# On Interference Alignment with Imperfect CSI: Characterizations of Outage Probability, Ergodic Rate and SER

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Abstract—In this paper, we give a unified performance analysis of interference alignment (IA) over MIMO interference channels. Rather than the asymptotic characterization, i.e. degree of freedom (DOF) at high signal-to-noise ratio (SNR), we focus on the other practical performance metrics, namely outage probability, ergodic rate and symbol error rate (SER). In particular, we consider imperfect IA due to the fact that the transmitters usually have only imperfect channel state information (CSI) in practical scenario. By characterizing the impact of imperfect CSI, we derive the exact closed-form expressions of outage probability, ergodic rate and SER in terms of CSI accuracy, transmit SNR, channel condition, number of antennas, and the number of data streams of each communication pair. Furthermore, we obtain some important guidelines for performance optimization of IA under imperfect CSI by minimizing the performance loss over IA with perfect CSI. Finally, our theoretical claims are validated by simulation results.

*Index Terms*—Interference alignment, MIMO interference channel, imperfect CSI, performance analysis.

### I. INTRODUCTION

Interference is always the bottleneck for performance improvement in an interference networks, e.g. multi-cell network [1] [2] and ad-hoc network [3] [4]. Various advanced interference mitigation techniques, such as coordinated beamforming over MIMO interference channel [5]-[7] (and the references therein), have been proposed to optimize the system performance. In particular, interference alignment (IA) receives considerable attentions, as it is able to exploit the maximum degrees of freedom (DOF) by aligning all the interferences into a specific area, so as to support higher number of concurrent data streams that will be free from interference [8]-[10].

### A. Related Works

Performance analysis is an important precondition for the design of system parameters, and hence provides useful guidelines for performance optimization [11] [12]. Since the channel capacity of interference channel is still unknown, performance analysis for IA becomes challenging. In this context, most works analyze the multiplexing gain at high signal-to-noise ratio (SNR), namely degree of freedom (DOF) [13]-[15]. It is proved that IA can achieve at most KM/2 DOFs over MIMO interference channels with K transmitter-receiver pairs each employing M antennas [8]. Then, the generalized DOF region is obtained for the general case of the MIMO Gaussian interference channel with arbitrary number of antennas at each node and the SNRs vary with arbitrary exponents to a nominal SNR [16]. However, DOF is an asymptotic performance metric, which is not able to make accurate predictions about the behavior of channel capacity at low and moderate SNRs under practical range of operation. Aiming to solve this problem, the authors in [17] proved that IA can achieve the capacity of the real, time-invariant, frequency-flat Gaussian X-channel to within a constant gap. In fact, for a system designer, the real concerns are outage probability, ergodic rate and symbol error rate (SER) at an arbitrary SNR on the time-varying fading channel. To the best of the authors' knowledge, there is no literature in solving this problem.

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On the other hand, channel state information (CSI) at the transmitters have a great impact on the performance of IA. Most previous work analyze the performance assuming that perfect CSI is available. However, it is difficult to obtain perfect CSI in practical systems, especially the CSI of the interference channels. A common way to solve this problem is to exchange CSI between the transmitters, e.g. CSI is conveyed between the base stations via a backhaul link in a multi-cell MIMO system. The issues related to CSI conveyance for IA, including channel estimation, limited CSI feedback and cooperation, had been discussed in [18]. For interference channels, there are two CSI exchange modes, namely digital [19] and analog [20] transmissions. Digital mode is more popular in IA, due to its flexibility by adjusting the size of quantization codebook [21]-[23]. However, considering limited feedback bandwidth, the transmitters can only have partial and imperfect CSI regardless digital or analog feedback mode. This results in imperfect IA, and hence complicate the performance analysis.

Existing literature [24] showed that even with limited CSI feedback, the full DOFs of the interference channel can still be achieved. The average residual interference caused by imperfect IA in terms of CSI exchange amount was analyzed in [25], when there is only one data stream for each transmitter-receiver pair. Then, it was generalized to the case of multiple data streams [22], where the performance loss resulting from imperfect CSI was investigated and the corresponding performance optimization scheme was given. The maximum DOF and an upper bound on the sum-rate performance loss due to limited differential feedback rate were given in [26]. A similar case over a two-cell interfering MIMO-MAC was considered in [27]. Moreover, the impact of feedback rate on the total

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rate loss of IA for a MIMO interference channel was analyzed in [28], while a feedback allocation scheme was proposed to maximize the rate. Furthermore, the achievable rate of IA in presence of imperfect CSI was investigated in [29], and several optimization schemes were proposed to maximize the rate afterwards. However, a comprehensive performance analysis of outage probability, ergodic rate and SER for IA under imperfect CSI, is still unsolved.

#### B. Main Contributions

Motivated by the above observations, this paper gives a comprehensive performance analysis of IA under imperfect CSI over a general MIMO interference channel. To be precise, each communication pair undergoes different propagation environment and has distinct number of data streams. We reveal the accurate relation between the performances and the amount of available CSI. The major contributions of this paper can be summarized as follows:

- We present a performance analysis framework for IA under imperfect CSI over a general MIMO interference channel, and derive the closed-form expressions of outage probability, ergodic rate and SER in terms of the amount of CSI exchanged and other system parameters.
- 2) We analyze the performance loss due to imperfect CSI, and reveal the impact of CSI accuracy on the system performance.
- We obtain clear insights on the system performance, which provide useful guidelines for the system designer.

## C. Paper Organization

The rest of this paper is organized as follows: Section II gives a brief introduction of the considered MIMO interference network employing IA under imperfect CSI. Section III focuses on the performance analysis and derive the closed-form expressions of outage probability, ergodic rate and SER. Section IV investigates the performance loss caused by imperfect CSI and get some useful optimization guidelines. Section V presents several numerical results to validate the theoretical claims, and finally Section VI concludes the whole paper.

*Notations*: We use bold upper (lower) letters to denote matrices (column vectors),  $(\cdot)^H$  to denote conjugate transpose,  $E[\cdot]$  to denote expectation,  $\|\cdot\|$  to denote the  $L_2$  norm of a vector,  $|\cdot|$  to denote the absolute value,  $(a)^+$  to denote max(a, 0),  $\lceil a \rceil$  to denote the smallest integer not less than a,  $\lfloor a \rfloor$  to denote the largest integer not greater than a, and  $\stackrel{d}{=}$  to denote the equality in distribution. The acronym i.i.d. means "independent and identically distributed", pdf means "probability density function" and cdf means "cumulative distribution function".

#### II. SYSTEM MODEL

We consider a MIMO interference network including K transmitter-receiver pairs, as shown in Fig.1. For analytical convenience, we assume an interference network where all the transmitters and receivers are equipped with  $N_t$  and  $N_r$  antennas, respectively. Transmitter k sends message(s) to its



Fig. 1. A diagram of interference alignment with imperfect CSI over a MIMO interference channel.

intended receiver k, while it also creates interference to other K-1 unintended receivers. We use  $\alpha_{k,i}^{1/2} \mathbf{H}_{k,i}$  to denote the MIMO channel from transmitter i to receiver k, where  $\alpha_{k,i}$  represents the path loss and  $\mathbf{H}_{k,i}$  is the  $N_r \times N_t$  channel small scale fading matrix with i.i.d. zero mean and unit variance complex Gaussian entries. Transmitter k has  $d_k$  independent data streams to be transmitted. It is worth pointing out that due to the limitation of spatial DOF, the values of  $d_k$ s must fulfill the feasibility conditions of IA [30] [31]. In what follows, we assume IA is feasible by choosing  $d_k$ s carefully. Thus, the received signal at receiver k can be expressed as

$$\mathbf{y}_{k} = \sum_{i=1}^{K} \sqrt{\frac{P\alpha_{k,i}}{d_{i}}} \mathbf{H}_{k,i} \sum_{l=1}^{d_{i}} \mathbf{w}_{i,l} s_{i,l} + \mathbf{n}_{k}, \qquad (1)$$

where  $\mathbf{y}_k$  is the  $N_r$  dimensional received signal vector,  $\mathbf{n}_k$  is the additive Gaussian white noise with zero mean and variance matrix  $\sigma^2 \mathbf{I}_{N_r}$ ,  $s_{i,l}$  denotes the *l*th normalized data stream from transmitter *i*, and  $\mathbf{w}_{i,l}$  is the corresponding  $N_t$  dimensional beamforming vector. *P* is the total transmit power at each transmitter, which is equally allocated to the data streams. Receiver *k* uses the received vector  $\mathbf{v}_{k,j}$  of unit norm to detect its *j*th data stream, which is given by

$$\hat{s}_{k,j} = \mathbf{v}_{k,j}^{H} \mathbf{y}_{k}$$

$$= \sqrt{\frac{P\alpha_{k,k}}{d_{k}}} \mathbf{v}_{k,j}^{H} \mathbf{H}_{k,k} \mathbf{w}_{k,j} s_{k,j} + \sqrt{\frac{P\alpha_{k,k}}{d_{k}}} \sum_{l=1, l \neq j}^{d_{k}}$$

$$\mathbf{v}_{k,j}^{H} \mathbf{H}_{k,k} \mathbf{w}_{k,l} s_{k,l} + \sum_{i=1, i \neq k}^{K} \sqrt{\frac{P\alpha_{k,i}}{d_{i}}} \sum_{l=1}^{d_{i}}$$

$$\mathbf{v}_{k,j}^{H} \mathbf{H}_{k,i} \mathbf{w}_{i,l} s_{i,l} + \mathbf{v}_{k,j}^{H} \mathbf{n}_{k}, \qquad (2)$$

where the first term at the right side of (2) is the desired signal, the second one is the inter-stream interference caused by the same transmitter, and the third one is the inter-link interference resulting from other transmitters. In order to mitigate these interferences and improve the performance, IA is performed accordingly. If perfect CSI is available at all nodes, according to the principle of IA, we have

$$\mathbf{v}_{k,j}^{H}\mathbf{H}_{k,k}\mathbf{w}_{k,l} = 0, \quad l \neq j, \forall k \in [1, K], \forall l, j \in [1, d_k].$$
(3)

and

$$\mathbf{v}_{k,j}^{H}\mathbf{H}_{k,i}\mathbf{w}_{i,l} = 0, \quad i \neq k, \forall k, i \in [1, K], \\ \forall l \in [1, d_i], \forall j \in [1, d_k].$$
(4)

In brief, inter-stream and inter-link interferences will be canceled completely if perfect CSI is available. However, in frequency division duplex (FDD) systems, it is difficult for the transmitters to obtain perfect CSI, especially the interference CSI. Similar to previous analogous works, this paper adopts quantization codebook based CSI conveyance scheme to inform the transmitters the interference CSI. Specifically, receiver k first quantizes the channel direction vectors  $\tilde{\mathbf{h}}_{k,i}, \forall i \in [1, K]$  with different codebooks, where  $\tilde{\mathbf{h}}_{k,i} = \frac{\mathbf{h}_{k,i}}{\|\mathbf{h}_{k,i}\|}$ and  $\mathbf{h}_{k,i} = \text{vec}(\mathbf{H}_{k,i})$  is the vectorization of  $\mathbf{H}_{k,i}$ . Then, the quantized CSI is conveyed to the corresponding transmitter. Due to limited feedback, the transmitters have imperfect CSI  $\hat{\mathbf{h}}_{k,i}$ , and the relation between the perfect and the imperfect CSI can be expressed as [32]

$$\tilde{\mathbf{h}}_{k,i} = \sqrt{1 - \rho_{k,i}} \hat{\mathbf{h}}_{k,i} + \sqrt{\rho_{k,i}} \mathbf{e}_{k,i}, \tag{5}$$

where  $\mathbf{e}_{k,i}$  is the quantization error vector with i.i.d. zero mean and unit variance complex Gaussian entries, and is independent of  $\hat{\mathbf{h}}_{k,i}$ .  $\rho_{k,i}$ , scaling from 0 to 1, indicates the CSI accuracy. If  $\rho_{k,i} = 0$ , then transmitter *i* has perfect CSI. Intuitively,  $\rho_{k,i}$  is related to the codebook size  $2^{B_{k,i}}$  or the CSI exchange amount  $B_{k,i}$ , and can be approximated as  $2^{-\frac{B_{k,i}}{N_t N_r - 1}}$  [32]. If the CSI exchange amount tends to infinity, the transmitter can obtain perfect CSI.

#### **III. PERFORMANCE ANALYSIS**

In this section, we set off the performance analysis for IA in a MIMO interference network with imperfect CSI. We put the focus on three practical performance metrics, namely outage probability, ergodic rate and SER. Note that we take the *j*th data stream of the *k*th pair as an example, but the analysis is applicable to all data streams.

## A. Outage Probability

Although IA is applied, (3) and (4) do not hold true any more due to limited CSI exchange. This results in residual interference, which is also called interference leakage. The signal to interference and noise ratio (SINR) related to the *j*th data stream of the *k*th transmitter-receiver pair can be expressed as

$$\gamma_{k,j} = \frac{P\alpha_{k,k}}{d_k} \left| \mathbf{v}_{k,j}^H \mathbf{H}_{k,k} \mathbf{w}_{k,j} \right|^2 / \left( \frac{P\alpha_{k,k}}{d_k} \sum_{l=1, l \neq j}^{d_k} \left| \mathbf{v}_{k,j}^H \mathbf{H}_{k,k} \mathbf{w}_{k,l} \right|^2 + \sum_{i=1, i \neq k}^{K} \frac{P\alpha_{k,i}}{d_i} \sum_{l=1}^{d_i} \left| \mathbf{v}_{k,j}^H \mathbf{H}_{k,i} \mathbf{w}_{i,l} \right|^2 + \sigma^2 \right)$$
$$= \frac{\kappa_{k,k} \left| \mathbf{h}_{k,k}^H \mathbf{T}_{j,j}^{(k,k)} \right|^2}{I_{k,j} + 1}, \qquad (6)$$

where  $\kappa_{k,k} = \frac{P\alpha_{k,k}}{d_k\sigma^2}$ ,  $\mathbf{T}_{j,j}^{(k,k)} = \mathbf{w}_{k,i} \otimes \mathbf{v}_{k,j}^H$ ,  $\otimes$  represents the Kronecker product, and  $I_{k,j}$  is the normalized total residual interference to the *j*th data stream of the *k*th pair. Through IA, the residual interference can be expressed as

$$I_{k,j} = \kappa_{k,k} \sum_{l=1, l \neq j}^{d_{k}} |\mathbf{v}_{k,j}^{H} \mathbf{H}_{k,k} \mathbf{w}_{k,l}|^{2} + \sum_{i=1, i \neq k}^{K} \kappa_{k,i} \sum_{l=1}^{d_{i}} |\mathbf{v}_{k,j}^{H} \mathbf{H}_{k,i} \mathbf{w}_{i,l}|^{2} = \kappa_{k,k} \sum_{l=1, l \neq j}^{d_{k}} |\mathbf{h}_{k,k}^{H} \mathbf{T}_{j,l}^{(k,k)}|^{2} + \sum_{i=1, i \neq k}^{K} \kappa_{k,i} \sum_{l=1}^{d_{i}} |\mathbf{h}_{k,i}^{H} \mathbf{T}_{j,l}^{(k,i)}|^{2} = \kappa_{k,k} \rho_{k,k} ||\mathbf{h}_{k,k}||^{2} \sum_{l=1, l \neq j}^{d_{k}} |\mathbf{e}_{k,k}^{H} \mathbf{T}_{j,l}^{(k,k)}|^{2} + \sum_{i=1, i \neq k}^{K} \kappa_{k,i} \rho_{k,i} ||\mathbf{h}_{k,i}||^{2} \sum_{l=1}^{d_{i}} |\mathbf{e}_{k,i}^{H} \mathbf{T}_{j,l}^{(k,i)}|^{2}, (7)$$

where (7) holds true since only partial interference is canceled based on the IA principle (3) and (4) due to imperfect CSI. Then, the outage probability is given by

$$P_{k,j}^{out} = P_r(\gamma_{k,j} \le \gamma_{th})$$
  
= 
$$\int_0^\infty F(\gamma_{th}(x+1)) g(x) dx, \qquad (8)$$

where  $\gamma_{th}$  is the SINR threshold,  $F_S(x)$  is the cdf of the desired signal quality  $\kappa_{k,k} \left| \mathbf{h}_{k,k}^H \mathbf{T}_{j,j}^{(k,k)} \right|^2$ , and g(x) is the pdf of the residual interference  $I_{k,j}$ . First, we check the distribution of the desired sinal quality. Since  $\mathbf{T}_{j,j}^{(k,k)}$  is designed independently of  $\mathbf{h}_{k,k}$  according to the principle of IA,  $\left| \mathbf{h}_{k,k}^H \mathbf{T}_{j,j}^{(k,k)} \right|^2$  is  $\chi^2(2)$  distributed, so the cdf of  $\kappa_{k,k} \left| \mathbf{h}_{k,k}^H \mathbf{T}_{j,j}^{(k,k)} \right|^2$  can be expressed as

$$F(x) = 1 - \exp\left(-\frac{x}{\kappa_{k,k}}\right).$$
(9)

Then, we analyze the distribution of the residual interference. According to the theory of quantization cell approximation [32],  $\rho_{k,i} \|\mathbf{h}_{k,i}\|^2$  is  $\Gamma(N_t N_r - 1, 2^{-\frac{B_{k,i}}{N_t N_r - 1}})$  distributed. Moreover,  $|\mathbf{e}_{k,i}^H \mathbf{T}_{j,l}^{(k,i)}|^2$  for  $i = 1, \cdots, K$  are i.i.d.  $\beta(1, N_t N_r - 2)$  distributed, since  $\mathbf{T}_{j,l}^{(k,i)}$  of unit norm is independent of  $\mathbf{e}_{k,i}$ . For the product of a  $\Gamma(N_t N_r - 1, 2^{-\frac{B_{k,i}}{N_t N_r - 1}})$  distributed random variable and a  $\beta(1, N_t N_r - 2)$  distributed random variable and a  $\beta(1, N_t N_r - 2)$  distributed random variable, it is equal to  $2^{-\frac{B_{k,i}}{N_t N_r - 1}} \chi^2(2)$  in distribution [33]. Based on the fact that the sum of M i.i.d.  $\chi^2(2)$  distributed random variables is  $\chi^2(2M)$  distributed,  $\rho_{k,i} \sum_{l=1}^{d_i} \|\mathbf{h}_{k,i}\|^2 |\mathbf{s}_{k,i}^H \mathbf{T}_{j,l}^{(k,i)}|^2$  is  $2^{-\frac{B_{k,i}}{N_t N_r - 1}} \chi^2(2d_i)$  distributed. Hence,  $I_{k,j}$  is a nested finite

weighted sum of K Erlang pdfs, whose pdf is given by [34]

$$g(x) = \sum_{i=1}^{K} \sum_{t=1}^{\eta_{k,i}} \Xi_K \left( i, t, \{\eta_{k,q}\}_{q=1}^K, \left\{ \frac{\varrho_{k,q}}{\eta_{k,q}} \right\}_{q=1}^K, \\ \{l_{k,q}\}_{q=1}^{K-2} \right) v \left( x, t, \frac{\varrho_{k,i}}{\eta_{k,i}} \right),$$
(10)

where  $\eta_{k,i} = d_i$ ,  $\varrho_{k,i} = \kappa_{k,i} 2^{-\frac{B_{k,i}}{N_t N_r - 1}}$  for  $i \neq k$ ,  $\eta_{k,k} = d_k - 1$ ,  $\varrho_{k,k} = \kappa_{k,k} 2^{-\frac{B_{k,k}}{N_t N_r - 1}}$ ,  $v(x, y, z) = \frac{x^{y-1} \exp(-x/z)}{z^y \Gamma(y)}$ ,  $\forall i$ , and  $\{x_i\}_{i=1}^N$  denotes the set of  $x_i, i = 1, \cdots, N$ . The weights  $\Xi_K$  are defined as

$$\begin{split} \Xi_{K} & \left(i, t, \left\{\eta_{k,q}\right\}_{q=1}^{K}, \left\{\varrho_{k,q}\right\}_{q=1}^{K}, \left\{l_{k,q}\right\}_{q=1}^{K-2}\right) \\ &= \sum_{l_{k,1}=t}^{\eta_{k,i}} \sum_{l_{k,2}=t}^{l_{k,1}} \cdots \sum_{l_{k,K-2}=t}^{l_{k,K-3}} \left[\frac{(-1)^{T_{k}-\eta_{k,i}} \varrho_{k,i}^{t}}{\prod_{h=1}^{K} \varrho_{k,h}^{\eta_{k,h}}} \right] \\ &\times \frac{\Gamma(\eta_{k,i}+\eta_{k,1+U(1-i)}-l_{k,1})}{\Gamma(\eta_{k,1+U(1-i)})\Gamma(\eta_{k,i}-l_{k,1}+1)} \\ &\times \left(\frac{1}{\varrho_{k,i}} - \frac{1}{\varrho_{k,1+U(1-i)}}\right)^{l_{k,1}-\eta_{k,i}-\eta_{k,1+U(1-i)}} \\ &\times \frac{\Gamma(l_{k,K-2}+\eta_{k,K-1+U(K-1-i)}-t)}{\Gamma(\eta_{k,L-1+U(K-1-i)})\Gamma(l_{k,K-2}-t+1)} \\ &\times \left(\frac{1}{\varrho_{k,i}} - \frac{1}{\varrho_{k,K-1+U(K-1-i)}}\right)^{t-l_{k,K-2}-\eta_{k,K-1+U(K-1-i)}} \\ &\times \prod_{s=1}^{K-3} \frac{\Gamma(l_{k,s}+\eta_{k,s+1+U(s+1-i)}-l_{k,s+1})}{\Gamma(\eta_{k,s+1+U(s+1-i)})\Gamma(l_{k,s}-l_{k,s+1}+1)} \\ &\times \left(\frac{1}{\varrho_{k,i}} - \frac{1}{\varrho_{k,s+1+U(s+1-i)}}\right)^{l_{k,s+1}-l_{k,s}-\eta_{k,s+1+U(s+1-i)}} \right], \end{split}$$

where  $T_k = \sum_{i=1}^{K} \eta_{k,i}$  and U(x) is the well-known unit step function defined as  $U(x \ge 0) = 1$  and zero otherwise. Note that the weights  $\Xi_K$  are constant when given  $\eta_{k,i}$  and  $\varrho_{k,i}$ . Substituting (9) and (11) into (8), we have

$$P_{k,j}^{out} = 1 - \sum_{i=1}^{K} \sum_{t=1}^{\eta_{k,i}} \Xi_K \left( i, t, \{\eta_{k,q}\}_{q=1}^K, \left\{ \frac{\varrho_{k,q}}{\eta_{k,q}} \right\}_{q=1}^K, \\ \{ l_{k,q} \}_{q=1}^{K-2} \right) \int_0^\infty \exp\left( -\frac{\gamma_{th}}{\kappa_{k,k}} (x+1) \right) \\ \times \frac{x^{t-1} \exp(-x/(\varrho_{k,q}/\eta_{k,q}))}{(\varrho_{k,q}/\eta_{k,q})^t \Gamma(t)} dx \\ = 1 - \exp\left( -\frac{\gamma_{th}}{\kappa_{k,k}} \right) \sum_{i=1}^K \sum_{t=1}^{\eta_{k,i}} \Xi_K \left( i, t, \{\eta_{k,q}\}_{q=1}^K, \\ \left\{ \frac{\varrho_{k,q}}{\eta_{k,q}} \right\}_{q=1}^K, \{ l_{k,q} \}_{q=1}^{K-2} \right) \left( 1 + \frac{\varrho_{k,q}\gamma_{th}}{\kappa_{k,k}\eta_{k,q}} \right)^{-t}.$$
(12)

Thus, we derive the outage probability for the *j*th data stream of the *k*th pair as a function of the number of data streams  $d_k$ , the amount of CSI exchange  $B_{k,i}$ , and the channel condition  $\kappa_{k,j}$ . Note that if each pair has only one data stream, the result can be further simplified. In this case, there is no

inter-link interference, and  $I_{k,j}$  is a weighted sum of K-1  $\chi^2(2)$  pdfs, whose pdf is given by [35]

$$p_I(x) = \left[\prod_{t=1, t \neq k}^K \lambda_{k,t}\right] \sum_{i=1, i \neq k}^K \frac{\exp(-\lambda_{k,i}x)}{\prod_{l=1, l \neq k, l \neq i}^K (\lambda_{k,l} - \lambda_{k,i})},$$
(13)

where  $\lambda_{k,i} = \frac{1}{\varrho_{k,i}}$ . Then, submitting (9) and (11) into (8), the outage probability in the case of single data stream for each pair is given by

$$P_{k,j}^{out} = 1 - \left[\prod_{t=1, t \neq k}^{K} \lambda_{k,t}\right]$$

$$\times \sum_{i=1, i \neq k}^{K} \int_{0}^{\infty} \frac{\exp\left(-\frac{\gamma_{k,j}}{\kappa_{k,k}}(x+1)\right) \exp(-\lambda_{k,i}x)}{\prod_{l=1, l \neq k, l \neq i}(\lambda_{k,l} - \lambda_{k,i})} dx$$

$$= 1 - \exp\left(-\frac{\gamma_{k,j}}{\kappa_{k,k}}\right) \left[\prod_{t=1, t \neq k}^{K} \lambda_{k,t}\right]$$

$$\times \sum_{i=1, i \neq k}^{K} \frac{\left(\frac{\gamma_{th}}{\kappa_{k,k}} + \lambda_{k,i}\right)^{-1}}{\prod_{l=1, l \neq k, l \neq i}(\lambda_{k,l} - \lambda_{k,i})}$$

$$= 1 - \exp\left(-\frac{\gamma_{k,j}}{\kappa_{k,k}}\right) \left[\prod_{t=1, t \neq k}^{K} \lambda_{k,t}\right]$$

$$\times \sum_{i=1, i \neq k}^{K} \frac{\left(1 + \frac{\varrho_{k,i}}{\kappa_{k,k}} \gamma_{th}\right)^{-1}}{\lambda_{k,i} \prod_{l=1, l \neq k, l \neq i}^{K}(\lambda_{k,l} - \lambda_{k,i})}.$$
(14)

Note that (14) is similar to (12) by letting all  $d_i = 1$ . Therefore, given the channel condition  $\kappa_{k,i}$  and amount of CSI exchange  $B_{k,i}$ , the outage probability can be computed according to (12) if an arbitrary  $d_i$  (namely  $\eta_{k,i}$ ) > 1, and (14) for all  $d_i = 1$ .

*Remark*: It is found that the outage probability in (12) is independent to the index of data stream j, this is because different data streams of the same pair have the same signal quality, and undergo the same interference in the statistical sense.

#### B. Ergodic Rate

For the *j*th data stream of the *k*th pair, the ergodic rate based on IA under imperfect CSI can be expressed as

$$R_{k,j} = E[\log_2(1+\gamma_{k,j})]$$

$$= \frac{1}{\ln(2)} \left( E\left[ \ln\left(\kappa_{k,k} \left| \mathbf{h}_{k,k}^H \mathbf{T}_{j,j}^{(k,k)} \right|^2 + I_{k,j} + 1 \right) \right]$$

$$-E\left[ \ln\left(I_{k,j} + 1 \right) \right] \right). \tag{15}$$

For analytical convenience, we use  $W_1$  and  $W_2$  to denote the first and the second expectation terms in the bracket at the right side of (15). As discussed above,  $\left|\mathbf{h}_{k,k}^H \mathbf{T}_{j,j}^{(k,k)}\right|^2$  is  $\chi^2(2)$  distributed, so  $\kappa_{k,k} \left| \mathbf{h}_{k,k}^H \mathbf{T}_{j,j}^{(k,k)} \right|^2 + I_{k,j}$  is a nested finite weighted sum of K + 1 Erlang pdfs, whose pdf is given by [34]

$$q(x) = \sum_{i=1}^{L} \sum_{t=1}^{\omega_{k,i}} \Xi_L \left( i, t, \{\omega_{k,q}\}_{q=1}^{L}, \left\{ \frac{\varrho_{k,q}}{\omega_{k,q}} \right\}_{q=1}^{L}, \{l_{k,q}\}_{q=1}^{L-2} \right) v \left( x, t, \frac{\varrho_{k,i}}{\omega_{k,i}} \right),$$
(16)

where L = K + 1,  $\omega_{k,i} = d_i$ ,  $\forall i \neq k$ ,  $\omega_{k,k} = d_k - 1$ ,  $\omega_{k,L} = 1$ and  $\varrho_{k,L} = \kappa_{k,k}$ . Hence,  $W_1$  can be computed as

$$W_{1} = \sum_{i=1}^{L} \sum_{t=1}^{\omega_{k,i}} \Xi_{L} \left( i, t, \{\omega_{k,q}\}_{q=1}^{L}, \left\{ \frac{\varrho_{k,q}}{\omega_{k,q}} \right\}_{q=1}^{L}, \{l_{k,q}\}_{q=1}^{L-2} \right) \\ \times \int_{0}^{\infty} \ln(x+1) \frac{x^{t-1} \exp\left(-\frac{x}{(\varrho_{k,q}/\omega_{k,q})}\right)}{(\varrho_{k,q}/\omega_{k,q})^{t} \Gamma(t)} dx \\ = \sum_{i=1}^{L} \sum_{t=1}^{\omega_{k,i}} \Xi_{L} \left( i, t, \{\omega_{k,q}\}_{q=1}^{L}, \left\{ \frac{\varrho_{k,q}}{\omega_{k,q}} \right\}_{q=1}^{L}, \{l_{k,q}\}_{q=1}^{L-2} \right) \\ \times Z \left( t, \frac{\varrho_{k,q}}{\omega_{k,q}} \right),$$
(17)

where

$$Z\left(t,\frac{\varrho_{k,q}}{\omega_{k,q}}\right) = \sum_{\vartheta=0}^{t-1} \frac{1}{\Gamma(t-\vartheta)} \left((-1)^{t-\vartheta-2} \left(\frac{\omega_{k,q}}{\varrho_{k,q}}\right)^{t-\vartheta-1} \times \exp\left(\frac{\omega_{k,q}}{\varrho_{k,q}}\right) \operatorname{E}_{i}\left(-\frac{\omega_{k,q}}{\varrho_{k,q}}\right) + \sum_{\nu=1}^{t-\vartheta-1} \Gamma(\nu) \left(-\frac{\omega_{k,q}}{\varrho_{k,q}}\right)^{t-\vartheta-\nu-1}\right), \quad (18)$$

and  $E_i(x) = \int_{-\infty}^x \frac{\exp(t)}{t} dt$  is the exponential integral function. (17) is obtained based on [36, Eq.4.337.5]. Similarly, we can get  $W_2$  as follows:

$$W_{2} = \sum_{i=1}^{K} \sum_{t=1}^{\eta_{k,i}} \Xi_{K} \left( i, t, \{\eta_{k,q}\}_{q=1}^{K}, \left\{ \frac{\varrho_{k,q}}{\eta_{k,q}} \right\}_{q=1}^{K}, \{l_{k,q}\}_{q=1}^{K-2} \right)^{\text{ff}} \mathbf{S}$$

$$\times \int_{0}^{\infty} \ln(x+1) \frac{x^{t-1} \exp\left(-\frac{x}{(\varrho_{k,q}/\eta_{k,q})}\right)}{(\varrho_{k,q}/\eta_{k,q})^{t} \Gamma(t)} dx$$

$$= \sum_{i=1}^{K} \sum_{t=1}^{\eta_{k,i}} \Xi_{K} \left( i, t, \{\eta_{k,q}\}_{q=1}^{K}, \left\{ \frac{\varrho_{k,q}}{\eta_{k,q}} \right\}_{q=1}^{K}, \{l_{k,q}\}_{q=1}^{K-2} \right)^{\mathbf{S}}$$

$$\times Z \left( t, \frac{\varrho_{k,q}}{\eta_{k,q}} \right). \tag{19}$$

Based on (17) and (19), the ergodic rate is given by

$$R_{k,j} = \frac{1}{\ln(2)} \left( \sum_{i=1}^{L} \sum_{t=1}^{\omega_{k,i}} \Xi_L \left( i, t, \{\omega_{k,q}\}_{q=1}^L, \left\{ \frac{\varrho_{k,q}}{\omega_{k,q}} \right\}_{q=1}^L \right), \\ \{l_{k,q}\}_{q=1}^{L-2} \right) Z \left( t, \frac{\varrho_{k,q}}{\omega_{k,q}} \right) \\ - \sum_{i=1}^{K} \sum_{t=1}^{\eta_{k,i}} \Xi_K \left( i, t, \{\eta_{k,q}\}_{q=1}^K, \left\{ \frac{\varrho_{k,q}}{\eta_{k,q}} \right\}_{q=1}^K \right), \\ \{l_{k,q}\}_{q=1}^{K-2} \right) Z \left( t, \frac{\varrho_{k,q}}{\eta_{k,q}} \right) \right).$$
(20)

Additionally, in the case of single data stream for each pair, the ergodic rate can be computed as

$$R_{k,j} = E\left[\log_{2}\left(1 + \frac{\kappa_{k,k}\left|\mathbf{h}_{k,k}^{H}\mathbf{T}_{j,j}^{(k,k)}\right|^{2}}{I_{k,j}+1}\right)\right]$$

$$= \frac{1}{\ln(2)}\left(E\left[\ln\left(\kappa_{k,k}\left|\mathbf{h}_{k,k}^{H}\mathbf{T}_{j,j}^{(k,k)}\right|^{2}+I_{k,j}+1\right)\right]\right)$$

$$-E[\ln(I_{k,j}+1)]\right)$$

$$= \frac{1}{\ln(2)}\left(\left[\prod_{t=1}^{K}\lambda_{k,t}\right]\sum_{i=1}^{K}\frac{\int_{0}^{\infty}\ln(x+1)\exp(-\lambda_{k,i}x)dx}{\prod_{l=1,l\neq i}^{K}(\lambda_{k,l}-\lambda_{k,i})}\right]$$

$$-\left[\prod_{t=1,t\neq k}^{K}\lambda_{k,t}\right]\sum_{i=1,i\neq k}^{K}\frac{\int_{0}^{\infty}\ln(x+1)\exp(-\lambda_{k,i}x)dx}{\prod_{l=1,l\neq k}^{K}(\lambda_{k,l}-\lambda_{k,i})}\right]$$

$$= \frac{1}{\ln(2)}\left(\left[\prod_{t=1}^{K}\lambda_{k,t}\right]\sum_{i=1}^{K}\frac{\exp(\lambda_{k,i})E_{i}(\lambda_{k,i})}{\lambda_{k,i}\prod_{l=1,l\neq i}^{K}(\lambda_{k,l}-\lambda_{k,i})}\right]$$

$$-\left[\prod_{t=1,t\neq k}^{K}\lambda_{k,t}\right]\sum_{i=1,i\neq k}^{K}\frac{\exp(\lambda_{k,i})E_{i}(\lambda_{k,i})}{\lambda_{k,i}\prod_{l=1,l\neq i}^{K}(\lambda_{k,l}-\lambda_{k,i})}\right],$$
(21)

where (21) is obtained based on [36, Eq.4.337.2]. Note that (21) is equivalent to (20) because of  $\lambda_{k,i} = \frac{1}{\varrho_{k,i}}$  when t = 1.

## C. SER

In addition to outage probability and ergodic rate, SER is also an important performance metric, which indicates the reliability of a wireless communication system. For a given SINR  $\gamma$ , the SER is given by [37]

$$\begin{cases} \frac{1}{\pi} \int_{0}^{\frac{(M-1)\pi}{M}} \exp\left(-\frac{g_{\text{PSK}\gamma}}{\sin^{2}(x)}\right) dx, & \text{for } M\text{-PSK} \\ \frac{2}{\pi} \frac{M-1}{M} \int_{0}^{\frac{\pi}{2}} \exp\left(-\frac{g_{\text{PAM}\gamma}}{\sin^{2}(x)}\right) dx, & \text{for } M\text{-PAM} \\ \frac{4}{\pi} \left(1 - \frac{1}{\sqrt{M}}\right) & \left[\frac{1}{\sqrt{M}} \int_{0}^{\frac{\pi}{4}} \exp\left(-\frac{g_{\text{QAM}\gamma}}{\sin^{2}(x)}\right) dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \exp\left(-\frac{g_{\text{QAM}\gamma}}{\sin^{2}(x)}\right) dx \right], & \text{for } M\text{-QAM} \end{cases}$$

$$(22)$$

where M is the modulation order,  $g_{\text{PSK}} = \sin^2 \frac{\pi}{M}$ ,  $g_{\text{PAM}} = \frac{3}{M^2 - 1}$  and  $g_{\text{AQM}} = \frac{3}{2(M-1)}$ . For analytical convenience, we use  $G(M, a \exp(-b\gamma))$  to denote a general SER function, where a and b depend on the modulation format. Then, the average SER of the *j*th data stream of the *k*th pair can be computed as

$$P_{k,j}^{ser} = \int_0^\infty \int_0^\infty G\left(M, a \exp\left(-b\frac{x}{y+1}\right)\right)$$
$$\times f(x)g(y)dxdy$$
$$= \int_0^\infty G\left(M, \frac{ay+a}{y+b\kappa_{k,k}+1}\right)g(y)dy \quad (23)$$

$$= G\left(M, \sum_{i=1}^{K} \sum_{t=1}^{\eta_{k,i}} \Xi_{K}\left(i, t, \{\eta_{k,q}\}_{q=1}^{K}, \{\frac{\varrho_{k,q}}{\eta_{k,q}}\}_{q=1}^{K}, \{l_{k,q}\}_{q=1}^{K-2}\right) \times a\left(1 - b\kappa_{k,k}(b\kappa_{k,k} + 1)^{t-1} \exp\left(\frac{(b\kappa_{k,k} + 1)\eta_{k,q}}{\varrho_{k,q}}\right) \times \Gamma\left(1 - t, \frac{(b\kappa_{k,k} + 1)\eta_{k,q}}{\varrho_{k,q}}\right)\right)\right),$$
(24)

where  $f(x) = \frac{1}{\kappa_{k,k}} \exp\left(-\frac{x}{\kappa_{k,k}}\right)$  is the pdf of  $\kappa_{k,k} \left| \mathbf{h}_{k,k}^H \mathbf{T}_{j,j}^{(k,k)} \right|^2$ . (23) follows the fact that  $G(M, a \exp(-b\gamma))$  is a linear function of  $a \exp\left(-b\frac{x}{y+1}\right)$ , and (24) is obtained based on [36, Eq.3.383.10].

Similarly, for the case of single data stream, the average SER can be reduced as

$$P_{k,j}^{ser} = \int_{0}^{\infty} \int_{0}^{\infty} G\left(M, a \exp\left(-b\frac{x}{y+1}\right)\right)$$

$$\times f_{S}(x)p_{I}(y)dxdy$$

$$= \int_{0}^{\infty} G\left(M, \frac{ay+a}{y+1+b\kappa_{k,k}}\right)p_{I}(y)dy$$

$$= G\left(M, \left[\prod_{t=1,t\neq k}^{K} \lambda_{k,t}\right]\sum_{i=1,i\neq k}^{K} a(1-b\kappa_{k,k})$$

$$\times \exp\left((b\kappa_{k,k}+1)\lambda_{k,i}\right)\Gamma(0, (b\kappa_{k,k}+1)\lambda_{k,i})\right)$$

$$/(\lambda_{k,i}\prod_{l=1,l\neq k,l\neq i}^{K} (\lambda_{k,l}-\lambda_{k,i}))\right).$$
(25)

Note that (25) is also equivalent to (24) by letting  $t = \eta_{k,i} = 1$ and  $\lambda_{k,i} = 1/\varrho_{k,i}$ . Submitting (24) and (25) into (22), we could obtain the expression of SER. As a simple example, for *M*-PSK modulation,  $a = \frac{1}{\pi}$  and  $b = \frac{g_{\text{PSK}}}{\sin^2(x)}$ , so the average SER is given by

$$P_{k,j}^{ser} = \frac{1}{\pi} \int_{0}^{\frac{(M-1)\pi}{M}} \sum_{i=1}^{K} \sum_{t=1}^{\eta_{k,i}} \Xi_{K} \left( i, t, \{\eta_{k,q}\}_{q=1}^{K}, \{\frac{\varrho_{k,q}}{\eta_{k,q}}\}_{q=1}^{K}, \{l_{k,q}\}_{q=1}^{K-2} \right) \\ \times \left( 1 - \frac{g_{\text{PSK}}\kappa_{k,k}}{\sin^{2}(x)} \left( \frac{g_{\text{PSK}}\kappa_{k,k} + \sin^{2}(x)}{\sin^{2}(x)} \right)^{t-1} \right) \\ \times \exp\left( \frac{(g_{\text{PSK}}\kappa_{k,k} + \sin^{2}(x))\eta_{k,q}}{\sin^{2}(x)\varrho_{k,q}} \right) \\ \times \Gamma\left( 1 - t, \frac{(g_{\text{PSK}} + \sin^{2}(x))\kappa_{k,k}\eta_{k,q}}{\sin^{2}(x)\varrho_{k,q}} \right) \right) dx.$$
(26)

Given channel condition and amount of CSI exchange,  $P_{k,j}^{ser}$  can be computed by numerical integration.

#### IV. PERFORMANCE LOSS FROM IMPERFECT CSI

Due to limited CSI feedback, there exists a certain performance loss with respect to the case of perfect CSI due to imperfect IA. In order to reveal the impact of CSI and obtain insightful guidelines for system design and performance optimization, we investigate the performance loss resulting from imperfect CSI from the standpoints of outage probability, ergodic rate and SER, respectively. For the convenience of analysis, we use  $\Upsilon = \frac{P}{\sigma^2}$  to denote the transmit SNR.

## A. Outage Probability

If the transmitters have perfect CSI, the interference can be canceled completely, and then the SINR of the jth data stream of the kth pair is transformed as

$$\gamma_{k,j}^{perfect} = \kappa_{k,k} \left| \mathbf{h}_{k,k}^{H} \mathbf{T}_{j,j}^{(k,k)} \right|^{2}.$$
(27)

In this context, the outage probability of the *j*th data stream of the *k*th pair related to a given SNR threshold  $\gamma_{th}$  can be computed as

$$P_{k,j}^{out,perfect} = P_r(\gamma_{k,j}^{perfect} \le \gamma_{th})$$
  
=  $F(\gamma_{th})$   
=  $1 - \exp\left(-\frac{\gamma_{th}}{\kappa_{k,k}}\right).$  (28)

Based on (12) and (28), the performance loss due to imperfect CSI is given by:

$$\Delta P_{k,j}^{out} = P_{k,j}^{out} - P_{k,j}^{out,full}$$

$$= \exp\left(-\frac{\gamma_{th}}{\kappa_{k,k}}\right) \left(1 - \sum_{i=1}^{K} \sum_{t=1}^{\eta_{k,i}} \Xi_K\left(i, t, \{\eta_{k,q}\}_{q=1}^K, \left\{\frac{\varrho_{k,q}}{\eta_{k,q}}\right\}_{q=1}^K, \{l_{k,q}\}_{q=1}^{K-2}\right) \left(1 + \frac{\varrho_{k,q}\gamma_{th}}{\kappa_{k,k}\eta_{k,q}}\right)^{-t}\right) (29)$$

$$= \exp\left(-\frac{\gamma_{th}}{\Upsilon\alpha_{k,k}}\right) \left(1 - \sum_{i=1}^{K} \sum_{t=1}^{\eta_{k,i}} \Xi_K\left(i, t, \{\eta_{k,q}\}_{q=1}^K, \left\{\frac{\varrho_{k,q}}{\eta_{k,q}}\right\}_{q=1}^K, \{l_{k,q}\}_{q=1}^{K-2}\right) \left(1 + \frac{\alpha_{k,q}\rho_{k,q}\gamma_{th}}{\alpha_{k,k}\eta_{k,q}}\right)^{-t}\right) (30)$$

(30) provides a general outage probability loss due to imperfect CSI in terms of CSI accuracy, transmit SNR, and channel condition. However, it is difficult to get some clear insights from (30). To address this problem, we perform asymptotic analysis under some extreme scenario. First, if SNR  $\Upsilon$  is low enough, we have the following theorem:

*Theorem 1*: At low SNR, the CSI exchange is useless, and the performance loss due to imperfect CSI is negligible.

*Proof:* The proof is straightforward. When SNR  $\Upsilon \rightarrow 0$ , the term  $\exp\left(-\frac{\gamma_{th}}{\Upsilon \alpha_{k,k}}\right)$  in (30) asymptotically approaches zero, so there is no performance gap between the two cases, and hence the CSI is useless.

*Remark*: If SNR is quite low, the interference term becomes negligible with respect to the noise. So the CSI exchange is not needed, as well as the IA. In this context, maximum ratio transmission (MRT) can obtain the maximum DOFs, and hence achieves the optimal performance.

On the other hand, if SNR  $\Upsilon$  is sufficiently high, the performance loss in terms of outage probability has the following property: Theorem 2: At high SNR, there is always a performance floor in terms of outage probability for IA with imperfect CSI. The performance loss becomes larger as SNR increases, and the maximum performance loss is given by  $\Delta P_{k,j}^{out,\max} = 1 - 1$ 

$$\sum_{i=1}^{K} \sum_{t=1}^{\eta_{k,i}} \Xi_{K} \left( i, t, \{\eta_{k,q}\}_{q=1}^{K}, \left\{ \frac{\varrho_{k,q}}{\eta_{k,q}} \right\}_{q=1}^{K}, \{l_{k,q}\}_{q=1}^{K-2} \right) \\ \left( 1 + \frac{\alpha_{k,q}\rho_{k,q}\gamma_{th}}{\alpha_{k,k}\eta_{k,q}} \right)^{-t}.$$

*Proof:* Please refer to Appendix I.

From Theorem 2, it is known that the performance gap of outage probability caused by imperfect CSI is an increasing function of  $\Upsilon$  and  $\rho_{k,i}$ , so the performance gap can be reduced by adding the amount of CSI exchange. Furthermore, for the sake of keeping a constant gap as SNR increases, the amount of CSI exchange should satisfy the following proposition:

Proposition 1: In order to avoid the performance floor and keep a constant gap between imperfect and perfect CSI, the total amount of CSI exchange from the kth receiver should fulfill the condition of  $B_k \sim K(N_t N_r - 1) \log_2(\Upsilon)$ , where  $\sim$  denotes proportional to.

*Proof:* Please refer to Appendix II.

Based on Proposition 1, we further obtain a useful guideline for performance optimization of IA under imperfect CSI as follows

*Proposition 2*: In order to keep a constant gap of outage probability between imperfect and perfect CSI, the total amount of CSI exchange B should be added as the number of antennas increases.

**Proof:** The proof can be dervied from Proposition 1 directly. Because of  $B_k \sim K(N_tN_r - 1)\log_2(\Upsilon)$ ,  $B_k$  should be added as the number of antennas increases, which is also consistent with the intuition that for a given  $B_k$ , increasing the number of antennas results in the reduction of CSI quantization accuracy, and hence leads to performance degradation.

#### B. Ergodic Rate

In this subsection, we turn the attention to the rate loss caused by imperfect CSI, which is given by

$$\Delta R_{k,j} = E \left[ \log_2 \left( 1 + \gamma_{k,j}^{perfect} \right) \right] - E[\log_2 \left( 1 + \gamma_{k,j} \right)]$$
$$= \frac{1}{\ln(2)} \left( \exp \left( \frac{1}{\kappa_{k,k}} \right) E_i \left( \frac{1}{\kappa_{k,k}} \right) \right) - R_{k,j}. (31)$$

Similarly, we perform asymptotic analysis on  $\Delta R_{k,j}$  to get some clear insights. First, if SNR  $\Upsilon$  is low enough, we have the following theorem:

*Theorem 3*: At low SNR, the CSI exchange is useless. The ergodic rates in the cases of both perfect and imperfect CSI approach 0, so the rate loss is negligible.

*Proof:* The proof is intuitive, as SNR tends to 0, the interference term is negligible with respect to the noise term, then the ergodic rate with imperfect CSI is equivalent to that with perfect CSI, and hence the performance gap becomes zero.

On the other hand, for the high SNR case, the rate loss due to imperfect CSI has the following theorem:

*Theorem 4*: At high SNR, there is always a performance ceiling in terms of ergodic rate for IA with imperfect CSI,

and the performance loss with respect to the ideal case of perfect CSI enlarges logarithmically with the SNR.

*Proof:* Please refer to Appendix III.

As analyzed above, the performance ceiling is a decreasing function of  $\rho_{k,i}$ . To keep a constant gap, the amount of CSI exchange should be added as SNR increases. It is assumed that the CSI exchange amount is the same and quite large in the high SNR region, then we have the following proposition:

*Proposition 3*: At high SNR with large amount of CSI exchange, the performance ceiling is a linear function of the amount of CSI exchange, and the performance gain by adding additional amount of CSI exchange is equal to  $\frac{1}{N_t N_r - 1}$  times the incremental amount of CSI.

*Proof:* Please refer to Appendix IV.

From Theorem 4, it is known that the ergodic rate with perfect CSI at high SNR is  $R_{k,j}^{perfect,high} = \frac{\ln(\Upsilon \alpha_{k,k}) - C}{\ln(2)}$ , so the performance loss at high SNR when *B* is sufficiently large can be approximated as

$$\Delta R_{k,j}^{high,large} = \log_2(\Upsilon) - \frac{B}{N_t N_r - 1} + \frac{1}{\ln(2)} \sum_{i=1}^K \sum_{t=1}^{\eta_{k,i}} \Xi_K \left( i, t, \{\eta_{k,q}\}_{q=1}^K, \left\{ \frac{\xi_{k,q}}{\eta_{k,q}} \right\}_{q=1}^K, \{l_{k,q}\}_{q=1}^{K-2} \right) \times (\psi(t) + \ln(\alpha_{k,q}) - \ln(\eta_{k,q})).$$
(32)

As mentioned in Theorem 4, the performance loss with respect to the ideal case of perfect CSI enlarges logarithmically with the SNR. Based on (32), we have the following proposition:

Proposition 4: At high SNR, in order to keep a constant performance gap, B should be added proportionally to  $(N_t N_r - 1) \log_2(\Upsilon)$ .

### C. SER

Intuitively, imperfect CSI would lead to SER performance degradation inevitably. In this subsection, we give a quantitative analysis of the performance loss. Substituting (27) into (22), the average SER with perfect CSI can be computed as

$$P_{k,j}^{ser,perfect} = \int_0^\infty G\left(M, a \exp(-bx)\right) \frac{1}{\kappa_{k,k}} \exp\left(-\frac{x}{\kappa_{k,k}}\right) dx$$
$$= G\left(M, \frac{a}{1+b\kappa_{k,k}}\right). \tag{33}$$

Thereby, the performance gap is given by

$$\Delta P_{k,j}^{ser} = G\left(M, \sum_{i=1}^{K} \sum_{t=1}^{\eta_{k,i}} \Xi_K \left(i, t, \{\eta_{k,q}\}_{q=1}^K, \left\{\frac{\varrho_{k,q}}{\eta_{k,q}}\right\}_{q=1}^K, \\ \{l_{k,q}\}_{q=1}^{K-2}\right) a \left(1 - b\kappa_{k,k}(b\kappa_{k,k}+1)^{t-1} \\ \times \exp\left(\frac{(b\kappa_{k,k}+1)\eta_{k,q}}{\varrho_{k,q}}\right) \\ \times \Gamma\left(1 - t, \frac{(b\kappa_{k,k}+1)\eta_{k,q}}{\varrho_{k,q}}\right)\right) - \frac{a}{1 + b\kappa_{k,k}}\right),$$
(34)

where (34) holds true since G(x, y) is a linear function of y. Similarly, we carry out asymptotic analysis on the performance

TABLE I PARAMETER TABLE FOR  $\alpha_{i,j}, \forall i, j \in [1,3].$ 

j	1	2	3
1	1.000	0.050	0.005
2	0.055	1.000	0.045
3	0.004	0.060	1.000

loss to get some clear insights. First, if SNR is low enough, we have the following theorem:

*Theorem 5*: At low SNR, the CSI exchange is useless, the average SER in the cases of both perfect and imperfect CSI approaches 0.5, so the performance gap is negligible.

*Proof:* The proof is similar to that of Theorem 1. Furthermore, when SNR asymptotically approaches infinity, the performance loss has the following theorem:

*Theorem* 6: At high SNR, there is a SER performance floor for IA with imperfect CSI. The performance gap becomes larger as SNR increases.

The floor appears since the SINR is saturated once SNR exceeds a specific value, so it is necessary to add the amount of CSI exchange logarithmically proportional to SNR in order to avoid the floor.

### V. NUMERICAL RESULTS

To verify the performance analysis results for IA with imperfect CSI in a MIMO interference network, we present several numerical results under different scenarios. For convenience, we set  $N_t = 4$ ,  $N_r = 2$ , K = 3,  $\sigma^2 = 1$ ,  $\gamma_{th} = 0$ dB,  $d_1 = d_2 = d_3 = d = 1$  and  $\alpha_{i,j}$  given in Tab.I for all simulation scenarios without explicit explanation. Note that we consider an interference channel model as shown in Fig.1, the propagation distances for the desired links are the same, and the distance between the interference transmitter to the receiver is larger than that between the desired transmitter to the receiver. Mathematically, we have  $\alpha_{i,i} > \alpha_{i,j}$ , if  $i \neq j$ . In convenience, we normalize  $\alpha_{i,i}$  as 1, so we have  $\alpha_{i,j} < 1$ for  $i \neq j$ . In addition, we use SNR (in dB) to represent  $10 \log_{10} \frac{P}{\sigma^2}$ , and B (in bit) to denote the same amount CSI exchange for each pair. Without loss of generality, we take the performance of the 1st data stream of 1st pair as an example.

Firstly, we investigate the outage probability of IA with imperfect CSI. As seen in Fig.2, the theoretical results are quite consistent with the simulation results throughout the whole SNR region under different amount of CSI exchange B, which validates the high accuracy of our analysis. It is found that at low SNR, the outage probabilities with different B are nearly the same, which reconfirms the claim of Theorem 1 that the CSI exchange in the low SNR region is useless. With the increase of the SNR, as Theorem 2 claims, there always exist a performance floor, which decreases as B adds. In addition, as shown in Fig.3, the performance loss due to imperfect CSI becomes larger as SNR increases. In order to keep a constant gap, B should be added logarithmically and linearly proportional to the SNR and the number of antennas respectively, as proved in Proposition 1 and 2.

Next, we show the ergodic rate performance of IA with imperfect CSI. Similarly, the CSI exchange is useless for



Fig. 2. Theoretical and simulation outage probability with different CSI exchange amount.



Fig. 3. Outage probability loss due to imperfect CSI.

performance improvement at low SNR, as seen in Fig.4, which confirms the Theorem 3. As SNR increases, the performance ceiling appears, if there is finite CSI exchange amount, resulting an obvious performance gap with respect to the case of perfect CSI  $(B = \infty)$ , as shown in Fig.5. Note that the number of data stream for each pair also has a great impact on the ergodic rate. The larger number of data streams, the more spatial multiplexing gains, but also leads to higher residual interference. As illustrated in Fig.6, at high SNR, the case of d = 1 has a significant advantage over that of d = 2, since under interference-limited condition, a small spatial multiplexing is beneficial, which has also been observed in a multiuser downlink network[38]. Although it is proven that IA can achieve full DOF at high SNR, but it is optimal to select single data stream from the perspective of maximizing the ergodic rate under the case of imperfect CSI. Furthermore, Fig.7 shows the ergodic rate at high SNR with large B. It is found that the ergodic rate is a linear function of B, which reconfirms the claim of proposition 3.

Finally, we discuss the SER performance of IA with imperfect CSI. Clearly, for the three different modulation formats, namely 8PSK, 8PAM and 8QAM, the theoretical analysis



Fig. 4. Theoretical and simulation ergodic rate performance with different CSI exchange amount.



Fig. 5. Ergodic rate performance loss due to imperfect CSI.

nicely coincides with the numerical simulation throughout the whole SNR region, as shown in Fig.8. Imperfect IA caused by imperfect CSI inevitably leads to error floor, and the performance gap with respect to the ideal case of perfect CSI becomes larger as SNR increases, as seen in Fig.9. On the other hand, the gap at low SNR is negligible, which proves that the CSI exchange is useless in such a scenario.

#### VI. CONCLUSION

This paper comprehensively analyzes the performance of IA with imperfect CSI over a general MIMO interference channel, including outage probability, ergodic rate and SER. Thus, the exact performance can be evaluated for a given transmit SNR, channel condition, and amount of CSI exchange. Moreover, through asymptotic analysis on the performance loss due to imperfect CSI, we get some important design guidelines. For example, CSI exchange is useless at low SNR, and the amount of CSI exchange should be increased logarithmically to the SNR and linearly proportional to the number of antennas at high SNR in order to avoid performance saturation. Though the number of data streams *d* should be large from the DoF



Fig. 6. Ergodic rate performance with different numbers of data streams.



Fig. 7. Ergodic rate performance at high SNR with large B.

point of view, we show that having d = 1 is optimal at high SNR, as there exists residual interference under imperfect CSI.

## APPENDIX A Proof of Theorem 2

Firstly, we prove there always exists a performance floor in terms of outage probability for IA with imperfect CSI. As  $\Upsilon \to \infty$ , the corresponding outage probability in (12) is transformed as

$$P_{k,j}^{out,high} = 1 - \sum_{i=1}^{K} \sum_{t=1}^{\eta_{k,i}} \Xi_K \left( i, t, \{\eta_{k,q}\}_{q=1}^K, \left\{ \frac{\varrho_{k,q}}{\eta_{k,q}} \right\}_{q=1}^K, \left\{ l_{k,q} \right\}_{q=1}^{K-2} \right) \left( 1 + \frac{\alpha_{k,q}\rho_{k,q}\gamma_{th}}{\alpha_{k,k}\eta_{k,q}} \right)^{-t}.$$
 (35)

It is found that  $P_{k,j}^{out,high}$  is independent of SNR, so given CSI accuracy and channel condition, there is always a performance floor.

On the other hand, examining the outage probability with perfect CSI in (28), it is a decreasing function of SNR, so the performance loss becomes larger as SNR increase. When  $\Upsilon \rightarrow$ 





Fig. 8. Theoretical and simulation SER performance with different modulation formats and B = 6.



Fig. 9. SER performance loss due to imperfect CSI with 8QAM modulation format.

 $\infty$ ,  $P_{k,j}^{out,perfect}$  is equal to zero, so the maximum performance loss is given by  $\Delta P_{k,j}^{out,\max} = P_{k,j}^{out,high}$ , which proves the Theorem 2.

## APPENDIX B Proof of Proposition 1

As seen in (29), in order to keep a constant gap, the only way is to add the CSI exchange amount, so that  $\Upsilon \rho_{k,q}$  is constant. Then, as  $\Upsilon \to \infty$ , the term  $\left(\frac{\varrho_{k,q}\gamma_{th}}{\kappa_{k,k}\eta_{k,q}}\right)^{-t}$  tends to zero, and hence  $\Delta P_{k,j}^{out,high}$  is equal to zero. In other words,  $2^{\frac{B_{k,q}}{N_tN_r-1}}$  should at least increase linearly proportional to  $\Upsilon$ , namely  $B_{k,q} \sim (N_tN_r - 1)\log_2(\Upsilon)$ . So we have  $B_k = \sum_{q=1}^{K} B_{k,q} \sim K(N_tN_r - 1)\log_2(\Upsilon)$ .

## APPENDIX C Proof of Theorem 4

If SNR is high enough, the constant 1 of  $\gamma_{k,j}$  in (6) is negligible, so the ergodic rate at high SNR is transformed as

R

4

$$\begin{split} {}^{high}_{k,j} &= \frac{1}{\ln(2)} \left( E \left[ \ln \left( \kappa_{k,k} \left| \mathbf{v}_{k,j}^{H} \mathbf{H}_{k,k} \mathbf{w}_{k,j} \right|^{2} + I_{k,j} \right) \right] \\ &- E \left[ \ln \left( I_{k,j} \right) \right] \right) \\ &= \frac{1}{\ln(2)} \left( \sum_{i=1}^{L} \sum_{t=1}^{\omega_{k,i}} \Xi_{L} \left( i, t, \{\omega_{k,q}\}_{q=1}^{L}, \left\{ \frac{\xi_{k,q}}{\omega_{k,q}} \right\}_{q=1}^{L}, \left\{ l_{k,q} \right\}_{q=1}^{L-2} \right) (\psi(t) + \ln(\omega_{k,q}) - \ln(\xi_{k,q})) \\ &- \sum_{i=1}^{K} \sum_{t=1}^{\eta_{k,i}} \Xi_{K} \left( i, t, \{\eta_{k,q}\}_{q=1}^{K}, \left\{ \frac{\xi_{k,q}}{\eta_{k,q}} \right\}_{q=1}^{K}, \left\{ l_{k,q} \right\}_{q=1}^{K-2} \right) (\psi(t) + \ln(\eta_{k,q}) - \ln(\xi_{k,q})) \right), \end{split}$$
(36)

where  $\xi_{k,i} = \alpha_{k,i}\rho_{k,i}/d_i$ ,  $\forall i \in [1, K]$ ,  $\xi_{k,L} = \alpha_{k,k}/d_k$  and  $\psi(x) = \frac{d\ln(x)}{dx}$  is the Euler's function. (36) is obtained based on [36, Eq.4.352.1]. Clearly, at high SNR, the ergodic rate  $R_{k,j}^{high}$  is independent of  $\Upsilon$ , so there is a performance ceiling. Additionally, the ergodic rate with perfect CSI at high SNR can be expressed as

$$R_{k,j}^{perfect,high} = \frac{1}{\ln(2)} E \left[ \ln \left( 1 + \Upsilon \alpha_{k,k} \left| \mathbf{v}_{k,j}^{H} \mathbf{H}_{k,k} \mathbf{w}_{k,j} \right|^{2} \right) \right]$$
$$\approx \frac{1}{\ln(2)} E \left[ \ln \left( \Upsilon \alpha_{k,k} \left| \mathbf{v}_{k,j}^{H} \mathbf{H}_{k,k} \mathbf{w}_{k,j} \right|^{2} \right) \right] (37)$$
$$= \frac{\ln(\Upsilon \alpha_{k,k}) - C}{\ln(2)}, \qquad (38)$$

where C = 0.57721566490 is the Euler's constant. (37) drops the constant 1 and (38) is obtained based on [36, Eq.4.331.1]. Then the performance loss at high SNR is given by

$$\Delta R_{k,j}^{high} = R_{k,j}^{full,high} - R_{k,j}^{high}$$

$$= \log_2(\Upsilon \alpha_{k,k}) - C - \frac{1}{\ln(2)} \left( \sum_{i=1}^{L} \sum_{t=1}^{\omega_{k,i}} \Xi_L \left( i, t, \{\omega_{k,q}\}_{q=1}^L, \left\{ \frac{\xi_{k,q}}{\omega_{k,q}} \right\}_{q=1}^L, \{l_{k,q}\}_{q=1}^{L-2} \right) \times (\psi(t) + \ln(\xi_{k,q}) - \ln(\omega_{k,q})) + \sum_{i=1}^{K} \sum_{t=1}^{\eta_{k,i}} \Xi_K \left( i, t, \{\eta_{k,q}\}_{q=1}^K, \left\{ \frac{\xi_{k,q}}{\eta_{k,q}} \right\}_{q=1}^K, \{l_{k,q}\}_{q=1}^{K-2} \right) \times (\psi(t) + \ln(\xi_{k,q}) - \ln(\eta_{k,q})) \right).$$
(39)

The performance loss enlarges logarithmically with  $\Upsilon$ , which proves the Theorem 4.

### APPENDIX D PROOF OF PROPOSITION 3

Let B be the same CSI exchange amount, if both  $\Upsilon$  and B are large enough, the ergodic rate can be approximated as

$$R_{k,j}^{high} \approx \frac{1}{\ln(2)} \left( E \left[ \ln \left( \frac{\kappa_{k,k} \left| \mathbf{v}_{k,j}^{H} \mathbf{H}_{k,k} \mathbf{w}_{k,j} \right|^{2}}{I_{k,j}} \right) \right] \right) \right) \\= \frac{1}{\ln(2)} \left( E \left[ \ln \left( \alpha_{k,k} \left| \mathbf{v}_{k,j}^{H} \mathbf{H}_{k,k} \mathbf{w}_{k,j} \right|^{2} \right) \right] \\- E \left[ \ln \left( \alpha_{k,k} \rho_{k,k} \left\| \mathbf{h}_{k,k} \right\|^{2} \sum_{l=1, l \neq j}^{d_{k}} \left| \mathbf{e}_{k,k}^{H} \mathbf{T}_{j,l}^{(k,k)} \right|^{2} \right) \right] \\+ \sum_{i=1, i \neq k}^{K} \alpha_{k,i} \rho_{k,i} \left\| \mathbf{h}_{k,i} \right\|^{2} \sum_{l=1}^{d_{i}} \left| \mathbf{e}_{k,i}^{H} \mathbf{T}_{j,l}^{(k,i)} \right|^{2} \right) \right] \right) \\= \frac{\ln(\alpha_{k,k}) - C}{\ln(2)} - \frac{1}{\ln(2)} \sum_{i=1}^{K} \sum_{t=1}^{\eta_{k,i}} \left[ \Xi_{K} \left( i, t, \{\eta_{k,q}\}_{q=1}^{K}, \left\{ \frac{\xi_{k,q}}{\eta_{k,q}} \right\}_{q=1}^{K}, \{l_{k,q}\}_{q=1}^{K-2} \right) \\\times (\psi(t) + \ln(\xi_{k,q}) - \ln(\eta_{k,q})) \\= \frac{B}{N_{t}N_{r} - 1} + \frac{\ln(\alpha_{k,k}) - C}{\ln(2)} - \frac{1}{\ln(2)} \sum_{i=1}^{K} \sum_{t=1}^{\eta_{k,i}} \sum_{i=1}^{\xi_{k,i}} \left[ \Xi_{K} \left( i, t, \{\eta_{k,q}\}_{q=1}^{K-1}, \left\{ \frac{\xi_{k,q}}{\eta_{k,q}} \right\}_{q=1}^{K}, \{l_{k,q}\}_{q=1}^{K-2} \right) \\\times (\psi(t) + \ln(\alpha_{k,q}) - \ln(\eta_{k,q})), \quad (40)$$

where (40) holds true because of  $\xi_{k,q} = \alpha_{k,q}\rho_{k,q}/d_q$  and follows the fact of  $\sum_{i=1}^{K} \sum_{t=1}^{\eta_{k,i}} \Xi_K \left( i, t, \{\eta_{k,q}\}_{q=1}^K, \{\frac{\xi_{k,q}}{\eta_{k,q}}\}_{q=1}^K, \{l_{k,q}\}_{q=1}^{K-2} \right) = 1$ . So the performance ceiling is a linear function of B. Given the channel conditions, the performance gain by adding the CSI exchange amount from  $B_1$  to  $B_2$  is equal to  $\frac{B_2 - B_2}{N_t N_r - 1}$ , which is linear proportionally to the increment of CSI.

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