eq. (11) for the high-SNR regime, which may offer us insights into the achievable diversity performance. We introduce $\bar{\gamma}_{ij} = u_{ij}\bar{\gamma}$ to take into account the pathloss of the link between node i and j. For simplicity of exposure, we investigate the different components in Eq. (11) separately.

For a sufficiently high SNR of $\bar{\gamma} \to +\infty$ with the aid of the series expansion, we may write:

$$\begin{aligned} P_{o,k}^{DT} &= \left(1 - e^{-\frac{2^{R_k}-1}{u_{s_k d_k}\bar{\gamma}}}\right)\left(1 - e^{-\frac{2^{R_k}-1}{u_{s_k r}\bar{\gamma}}}\right) \\ &= \frac{\left(2^{R_k}-1\right)^2}{u_{s_k d_k} u_{s_k r}} \frac{1}{\bar{\gamma}^2} + O\left(\frac{1}{\bar{\gamma}^2}\right), \end{aligned} \quad (12)$$

$$\begin{aligned} P_e^{low}(F_s) &= 1 - e^{-\alpha} - \alpha e^{-\beta} \frac{e^{\beta-\alpha} - 1}{\beta - \alpha} \\ &= \frac{\left(2^{R_k}-1\right)^2}{2u_{s_k d_k} u_{min}(F_s)} \frac{1}{\bar{\gamma}^2} + O\left(\frac{1}{\bar{\gamma}^2}\right), \end{aligned} \quad (13)$$

$$\begin{aligned} \Pr\{F_s\} &= \prod_{S_i \in F_s} e^{-\frac{2^{R_i}-1}{u_{s_i r}\bar{\gamma}}} \prod_{S_j \notin F_s} \left(1 - e^{-\frac{2^{R_j}-1}{u_{s_j r}\bar{\gamma}}}\right) \\ &= \left(\prod_{S_j \notin F_s} \frac{2^{R_j}-1}{u_{s_j r}} \right) \frac{1}{\bar{\gamma}^{M-|F_s|}} + O\left(\frac{1}{\bar{\gamma}^{M-|F_s|}}\right), \end{aligned} \quad (14)$$

where $u_{min}(F_s) = \left(\frac{1}{u_{rd_k/|F_s|}} + \sum_{S_i \in F_s, i \neq k} \frac{1}{u_{s_i d_k}} \right)^{-1}$ is conditioned on the forwarding set F_s, and $O\left(\gamma^{-K}\right)$ represents the components having diversity orders higher than K. By substituting Eq. (12) to Eq. (14) into Eq. (11), we may arrive at the ***high-SNR expression of the lower-bound***:

$$\begin{aligned} & P_{o,k}^{low} \\ =& \frac{\left(2^{R_k}-1\right)^2}{u_{s_k d_k} u_{s_k r}} \frac{1}{\bar{\gamma}^2} + \frac{\left(2^{R_k}-1\right)^2}{2u_{s_k d_k} u_{min}(S_t)} \frac{1}{\bar{\gamma}^2} + O\left(\frac{1}{\bar{\gamma}^2}\right). \end{aligned} \quad (15)$$

Therefore, we can see that the lower bound of the outage probability is indicative of a the diversity order of 2.

The first term in Eq. (15) indicates the high-SNR outage probability contributed by the specific scenario, when RN fails to decode the signal of $S_k (S_k \notin F_s)$, and hence D_k relies purely on the direct link for decoding. The second term in Eq. (15) represents the high-SNR outage probability contributed by the particular scenario, when RN successfully decodes the signals received from all the SNs $(F_s = S_t)$ and we have

$$u_{min}(S_t) = \left(\frac{M}{u_{rd_k}} + \sum_{i=1, i \neq k}^{M} \frac{1}{u_{s_i d_k}} \right)^{-1}.$$

Therefore, as the number M of SN-DN pairs increases, $u_{min}(S_t)$ decreases and this results in an increased outage probability in Eq. (15).

2) Diversity Order of the Outage Probability Upper-bound: We may also derive an upper-bound for the exact outage probability in Eq. (3) by using the following inequality,

$$\begin{aligned} \gamma_k^2 &= \frac{1}{\frac{1}{\gamma_{rd_k/|F_s|}} + \sum_{S_i \in F_s, i \neq k} \frac{1}{\gamma_{s_i d_k}}} \\ &\geq \frac{1}{M \times max_{S_i \in F_s, i \neq k}\left(\frac{1}{\gamma_{rd_k/|F_s|}}, \frac{1}{\gamma_{s_i d_k}} \right)} \\ &= \frac{1}{M} \times min_{S_i \in F_s, i \neq k}\left(\gamma_{rd_k/|F_s|}, \gamma_{s_i d_k} \right) = \frac{\gamma_{min}}{M}. \end{aligned} \quad (16)$$

By using the relationship of $\gamma_k^2 \geq \frac{\gamma_{min}}{M}$ in Eq. (3), we may arrive at the ***upper-bound*** $P_e^{up}(F_s)$ ***of*** $P_e(F_s)$, which may be expressed as:

$$\begin{aligned} P_e(F_s) =& \Pr\left\{ \log_2\left(1 + \gamma_k^1 + \gamma_k^2\right) < R_k | F_s \right\}. \\ P_e(F_s) \leq& P_e^{up}(F_s) = \Pr\left\{ \log_2\left(1 + \gamma_k^1 + \frac{\gamma_{min}}{M}\right) < R_k | F_s \right\}. \end{aligned} \quad (17)$$

Similar to the derivation of the outage probability lower-bound, we may readily deduce the ***high-SNR expression of the outage probability upper-bound*** as:

$$\begin{aligned} & P_{o,k}^{up} \\ =& \frac{\left(2^{R_k}-1\right)^2}{u_{s_k d_k}} \left(\frac{1}{u_{s_k r}} + \frac{M}{2u_{min}(S_t)} \right) \frac{1}{\bar{\gamma}^2} + O\left(\frac{1}{\bar{\gamma}^2}\right). \end{aligned} \quad (18)$$

Since both the lower-bound in Eq. (15) and the upper-bound in Eq. (18) give a diversity order of 2, we may conclude that the exact diversity order is 2.

D. Discussions

Firstly, regardless of the number of SN-DN pairs in the network, a diversity order of 2 is achieved by the SPM-NC-CC scheme. As the number of SN-DN pairs increases, the spectral efficiency improvement is represented by the $\frac{M+1}{M}$ time slots per user, which approaches 1 time slot per user, as in the traditional TDMA scheme, while maintaining the second-order diversity gain.

However, Eq. (7) also reveals the fact that the SNR gain achieved during the relaying phase is limited by the quality of the worst overheated link spanning from $S_i (i \neq k)$ to D_k. This effect will be illustrated by simulations in Fig. 6 of Section V. Therefore, in order to exploit the spectral efficiency improvements of our SPM-NC-CC scheme, it is important to carefully construct the SPM-NC-CC group for the sake of mitigating the performance loss encountered by the overheard S-D channels. On the other hand, when the group formation has been completed, even though the number of available relays is larger than 1, the achievable diversity order would not improve with the aid of dynamic techniques, such as opportunistic relay selection [20]. This is due to the fact that the SNR gain of the relaying phase would be dominated by the worst overheard links. Therefore, we may adopt a simple relay selection scheme using the analytical outage expressions in Eq. (11), which relies on a single relay during the entire session, whilst achieving a diversity order of 2 and maintaining the minimum outage probability.

IV. Applications: Relay Selection

In this section, we consider a scenario, where multiple SN-DN pairs intend to share a RN for creating a SPM-NC-CC arrangement under the assumption that multiple RNs are available in the region.

The candidate RN set is denoted as $\{R_1, R_2, ..., R_N\}$, where we consider that all the RNs contend during the relay selection stage and the best RN is selected for transmission during the data transmission stage. The criterion for selecting the best RN is that of the minimum sum outage probability of all SN-DN pairs. With the aid of the analytical lower-bound of the outage probability derived in Eq. (11), we may design a low-complexity relay selection method by avoiding time consuming Monte-Carlo simulations for evaluating the sum of the resultant outage probabilities.

The relay selection stage is designed to have two phases: the channel measurement phase and the relay contention phase. During the channel measurement phase, each RN R_n evaluates the sum outage probability, if R_n itself is selected for relaying. In the relay contention phase, a distributed method relying on local timers [20] is adopted. The design of the relay selection stage is detailed in the following sections.

A. Channel measurement phase

In the channel measurement phase, each RN R_n would evaluate the sum outage probability of $P^n_{o,sum} = \sum_{k=1}^{M} P^n_{o,k}$. According to Eq. (11), in order to evaluate the outage probability of the k-th SN-DN pair via R_n, which is denoted as $P^n_{o,k}$, the knowledge of the CSI is required and the related process works as follows:

- The average SNR of the channels between R_n and each SN is denoted as $\bar{\gamma}_{sr_n} = \{\bar{\gamma}_{s_i r_n} | i = 1, .., M\}$.
- The average SNR of the channels between R_n and D_k is denoted as $\bar{\gamma}_{r_n d_k}$. Hence, in order to evaluate the sum outage probability, the knowledge of the average SNR of the channels between R_n and each DN, denoted as $\bar{\gamma}_{r_n d} = \{\bar{\gamma}_{r_n d_k} | k = 1, ..., M\}$ is required.
- The knowledge of the $\bar{\gamma}_{sd_k} = \{\bar{\gamma}_{s_i d_k} | i = 1, .., M\}$ average SNR of the channels between each SN and D_k is necessitated. Hence, in order to evaluate the sum outage probability, the average SNR $\bar{\gamma}_{sd} = \{\bar{\gamma}_{s_i d_k} | i = 1, .., M, k = 1, ..., M\}$ of the channels between each SN-DN pair is required .

In order to evaluate the required CSI, the pilot and feedback are designed as follows:

1) Each SN S_i broadcasts a pilot signal in orthogonal time slots, where each RN R_n and each DN D_k would estimate both $\bar{\gamma}_{sr_n}$and $\bar{\gamma}_{sd_k}$, respectively.
2) Each RN R_n broadcasts a pilot signal in orthogonal time slots, where each DN estimates the $\bar{\gamma}_{r_n d_k}$.
3) Each DN D_k broadcasts a feedback signal coupling the information of $\bar{\gamma}_{r_n d_k}, n = 1, ..., N$ and $\bar{\gamma}_{sd_k}$.

Then, each RN R_n becomes capable of acquiring the CSI of $\bar{\gamma}_{sr_n}$,$\bar{\gamma}_{r_n d_k}$and $\bar{\gamma}_{sd}$ and hence it is capable of evaluating the approximate sum outage probability $\hat{P}^n_{o,sum} = \sum_{k=1}^{M} P^{low}_{o,k}$ with the aid of Eq. (11).

B. Relay contention phase

In the relay contention phase, the most suitable RN would be selected. The specific node within the RN set R_{n^*}, which has the minimum sum outage probability $\hat{P}^{n^*}_{o,sum}$ is selected as the relaying node. This may be implemented without any coordination by a central controller, when the distributed-timer-based technique of [20] is adopted. Specifically, the contention phase begins with a request-to-send (RTS) signal transmitted by a SN, when each RN listens. Upon receiving the RTS signal, each RN R_n starts a local timer having an expiration duration proportional to the sum outage probability $\hat{P}^n_{o,sum}$. Therefore, the timer at R_{n^*} having the minimum $\hat{P}^{n^*}_{o,sum}$ would expire first, which hence broadcasts a clear-to-send (CTS) signal, claiming its occupation of the data transmission stage. Upon receiving the CTS signal by R_{n^*}, the other RNs would remain silent during the data transmission stage.

The relay selection stage may be invoked again, when the topology of the network changes, for example, when some new SN-DN pairs join or leave the SPM-NC-CC group. The performance of the relay selection scheme will be illustrated in Section V-B.

V. Simulation Results and Discussions

A. Outage Probability lower-bound

In this section, we consider both a symmetric and an asymmetric topology. We assume that all nodes in the network use an identical transmit power P_t. The time-averaged SNR of the SN-DN channel is denoted as $\bar{\gamma}_{s,d} = P_t/(N_0 \times d^{\delta}_{s,d})$, where the noise power N_0 is set to -80 dBm, while δ is the channel's pathloss exponent. Therefore, the average SNR of the channel between any two nodes is $\bar{\gamma}_{i,j} = P_t/(N_0 \times d^{\delta}_{i,j}) = (d_{i,j}/d_{s,d})^{-\delta} \bar{\gamma}_{s,d}$, where $d_{i,j}$ denotes the distance between node i and node j. We assume $\delta = 3$ for our simulations.

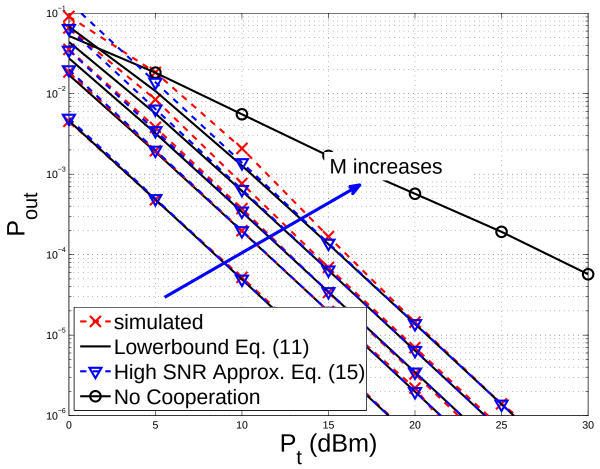

Figure 2: Outage probability versus the transmit power for Diversity Combining over block Rayleigh fading channels. The numbers of SN-DN pairs are $M = 1$, 2, 3, 5 and 10.

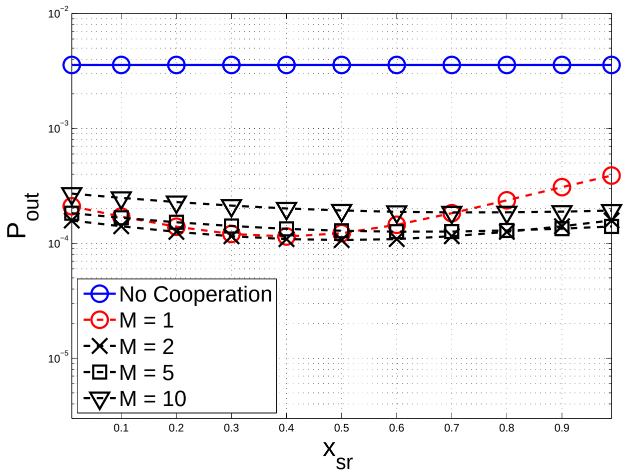

Figure 3: Outage probability versus the relay position over block Rayleigh fading channels with identical e2e throughput.

1) Example 1: Firstly, we consider a symmetric topology comprising M SN-DN pairs, where the average distance between a SN-DN pair is $d = 200$ meters. A RN is placed halfway along the line between any SN-DN pair. The transmission rate of each SN is set to 2 bits/s/Hz. In Fig. 2, we compare the analytical lower-bounds for the DC scheme to that acquired by Monte-Carlo simulations. The dashed lines in Fig. 2 represent the simulated results, while the solid curves rely on the lower-bounds of Eq. (11) derived in this paper. The high-SNR approximations derived in Eq. (15) is also illustrated. It is shown that for $M = 1, 2, 3, 5, 10$ SN-DN pairs, the lower-bounds match the simulated results quite closely. Furthermore, it is shown that as the number of sessions or SN-DN pairs simultaneously accessing a RN increases, the performance degrades because of the increased NC noise [16]. However, regardless of the number of cooperative SNs, all SPM-NC-CC schemes achieve a diversity of 2.

2) Example 2: In Fig. 2, each SN is assumed to have a constant transmission rate R, regardless of the number of the SN-DN pairs in the SPM-NC-CC group. From the perspective of the achievable end-to-end (e2e) throughput, the actual throughput of each SN-DN pair should be $R_{e2e} = \frac{M}{M+1}R$. Therefore, the outage performance comparison of the SPM-NC-CC schemes may be unfair, since they achieve a higher e2e throughput by using less time slots, compared to the traditional scheme relying on $M = 1$.

In this case, we assume that in order to achieve a certain e2e throughput R_{e2e}, the SNs are capable of adjusting the physical layer transmission rate R, according to the number of SN-DN pairs M. In Fig. 3, we fixed $R_{e2e} = 2$ bits/sec/Hz as well as $P_t = 15$ dBm and compared the outage performance of the proposed scheme for different values of M. Fig. 3 illustrates the effects of the RN position, which is determined by $X_{sr} = \frac{d_{sr}}{d_{sd}}$.

Based on Fig. 3, we may first revisit the outage performance recorded for different values of M for the scenario of $X_{sr} = 0.5$, when the RN is positioned half-way between each SN-DN pair. When $M = 2$, the outage performance becomes better than that of the traditional $M = 1$ scheme operating without SPC-NC-CC, while the outage performance of the $M = 10$ scheme is slightly degraded.

However, observe in Fig. 3 that when the RN position changes, the outage performance of $M = 10$ remains almost unaffected and becomes better than that of the $M = 1$ scheme, when the RN is closer to the DN. For the symmetric scenario considered in this example, the $M = 2$ scheme achieves the best outage performance for arbitrary RN positions. Interestingly, the $M = 2$ scheme also exhibits both the lowest implementation complexity and the lowest e2e delay. Therefore, in order to exploit the benefits of the SPC-NC-CC scheme, the RN position and the number of SN-DN pairs should be carefully considered in the light of the analytical results given in Eq. (11). As a final remark, compared to the non-cooperation scenario, the proposed schemes achieve significant improvements as a benefit of their increased diversity gain.

3) Example 3: In order to validate the analytical results in the non-symmetric network topology, we compare the lower-bounds to our simulation results. However, for reasons of space economy, we only consider a simple example topology comprised of $M = 4$ SN-DN pairs, where the nodes are assumed to be placed as in Fig. 4. We compare the outage probability of each SN-DN pair using simulations to the lower bounds derived in Eq. (11). It is shown in Fig. 5 that the lower-bounds closely match the simulated results for each SN-DN pair, which indicates that the lower-bound deduced is tight. The high-SNR expressions derived in Eq. (15) also match with the simulated results quite closely at high SNR, which verifies that the diversity order of 2 is indeed achieved by the SPM-NC-CC scheme.

4) Example 4: In Fig. 6 we illustrate that the quality of the the worst overheard link dominates the outage performance. In a network supporting 5 SN-DN pairs, we observe the outage probability of the session S_1-D_1, where we fixed the received SNR of the overheard link spanning from S_2 to D_1 and gradually improve both the other overheard links at D_1 as well as the relaying link, namely, the links spanning from S_3, S_4, S_5 and the RN to D_1. As shown in Fig.6, although we improve both the relaying links and the overheard links towards D_1, the outage performance of S_1-D_1 improves only marginally as long as the SNR of the worst overheard link is fixed, indicating that the outage performance is dominated by the quality of the worst overheard link.

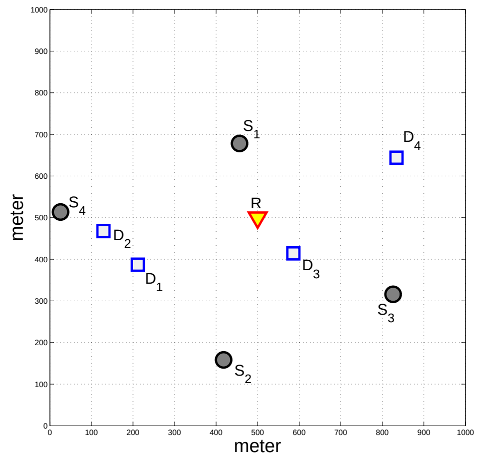

Figure 4: Network topology for $M = 4$ SN-DN pairs and a single RN.

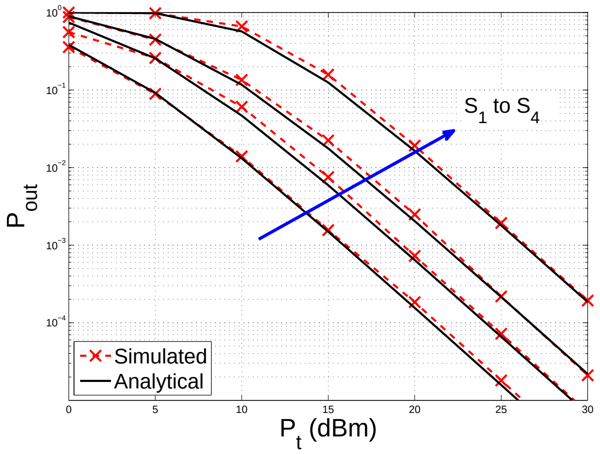

Figure 5: Outage probability versus the transmit power for Diversity Combining at the DNs over block Rayleigh fading channels. The network topology is illustrated in Fig. 4.

B. Relay Selection

1) Example 5: Let us now consider a square-shaped region with an edge-length of 400 meters, where 10 RNs are available. We vary the number of SN-DN pairs and compare the sum outage probability of the proposed relay selection scheme in Section IV to that of a conventional random relay selection scheme, where the RN is chosen randomly. The outage performance was averaged over 100 network instances. Observe in Fig. 7 that the proposed relay selection achieves a better outage performance than the benchmark scheme across the entire range of transmit powers. It is also observed that as the number of SN-DN pairs increases, the outage performance degrades.

VI. Conclusions

A cooperative network was considered, where multiple SN-DN pairs communicate with the aid of a single RN. The lower bounds of the outage probability were derived, which matches tightly with the simulation results. The results explicitly quantified the detrimental effects of NC noise imposed on SPM-NC-CC schemes. Additionally, we designed a relay selection approach for our SPM-NC-CC scheme using the closed form outage expression derived in Eq. (11), which avoids the excessive computational burden required by Monte-Carlo simulations.

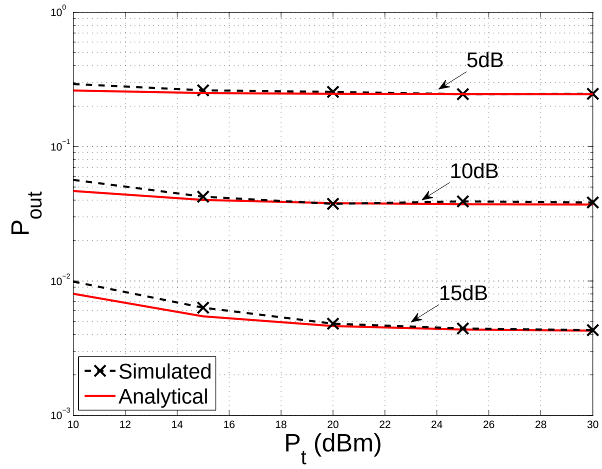

Figure 6: Outage probability of $S_1 - D_1$ versus the transmit power. The SNR of the direct link between S_1 and D_1 is fixed to 20dB. The SNR of the link between S_2 and D_1 is fixed to 5dB, 10dB and 15dB, respectively. While the other overheard links and the relaying links are improved gradually by increasing the transmit power.

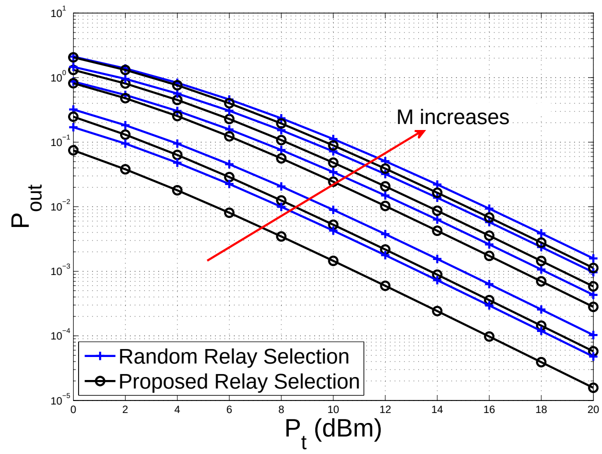

Figure 7: Sum outage probability versus the transmit power for different number of SN-DN pairs $M = 1, 2, 4, 6, 8$. The number of available RNs is 10. The outage performance is averaged over 100 network instances.

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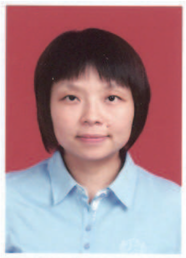

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