New Iterative Detector of MIMO Transmission Using Sparse Decomposition

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Institut Télécom; Télécom Bretagne; UMR CNRS 6285 Lab-STICC
Technopôle Brest Iroise CS 83818 29238 Brest, France
Université européenne de Bretagne
Email: Firstname.Lastname@telecom-bretagne.eu

***Abstract*—This paper addresses the problem of decoding in large scale MIMO systems. In this case, the optimal maximum likelihood detector becomes impractical due to an exponential increase of the complexity with the signal and the constellation dimensions. Our work introduces an iterative decoding strategy with a tolerable complexity order. We consider a MIMO system with finite constellation and model it as a system with sparse signal sources. We propose an ML relaxed detector that minimizes the euclidean distance with the received signal while preserving a constant ℓ_1-norm of the decoded signal. We also show that the detection problem is equivalent to a convex optimization problem which is solvable in polynomial time. Two applications are proposed, and simulation results illustrate the efficiency of the proposed detector.**

***Index Terms*—MIMO systems, low-complexity detector, convex optimization, sparse decomposition.**

I. Introduction

Due to its ability to increase the achievable capacity in a point-to-point communication, research interests have been steered towards Multiple Input Multiple Output (MIMO) transmission. The performance of MIMO systems is nonetheless conditioned to a wise management of the multi-antenna interference [1]. Maximum likelihood (ML) joint detection is an optimal strategy that decodes at once the transmitted signals [2]. It has been proved to minimize the probability of error for medium to high signal-to-noise ratio (SNR) values. However, its complexity grows exponentially with the antenna dimensions and the constellation size, which makes it impractical. Linear equalizers such as minimum mean square error (MMSE) and zero-forcing (ZF) present rather low complexity at the expense of a high performance-loss, specially when used in underdetermined[1] MIMO systems. Alternative solutions have been proposed, among which the sphere decoder (SD) achieves near-optimal performance provided that the spherical search is well defined [3]. However, SD suffer from a variable computational complexity that depends heavily on the SNR value, the signal dimensions, and the sphere radius initialization. The computational complexity order has been upper-bounded by $\mathcal{O}(M^{\gamma N})$, where $\gamma \in (0, 1]$, N is the transmit antennas number, and M is the constellation size [4], [5].

[1]The number of transmit antennas is larger than the number of receive antennas

The aforementioned optimal detectors are based on an exhaustive search of the desired signal. This implies a high computational complexity order that does not suit practical systems. In this paper, we address the problem of detection for MIMO systems with finite constellation size, and propose an ML detector with relaxed constraints. This detector is based on an iterative strategy, and is proposed for both cases: determined and underdetermined systems. This iterative strategy of decoding aims to maintain a low computational cost even with the increase of the signal and/or constellation size. We rewrite the MIMO channel model with inputs selected from a finite alphabet set, as a MIMO channel with sparse inputs belonging to the binary set $\{0, 1\}$. Then, we formulate the ML decoding problem as a minimization problem of a quadratic objective function under well-defined constraints. One of the constraints consists in maintaining a constant ℓ_0-norm, which makes the problem NP-hard. Exploiting the sparsity property of the signal to decode, we relax the ℓ_0-norm by the ℓ_1-norm in order to reduce the computational cost. This relaxation requires a solution belonging to the intersection of a lozenge with unit diameter and an explicit plan. However, the equality constraint involving the ℓ_1-norm is not convex. Therefore, we prove that for our particular problem the relaxed ℓ_1-norm constraint amounts to ensuring that all components of the variable vector are positive. This reduces our problem to a quadratic minimization problem under linear equality constraints and positive variable constraints, which define a convex set. Such a problem can be solved iteratively using first order optimization algorithms (i.e. gradient descent) or polynomial time algorithms e.g. primal-dual point interior method [6].

Related works have been presented in [7], where the authors have proposed a detector based on the minimization of an ℓ_1-norm iteratively within a sphere of radius ϵ. The dependency on ϵ makes the problem harder to solve, specially that the performance are far from the optimal as shown in our simulation results hereafter. In the problem proposed herein, we ensure that detection performance only depends on the equivalence between the ℓ_1-norm and the ℓ_0-norm.

It is shown hereafter, that our detector is reliable when applied to the underdetermined MIMO systems as well as the determined MIMO systems, which is not the case for the linear detector. The main points are threefold: i) Transforming the

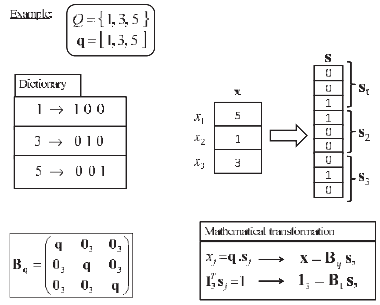

Fig. 1: Sparse decomposition of the vector with symbols belonging to a finite alphabet set.

MIMO decoding problem into a sparse input recovering problem, ii) Detecting the desired signal via quadratic minimization under convex constraints, iii) Applying the proposed detector to large-scale antenna on non-selective channels and to MIMO systems on frequency selective channel.

The remaining of the paper is organized as follows. Section II describes the MIMO transmission model with finite constellation size and its transformation into a MIMO model with sparse input. Section III proposes the iterative decoding scheme that minimizes the euclidean distance with the received signal preserving a constant norm ℓ_1. In Section V, the simulation results enable the evaluation of our contribution. Finally, Section VI concludes the paper.

Notations: boldface upper case letters and boldface lower case letters denote matrices and vectors, respectively. For the transpose, transpose conjugate and conjugate matrices we use $(.)^T$, $(.)^H$ and $(.)^*$, respectively. $||.||_p$ denotes the norm ℓ_p, and $\otimes$ is defined as the Kronecker product. $\boldsymbol{I}_p$ is the $p \times p$ identity matrix and $\mathbf{1}_p$ is the ones vector of length p.

II. SYSTEM MODEL

We first consider a MIMO transmission over a flat fading channel, where the transmitter and the receiver are equipped with N and n antennas, respectively. We assume no prior knowledge of the channel state information (CSI) at the transmitter and a perfect CSI knowledge at the receiver.

The received signal in MIMO single user channel is defined as

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{z}, \tag{1}$$

where $\boldsymbol{H}$ is an $n \times N$ random channel matrix ($n \leq N$), $\boldsymbol{x}$ is the $N \times 1$ data vector, and $\boldsymbol{z}$ is the $n \times 1$ complex circularly symmetric additive white Gaussian noise vector with zero mean and covariance matrix equals to $\sigma^2 \boldsymbol{I}_n$. The components of $\boldsymbol{x}$ belong to a finite alphabet defined as $\mathcal{Q} = \{q_1, q_2, \cdots, q_M\}$. For example, a 4-QAM constellation is defined as $M = 4$ and $\mathcal{Q} = \{\frac{1+i}{\sqrt{2}}, \frac{1-i}{\sqrt{2}}, \frac{-1+i}{\sqrt{2}}, \frac{-1-i}{\sqrt{2}}\}$.

The goal of our work is to find out an efficient detection scheme of the received data samples with moderate computational complexity order over the whole SNR region. This requires a priori knowledge on the transmitted information. Therefore, we exploit the fact that the original symbols belong to a finite alphabet, and we decompose each symbol on the basis of the vector space in which the finite alphabet vector $\boldsymbol{q} = [q_1, q_2, \cdots, q_M]$ can be cast. The decomposition of the vector with symbols belonging to the finite alphabet set is illustrated in Fig. 1. In this example, we assume a constellation set $\mathcal{Q}$ with three elements $\{q_1, q_2, q_3\}$, and we define the dictionary that matches each element of $\mathcal{Q}$ to its proper sub-vector in the set $\{(0\ 0\ 1)^T, (0\ 1\ 0)^T, (1\ 0\ 0)^T\}$. Each sub-vector in the dictionary associated to an element q_i $\forall i \in \{1, 2, 3\}$ and denoted by $\boldsymbol{s}_i$ contains only one non-zero component at the position i. This means that any element q_i can be written as its associated sub-vector $\boldsymbol{s}_i$ projected on the finite alphabet vector $\boldsymbol{q}$, i.e. $q_i = \boldsymbol{q}\boldsymbol{s}_i$. Using this decomposition, any vector with symbols belonging to $\mathcal{Q}$ can be written as a block diagonal matrix $\boldsymbol{B}_q$, with $\boldsymbol{q}$ repeated on its diagonal, multiplied by a sparse vector. This latter is constructed by stacking all the sub-vectors associated to the transmitted symbols.

The model in (1) can be then viewed as an equivalent MIMO channel with sparse input vector. In other words, the data vector with N entries in the MIMO transmission can be modeled as an equivalent sparse data vector $\boldsymbol{s}$ with NM entries. The j^{th} component x_j of $\boldsymbol{x}$ is decomposed as

$$\begin{aligned} x_j &= \boldsymbol{q}\,\boldsymbol{s}_j, \\ \text{where } \boldsymbol{s}_i &= [\delta_{q_1}(x_j),\ \delta_{q_2}(x_j)),\ \cdots,\ \delta_{q_M}(x_j)]^T, \\ \text{and } \delta_{q_i}(x_j) &= \begin{cases} 1 & \text{if } x_j = q_i \\ 0 & \text{otherwise} \end{cases}. \end{aligned} \tag{2}$$

The vector $\boldsymbol{x}$ can be then written as a function of $\boldsymbol{s}$ as

$$\begin{aligned} \boldsymbol{x} &= \boldsymbol{B_q}\,\boldsymbol{s}, \\ \text{where } \boldsymbol{s} &= [\boldsymbol{s}_1^T,\ \boldsymbol{s}_2^T,\ \cdots,\ \boldsymbol{s}_N^T]^T, \text{ and } \boldsymbol{B_q} = \boldsymbol{I}_N \otimes \boldsymbol{q}. \end{aligned} \tag{3}$$

$\boldsymbol{B_q}$ is a block diagonal matrix with dimensions $N \times (NM)$. Substituting (3) into (1) yields

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{B_q}\,\boldsymbol{s} + \boldsymbol{z}. \tag{4}$$

The transmitted signal $\boldsymbol{x}$ in (1) can be detected by seeking the optimal solution $\boldsymbol{s}$ in (4). In next section, we formulate the ML detection problem as the minimization problem of the Euclidean distance between the received signal $\boldsymbol{y}$ and the received constellation $\boldsymbol{H}\boldsymbol{B}_q\boldsymbol{s}$ under linear and ℓ_0-norm constraints. Then, we relax the constraints of the equivalent ML minimization problem in such a way that first order optimization and polynomial algorithms can be used to detect the desired information.

III. DETECTION OF SPARSE TRANSFORMED MIMO VIA MINIMUM DISTANCE MINIMIZATION

A. ML detection problem relaxation

The ML detector requires an exhaustive search over all possible transmitted symbol vectors, and selects the solution that corresponds to the closest point to the received signal in the received constellation. In other words, it selects the symbol vector that minimizes the euclidean distance between $\boldsymbol{y}$ and $\boldsymbol{H}\boldsymbol{x}$. Hence, the ML detection problem is defined as

$$[\boldsymbol{ML}]:\ \arg\min_{\boldsymbol{x}} ||\boldsymbol{y} - \boldsymbol{H}\boldsymbol{x}||_2 \quad \text{subject to} \quad \boldsymbol{x} \in \mathcal{Q}^N. \tag{5}$$

The main drawback of the ML problem is that it suffers from a high computational complexity because of the exhaustive search required over the set $\mathcal{Q}^N$. Herein, we propose an equivalence to this constraint using the following proposition.

Proposition 1: The components of $\boldsymbol{x}$ belong to the finite alphabet constellation $\mathcal{Q}$ if and only if the following equality hold: $\boldsymbol{B}_1\boldsymbol{s} = \mathbf{1}_N$ and $||\boldsymbol{s}||_0 = N$, where the norm $||\boldsymbol{s}||_0$ represents the weight of $\boldsymbol{s}$ and the $N \times NM$ matrix $\boldsymbol{B}_1$ is defined as

$$\boldsymbol{B_1} \quad = \quad \boldsymbol{I}_N \otimes \mathbf{1}_M, \tag{6}$$

where $\mathbf{1}_M$ is the ones row vector of length M.

Proof: Assuming first that the components of a N-dimensional vector $\boldsymbol{x}$ belong to a finite alphabet set, $\boldsymbol{x}$ can be sparsely decomposed as $\boldsymbol{x} = \boldsymbol{B}_q\boldsymbol{s}$ (see section II), where $\boldsymbol{s}$ consists of N sub-vectors with only one non-zero element equals to one for each. This means that the sum over each sub-vector is equal to one i.e. $\boldsymbol{B}_1\boldsymbol{s} = \mathbf{1}_N$, and the total number of non-zero elements in $\boldsymbol{s}$ is equal to N i.e. $||\boldsymbol{s}||_0 = N$.
Let us now assume $\boldsymbol{B}_1\boldsymbol{s} = \mathbf{1}_N$ and $||\boldsymbol{s}||_0 = N$. The first equality $\boldsymbol{B}_1\boldsymbol{s} = \mathbf{1}_N$, i.e. $\sum_{p=1}^{M} s_{(j-1)M+p} = 1$ for all $j \in \{1, \cdots, N\}$, implies that at least one non-zero element exists in any sub-vector $j \in \{1, \cdots, N\}$, with a minimum total non-zero elements number N. The second equality $||\boldsymbol{s}||_0 = N$ imposes the total non-zero elements number to be equal to N, thus along with the first equality each sub-vector can contain only one element different from zero and equal to one. Thereby, the projection of the whole vector $\boldsymbol{s}$ onto the decomposition matrix $\boldsymbol{B}_q$ yields a vector $\boldsymbol{x} = \boldsymbol{B}_q\boldsymbol{s}$ in the finite alphabet constellation $\mathcal{Q}^N$. ■

Using proposition 1, the [ML] minimization problem in (5) becomes

$$\begin{aligned} \underset{\boldsymbol{s}\in \mathrm{R}^{NM}}{\arg\min} \quad & ||\boldsymbol{y}_k - \bar{\boldsymbol{H}}_k\boldsymbol{B_q}\boldsymbol{s}||_2 \\ \text{subject to} \quad & \boldsymbol{B_1}\boldsymbol{s} = \mathbf{1}_N, \ ||\boldsymbol{s}||_0 = N. \end{aligned} \tag{7}$$

The first constraint given by $\boldsymbol{B_1}\boldsymbol{s} = \mathbf{1}_N$ is linear and the second constraint given by $||\boldsymbol{s}||_0 = N$ is discrete i.e. $\boldsymbol{s}$ belongs to a finite discrete set, thereby making the problem NP-hard. Such problems necessitate exhaustive search to be solved yielding an exponential increase of the computational complexity with the signal dimension. To overcome this drawback, we exploit the fact that the new signal vector to decode $\boldsymbol{s}$ is sparse, and we propose to relax the ℓ_0-norm by the ℓ_1-norm which is widely used in the literature, e.g. [8]–[10]. The relaxed constraint yields $||\boldsymbol{s}||_1 = N$. Such a constraint is not convex, thus, a global optimum is not necessarily achieved. In order to transform the problem into a quadratic minimization problem subject to convex constraints, we introduce the following lemma:

Lemma 1: Let $\boldsymbol{s}$ an NM-dimensional real vector satisfying $\boldsymbol{B}_1\boldsymbol{s} = \mathbf{1}_N$. Then all components of $\boldsymbol{s}$ are positive if and only if its ℓ_1-norm equals N i.e. $\|\boldsymbol{s}\|_1 = N$.

Proof: Let $\boldsymbol{B}_1 = \boldsymbol{I}_N \otimes \mathbf{1}_M$. The non-zeros elements of the k^{th} row of $\boldsymbol{B}_1$ are all equal to one and are those whose indexes range from $(k-1)M + 1$ to kM. Thus $\boldsymbol{B}_1\boldsymbol{s} = \mathbf{1}_N$ implies

$$\sum_{p=1}^{M} s_{(k-1)M+p} = 1, \quad \forall k \in \{1, \cdots, N\} \tag{8}$$

By successive additions with respect to k, we obtain

$$\sum_{i=1}^{NM} s_i = \sum_{k=1}^{N}\sum_{p=1}^{M} s_{(k-1)M+p} = N. \tag{9}$$

Let us first assume that all components of $\boldsymbol{s}$ are positive. Then $\boldsymbol{s}_i = |\boldsymbol{s}_i|$ and using (9), we deduce that $\sum_{i=1}^{NM} |s_i| = N$, i.e. $\|\boldsymbol{s}\|_1 = N$.

Let us now assume that $\|\boldsymbol{s}\|_1 = N$. According to (9), we can thus write

$$\sum_{i=1}^{NM} (|s_i| - s_i) = 0. \tag{10}$$

Let $\mathcal{N}(s) \neq \emptyset$ denote the set of indexes corresponding to all nonzero negative elements of $\boldsymbol{s}$. Then $\sum_{i=1}^{NM}(|s_i| - s_i) = 2\sum_{i\in\mathcal{N}(s)} |s_i|$. It follows from (10) that $s_i = 0$ for every $i \in \mathcal{N}(s)$ which is in contradiction with $\mathcal{N}(s) \neq \emptyset$. We thus deduce that $\mathcal{N}(s) = \emptyset$ and all components of s are positive. ■

Using the lemma introduced above, the decoding problem becomes

$$\begin{aligned} [\textbf{Quad-min}]: \quad & \underset{\boldsymbol{s}\in \mathrm{R}^{NM}}{\arg\min} \ ||\boldsymbol{y} - \boldsymbol{H}\boldsymbol{B_q}\boldsymbol{s}||_2 \\ \text{subject to} \quad & \boldsymbol{B_1}\boldsymbol{s} = \mathbf{1}_N, \ \text{and } \boldsymbol{s} \geq 0. \end{aligned} \tag{11}$$

This new optimization model is a quadratic programming model with linear equality constraints and non-negative variables.

B. Computational complexity

The proposed detection problem is a convex quadratic relaxation of the original non-convex ML detection problem. It can be formulated as follows

$$\begin{aligned} \text{(P)} \qquad \underset{\boldsymbol{s}}{\text{minimize}} \quad & \frac{1}{2}\boldsymbol{s}^H\boldsymbol{G}\boldsymbol{s} + \boldsymbol{c}^H\boldsymbol{s} \\ \text{subject to} \quad & \boldsymbol{A}\boldsymbol{s} = \boldsymbol{b}, \\ & \boldsymbol{s} \geq 0, \\ & \boldsymbol{s} \in \mathbb{R}^L, \end{aligned} \tag{12}$$

where $\boldsymbol{A} = \boldsymbol{B}_1$, $\boldsymbol{b} = \mathbf{1}_N$, $\boldsymbol{c}^H = -\boldsymbol{y}^H\boldsymbol{H}\boldsymbol{B}_q$, and $\boldsymbol{G} = (\boldsymbol{H}\boldsymbol{B}_q)^H(\boldsymbol{H}\boldsymbol{B}_q)$, and L is the length of $\boldsymbol{s}$. Various techniques exist to solve the problem (P) among which the primal dual interior point methods (PDIP) are characterized by high accuracy level. Several algorithms have been proposed since the publication by Kramarkar et *al.* of a practical polynomial time interior projective algorithm [11]. Among the best known is the algorithm introduced by Golfarb et *al.* in [12]. It defines a reduced PDIP algorithm in which modified Newton steps are used. For L variables, the algorithm has a total complexity upper-bounded by $\mathcal{O}(L^3)$, where at most $\mathcal{O}(\sqrt{L})$ iterations are required and all computations at each iteration require $\mathcal{O}(L^{2.5})$ arithmetic operations (for more details see Section 5 in [12]). Applied to our problem, it yields a computational complexity of $\mathcal{O}(M^3N^3)$, where the variable vector consists of $N \times M$ elements s_i $\forall i \in \{1, \cdots, N \times M\}$ as detailed in (2) and (3). For a given constellation, M is constant. Since the constant multipliers do not affect the big $\mathcal{O}$ notation, the complexity order is of $\mathcal{O}(N^3)$ [13]. Other simple iterative algorithms than the PDIP can also be applied such as the projected gradient descent algorithm, however, such methods are much less accurate and require a high iteration number to converge.

Remark 1: The decomposition in Section II is important in the sense that it allows to replace the original variable vector in the ML problem with another binary variable vector in the set $\{0, 1\}$, and to replace the constraint in (5) by a constant ℓ_0-norm constraint and other linear constraints. Then, the ℓ_0-norm constraint can be relaxed by the ℓ_1-norm constraint thanks to the sparsity property of the new variable vector. This relaxation is widely used when dealing with sparse vectors and in some cases the equivalence holds as e.g. in [8]–[10].

IV. Applications

A. Massive MIMO systems on flat fading channels

We consider the uplink of a TDD transmission where K user equipments transmit simultaneously and in the same bandwidth to a base station. The assumptions are the following. No restriction is imposed on the user equipment antenna number, while the base station antenna number is assumed to be high. The user equipments cannot coordinate and transmit reference signals to the base station for channel state information (CSI) estimation purpose. The overall CSI is perfectly known at the receiver. We consider a large scale MIMO system [14] where the user number as well as the base station antenna number are high. In this respect, the optimal joint detection, based on an exhaustive search, exhibits a huge computation cost, which exponentially increases with the variable vector length N and makes it unfeasible in practice. On the other hand, the proposed detection problem [Quad-Min] is solvable in polynomial time, which is suitable for an implementation, even when applied to high-dimension MIMO systems, i.e. with tens to hundreds antennas.

B. Frequency Selective MIMO channel

Another context is the frequency selective MIMO channel where $L+1$ multipath interfere at every channel use. The transmitter and the receiver are equipped with N and n antennas, respectively. We consider a transmission over a frame with length T_f, and for every transmitted symbol $L+1$ copies are received through $L+1$ different paths. The frequency selective channel output at instant t is expressed as

$$\boldsymbol{y}(t) = \sum_{l=0}^{L} \boldsymbol{H}_l \boldsymbol{x}(t-l) + \boldsymbol{z} = \sum_{l=0}^{L} \boldsymbol{H}_l \boldsymbol{B}_q \boldsymbol{s}(t-l) + \boldsymbol{z}(t), \quad (13)$$

where $\boldsymbol{H}_l$ represents the l^{th} path in the frequency selective channel. In order to decode the original symbol vector, we

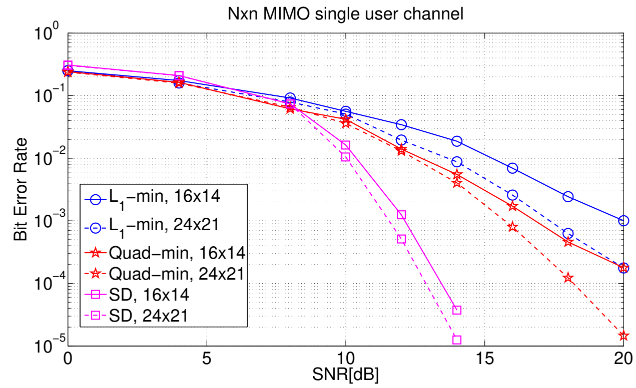

Fig. 2: BER performance comparison of the proposed detector versus the sphere decoder [18] and the [ℓ_1-min] detector

propose a joint detection over the whole frame[2]. Thus, instead of applying the decoding process T_f times per transmit antenna, we only decode at once a concatenated vector that consists of simultaneously transmitted vectors. The channel model is reformulated as in (14).

One can notice that the performance of the proposed detector is dependent on the number of multipath. When there is only one path (non frequency-selective channel), the channel matrix $\boldsymbol{H}_{fs}$ in (14) becomes block diagonal, and the problem (13) is then equivalent to decoding each $N \times 1$ data vector independently. In contrast, in presence of multipath, a joint detection over the whole (or part of the) frame must be applied. This implies an exploitation of the multipath diversity, and results in a performance improvement and a total complexity upperbounded by $\mathcal{O}(N^3)$ when M and T_f are constant.

V. Simulation results

In this section, we evaluate the Bit Error Rate (BER) and the computational complexity of the proposed detector based on quadratic minimization. We consider $N \times n$ MIMO systems, where N and n are the number of transmit and receive antennas, respectively. The channel coefficients are i.i.d. circularly symmetric complex Gaussian distributed with zero mean and unit variance, and the data symbols belong to a finite constellation. The number of transmitted symbols is equal to N at one channel use. For our proposed decoding scheme, we use the cvx toolbox, which is a Matlab-based modeling system for convex optimization [15], [16]. We use the Gurobi optimizer to solve the **[Quad-min]** problem under convex constraints [17]. This optimizer employs the PDIP methods for the quadratic programming. This method can be implemented using the algorithm proposed in [12]. We simulate this system using Matlab 7.10 on a processor Intel(R) Core(TM) i5-3317U CPU at 1.70GHz and memory 6GB RAM.

A. Comparison with optimal algorithms

For underdetermined large MIMO systems, Fig. 2 compares the BER performance of the proposed detection scheme

[2]The joint detection can also carried out over only a part of the frame as well.

$$
\begin{pmatrix} \boldsymbol{y}(1) \\ \vdots \\ \boldsymbol{y}(T_f+L) \end{pmatrix} = \begin{pmatrix} \boldsymbol{H}_0^T & \cdots & \boldsymbol{H}_L^T & \mathbf{0}_{n\times N}^T & \cdots & \mathbf{0}_{n\times N}^T \\ \mathbf{0}_{n\times N}^T & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \mathbf{0}_{n\times N}^T & \cdots & \mathbf{0}_{n\times N}^T & \boldsymbol{H}_0^T & \cdots & \boldsymbol{H}_L^T \end{pmatrix}^T \begin{pmatrix} \boldsymbol{x}(1) \\ \vdots \\ \boldsymbol{x}(T_f) \end{pmatrix} + \begin{pmatrix} \boldsymbol{z}(1) \\ \vdots \\ \boldsymbol{z}(T_f+L) \end{pmatrix}. \tag{14}
$$

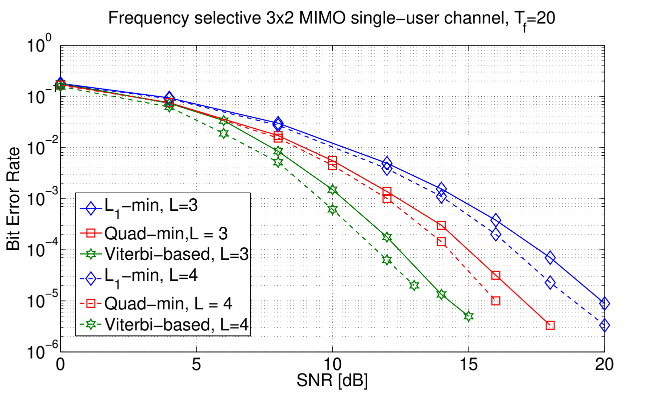

Fig. 3: BER performance comparison of the proposed detector versus the viterbi decoder and the [ℓ_1-min] detector when $L = 4$ and $L = 3$.

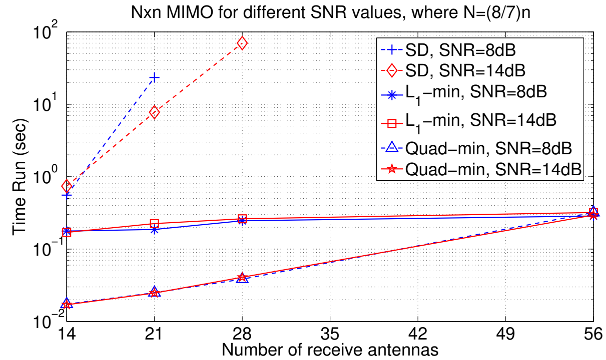

Fig. 5: Runtime evaluation of the proposed detector for different SNR values with QPSK modulation

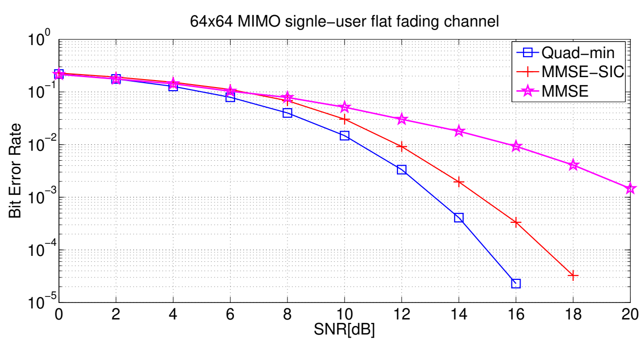

Fig. 4: BER performance comparison of the proposed detector with the MMSE and MMSE-SIC in a 64×64 MIMO channel.

[Quad-min] to the sphere decoder (SD), described in [18], and to the **[ℓ_1-min]** detector proposed in [7]. We assume a QPSK constellation mapping known at both the transmitter and the receiver. It can be observed that beyond 8dB, the SD outperforms the proposed scheme, e.g. at BER 10^{-2}, a gain about 2dB and 2.5dB is obtained when the dimensions are 16×14 and 24×21, respectively. Compared to the **[ℓ_1-min]** scheme in [7], the proposed scheme illustrates a gain over the whole SNR region, e.g. at BER 10^{-2}, the gain is between 1.5dB and 2.5dB for the system dimensions 16×14 and 24×21, respectively.

In a frequency selective channel context, Fig. 3 compares the BER performance of our proposed detector to the detector based on the Viterbi algorithm [19], and to the **[ℓ_1-min]** detector. The Viterbi algorithm is known to exploit the channel multi-paths, and achieves optimal performance over a frequency selective channel. We assume a frame length equal to $T_f = 20$, and a BPSK constellation. Over the entire SNR region, the Viterbi detector outperforms the **[Quad-min]** detector with 2dB to 3dB when $L = 3$ and $L = 4$, while preserving a reduced computational complexity. The **[ℓ_1-min]** detector, however, suffers from a higher loss. As shown in Fig. 3, at BER 10^{-2}, the gain of the Viterbi detector over the **[ℓ_1-min]** detector fluctuates between 1dB and 2.5dB, and at BER 10^{-3} the gain fluctuates between 3dB and 4dB, for different multi-paths number.

B. Comparison with low-complexity algorithms

In order to compare the proposed receiver to low-complexity receiver references in terms of error rate, we assume a determined 64×64 MIMO single user configuration. We compare in Fig 4 the proposed detector to the simple MMSE-based and the MMSE Successive interference cancellation (MMSE-SIC)-based described in [20]. One can observe that the **[Quad-min]** detector exploits the receive diversity better than the two other detectors. At BER 10^{-3}, the **[Quad-min]** detector outperforms the MMSE-SIC by about 1.5dB and the MMSE by 7dB. The gain compared to the MMSE-SIC increases to reach 2dB for a BER of 3.10^{-5}. On the other hand, all detectors use polynomial time algorithms, but their complexities differ from one detector to another, as will be shown in Section V-C.

C. Computational complexity evaluation

We evaluate the the computational complexity in Big$-\mathcal{O}$ notation. Big$-\mathcal{O}$ notation, also called Landau's symbol, is a well-understood symbolism widely used in complexity theory, computer science, and mathematics to describe the asymptotic behavior of functions [13], [21]. Basically, it tells how fast

	iteration number	computational cost per iteration	Total
MMSE	1	$\mathcal{O}(n^3)$	$\mathcal{O}(n^3)$
Basic MMSE-SIC	1	$\mathcal{O}(N^2n^2)+\mathcal{O}(Nn^3)+\mathcal{O}(N^4)$	$\mathcal{O}(N^2n^2)+\mathcal{O}(Nn^3)+\mathcal{O}(N^4)$
Improved MMSE-SIC	1	$\mathcal{O}(n^3)+\mathcal{O}(Nn^2)+\mathcal{O}(N^2n)$	$\mathcal{O}(n^3)+\mathcal{O}(Nn^2)+\mathcal{O}(N^2n)$
Quad-min	$\mathcal{O}(\sqrt{N})$	$\mathcal{O}(N^{2.5})$	$\mathcal{O}(N^3)$
SD	1	$\mathcal{O}(\sqrt{M^{\gamma N}})$	$\mathcal{O}(\sqrt{M^{\gamma N}})$
ML	1	$\mathcal{O}(\sqrt{M^N})$	$\mathcal{O}(\sqrt{M^N})$

TABLE I: Computational cost analysis

a function grows or declines. Table I summarizes the complexity order of our proposed detector, the simple MMSE, the basic and improved MMSE-SIC proposed in [20], and the ML optimal detector. The ML based detector is NP-hard, thus it is the least cost efficient. The simple MMSE-based detector consists of a complex inversion of $n \times n$ matrix, and some matrix multiplications and additions. As described in [22], the MMSE filter matrix can be computed using a QR decomposition, for which the complexity is given by $\mathcal{O}(n^3)$, thereby the MMSE computational complexity is of $\mathcal{O}(n^3)$. The computational complexity of the basic MMSE-SIC algorithm is of $\mathcal{O}(N^2n^2)+\mathcal{O}(Nn^3)+\mathcal{O}(N^4)$, which is equivalent to N^4 when $n=N$. This complexity order decreases to $\mathcal{O}(n^3)+\mathcal{O}(Nn^2)+\mathcal{O}(N^2n)$, for the improved MMSE-SIC i.e. to N^3 when $n=N$. Comparing the improved MMSE-SIC based detector to the **[Quad-min]** detector, similar complexity order is obtained for determined MIMO systems. However, in overdetermined MIMO configuration and for $n >> N$, the complexity order of the **[Quad-min]** becomes much lower than the MMSE-SIC. This is because the latter keeps a complexity of $\mathcal{O}(n^3)$, whereas the **[Quad-min]** complexity order, total and per iteration, depends only on N [12].

The theoretical results are also supported by the runtime of the proposed simulations that evaluate algorithms. The runtime represents the average duration that needs a processor to decode the received signal. Fig. 5 evaluates the runtime of the proposed detection scheme **[Quad-min]**. It can be observed that the runtime increases slightly with the system dimensions, and independently of the SNR level. E.g. the runtime of the **[Quad-min]** problem is equal to 0.017sec over the whole SNR region using a 16×14 system dimensions, and when the dimensions increase from 16×14 to 28×32, the runtime entails an increase of only 0.023sec. The same observations are made in the frequency selective case, where the proposed iterative detection yields a runtime that increases slightly with the number of multi-paths as shown in Fig. 6. For example, increasing the number of multipath from $L=3$ to $L=5$ results in an increasing runtime only from 0.02sec to 0.03sec.

Remark 2: It is important to mention that the relaxed constraints impose the subvector components to be in the interval [0,1], with their sum equal to one. Thus interpreting the solution subvector components as reliability values relative to the associated alphabet symbols, a preliminary analysis, which is still in progress, has shown that these output can be used to provide soft input to a channel decoder.

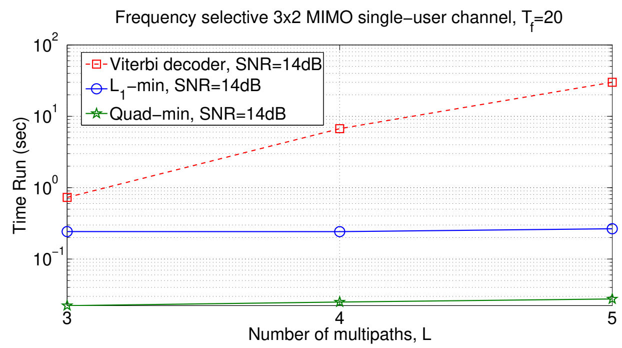

Fig. 6: Runtime evaluation of the proposed detector for SNR = 14dB, assuming BPSK modulation

VI. Conclusion

In this paper we have addressed the problem of decoding in high dimensional MIMO systems with finite constellation. We have exploited the fact that the original symbols belong to a finite alphabet, and we have modeled the transmission channel as a higher-dimensional MIMO channel with sparse input vector belonging to the binary set $\{0,1\}$. Using this decomposition, we have relaxed the ML problem with another minimization problem that can be solved using iterative method with polynomial complexity. Simulations carried out in the cases of large-scale MIMO systems on flat fading channel and MIMO on frequency selective channels, and have enhanced the relevance of the proposed detector for practical purposes. Beside the advantage of fitting underdetermined MIMO systems, the superiority in terms of error rate performance over other low-complexity receivers such as MMSE and MMSE-SIC has been highlighted.

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