Rate-Compatible Root-Protograph LDPC Codes for Quasi-Static Fading Relay Channels

Yi Fang, Yong Liang Guan, Guoan Bi, Lin Wang, and Francis C. M. Lau

Abstract—**We investigate the protograph low-density parity-check (LDPC) codes over quasi-static fading (QSF) relay channels with coded cooperation (CC) in this paper. A new construction method is proposed for designing the full-diversity rate-compatible root-protograph (RCRP) codes for such a relaying protocol. The asymptotic word-error-rate (WER) and bit-error-rate (BER) of the RCRP codes are analyzed by exploiting a new protograph extrinsic information transfer (PEXIT) algorithm. Furthermore, the expression of outage probability is also developed so as to characterize the error performance of RCRP codes. Both theoretical and simulated results suggest that the RCRP codes not only outperform the conventional regular rate-compatible protograph (RCP) codes in terms of error performance with code rates ranging from 0 to $1/2$, but also approach the outage limit.**

Index Terms—**Coded cooperation (CC), extrinsic information transfer (EXIT), outage probability, protograph LDPC codes, quasi-static fading (QSF) channels.**

I. Introduction

Fading is a major factor that deteriorates the quality of signal transmission in wireless communications. Spatial diversity is a useful technique to alleviate the fading effect and can be achieved by installing multiple antennas at a transmitter. However, the transmitters may not be able to support multiple antennas due to the constraints of size, complexity, power, cost, etc. Alternatively, cooperation between two single-antenna users can also yield spatial diversity so as to greatly increase the system reliability [1]. The relay channel [2], which consists of a source, a relay, and a destination, is the most elementary framework of cooperative communication systems. Recently, the fundamental theoretical-limits of the relay channel have been carefully studied in ergodic and quasi-static fading (QSF) channels [2], [3].

In a relay channel, the specific cooperative algorithm is dependent on the relaying strategy or protocol. As a classical protocol, the decode-and-forward (DF) has been first proposed in [1]. To achieve the capacity-approaching error performance in DF-based additive white Gaussian noise (AWGN) relay channels, bilayer LDPC and protograph codes have been designed by exploiting the extrinsic information transfer (EXIT) algorithms [4], [5]. However, the two types of codes and DF cannot attain full diversity in QSF scenarios. To overcome the limitation of DF protocol, the selection DF (SDF) and amplify-and-forward (AF) protocols, which can realize full diversity, have been proposed and analyzed [6]. For all the aforementioned methods, the relay repeats the received message via decoding or amplification. Recently, a more efficient protocol, namely coded cooperation (CC), has been proposed [7]. The basic idea of this technique is that the relay helps to transmit the incremental redundancy for the user rather than repeat the received message. Later, CC has been implemented using some powerful codes, e.g., convolutional codes [8], turbo codes [9], and LDPC codes [10], because of their superior error performance. Unfortunately, the LDPC codes designed particularly for ergodic fading relay channels [10] are unable to achieve full diversity and hence do not perform well over QSF relay channels. To address this problem, the full-diversity rate-compatible root-LDPC (RCR-LDPC) codes have been proposed [11], [12] for CC protocol with the help of root-LDPC codes [13]. Nevertheless, most RCR-LDPC codes are unstructured and hence possess relatively high encoding complexity. As one type of structured LDPC codes, the protograph codes can accomplish excellent error performance with linear encoding complexity as well as fast decoding [14]. Based on the above-mentioned advantages, the root-protograph (RP) codes have been proposed and optimized for block-fading (BF) channels [15] and they offer a new direction to implement CC in QSF relay channels.

In this work, we conduct an investigation on the protograph codes over half-duplex Nakagami QSF channels with CC protocol. We firstly analyze the outage probability which can be considered as the fundamental lower-limit of the word-error-rate (WER) of all error-correction codes (ECCs) for CC. Then, we propose the full-diversity rate-compatible root-protograph (RCRP) codes based on the structure of root-protograph (RP) codes. Also, the protograph EXIT (PEXIT) algorithm has been developed to predict the theoretical WER and bit-error-rate (BER) of the RCRP ensembles. The analytical and simulated results show that the proposed RCRP codes can provide excellent error performance with different code rates up to $1/2$. It should be noted that the PEXIT algorithm can facilitate the optimization of the RCRP codes for the slowly-varying fading relay channels.

II. System Model

Fig. 1 shows a half-duplex relay system with one source S, one relay R, and one destination D. In a QSF relay channel, each transmission period is divided into two time slots, i.e., the broadcast time slot and the cooperative time slot. Assume every K information bits of S are encoded into a protograph code C of length N, where the code rate $R_c = K/N$. In the CC framework, each codeword is then split into two successive sub-codewords (frames), i.e., C_1 of length N_1 and C_2 of length N_2, respectively, denoted by $C = (C_1, C_2)$. In the 1st time slot, S broadcasts the rate-R_1 ($R_1 > R_c$) frame C_1 to R and D. In the 2nd time slot, if R can decode C_1 successfully, R will

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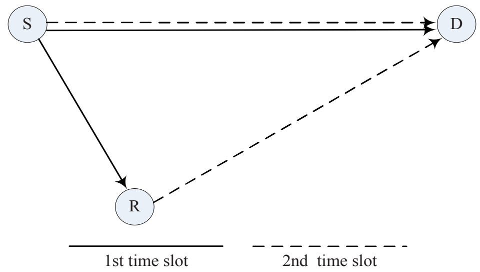


Fig. 1. The half-duplex relay system with CC protocol.

compute and transmit the other frame C_2 to D and at the same time S will remain idle; otherwise S transmits C_2 instead of R. In general, there are two different cooperative cases in the 2nd time slot for such a transmission technique. We respectively denote case 1 ($\Theta = 1$) as R decodes C_1 successfully and case 2 ($\Theta = 2$) as R fails to decode. The level of cooperation is defined as $\eta = N_2/N$. In this paper, η is set to $1/2$ (i.e., $N_1 = N_2 = N/2$) unless otherwise mentioned. For simplicity, we assume the BPSK modulation is adopted in our system. The received signal corresponding to j-th transmitted symbol $x_{\mathrm{A},j}$ of $\mathsf{A} \rightarrow \mathsf{B}$ link is given by

$$y_{\mathrm{B},j} = h_{\mathrm{AB}} x_{\mathrm{A},j} + n_{\mathrm{B},j} \tag{1}$$

where $\mathsf{A} \in \{\mathsf{S}, \mathsf{R}\}$ is the transmitter, $\mathsf{B} \in \{\mathsf{R}, \mathsf{D}\}$ is the corresponding receiver, $x_{\mathrm{A},j} \in \{+1, -1\}$ is the transmitted BPSK signal, and $n_{\mathrm{B},j} \sim \mathcal{N}(0, \sigma_n^2)$ is the Gaussian noise with zero mean and variance $\sigma_n^2 = N_0/2$. Also, $h_{\mathrm{AB}} = \alpha_{\mathrm{AB}}/d_{\mathrm{AB}}$ is the channel gain, where d_{AB} denotes the distance between A and B, and α_{AB} is the Nakagami fading parameter with $\Omega_{\mathrm{AB}} = \mathbb{E}(\alpha_{\mathrm{AB}}^2) = 1$. We assume that α_{AB} is constant for each transmission period and is an i.i.d. Nakagami random variable (RV) for different transmission periods. We also assume that the channels formed by different transmitter-receiver pairs are mutually independent and the channel state information (CSI) is available at each receiver. Additionally, the received signal-to-noise-ratio (SNR) per symbol of $\mathsf{A} \rightarrow \mathsf{B}$ link is defined as $\gamma_{\mathrm{AB}} = (E_s/N_0)h_{\mathrm{AB}}^2 = ((E_b/N_0)\alpha_{\mathrm{AB}}^2)/(R_c d_{\mathrm{AB}}^2)$, where E_s and E_b are the average energy per transmitted symbol and per bit, respectively. Accordingly, one can easily obtain that γ_{AB} follows a gamma distribution as

$$\begin{aligned}\gamma_{\mathrm{AB}} &\sim G\left(m_{\mathrm{AB}}, (E_s/N_0)\Omega_{\mathrm{AB}}/(m_{\mathrm{AB}} d_{\mathrm{AB}}^2)\right) \\ &= G\left(m_{\mathrm{AB}}, \bar{\gamma}_{\mathrm{AB}}/m_{\mathrm{AB}}\right)\end{aligned} \tag{2}$$

where $\bar{\gamma}_{\mathrm{AB}} = \mathbb{E}(\gamma_{\mathrm{AB}}) = ((E_s/N_0)\Omega_{\mathrm{AB}})/d_{\mathrm{AB}}^2$. The PDF of the well-known gamma-distributed RV $X \sim G(m, \bar{\gamma}/m)$ is $f_X(x) = [x^{m-1}\exp(-x/(\bar{\gamma}/m))]/[(\bar{\gamma}/m)^m \Gamma(m)]$, where $m \geq 1/2$ is the fading depth of the Nakagami fading channel [9].

Definition 1. In QSF relay channels, an error-correction code C is said to have full diversity if $d_c = N_u$, where d_c denotes the diversity order of C and N_u represents the total number of source and relays.

According to the Singleton bound [16], the code rate of full-diversity code should be satisfying $R_c \leq 1/d_c = 1/N_u$. In our system, $N_u = 2$ and we have $R_c \leq 1/2$. In other words, $R_{c,\max} = 1/2$ is the maximum code rate to achieve full diversity.

III. Outage Probability Analysis

The outage probability of CC protocol has been analyzed for the 2-user cooperative communication systems with different types of modulations [7], [12]. We now conduct the outage analysis for our system with BPSK modulation. Firstly, consider the scenario of non-cooperative direct transmission between a source and a destination. With quasi-static fading, the outage probability is defined as

$$P_{\mathrm{out}}(\gamma) = \Pr(I(\gamma) < R_c)$$

where $\gamma = (E_s/N_0)\alpha^2$ is the received SNR, R_c is the code rate, $I(\gamma) = -\int_{-\infty}^{+\infty} \varphi(\tau,\gamma)\log_2[\varphi(\tau,\gamma)]\mathrm{d}\tau - (1/2)\log_2[(\pi e)/\gamma]$ is the mutual information (MI) between the input and output of an AWGN channel, and $\varphi(\tau,\gamma) = (1/(2\sqrt{\pi/\gamma}))[\exp(-\gamma(\tau+1)^2) + \exp(-\gamma(\tau-1)^2)]$. Moreover, $I(\gamma)$ is a non-decreasing function of γ.

As discussed in Sect. II, there are totally two possible cases for the second-frame transmission of CC, dependent on whether the first-frame transmission is successfully or not. We can deduce the conditional MIs and the outage events for the two cases as follows.

Case 1 ($\boldsymbol{\Theta = 1}$): R correctly decodes C_1 in the 1st time slot, which corresponds to the event $I(\gamma_{\mathrm{SR}}) \geq R_1$. We subsequently get the convergence domain of such an event as $\gamma_{\mathrm{SR}} \geq \gamma_{\mathrm{out,SR,th}}$, where $I(\gamma_{\mathrm{out,SR,th}}) = R_1$. In the 2nd time slot, R transmits the second frame C_2 to D. Generally, D receives the two frames through two independent fading channels, which can be considered as two parallel channels. Therefore, the totally conditional MI equals to the summation of the two MIs using the fractions $1-\eta$ and η, respectively. The corresponding outage event and outage region for S are $I(\gamma_{\mathrm{SD}}, \gamma_{\mathrm{RD}}|\Theta = 1) = (1-\eta)I(\gamma_{\mathrm{SD}}) + \eta I(\gamma_{\mathrm{RD}}) < R_c$ and $\bar{Q}_{\mathrm{D1}} = \{(\gamma_{\mathrm{SD}}, \gamma_{\mathrm{RD}}) \in \mathbb{R}_+^2 \mid I(\gamma_{\mathrm{SD}}, \gamma_{\mathrm{RD}}|\Theta = 1) < R_c\}$, respectively.

Case 2 ($\boldsymbol{\Theta = 2}$): R fails to decode C_1 in the 1st time slot, which corresponds to the event $I(\gamma_{\mathrm{SR}}) < R_1$ and the outage domain $0 \leq \gamma_{\mathrm{SR}} < \gamma_{\mathrm{out,SR,th}}$. In this case, D receives both frames through the same QSF channel of $\mathsf{S} \rightarrow \mathsf{D}$ link. We then obtain the corresponding outage event and outage domain for S as $I(\gamma_{\mathrm{SD}}|\Theta = 2) = I(\gamma_{\mathrm{SD}}) < R_c$ and $0 \leq \gamma_{\mathrm{SD}} < \gamma_{\mathrm{out,SD,th}}$, respectively, where $I(\gamma_{\mathrm{out,SD,th}}) = R_c$.

Since γ_{SR}, γ_{RD}, and γ_{SD} are mutually independent, the overall outage probability is calculated as (3), which is given at the top of the following page. Here, $\Pr(\Theta = \mu)$ ($\mu = 1, 2$) is the probability of case μ in the information-theoretical sense. Combining (2) and (3) yields the outage probability.

IV. RCRP codes

A protograph is a Tanner graph with a relatively small number of nodes [14]. A protograph $\mathcal{G} = (\mathcal{V}, \mathcal{C}, \mathcal{E})$ consists of three sets $\mathcal{V}$, $\mathcal{C}$, and $\mathcal{E}$, which include N_{P} variable nodes (VNs), M_{P} check nodes (CNs), and the edges, respectively. Each edge $e_{i,j} \in \mathcal{E}$ connects a VN $v_j \in \mathcal{V}$ to a CN $c_i \in \mathcal{C}$. Parallel edges are allowed in a protograph. The derived graph corresponding to a protograph code can be obtained by performing a "copy-and-permute" operation on the protograph. A protograph can also be represented by a base matrix $\mathbf{B} = (b_{i,j})$ of size $M_{\mathrm{P}} \times N_{\mathrm{P}}$, where $b_{i,j}$ denotes the number of edges connecting v_j to c_i. We define P_j as the punctured label of v_j, where $P_j = 0$ if v_j is punctured and $P_j = 1$ otherwise.

A. RP Codes

In a BF channel with two independent fading gains (i.e., α_1 and α_2), a rate-1/2 full-diversity RP code C_{RP} can be constructed by a protograph which contains 2 different types of rootchecks. Rootcheck has been proposed in the root LDPC codes [13]. A type-l ($l = 1, 2$) rootcheck with degree ρ_i ($\rho_i \geq 3$) is defined as a degree-ρ_i check node with one edge connected to an information bit affected by α_l and the remaining $\rho_i - 1$ edges connected to other information/parity bits affected by α_l' ($l' \neq l$). The number of rootchecks in each type, i.e., M_{R}, should be identical so as to maintain the symmetric property of RP code. As a consequence, the generalized $M_{\mathrm{P}} \times N_{\mathrm{P}}$ base matrix

$$\begin{aligned}P_{\mathrm{out}} &= \Pr(\Theta=1)P_{\mathrm{out}}(\gamma_{\mathrm{SD}},\gamma_{\mathrm{RD}}|\Theta=1)+\Pr(\Theta=2)P_{\mathrm{out}}(\gamma_{\mathrm{SD}}|\Theta=2)\\ &= \Pr(I(\gamma_{\mathrm{SR}})\geq R_1)\Pr(I(\gamma_{\mathrm{SD}},\gamma_{\mathrm{RD}}|\Theta=1)<R_c)+\Pr(I(\gamma_{\mathrm{SR}})<R_1)\Pr(I(\gamma_{\mathrm{SD}}|\Theta=2)<R_c)\\ &= \int_{\gamma_{\mathrm{out,SR,th}}}^{+\infty} f(\gamma_{\mathrm{SR}})\mathrm{d}\gamma_{\mathrm{SR}}\iint_{\bar{Q}_{\mathrm{D1}}} f(\gamma_{\mathrm{SD}})f(\gamma_{\mathrm{RD}})\mathrm{d}\gamma_{\mathrm{SD}}\mathrm{d}\gamma_{\mathrm{RD}}+\int_0^{\gamma_{\mathrm{out,SR,th}}} f(\gamma_{\mathrm{SR}})\mathrm{d}\gamma_{\mathrm{SR}}\int_0^{\gamma_{\mathrm{out,SD,th}}} f(\gamma_{\mathrm{SD}})\mathrm{d}\gamma_{\mathrm{SD}}\end{aligned} \quad (3)$$

$$\mathbf{B}_{\mathrm{RP}} = \begin{array}{c} \begin{array}{cccc} \mathcal{V}_{\mathrm{i1}} & \mathcal{V}_{\mathrm{p1}} & \mathcal{V}_{\mathrm{i2}} & \mathcal{V}_{\mathrm{p2}} \end{array} \\ \begin{pmatrix} \mathbf{I} & \mathbf{0} & \mathbf{H}_{\mathrm{i2}} & \mathbf{H}_{\mathrm{p2}} \\ \mathbf{H}_{\mathrm{i1}} & \mathbf{H}_{\mathrm{p1}} & \mathbf{I} & \mathbf{0} \end{pmatrix} \end{array} \begin{array}{c} \\ \mathcal{C}_1 \\ \mathcal{C}_2 \end{array}$$

Fig. 2. Generalized base matrix of the rate-$1/2$ RP codes.

$$\mathrm{B} = \begin{array}{c} \begin{array}{ccccccc} \mathcal{V}_{\mathrm{i1}} & \mathcal{V}_{\mathrm{p1}} & \mathcal{V}_{\mathrm{p1'}} & & \mathcal{V}_{\mathrm{i2}} & \mathcal{V}_{\mathrm{p2}} & \mathcal{V}_{\mathrm{p2'}} \end{array} \\ \begin{pmatrix} \mathrm{I} & 0 & 0_2 & | & \mathrm{H}_{\mathrm{i2}} & \mathrm{H}_{\mathrm{p2}} & 0_2 \\ \mathrm{H}_{\mathrm{i1}} & \mathrm{H}_{\mathrm{p1}} & 0_2 & | & \mathrm{I} & 0 & 0_2 \\ & \mathrm{B}_{1\mathrm{S}} & & | & & 0_1 & \\ & 0_1 & & | & & \mathrm{B}_{1\mathrm{R}} & \end{pmatrix} \end{array} \begin{array}{c} \\ \mathcal{C}_1 \\ \mathcal{C}_2 \\ \mathcal{C}_3 \\ \mathcal{C}_4 \end{array}$$

Fig. 3. Generalized base matrix of the rate-R_c RCRP codes.

of the rate-$1/2$ RP codes is defined in Fig. 2. In this figure, $N_{\mathrm{P}} = 2M_{\mathrm{P}} = 4M_{\mathrm{R}}$. $\mathbf{I}$ and $\mathbf{0}$ are the identity matrix and zero matrix of size $M_{\mathrm{R}} \times M_{\mathrm{R}}$, respectively. $\mathbf{H}_{\mathrm{i}l}$ and $\mathbf{H}_{\mathrm{p}l}$ are the sub-matrices of size $M_{\mathrm{R}} \times M_{\mathrm{R}}$. As seen, the VNs are divided into four subsets: two subsets correspond to the information bits, i.e., $\mathcal{V}_{\mathrm{i1}}$ and $\mathcal{V}_{\mathrm{i2}}$, and the other two subsets correspond to the parity bits, i.e., $\mathcal{V}_{\mathrm{p1}}$ and $\mathcal{V}_{\mathrm{p2}}$. The VNs $v_j \in \mathcal{V}_{\mathrm{i}l} \bigcup \mathcal{V}_{\mathrm{p}l}$ are transmitted on α_l. The CNs are divided into two subsets: $\mathcal{C}_1$ and $\mathcal{C}_2$, where $\mathcal{C}_l$ denotes the subset of type-l rootchecks. The weight per-row in a combined sub-matrix $\mathbf{B}_{\mathrm{t}l} = [\mathbf{H}_{\mathrm{i}l}\ \mathbf{H}_{\mathrm{p}l}]$ is greater than or equal to 2. We assume $0 \leq b_{i,j} \leq 3$ to retain the low encoding complexity. The full-diversity RP code can be produced via the derived graph corresponding to $\mathbf{B}_{\mathrm{RP}}$.

B. RCRP Codes

Based on the RP codes, we propose the new full-diversity RCRP codes by extending the RP codes over QSF relay channels with CC protocol. According to [17], a rate-R_c ($R_c \leq 1/2$) RCRP codes can be constructed by adding the same number of VNs and CNs into the base matrix $\mathbf{B}_{\mathrm{RP}}$, i.e., by adding extra parity bits to the RP codes while keeping the information bits unchanged. Assuming that the VNs $v_j \in \mathcal{V}_{\mathrm{t}l}$ ($\mathcal{V}_{\mathrm{t}l} = \mathcal{V}_{\mathrm{i}l} \bigcup \mathcal{V}_{\mathrm{p}l}$) of a RP code are transmitted in the l-th time slot. To protect these VNs, we use two rate-R_1 protograph codes of length N_1, which corresponds to base matrices $\mathbf{B}_{1\mathrm{S}}$ and $\mathbf{B}_{1\mathrm{R}}$, to re-encode $\mathcal{V}_{\mathrm{t1}}$ and $\mathcal{V}_{\mathrm{t2}}$ and form $C_1 = (\mathcal{V}_{\mathrm{t1}}, \mathcal{V}_{\mathrm{p1'}})$ and $C_2 = (\mathcal{V}_{\mathrm{t2}}, \mathcal{V}_{\mathrm{p2'}})$, respectively. According to the structures of C_1 and C_2, $\mathcal{V}_{\mathrm{p1'}}$ and $\mathcal{V}_{\mathrm{p2'}}$ are indeed new parity bits determined only by $\mathbf{B}_{1\mathrm{S}}$ and $\mathbf{B}_{1\mathrm{R}}$, respectively. As discussed above, the generalized $M \times N$ base matrix of the overall rate-R_c RCRP codes can be obtained by combining $\mathbf{B}_{\mathrm{RP}}$, $\mathbf{B}_{1\mathrm{S}}$ and $\mathbf{B}_{1\mathrm{R}}$, as shown in Fig. 3.

Referring to this figure, $R_c = R_1/2$, $N_1 = N/2$, and $M = (1 - R_1/2)N$. $\mathbf{B}_{\mathrm{RP}}$ is the $(R_1N/2) \times R_1N$ (i.e., $M_{\mathrm{P}} = R_1N/2$, $N_{\mathrm{P}} = R_1N$) base matrix of C_{RP}, $\mathbf{B}_{1\mathrm{S}}$ and $\mathbf{B}_{1\mathrm{R}}$ are the $((1-R_1)N/2) \times N/2$ base matrices of sub-codewords C_1 and C_2, respectively. $\mathbf{0}_1$ and $\mathbf{0}_2$ are the $((1-R_1)N/2) \times N/2$ and $(R_1N/4) \times ((1-R_1)N/2)$ zero matrices, respectively. The two new subsets of CNs, i.e., $\mathcal{C}_3$ and $\mathcal{C}_4$, which associate to the two sub-codewords C_1 and C_2, respectively, are not rootchecks.

In the relay channel, the information bits $v_j \in \mathcal{V}_{\mathrm{i}}$ ($\mathcal{V}_{\mathrm{i}} = \mathcal{V}_{\mathrm{i1}} \bigcup \mathcal{V}_{\mathrm{i2}}$) are firstly encoded into $C_{\mathrm{RP}} = (\mathcal{V}_{\mathrm{i1}}, \mathcal{V}_{\mathrm{p1}}, \mathcal{V}_{\mathrm{i2}}, \mathcal{V}_{\mathrm{p2}})$ by $\mathbf{B}_{\mathrm{RP}}$. Afterwards, two sub-codewords, i.e., C_1 and C_2, of C can be constructed utilizing C_{RP}, $\mathbf{B}_{1\mathrm{S}}$ and $\mathbf{B}_{1\mathrm{R}}$. C_1 and C_2 will be transmitted by S in the 1st time slot and transmitted by R or S (determined by the decoding result) in the 2nd time slot, respectively. In the 1st time slot, R tries to decode C_1 using the parity-check matrix corresponding to $\mathbf{B}_{1\mathrm{S}}$ and retrieve $v_j \in \mathcal{V}_{\mathrm{t2}}$ using the parity-check matrix corresponding to $\mathbf{B}_{\mathrm{RP}}$, while D will exploit the overall parity-check matrix corresponding to $\mathbf{B}$ to decode the whole codeword C in the 2nd time slot. For ease of analysis, we assume that both C_1 and C_2 belong to the same regular column-weight-3 protograph ensemble such that $\mathbf{B}_{1\mathrm{S}} = \mathbf{B}_{1\mathrm{R}} = \mathbf{B}_1$.[1] In the RCRP codes, the information bits $v_j \in \mathcal{V}_{\mathrm{i}}$ and the check bits $v_j \in \mathcal{V}_{\mathrm{p}}$ ($\mathcal{V}_{\mathrm{p}} = \mathcal{V}_{\mathrm{p1}} \bigcup \mathcal{V}_{\mathrm{p1'}} \bigcup \mathcal{V}_{\mathrm{p2}} \bigcup \mathcal{V}_{\mathrm{p2'}}$) are clearly distinguished as in RP codes. It can be easily proven that the information bits of the RCRP codes exhibit full diversity under belief propagation (BP) decoding based on [13]. Consequently, we describe the RCRP code as a $\mathbf{B} = (\mathbf{B}_1, \mathbf{B}_{\mathrm{RP}})$ protograph code. The cooperative level is $1/2$ exploiting such a coding method. We now present two examples of the RCRP codes.

Example 1 -Rate-$1/3$ regular RCRP code. Consider a rate-$2/3$ regular column-weight-3 protograph code C_1 and a rate-$1/2$ regular RP code C_{RP}, we can get the base matrix $\mathbf{B} = (\mathbf{B}_1, \mathbf{B}_{\mathrm{RP}})$ of the rate-$1/3$ regular RCRP code C according to Fig. 3. The base matrices $\mathbf{B}_1, \mathbf{B}_{\mathrm{RP}}$, and $\mathbf{B}$ are given by

$$\mathbf{B}_1 = \begin{pmatrix} 3\,3\,3 \end{pmatrix}, \quad \mathbf{B}_{\mathrm{RP}} = \begin{pmatrix} 1\,0\,2\,3 \\ 2\,3\,1\,0 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 1\,0\,0\,2\,3\,0 \\ 2\,3\,0\,1\,0\,0 \\ 3\,3\,3\,0\,0\,0 \\ 0\,0\,0\,3\,3\,3 \end{pmatrix}. \quad (4)$$

Example 2 -Rate-$2/5$ regular RCRP code. In order to construct the rate-$2/5$ regular RCRP code C, we adopt a rate-$4/5$ regular column-weight-3 protograph code C_1 and an extended rate-$1/2$ regular RP code C_{RP}. The base matrices of the two constituent codes and the overall code are expressed as

$$\mathbf{B}_1 = \begin{pmatrix} 3\,3\,3\,3\,3 \end{pmatrix},$$
$$\mathbf{B}_{\mathrm{RP}} = \begin{pmatrix} 1\,0\,0\,0\,1\,1\,2\,1 \\ 0\,1\,0\,0\,1\,1\,1\,2 \\ 1\,1\,1\,2\,1\,0\,0\,0 \\ 1\,1\,2\,1\,0\,1\,0\,0 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 1\,0\,0\,0\,0\,1\,1\,2\,1\,0 \\ 0\,1\,0\,0\,0\,1\,1\,1\,2\,0 \\ 1\,1\,1\,2\,0\,1\,0\,0\,0\,0 \\ 1\,1\,2\,1\,0\,0\,1\,0\,0\,0 \\ 3\,3\,3\,3\,3\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,3\,3\,3\,3\,3 \end{pmatrix}. \quad (5)$$

In summary, given a fixed code rate $R_{\mathrm{p}} = 1/2$ of $\mathbf{B}_{\mathrm{RP}}$, the base matrix of a RCRP code with a code rate $R_c = R_1/2$ ranging from 0 to $1/2$ can be produced by varying the code rate R_1 of $\mathbf{B}_1$. Moreover, the irregular RCRP codes can also be generated if we use irregular RP codes.

V. PEXIT Algorithm of RCRP Codes

In this section, we extend the PEXIT algorithm [18]–[20] for the RCRP codes over QSF channels with minor modifications, taking

[1]This assumption will not affect any of the derivations in QSF relay systems.

into consideration the property of unequal error protection (UEP) of the RCRP codes. Based on such an algorithm, the theoretical WER and BER expressions are further derived.

A. Sub-codeword C_1 of $S \to R$ Link

To determine the convergence performance of protograph code C_1 of $\mathsf{S} \to \mathsf{R}$ link in the 1st time slot, we only need to modify the initialization step of PEXIT algorithm in [18] as follows.

Initialization: For $j = 1, 2, \ldots, N_1$, the modified variance of channel initial LLR corresponding to v_j is $\sigma^2_{\mathrm{SR,ch},j} = 8P_j\gamma_{\mathrm{SR}}$. Using this variance, the decoding thresholds for v_j and C_1 of $\mathsf{S} \to \mathsf{R}$ link, denoted as $\gamma_{\mathrm{SR,th},j}$ and $\gamma_{\mathrm{SR,th}}$, are formulated, respectively. Accordingly, in the threshold sense, the probabilities that case 1 and case 2 occurred are yielded as

$$P(\Theta = 1) = \int_{\gamma_{\mathrm{SR,th}}}^{\infty} f(\gamma_{\mathrm{SR}})\mathrm{d}\gamma_{\mathrm{SR}}, \tag{6a}$$

$$P(\Theta = 2) = \int_{0}^{\gamma_{\mathrm{SR,th}}} f(\gamma_{\mathrm{SR}})\mathrm{d}\gamma_{\mathrm{SR}}. \tag{6b}$$

Moreover, the conditional BER of v_j is $P_{\mathrm{SR,b},j}(e|\gamma_{\mathrm{SR}}) \approx (1/2)\mathrm{erfc}((J^{-1}(I_{\mathrm{SR,app}}(j)))/(2\sqrt{2}))$, where $I_{\mathrm{SR,app}}(j)$ is the a-posteriori MI of v_j [18], and the function $J^{-1}(\cdot)$ is defined as [19]

$$J^{-1}(x) = \begin{cases} \lambda_1 x^2 + \lambda_2 x + \lambda_3\sqrt{x} & \text{if } 0 \le x \le 0.3646, \\ \lambda_4 \ln[\lambda_5(1-x)] + \lambda_6 x & \text{otherwise,} \end{cases} \tag{7}$$

where $\lambda_1 = 1.09542, \lambda_2 = 0.214217, \lambda_3 = 2.33737, \lambda_4 = -0.706692, \lambda_5 = 0.386013$, and $\lambda_6 = 1.75017$.

Therefore, the average BER can be obtained by integrating the conditional BER over the fading distribution and averaging the corresponding result over the N_1 VNs, resulting in

$$P_{\mathrm{SR,b}}(e) = \frac{1}{N_1}\sum_{j=1}^{N_1}\int_{0}^{\gamma_{\mathrm{SR,th},j}} P_{\mathrm{SR,b},j}(e|\gamma_{\mathrm{SR}}) f(\gamma_{\mathrm{SR}})\,\mathrm{d}\gamma_{\mathrm{SR}}. \tag{8}$$

B. Overall Codeword C at D

For the overall codeword C with $\mathbf{B} = (\mathbf{B}_1, \mathbf{B}_{\mathrm{RP}})$ at D in the 2nd time slot, we should separately discuss the two cooperative cases. The major differences between the PEXIT algorithm here and that in [18] are the initialization step and the finalization step.

Case 1 $(\boldsymbol{\Theta = 1})$**:** D receives the two frames of the whole codeword C via two independent fading channels, i.e., C_1 via link $\mathsf{S} \to \mathsf{D}$ and C_2 via link $\mathsf{R} \to \mathsf{D}$. Equivalently, we can treat it as the scenario that C is transmitted over a point-to-point BF channel, with C_1 affected by α_{SD} and C_2 affected by α_{RD}. Based on such a transmission scheme, we re-describe the two steps as following.

Initialization: For $j = 1, 2, \ldots, N$, the variance of channel initial LLR corresponding to v_j becomes

$$\sigma^2_{\mathrm{D1,ch},j} = \begin{cases} 8P_j\gamma_{\mathrm{SD}} & j = 1, 2, \ldots, N/2, \\ 8P_j\gamma_{\mathrm{RD}} & \text{otherwise.} \end{cases}$$

Finalization: For $j = 1, 2, \ldots, N$, we measure the a-posteriori MI $I_{\mathrm{D1,app}}(j)$ of v_j. If $I_{\mathrm{D1,app}}(j) = 1$ for $j = 1, \ldots, R_1N/4; N/2 + 1, \ldots, (2 + R_1)N/4$ (i.e.,$v_j \in \mathcal{V}_{\mathrm{i}}$), the iterative decoder converges successfully and generates the SNR threshold pair $(\gamma_{\mathrm{SD,th}}, \gamma_{\mathrm{RD,th}})$.

We also obtain the outage region [20], [21] of v_j utilizing this algorithm as $\bar{D}_{\mathrm{D1},j} = \{(\gamma_{\mathrm{SD}}, \gamma_{\mathrm{RD}}) \in \mathbb{R}^2_+ \mid \exists\zeta > 0, I_{\mathrm{D1,app}}(j) < 1-\zeta\}$, in which v_j cannot converge successfully. Similarly, the outage region of a given RCRP ensemble is $\bar{D}_{\mathrm{D1}} = \{(\gamma_{\mathrm{SD}}, \gamma_{\mathrm{RD}}) \in \mathbb{R}^2_+ \mid \exists v_j \in \mathcal{V}_{\mathrm{i}} \;\&\; \exists\zeta > 0, I_{\mathrm{D1,app}}(j) < 1-\zeta\}$.

Thus, the theoretical WER of C is expressed by

$$P_{\mathrm{w}}(e|\Theta = 1) = \iint_{\bar{D}_{\mathrm{D1}}} f(\gamma_{\mathrm{SD}})f(\gamma_{\mathrm{RD}})\,\mathrm{d}\gamma_{\mathrm{SD}}\mathrm{d}\gamma_{\mathrm{RD}}, \tag{9}$$

and the corresponding conditional BER of v_j is $P_{\mathrm{D1,b},j}(e|\gamma_{\mathrm{SD}}, \gamma_{\mathrm{RD}}) \approx (1/2)\mathrm{erfc}((J^{-1}(I_{\mathrm{D1,app}}(j)))/(2\sqrt{2}))$, where $I_{\mathrm{D1,app}}(j)$ is the a-posteriori MI of v_j for case 1. Finally, we calculate the average BER of C as

$$P_{\mathrm{D1,b}}(e) = \frac{1}{R_cN}\sum_{v_j\in\mathcal{V}_{\mathrm{i}}}\iint_{\bar{D}_{\mathrm{D1},j}} P_{\mathrm{D1,b},j}(e|\gamma_{\mathrm{SD}}, \gamma_{\mathrm{RD}}) \times f(\gamma_{\mathrm{SD}})f(\gamma_{\mathrm{RD}})\,\mathrm{d}\gamma_{\mathrm{SD}}\mathrm{d}\gamma_{\mathrm{RD}}. \tag{10}$$

Case 2 $(\boldsymbol{\Theta = 2})$**:** D receives both the two frames of C through the same QSF channel of $\mathsf{S} \to \mathsf{D}$ link. In this case, the variance of channel initial LLR corresponding to v_j is given by $\sigma^2_{\mathrm{D2,ch},j} = 8P_j\gamma_{\mathrm{SD}}$. In the finalization step, we get the decoding threshold $\gamma_{\mathrm{SD,th}}$ if the a-posteriori MI values $I_{\mathrm{D2,app}}(j) = 1$ for all $v_j \in \mathcal{V}_{\mathrm{i}}$. Given a fixed VN v_j, We also get the corresponding threshold $\gamma_{\mathrm{SD,th},j}$.

Likewise, the WER and average BER of C are formulated as

$$P_{\mathrm{w}}(e|\Theta = 2) = \int_{0}^{\gamma_{\mathrm{SD,th}}} f(\gamma_{\mathrm{SD}})\mathrm{d}\gamma_{\mathrm{SD}}, \tag{11}$$

$$P_{\mathrm{D2,b}}(e) = \frac{1}{R_cN}\sum_{v_j\in\mathcal{V}_{\mathrm{i}}}\int_{0}^{\gamma_{\mathrm{SD,th},j}} P_{\mathrm{D2,b},j}(e|\gamma_{\mathrm{SD}})f(\gamma_{\mathrm{SD}})\mathrm{d}\gamma_{\mathrm{SD}} \tag{12}$$

where $P_{\mathrm{D2,b},j}(e|\gamma_{\mathrm{SD}}) \approx (1/2)\mathrm{erfc}((J^{-1}(I_{\mathrm{D2,app}}(j)))/(2\sqrt{2}))$ is the conditional BER of v_j and $I_{\mathrm{D2,app}}(j)$ is the a-posteriori MI of v_j for case 2.

Combining (6a), (6b), (9), and (11) yields the total WER of a RCRP ensemble in the relay channel

$$P_{\mathrm{w}}(e) = P(\Theta = 1)P_{\mathrm{w}}(e|\Theta = 1) + P(\Theta = 2)P_{\mathrm{w}}(e|\Theta = 2). \tag{13}$$

Moreover, we obtain the total BER of a RCRP ensemble using (8), (10), and (12), as

$$P_{\mathrm{b}}(e) = (1 - P_{\mathrm{SR,b}}(e))P_{\mathrm{D1,b}}(e) + P_{\mathrm{SR,b}}(e)P_{\mathrm{D2,b}}(e). \tag{14}$$

Remark: Only the a-posteriori MIs of information bits $v_j \in \mathcal{V}_i$ are calculated to determine the convergence performance of a RCRP ensemble because of the UEP property.

VI. Numerical Results

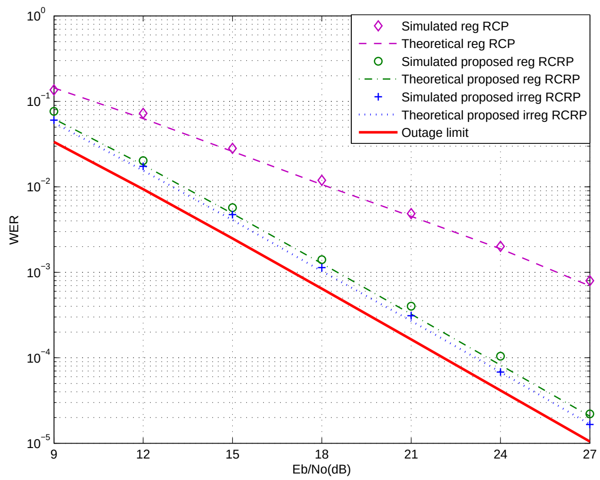
In the following, both the asymptotic and simulated error rates of the RCRP codes are presented so as to verify (13) and (14). We consider two different transmission scenarios: (1) $d_{\mathrm{SR}} : d_{\mathrm{RD}} : d_{\mathrm{SD}} = 0.8 : 1 : 1$ and (2) $d_{\mathrm{SR}} : d_{\mathrm{RD}} : d_{\mathrm{SD}} = 0.4 : 0.6 : 1$ (i.e., R lies on the straight line joining S and D). In scenario 1, we test the rate-$1/3$ regular RCRP code (4). For comparison, we also simulate the rate-$1/3$ irregular RCRP code with $\mathbf{B}_{\mathrm{RP-irreg}}$ and the conventional regular RCP code with $\mathbf{B}_{\mathrm{RP-conv}}$, expressed as

$$\mathbf{B}_{\mathrm{RP-irreg}} = \begin{pmatrix} 1\,0\,2\,2 \\ 3\,3\,1\,0 \end{pmatrix}, \quad \mathbf{B}_{\mathrm{RP-rcp}} = \begin{pmatrix} 1\,1\,2\,2 \\ 2\,2\,1\,1 \end{pmatrix}.$$

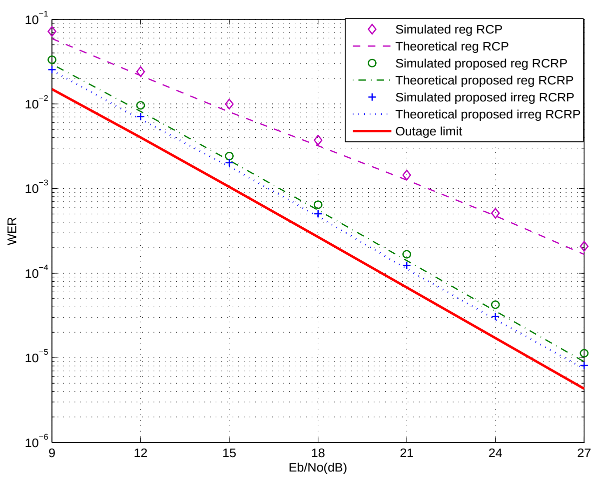
In scenario 2, we test rate-$2/5$ regular RCRP code (5), the irregular RCRP code which is based on a irregular RP code, and the conventional regular RCP code, where

$$\mathbf{B}_{\mathrm{RP-irreg}} = \begin{pmatrix} 1\,0\,0\,0\,2\,1\,2\,0 \\ 0\,1\,0\,0\,3\,0\,3\,1 \\ 1\,0\,1\,2\,1\,0\,0\,0 \\ 0\,2\,2\,1\,0\,1\,0\,0 \end{pmatrix}, \quad \mathbf{B}_{\mathrm{RP-rcp}} = \begin{pmatrix} 1\,1\,0\,0\,1\,1\,1\,1 \\ 1\,1\,1\,1\,0\,0\,1\,1 \\ 1\,1\,1\,1\,1\,1\,0\,0 \\ 0\,0\,1\,1\,1\,1\,1\,1 \end{pmatrix}.$$

Remark: For a given code rate R_c, all the RCRP and RCP codes share the same $\mathbf{B}_1$. Besides the existing protograph LDPC codes, we consider the regular root-LDPC codes, which possess the full-diversity and show excellent error performance in QSF relay channels [12].



(a)



(b)

Fig. 4. WER results of the regular RCRP code, irregular RCRP code, and regular RCP code with code rates (a) $R_c = 1/3$ (scenario 1) and (b) $R_c = 2/5$ (scenario 2) in a Nakagami QSF relay channel with CC protocol.

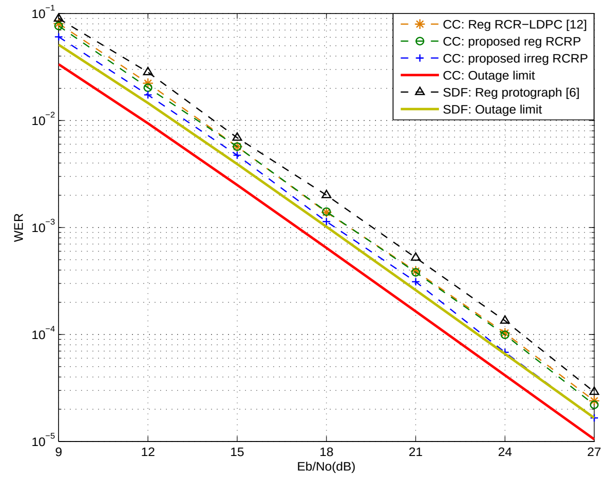


Fig. 5. WER results of the rate-$1/3$ (scenario 1) regular RCRP code, irregular RCRP code, and root-LDPC code in a Nakagami QSF relay channel. The relaying protocols used are CC and SDF.

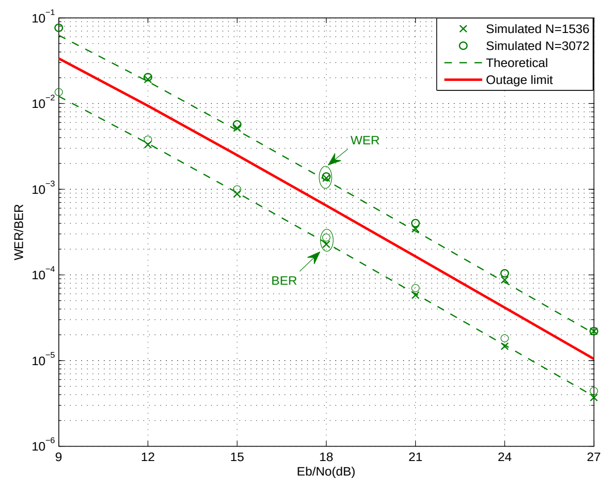


Fig. 6. BER and WER results of the rate-$1/3$ (scenario 1) regular RCRP codes with different codeword lengths in a Nakagami QSF relay channel with CC protocol.

Unless otherwise stated, we assume that the information length is $K = 1024$ in both scenarios. Hence, the codeword lengths are $N = 3072$ and $N = 2560$ for $R_c = 1/3$ and $R_c = 2/5$, respectively. The channel being considered is the Nakagami QSF relay channel with fading depth $m = 1$.

Fig. 4 plots the WER results of the proposed RCRP codes and the regular RCP code with code rates $R_c = 1/3$ (scenario 1) and $R_c = 2/5$ (scenario 2). Referring to Fig. 4(a), the simulated WER curves are in good agreement with the theoretical ones for all the codes. The irregular RCRP code performs slightly better than the regular RCRP code, which is remarkably superior to the conventional regular RCP code. Moreover, both the irregular and regular RCRP codes exhibit the outage-limit-approaching error performance, showing gaps within 1.5 dB to the outage limit. Similar observations are also obtained from the results for $R_c = 2/5$ (scenario 2) in Fig. 4(b).

Fig. 5 depicts the simulated WER results of the rate-$1/3$ (scenario 1) proposed RCRP codes and regular RCR-LDPC code. We observe that the WER performance of the proposed regular RCRP code, which possesses lower encoding complexity, is comparable to that of regular root-LDPC code. In the same figure, we also present the outage limit and the WER for SDF protocol [6], and observe that the corresponding outage limit and WER are inferior to those for CC protocol. The outage probability, theoretical WER, and BER expressions for SDF with BPSK modulation are briefly derived in Appendix.

Fig. 6 illustrates the BER and WER curves of the rate-$1/3$ regular RCRP code (scenario 1) with different codeword lengths. As seen from this figure, the error performance of the code is insensitive to the codeword length, which is consistent with other near-outage-limit ECCs [12], [13], [21]. We have also performed the simulations for the irregular RCRP codes and have obtained similar observations.

VII. Conculsions

The performance of the protograph codes in QSF relay channels with CC protocol has been studied in this paper. Based on the structure of RP codes, we have proposed the new RCRP codes which can realize full diversity and rate compatibility. The outage probability and the PEXIT algorithm have been developed in such channels so as to characterize the fundamental lower-limit and the asymptotic error performance of RCRP codes. Simulations have been performed to verify the tightness of the asymptotic performance and the superiority of the RCRP codes with arbitrary code rates upper bounded by $1/2$. Furthermore, we have shown that CC performs better than SDF in our system. It should be noted that the RCRP

$$
\begin{aligned}
P_{\mathrm{DF,out}} &= \Pr(I(\gamma_{\mathrm{SR}}) \geq 2R_c) \Pr(I(\gamma_{\mathrm{SD}}+\gamma_{\mathrm{RD}}) < 2R_c) + \Pr(I(\gamma_{\mathrm{SR}}) < 2R_c) \Pr(I(2\gamma_{\mathrm{SD}}) < 2R_c) \\
&= \int_{\gamma_{\mathrm{out,SR,th}}}^{+\infty} f(\gamma_{\mathrm{SR}}) \mathrm{d}\gamma_{\mathrm{SR}} \iint_{\bar{Q}_{\mathrm{DF,D1}}} f(\gamma_{\mathrm{SD}}) f(\gamma_{\mathrm{RD}}) \mathrm{d}\gamma_{\mathrm{SD}} \mathrm{d}\gamma_{\mathrm{RD}} + \int_0^{\gamma_{\mathrm{out,SR,th}}} f(\gamma_{\mathrm{SR}}) \mathrm{d}\gamma_{\mathrm{SR}} \int_0^{\gamma_{\mathrm{out,DF,SD,th}}} f(\gamma_{\mathrm{SD}}) \mathrm{d}\gamma_{\mathrm{SD}}. \quad (15)
\end{aligned}
$$

codes not only can be directly utilized to accomplish outage-limit-approaching error performance in the 2-user cooperation scenario, but also can be exploited to achieve full diversity in the multi-relay scenario with the use of maximal-ratio combining (MRC) [9]. Thanks to the outstanding performance and low complexity, the RCRP codes are very attractive for the slowly-varying fading wireless cooperative communication applications.

APPENDIX
OUTAGE PROBABILITY AND PEXIT ALGORITHM FOR SDF

According to [7], CC is integrated with channel coding inherently. For a fair comparison, a regular protograph code C_1 with rate $R_c' = 2R_c$ is employed for SDF, resulting in an overall rate R_c due to the fact that the repetition is made in the 2nd time slot. The only difference between SDF and CC is that C_1 (first frame) is repeated by R or S in the 2nd time slot. Consequently, there are the same two cooperative cases determined by the first-frame transmission. Since all the probabilities and events of $\mathsf{S} \to \mathsf{R}$ link in the 1st time slot remain the same as those for CC, we only derive the corresponding formulas at D in the 2nd time slot. We assume that MRC is utilized at D.

1) Outage Probability: The conditional MIs and outage events for S at D for case 1 and case 2 become $I(\gamma_{\mathrm{SD}}, \gamma_{\mathrm{RD}}|\Theta = 1) = I(\gamma_{\mathrm{SD}} + \gamma_{\mathrm{RD}}) < 2R_c$ and $I(\gamma_{\mathrm{SD}}|\Theta = 2) = I(2\gamma_{\mathrm{SD}}) < 2R_c$, respectively. Therefore, the corresponding outage region and domain are $\bar{Q}_{\mathrm{DF,D1}} = \{(\gamma_{\mathrm{SD}}, \gamma_{\mathrm{RD}}) \in \mathbb{R}_+^2 \mid I(\gamma_{\mathrm{SD}} + \gamma_{\mathrm{RD}}) < 2R_c\}$ and $0 \leq \gamma_{\mathrm{SD}} < \gamma_{\mathrm{out,DF,SD,th}}$, respectively, where $I(2\gamma_{\mathrm{out,DF,SD,th}}) = 2R_c$. Based on the above-mentioned discussion, the outage probability for SDF protocol is calculated as (15), which is shown at the top of this page.

2) PEXIT algorithm: For SDF, the PEXIT algorithm for codeword C_1 at D is described as follows.

Case 1 ($\boldsymbol{\Theta = 1}$): Since D receives the same codeword C_1 twice through $\mathsf{S} \to \mathsf{D}$ link and $\mathsf{R} \to \mathsf{D}$ link, the received SNR at D is $\gamma_{\mathrm{DF,D1}} = \gamma_{\mathrm{SD}} + \gamma_{\mathrm{RD}}$. Then, we have $\sigma^2_{\mathrm{DF,D1,ch},j} = 8P_j\gamma_{\mathrm{DF,D1}}$. Exploiting the variance to the PEXIT algorithm, we can attain the corresponding outage regions of v_j and C_1 as $\bar{D}_{\mathrm{DF,D1},j} = \{(\gamma_{\mathrm{SD}}, \gamma_{\mathrm{RD}}) \in \mathbb{R}_+^2 \mid \exists\zeta > 0, I_{\mathrm{DF,D1,app}}(j) < 1-\zeta\}$ and $\bar{D}_{\mathrm{DF,D1}} = \{(\gamma_{\mathrm{SD}}, \gamma_{\mathrm{RD}}) \in \mathbb{R}_+^2 \mid \exists v_j \in \mathcal{V} \ \&\ \exists\zeta > 0, I_{\mathrm{DF,D1,app}}(j) < 1-\zeta\}$, respectively.

Case 2 ($\boldsymbol{\Theta = 2}$): D receives two copies of codeword C_1 through the same QSF channel of $\mathsf{S} \to \mathsf{D}$ link, hence $\gamma_{\mathrm{DF,D2}} = 2\gamma_{\mathrm{SD}}$ and $\sigma^2_{\mathrm{DF,D2,ch},j} = 8P_j\gamma_{\mathrm{DF,D2}} = 16P_j\gamma_{\mathrm{SD}}$. Afterwards, one can perform the PEXIT algorithm to obtain the decoding threshold of v_j and C_1 as $\gamma_{\mathrm{DF,SD,th},j}$ and $\gamma_{\mathrm{DF,SD,th}}$, respectively.

Thus, the corresponding WERs and average BERs for the two cases can be respectively measured in a similar way to those of CC in Sect. V.

Remark: One should investigate the a-posteriori MIs of all the VNs to determine outage region or threshold of C_1 for SDF because the information bits and check bits can not be distinguished here.

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