Hybrid Centralized-Distributed Resource Allocation for Device-to-Device Communication Underlaying Cellular Networks

Setareh Maghsudi and Sławomir Stańczak, Senior Member, IEEE

Abstract

The basic idea of device-to-device (D2D) communication is that pairs of suitably selected wireless devices reuse the cellular spectrum to establish direct communication links, provided that the adverse effects of D2D communication on cellular users is minimized and cellular users are given a higher priority in using limited wireless resources. Despite its great potential in terms of coverage and capacity performance, implementing this new concept poses some challenges, in particular with respect to radio resource management. The main challenges arise from a strong need for distributed D2D solutions that operate in the absence of precise channel and network knowledge. In order to address this challenge, this paper studies a resource allocation problem in a single-cell wireless network with multiple D2D users sharing the available radio frequency channels with cellular users. We consider a realistic scenario where the base station (BS) is provided with strictly limited channel knowledge while D2D and cellular users have no information. We prove a lower-bound for the cellular aggregate utility in the downlink with fixed BS power, which allows for decoupling the channel allocation and D2D power control problems. An efficient graph-theoretical approach is proposed to perform the channel allocation, which offers flexibility with respect to allocation criterion (aggregate utility maximization, fairness, quality of service guarantee). We model the power control problem as a multi-agent learning game. We show that the game is an exact potential game with noisy rewards, defined on a discrete strategy set, and characterize the set of Nash equilibria. Q-learning better-reply dynamics is then used to achieve equilibrium.

Index Terms

A short version of this paper is accepted to appear at the IEEE International Conference on Communications, 2014. The work was supported by the German Research Foundation (DFG) under grant STA 864/3-3. The authors are with the Communications and Information Theory Group, Technical University of Berlin. S. Stańczak is also with the Fraunhofer Institute for Telecommunications Heinrich Hertz Institute, Berlin, Germany (e-mail: setareh.maghsudi@tu-berlin.de, slawomir.stanczak@hhi.fraunhofer.de).

Channel allocation, game theory, graph theory, power control, Q-learning, underlay device-to-device communication.

I. INTRODUCTION

A. Related Works

Device-to-device (D2D) communication as an underlay to cellular networks is regarded as one of the key technologies for enhancing the performance of future cellular networks [1]. The basic idea is to reuse cellular spectrum resources by allowing nearby wireless devices to establish direct communication links. This concept not only improves the efficiency of spectrum usage [2], but also has a great potential for enhancing the network performance expressed in terms of capacity, coverage, energy efficiency and end-to-end delays [3]. In order to realize networkedcontrolled D2D communication as an underlay to cellular networks, a system designer faces some challenges, which mainly arise due to the lack of reliable channel state information (CSI) at base stations (BS). In particular, efficient feedback is the key to obtaining CSI; nonetheless, while CSI for cellular users¹ can be efficiently acquired at a serving BS, such information is in general not available for D2D channels. The reason is the separation of the user/data plane from the control plane in the case of network-controlled D2D communication. An immediate consequence of this separation is that, in contrast to cellular users, D2D users cannot directly utilize pilot signals broadcasted by BSs for estimation of D2D channels. In addition, local transmissions of distinct pilot signals by each D2D user are infeasible and would not solve the problem due to pilot contamination.² Since strategies for suppressing pilot contamination in D2D scenarios suffer from the need for increased feedback and control overhead, it is reasonable to assume that allocation of resources to D2D users has to be performed in a distributed manner under strictly limited CSI. Moreover, it is of utmost importance that direct transmissions among devices are coordinated to ensure that they do not have a detrimental impact on the performance of cellular users. Such coordination must involve a careful power-controlled allocation of D2D users to available radio frequency channels, primarily used by a BS (downlink frequencies)

¹In this paper, D2D user/link is used to refer to any pair of wireless devices that communicate directly, while any wireless device that operates in the traditional cellular mode is called a cellular user.

²Pilot contamination refers to a situation, in which the use of a large number of pilot signals leads to a relatively strong interference that may deteriorate the quality of channel estimation.

and/or cellular users (uplink frequencies). This problem, which is difficult to solve even in a centralized manner, is further aggravated in D2D setting by the need for distributed solutions.

To date, numerous resource allocation schemes are developed for underlay D2D communication systems. Many of them, however, are only applicable to networks with limited number of D2D and/or cellular users. For instance, Reference [4] studies the optimal channel allocation and power control where one cellular and two D2D users share wireless resources. Similarly, in [5], the system model includes one cellular and two D2D users, and a game-theoretical approach (reverse auction) is proposed to solve the resource sharing problem. References [6] and [7] study a system with multiple D2D users; however, in every time slot, only one D2D user is allowed to transmit in a channel that is primarily allocated to a cellular user. Similar examples include [8], [9] and [10], among many others.

Moreover, many works propose centralized resource allocation schemes for hybrid D2D and cellular communication. The schemes are mainly developed under the assumption that a central controller has access to the global channel and network knowledge, and therefore is capable of making coordination and resource allocation decisions not only for cellular users, but also for D2D users. For instance, Reference [11], formulates the joint channel allocation and power control problem as a mixed integer programming, which is solved using column generation method. Similarly, in [12], an energy-efficient uplink resource allocation scheme is proposed and analyzed by using mixed integer programming. The authors of [13] assume that a BS is able to perfectly coordinate the interference among cellular and D2D users. As another example, Reference [14] formulates a joint density and power allocation problem as a non-convex optimization problem using stochastic geometry, and proposes an algorithm to solve the problem. A joint resource allocation and mode selection mechanism based on particle swarm optimization is developed in [15]. See also [16], [17] and [18] for further examples.

In addition, in many research studies, some prior knowledge (such as information about utility functions) is assumed to be known to D2D users. In most cases, the problem is then solved using game-theoretical approaches such as pricing [19], [20], auctions [21] or coalition formation [22], [23], [24], [25]. Moreover, Reference [26] proposes a resource allocation mechanism based on contract design. Besides requiring prior knowledge at the node level, most game-theoretical solutions impose large overhead due to the need for heavy information exchange in terms of bids, or prices and demands.

B. Our Contribution

The system model considered in this paper generalizes existing works in the following important directions:

- There is *no limit* on the number of cellular and D2D users that coexist in the network.
- *Multiple* D2D users might be allowed to share a given channel with a cellular user.
- The BS is only aware of *statistical* channel knowledge of *cellular users* and *geographical locations* of D2D users. This information can be simply acquired by using pilot signals for cellular users and GPS (Global Positioning System) data of D2D users. This means that implementing D2D transmissions do not impose any overhead.
- D2D and cellular users do not have any channel knowledge.

We first prove a lower-bound on the aggregate utility of cellular users. Based on this lowerbound, while taking the higher priority of cellular users into account, we decompose the resource allocation problem into two cascaded problems related to channel allocation and D2D power control. The former problem, which deals with maximizing the utility sum of cellular users, is a multi-objective combinatorial optimization problem that is very costly to solve with respect to the time and computational complexity. Therefore we propose a suboptimal, but efficient, graph-theoretical heuristic solution that involves maximum-weighted bipartite matching [27], [28] and minimum-weighted graph partitioning [29], [30]. The problem can be then solved in a centralized manner by the BS, since the solution relies only on strictly limited information. The approach also offers high flexibility in terms of performance criteria, since quality of service or fairness can be also taken into account. The latter problem, in turn, deals with maximizing the aggregate utility of D2D users by means of power control, desirably in a distributed manner. We model the power control problem as a game with incomplete information, which, in contrast to most previous studies, is defined on a discrete strategy set. We show that this game is an exact potential game [31] and characterize the set of Nash equilibria. Furthermore, we use Q-learning better-reply dynamics [32] in order to converge to Nash equilibrium. Finally, extensive numerical analysis is performed to evaluate the performance of the proposed approach in practical cases.

C. Organization

The paper is organized as follows. In Section II, we introduce the network model and formulate the resource allocation problem. Section III is devoted to the first stage of the formulated problem, i.e., centralized channel allocation. Section IV deals with the second stage of the problem, i.e., distributed power control. Section V presents numerical evaluations, while Section VI completes the paper.

D. Notation

Throughout the paper we denote a set and its cardinality by a unique letter, and distinguish them by using calligraphic and italic fonts, such as \mathcal{A} and \mathcal{A} , respectively. Matrices are shown by bold upper case letters, for instance **A**. Moreover, \mathbf{A}_l denotes the *l*-th column of matrix **A**. Vectors are shown by bold lower case letters, for example **a**.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. System Model

1) Network Model: We consider the downlink of a single-cell network with one BS denoted by b, and a set \mathcal{L} consisting of L single-antenna cellular users, each denoted by l. The cell is provided with a set Q of Q = L orthogonal frequency channels that are referred to by q. Throughout the paper, by the term D2D user we refer to a pre-defined pair of one singleantenna transmitter and one single-antenna receiver, which is represented either by k or by the pair (k, k'). Note that a single device can be either transmitter or receiver. We use \mathcal{K} to denote the set of K D2D users. The BS is able to communicate with multiple cellular users simultaneously, possibly by means of multiple antennas. The data stream intended to any given cellular user is transmitted with fixed average power p_c . Each D2D user selects a power level from the set $\mathcal{M} = \left\{ p_d^{(1)}, p_d^{(2)}, ..., p_d^{(M)} \right\}, \text{ where } 1 < p_d^{(1)} < p_d^{(2)} < \dots < p_d^{(M)}. \text{ We assume that } p_d^{(M)} \ll p_c,$ since in general the BS has access to larger energy resources in comparison with user devices. Each *downlink* frequency channel q is used i) by the BS in order to transmit to some set $\mathcal{L}_q \subseteq \mathcal{L}$ of L_q cellular users, and ii) by a set $\mathcal{K}_q \subseteq \mathcal{K}$ of K_q D2D users for direct communication. We assume that $L_q = 1 \forall q \in Q$; that is, each channel is assigned to exactly one cellular user and therefore no vacant channel exists. This assumption is made in order to protect cellular users from an excessive interference due to a high BS power. We use $\mathbf{p}_{d,q} = (p_1, ..., p_{K_q})$ to denote the vector of transmit powers of the D2D users that transmit through channel q. Throughout the paper, $h_{uv,q} > 0$ is the average gain of channel q from transmitter u to receiver v. We assume that $h_{uv,q} = f_{uv,q}g_{uv}$, where $0 < f_{uv,q} \le 1$ and $0 < g_{uv} \le 1$ stand for fast fading and

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path loss, respectively. We assume that the channel gains of any given link are drawn from a stationary distribution. Moreover, due to channel reciprocity, we have $h_{uv,q} = h_{vu,q}$. Signalto-interference ratio (SIR) is denoted by γ . We consider a high SIR regime where $1 < \gamma$, so that $\log(1 + \gamma) \approx \log(\gamma)$. When treating interference as noise, $\log(\gamma)$ represents the achievable transmission rate of interference-limited point to point transmission.

2) Utility Model: The utility of cellular user $l \in \mathcal{L}_q$ that occupies channel q is defined as³

$$R_l(q, \mathbf{p}_{d,q}) = \log\left(\frac{p_c h_{bl,q}}{1 + \sum_{k \in \mathcal{K}_q} p_k h_{kl,q}}\right),\tag{1}$$

which corresponds to the achievable transmission rate, as described before. Note that this utility model is widely used in litrature; see for example [33].

Since D2D users are subject to power control in addition to channel allocation, the utility of any D2D user $k \in \mathcal{K}_q$ is defined to be

$$R_k(q, \mathbf{p}_{d,q}) = \log\left(\frac{p_k h_{kk',q}}{1 + \sum_{j \in \mathcal{K}_q, j \neq k} p_j h_{jk',q} + p_c h_{bk',q}}\right) - cp_k,\tag{2}$$

where c is a fixed price factor to penalize excessive power usage [34]. By definition, the utility of a D2D user corresponds to its transmission rate (see above) minus a cost that is paid to the cellular user in order to reimburse the adverse effects of spectrum sharing. The price factor can be either equal for all D2D users (as in (2)) or selected proportional to the channel gain (or distance) between a D2D user and the cellular user transmitting in the same channel [35]. Our analysis holds for both cases.

3) Information Model: We consider a model with strictly limited information, as described in the following assumption.

Assumption A1. Each of the following is assumed throughout the paper.

- a) The BS has knowledge of i) geographical locations of cellular and D2D users and the path loss exponent, thereby $g_{lk} \forall l \in \mathcal{L}, k \in \mathcal{K}$, and ii) the average fading gain of cellular to BS links, i.e., $h_{bl,q} \forall l \in \mathcal{L}, q \in Q$.
- b) The BS has no information about the fast fading component of cellular to cellular or D2D to D2D links.

³Throughout the paper, all logarithms are natural.

B. Problem Formulation

Network aggregate utility is conventionally regarded as a measure for evaluating the performance of resource management protocols [36], [37], [38], [39]. Based on this criterion, the problem is to allocate channels and power levels to cellular and D2D users so as to maximize the network aggregate utility. With (1) and (2) in hand, this problem can be stated formally as

$$\underset{\mathcal{L}_{q},\mathcal{K}_{q},\mathbf{p}_{d,q}}{\text{maximize}} \quad \sum_{q=1}^{Q} \left(\sum_{l \in \mathcal{L}_{q}} R_{l}\left(q,\mathbf{p}_{d,q}\right) + \sum_{k \in \mathcal{K}_{q}} R_{k}\left(q,\mathbf{p}_{d,q}\right) \right),$$
(3)

where $\mathcal{L}_q \subseteq \mathcal{L}$, $\mathcal{K}_q \subseteq \mathcal{K}$, $\mathbf{p}_{d,q} \in \bigotimes_{k=1}^{K_q} \left\{ p_d^{(1)}, \dots, p_d^{(M)} \right\}$ and \bigotimes denotes the Cartesian product. Note that unlike some previous works such as [40] and [41], the utility functions defined here are *user-specific*, i.e., the reward of any given channel differs to different users. As a result, the *set* of D2D and cellular users allocated to each channel is required to be determined, and not just the *number* of users.

Such formulation however does not comply with the underlay D2D concept, and suffers from the following drawbacks that make it difficult or even impossible to deal with: i) The objective function in (3) is not available at the BS due to the lack of information (see Assumption A1), ii) The higher priority of cellular users is not taken into account, and iii) The objective function depends on both channel and power allocations that are mutually dependent. Therefore a solution to (3) is difficult to obtain and is expected to be not amenable to distributed implementation. Our goal is therefore to develop a sophisticated heuristic approach. To this end, we first prove a lower-bound on the aggregate utility of cellular users that enables us to decouple the channel allocation and power control problems.

Proposition 1. For any $p_{d,q}$, p_c and channel gains, we have

$$\sum_{q=1}^{Q} \sum_{l \in \mathcal{L}_{q}} R_{l}(q, \mathbf{p}_{d,q}) > \sum_{q=1}^{Q} \sum_{l \in \mathcal{L}_{q}} \log(p_{c}h_{bl,q}) - \sum_{q=1}^{Q} \sum_{l \in \mathcal{L}_{q}} \sum_{k \in \mathcal{K}_{q}} p_{d}^{(M)} g_{kl}.$$
 (4)

Proof: See Appendix VII-A.

In words, the lower-bound in (4) corresponds to the worst-case scenario, in which all D2D users transmit at the maximum available power and the fast fading component of all D2D to

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cellular links equals one, thereby causing the maximum interference. Thus, for any realization of channel gains, the accuracy of the bound depends strongly on the range of the set of power levels \mathcal{M} , i.e., $p_d^{(M)} - p_d^{(1)}$. Apart from this, as the bound does not depend on D2D power allocation and relies on the available information at the BS, it can serve as a basis for resource management.

Since cellular users are assumed to have a higher priority and should be served first, we propose a two-step resource allocation strategy. In the first step, the objective is to maximize the lower-bound in (4) on the aggregate utility of cellular users. More precisely, given $p_d^{(M)}$, p_c and imperfect channel knowledge, we aim at assigning channels to cellular and D2D users so as

$$\underset{\mathcal{L}_{q},\mathcal{K}_{q}}{\text{maximize}} \quad \sum_{q=1}^{Q} \sum_{l \in \mathcal{L}_{q}} \log\left(p_{c}h_{bl,q}\right) - \sum_{q=1}^{Q} \sum_{l \in \mathcal{L}_{q}} \sum_{k \in \mathcal{K}_{q}} p_{d}^{(M)} g_{kl},$$
(5)

subject to

$$L_q = 1, \quad \forall \ q \in \mathcal{Q}. \tag{6}$$

This problem is investigated in Section III.

Once channels are allocated, in the second step we address the power control problem for D2D users, with the goal of maximizing the aggregate utility of D2D users as formalized below.

$$\underset{\mathbf{p}_{d,q}\in\bigotimes_{k=1}^{K_{q}}\left\{p_{d}^{(1)},\ldots,p_{d}^{(M)}\right\}}{\text{maximize}} \quad \sum_{q=1}^{Q}\sum_{k\in\mathcal{K}_{q}}R_{k}\left(q,\mathbf{p}_{d,q}\right).$$

$$\tag{7}$$

Section IV is devoted to this problem.

Summarizing, the resource allocation problem is decomposed into a channel allocation problem for all users followed by a power control problem for D2D users. As we see later, while the first problem is solved by the BS using a centralized method, the second problem is solved by D2D users in a distributed manner. Using such a two-stage scheme, not only a higher priority of cellular users is taken into account, but also D2D users utilize the assigned channels efficiently. Moreover, the limited available information is exploited with low computational effort.

III. CHANNEL ALLOCATION

This section deals with the first step of resource management, i.e., channel assignment with the goal of optimizing the performance of cellular users in terms of (5).

A. The Channel Allocation Scheme

We notice that the first and second terms in (5) are *proportional* to the sum of the desired signals and interferences over all cellular users, respectively. Moreover, while the first term depends only on cellular users, the second term depends on D2D users as well. Roughly speaking, the problem in (5) can be rephrased as maximize f(x) - g(x, y), where x and y respectively denote the cellular and D2D channel assignments. This problem is a multi-objective combinatorial optimization problem that is NP-hard and hence notoriously difficult to solve. Therefore we propose the following suboptimal, but simple and efficient, heuristic approach: At the beginning, we maximize the first term (weighted signal sum) so that the sets \mathcal{L}_q , $q \in \mathcal{Q}$, are defined. Afterwards, given \mathcal{L}_q , we allocate D2D users to frequency channels in a way that the second term (interference sum) is minimized. Formally,

$$\underset{\mathcal{L}_{q}}{\text{maximize}} \quad \sum_{q=1}^{Q} \sum_{l \in \mathcal{L}_{q}} \log\left(p_{c} h_{bl,q}\right) \tag{8}$$

subject to (6), and

$$\underset{\mathcal{K}_{q}}{\text{minimize}} \quad \sum_{q=1}^{Q} \sum_{l \in \mathcal{L}_{q}} \sum_{k \in \mathcal{K}_{q}} p_{d}^{(M)} g_{kl}.$$
(9)

We call (8) and (9) as *assignment* and *clustering* problems, respectively. In the next two subsections, we show that these problems boil down to two classic graph-theoretical problems on the induced network graph, namely *maximum-weighted bipartite matching* and *minimum-weighted partitioning*.

1) Assignment Problem: In the following, we show that problem (8) can be formulated as a weighted bipartite matching, defined below.

Definition 1 (Weighted Bipartite Matching). Let $G = (\mathcal{V}, \mathcal{E})$ be a weighted bipartite graph where $\mathcal{V} = \mathcal{V}_1 \cup \mathcal{V}_2$, $\mathcal{V}_1 \cap \mathcal{V}_2 = \emptyset$ and $\mathcal{E} \subseteq \mathcal{V}_1 \times \mathcal{V}_2$. Each edge $e \in \mathcal{E}$ connecting any two vertices $x \in \mathcal{V}_1$ and $y \in \mathcal{V}_2$ is associated with some weight w_{xy} . The weights are gathered in the $V_1 \times V_2$ graph matrix denoted by $\mathbf{W} = [w_{xy}]$.

Matching: A matching is a subset $\mathcal{M} \subseteq \mathcal{E}$ such that $\forall v \in \mathcal{V}$ at most one edge in \mathcal{M} is incident upon v.

Maximum Matching: A matching \mathcal{M} such that every other matching \mathcal{M}' satisfies $W_{\mathcal{M}'} \leq W_{\mathcal{M}}$,

where $W_{\mathcal{M}}$ denotes the total weight of the selected edges for some matching \mathcal{M} . Minimum Matching: A matching \mathcal{M} such that every other matching \mathcal{M}' satisfies $W_{\mathcal{M}} \leq W_{\mathcal{M}'}$.

Based on Definition 1, consider a bipartite graph $G_L(\mathcal{V}, \mathcal{E})$, with $\mathcal{V}_1 = \mathcal{L}$ (the set of cellular users) and $\mathcal{V}_2 = \mathcal{Q}$ (the set of channels). The weight of the edge connecting $l \in \mathcal{L}$ and $q \in \mathcal{Q}$, w_{lq} , is defined as the weighted average gain of channel q between the cellular user l and the BS, i.e., $\log(p_c h_{bl,q})$. The problem is then to assign each cellular user a channel so that (6) and (8) are satisfied. Let the assignment be presented by an $L \times Q$ assignment matrix $\mathbf{A} = [a_{lq}]$, where

$$a_{lq} = \begin{cases} 1 & \text{if } l \in \mathcal{L}_q \\ 0 & \text{otherwise} \end{cases}$$
(10)

Therefore A satisfies the following constraints:

$$\sum_{l=1}^{L} a_{lq} \le 1 \quad , \ q \in \{1, 2, ..., Q\},$$
(11)

$$\sum_{q=1}^{Q} a_{lq} = 1 \quad , \ l \in \{1, 2, ..., L\},$$
(12)

$$a_{lq} \in \{0, 1\}$$
 , $\forall l, q.$ (13)

While (11) implies that each channel serves at most one cellular user, (12) means that each cellular user is served by exactly one channel. Note that equality holds in (11) as we assume Q = L (see Section II-A1). The sum of edges' weights yields

$$\sum_{q=1}^{Q} \sum_{l \in \mathcal{L}} w_{lq} a_{lq} = \sum_{q=1}^{Q} \sum_{l \in \mathcal{L}_q} w_{lq}.$$
(14)

Thus, the problem in (8) subject to (6) is equivalent to maximizing (14), subject to (11), (12), and (13), i.e., it corresponds to the maximum matching of G_L .

2) *Clustering Problem:* This step consists of allocating channels to D2D users with the goal of minimizing the total interference to the cellular users over all channels. In order to address this problem we need to define the network graph.

Definition 2 (Network Graph). The network graph for any channel $q \in Q$ is an undirected graph $G_N = (\mathcal{V}, \mathcal{E})$ with $\mathcal{V} = \mathcal{V}_1 \cup \mathcal{V}_2$, where \mathcal{V}_1 and \mathcal{V}_2 represent the set of K D2D transmitters and L cellular receivers, respectively. The weight of an edge between any pair of graph vertices

(x, y) is denoted by w_{xy} , where w_{xy} is equal to the average gain of channel q between x and y.

However, by Assumption A1, only limited CSI is available at the BS; therefore the network graph cannot be constructed. As a result, we define the *estimated* network graph, which can be reproduced by the BS using the available information.

Definition 3 (Estimated Network Graph). Estimated network graph is an undirected graph $G_E = (\mathcal{V}, \mathcal{E})$ with $\mathcal{V} = \mathcal{V}_1 \cup \mathcal{V}_2$, where \mathcal{V}_1 and \mathcal{V}_2 represent the set of K D2D transmitters and L cellular receivers, respectively. The weight of an edge between any D2D transmitter k and cellular receiver l is defined as $w_{kl} = p_d^{(M)}g_{kl}$. The weight of the edge between any two cellular users and any two D2D users are respectively equal to some constant $C > Kp_d^{(M)}$ and zero.⁴

Next we show that problem (9) can be rephrased as Q-way minimum-weighted graph partitioning on the estimated network graph G_E .

Definition 4 (Q-way Weighted Partitioning). Let $G = (\mathcal{V}, \mathcal{E})$ be a weighted graph where each edge $e \in \mathcal{E}$ connecting any two vertices x and y is associated with some weight w_{xy} . The weights are gathered in a $V \times V$ matrix denoted by $\mathbf{W} = [w_{xy}]$. The minimum-weighted Q-way partitioning problem divides the set of vertices into Q disjoint subsets in a way that the sum weights of edges whose incident vertices fall into the same subset is minimized.

Now consider the estimated network graph, G_E . Then solving (9) is equivalent to finding some $(L + K) \times Q$ assignment matrix $\mathbf{B} = [b_{jq}]$ that is defined to be

$$b_{jq} = \begin{cases} 1 & \text{if } j \in \mathcal{L}_q \cup \mathcal{K}_q \\ 0 & \text{otherwise} \end{cases}$$
(15)

Thus each column in **B**, e.g., $\mathbf{B}_q = \begin{bmatrix} b_{1q}, b_{2q}, ..., b_{(L+K)q} \end{bmatrix}^T$, $q \in \{1, 2, ..., Q\}$, is an indicator describing cluster q. Therefore b_{jq} satisfies the following constraints:

$$\sum_{j=1}^{L+K} b_{jq} = L_q + K_q \quad , \ q \in \{1, 2, ..., Q\},$$
(16)

 $^{^{4}}$ Later we see that this definition results in some form of clustering by which the cellular to cellular and also the D2D to cellular interferences decrease. D2D to D2D interference is however neglected. This implies that in the absence of full and precise channel knowledge the priority is to protect cellular users.

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$$\sum_{q=1}^{Q} b_{jq} = 1 \quad , \ j \in \{1, 2, ..., L+K\},$$
(17)

and

$$b_{jq} \in \{0, 1\}$$
 , $\forall j, q.$ (18)

The sum of edges' weights connecting users in cluster q hence follows as

$$\frac{1}{2} \sum_{j \in \mathcal{L} \cup \mathcal{K}} \sum_{j' \in \mathcal{L} \cup \mathcal{K}} w_{jj'} b_{jq} b_{j'q} = \frac{1}{2} \mathbf{B}_q^T \mathbf{W}_E \mathbf{B}_q,$$
(19)

where W_E is the weight matrix of G_E . As a result, the total sum-weight of edges that are not cut by the Q-way partitioning of G_E yields

$$\frac{1}{2} \sum_{q=1}^{Q} \sum_{j \in \mathcal{L} \cup \mathcal{K}} \sum_{j' \in \mathcal{L} \cup \mathcal{K}} w_{jj'} b_{jq} b_{j'q} =$$

$$\frac{1}{2} \sum_{q=1}^{Q} \sum_{j \in \mathcal{K}} \sum_{j' \in \mathcal{K}} w_{jj'} b_{jq} b_{j'q} + \frac{1}{2} \sum_{q=1}^{Q} \sum_{j \in \mathcal{L}} \sum_{j' \in \mathcal{L}} w_{jj'} b_{jq} b_{j'q} +$$

$$+2 \times \frac{1}{2} \sum_{q=1}^{Q} \sum_{j \in \mathcal{L}} \sum_{j' \in \mathcal{K}} w_{jj'} b_{jq} b_{j'q}.$$
(20)

The first term on the right-hand side of (20) is zero by the definition of G_E . Also, by the following proposition, the second term equals zero as well, since any minimum-weighted partitioning assigns exactly one cellular user to each cluster.

Proposition 2. Any minimum-weighted Q-way partitioning of the estimated network graph G_E assigns exactly one cellular user to each cluster, that is $L_q = 1 \ \forall q \in Q$.

Proof: See Appendix VII-B.

By Proposition 2 and comparing (19) with (20) we have

$$\frac{1}{2} \sum_{q=1}^{Q} \mathbf{B}_{q}^{T} \mathbf{W}_{E} \mathbf{B}_{q} = \sum_{q=1}^{Q} \sum_{j \in \mathcal{L}} \sum_{j' \in \mathcal{K}} w_{jj'} b_{jq} b_{j'q}$$

$$= \sum_{q=1}^{Q} \sum_{j \in \mathcal{L}_{q}} \sum_{j' \in \mathcal{K}_{q}} w_{jj'}.$$
(21)

By comparing (21) with (9) and by using the definition of G_E , it can be concluded that (9) is equivalent to the minimum-weighted Q-way partitioning of G_E .

Remark 1. As described in Section II-A1, D2D user is referred to a pair of one single-antenna transmitter and one single-antenna receiver. Also, as described before, after clustering, any transmitter-receiver pair, which represents a D2D user, belong to a single cluster. As a result, i) no D2D transmitter communicates simultaneously with multiple receivers, and ii) no intercluster communication takes place; that is, communication occurs only between devices in the same cluster.

B. Some Notes on Complexity

In principal, the proposed channel allocation scheme solves two problems, namely maximumweighted matching and minimum-weighted partitioning. The latter problem, however, can be itself reformulated as a minimum-weighted matching, due to the special characteristics of the defined estimated network graph. This is described formally in the following proposition.

Proposition 3. Define a bipartite graph $G'(\mathcal{V}, \mathcal{E})$ where $\mathcal{V}_1 = \mathcal{K}$ and \mathcal{V}_2 is produced by K times replicating \mathcal{L} , i.e., $\mathcal{V}_2 = \underbrace{\mathcal{L} \cup \mathcal{L} ... \cup \mathcal{L}}_{k}$. The weight of any edge connecting some D2D user $k \in \mathcal{V}_1$ to each copy $l_j \in \mathcal{V}_2$ $(j \in \{1, ..., K\})$ of some cellular user $l \in \mathcal{L}$ is w_{lk} , that is, equal to the weight of the edge connecting k and l in the estimated network graph, G_E . Then the minimum-weighted Q-way partitioning of G_E is equivalent to a minimum-weighted bipartite matching of G'.

Proof: See Appendix VII-C.

Therefore the algorithm is required to solve two (parallel) weighted matching problems. Weighted matching is a classic graph-theoretical problem for which numerous efficient algorithmic solutions exist. A well-known solution is the Hungarian algorithm [27]. For a bipartite graph $G(\mathcal{V}, \mathcal{E})$, the space complexity of Hungarian algorithm yields $O(V^2 E)$ with V =max $\{V_1, V_2\}$,⁵ that is polynomial in the number of vertices and also in the number of edges. The running time is $O(V^3)$, which is also polynomial in the number of vertices. In our model, for the first matching we have V = L and $E = L^2$, by the definition of G_L .⁶ For the second matching, on the other hand, we have V = KL and $E = (KL)^2$, by the definition of G_E and

⁵In case $V_1 \neq V_2$, dummy vertices are added. See [27] for details.

⁶This number of edges corresponds to the worst-case scenario where the bipartite graph is complete, i.e., there exists an edge between any pair $x \in \mathcal{V}_1$ and $y \in \mathcal{V}_2$.

Proposition 3. Note that the two problems can be solved simultaneously; hence the running times do not add up. More algorithmic solutions can be found in [28] and [42] for instance.

C. Quality of Service Guarantee and Fairness

Despite being suboptimal, the decoupling approach described in Section III-A provides the possibility of solving the channel allocation problem efficiently under different constraints. Two examples are given below.

• Quality of service (QoS) requirement for cellular users: By problem (5), the goal of channel allocation is to provide *every* D2D user with some transmission channel in a way that the aggregate utility of cellular users is maximized, thereby ignoring the *individual* performances of cellular users. In many networks, however, cellular users require some specific QoS that restricts the amount of tolerable interference. Assume that each cellular user *l* requires some minimum utility, $R_{l,\min}$, by which its QoS is guaranteed. After solving problem (8), each cellular user is assigned a channel. Therefore, the nominator of (1) is known. As a result, the maximum tolerable interference of each cellular user *l*, say $I_{l,\max}$, can be calculated based on $R_{l,\min}$. We construct a bipartite graph with $V_1 = \mathcal{K}$ and $V_2 = \mathcal{L}$. The problem is then to assign *as many as possible* D2D users to cellular users (thus to channels) so that no interference experienced by any cellular user exceeds the maximum tolerable value. Formally, the problem is to find an $K \times L$ assignment matrix $\mathbf{X} = [x_{kl}]$ so that

maximize
$$\sum_{l=1}^{L} \sum_{k=1}^{K} x_{kl},$$
 (22)

subject to the following constraints:

$$\sum_{k \in \mathcal{K}} w_{kl} \ x_{kl} \le I_{l,\max}, \quad \forall l \in \mathcal{L},$$
(23)

$$\sum_{l=1}^{L} x_{kl} \le 1, \quad \forall k \in \mathcal{K},$$
(24)

and

$$x_{kl} \in \{0, 1\}, \quad \forall l, k.$$
 (25)

Note that by the definition of estimated network graph, $w_{kl} = p_d^{(M)}g_{kl}$, i.e., it is an upper-

bound of the interference experienced by cellular user l due to D2D user k. This problem is known as the *generalized assignment problem* which is NP-hard; nonetheless, efficient approximate solutions exist. See [43] as an example.

Fairness requirement: Here the problem is similar to the partitioning problem described in Section III-A2, with the additional requirement that the resulted clusters are balanced, in the sense that the interference experienced by cellular users due to D2D users are *almost* equal. Formally, desired is to solve (9), subject to (16), (17) and (18), so that ∑_{l∈L1}∑_{k∈K1} w_{kl} ≈ ∑_{l∈L2}∑_{k∈K2} w_{kl} ≈ ... ≈ ∑_{l∈L2}∑_{k∈K2} w_{kl}. It should be emphasized that in this context, the burden of D2D communication is divided (almost) equally among cellular users, which does not necessarily result in achieving equal utilities by all of them.

IV. POWER CONTROL

This section deals with the second step of resource assignment, i.e., D2D power control, which aims at optimizing the performance of D2D users.

A. Power Control Game

As described in the foregoing section, while performing the channel assignment, the BS ignores the potential interferences that might arise among D2D users, due to the lack of information and also their lower priority. In essence, D2D users are partitioned into clusters and each cluster is assigned a single channel. Given no information, each D2D user therefore intends to maximize its own utility, thereby causing interference to the users with whom it shares a channel. By power control, however, interference can be managed so that the channel assigned to each cluster is utilized efficiently. We model the power control problem as a game with incomplete information, defined on a discrete strategy set. We show that the game is potential and characterize the set of Nash equilibria. To this end, we define (exact) potential games [44] and Nash equilibrium [45].

Definition 5 (Potential Game). Consider a strategic game $\mathfrak{G} = \{\mathcal{K}, \mathcal{I}, \{R_k\}_{k \in \mathcal{K}}\}$, where \mathcal{K} is the set of K players, \mathcal{I} is the set of pure-strategy joint action profiles of all players, and $R_k : \mathcal{I} \to \mathbb{R}^+$ denotes the payoff function of player k. Then \mathfrak{G} is an exact potential game if there exists a function $v : \mathcal{I} \to \mathbb{R}^+$ such that for all $k \in \mathcal{K}$ we have

$$R_k(i_k, \mathbf{i}_{-k}) - R_k(i'_k, \mathbf{i}_{-k}) = v(i_k, \mathbf{i}_{-k}) - v(i'_k, \mathbf{i}_{-k}),$$
(26)

Definition 6 (Nash equilibrium). A joint strategy profile $\mathbf{i} = (i_1, ..., i_k, ..., i_K)$ is called a purestrategy Nash equilibrium if for all $k \in \mathcal{K}$ and all actions i'_k , the joint strategy profile $\mathbf{i}' = (i_1, ..., i'_k, ..., i_K)$ yields $R_k(\mathbf{i}') \leq R_k(\mathbf{i})$.

As clusters are assigned orthogonal channels, the actions of D2D users inside any given cluster do not affect the utilities of the users outside that cluster. Therefore the power allocation problem in any cluster $q \in \{1, ..., Q\}$ can be defined as a game among K_q D2D users.

Definition 7 (Cluster Power Allocation Game). The power allocation game of cluster $q \in \{1, ..., Q\}$ is a strategic game defined as $\mathfrak{G}_{\mathfrak{q}} = \{\mathcal{K}_q, \mathcal{I}, \{R_k\}_{k \in \mathcal{K}_q}\}$, where \mathcal{K}_q is the set of D2D users assigned to channel $q, \mathcal{I} = \bigotimes_{k=1}^{K_q} \{p_d^{(1)}, p_d^{(2)}, ..., p_d^{(M)}\}$ is the set of joint actions with realizations $\mathbf{p}_{d,q} = (p_1, ..., p_{K_q})$, and $R_k : \mathcal{I} \to \mathbb{R}^+$ is the payoff function of player $k \in \{1, ..., K_q\}$ defined in (2) (Section II-A2).

The main difference between the cluster power allocation game and the standard power control games investigated in other studies including [34] is that the strategy set of players is here extracted from a *discrete space*, while in the previous contributions the strategy space is continuous. Consequently, most of the existing results do not hold, and hence we proceed to the following theorem.

Theorem 1. *a) The cluster power allocation game (Definition 7) is an exact potential game with potential*

$$v(\mathbf{p}_{d,q}) = \sum_{k \in \mathcal{K}_q} \log(p_k) - \sum_{k \in \mathcal{K}_q} cp_k.$$
(27)

b) Denote the set of potential maximizers by \mathcal{V}_{max} . Then, a joint action profile $\mathbf{p}_{d,q}$ is a Nash equilibrium if and only if $\mathbf{p}_{d,q} \in \mathcal{V}_{max}$.

Proof: See Appendix VII-D.

1) Quality of Service Guarantee: In Definition 7, we assume that D2D users have no strict QoS requirement, and only aim at maximizing some reward, expressed in terms of SIR and cost. As a result, the set of joint strategies yields $\mathcal{I} = \bigotimes_{k=1}^{K_q} \left\{ p_d^{(1)}, p_d^{(2)}, ..., p_d^{(M)} \right\}$. While this

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formulation holds for many problems, there are some cases where D2D users need to meet some specific QoS requirements, expressed for instance in terms of some minimum SIR value. In such scenarios, each player tries to selfishly solve the following problem

$$\underset{p_k \in \mathcal{A}_k(\mathbf{p}_{-k})}{\text{minimize}} p_k \tag{28}$$

where A_k is the set of strategies for player k, which depends on the joint strategy profile of its opponents, \mathbf{p}_{-k} , and is given by

$$\mathcal{A}_k = \{ p_k \in \mathcal{M} : \gamma_k \ge \Gamma_k \} \,. \tag{29}$$

Here Γ_k is the minimum required SIR for D2D user k to meet its QoS target. In other words, the players' strategy sets are correlated so that any player plays only the actions that satisfy its QoS constraint, given the actions of opponents. It is known that the problem in (28) can be modeled as a strategic game, where the utility of each player k is defined as $R_k = -p_k$ [33] or $R_k = -\log(p_k)$ [34]. Along similar lines with Theorem 1, it is straightforward to show that the game is an exact potential game with potential $v(\mathbf{p}_{d,q}) = \sum_{k=1}^{K_q} R_k(\mathbf{p}_{d,q})$, provided that the original problem (28) is feasible.

B. Q-Learning Better-Reply Dynamics

According to the system model, in the cluster power allocation game (Definition 7), the utility functions are not known by players (D2D users) in advance. Therefore they require interacting with the environment in order to i) learn the reward functions, and ii) achieve equilibrium. We consider the cluster power allocation game to be a game with noisy payoffs. In such games, for each joint action profile $\mathbf{i} \in \mathcal{I}$ of K_q players, the utility achieved by player k at each interaction can be written as $R_k = \bar{R}_k(\mathbf{i}) + e_k$, where \bar{R}_k is the true expected value of the utility function R_k and e_k is a random fluctuation with zero mean and bounded variance, independent from all other random variables. During the learning process, each player faces a trade-off between gathering information (learning) on the one hand and using information to achieve higher utility (control) on the other hand. This trade-off is known as exploration-exploitation dilemma. In order to deal with this dilemma and also to achieve equilibrium in a distributed manner, we use Qlearning better-reply dynamics [32]. This strategy consists of three main steps that are performed recursively: 1) Observe the personal reward and also the actions of opponents.⁷ 2) Update the Q-values of the played joint action profile. 3) With a small probability, $\epsilon \ll 1$, select an action uniformly at random, while with a large probability, $1 - \epsilon$, play according to the better-reply dynamics that is described in the following definition.

Definition 8 (Better-Reply Dynamics [32]). Assume that at some trial t - 1, a player k plays with action $p_{k,t-1}$. Then, at trial t, with probability ζ_k , the player selects the same action as in the previous trial, t - 1, i.e., $p_{k,t} = p_{k,t-1}$. With probability $1 - \zeta_k$, however, the player selects an action according to a distribution that puts positive probabilities only on actions that are better replies to its (finite) memory than $p_{k,t-1}$. For instance, it selects an action according a uniform distribution over all better-replies.

For readers' convenience, the detailed strategy is described in Algorithm 1 for some player $k \in \mathcal{K}_q$.

Theorem 2 ([32]). The Q-learning better-reply dynamics (Algorithm 1), with ε^t and λ^t given by (30) and (32) respectively, converges to a pure Nash equilibrium in games with noisy unknown rewards that are generic and admit a potential function.

Corollary 1. By using *Q*-learning better-reply dynamics, the cluster power allocation game (Definition 7) converges to a pure Nash equilibrium that maximizes the potential function.

Proof: The proof directly follows from Theorem 1 and Theorem 2.

Remark 2. Let $\alpha = O(M^{K_q})$ be the size of the normal form representation of the cluster power allocation game. Similar to any other equilibrium-learning strategy, Algorithm 1 follows a better-reply path to a pure Nash equilibrium, whose length grows exponentially in α [48]. On the other hand, as for Q-learning, the Q-value of all joint action profiles (that is equal to α) must be learned. As a result, the running time is at least exponential in the size of the game,

⁷When using multi-agent Q-learning algorithms, conventionally it is assumed that every agent observes the state of the environment and/or the actions of its opponents [46]. In our model, players are therefore required to announce their transmit powers, for example by broadcasting in a specific time period, borrowed from the total transmission time. This overhead, however, is much less than that of the frequent and pairwise data exchange, for which usually a control channel is allocated [47]. The reason is that after convergence, which is achieved relatively fast, the transmit powers of players remain fixed. Therefore no more broadcasting is required and the borrowed time period is again available for data transmission. We also assume that the players have a finite memory of length m; that is, at each trial, each player remembers the played joint action profiles of exactly m past trials.

Algorithm 1 Q-Learning Better-Reply Dynamics [32]

- 1: Select arbitrary positive constants c_{λ} and c_{ε} .
- 2: Select learning parameters $\rho_{\lambda} \in \left\lfloor \frac{1}{2}, 1 \right\rfloor$.
- 3: Let $\delta_{k,t}$ be the mixed strategy of player k at time t. Let $\delta_{k,1}$ be the uniform distribution over all actions (power levels).
- 4: Select an action, $p_{k,t}$, using $\delta_{k,1}$. Play and observe the reward.
- 5: for t = 2, ..., T do
- 6: Let

$$\varepsilon_t = c_{\varepsilon} t^{\frac{-1}{K_q}}.$$
(30)

- 7: With probability ε_t , let $\delta_{k,t}$ be the uniform distribution over all actions.
 - With probability $1 \varepsilon_t$, perform the following (better-reply dynamics):
 - With probability ζ_k , let $\delta_{k,t}$ be the Dirac probability distribution on $p_{k,t-1}$.
 - With probability $1 \zeta_k$, let $\delta_{k,t}$ be the uniform distribution over all actions that are better replies to the full (finite) memory than $p_{k,t-1}$.
- Using $\delta_{k,t}$, select the action of time t, $p_{k,t}$, and play. 8:
- Announce the selected action. Moreover, observe the played joint action profile of other players, $\mathbf{p}_{-k,t}$, and 9: also the achieved reward, $R_k(\mathbf{p}_{d,q}^{(t)})$, where $\mathbf{p}_{d,q}^{(t)} = (p_{k,t}, \mathbf{p}_{-k,t}) = (p_{1,t}, ..., p_{k,t}, ..., p_{K_q,t})$. Update the Q-value of the played joint action profile as
- 10:

$$Q_{k,t+1}(\mathbf{p}_{d,q}^{(t)}) = Q_{k,t}(\mathbf{p}_{d,q}^{(t)}) + \lambda_t \left(R_k(\mathbf{p}_{d,q}^{(t)}) - Q_{k,t}(\mathbf{p}_{d,q}^{(t)}) \right) \mathbf{1}_{\mathbf{p}_{d,q}^{(t)}},\tag{31}$$

with

$$\lambda_t = \left(c_\lambda + \#^t[\mathbf{p}_{d,q}^{(t)}]\right)^{-\rho_\lambda},\tag{32}$$

where $\#^t[\mathbf{p}_{d,q}^{(t)}]$ denotes the number of trials in which $\mathbf{p}_{d,q}^{(t)}$ is played while $\mathbf{1}_{\mathbf{p}_{d,q}^{(t)}}$ is the indicator function. 11: end for

i.e., $O(c^{\alpha})$ for some constant c > 1. Thus, for a specific number of players (which is determined by clustering), smaller M (number of power levels) yields faster convergence, as one expects intuitively. Similarly, smaller M yields lower computational complexity.

Remark 3. As described before, in any game, complexity and convergence speed to equilibrium depends dramatically on the size of the game. This dependency becomes even stronger for games with incomplete information, as the reward of all joint action profiles must be learned through successive interactions. As a result, it is of utmost importance to reduce the size of the game and/or to use any available information. The designed two-stage resource allocation mechanism strictly follows this policy, as by excluding cellular users from the set of players, and channels from the set of actions, the game size reduces abruptly in comparison with a one-stage game, while the available information at the BS is used efficiently. Additionally, it allows taking the priority of cellular users into account, which is not possible in a one-stage game.

C. Efficiency of Equilibrium

According to Theorem 2, for the cluster power allocation game, any pure-strategy Nash equilibrium maximizes the potential function, given by (27). It should be however noted that here the potential function is not equal to social welfare, $f(\mathbf{p}_{d,q}) = \sum_{k=1}^{K_q} R_k(\mathbf{p}_{d,q})$. Therefore, the pure-strategy Nash equilibrium does not necessarily maximizes the sum utilities of all players, although such a solution is desired. The inefficiency of equilibrium is formalized by *price of stability*, defined below.

Definition 9 (Price of Stability [49]). Let $f(\mathbf{p}_{d,q})$ be an objective function such as social welfare, which we wish to maximize. Moreover, let \mathcal{N} denote the set of pure Nash equilibriums of the cluster power allocation game. Then the price of stability (PoS) is defined as

$$\operatorname{PoS} = \frac{\max f(\mathbf{p}_{d,q})}{\max_{\mathbf{p}_{d,q} \in \mathcal{N}} f(\mathbf{p}_{d,q})}.$$
(33)

Note that the objective function to be optimized and the solution set being evaluated might vary. For instance, the objective function could be the minimum reward (so that the optimization problem corresponds to max-min fairness criterion), or the set of solution might also include mixed-strategy equilibria. The following proposition provides an upper-bound for the inefficiency of pure-strategy Nash equilibrium in the cluster power allocation game.

Proposition 4. For the cluster power allocation game described in Definition 7, define

$$\gamma_{\min} := \min_{k \in \mathcal{K}} \frac{p_d^{(1)} h_{kk',q}}{1 + \sum_{j \in \mathcal{K}_q, j \neq k} p_d^{(M)} h_{jk',q} + p_c h_{bk',q}}.$$
(34)

Then we have $1 \leq \operatorname{PoS} \leq \frac{\log(p_d^{(M)})}{\log(\gamma_{\min})}$.

Proof: See Appendix VII-E.

Although the bound provided by Proposition 4 is loose, in general it clearly shows that a larger $p_d^{(M)} - p_d^{(1)}$ value of (range of the set of power levels, \mathcal{M}) may yield higher inefficiency of pure Nash equilibrium. Recall that large range of \mathcal{M} has also an adverse effect on the lower-bound given by (4). Therefore the two-stage resource allocation mechanism is particularly suitable for \mathcal{M} with small ranges. It is worth mentioning that for games with multiple equilibriums, the inefficiency of the worst Nash equilibrium is formalized by *price of anarchy* (PoA) [50].

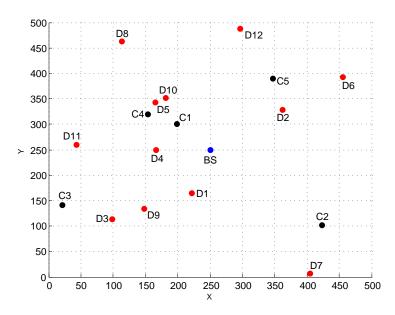


Fig. 1. Network model consisting of D2D transmitters $(Di, i \in \{1, ..., 12\})$ and cellular receivers $(Ci, i \in \{1, ..., 5\})$.

Calculating PoA is mathematically involved and lies out of the scope of this paper.

V. NUMERICAL ANALYSIS

We consider an underlay D2D communication system, consisting of twelve D2D users (K = 12) and five cellular users (L = 5), as depicted in Figure 1. Note that only the *transmitter side* of D2D users are shown in the figure, as receivers do not cause any interference to cellular users and therefore do not impact the channel allocation (see also the definition of estimated network graph in Section III-A2). Also note that for numerical analysis, the locations of cellular and D2D users, as well as channel gains, are selected randomly. According to the system model (Section II-A1), there exist five orthogonal channels (Q = 5). Each D2D user $k \in \mathcal{K}$ selects a transmit power from the set of power levels, $\mathcal{M} = \{2, 4\}$. Moreover, the transmit power of the BS to the cellular users is $p_c = 7$.

A. Channel Allocation

Table I includes $h_{bl,q}$ (cellular-BS average channel gains) for $l, q \in \{1, ..., 5\}$, which is assumed to be known by the BS together with the network topology (Figure 1), according to Assumption A1 (Section II). Based on this information and by using the graph-theoretical channel allocation scheme described in Section III, the BS assigns each (cellular and D2D) user a channel, as summarized in Table II(a). Based on Table I and Figure 1, it can be concluded that by the channel allocation given in Table II(a), both (8) and (9) are satisfied.

Channel User	1	2	3	4	5
C1	0.04	0.01	0.27	0.12	0.04
C2	0.29	0.06	0.15	0.18	0.26
C3	0.31	0.46	0.24	0.19	0.06
C4	0.12	0.06	0.29	0.34	0.16
C 5	0.24	0.08	0.23	0.41	0.07

TABLE I BS TO CELLULAR AVERAGE CHANNEL GAINS

As discussed in Section III-C, it is also possible to change the criterion of channel allocation from maximizing the social welfare to address the QoS guarantee or fairness issues (of cellular users). Assume that the required QoS of any cellular user $l \in \mathcal{L}$ is satisfied if it achieves some minimum utility, say $R_{l,\min} = 3.5$.⁸ Therefore by using the data given in Table I, the maximum tolerable interference of each cellular user can be simply calculated. A channel allocation that guarantees the QoS satisfaction of all cellular users is summarized in Table II(b). Moreover, the result of channel assignment based on fairness among cellular users is shown in Table II(c).⁹

The achieved average rewards of cellular users under all three criteria are shown in Figure 2. It can be seen that to achieve the highest utility sum, some cellular users do not experience any interference, while some others are strongly disturbed. In case of QoS guarantee, users with higher channel gains experience more interference and vice versa, so that at the end all cellular users are satisfied. Moreover, by Table II(b), in the current setting, all D2D users can be served without violating the QoS requirement of cellular users.¹⁰ In the last criterion, all cellular users experience almost equal amounts of interference, regardless of their achieved utilities.

⁸Note that the QoS requirements of cellular users do not need to be necessarily similar.

⁹Note that the solutions are approximately-optimal and also not unique.

¹⁰Clearly, this might not be always the case. In fact, given a specific QoS requirement of cellular users, the number of D2D users that can be served depends strongly on network topology, channel quality and the required QoS.

 TABLE II

 CHANNEL ALLOCATION BASED ON DIFFERENT PERFORMANCE CRITERIA OF CELLULAR USERS

(a) Maximum Aggregate Utility				
Channel	User			
1	C5,D1,D3,D9			
2	C3,D2,D6,D7,D12			
3	C1			
4	C4			
5	C2,D4,D5,D8,D10,D11			

(b) QoS guarantee				
Channel	User			
1	C5,D3,D9			
2	C3,D1,D2,D11,D12			
3	C1,D8,D10			
4	C4,D6			
5	C2,D4			

(c) Fairness				
Channel	User			
1	C5,D3,D9,D11			
2	C3,D2,D12			
3	C1,D1,D8			
4	C4,D6,D7			
5	C2,D4,D5,D10			

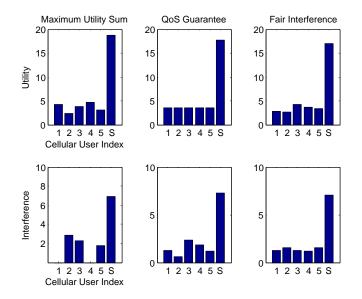


Fig. 2. Average utility and interference experienced by cellular users under three criteria (S:Sum).

For our primary channel allocation criterion, i.e., maximizing the aggregate utility of cellular users, it is of interest to investigate the performance loss of cellular users, caused by sharing resources with D2D users. The performance degradation is shown in Figure 3, where the achievable utilities of cellular users without any interference (no channel sharing) are shown

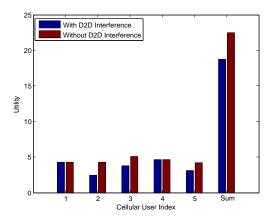


Fig. 3. Performance loss of cellular users due to channel sharing with the allocation criterion being the maximization of cellular utility sum.

in comparison with the case where *all* D2D users are assigned some channel. From this figure, it can be concluded that in the current setting, serving all D2D users costs approximately 15% performance loss to cellular users.

B. Power Control

From Table II(a), it can be observed that minimum-weighted partitioning divides the D2D and cellular users into five clusters, each allocated a frequency channel. In this section, we investigate the power control game of the first cluster, i.e., the cluster that includes three D2D users (D1, D3 and D9) and is assigned channel one. The games of other clusters are similar. The game horizon and price factor are considered to be $T = 2 \times 10^3$ and c = 0.1, respectively. The joint action profiles of the three users as well as their average rewards are given by Table III. From

TABLE III Joint Reward Table

Joint Action	Joint Reward	Joint Action	Joint Reward
(2, 2, 2)	(2.60, 2.36, 2.10)	(4, 4, 2)	(2.80, 2.54, 0.30)
(2, 4, 2)	(1.80, 3.36, 1.30)	(2,4,4)	(1.22, 2.54, 2.28)
(4, 2, 2)	(3.58, 1.56, 1.28)	(4, 2, 4)	(2.80, 0.98, 2.28)
(2, 2, 4)	(1.80, 1.56, 3.08)	(4, 4, 4)	(2.20, 1.98, 1.90)

this table, the action profile (4, 4, 4), i.e., $(p_d^{(2)}, p_d^{(2)}, p_d^{(2)})$, is the unique Nash equilibrium, which

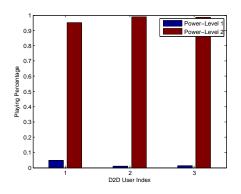


Fig. 4. Fraction of trials in which any given action is played by D2D users.

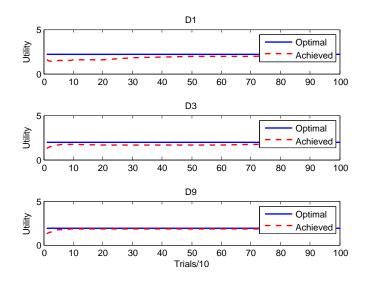


Fig. 5. Utilities achieved by D2D users versus utility values at equilibrium.

maximizes the potential function. Hence the game converges theoretically to this point. Figure 4 describes the frequency in which any given action is played by each D2D user. It can be seen that the equilibrium strategy is played almost all the time. Figure 5 depicts the average utility of D2D users versus the equilibrium reward, confirming that in a short time the average reward of every player converges to that of equilibrium point.

C. Overall Performance

In order to evaluate the overall performance of the proposed resource allocation scheme, we compare it with three other strategies that are described below.

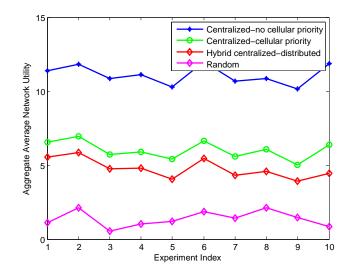


Fig. 6. Overall performance of the proposed scheme compared to some other strategies.

- Centralized approach that is based on the exhaustive search given global information. In accordance with the concept of underlay D2D networks, the priority is here granted to the cellular users. Formally, the selected joint channel and power allocation vector maximizes $\sum_{l=1}^{L} R_l$, and ties are broken in favor of the allocation vector that yields higher aggregate D2D utility, i.e., larger $\sum_{k=1}^{K} R_k$.
- Centralized approach that is based on the exhaustive search given global information, but *without* considering the priority of cellular users. Formally, the algorithm searches for the joint channel and power allocation vector that maximizes $\sum_{l=1}^{L} R_l + \sum_{k=1}^{K} R_k$.
- Random resource allocation, where the channel and power levels are assigned using uniform distribution.

As applying the exhaustive search approach to the large network investigated before (Figure 1) yields excessive complexity $(5^{16} \times 2^{12} \text{ cases should be searched})$, we turn to a smaller network with L = Q = M = 2 and K = 6. Ten experiments are performed. For *each experiment*, independent from others, average channel gains and users' locations are selected randomly. In other words, ten random simulation settings are selected. For *each experiment*, the sum of average rewards of all (cellular and D2D) users is simulated over $T = 10^3$ trials. Results are depicted in Figure 6. From this figure, it can be concluded that the utility achieved by our proposed resource allocation scheme is almost equal to the highest possible aggregate network utility when taking

the priority of cellular users into account. Note that the difference is due to i) bounding and decomposition techniques that are used in Section III, and ii) the inefficiency of equilibrium that is described in Section IV. Hence the performance gap is in fact the cost of i) absence of a coordinator, ii) lack of information, and iii) low time and computational complexity tolerance. It is also worth noting that larger network utility sum can be achieved by neglecting cellular priority; nevertheless, such setting does not comply with the concept of underlay D2D communication, since cellular users might be extremely disturbed. It is also worth mentioning that for larger number of D2D and cellular users, the number of possible channel and power allocation vectors grows exponentially, and hence centralized resource allocation based on exhaustive search yields excessive cost in terms of time and computational complexity, as well as a large overhead that is required for information acquisition. Our approach, in contrast, offers low complexity and overhead; hence it is specifically suitable for large networks.

VI. CONCLUSION AND REMARKS

We studied an underlay D2D communication system, and proposed a two-stage resource allocation strategy that takes the priority of cellular users into account, and relies on strictly limited information. In the first stage, centralized channel allocation is performed by using a graph-theoretical method. The method offers high flexibility for selecting the allocation criteria, for instance aggregate utility, fairness or QoS guarantee. The complexity was shown to be polynomial in the number of users. In the second stage, power control problem is modeled as a game with incomplete information. We showed that the game is an exact potential game defined on a discrete strategy set, and therefore Q-learning better-reply dynamics can be used by players to achieve a pure strategy Nash equilibrium in a distributed manner. The set of Nash equilibria was shown to be equivalent to the set of potential maximizers, and the inefficiency of Nash equilibrium was discussed. Extensive numerical analysis demonstrated the applicability of our approach, specifically in the context of large-scale networks. Moreover, the results showed that the number of D2D users that can be served depends on QoS requirement of cellular users. If no QoS requirement exists, serving all D2D users causes degradation of the cellular aggregate utility, depending on the channel qualities as well as the number of D2D users. In addition, it was concluded that using Q-learning better-reply dynamics results in a fast convergence to equilibrium.

VII. APPENDIX

A. Proof of Proposition 1

According to our system model, $p_k \leq p_d^{(M)} \forall k \in \mathcal{K}$. Moreover, $h_{uv,q} = f_{uv,q}g_{uv}$ with $0 < f_{uv,q} \leq 1$ and $0 < g_{uv} \leq 1$. Hence,

$$\sum_{q=1}^{Q} \sum_{l \in \mathcal{L}_q} \log\left(\frac{p_c h_{bl,q}}{1 + \sum_{k \in \mathcal{K}_q} p_k h_{kl,q}}\right) \ge \sum_{q=1}^{Q} \sum_{l \in \mathcal{L}_q} \log\left(\frac{p_c h_{bl,q}}{1 + \sum_{k \in \mathcal{K}_q} p_d^{(M)} g_{kl}}\right).$$
(35)

By basic properties of the logarithm, the right-hand side of (35) can be written as

$$\sum_{q=1}^{Q} \sum_{l \in \mathcal{L}_{q}} \log \left(p_{c} h_{bl,q} \right) - \sum_{q=1}^{Q} \sum_{l \in \mathcal{L}_{q}} \log \left(1 + \sum_{k \in \mathcal{K}_{q}} p_{d}^{(M)} g_{kl} \right) >$$

$$\sum_{q=1}^{Q} \sum_{l \in \mathcal{L}_{q}} \log \left(p_{c} h_{bl,q} \right) - \sum_{q=1}^{Q} \sum_{l \in \mathcal{L}_{q}} \sum_{k \in \mathcal{K}_{q}} p_{d}^{(M)} g_{kl},$$
(36)

where the inequality follows from the standard logarithm inequality, $\frac{a}{1+a} \le \log(1+a) \le a, \forall a > -1$ [51].

B. Proof of Proposition 2

We proceed by contraposition, i.e., we show that if $\{q \in Q | L_q \neq 1\} \neq \emptyset$ then the partitioning is suboptimal.

Let C be the set of all possible Q-way partitioning forms of L + K vertices of G_E . Assume that there exists some partitioning $c \in C$, by which the graph is partitioned into Q_a clusters with $L_q > 1$. As L = Q (see Section II-A1), there remain $Q_b = Q - Q_a$ clusters with $L_q = 0$. In what follows, we show that partitioning c is suboptimal, by constructing another partitioning whose cost is less than that of c.

Index Q_a and Q_b clusters of partitioning c by $1, ..., Q_a$ and $Q_a+1, ..., Q$, respectively. Moreover, let T_a and T_b correspondingly denote the aggregate sum weight of edges inside all clusters with and without cellular users. Thus we have

$$T_a = \sum_{q=1}^{Q_a} \sum_{l \in \mathcal{L}_q} \left(\sum_{j \in \mathcal{L}_q, j \neq l} w_{jl} + \sum_{k \in \mathcal{K}_q} w_{kl} \right),$$
(37)

and $T_b = 0$ by Definition 3. Let T_c denote the total cost of partitioning c. In order to establish that partitioning c is suboptimal, we show that

$$T_c = T_a + T_b > \min_{\mathcal{C}} \sum_{q=1}^{Q_a} \sum_{l \in \mathcal{L}_q} \left(\sum_{j \in \mathcal{L}_q, j \neq l} w_{jl} + \sum_{k \in \mathcal{K}_q} w_{kl} \right).$$
(38)

To this end, we construct some partitioning c' with $T_{c'} < T_c$. Assume that we change only one cluster of c, say cluster $r \in \{1, ..., Q_a\}$ with $L_q > 1$, by removing a cellular user $J \in \mathcal{L}_r$. Since all vertices must be included in the partitioning, J is added in some cluster $r' \in \{1, ..., Q\} - \{r\}$. Therefore, one of the following holds:

- $r' \in \{1, ..., Q_a\} \{r\}$, or
- $r' \in \{Q_a + 1, ..., Q\}.$

It is clear that the first case results in the original problem. Hence, we assume that the cellular user J is included in $r' \in \{Q_a + 1, ..., Q\}$, and refer to the new partitioning by c'. Then we have

$$T_{c'} = T_c - \sum_{j \in \mathcal{L}_r} w_{jJ} - \sum_{k \in \mathcal{K}_r} w_{kJ} + \sum_{k \in \mathcal{K}_{r'}} w_{kJ}.$$
(39)

Since $0 \le w_{kJ} \le p_d^{(M)}$, we have $0 \le \sum_{k \in \mathcal{K}_x} w_{kJ} \le K p_d^{(M)}$, for any clusters x. Moreover, as $\mathcal{L}_r > 1$ and $w_{jJ} = C$ for $j, J \in \mathcal{L}$, then $\sum_{j \in \mathcal{L}_r} w_{jJ} \ge C$ (see also Definition 3). Hence the worst-case occurs when: i) $\sum_{k \in \mathcal{K}_r} w_{kJ} = 0$, which means that in cluster r, no D2D user causes interference to the cellular user J, ii) $\sum_{k \in \mathcal{K}_{r'}} w_{kJ} = K p_d^{(M)}$, that is, cluster r' includes all D2D users that cause the maximum interference to the cellular user J, and iii) $\sum_{j \in \mathcal{L}_r} w_{jJ} = C$, i.e., $L_r = 2$. As a result,

$$T_{c'} \le T_c - C + K p_d^{(M)} < T_c,$$
(40)

as we assume $C > K p_d^{(M)}$ by Definition 3. Therefore by (40) partitioning c is suboptimal, which is the contraposition and hence the proof is complete.

C. Proof of Proposition 3

By Proposition 2, any optimal partitioning of the estimated network graph G_E includes exactly one cellular user in each cluster; therefore we can assume that $w_{ij} = 0$, $\forall i, j \in \mathcal{L}$. Moreover, by Definition 3, $w_{ij} = 0 \ \forall i, j \in \mathcal{K}$. Therefore we define a complete bipartite graph G with $\mathcal{V}_1 = \mathcal{K}$ and $\mathcal{V}_2 = \mathcal{L}$. The weight of the edge connecting $k \in \mathcal{K}$ and $l \in \mathcal{L}$ is equal to the corresponding edge in G_E , i.e., w_{kl} . We then augment \mathcal{V}_2 by K times replicating each node $l \in \mathcal{L}$, resulting in a set $\mathcal{L}' = \underbrace{\mathcal{L} \cup \mathcal{L} \ldots \cup \mathcal{L}}_{\times K}$. Using this set, a bipartite graph G' is constructed, where $\mathcal{V}_1 = \mathcal{K}$ and $\mathcal{V}_2 = \mathcal{L}'$. The weight of an edge connecting any pair $k \in \mathcal{K}$ to every copy $l' \in \mathcal{L}'$ of some lis $w_{kl'} = w_{kl}$. On graph G', a bipartite minimum-weighted matching results in a $K \times (K \times L)$ assignment matrix $\mathbf{B} = [b_{kl'}]$, so that the sum

$$\sum_{k\in\mathcal{K}}\sum_{l\in\mathcal{L}'}w_{kl'}b_{kl'}\tag{41}$$

is minimized. For each l, let the set of its copies be denoted by \mathcal{U}_l . Moreover, the set of all users $k \in \mathcal{K}$ that are assigned to any copy of l is denoted by \mathcal{A}_l . Thus (41) can be reformulated as

$$\sum_{l=1}^{L} \sum_{j \in \mathcal{U}_l} \sum_{j' \in \mathcal{A}_l} b_{jl} w_{jj'} b_{j'l},\tag{42}$$

which is identical to (21). Hence the proposition follows.

D. Proof of Theorem 1

1) Some Auxiliary Definitions and Results: The proof is based on some auxiliary definitions and results that are briefly stated in the following.

In what follows, v stands for a function defined on a discrete set $\mathcal{X} \subseteq \mathbb{Z}^{I}$ where $\mathcal{X} = \prod_{i \in I} \mathbf{x}_{i}$, $\mathbf{x}_{i} = \{x_{i} \in \mathbb{Z} : \underline{x}_{i} \leq x_{i} \leq \overline{x}_{i}\} \subseteq \mathbb{Z}$, and $\underline{x}_{i}, \overline{x}_{i} \in \mathbb{Z}$. Moreover, $\|\mathbf{x}\| = \sum_{i} |x_{i}|$ denotes the l_{1} -norm of a vector $\mathbf{x} \subseteq \mathbb{Z}^{I}$.

Definition 10 (Larger Midpoint Property (LMP)). We say that a function $v : \mathcal{X} \to \mathbb{R}$ satisfies the larger midpoint property (LMP) if, for any $\mathbf{x}, \mathbf{y} \in \mathcal{X}$ with $\|\mathbf{x} - \mathbf{y}\| = 2$,

$$\underset{\mathbf{z}\in\mathcal{X}:\|\mathbf{x}-\mathbf{z}\|=\|\mathbf{y}-\mathbf{z}\|=1}{\operatorname{maximum}} \quad f(\mathbf{z}) \ge tf(\mathbf{x}) + (1-t)f(\mathbf{y}) \quad (\exists \ t \in (0,1)),$$
(43)

or

$$\max_{\mathbf{z}\in\mathcal{X}:\|\mathbf{x}-\mathbf{z}\|=\|\mathbf{y}-\mathbf{z}\|=1} f(\mathbf{z}) \begin{cases} >\min\{f(\mathbf{x}), f(\mathbf{y})\} & \text{if } f(\mathbf{x}) \neq f(\mathbf{y}) \\ \ge f(\mathbf{x}) = f(\mathbf{y}) & \text{o.w.} \end{cases}.$$
(44)

Definition 11 (Separable Concave Function). A function $v : \mathcal{X} \to \mathbb{R}$ is separable concave if it can be written in the form $v(\mathbf{x}) = \sum_{i \in I} v_i(x_i)$, where $v_i(x_i) \ge \frac{v_i(x_i-1)+v_i(x_i+1)}{2}$ for all $x_i \neq \underline{x}_i, \overline{x}_i$.

Lemma 1 ([31]). If $v : \mathcal{X} \to \mathbb{R}$ is a separable concave function, then (43) holds, and therefore v satisfies the larger midpoint property.

Proposition 5 ([31]). Let \mathfrak{G} be an exact potential game with a potential function v that satisfies the LMP property. Then $\mathbf{i} \in \mathcal{I}$ maximizes v if and only if it is a Nash equilibrium.

2) *Proof of Theorem 1:* The proof consists of two parts. First we show that the power allocation game defined in Definition 7 is an exact potential game by deriving a potential function. This will prove the first part of Theorem 1. Afterwards we establish that the potential function satisfies the LMP property, and we characterize the set of Nash equilibria using Proposition 5. This will prove the second part of the theorem.

Part One

By Definition 5, we need to find a function $v : \mathcal{I} \to \mathbb{R}^+$ that satisfies (26). With $R_k(\mathbf{i})$ given by (2) we have

$$R_{k}(p_{k}, \mathbf{p}_{-k}) - R_{k}(p_{k}', \mathbf{p}_{-k}) = \log\left(\frac{p_{k}}{p_{k}'}\right) - c(p_{k} - p_{k}')$$
(45)

Define

$$v(\mathbf{p}_{d,q}) = \sum_{k \in \mathcal{K}_q} \log(p_k) - \sum_{k \in \mathcal{K}_q} cp_k.$$
(46)

Then By simple calculus it follows that

$$v(p_k, \mathbf{p}_{-k}) - v(p'_k, \mathbf{p}_{-k}) = \log\left(\frac{p_k}{p'_k}\right) - c(p_k - p'_k).$$

$$(47)$$

Therefore, according to Definition 5 and by comparing (47) with (45), it can be concluded that the power allocation game is an exact potential game with potential function defined in (46).

Part Two

Lemma 2. The potential function of the cluster power allocation game (given by (46)) is separable concave.

Proof: Clearly, the potential function can be written as $v(\mathbf{p}_{d,q}) = \sum_{k \in \mathcal{K}_q} v_k(p_k)$ with

$$v_k(p_k) = \log(p_k) - cp_k. \tag{48}$$

Thus, by the assumption $p_k > 1$ (see Section II-A1), we have

$$\frac{v_k(p_k+1) + v_k(p_k-1)}{2} = \frac{\log(p_k^2 - 1) - 2cp_k}{2}$$

$$\leq \frac{\log(p_k^2) - 2cp_k}{2}$$

$$= \log(p_k) - cp_k.$$
(49)

Therefore, by Definition 11, the function is separable concave.

Lemma 3. The potential function of the cluster power allocation game (given by (46)) satisfies the larger midpoint property.

Proof: The proof directly follows from Lemma 1 and Lemma 2.

Therefore, since the potential function satisfies the LMP property, the second part of Theorem 1 follows directly from Proposition 5.

E. Proof of Proposition 4

By Definition 9, $1 \leq \text{PoS}$. Hence we only need to show that $\text{PoS} \leq \frac{\log(p_d^{(M)})}{\log(\gamma_{\min})}$. To this end, we need the following theorem.

Theorem 3 ([52]). Let $\mathfrak{G} = \{\mathcal{K}, \mathcal{I}, \{R_k\}_{k \in \mathcal{K}}\}$ be a potential game with some potential function v (i). Also, let $f(\mathbf{i}) = \sum_{k=1}^{K} R_k(\mathbf{i})$. Assume that for any joint action profile \mathbf{i} ,

$$\frac{1}{\alpha}f(\mathbf{i}) \le v\left(\mathbf{i}\right) \le \beta f\left(\mathbf{i}\right),\tag{50}$$

for some positive constants α and β . Then PoS is at most $\alpha\beta$.

For the cluster power allocation game, we have $\mathbf{i} := \mathbf{p}_{d,q}$, and $v(\mathbf{p}_{d,q})$ is given by (27). Also, by the definition of utility function given in (2), we have $f(\mathbf{p}_{d,q}) = \sum_{k \in \mathcal{K}_q} \log(\gamma_i) - \sum_{k \in \mathcal{K}_q} cp_k$. Besides, as $0 < f_{uv,q} \le 1$ and $0 < g_{uv} \le 1$ (see Section II-A1), at each trial, for any selected transmit power $p_k \in \mathcal{M}$ and any player $k \in \mathcal{K}$, we have $\gamma_{min} \le \gamma_k \le p_k$. Therefore, for any $\mathbf{p}_{d,q}$,

$$\frac{v(\mathbf{p}_{d,q})}{f(\mathbf{p}_{d,q})} \ge 1. \tag{51}$$

On the other hand,

$$\frac{v(\mathbf{p}_{d,q})}{f(\mathbf{p}_{d,q})} = \frac{\sum_{k \in \mathcal{K}_q} \log(p_k) - \sum_{k \in \mathcal{K}_q} cp_k}{\sum_{k \in \mathcal{K}_q} \log(\gamma_k) - \sum_{k \in \mathcal{K}_q} cp_k} < \frac{\sum_{k \in \mathcal{K}_q} \log(p_k)}{\sum_{k \in \mathcal{K}_q} \log(\gamma_k)} < \frac{\log\left(p_d^{(M)}\right)}{\log\left(\gamma_{\min}\right)},$$
(52)

where the first inequality is concluded from (51). Thus, by Theorem 3, the result follows.

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