Achievable Rates of Underlay-Based Cognitive Radio Operating Under Rate Limitation

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***Abstract*—A new information-theoretic model is proposed for underlay-based cognitive radio (CR), which imposes rate limitation on the secondary user (SU), whereas the traditional systems impose either interference or transmit power limitations. The channel is modeled as a twin-user interference channel constituted by the primary user (PU) and the SU. The achievable rate of the SU is derived based on the inner bound formulated by Han and Kobayashi, where the PU achieves the maximum attainable rate of the single-user point-to-point link. We show that it is necessary for the SU to allocate its full power for the "public" message that can be decoded both by the SU and by the PU. We also demonstrate that it is optimal for the PU to allocate its full power for the "private" message that can only be decoded by the PU if the level of interference imposed by the PU on the SU is "ergodically strong." Similarly, it is optimal for the PU to allocate its full power for the public message that can be decoded both by the SU and PU if this interference is "ergodically weak." These findings suggest that this power allocation is independent of the level of interference imposed by the SU on the PU. Furthermore, the achievable rate is analyzed as a function of the average level of interference. An interesting observation is that if the level of interference imposed by the SU on the PU is "ergodically weak," the achievable rate becomes a monotonically increasing function of this interference, and it is independent of the level of interference imposed by the PU on the SU. Furthermore, we analyze the realistic imperfect channel estimation scenario and demonstrate that the channel estimation errors will not affect the optimal nature of the SU's power allocation.**

***Index Terms*—Cognitive radio (CR), interference limitation, rate limitation, underlay.**

I. INTRODUCTION

THE conventional fixed spectrum allocation policy of wireless transmissions has led to much of the spectrum being underutilized, whereas some bands are becoming overcrowded due to the avalanche-like proliferation of wireless devices [1]. Cognitive radio (CR)-based spectrum sharing is seen as a possible solution to the problem of inefficient spectrum utilization [2]–[4]. There are various notions of spectrum sharing. One of the most popular versions is the underlay-based spectrum sharing [5]–[14]. In underlay, the basic cognition is associated with near-instantaneously estimating the interfering link's gain at the receivers but, in the advanced scenario, interfering link's gain at the transmitters is also included. Moreover, the traditional approach of underlay-based CR introduces a new parameter for characterizing the interference temperature defined in [3], which limits the aggregate interference that the CRs may inflict upon the primary user (PU), so that the PU still achieves data rates that satisfy its quality-of-service requirement. This interference temperature limit can either be imposed as a peak interference constraint or as an average interference constraint. These constraints directly translate to the corresponding peak transmit power or average transmit power constraints to be assigned at the transmitters.

The objective of this paper is to quantify the achievable rates of the secondary user (SU) without inflicting any rate loss upon the PU. This requires us to consider the PU–SU system from an information-theoretic perspective. In contrast to the traditional interference limitation or transmit power limitation constraints imposed on the SU in [5], [7], [8], [12], and [13], we impose a rate constraint on the SU. This constrained rate would be the maximum rate that the SU is capable of achieving *without affecting the PU's transmission rate*, namely the rate at which the PU is capable of reliably transmitting in the single-user point-to-point scenario. Indeed, a rate constraint has been imposed on the SU also in some of previous contributions [15], [16]; however, the aim in those prior contributions was to maximize the SU's rate over the different possible beamforming vectors, whereas the interference imposed both on the SU and PU was assumed additive noise. The information-theoretic literature routinely exploits that when the interference level is high, it can be readily canceled. Hence, in this CR scenario, this assumption would imply that both the PU and the SU succeed in partially canceling the interference and thereby become capable of increasing their individual rates. This line of thought was adapted for example in [6], albeit the authors' aim was to quantify the penalty that had to be tolerated by the PU when subjected to the interference imposed by the SU. In other contributions [9]–[11], [17], an interference temperature constraint was imposed, which led to a more meaningful outage constraint that had to be satisfied by the PU.

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The proposed rate limitation differs from the existing interference temperature and outage constraint model in terms of the following five aspects.

- The rate limitation observed by the SU allows the PU to communicate at the full rate of the point-to-point scenario, which is not possible when an interference constraint is imposed, as explicitly noted in [6].
- The rate limitation approach relies on the idealized simplifying assumption of using perfect capacity-achieving coding techniques at both the SU and the PU, which allows us to detect, decode, and subtract the interference at both the SU and PU. By contrast, in the case of the interference-limited approach, this interference removal is not exploited since the interference is treated as noise [5], [8]; hence, the advantages of the aforementioned sophisticated coding techniques cannot be readily exploited for interference cancelation. However, in contrast to the overlay CR concept [14], [18] no causal or noncausal message of the PU is available at the SU.
- It will be shown that this approach allows for the SU rate to vary according to the average interference levels, even when the channel information is unknown at the transmitter. By contrast this is not possible in the interference-temperature-based model, which treats both the PU and SU channels as an additive white Gaussian noise channel and treats the interference as additional noise.
- By contrast, our approach of limiting the rate allows us to evaluate the simultaneously achievable rates of the PU and SU. In contrast to most existing contributions on underlay-based CR, which do not consider the effect of any ongoing PU transmission at the SU receiver [13], [19], we are able to do so. This is also another beneficial feature of our solution.
- In contrast to the outage constraint, the PU always maintains a reliable ergodic achievable rate in the context of the rate-limited model.

To quantify the achievable rates of the SU, the Han–Kobayshi achievable rate region [20], [21] is invoked. This rate region was derived for a scenario having fixed channel coefficients, which is also in line with the capacity estimates of [22], [23]. Moreover, in all the regimes where either the capacity [26], [27] or the sum capacity is known [28], this achievable rate region turns out to be tight. For the fading scenario, the optimality of many of the results remains an open challenge to prove analytically. However, the results in [29] and [30] indicate that the Han–Kobayashi region extended to the fading case may be approximately optimal in various scenarios.

In light of these discussions, the major contributions of this paper are as follows.

- The achievable rates are determined for the SU without inflicting any rate loss upon the PU.
- It is shown that, in the specific scenarios, when the interference imposed by the PU on the SU is ergodically strong, regardless of the level of interference inflicted by the SU on the PU, then it is optimal to detect, demodulate, and cancel the interference imposed by the SU on the PU. By contrast, in the opposite scenario, it is better to treat this interference as noise.
- It is also shown that the achievable rate of the SU is an increasing function of the interference imposed by the SU on the PU, when the level of this interference is ergodically weak[1] and that the SU rate is independent of the level of interference imposed by the PU on the SU. If, however, the level of interference imposed by the SU on the PU is ergodically strong, the achievable rate of the SU is shown to be a decreasing function of the level of interference imposed by the PU on the SU, provided that the PU interference is ergodically weak. The opposite trend prevails if this interference is ergodically strong.
- Analysis for the case when there is error in the channel state estimation process is also studied. It is shown that the conditions under which it is optimal to detect, demodulate, and cancel the interference imposed by the SU on the PU in the case with error in estimation is the same as when there is no error. The only difference that arises is in the structure of the achievable rates in certain regimes (described in detail later) and in the effective noise variances at the PU and the SU receiver that appear in the expressions of the achievable rates.

This paper is structured as follows. Section II describes the system model and introduces the problem followed by our main results presented in Section III. In Section IV, the analysis of the derived results sheds light on their nature. In Section V analyzes the achievable rate when there is error in channel state information. Finally, we conclude in Section V.

II. System Model and Problem Statement

Let us consider an underlay CR system, where the PU is transmitting at random instants, where p is the probability that the PU is silent. The SU transmits at a *low rate*, so that the PU and SU can communicate simultaneously without the PU having to reduce its transmission rate.

The channel is shown in Fig. 1, which is modeled as follows:

$$Y_p = H_{pp}S_pX_p + H_{sp}X_s + Z_p \tag{1}$$

$$Y_s = H_{ps}S_pX_p + H_{ss}X_s + Z_p \tag{2}$$

where Y_p and Y_s are the outputs at the PU and the SU receivers, respectively, in response to the inputs X_p at the PU and X_s at the SU. The power constraints of the PU and SU on their transmit rate are $\mathbb{E}[|X_p|^2] \leq P_p$ and $\mathbb{E}[|X_ps^2] \leq P_s$. The random variable (RV) $S_p = \{0, 1\}$ indicates whether the PU transmission is ON or OFF, with $S_p = 1$ indicating that the transmission is ON. Hence, we have $\Pr[S_p = 1] = 1 - p$. The value of S_p is not known at the SU transmitter and receiver. The instantaneous channel coefficient of the PU-to-PU link is

[1]Ergodically weak interference is said to be imposed by the SU on the PU if the average value of this interfering link is below unity. By contrast, the interference is deemed to be ergodically strong if it is higher than unity. A precise definition is provided in the system model.

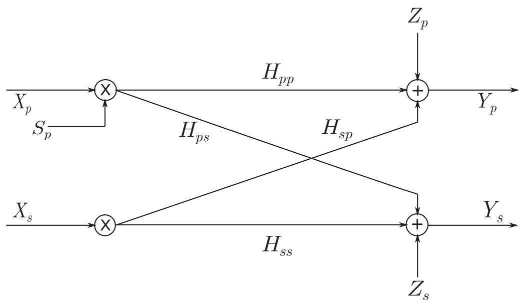

Fig. 1. Underlay channel scenario. Here, $\mathbb{E}[\|H_{pp}\|^2] = 1$, $\mathbb{E}[|H_{ss}|^2] = 1$, $\mathbb{E}[|H_{sp}|^2] = b^2$, and $\mathbb{E}[|H_{ps}|^2] = a^2$. The noise $Z_p \sim \mathcal{N}(0, 1)$, and $Z_s \sim \mathcal{N}(0, 1)$. The input $\mathbb{E}[|X_p|^2] = P_p$, and $\mathbb{E}[|X_s|^2] = P_s$.

denoted by the RV H_{pp}, that of the SU-to-SU link by H_{ss}, that of the interfering PU-to-SU link by H_{ps}, and that of the interfering SU-to-PU link by H_{sp}. All these value are complex. We assume that all the instantaneous channel coefficients are known at the PU and SU receivers and the distribution of these are known at the PU and SU transmitter in conjunction with $\mathbb{E}[|H_{pp}|^2] = 1$, $\mathbb{E}[|H_{ss}|^2] = 1$, $\mathbb{E}[|H_{sp}|^2] = b^2$, and $\mathbb{E}[|H_{ps}|^2] = a^2$. The noise is denoted by the RVs Z_p and Z_s, which are zero-mean unit-variance Gaussian RVs. Both the fading and the noise RVs are assumed to be independent and identically distributed (i.i.d.) over time.

We state that the PU's receiver faces ergodically strong interference from the SU if $b > 1$, whereas it faces ergodically weak interference if $b \leq 1$. Similarly, the SU receiver faces ergodically strong interference from the PU if $a > 1$, and it faces ergodically weak interference if $a \leq 1$.

The question that we ask now is as follows: What rates can be achieved for the SU subject to the fact that the PU rate is the same as that in the point-to-point single-link case, when no interference arrives from the SU? The answer to this is derived from the Han–Kobayashi achievable region [20], [21], [23], [30] for the twin-user interference channel. The two users of the interference channel in our case are the PU and the SU. The scheme proposed by Han and Kobayashi [20], [23] involves splitting of the messages of both the PU and SU into two parts, namely the part which is decoded at both the receivers and the other which is only decoded at its respective desired receivers. The messages that are decoded at both the receivers are referred to as "public" messages, whereas those that are decoded only at the respective receiver are termed as the "private" message. Accordingly, the PU assigns a fraction α of the power P_p to its private message, whereas the SU dedicates a fraction β of the power P_s to its private messages. The fractions α and β are referred to as rate sharing parameters. For the PU to achieve its full single-user transmission rate, the PU should be able to perfectly decode the interference; hence, all the SU messages should be public messages. This requires that the rate sharing parameter at the SU be zero, i.e., $\beta = 0$. We now formulate the following proposition that quantifies the Han–Kobayashi achievable rate region for $\beta = 0$. The complete rate region with partial side information is given in [30].

Proposition 1: The Han–Kobayashi achievable rate region of a two-user Gaussian fading interference channel is characterized in [30], which is reproduced for $\beta = 0$ using the following notation:

$$R_p \leq \mathbb{E}_{(|H_{pp}|)}\left[\log\left(1 + |H_{pp}|^2 P_p\right)\right] \tag{3}$$

$$R_s \leq \mathbb{E}_{(|H_{ss}|,|H_{ps}|)}\left[\log\left(1 + \frac{|H_{ss}|^2 P_s}{\alpha|H_{ps}|^2 P_p + 1}\right)\right] \tag{4}$$

$$R_p + R_s \leq \mathbb{E}_{(|H_{pp}|)}\left[\log\left(1 + \alpha|H_{pp}|^2 P_p\right)\right] + \mathbb{E}_{(|H_{ss}|,|H_{ps}|)}\left[\log\left(\frac{1+|H_{ss}|^2 P_s + |H_{ps}|^2 P_p}{\alpha|H_{ps}|^2 P_p + 1}\right)\right] \tag{5}$$

$$R_p + R_s \leq \mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(1 + |H_{pp}|^2 P_p + |H_{sp}|^2 P_s\right)\right] \tag{6}$$

$$R_p + R_s \leq \mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(1 + \alpha|H_{pp}|^2 P_p + |H_{sp}|^2 P_s\right)\right] + \mathbb{E}_{(|H_{ps}|)}\left[\log\left(\frac{1 + |H_{ps}|^2 P_p}{\alpha|H_{ps}|^2 P_p + 1}\right)\right] \tag{7}$$

$$2R_p + R_s \leq \mathbb{E}_{(|H_{pp}|)}\left[\log\left(1 + \alpha|H_{pp}|^2 P_p\right)\right] + \mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(1 + |H_{pp}|^2 P_p + |H_{sp}|^2 P_s\right)\right] + \mathbb{E}_{(|H_{ps}|)}\left[\log\left(\frac{1 + |H_{ps}|^2 P_p}{\alpha|H_{ps}|^2 P_p + 1}\right)\right] \tag{8}$$

$$R_p + 2R_s \leq \mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(1 + \alpha|H_{pp}|^2 P_p + |H_{sp}|^2 P_s\right)\right] + \mathbb{E}_{(|H_{ss}|,|H_{ps}|)}\left[\log\left(\frac{1+|H_{ss}|^2 P_s + |H_{ps}|^2 P_p}{\alpha|H_{ps}|^2 P_p + 1}\right)\right]. \tag{9}$$

Let us now provide an interpretation of (3)–(9), where (3) and (4) describe the individually achievable rates of the PU and SU, respectively. This is followed by the three sum-rate constraints $(R_p + R_s)$ in (5)–(7), where the first term in (5) represents the public message of the PU decoded at the PU receiver, whereas the second term represents the private message of the PU and the complete message (public and private both) of the SU decoded at the SU. The sum rate constraint in (6) represents the complete message decoding process of both the PU and the SU at the PU receiver. In (7), the first term represents the private message of the PU and the complete message of the SU decoded at the PU receiver, whereas the second term represents the public message of the PU decoded at the SU receiver. The first term of the constraint in (8) represents the private message of the PU decoded at the PU receiver, the second term represents the complete message of both the PU and the SU decoded at the PU receiver, and the third term represents the public message of the PU decoded at the SU receiver, resulting in a rate of $(2R_p + R_s)$. Finally, in (9) the first term represents the private message decoding process of the PU and the complete message decoding of the SU at the PU receiver, whereas the second term represents the public message decoding process of the PU and the complete message decoding process of the SU at the SU receiver, resulting in the rate of $(R_p + 2R_s)$. All the PU rate constraints R_p arise either because the PU decodes its private message at its receiver and its public message at the SU receiver or because it decodes its complete message at its receiver. However, the SU rate constraint R_s is a consequence of the PU ability to decode the full message of the SU at its receiver.

Our aim is to find what is the maximum achievable SU rate C_{sm} subject to the PU rate given in (3) and to find the corresponding rate sharing parameter at the PU that achieves this. The solution is obtained by solving the following proposition.

Proposition 2: The achievable rate C_{sm} of the SU is given by

$$C_{sm} = \min\left(r_3,\ \max_{\alpha\in[0,1]} \{\min(r_1, r_2, r_4, r_5, r_6)\}\right)$$

where r_i, $i = \{1, 2, 3, 4, 5, 6\}$, are as given in the following:

$$r_1 = \mathbb{E}_{(|H_{ss}|,|H_{ps}|)}\left[\log\left(1 + \frac{|H_{ss}|^2 P_s}{\alpha|H_{ps}|^2 P_p + 1}\right)\right] \tag{10}$$

$$\begin{aligned} r_2 = {} & \mathbb{E}_{(|H_{pp}|)}\left[\log\left(\frac{1 + \alpha|H_{pp}|^2 P_p}{1 + |H_{pp}|^2 P_p}\right)\right] \\ & + \mathbb{E}_{(|H_{ss}|,|H_{ps}|)}\left[\log\left(\frac{1+|H_{ss}|^2 P_s+|H_{ps}|^2 P_p}{\alpha|H_{ps}|^2 P_p+1}\right)\right] \end{aligned} \tag{11}$$

$$r_3 = \mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(1 + \frac{|H_{sp}|^2 P_s}{1 + |H_{pp}|^2 P_p}\right)\right] \tag{12}$$

$$\begin{aligned} r_4 = {} & \mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(\frac{1 + \alpha|H_{pp}|^2 P_p + |H_{sp}|^2 P_s}{1 + |H_{pp}|^2 P_p}\right)\right] \\ & + \mathbb{E}_{(|H_{ps}|)}\left[\log\left(\frac{1 + |H_{ps}|^2 P_p}{\alpha|H_{ps}|^2 P_p + 1}\right)\right] \end{aligned} \tag{13}$$

$$\begin{aligned} r_5 = {} & \mathbb{E}_{(|H_{pp}|)}\left[\log\left(\frac{1 + \alpha|H_{pp}|^2 P_p}{1 + |H_{pp}|^2 P_p}\right)\right] \\ & + \mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(1 + \frac{|H_{sp}|^2 P_s}{1 + |H_{pp}|^2 P_p}\right)\right] \\ & + \mathbb{E}_{(|H_{ps}|)}\left[\log\left(\frac{1 + |H_{ps}|^2 P_p}{\alpha|H_{ps}|^2 P_p + 1}\right)\right] \end{aligned} \tag{14}$$

$$\begin{aligned} r_6 = {} & \frac{1}{2}\left(\mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(\frac{1 + \alpha|H_{pp}|^2 P_p + |H_{sp}|^2 P_s}{1 + |H_{pp}|^2 P_p}\right)\right]\right) \\ & + \frac{1}{2}\left(\mathbb{E}_{(|H_{ss}|,|H_{ps}|)}\left[\log\left(\frac{1+|H_{ss}|^2 P_s+|H_{ps}|^2 P_p}{\alpha|H_{ps}|^2 P_p+1}\right)\right]\right). \end{aligned} \tag{15}$$

Proof: All the rate expressions r_i, $i = \{1, \ldots, 6\}$ are obtained by substituting $R_p = \mathbb{E}_{(|H_{pp}|)}[\log(1 + |H_{pp}|^2 P_p)]$ into (3)–(8) in the same order and then simplifying the resultant expressions. The value of C_{sm} is then optimized by maximizing it over all possible values of $\alpha \in [0, 1]$. ■

Note that the interpretations of (10)–(15) remain similar to those mentioned earlier regarding (3)–(8).

The achievable rate of our underlay CR system then becomes

$$R_p \leq (1 - p)\mathbb{E}_{(|H_{pp}|)}\left[\log\left(1 + |H_{pp}|^2 P_p\right)\right] \tag{16}$$

$$R_s \leq C_{sm}. \tag{17}$$

The term $(1 - p)$ in the PU rate is a result of the fact that the PU is not always active. However, if the PU were to be always active, i.e., if $p = 0$, then the rate of the PU would be $R_p \leq \mathbb{E}_{(|H_{pp}|)}[\log(1 + |H_{pp}|^2 P_p)]$. This would not affect the SU rate since the basic premise of underlay CR is the assumption of having no spectrum sensing at the SU transmitter and hence being unaware of the PU presence. In our system model, this situation is taken into account by assuming that the SU transmitter and receiver are unaware of S_p.

In the following, we discuss and characterize our main results in more detail.

III. Main Results

Our main result is essentially derived from the Han–Kobayshi achievable rate region [20], [21], which is known to be tight in all those interference regimes where the capacity is known.

As noted earlier, a necessary condition for operating at the full single-user rate for the PU is that the rate sharing parameter at the SU is chosen to be $\beta = 0$, i.e., the SU has to assign all of its power for the public message that can be perfectly decoded, demodulated, and canceled out not only at the SU receiver but also at the PU receiver. We will now demonstrate that the rate sharing parameter α of the PU also has a simple structure.

Theorem 1: If $a \leq 1$, then it is optimal to select $\alpha = 1$, whereas if $a > 1$, then it is optimal to select $\alpha = 0$.

Proof: See Appendix B. ■

It is thus clear that the value of β is zero (as dictated by the requirement of achieving the full rate for the PU) and that of α is unity if the interference imposed by the PU on the SU is ergodically weak (i.e., $a \leq 1$), and it is zero if the interference is ergodically strong ($a > 1$). This implies that if the interference at the SU is weak, then treating the interference as noise is best; hence, the interference is not canceled. However, when the interference at the SU is strong, the interference is perfectly canceled out. An important point to note is that the result does not have any generic structure for α, such as $\alpha = \alpha^*$, where $\alpha^* \in (0, 1)$ represents the optimal rate sharing parameter at the PU that maximizes the SU rate. This implies that partial cancelation of the interference is not optimal in any case. In the following, we quantify the achievable rates associated with $\alpha = 0$ or 1 and $\beta = 0$.

Theorem 2: The achievable rate of the SU, which is subject to the condition that the required rate of the PU of $\mathbb{E}_{(|H_{pp}|)}[\log(1 + |H_{pp}|^2 P_p)]$ is met, is given by

$$R_s \leq C_{sm} \tag{18}$$

where C_{sm} is formulated as follows:

$$C_{sm} = \begin{cases} \min(C_{s1}, C_{s2}), & \text{if } a \leq 1 \text{ and } b > 1 \\ \min(C_{s1}, C_{s3}, C_{s4}), & \text{if } a > 1 \text{ and } b > 1 \\ C_{s1}, & \text{if } b \leq 1 \end{cases}$$

where, we have

$$C_{s1} = \mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(1 + \frac{|H_{sp}|^2 P_s}{1 + |H_{pp}|^2 P_p}\right)\right] \tag{19}$$

$$C_{s2} = \mathbb{E}_{(|H_{ss}|,|H_{ps}|)}\left[\log\left(1 + \frac{|H_{ss}|^2 P_s}{1 + |H_{ps}|^2 P_p}\right)\right] \tag{20}$$

$$C_{s3} = \mathbb{E}_{(|H_{ss}|)}\left[\log\left(1 + |H_{ss}|^2 P_s\right)\right] \tag{21}$$

$$C_{s4} = \mathbb{E}_{(|H_{ss}|,|H_{ps}|)}\left[\log\left(\frac{1 + |H_{ps}|^2 P_p + |H_{ss}|^2 P_s}{1 + |H_{pp}|^2 P_p}\right)\right]. \tag{22}$$

TABLE I
SU ACHIEVABLE RATE IN UNDERLAY CR FOR THE DIFFERENT REGIMES OF AVERAGE INTERFERENCE LEVELS

Regime → / Parameter ↓	I - $b \leq 1$	II - $b>1$ and $a \leq a_1$	III - $b>1$ and $a_1<a\leq 1$	IV - $b>1$ and $1<a\leq a_2$	V - $b>1$ and $a_2<a\leq a_3$	VI - $b>1$ and $a>a_3$
Average interference coefficient PU-SU link a	Constant behaviour	Constant behaviour	Decreases with a as interferece from the PU is treated as noise	Increases with a as interferece from the PU is decoded out. More interference more information is decoded	Constant behaviour	Constant behaviour
Average interference coefficient SU-PU link b	Increases with b. The rate is dictated by how much PU is able to decode out at its receiver	Increases with b. The rate is dictated by how much PU is able to decode out at its receiver	Constant behaviour	Constant behaviour	Increases with b. The rate is dictated by how much PU is able to decode out at its receiver	Constant behaviour
Transmit power constraint at PU P_p	Decreases with P_p with a rate s_1 (say). At PU receiver the PU message is treated as noise to decode the SU common message	Decreases with P_p with a rate s_1. At PU receiver the PU message is treated as noise to decode the SU common message	Decreases with P_p witha rate $s_2 < s_1$. At SU receiver the PU message is treated as noise to decode the SU common message	Decreases for values of a near unity and may possibly increase at large values of a, depending upon the value of b	Decreases with P_p with a rate $s_3 > s_1$. At PU receiver the PU message is treated as noise to decode the SU common message	Constant behaviour
Transmit power constraint at SU P_s	Increases with P_s with a rate s_4 (say). At PU receiver the PU message is treated as noise to decode the SU common message	Increases with P_s with a rate $s_5 > s_4$. At PU receiver the PU message is treated as noise to decode the SU common message	Increases with P_s with a rate $s_5 > s_4$. At PU receiver the PU message is treated as noise to decode the SU common message	Increases with P_s with a rate $s_6 < s_5$. At SU receiver simultaneous decoding is performed by the SU followed by complete interference cancellation	Increases with P_s with a rate $s_7 > s_6$. At PU receiver simultaneous decoding is performed by the PU.	Increases with P_s with a rate $s_8 > s_7$. At SU receiver simultaneous decoding is performed by the SU followed by complete interference cancellation.

Proof: See Appendix C. ■

IV. DISCUSSIONS

To quantify the SU rate associated with various parameters, we structure our analysis based on the value of average interference coefficients in Table I as follows:

- The interference at the PU is ergodically weak, i.e., we have $b \leq 1$. We refer to this as Regime I in Table I.
- The interference at the PU is ergodically strong and that at the SU is ergodically very weak, i.e., we have $b > 1$ and $a \leq a_1$, where for a given b, a_1 is that specific value of a, where $C_{s1} = C_{s2}$. We refer to this as Regime II in Table I.
- The interference at the PU is ergodically strong and that at the SU is ergodically weak, i.e., we have $b > 1$ and $a_1 < a \leq 1$. We refer to this as Regime III in Table I.
- The interference at the PU is ergodically strong and that at the SU is also ergodically strong, i.e., we have $b > 1$ and $1 < a \leq a_2$, where for a given b, a_2 is that specific value of a, where $C_{s1} = C_{s4}$. We refer to this as Regime IV in Table I.
- The interference at the PU is ergodically strong, and that at the SU is ergodically moderately strong, i.e., we have $b > 1$ and $a_2 < a \leq a_3$, where for a given b, a_3 is that specific value of a, where $C_{s4} = C_{s3}$. We refer to this as Regime V in Table I.
- The interference at the PU is ergodically strong, and that at the SU is ergodically very strong, i.e., $b > 1$ and $a > a_3$. We refer to this as Regime VI in Table I.

We now analyze the behavior of the achievable rate in each regime. The achievable rate C_{sm} of the SU obeys the following trend:

1) Regime I of Table I: For $b \leq 1$, the value of C_{sm} is increasing with b, and it is constant for a given a. We have shown mathematically as to why C_{s1} holds in this regime. From a conceptual perspective, we try to understand this by dividing this regime into two parts: 1) $a \leq 1$, and 2) $a > 1$. Since the interference is ergodically weak for $a < 1$, we imagine a compound channel [23] from the SU's perspective. Both the PU and the SU receivers want to recover the SU message and hence treat the PU message as noise. Since we have $a \leq 1$ and $b \leq 1$, the SU–PU link is more noisy than the SU–SU link; hence, the SU–PU link determines the achievable rate. On the other hand, for $a > 1$ imagine a pair of multiple access channels, namely MAC1 comprised of the PU–SU and SU–SU links, and MAC2 comprised of the PU–PU and SU–SU links. Fig. 2(a) shows the capacity region for these MACs. It is clear from Fig. 2(a) that the capacity region of MAC2 is completely contained within that of MAC1 if $a > 1$ and $b \leq 1$. Hence, again, C_{s1} is a corner point of the MAC1 capacity region where PU achieves its full rate. Hence, for $b \leq 1$, C_{sm} is a monotonically increasing function of b.
2) Regime II of Table I: Based on the compound channel explanation above for $b > 1$ and $a \leq a_1 < 1$, the weak link is the SU–PU link; hence, C_{s1} is cached. Hence, the PU receiver perfectly decoding the SU message completely by treating its own message as noise is the determining achievable rate.

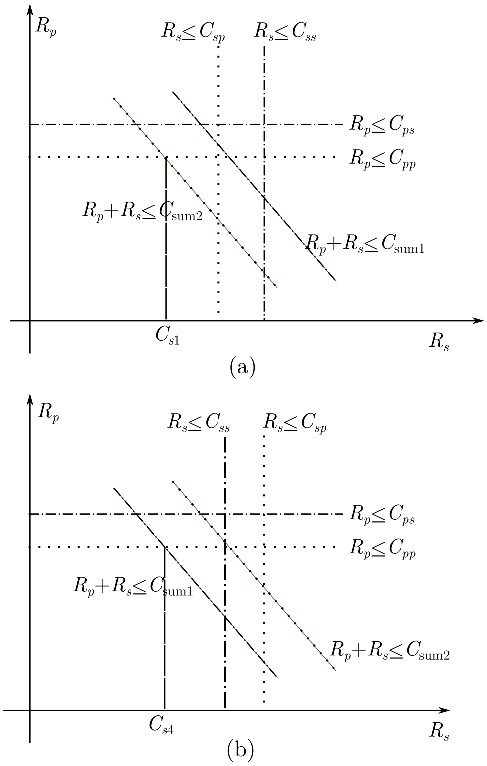

Fig. 2. Two scenarios are as follows. (a) Scenario for Regime I when $a > 1$; and (b) scenario for Regime IV. Here, $C_{pp} = \mathbb{E}_{|H_{pp}|}[\log(1 + |H_{pp}|^2 P_p)]$, $C_{ss} = \mathbb{E}_{|H_{ss}|}[\log(1 + |H_{ss}|^2 P_s)]$, $C_{sp} = \mathbb{E}_{|H_{sp}|}[\log(1 + |H_{sp}|^2 P_s)]$, $C_{pp} = \mathbb{E}_{|H_{ps}|}[\log(1 + |H_{ps}|^2 P_p)]$, $C_{\text{sum1}} = \mathbb{E}_{|H_{pp}|,|H_{sp}|}[\log(1 + |H_{pp}|^2 P_p) + |H_{sp}|^2 P_s]$, and $C_{\text{sum2}} = \mathbb{E}_{|H_{ss}|,|H_{ps}|}[\log(1 + |H_{ps}|^2 P_p) + |H_{ss}|^2 P_s]$.

3) Regime III of Table I: For $b > 1$ and $a_1 < a \le 1$, again, based on the above compound channel explanation, the weak link the is SU–SU link; hence, C_{s2} holds. Hence, the SU receiver decoding the SU message by treating the PU message as noise determines the achievable rate.
4) Regime IV of Table I: For $b > 1$ and $1 < a \le a_2$, again, imagine the same two aforementioned MACs. Fig. 2(b) shows the capacity region for these two MACs. Unlike for the case above, the MAC2 capacity region is not completely contained in MAC1, as shown in Fig. 2(b). In fact, for this regime, we have to consider the intersection of the two MACs. This turns out to be the achievable point-to-point rate for both the SU and the PU, which constitutes as their individual constraint and the sum constraint arising from MAC1 (because $1 < a \le a_2$). Hence, the constraint C_{s4} holds, which is the corner point of this region obtained by the specific intersection where the PU attains its full rate and the SU gets C_{s4}.
5) Regime V of Table I-$b > 1$ and $a_2 < a \le a_3$: The same discussions as above are valid, with the individual rate constraints being the same but with the only difference being that the sum rate constraint is now due to MAC2 and not MAC1 (because $a_2 < a \le a_3$). Hence, the constraint C_{s1} holds, which is the corner point of this region obtained by intersection, where the PU attains full rate, and the SU gets C_{s1}.
6) Regime VI of Table I-$b > 1$ and $a > a_3$: This regime is ergodically very strong; hence, the sum-rate constraints are not binding. Each channel behaves as if it was interference free. Hence, both the PU and SU both achieve their full single-user rate.

A summary of the discussion above about the behavior of achievable rate of SU with various parameters is provided in Table I.

Fig. 3 plots the different regimes for an uncorrelated Rayleigh fading channel. For a given SNR at the PU and SU, we plot C_{sm} for different values of $a \times b \in [0.2, 2] \times [0.2, 2]$, as shown in Fig. 3. Observe that the system's behavior with respect to a and b is as characterized in Table I. The curves recorded for $a = a_1$ and $a = a_2$ are marked on the plot. The curve for $a = a_3$ occurs at very strong interference levels; hence, it is not visible in the selected range of a and b values. The curve a_1 can be seen to be a monotonically decreasing function of b; this is because when the value of b increases, the values of a for which $C_{s1} < C_{s2}$ also decreases. Similarly, a_2 is an increasing function of b because when the value of b increases the value of a for which we have $C_{s4} < C_{s1}$ increases.

V. Achievable Rates Under Imperfect Channel State Estimation

Earlier, the idealized simplifying assumption of having perfect channel knowledge of all the links at all the receivers was assumed. Naturally, in practice, this is not the case. The receivers in practice use m training symbols for estimating the channel. This technique implicitly assumes that the channel's envelope remains constant not only over the m pilot symbol duration but also during the entire transmission burst to be detected. This process is then repeated for all new bursts. Having said this, powerful decision-directed joint iterative channel and data estimators are capable of operating close to the perfect-channel scenario for the desired link, as documented in [24] and [25].

Accordingly, we consider two specific cases, namely: 1) when an estimation error is imposed only on the interfering links; and 2) when the estimation error contaminates all the links. The error in the cross links is modeled as follows. Let $\hat{H}_{ps}$ and $\hat{H}_{sp}$ represent the estimates of H_{ps} and H_{sp}, namely, that of the link between the PU and the SU and *vice versa*, respectively. Let furthermore E_{ps} and E_{sp} be the errors associated with a single channel use. Then, by performing maximum likelihood (ML) estimation over a block of m symbol duration and by applying the central limit theorem, we have [31]

$$\hat{H}_{ps} = H_{ps} + \frac{1}{\sqrt{mP_p}} E_{ps} \tag{23}$$

$$\hat{H}_{sp} = H_{sp} + \frac{1}{\sqrt{mP_s}} E_{sp}. \tag{24}$$

Note that the both E_{ps} and E_{sp} are zero-mean and unit-variance standard Gaussian RVs, i.e., they are distributed as $\mathcal{N}(0, 1)$. The error scaled by $1/\sqrt{mP}$ suggests that performing the estimation over multiple symbol duration and relying on an increased training sequence power reduces the effects of

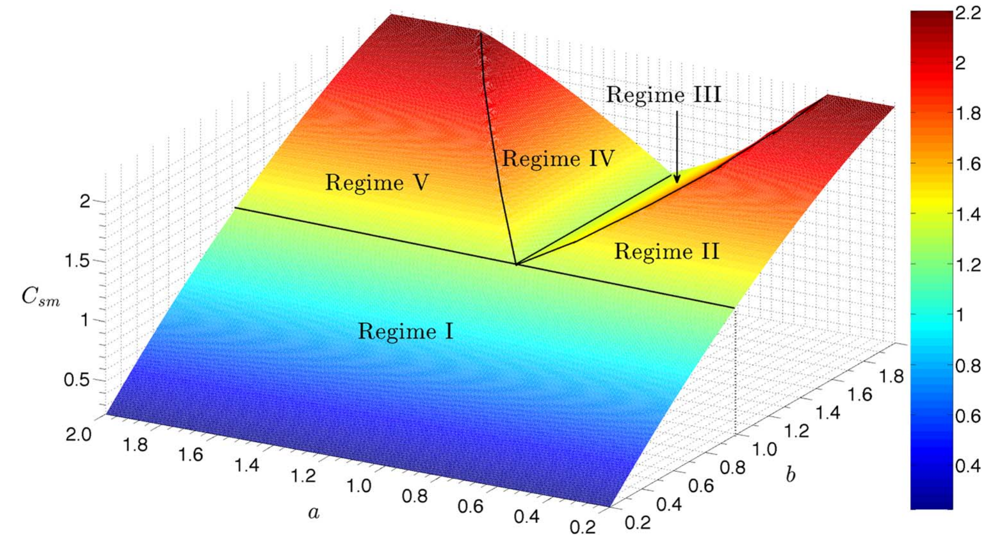

Fig. 3. Variation of the SU achievable rate C_{sm} as a function of a and b for $P_p = 200$ and $P_s = 100$.

estimation error. Thus, the baseband equations that we have are the following:

$$Y_p = H_{pp}X_p + H_{sp}X_s + Z_{pe1} \tag{25}$$

$$Y_s = H_{ss}X_s + H_{ps}X_p + Z_{se1} \tag{26}$$

where $Z_{pe1} \sim \mathcal{N}(0, 1+(1/\sqrt{mP_s}))$ and where $Z_{se1} \sim \mathcal{N}(0, 1+(1/\sqrt{mP_p}))$. This suggests that the effect of channel estimation errors simply increases the effective noise. The impact of these errors will depend upon the average transmit powers of the PU and the SU. Let $N_{p1} = 1 + (1/\sqrt{mP_s})$ and $N_{s1} = 1 + (1/\sqrt{mP_p})$.

Similarly, if there are estimation errors in all the four links, then, in addition to (23) and (24), for the direct links, we have

$$\hat{H}_{pp} = H_{pp} + \frac{1}{\sqrt{mP_p}}E_{pp} \tag{27}$$

$$\hat{H}_{ss} = H_{ss} + \frac{1}{\sqrt{mP_s}}E_{ss}. \tag{28}$$

Similar to E_{ps} and E_{sp}, E_{pp} and E_{ss} are also zero-mean and unit-variance standard Gaussian RVs, i.e., they are distributed as $\mathcal{N}(0, 1)$. Thus, the baseband equations that we have are the following:

$$Y_p = H_{pp}X_p + H_{sp}X_s + Z_{pe2} \tag{29}$$

$$Y_s = H_{ss}X_s + H_{ps}X_p + Z_{se2} \tag{30}$$

where $Z_{pe1} \sim \mathcal{N}(0, 1+(1/\sqrt{mP_s}) + (1/\sqrt{mP_p}))$, and $Z_{se1} \sim \mathcal{N}(0, 1+(1/\sqrt{mP_p})+(1/\sqrt{mP_s}))$. Let $N_{p2} = 1+(1/\sqrt{mP_s})+(1/\sqrt{mP_p})$ and $N_{s2} = 1 + (1/\sqrt{mP_p}) + (1/\sqrt{mP_s})$. Thus, $N_{s2} = N_{p2}$.

This increase in noise power requires us to characterize the achievable rates described in (3)–(9) in terms of the noise. Let N_p and N_s be the noise variance at the PU and the SU. To formulate the achievable rate regions, we replace the unit variance of the noise by N_p if the rate constraint was due to decoding at the PU and by N_s, if the rate constraint was due to decoding at the SU. Then, the achievable region is formulated as

$$R_p \le \mathbb{E}_{(|H_{pp}|)}\left[\log\left(1 + \frac{|H_{pp}|^2 P_p}{N_p}\right)\right] \tag{31}$$

$$R_s \le \mathbb{E}_{(|H_{ss}|,|H_{ps}|)}\left[\log\left(1 + \frac{|H_{ss}|^2 P_s}{\alpha|H_{ps}|^2 P_p + N_s}\right)\right] \tag{32}$$

$$R_p + R_s \le \mathbb{E}_{(|H_{pp}|)}\left[\log\left(1 + \frac{\alpha|H_{pp}|^2 P_p}{N_p}\right)\right] + \mathbb{E}_{(|H_{ss}|,|H_{ps}|)}\left[\log\left(\frac{1+|H_{ss}|^2 P_s + |H_{ps}|^2 P_p}{\alpha|H_{ps}|^2 P_p + N_s}\right)\right] \tag{33}$$

$$R_p + R_s \le \mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(1 + \frac{|H_{pp}|^2 P_p + |H_{sp}|^2 P_s}{N_p}\right)\right] \tag{34}$$

$$R_p + R_s \le \mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(1 + \frac{\alpha|H_{pp}|^2 P_p + |H_{sp}|^2 P_s}{N_p}\right)\right] + \mathbb{E}_{(|H_{ps}|)}\left[\log\left(\frac{1+|H_{ps}|^2 P_p}{\alpha|H_{ps}|^2 P_p + N_s}\right)\right] \tag{35}$$

$$2R_p + R_s \le \mathbb{E}_{(|H_{pp}|)}\left[\log\left(1 + \frac{\alpha|H_{pp}|^2 P_p}{N_p}\right)\right] + \mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(1 + \frac{|H_{pp}|^2 P_p + |H_{sp}|^2 P_s}{N_p}\right)\right] + \mathbb{E}_{(|H_{ps}|)}\left[\log\left(\frac{1+|H_{ps}|^2 P_p}{\alpha|H_{ps}|^2 P_p + N_s}\right)\right] \tag{36}$$

$$R_p + 2R_s \le \mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(1 + \frac{\alpha|H_{pp}|^2 P_p + |H_{sp}|^2 P_s}{N_p}\right)\right] + \mathbb{E}_{(|H_{ss}|,|H_{ps}|)}\left[\log\left(\frac{1+|H_{ss}|^2 P_s + |H_{ps}|^2 P_p}{\alpha|H_{ps}|^2 P_p + N_s}\right)\right]. \tag{37}$$

Consequently, the expressions for r_i, $i = \{1, \ldots, 6\}$ are as follows:

$$r_1 = \mathbb{E}_{(|H_{ss}|,|H_{ps}|)}\left[\log\left(1 + \frac{|H_{ss}|^2 P_s}{\alpha|H_{ps}|^2 P_p + N_s}\right)\right] \quad (38)$$

$$r_2 = \mathbb{E}_{(|H_{pp}|)}\left[\log\left(\frac{N_p + \alpha|H_{pp}|^2 P_p}{N_p + |H_{pp}|^2 P_p}\right)\right] + \mathbb{E}_{(|H_{ss}|,|H_{ps}|)}\left[\log\left(\frac{N_s+|H_{ss}|^2 P_s+|H_{ps}|^2 P_p}{\alpha|H_{ps}|^2 P_p+N_s}\right)\right] \quad (39)$$

$$r_3 = \mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(\frac{N_p+|H_{pp}|^2 P_p+|H_{sp}|^2 P_s}{N_s+|H_{pp}|^2 P_p}\right)\right] \quad (40)$$

$$r_4 = \mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(\frac{N_p + \alpha|H_{pp}|^2 P_p + |H_{sp}|^2 P_s}{N_p + |H_{pp}|^2 P_p}\right)\right] + \mathbb{E}_{(|H_{ps}|)}\left[\log\left(\frac{N_s + |H_{ps}|^2 P_p}{\alpha|H_{ps}|^2 P_p + N_s}\right)\right] \quad (41)$$

$$r_5 = \mathbb{E}_{(|H_{pp}|)}\left[\log\left(\frac{N_p + \alpha|H_{pp}|^2 P_p}{N_p + |H_{pp}|^2 P_p}\right)\right] + \mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(\frac{N_p + |H_{pp}|^2 P_p + |H_{sp}|^2 P_s}{N_p + |H_{pp}|^2 P_p}\right)\right] + \mathbb{E}_{(|H_{ps}|)}\left[\log\left(\frac{N_s + |H_{ps}|^2 P_p}{\alpha|H_{ps}|^2 P_p + N_s}\right)\right] \quad (42)$$

$$r_6 = \frac{1}{2}\left(\mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(\frac{N_p+\alpha|H_{pp}|^2 P_p+|H_{sp}|^2 P_s}{N_p+|H_{pp}|^2 P_p}\right)\right]\right) + \frac{1}{2}\left(\mathbb{E}_{(|H_{ss}|,|H_{ps}|)}\left[\log\left(\frac{N_s+|H_{ss}|^2 P_s+|H_{ps}|^2 P_p}{\alpha|H_{ps}|^2 P_p+N_s}\right)\right]\right). \quad (43)$$

Now, since $N_{p2} = N_{s2}$, when there are estimation errors on each link then $N_p = N_{p2} = N_s = N_{s2}$. Hence, we recover the results mentioned in Theorems 1 and 2 with only a small change in Theorem 2 as described in the following.

Theorem 3: The achievable rate of the SU, i.e., subject to the condition that the required rate of the PU of $\mathbb{E}_{(|H_{pp}|)}[\log(1 + ((|H_{pp}|^2 P_p)/N_{p2}))]$ is met under imperfect channel estimation on all four links, is given by

$$R_s \leq C_{sma} \quad (44)$$

where C_{sma} is formulated as follows:

$$C_{sma} = \begin{cases} \min(C_{s1a}, C_{s2a}), & \text{if } a \leq 1 \text{ and } b > 1 \\ \min(C_{s1a}, C_{s3a}, C_{s4a}), & \text{if } a > 1 \text{ and } b > 1 \\ C_{s1a}, & \text{if } b \leq 1 \end{cases}$$

where, we have

$$C_{s1a} = \mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(1+\frac{|H_{sp}|^2 P_s}{N_{p2}+|H_{pp}|^2 P_p}\right)\right] \quad (45)$$

$$C_{s2a} = \mathbb{E}_{(|H_{ss}|,|H_{ps}|)}\left[\log\left(1+\frac{|H_{ss}|^2 P_s}{N_{s2}+|H_{ps}|^2 P_p}\right)\right] \quad (46)$$

$$C_{s3a} = \mathbb{E}_{(|H_{ss}|)}\left[\log\left(1 + \frac{|H_{ss}|^2 P_s}{N_{s2}}\right)\right] \quad (47)$$

$$C_{s4a} = \mathbb{E}_{(|H_{ss}|,|H_{ps}|)}\left[\log\left(\frac{N_{s2}+|H_{ps}|^2 P_p+|H_{ss}|^2 P_s}{N_{p2}+|H_{pp}|^2 P_p}\right)\right]. \quad (48)$$

Proof: The proof follows from the proof of Theorem 2. This is because all the results in Lemmas 1, 2, and 3 and the proof for Theorem 1 do not depend upon the ordering or the value of N_p and N_s. ■

When only the cross links are contaminated by the channel estimation error, then there are two possibilities: Either $N_{p1} \leq N_{s1}$ or $N_{p1} > N_{s1}$. The condition $N_{p1} \leq N_{s1}$ translates to $P_p \geq P_s$, which can be assumed to be reasonable. In this case, again, the results of Theorems 1 and 2 hold.

Theorem 4: The achievable rate of the SU, subject to the condition that the required rate of the PU of $\mathbb{E}_{(|H_{pp}|)}[\log(1 + ((|H_{pp}|^2 P_p)/N_{p2}))]$ is met under imperfect channel estimation only on the interfering links with $P_p \geq P_s$, is given by

$$R_s \leq C_{smi} \quad (49)$$

where C_{smi} is formulated as follows:

$$C_{smi} = \begin{cases} \min(C_{s1i}, C_{s2i}), & \text{if } a \leq 1 \text{ and } b > 1 \\ \min(C_{s1i}, C_{s3i}, C_{s4i}), & \text{if } a > 1 \text{ and } b > 1 \\ C_{s1i}, & \text{if } b \leq 1 \end{cases}$$

where, we have

$$C_{s1i} = \mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(1+\frac{|H_{sp}|^2 P_s}{N_{p1}+|H_{pp}|^2 P_p}\right)\right] \quad (50)$$

$$C_{s2i} = \mathbb{E}_{(|H_{ss}|,|H_{ps}|)}\left[\log\left(1+\frac{|H_{ss}|^2 P_s}{N_{s1}+|H_{ps}|^2 P_p}\right)\right] \quad (51)$$

$$C_{s3i} = \mathbb{E}_{(|H_{ss}|)}\left[\log\left(1+\frac{|H_{ss}|^2 P_s}{N_{s1}}\right)\right] \quad (52)$$

$$C_{s4i} = \mathbb{E}_{(|H_{ss}|,|H_{ps}|)}\left[\log\left(\frac{N_{s1}+|H_{ps}|^2 P_p+|H_{ss}|^2 P_s}{N_{p1}+|H_{pp}|^2 P_p}\right)\right]. \quad (53)$$

Proof: The proof follows from the proof of Theorem 2 and the fact that the conditions $r_2|_{\alpha=1} > r_3$ for $a, b \leq 1$, and $r_2|_{\alpha=0} > r_3$ for $a > 1$, $b \leq 1$ are satisfied only when $N_{p1} \leq N_{s1}$. ■

For the case when we have $N_{p1} < N_{s1}$, the conditions $r_2|_{\alpha=1} > r_3$ for $a, b \leq 1$, and $r_2|_{\alpha=0} > r_3$ for $a > 1$ and $b \leq 1$ are not necessarily true. Hence, we have the following result.

Theorem 5: The achievable rate of the SU, subject to the condition that the required rate of the PU of $\mathbb{E}_{(|H_{pp}|)}[\log(1 + ((|H_{pp}|^2 P_p)/N_{p2}))]$ is met under having imperfect channel estimation only for the interfering links with $P_p < P_s$ is given by

$$R_s \leq C_{sme} \quad (54)$$

where C_{sme} is formulated as follows:

$$C_{sme} = \begin{cases} \min(C_{s1e}, C_{s2e}), & \text{if } a \leq 1 \\ \min(C_{s1e}, C_{s3e}, C_{s4e}), & \text{if } a > 1 \text{ and } b > 1 \\ \min(C_{s1e}, C_{s4e}), & \text{if } a > 1 \text{ and } b \leq 1 \end{cases}$$

where we have

$$C_{s1e} = \mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(1 + \frac{|H_{sp}|^2 P_s}{N_{p1} + |H_{pp}|^2 P_p}\right)\right] \quad (55)$$

$$C_{s2e} = \mathbb{E}_{(|H_{ss}|,|H_{ps}|)}\left[\log\left(1 + \frac{|H_{ss}|^2 P_s}{N_{s1} + |H_{ps}|^2 P_p}\right)\right] \quad (56)$$

$$C_{s3e} = \mathbb{E}_{(|H_{ss}|)}\left[\log\left(1 + \frac{|H_{ss}|^2 P_s}{N_{s1}}\right)\right] \quad (57)$$

$$C_{s4e} = \mathbb{E}_{(|H_{ss}|,|H_{ps}|)}\left[\log\left(\frac{N_{s1} + |H_{ps}|^2 P_p + |H_{ss}|^2 P_s}{N_{p1} + |H_{pp}|^2 P_p}\right)\right]. \quad (58)$$

Proof: The expressions of the achievable rates under $b \leq 1$ and $b > 1$ turn out to be the same, which is the minimum of $\min(C_{s1e}, C_{s2e})$. Hence, unlike the previous results in Theorems 2–4, the achievable rate for $b \leq 1$ does not have the same expression, whereas now for $a \leq 1$, the characterization is the same. ■

Hence, the effect of channel estimation errors *does not* change the optimal structure of the rate sharing parameter described in Theorem 1. Moreover, when all the links have estimation errors and when only the cross-links have estimation error associated with $P_s \geq P_p$, then the formulation of the achievable rate remains similar to that of the perfect estimation scenario, with the only difference being the addition of the general noise variance terms of N_p and N_s instead of unity. When only the cross-links have an estimation error associated with $P_s \geq P_p$, then the description of the achievable rate changes in the regimes of $a \leq 1$, $b > 1$, and $a > 1$, $b \leq 1$ regimes.

Note that the extra terms in the variance, i.e., $(1/\sqrt{mP_p}) + (1/\sqrt{mP_s})$ that arise are quite small, particularly when the value of m is high. However, a high-Doppler fading channel will change substantially for a large value of m. Nevertheless, if the average transmit power values P_p and P_s are high enough, the impact of channel estimation errors can be reduced to a small value. By contrast, if the transmit power values are insufficiently high and they are combined with a small value of m, this might affect the achievable rates significantly.

VI. Conclusion

In this paper, a new information-theoretic model was con ceived for underlay-based CR. By extending the Han–Kobayashi achievable rate region to fading interference channels, we determined the optimal rate sharing parameters for both the SU and the PU that satisfy the relevant constraints and maximize the achievable rates. Furthermore, we provided a detailed analysis of the binding constraints accompanied by their conceptual interpretation. Then, we provided an analysis of the realistic imperfect channel estimation scenario. It was demonstrated that, despite having channel estimation errors, the optimal structure of the rate sharing parameter remains the same.

Appendix A
Supporting Lemmas

Lemma 1: r_1 is a monotonically decreasing function of α for all a, whereas r_2 and r_5 are monotonically decreasing functions of α for $a > 1$ and are monotonically increasing functions of α for $a \leq 1$.

Proof: This follows from the fact that the $\log(1+x)$ function is a strictly increasing function of x. Hence, for a pair of bounded RVs X and Y, if $\mathbb{E}[X] > \mathbb{E}[Y]$ is satisfied, then we have $\mathbb{E}[\log(1+X)] > \mathbb{E}[\log(1+Y)]$. A rigorous proof involving differentiations can be provided for any of the known fading distributions. ■

Lemma 2: From (10)–(15), it is sufficient to consider only the three rate constraints r_2, r_3, and r_5 for $a < 1$ and four rate constraints r_1, r_2, r_3, and r_5 for $a > 1$.

Proof: We have to show that the constraint of r_1 for $a < 1$ is redundant, whereas the constraints of r_4 and r_6 are always redundant.

For r_1, we show that, if we have $a < 1$, then $r_1 \geq r_2$.

From Lemma 1, if $a < 1$, then r_2 is a monotonically increasing function of α, whereas r_1 is always a monotonically decreasing function of α. Furthermore, we have $r_1|_{\alpha=1} = r_2|_{\alpha=1}$. Hence, for $a < 1$, $r_1 \geq r_2$ is satisfied.

For r_4, we show that $r_4 \geq r_5$ is valid for all a since we have

$$\begin{aligned} r_4 - r_5 &= \mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(\frac{1+\alpha|H_{pp}|^2P_p+|H_{sp}|^2P_s}{1+\alpha|H_{pp}|^2P_p}\right)\right] \\ &\quad - \mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(1+\frac{|H_{sp}|^2P_s}{1+|H_{pp}|^2P_p}\right)\right] \\ &= \mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(1+\frac{|H_{sp}|^2P_s}{1+\alpha|H_{pp}|^2P_p}\right)\right] \\ &\quad - \mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(1+\frac{|H_{sp}|^2P_s}{1+|H_{pp}|^2P_p}\right)\right] \\ &\geq 0. \end{aligned} \tag{59}$$

Thus, $r_4 \geq r_5$ is satisfied.

For r_6, we show that $r_6 \geq \min(r_2, r_3)$ is satisfied for all a. Observing that

$$\begin{aligned} r_6 - \frac{r_2}{2} &= \frac{1}{2}\mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(\frac{1+\alpha|H_{pp}|^2P_p+|H_{sp}|^2P_s}{1+|H_{pp}|^2P_p}\right)\right] \\ &\quad - \mathbb{E}_{(|H_{pp}|)}\left[\log\left(\frac{1+\alpha|H_{pp}|^2P_p}{1+|H_{pp}|^2P_p}\right)\right] \end{aligned} \tag{60}$$

$$\begin{aligned} \text{or } r_6 &= \frac{r_2}{2} + \frac{1}{2}\mathbb{E}_{(|H_{pp}|,|H_{sp}|)} \\ &\quad \times \left[\log\left(\frac{1+\alpha|H_{pp}|^2P_p+|H_{sp}|^2P_s}{1+\alpha|H_{pp}|^2P_p}\right)\right] \end{aligned} \tag{61}$$

$$= \frac{r_2}{2} + \frac{1}{2}\mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(1+\frac{|H_{sp}|^2P_s}{1+\alpha|H_{pp}|^2P_p}\right)\right] \tag{62}$$

$$\geq \frac{r_2}{2} + \frac{r_3}{2} = \frac{r_2+r_3}{2} \geq \min\left(r_2, r_3\right). \tag{63}$$

Lemma 2 is proven. ■

Appendix B
Proof of Theorem 1

From Lemma 2, we established that, for $a<1$, only the rate constraints r_2, r_3, and r_5 are binding. Hence, we have

$$C_{sm} = \min\left(r_3, \max_{\alpha\in[0,1]}\{\min(r_2, r_5,)\}\right). \tag{64}$$

From Lemma 1, we note that functions r_2 and r_5 are monotonically increasing functions of α if $a \leq 1$. Hence, we have

$$\arg\max_{\alpha\in[0,1]}\{\min(r_2, r_5,)\} = 1.$$

Since r_3 is independent of α, if the constraint r_3 is binding, we can select $\alpha = 1$ as the default value. Hence, $\alpha = 1$ is optimal for $a \leq 1$.

Following the same line of argument, we can establish that $\alpha = 0$ is optimal for $a > 1$. ■

APPENDIX C
PROOF OF THEOREM 2

For the condition of $a > 1$ and $b > 1$, the value of C_{sm} is obtained by selecting the minimum of r_1, r_2, r_3 and r_5 evaluated at $\alpha = 0$. It can be shown that $r_5|_{\alpha=0} > r_3$ for $a > 1$. Hence, for $a > 1$ and $b > 1$, we have $C_{sm} = \min(r_1|_{\alpha=0}, r_2|_{\alpha=0}, r_3)$.

For the condition of $a \le 1$ and $b > 1$, the value of C_{sm} is obtained by taking the minimum of r_2, r_3 and r_5 evaluated at $\alpha = 1$. Since, we have $r_5|_{\alpha=1} = r_3$, hence, for $a \le 1$ and $b > 1$, we arrive at $C_{sm} = \min(r_2|_{\alpha=1}, r_3)$.

For the condition of $b \le 1$ and $a \le 1$, $r_2|_{\alpha=1} \ge r_3$ holds. Hence, $C_{sm} = r_3$.

For the condition of $b \le 1$ and $a > 1$, $r_1|_{\alpha=0} > r_3$ hold. The only fact that remains to be shown is that $r_2|_{\alpha=0} > r_3$. To show this, we demonstrate that

$$\mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(\frac{1+|H_{pp}|^2P_p+|H_{sp}|^2P_s}{1+|H_{ps}|^2P_p+|H_{ss}|^2P_s}\right)\right] < 0.$$

To show this, we observe that

$$\mathbb{E}_{(|H_{pp}|,|H_{sp}|,|H_{ss}|,|H_{ps}|)}\left[\log\left(\frac{1+|H_{pp}|^2P_p+|H_{sp}|^2P_s}{1+|H_{ps}|^2P_p+|H_{ss}|^2P_s}\right)\right]$$
$$\le \mathbb{E}_{(|H_{pp}|,|H_{sp}|,|H_{ss}|)}\left[\log\left(\frac{1+|H_{pp}|^2P_p+|H_{sp}|^2P_s}{1+|H_{pp}|^2P_p+|H_{ss}|^2P_s}\right)\right] \tag{65}$$
$$= \mathbb{E}_{(|H_{pp}|,|H_{sp}|,|H_{ss}|)}\left[\log\left(\frac{1+\frac{|H_{sp}|^2P_s}{1+|H_{pp}|^2P_p}}{1+\frac{|H_{ss}|^2P_s}{1+|H_{pp}|^2P_p}}\right)\right] \tag{66}$$
$$\le 0. \tag{67}$$

■

REFERENCES

[1] FCC, "Report of the spectrum efficiency working group," FCC Spectrum Policy Task Force, Washington, DC, USA Tech. Rep., 2002.

[2] J. Mitola and G. Q. Maguire Jr., "Cognitive radio: Making software radios more personal," *IEEE Pers. Commun.*, vol. 6, no. 4, pp. 13–18, Aug. 1999.

[3] S. Haykin, "Cognitive radio: Brain-empowered wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 23, no. 2, pp. 201–220, Feb. 2005.

[4] I. F. Akyildiz, W.-Y. Lee, M. C. Vuran, and S. Mohanty, "Next generation/ dynamic spectrum access/cognitive radio wireless networks: A survey," *Comput. Netw.*, vol. 50, no. 13, pp. 2127–2159, Sep. 2006.

[5] K. N. Mohammad, G. Khoshkholgh, and H. Yanikomeroglu, "Access strategies for spectrum sharing in fading environment: Overlay, underlay, and mixed," *IEEE Trans. Mobile Comput.*, vol. 9, no. 12, pp. 1780–1793, Mar. 2010.

[6] N. Yi, Y. Ma, and R. Tafazolli, "Underlay cognitive radio with full or partial channel quality information," *Int. J. Navigat. Observ.*, vol. 2010, 2010, Art. ID 105723.

[7] G. Amir and S. S. Elvino., "Fundamental limits of spectrum-sharing in fading environments," *IEEE Trans. Wireless Commun.*, vol. 6, no. 2, pp. 649–658, Feb. 2007.

[8] L. B. Le and E. Hossain, "Resource allocation for spectrum underlay in cognitive radio networks," *IEEE Trans. Wireless Commun.*, vol. 7, no. 12, pp. 5306–5315, Dec. 2008.

[9] M. Filippou, D. Gesbert, and G. Ropokis, "Underlay versus interweaved cognitive radio networks: A performance comparison study," in *Proc. 9th Int. Conf. CROWNCOM*, Jun. 2014, pp. 226–231.

[10] M. C. Filippou, D. Gesbert, and G. A. Ropokis, "A comparative performance analysis of interweaved and underlay multi-antenna cognitive radio networks," *IEEE Trans. Wireless Commun.*, vol. 14, no. 5, pp. 2911–2925, Jan. 2015.

[11] L. Musavian and S. Aïssa, "Fundamental capacity limits of cognitive radio in fading environments with imperfect channel information," *IEEE Trans. Commun.*, vol. 57, no. 11, pp. 3472–3480, Nov. 2009.

[12] D. Xu, Z. Feng, and P. Zhang, "On the impacts of channel estimation errors and feedback delay on the ergodic capacity for spectrum sharing cognitive radio," *Wireless Pers. Commun.*, vol. 72, no. 4, pp. 1875–1887, Oct. 2013.

[13] L. Sboui, Z. Rezki, and M.-S. Alouini, "A unified framework for the ergodic capacity of spectrum sharing cognitive radio systems," *IEEE Trans. Wireless Commun.*, vol. 12, no. 2, pp. 877–887, Feb. 2013.

[14] A. Goldsmith, S. Jafar, I. Maric, and S. Srinivasa, "Breaking spectrum gridlock with cognitive radios: An information theoretic perspective," *Proc. IEEE*, vol. 97, no. 5, pp. 894–914, May 2009.

[15] M. C. Filippou, G. A. Ropokis, and D. Gesbert, "A team decisional beamforming approach for underlay cognitive radio networks," in *Proc. IEEE 24th Int. Symp. PIMRC*, Sep. 2013, pp. 575–579.

[16] P. de Kerret, M. Filippou, and D. Gesbert, "Statistically coordinated precoding for the miso cognitive radio channel," in *Proc. 48th Asilomar Conf. Signals, Syst. Comput.*, Nov. 2014, pp. 1083–1087.

[17] X. Kang, R. Zhang, Y.-C. Liang, and H. K. Garg, "Optimal power allocation strategies for fading cognitive radio channels with primary user outage constraint," *IEEE J. Sel. Areas Commun.*, vol. 29, no. 2, pp. 374–383, Feb. 2011.

[18] N. Devroye, P. Mitran, and V. Tarokh, "Achievable rates in cognitive radio channels," *IEEE Trans. Inf. Theory*, vol. 52, no. 5, pp. 1813–1827, May 2006.

[19] L. Sboui, Z. Rezki, and M.-S. Alouini, "Achievable rate of spectrum sharing cognitive radio systems over fading channels at low-power regime," *IEEE Trans. Wireless Commun.*, vol. 13, no. 11, pp. 6461–6473, Nov. 2014.

[20] T. Han and K. Kobayashi, "A new achievable rate region for the interference channel," *IEEE Trans. Inf. Theory*, vol. IT-27, no. 1, pp. 49–60, Jan. 1981.

[21] H.-F. Chong, M. Motani, H. K. Garg, and H. El Gamal, "On the Han Kobayashi region for the interference channel," *IEEE Trans. Inf. Theory*, vol. 54, no. 7, pp. 3188–3195, Jul. 2008.

[22] R. Etkin, D. Tse, and H. Wang, "Gaussian interference channel capacity to within one bit," *IEEE Trans. Inf. Theory*, vol. 54, no. 12, pp. 5534–5562, Dec. 2008.

[23] A. El Gamal, Y.-H. Kim, *Network Information Theory*. Cambridge, U.K.: Cambridge Univ. Press, 2012.

[24] L. Hanzo, M. Münster, B. J. Choi, and T. Keller, *OFDM and MC-CDMA for Broadband Multi-User Communications, WLANs and Broadcasting*, Hoboken, NJ, USA: Wiley, Jul. 2003

[25] L. Hanzo, J. Akhtman, L. Wang, and M. Jiang, *MIMO-OFDM for LTE, WIFI and WIMAX: Coherent Versus Non-Coherent and Cooperative Turbo-Transceivers*. Hoboken, NJ, USA: IEEE Press—Wiley, Mar. 2010,

[26] H. Sato, "The capacity of the Gaussian interference channel under strong interference," *IEEE Trans. Inf. Theory*, vol. IT-27, no. 6, pp. 786–788, Nov. 1981.

[27] A. Carleial, "A case where interference does not reduce capacity," *IEEE Trans. Inf. Theory*, vol. IT-21, no. 5, pp. 569–570, Sep. 1975.

[28] V. Annapureddy and V. Veeravalli, "Gaussian interference networks: Sum capacity in the low-interference regime and new outer bounds on the capacity region," *IEEE Trans. Inf. Theory*, vol. 55, no. 7, pp. 3032–3050, Jul. 2009.

[29] L. Sankar, X. Shang, E. Erkip, and H. Poor, "Ergodic fading interference channels: Sum-capacity and separability," *IEEE Trans. Inf. Theory*, vol. 57, no. 5, pp. 2605–2626, May 2011.

[30] R. Farsani, "The capacity region of the wireless ergodic fading interference channel with partial CSIT to within one bit," in *Proc. IEEE ISIT*, Jul. 2013, pp. 759–763.

[31] M. Khan, "Achieving exponential diversity with spatiotemporal power allocation with imperfect channel state information," in *Proc. NCC*, Jan. 2011, pp. 1–5.

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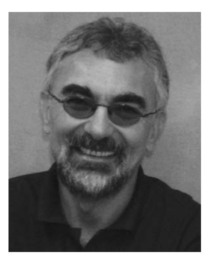

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AUTHOR QUERIES

AUTHOR PLEASE ANSWER ALL QUERIES

AQ1 = The sentence was modified for clarity. Please check if the following changes are appropriate. If not, kindly provide the necessary corrections.

AQ2 = Please provide specific year when the degrees were received by author "S. N. Merchant."

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Achievable Rates of Underlay-Based Cognitive Radio Operating Under Rate Limitation

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***Abstract*—A new information-theoretic model is proposed for underlay-based cognitive radio (CR), which imposes rate limitation on the secondary user (SU), whereas the traditional systems impose either interference or transmit power limitations. The channel is modeled as a twin-user interference channel constituted by the primary user (PU) and the SU. The achievable rate of the SU is derived based on the inner bound formulated by Han and Kobayashi, where the PU achieves the maximum attainable rate of the single-user point-to-point link. We show that it is necessary for the SU to allocate its full power for the "public" message that can be decoded both by the SU and by the PU. We also demonstrate that it is optimal for the PU to allocate its full power for the "private" message that can only be decoded by the PU if the level of interference imposed by the PU on the SU is "ergodically strong." Similarly, it is optimal for the PU to allocate its full power for the public message that can be decoded both by the SU and PU if this interference is "ergodically weak." These findings suggest that this power allocation is independent of the level of interference imposed by the SU on the PU. Furthermore, the achievable rate is analyzed as a function of the average level of interference. An interesting observation is that if the level of interference imposed by the SU on the PU is "ergodically weak," the achievable rate becomes a monotonically increasing function of this interference, and it is independent of the level of interference imposed by the PU on the SU. Furthermore, we analyze the realistic imperfect channel estimation scenario and demonstrate that the channel estimation errors will not affect the optimal nature of the SU's power allocation.**

***Index Terms*—Cognitive radio (CR), interference limitation, rate limitation, underlay.**

I. Introduction

THE conventional fixed spectrum allocation policy of wireless transmissions has led to much of the spectrum being underutilized, whereas some bands are becoming overcrowded due to the avalanche-like proliferation of wireless devices [1]. Cognitive radio (CR)-based spectrum sharing is seen as a possible solution to the problem of inefficient spectrum utilization [2]–[4]. There are various notions of spectrum sharing. One of the most popular versions is the underlay-based spectrum sharing [5]–[14]. In underlay, the basic cognition is associated with near-instantaneously estimating the interfering link's gain at the receivers but, in the advanced scenario, interfering link's gain at the transmitters is also included. Moreover, the traditional approach of underlay-based CR introduces a new parameter for characterizing the interference temperature defined in [3], which limits the aggregate interference that the CRs may inflict upon the primary user (PU), so that the PU still achieves data rates that satisfy its quality-of-service requirement. This interference temperature limit can either be imposed as a peak interference constraint or as an average interference constraint. These constraints directly translate to the corresponding peak transmit power or average transmit power constraints to be assigned at the transmitters.

The objective of this paper is to quantify the achievable rates of the secondary user (SU) without inflicting any rate loss upon the PU. This requires us to consider the PU–SU system from an information-theoretic perspective. In contrast to the traditional interference limitation or transmit power limitation constraints imposed on the SU in [5], [7], [8], [12], and [13], we impose a rate constraint on the SU. This constrained rate would be the maximum rate that the SU is capable of achieving *without affecting the PU's transmission rate*, namely the rate at which the PU is capable of reliably transmitting in the single-user point-to-point scenario. Indeed, a rate constraint has been imposed on the SU also in some of previous contributions [15], [16]; however, the aim in those prior contributions was to maximize the SU's rate over the different possible beamforming vectors, whereas the interference imposed both on the SU and PU was assumed additive noise. The information-theoretic literature routinely exploits that when the interference level is high, it can be readily canceled. Hence, in this CR scenario, this assumption would imply that both the PU and the SU succeed in partially canceling the interference and thereby become capable of increasing their individual rates. This line of thought was adapted for example in [6], albeit the authors' aim was to quantify the penalty that had to be tolerated by the PU when subjected to the interference imposed by the SU. In other contributions [9]–[11], [17], an interference temperature constraint was imposed, which led to a more meaningful outage constraint that had to be satisfied by the PU.

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The proposed rate limitation differs from the existing interference temperature and outage constraint model in terms of the following five aspects.

- The rate limitation observed by the SU allows the PU to communicate at the full rate of the point-to-point scenario, which is not possible when an interference constraint is imposed, as explicitly noted in [6].
- The rate limitation approach relies on the idealized simplifying assumption of using perfect capacity-achieving coding techniques at both the SU and the PU, which allows us to detect, decode, and subtract the interference at both the SU and PU. By contrast, in the case of the interference-limited approach, this interference removal is not exploited since the interference is treated as noise [5], [8]; hence, the advantages of the aforementioned sophisticated coding techniques cannot be readily exploited for interference cancelation. However, in contrast to the overlay CR concept [14], [18] no causal or noncausal message of the PU is available at the SU.
- It will be shown that this approach allows for the SU rate to vary according to the average interference levels, even when the channel information is unknown at the transmitter. By contrast this is not possible in the interference-temperature-based model, which treats both the PU and SU channels as an additive white Gaussian noise channel and treats the interference as additional noise.
- By contrast, our approach of limiting the rate allows us to evaluate the simultaneously achievable rates of the PU and SU. In contrast to most existing contributions on underlay-based CR, which do not consider the effect of any ongoing PU transmission at the SU receiver [13], [19], we are able to do so. This is also another beneficial feature of our solution.
- In contrast to the outage constraint, the PU always maintains a reliable ergodic achievable rate in the context of the rate-limited model.

To quantify the achievable rates of the SU, the Han–Kobayshi achievable rate region [20], [21] is invoked. This rate region was derived for a scenario having fixed channel coefficients, which is also in line with the capacity estimates of [22], [23]. Moreover, in all the regimes where either the capacity [26], [27] or the sum capacity is known [28], this achievable rate region turns out to be tight. For the fading scenario, the optimality of many of the results remains an open challenge to prove analytically. However, the results in [29] and [30] indicate that the Han–Kobayashi region extended to the fading case may be approximately optimal in various scenarios.

In light of these discussions, the major contributions of this paper are as follows.

- The achievable rates are determined for the SU without inflicting any rate loss upon the PU.
- It is shown that, in the specific scenarios, when the interference imposed by the PU on the SU is ergodically strong, regardless of the level of interference inflicted by the SU on the PU, then it is optimal to detect, demodulate, and cancel the interference imposed by the SU on the PU. By contrast, in the opposite scenario, it is better to treat this interference as noise.
- It is also shown that the achievable rate of the SU is an increasing function of the interference imposed by the SU on the PU, when the level of this interference is ergodically weak[1] and that the SU rate is independent of the level of interference imposed by the PU on the SU. If, however, the level of interference imposed by the SU on the PU is ergodically strong, the achievable rate of the SU is shown to be a decreasing function of the level of interference imposed by the PU on the SU, provided that the PU interference is ergodically weak. The opposite trend prevails if this interference is ergodically strong.
- Analysis for the case when there is error in the channel state estimation process is also studied. It is shown that the conditions under which it is optimal to detect, demodulate, and cancel the interference imposed by the SU on the PU in the case with error in estimation is the same as when there is no error. The only difference that arises is in the structure of the achievable rates in certain regimes (described in detail later) and in the effective noise variances at the PU and the SU receiver that appear in the expressions of the achievable rates.

This paper is structured as follows. Section II describes the system model and introduces the problem followed by our main results presented in Section III. In Section IV, the analysis of the derived results sheds light on their nature. In Section V analyzes the achievable rate when there is error in channel state information. Finally, we conclude in Section V.

II. System Model and Problem Statement

Let us consider an underlay CR system, where the PU is transmitting at random instants, where p is the probability that the PU is silent. The SU transmits at a *low rate*, so that the PU and SU can communicate simultaneously without the PU having to reduce its transmission rate.

The channel is shown in Fig. 1, which is modeled as follows:

$$Y_p = H_{pp} S_p X_p + H_{sp} X_s + Z_p \tag{1}$$

$$Y_s = H_{ps} S_p X_p + H_{ss} X_s + Z_p \tag{2}$$

where Y_p and Y_s are the outputs at the PU and the SU receivers, respectively, in response to the inputs X_p at the PU and X_s at the SU. The power constraints of the PU and SU on their transmit rate are $\mathbb{E}[|X_p|^2] \leq P_p$ and $\mathbb{E}[|X_p s^2] \leq P_s$. The random variable (RV) $S_p = \{0, 1\}$ indicates whether the PU transmission is ON or OFF, with $S_p = 1$ indicating that the transmission is ON. Hence, we have $\Pr[S_p = 1] = 1 - p$. The value of S_p is not known at the SU transmitter and receiver. The instantaneous channel coefficient of the PU-to-PU link is

[1]Ergodically weak interference is said to be imposed by the SU on the PU if the average value of this interfering link is below unity. By contrast, the interference is deemed to be ergodically strong if it is higher than unity. A precise definition is provided in the system model.

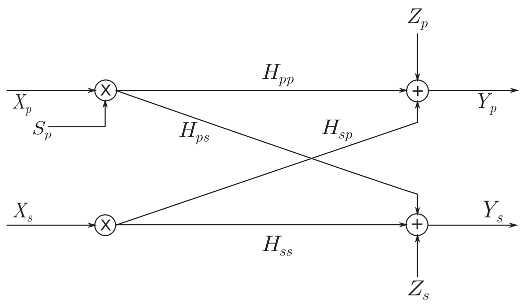

Fig. 1. Underlay channel scenario. Here, $\mathbb{E}[\|H_{pp}\|^2] = 1$, $\mathbb{E}[|H_{ss}|^2] = 1$, $\mathbb{E}[|H_{sp}|^2] = b^2$, and $\mathbb{E}[|H_{ps}|^2] = a^2$. The noise $Z_p \sim \mathcal{N}(0, 1)$, and $Z_s \sim \mathcal{N}(0, 1)$. The input $\mathbb{E}[|X_p|^2] = P_p$, and $\mathbb{E}[|X_s|^2] = P_s$.

denoted by the RV H_{pp}, that of the SU-to-SU link by H_{ss}, that of the interfering PU-to-SU link by H_{ps}, and that of the interfering SU-to-PU link by H_{sp}. All these value are complex. We assume that all the instantaneous channel coefficients are known at the PU and SU receivers and the distribution of these are known at the PU and SU transmitter in conjunction with $\mathbb{E}[|H_{pp}|^2] = 1$, $\mathbb{E}[|H_{ss}|^2] = 1$, $\mathbb{E}[|H_{sp}|^2] = b^2$, and $\mathbb{E}[|H_{ps}|^2] = a^2$. The noise is denoted by the RVs Z_p and Z_s, which are zero-mean unit-variance Gaussian RVs. Both the fading and the noise RVs are assumed to be independent and identically distributed (i.i.d.) over time.

We state that the PU's receiver faces ergodically strong interference from the SU if $b > 1$, whereas it faces ergodically weak interference if $b \leq 1$. Similarly, the SU receiver faces ergodically strong interference from the PU if $a > 1$, and it faces ergodically weak interference if $a \leq 1$.

The question that we ask now is as follows: What rates can be achieved for the SU subject to the fact that the PU rate is the same as that in the point-to-point single-link case, when no interference arrives from the SU? The answer to this is derived from the Han–Kobayashi achievable region [20], [21], [23], [30] for the twin-user interference channel. The two users of the interference channel in our case are the PU and the SU. The scheme proposed by Han and Kobayashi [20], [23] involves splitting of the messages of both the PU and SU into two parts, namely the part which is decoded at both the receivers and the other which is only decoded at its respective desired receivers. The messages that are decoded at both the receivers are referred to as "public" messages, whereas those that are decoded only at the respective receiver are termed as the "private" message. Accordingly, the PU assigns a fraction α of the power P_p to its private message, whereas the SU dedicates a fraction β of the power P_s to its private messages. The fractions α and β are referred to as rate sharing parameters. For the PU to achieve its full single-user transmission rate, the PU should be able to perfectly decode the interference; hence, all the SU messages should be public messages. This requires that the rate sharing parameter at the SU be zero, i.e., $\beta = 0$. We now formulate the following proposition that quantifies the Han–Kobayashi achievable rate region for $\beta = 0$. The complete rate region with partial side information is given in [30].

Proposition 1: The Han–Kobayashi achievable rate region of a two-user Gaussian fading interference channel is characterized in [30], which is reproduced for $\beta = 0$ using the following notation:

$$R_p \leq \mathbb{E}_{(|H_{pp}|)}\left[\log\left(1 + |H_{pp}|^2 P_p\right)\right] \tag{3}$$

$$R_s \leq \mathbb{E}_{(|H_{ss}|,|H_{ps}|)}\left[\log\left(1 + \frac{|H_{ss}|^2 P_s}{\alpha|H_{ps}|^2 P_p + 1}\right)\right] \tag{4}$$

$$\begin{aligned} R_p + R_s \leq\, & \mathbb{E}_{(|H_{pp}|)}\left[\log\left(1 + \alpha|H_{pp}|^2 P_p\right)\right] \\ & + \mathbb{E}_{(|H_{ss}|,|H_{ps}|)}\left[\log\left(\frac{1+|H_{ss}|^2 P_s+|H_{ps}|^2 P_p}{\alpha|H_{ps}|^2 P_p+1}\right)\right] \end{aligned} \tag{5}$$

$$R_p + R_s \leq \mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(1 + |H_{pp}|^2 P_p + |H_{sp}|^2 P_s\right)\right] \tag{6}$$

$$\begin{aligned} R_p + R_s \leq\, & \mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(1 + \alpha|H_{pp}|^2 P_p + |H_{sp}|^2 P_s\right)\right] \\ & + \mathbb{E}_{(|H_{ps}|)}\left[\log\left(\frac{1 + |H_{ps}|^2 P_p}{\alpha|H_{ps}|^2 P_p + 1}\right)\right] \end{aligned} \tag{7}$$

$$\begin{aligned} 2R_p + R_s \leq\, & \mathbb{E}_{(|H_{pp}|)}\left[\log\left(1 + \alpha|H_{pp}|^2 P_p\right)\right] \\ & + \mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(1 + |H_{pp}|^2 P_p + |H_{sp}|^2 P_s\right)\right] \\ & + \mathbb{E}_{(|H_{ps}|)}\left[\log\left(\frac{1 + |H_{ps}|^2 P_p}{\alpha|H_{ps}|^2 P_p + 1}\right)\right] \end{aligned} \tag{8}$$

$$\begin{aligned} R_p + 2R_s \leq\, & \mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(1 + \alpha|H_{pp}|^2 P_p + |H_{sp}|^2 P_s\right)\right] \\ & + \mathbb{E}_{(|H_{ss}|,|H_{ps}|)}\left[\log\left(\frac{1+|H_{ss}|^2 P_s+|H_{ps}|^2 P_p}{\alpha|H_{ps}|^2 P_p + 1}\right)\right]. \end{aligned} \tag{9}$$

Let us now provide an interpretation of (3)–(9), where (3) and (4) describe the individually achievable rates of the PU and SU, respectively. This is followed by the three sum-rate constraints $(R_p + R_s)$ in (5)–(7), where the first term in (5) represents the public message of the PU decoded at the PU receiver, whereas the second term represents the private message of the PU and the complete message (public and private both) of the SU decoded at the SU. The sum rate constraint in (6) represents the complete message decoding process of both the PU and the SU at the PU receiver. In (7), the first term represents the private message of the PU and the complete message of the SU decoded at the PU receiver, whereas the second term represents the public message of the PU decoded at the SU receiver. The first term of the constraint in (8) represents the private message of the PU decoded at the PU receiver, the second term represents the complete message of both the PU and the SU decoded at the PU receiver, and the third term represents the public message of the PU decoded at the SU receiver, resulting in a rate of $(2R_p + R_s)$. Finally, in (9) the first term represents the private message decoding process of the PU and the complete message decoding of the SU at the PU receiver, whereas the second term represents the public message decoding process of the PU and the complete message decoding process of the SU at the SU receiver, resulting in the rate of $(R_p + 2R_s)$. All the PU rate constraints R_p arise either because the PU decodes its private message at its receiver and its public message at the SU receiver or because it decodes its complete message at its receiver. However, the SU rate constraint R_s is a consequence of the PU ability to decode the full message of the SU at its receiver.

Our aim is to find what is the maximum achievable SU rate C_{sm} subject to the PU rate given in (3) and to find the corresponding rate sharing parameter at the PU that achieves this. The solution is obtained by solving the following proposition.

Proposition 2: The achievable rate C_{sm} of the SU is given by

$$C_{sm} = \min\left(r_3, \max_{\alpha\in[0,1]} \{\min(r_1, r_2, r_4, r_5, r_6)\}\right)$$

where r_i, $i = \{1, 2, 3, 4, 5, 6\}$, are as given in the following:

$$r_1 = \mathbb{E}_{(|H_{ss}|,|H_{ps}|)}\left[\log\left(1+\frac{|H_{ss}|^2P_s}{\alpha|H_{ps}|^2P_p+1}\right)\right] \tag{10}$$

$$\begin{aligned} r_2 = {}& \mathbb{E}_{(|H_{pp}|)}\left[\log\left(\frac{1+\alpha|H_{pp}|^2P_p}{1+|H_{pp}|^2P_p}\right)\right] \\ &+ \mathbb{E}_{(|H_{ss}|,|H_{ps}|)}\left[\log\left(\frac{1+|H_{ss}|^2P_s+|H_{ps}|^2P_p}{\alpha|H_{ps}|^2P_p+1}\right)\right] \end{aligned} \tag{11}$$

$$r_3 = \mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(1+\frac{|H_{sp}|^2P_s}{1+|H_{pp}|^2P_p}\right)\right] \tag{12}$$

$$\begin{aligned} r_4 = {}& \mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(\frac{1+\alpha|H_{pp}|^2P_p+|H_{sp}|^2P_s}{1+|H_{pp}|^2P_p}\right)\right] \\ &+ \mathbb{E}_{(|H_{ps}|)}\left[\log\left(\frac{1+|H_{ps}|^2P_p}{\alpha|H_{ps}|^2P_p+1}\right)\right] \end{aligned} \tag{13}$$

$$\begin{aligned} r_5 = {}& \mathbb{E}_{(|H_{pp}|)}\left[\log\left(\frac{1+\alpha|H_{pp}|^2P_p}{1+|H_{pp}|^2P_p}\right)\right] \\ &+ \mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(1+\frac{|H_{sp}|^2P_s}{1+|H_{pp}|^2P_p}\right)\right] \\ &+ \mathbb{E}_{(|H_{ps}|)}\left[\log\left(\frac{1+|H_{ps}|^2P_p}{\alpha|H_{ps}|^2P_p+1}\right)\right] \end{aligned} \tag{14}$$

$$\begin{aligned} r_6 = {}& \frac{1}{2}\left(\mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(\frac{1+\alpha|H_{pp}|^2P_p+|H_{sp}|^2P_s}{1+|H_{pp}|^2P_p}\right)\right]\right) \\ &+ \frac{1}{2}\left(\mathbb{E}_{(|H_{ss}|,|H_{ps}|)}\left[\log\left(\frac{1+|H_{ss}|^2P_s+|H_{ps}|^2P_p}{\alpha|H_{ps}|^2P_p+1}\right)\right]\right). \end{aligned} \tag{15}$$

Proof: All the rate expressions r_i, $i = \{1, \ldots, 6\}$ are obtained by substituting $R_p = \mathbb{E}_{(|H_{pp}|)}[\log(1+|H_{pp}|^2P_p)]$ into (3)–(8) in the same order and then simplifying the resultant expressions. The value of C_{sm} is then optimized by maximizing it over all possible values of $\alpha \in [0, 1]$. ■

Note that the interpretations of (10)–(15) remain similar to those mentioned earlier regarding (3)–(8).

The achievable rate of our underlay CR system then becomes

$$R_p \le (1-p)\mathbb{E}_{(|H_{pp}|)}\left[\log\left(1+|H_{pp}|^2P_p\right)\right] \tag{16}$$

$$R_s \le C_{sm}. \tag{17}$$

The term $(1-p)$ in the PU rate is a result of the fact that the PU is not always active. However, if the PU were to be always active, i.e., if $p = 0$, then the rate of the PU would be $R_p \le \mathbb{E}_{(|H_{pp}|)}[\log(1+|H_{pp}|^2P_p)]$. This would not affect the SU rate since the basic premise of underlay CR is the assumption of having no spectrum sensing at the SU transmitter and hence being unaware of the PU presence. In our system model, this situation is taken into account by assuming that the SU transmitter and receiver are unaware of S_p.

In the following, we discuss and characterize our main results in more detail.

III. Main Results

Our main result is essentially derived from the Han–Kobayshi achievable rate region [20], [21], which is known to be tight in all those interference regimes where the capacity is known.

As noted earlier, a necessary condition for operating at the full single-user rate for the PU is that the rate sharing parameter at the SU is chosen to be $\beta = 0$, i.e., the SU has to assign all of its power for the public message that can be perfectly decoded, demodulated, and canceled out not only at the SU receiver but also at the PU receiver. We will now demonstrate that the rate sharing parameter α of the PU also has a simple structure.

Theorem 1: If $a \le 1$, then it is optimal to select $\alpha = 1$, whereas if $a > 1$, then it is optimal to select $\alpha = 0$.

Proof: See Appendix B. ■

It is thus clear that the value of β is zero (as dictated by the requirement of achieving the full rate for the PU) and that of α is unity if the interference imposed by the PU on the SU is ergodically weak (i.e., $a \le 1$), and it is zero if the interference is ergodically strong $(a > 1)$. This implies that if the interference at the SU is weak, then treating the interference as noise is best; hence, the interference is not canceled. However, when the interference at the SU is strong, the interference is perfectly canceled out. An important point to note is that the result does not have any generic structure for α, such as $\alpha = \alpha^*$, where $\alpha^* \in (0, 1)$ represents the optimal rate sharing parameter at the PU that maximizes the SU rate. This implies that partial cancelation of the interference is not optimal in any case. In the following, we quantify the achievable rates associated with $\alpha = 0$ or 1 and $\beta = 0$.

Theorem 2: The achievable rate of the SU, which is subject to the condition that the required rate of the PU of $\mathbb{E}_{(|H_{pp}|)}[\log(1+|H_{pp}|^2P_p)]$ is met, is given by

$$R_s \le C_{sm} \tag{18}$$

where C_{sm} is formulated as follows:

$$C_{sm} = \begin{cases} \min(C_{s1}, C_{s2}), & \text{if } a \le 1 \text{ and } b > 1 \\ \min(C_{s1}, C_{s3}, C_{s4}), & \text{if } a > 1 \text{ and } b > 1 \\ C_{s1}, & \text{if } b \le 1 \end{cases}$$

where, we have

$$C_{s1} = \mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(1+\frac{|H_{sp}|^2P_s}{1+|H_{pp}|^2P_p}\right)\right] \tag{19}$$

$$C_{s2} = \mathbb{E}_{(|H_{ss}|,|H_{ps}|)}\left[\log\left(1+\frac{|H_{ss}|^2P_s}{1+|H_{ps}|^2P_p}\right)\right] \tag{20}$$

$$C_{s3} = \mathbb{E}_{(|H_{ss}|)}\left[\log\left(1+|H_{ss}|^2P_s\right)\right] \tag{21}$$

$$C_{s4} = \mathbb{E}_{(|H_{ss}|,|H_{ps}|)}\left[\log\left(\frac{1+|H_{ps}|^2P_p+|H_{ss}|^2P_s}{1+|H_{pp}|^2P_p}\right)\right]. \tag{22}$$

TABLE I
SU ACHIEVABLE RATE IN UNDERLAY CR FOR THE DIFFERENT REGIMES OF AVERAGE INTERFERENCE LEVELS

Regime → / Parameter ↓	I - $b \leq 1$	II - $b>1$ and $a \leq a_1$	III - $b>1$ and $a_1<a \leq 1$	IV - $b>1$ and $1<a \leq a_2$	V - $b>1$ and $a_2<a \leq a_3$	VI - $b>1$ and $a>a_3$
Average interference coefficient PU-SU link a	Constant behaviour	Constant behaviour	Decreases with a as interferece from the PU is treated as noise	Increases with a as interferece from the PU is decoded out. More interference more information is decoded	Constant behaviour	Constant behaviour
Average interference coefficient SU-PU link b	Increases with b. The rate is dictated by how much PU is able to decode out at its receiver	Increases with b. The rate is dictated by how much PU is able to decode out at its receiver	Constant behaviour	Constant behaviour	Increases with b. The rate is dictated by how much PU is able to decode out at its receiver	Constant behaviour
Transmit power constraint at PU P_p	Decreases with P_p with a rate s_1 (say). At PU receiver the PU message is treated as noise to decode the SU common message	Decreases with P_p with a rate s_1. At PU receiver the PU message is treated as noise to decode the SU common message	Decreases with P_p witha rate $s_2 < s_1$. At SU receiver the PU message is treated as noise to decode the SU common message	Decreases for values of a near unity and may possibly increase at large values of a, depending upon the value of b	Decreases with P_p with a rate $s_3 > s_1$. At PU receiver the PU message is treated as noise to decode the SU common message	Constant behaviour
Transmit power constraint at SU P_s	Increases with P_s with a rate s_4 (say). At PU receiver the PU message is treated as noise to decode the SU common message	Increases with P_s with a rate $s_5 > s_4$. At PU receiver the PU message is treated as noise to decode the SU common message	Increases with P_s with a rate $s_5 > s_4$. At PU receiver the PU message is treated as noise to decode the SU common message	Increases with P_s with a rate $s_6 < s_5$. At SU receiver simultaneous decoding is performed by the SU followed by complete interference cancellation	Increases with P_s with a rate $s_7 > s_6$. At PU receiver simultaneous decoding is performed by the PU.	Increases with P_s with a rate $s_8 > s_7$. At SU receiver simultaneous decoding is performed by the SU followed by complete interference cancellation.

Proof: See Appendix C. ∎

IV. DISCUSSIONS

To quantify the SU rate associated with various parameters, we structure our analysis based on the value of average interference coefficients in Table I as follows:

- The interference at the PU is ergodically weak, i.e., we have $b \leq 1$. We refer to this as Regime I in Table I.
- The interference at the PU is ergodically strong and that at the SU is ergodically very weak, i.e., we have $b > 1$ and $a \leq a_1$, where for a given b, a_1 is that specific value of a, where $C_{s1} = C_{s2}$. We refer to this as Regime II in Table I.
- The interference at the PU is ergodically strong and that at the SU is ergodically weak, i.e., we have $b > 1$ and $a_1 < a \leq 1$. We refer to this as Regime III in Table I.
- The interference at the PU is ergodically strong and that at the SU is also ergodically strong, i.e., we have $b > 1$ and $1 < a \leq a_2$, where for a given b, a_2 is that specific value of a, where $C_{s1} = C_{s4}$. We refer to this as Regime IV in Table I.
- The interference at the PU is ergodically strong, and that at the SU is ergodically moderately strong, i.e., we have $b > 1$ and $a_2 < a \leq a_3$, where for a given b, a_3 is that specific value of a, where $C_{s4} = C_{s3}$. We refer to this as Regime V in Table I.
- The interference at the PU is ergodically strong, and that at the SU is ergodically very strong, i.e., $b > 1$ and $a > a_3$. We refer to this as Regime VI in Table I.

We now analyze the behavior of the achievable rate in each regime. The achievable rate C_{sm} of the SU obeys the following trend:

1) Regime I of Table I: For $b \leq 1$, the value of C_{sm} is increasing with b, and it is constant for a given a. We have shown mathematically as to why C_{s1} holds in this regime. From a conceptual perspective, we try to understand this by dividing this regime into two parts: 1) $a \leq 1$, and 2) $a > 1$. Since the interference is ergodically weak for $a < 1$, we imagine a compound channel [23] from the SU's perspective. Both the PU and the SU receivers want to recover the SU message and hence treat the PU message as noise. Since we have $a \leq 1$ and $b \leq 1$, the SU–PU link is more noisy than the SU–SU link; hence, the SU–PU link determines the achievable rate. On the other hand, for $a > 1$ imagine a pair of multiple access channels, namely MAC1 comprised of the PU–SU and SU–SU links, and MAC2 comprised of the PU–PU and SU–SU links. Fig. 2(a) shows the capacity region for these MACs. It is clear from Fig. 2(a) that the capacity region of MAC2 is completely contained within that of MAC1 if $a > 1$ and $b \leq 1$. Hence, again, C_{s1} is a corner point of the MAC1 capacity region where PU achieves its full rate. Hence, for $b \leq 1$, C_{sm} is a monotonically increasing function of b.
2) Regime II of Table I: Based on the compound channel explanation above for $b > 1$ and $a \leq a_1 < 1$, the weak link is the SU–PU link; hence, C_{s1} is cached. Hence, the PU receiver perfectly decoding the SU message completely by treating its own message as noise is the determining achievable rate.

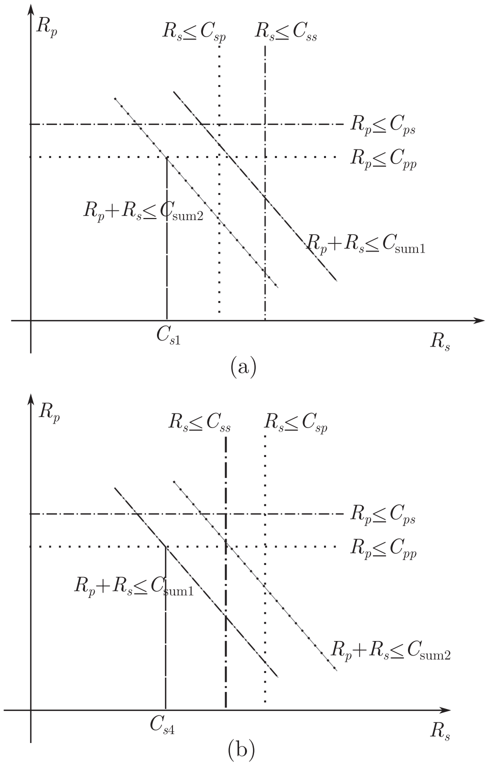

Fig. 2. Two scenarios are as follows. (a) Scenario for Regime I when $a > 1$; and (b) scenario for Regime IV. Here, $C_{pp} = \mathbb{E}_{|H_{pp}|}[\log(1 + |H_{pp}|^2 P_p)]$, $C_{ss} = \mathbb{E}_{|H_{ss}|}[\log(1 + |H_{ss}|^2 P_s)]$, $C_{sp} = \mathbb{E}_{|H_{sp}|}[\log(1 + |H_{sp}|^2 P_s)]$, $C_{pp} = \mathbb{E}_{|H_{ps}|}[\log(1 + |H_{ps}|^2 P_p)]$, $C_{\mathrm{sum1}} = \mathbb{E}_{|H_{pp}|,|H_{sp}|}[\log(1 + |H_{pp}|^2 P_p) + |H_{sp}|^2 P_s]$, and $C_{\mathrm{sum2}} = \mathbb{E}_{|H_{ss}|,|H_{ps}|}[\log(1 + |H_{ps}|^2 P_p) + |H_{ss}|^2 P_s]$.

3) Regime III of Table I: For $b > 1$ and $a_1 < a \le 1$, again, based on the above compound channel explanation, the weak link the is SU–SU link; hence, C_{s2} holds. Hence, the SU receiver decoding the SU message by treating the PU message as noise determines the achievable rate.

4) Regime IV of Table I: For $b > 1$ and $1 < a \le a_2$, again, imagine the same two aforementioned MACs. Fig. 2(b) shows the capacity region for these two MACs. Unlike for the case above, the MAC2 capacity region is not completely contained in MAC1, as shown in Fig. 2(b). In fact, for this regime, we have to consider the intersection of the two MACs. This turns out to be the achievable point-to-point rate for both the SU and the PU, which constitutes as their individual constraint and the sum constraint arising from MAC1 (because $1 < a \le a_2$). Hence, the constraint C_{s4} holds, which is the corner point of this region obtained by the specific intersection where the PU attains its full rate and the SU gets C_{s4}.

5) Regime V of Table I-$b > 1$ and $a_2 < a \le a_3$: The same discussions as above are valid, with the individual rate constraints being the same but with the only difference being that the sum rate constraint is now due to MAC2 and not MAC1 (because $a_2 < a \le a_3$). Hence, the constraint C_{s1} holds, which is the corner point of this region obtained by intersection, where the PU attains full rate, and the SU gets C_{s1}.

6) Regime VI of Table I-$b > 1$ and $a > a_3$: This regime is ergodically very strong; hence, the sum-rate constraints are not binding. Each channel behaves as if it was interference free. Hence, both the PU and SU both achieve their full single-user rate.

A summary of the discussion above about the behavior of achievable rate of SU with various parameters is provided in Table I.

Fig. 3 plots the different regimes for an uncorrelated Rayleigh fading channel. For a given SNR at the PU and SU, we plot C_{sm} for different values of $a \times b \in [0.2, 2] \times [0.2, 2]$, as shown in Fig. 3. Observe that the system's behavior with respect to a and b is as characterized in Table I. The curves recorded for $a = a_1$ and $a = a_2$ are marked on the plot. The curve for $a = a_3$ occurs at very strong interference levels; hence, it is not visible in the selected range of a and b values. The curve a_1 can be seen to be a monotonically decreasing function of b; this is because when the value of b increases, the values of a for which $C_{s1} < C_{s2}$ also decreases. Similarly, a_2 is an increasing function of b because when the value of b increases the value of a for which we have $C_{s4} < C_{s1}$ increases.

V. Achievable Rates Under Imperfect Channel State Estimation

Earlier, the idealized simplifying assumption of having perfect channel knowledge of all the links at all the receivers was assumed. Naturally, in practice, this is not the case. The receivers in practice use m training symbols for estimating the channel. This technique implicitly assumes that the channel's envelope remains constant not only over the m pilot symbol duration but also during the entire transmission burst to be detected. This process is then repeated for all new bursts. Having said this, powerful decision-directed joint iterative channel and data estimators are capable of operating close to the perfect-channel scenario for the desired link, as documented in [24] and [25].

Accordingly, we consider two specific cases, namely: 1) when an estimation error is imposed only on the interfering links; and 2) when the estimation error contaminates all the links. The error in the cross links is modeled as follows. Let $\hat{H}_{ps}$ and $\hat{H}_{sp}$ represent the estimates of H_{ps} and H_{sp}, namely, that of the link between the PU and the SU and *vice versa*, respectively. Let furthermore E_{ps} and E_{sp} be the errors associated with a single channel use. Then, by performing maximum likelihood (ML) estimation over a block of m symbol duration and by applying the central limit theorem, we have [31]

$$\hat{H}_{ps} = H_{ps} + \frac{1}{\sqrt{mP_p}} E_{ps} \tag{23}$$

$$\hat{H}_{sp} = H_{sp} + \frac{1}{\sqrt{mP_s}} E_{sp}. \tag{24}$$

Note that the both E_{ps} and E_{sp} are zero-mean and unit-variance standard Gaussian RVs, i.e., they are distributed as $\mathcal{N}(0, 1)$. The error scaled by $1/\sqrt{mP}$ suggests that performing the estimation over multiple symbol duration and relying on an increased training sequence power reduces the effects of

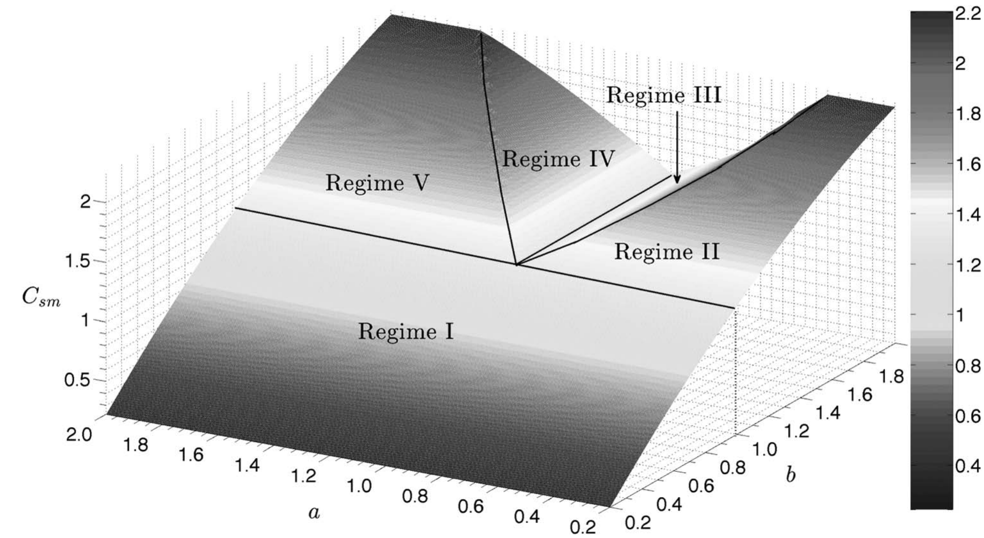

Fig. 3. Variation of the SU achievable rate C_{sm} as a function of a and b for $P_p = 200$ and $P_s = 100$.

estimation error. Thus, the baseband equations that we have are the following:

$$Y_p = H_{pp}X_p + H_{sp}X_s + Z_{pe1} \tag{25}$$

$$Y_s = H_{ss}X_s + H_{ps}X_p + Z_{se1} \tag{26}$$

where $Z_{pe1} \sim \mathcal{N}(0, 1+(1/\sqrt{mP_s}))$ and where $Z_{se1} \sim \mathcal{N}(0, 1+(1/\sqrt{mP_p}))$. This suggests that the effect of channel estimation errors simply increases the effective noise. The impact of these errors will depend upon the average transmit powers of the PU and the SU. Let $N_{p1} = 1 + (1/\sqrt{mP_s})$ and $N_{s1} = 1 + (1/\sqrt{mP_p})$.

Similarly, if there are estimation errors in all the four links, then, in addition to (23) and (24), for the direct links, we have

$$\hat{H}_{pp} = H_{pp} + \frac{1}{\sqrt{mP_p}}E_{pp} \tag{27}$$

$$\hat{H}_{ss} = H_{ss} + \frac{1}{\sqrt{mP_s}}E_{ss}. \tag{28}$$

Similar to E_{ps} and E_{sp}, E_{pp} and E_{ss} are also zero-mean and unit-variance standard Gaussian RVs, i.e., they are distributed as $\mathcal{N}(0,1)$. Thus, the baseband equations that we have are the following:

$$Y_p = H_{pp}X_p + H_{sp}X_s + Z_{pe2} \tag{29}$$

$$Y_s = H_{ss}X_s + H_{ps}X_p + Z_{se2} \tag{30}$$

where $Z_{pe1} \sim \mathcal{N}(0, 1+(1/\sqrt{mP_s}) + (1/\sqrt{mP_p}))$, and $Z_{se1} \sim \mathcal{N}(0, 1+(1/\sqrt{mP_p})+(1/\sqrt{mP_s}))$. Let $N_{p2} = 1+(1/\sqrt{mP_s})+(1/\sqrt{mP_p})$ and $N_{s2} = 1 + (1/\sqrt{mP_p}) + (1/\sqrt{mP_s})$. Thus, $N_{s2} = N_{p2}$.

This increase in noise power requires us to characterize the achievable rates described in (3)–(9) in terms of the noise. Let N_p and N_s be the noise variance at the PU and the SU. To formulate the achievable rate regions, we replace the unit variance of the noise by N_p if the rate constraint was due to decoding at the PU and by N_s, if the rate constraint was due to decoding at the SU. Then, the achievable region is formulated as

$$R_p \le \mathbb{E}_{(|H_{pp}|)}\left[\log\left(1 + \frac{|H_{pp}|^2P_p}{N_p}\right)\right] \tag{31}$$

$$R_s \le \mathbb{E}_{(|H_{ss}|,|H_{ps}|)}\left[\log\left(1 + \frac{|H_{ss}|^2P_s}{\alpha|H_{ps}|^2P_p + N_s}\right)\right] \tag{32}$$

$$R_p + R_s \le \mathbb{E}_{(|H_{pp}|)}\left[\log\left(1 + \frac{\alpha|H_{pp}|^2P_p}{N_p}\right)\right] + \mathbb{E}_{(|H_{ss}|,|H_{ps}|)}\left[\log\left(\frac{1+|H_{ss}|^2P_s+|H_{ps}|^2P_p}{\alpha|H_{ps}|^2P_p + N_s}\right)\right] \tag{33}$$

$$R_p + R_s \le \mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(1+\frac{|H_{pp}|^2P_p+|H_{sp}|^2P_s}{N_p}\right)\right] \tag{34}$$

$$R_p + R_s \le \mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(1+\frac{\alpha|H_{pp}|^2P_p+|H_{sp}|^2P_s}{N_p}\right)\right] + \mathbb{E}_{(|H_{ps}|)}\left[\log\left(\frac{1+|H_{ps}|^2P_p}{\alpha|H_{ps}|^2P_p + N_s}\right)\right] \tag{35}$$

$$2R_p + R_s \le \mathbb{E}_{(|H_{pp}|)}\left[\log\left(1 + \frac{\alpha|H_{pp}|^2P_p}{N_p}\right)\right] + \mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(1+\frac{|H_{pp}|^2P_p+|H_{sp}|^2P_s}{N_p}\right)\right] + \mathbb{E}_{(|H_{ps}|)}\left[\log\left(\frac{1+|H_{ps}|^2P_p}{\alpha|H_{ps}|^2P_p + N_s}\right)\right] \tag{36}$$

$$R_p + 2R_s \le \mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(1+\frac{\alpha|H_{pp}|^2P_p+|H_{sp}|^2P_s}{N_p}\right)\right] + \mathbb{E}_{(|H_{ss}|,|H_{ps}|)}\left[\log\left(\frac{1+|H_{ss}|^2P_s+|H_{ps}|^2P_p}{\alpha|H_{ps}|^2P_p + N_s}\right)\right]. \tag{37}$$

Consequently, the expressions for r_i, $i = \{1, \ldots, 6\}$ are as follows:

$$r_1 = \mathbb{E}_{(|H_{ss}|,|H_{ps}|)}\left[\log\left(1+\frac{|H_{ss}|^2P_s}{\alpha|H_{ps}|^2P_p+N_s}\right)\right] \tag{38}$$

$$\begin{aligned} r_2 &= \mathbb{E}_{(|H_{pp}|)}\left[\log\left(\frac{N_p+\alpha|H_{pp}|^2P_p}{N_p+|H_{pp}|^2P_p}\right)\right] \\ &\quad + \mathbb{E}_{(|H_{ss}|,|H_{ps}|)}\left[\log\left(\frac{N_s+|H_{ss}|^2P_s+|H_{ps}|^2P_p}{\alpha|H_{ps}|^2P_p+N_s}\right)\right] \end{aligned} \tag{39}$$

$$r_3 = \mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(\frac{N_p+|H_{pp}|^2P_p+|H_{sp}|^2P_s}{N_s+|H_{pp}|^2P_p}\right)\right] \tag{40}$$

$$\begin{aligned} r_4 &= \mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(\frac{N_p+\alpha|H_{pp}|^2P_p+|H_{sp}|^2P_s}{N_p+|H_{pp}|^2P_p}\right)\right] \\ &\quad + \mathbb{E}_{(|H_{ps}|)}\left[\log\left(\frac{N_s+|H_{ps}|^2P_p}{\alpha|H_{ps}|^2P_p+N_s}\right)\right] \end{aligned} \tag{41}$$

$$\begin{aligned} r_5 &= \mathbb{E}_{(|H_{pp}|)}\left[\log\left(\frac{N_p+\alpha|H_{pp}|^2P_p}{N_p+|H_{pp}|^2P_p}\right)\right] \\ &\quad + \mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(\frac{N_p+|H_{pp}|^2P_p+|H_{sp}|^2P_s}{N_p+|H_{pp}|^2P_p}\right)\right] \\ &\quad + \mathbb{E}_{(|H_{ps}|)}\left[\log\left(\frac{N_s+|H_{ps}|^2P_p}{\alpha|H_{ps}|^2P_p+N_s}\right)\right] \end{aligned} \tag{42}$$

$$\begin{aligned} r_6 &= \frac{1}{2}\left(\mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(\frac{N_p+\alpha|H_{pp}|^2P_p+|H_{sp}|^2P_s}{N_p+|H_{pp}|^2P_p}\right)\right]\right) \\ &\quad + \frac{1}{2}\left(\mathbb{E}_{(|H_{ss}|,|H_{ps}|)}\left[\log\left(\frac{N_s+|H_{ss}|^2P_s+|H_{ps}|^2P_p}{\alpha|H_{ps}|^2P_p+N_s}\right)\right]\right). \end{aligned} \tag{43}$$

Now, since $N_{p2} = N_{s2}$, when there are estimation errors on each link then $N_p = N_{p2} = N_s = N_{s2}$. Hence, we recover the results mentioned in Theorems 1 and 2 with only a small change in Theorem 2 as described in the following.

Theorem 3: The achievable rate of the SU, i.e., subject to the condition that the required rate of the PU of $\mathbb{E}_{(|H_{pp}|)}[\log(1+((|H_{pp}|^2P_p)/N_{p2}))]$ is met under imperfect channel estimation on all four links, is given by

$$R_s \leq C_{sma} \tag{44}$$

where C_{sma} is formulated as follows:

$$C_{sma} = \begin{cases} \min(C_{s1a}, C_{s2a}), & \text{if } a \leq 1 \text{ and } b > 1 \\ \min(C_{s1a}, C_{s3a}, C_{s4a}), & \text{if } a > 1 \text{ and } b > 1 \\ C_{s1a}, & \text{if } b \leq 1 \end{cases}$$

where, we have

$$C_{s1a} = \mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(1+\frac{|H_{sp}|^2P_s}{N_{p2}+|H_{pp}|^2P_p}\right)\right] \tag{45}$$

$$C_{s2a} = \mathbb{E}_{(|H_{ss}|,|H_{ps}|)}\left[\log\left(1+\frac{|H_{ss}|^2P_s}{N_{s2}+|H_{ps}|^2P_p}\right)\right] \tag{46}$$

$$C_{s3a} = \mathbb{E}_{(|H_{ss}|)}\left[\log\left(1+\frac{|H_{ss}|^2P_s}{N_{s2}}\right)\right] \tag{47}$$

$$C_{s4a} = \mathbb{E}_{(|H_{ss}|,|H_{ps}|)}\left[\log\left(\frac{N_{s2}+|H_{ps}|^2P_p+|H_{ss}|^2P_s}{N_{p2}+|H_{pp}|^2P_p}\right)\right]. \tag{48}$$

Proof: The proof follows from the proof of Theorem 2. This is because all the results in Lemmas 1, 2, and 3 and the proof for Theorem 1 do not depend upon the ordering or the value of N_p and N_s. ■

When only the cross links are contaminated by the channel estimation error, then there are two possibilities: Either $N_{p1} \leq N_{s1}$ or $N_{p1} > N_{s1}$. The condition $N_{p1} \leq N_{s1}$ translates to $P_p \geq P_s$, which can be assumed to be reasonable. In this case, again, the results of Theorems 1 and 2 hold.

Theorem 4: The achievable rate of the SU, subject to the condition that the required rate of the PU of $\mathbb{E}_{(|H_{pp}|)}[\log(1+((|H_{pp}|^2P_p)/N_{p2}))]$ is met under imperfect channel estimation only on the interfering links with $P_p \geq P_s$, is given by

$$R_s \leq C_{smi} \tag{49}$$

where C_{smi} is formulated as follows:

$$C_{smi} = \begin{cases} \min(C_{s1i}, C_{s2i}), & \text{if } a \leq 1 \text{ and } b > 1 \\ \min(C_{s1i}, C_{s3i}, C_{s4i}), & \text{if } a > 1 \text{ and } b > 1 \\ C_{s1i}, & \text{if } b \leq 1 \end{cases}$$

where, we have

$$C_{s1i} = \mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(1+\frac{|H_{sp}|^2P_s}{N_{p1}+|H_{pp}|^2P_p}\right)\right] \tag{50}$$

$$C_{s2i} = \mathbb{E}_{(|H_{ss}|,|H_{ps}|)}\left[\log\left(1+\frac{|H_{ss}|^2P_s}{N_{s1}+|H_{ps}|^2P_p}\right)\right] \tag{51}$$

$$C_{s3i} = \mathbb{E}_{(|H_{ss}|)}\left[\log\left(1+\frac{|H_{ss}|^2P_s}{N_{s1}}\right)\right] \tag{52}$$

$$C_{s4i} = \mathbb{E}_{(|H_{ss}|,|H_{ps}|)}\left[\log\left(\frac{N_{s1}+|H_{ps}|^2P_p+|H_{ss}|^2P_s}{N_{p1}+|H_{pp}|^2P_p}\right)\right]. \tag{53}$$

Proof: The proof follows from the proof of Theorem 2 and the fact that the conditions $r_2|_{\alpha=1} > r_3$ for $a, b \leq 1$, and $r_2|_{\alpha=0} > r_3$ for $a > 1, b \leq 1$ are satisfied only when $N_{p1} \leq N_{s1}$. ■

For the case when we have $N_{p1} < N_{s1}$, the conditions $r_2|_{\alpha=1} > r_3$ for $a, b \leq 1$, and $r_2|_{\alpha=0} > r_3$ for $a > 1$ and $b \leq 1$ are not necessarily true. Hence, we have the following result.

Theorem 5: The achievable rate of the SU, subject to the condition that the required rate of the PU of $\mathbb{E}_{(|H_{pp}|)}[\log(1+((|H_{pp}|^2P_p)/N_{p2}))]$ is met under having imperfect channel estimation only for the interfering links with $P_p < P_s$ is given by

$$R_s \leq C_{sme} \tag{54}$$

where C_{sme} is formulated as follows:

$$C_{sme} = \begin{cases} \min(C_{s1e}, C_{s2e}), & \text{if } a \leq 1 \\ \min(C_{s1e}, C_{s3e}, C_{s4e}), & \text{if } a > 1 \text{ and } b > 1 \\ \min(C_{s1e}, C_{s4e}), & \text{if } a > 1 \text{ and } b \leq 1 \end{cases}$$

where we have

$$C_{s1e} = \mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(1+\frac{|H_{sp}|^2P_s}{N_{p1}+|H_{pp}|^2P_p}\right)\right] \tag{55}$$

$$C_{s2e} = \mathbb{E}_{(|H_{ss}|,|H_{ps}|)}\left[\log\left(1+\frac{|H_{ss}|^2P_s}{N_{s1}+|H_{ps}|^2P_p}\right)\right] \tag{56}$$

$$C_{s3e} = \mathbb{E}_{(|H_{ss}|)}\left[\log\left(1+\frac{|H_{ss}|^2P_s}{N_{s1}}\right)\right] \tag{57}$$

$$C_{s4e} = \mathbb{E}_{(|H_{ss}|,|H_{ps}|)}\left[\log\left(\frac{N_{s1}+|H_{ps}|^2P_p+|H_{ss}|^2P_s}{N_{p1}+|H_{pp}|^2P_p}\right)\right]. \tag{58}$$

Proof: The expressions of the achievable rates under $b \leq 1$ and $b > 1$ turn out to be the same, which is the minimum of $\min(C_{s1e}, C_{s2e})$. Hence, unlike the previous results in Theorems 2–4, the achievable rate for $b \leq 1$ does not have the same expression, whereas now for $a \leq 1$, the characterization is the same. ■

Hence, the effect of channel estimation errors *does not* change the optimal structure of the rate sharing parameter described in Theorem 1. Moreover, when all the links have estimation errors and when only the cross-links have estimation error associated with $P_s \geq P_p$, then the formulation of the achievable rate remains similar to that of the perfect estimation scenario, with the only difference being the addition of the general noise variance terms of N_p and N_s instead of unity. When only the cross-links have an estimation error associated with $P_s \geq P_p$, then the description of the achievable rate changes in the regimes of $a \leq 1$, $b > 1$, and $a > 1$, $b \leq 1$ regimes.

Note that the extra terms in the variance, i.e., $(1/\sqrt{mP_p}) + (1/\sqrt{mP_s})$ that arise are quite small, particularly when the value of m is high. However, a high-Doppler fading channel will change substantially for a large value of m. Nevertheless, if the average transmit power values P_p and P_s are high enough, the impact of channel estimation errors can be reduced to a small value. By contrast, if the transmit power values are insufficiently high and they are combined with a small value of m, this might affect the achievable rates significantly.

VI. Conclusion

In this paper, a new information-theoretic model was conceived for underlay-based CR. By extending the Han–Kobayashi achievable rate region to fading interference channels, we determined the optimal rate sharing parameters for both the SU and the PU that satisfy the relevant constraints and maximize the achievable rates. Furthermore, we provided a detailed analysis of the binding constraints accompanied by their conceptual interpretation. Then, we provided an analysis of the realistic imperfect channel estimation scenario. It was demonstrated that, despite having channel estimation errors, the optimal structure of the rate sharing parameter remains the same.

Appendix A
Supporting Lemmas

Lemma 1: r_1 is a monotonically decreasing function of α for all a, whereas r_2 and r_5 are monotonically decreasing functions of α for $a > 1$ and are monotonically increasing functions of α for $a \leq 1$.

Proof: This follows from the fact that the $\log(1+x)$ function is a strictly increasing function of x. Hence, for a pair of bounded RVs X and Y, if $\mathbb{E}[X] > \mathbb{E}[Y]$ is satisfied, then we have $\mathbb{E}[\log(1+X)] > \mathbb{E}[\log(1+Y)]$. A rigorous proof involving differentiations can be provided for any of the known fading distributions. ■

Lemma 2: From (10)–(15), it is sufficient to consider only the three rate constraints r_2, r_3, and r_5 for $a < 1$ and four rate constraints r_1, r_2, r_3, and r_5 for $a > 1$.

Proof: We have to show that the constraint of r_1 for $a < 1$ is redundant, whereas the constraints of r_4 and r_6 are always redundant.

For r_1, we show that, if we have $a < 1$, then $r_1 \geq r_2$.

From Lemma 1, if $a < 1$, then r_2 is a monotonically increasing function of α, whereas r_1 is always a monotonically decreasing function of α. Furthermore, we have $r_1|_{\alpha=1} = r_2|_{\alpha=1}$. Hence, for $a < 1$, $r_1 \geq r_2$ is satisfied.

For r_4, we show that $r_4 \geq r_5$ is valid for all a since we have

$$\begin{aligned} r_4 - r_5 &= \mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(\frac{1+\alpha|H_{pp}|^2P_p+|H_{sp}|^2P_s}{1+\alpha|H_{pp}|^2P_p}\right)\right] \\ &\quad - \mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(1+\frac{|H_{sp}|^2P_s}{1+|H_{pp}|^2P_p}\right)\right] \\ &= \mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(1+\frac{|H_{sp}|^2P_s}{1+\alpha|H_{pp}|^2P_p}\right)\right] \\ &\quad - \mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(1+\frac{|H_{sp}|^2P_s}{1+|H_{pp}|^2P_p}\right)\right] \\ &\geq 0. \end{aligned} \tag{59}$$

Thus, $r_4 \geq r_5$ is satisfied.

For r_6, we show that $r_6 \geq \min(r_2, r_3)$ is satisfied for all a. Observing that

$$\begin{aligned} r_6 - \frac{r_2}{2} &= \frac{1}{2}\mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(\frac{1+\alpha|H_{pp}|^2P_p+|H_{sp}|^2P_s}{1+|H_{pp}|^2P_p}\right)\right] \\ &\quad - \mathbb{E}_{(|H_{pp}|)}\left[\log\left(\frac{1+\alpha|H_{pp}|^2P_p}{1+|H_{pp}|^2P_p}\right)\right] \end{aligned} \tag{60}$$

$$\text{or } r_6 = \frac{r_2}{2} + \frac{1}{2}\mathbb{E}_{(|H_{pp}|,|H_{sp}|)} \times\left[\log\left(\frac{1+\alpha|H_{pp}|^2P_p+|H_{sp}|^2P_s}{1+\alpha|H_{pp}|^2P_p}\right)\right] \tag{61}$$

$$= \frac{r_2}{2} + \frac{1}{2}\mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(1+\frac{|H_{sp}|^2P_s}{1+\alpha|H_{pp}|^2P_p}\right)\right] \tag{62}$$

$$\geq \frac{r_2}{2} + \frac{r_3}{2} = \frac{r_2+r_3}{2} \geq \min(r_2, r_3). \tag{63}$$

Lemma 2 is proven. ■

Appendix B
Proof of Theorem 1

From Lemma 2, we established that, for $a<1$, only the rate constraints r_2, r_3, and r_5 are binding. Hence, we have

$$C_{sm} = \min\left(r_3, \max_{\alpha\in[0,1]}\{\min(r_2, r_5,)\}\right). \tag{64}$$

From Lemma 1, we note that functions r_2 and r_5 are monotonically increasing functions of α if $a \leq 1$. Hence, we have

$$\arg\max_{\alpha\in[0,1]}\{\min(r_2, r_5,)\} = 1.$$

Since r_3 is independent of α, if the constraint r_3 is binding, we can select $\alpha = 1$ as the default value. Hence, $\alpha = 1$ is optimal for $a \leq 1$.

Following the same line of argument, we can establish that $\alpha = 0$ is optimal for $a > 1$. ■

APPENDIX C
PROOF OF THEOREM 2

For the condition of $a > 1$ and $b > 1$, the value of C_{sm} is obtained by selecting the minimum of r_1, r_2, r_3 and r_5 evaluated at $\alpha = 0$. It can be shown that $r_5|_{\alpha=0} > r_3$ for $a > 1$. Hence, for $a > 1$ and $b > 1$, we have $C_{sm} = \min(r_1|_{\alpha=0}, r_2|_{\alpha=0}, r_3)$.

For the condition of $a \leq 1$ and $b > 1$, the value of C_{sm} is obtained by taking the minimum of r_2, r_3 and r_5 evaluated at $\alpha = 1$. Since, we have $r_5|_{\alpha=1} = r_3$, hence, for $a \leq 1$ and $b > 1$, we arrive at $C_{sm} = \min(r_2|_{\alpha=1}, r_3)$.

For the condition of $b \leq 1$ and $a \leq 1$, $r_2|_{\alpha=1} \geq r_3$ holds. Hence, $C_{sm} = r_3$.

For the condition of $b \leq 1$ and $a > 1$, $r_1|_{\alpha=0} > r_3$ hold. The only fact that remains to be shown is that $r_2|_{\alpha=0} > r_3$. To show this, we demonstrate that

$$\mathbb{E}_{(|H_{pp}|,|H_{sp}|)}\left[\log\left(\frac{1+|H_{pp}|^2P_p+|H_{sp}|^2P_s}{1+|H_{ps}|^2P_p+|H_{ss}|^2P_s}\right)\right] < 0.$$

To show this, we observe that

$$\mathbb{E}_{(|H_{pp}|,|H_{sp}|,|H_{ss}|,|H_{ps}|)}\left[\log\left(\frac{1+|H_{pp}|^2P_p+|H_{sp}|^2P_s}{1+|H_{ps}|^2P_p+|H_{ss}|^2P_s}\right)\right]$$

$$\leq \mathbb{E}_{(|H_{pp}|,|H_{sp}|,|H_{ss}|)}\left[\log\left(\frac{1+|H_{pp}|^2P_p+|H_{sp}|^2P_s}{1+|H_{pp}|^2P_p+|H_{ss}|^2P_s}\right)\right] \quad (65)$$

$$= \mathbb{E}_{(|H_{pp}|,|H_{sp}|,|H_{ss}|)}\left[\log\left(\frac{1+\frac{|H_{sp}|^2P_s}{1+|H_{pp}|^2P_p}}{1+\frac{|H_{ss}|^2P_s}{1+|H_{pp}|^2P_p}}\right)\right] \quad (66)$$

$$\leq 0. \quad (67)$$

■

REFERENCES

[1] FCC, "Report of the spectrum efficiency working group," FCC Spectrum Policy Task Force, Washington, DC, USA Tech. Rep., 2002.

[2] J. Mitola and G. Q. Maguire Jr., "Cognitive radio: Making software radios more personal," *IEEE Pers. Commun.*, vol. 6, no. 4, pp. 13–18, Aug. 1999.

[3] S. Haykin, "Cognitive radio: Brain-empowered wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 23, no. 2, pp. 201–220, Feb. 2005.

[4] I. F. Akyildiz, W.-Y. Lee, M. C. Vuran, and S. Mohanty, "Next generation/dynamic spectrum access/cognitive radio wireless networks: A survey," *Comput. Netw.*, vol. 50, no. 13, pp. 2127–2159, Sep. 2006.

[5] K. N. Mohammad, G. Khoshkholgh, and H. Yanikomeroglu, "Access strategies for spectrum sharing in fading environment: Overlay, underlay, and mixed," *IEEE Trans. Mobile Comput.*, vol. 9, no. 12, pp. 1780–1793, Mar. 2010.

[6] N. Yi, Y. Ma, and R. Tafazolli, "Underlay cognitive radio with full or partial channel quality information," *Int. J. Navigat. Observ.*, vol. 2010, 2010, Art. ID 105723.

[7] G. Amir and S. S. Elvino., "Fundamental limits of spectrum-sharing in fading environments," *IEEE Trans. Wireless Commun.*, vol. 6, no. 2, pp. 649–658, Feb. 2007.

[8] L. B. Le and E. Hossain, "Resource allocation for spectrum underlay in cognitive radio networks," *IEEE Trans. Wireless Commun.*, vol. 7, no. 12, pp. 5306–5315, Dec. 2008.

[9] M. Filippou, D. Gesbert, and G. Ropokis, "Underlay versus interweaved cognitive radio networks: A performance comparison study," in *Proc. 9th Int. Conf. CROWNCOM*, Jun. 2014, pp. 226–231.

[10] M. C. Filippou, D. Gesbert, and G. A. Ropokis, "A comparative performance analysis of interweaved and underlay multi-antenna cognitive radio networks," *IEEE Trans. Wireless Commun.*, vol. 14, no. 5, pp. 2911–2925, Jan. 2015.

[11] L. Musavian and S. Aïssa, "Fundamental capacity limits of cognitive radio in fading environments with imperfect channel information," *IEEE Trans. Commun.*, vol. 57, no. 11, pp. 3472–3480, Nov. 2009.

[12] D. Xu, Z. Feng, and P. Zhang, "On the impacts of channel estimation errors and feedback delay on the ergodic capacity for spectrum sharing cognitive radio," *Wireless Pers. Commun.*, vol. 72, no. 4, pp. 1875–1887, Oct. 2013.

[13] L. Sboui, Z. Rezki, and M.-S. Alouini, "A unified framework for the ergodic capacity of spectrum sharing cognitive radio systems," *IEEE Trans. Wireless Commun.*, vol. 12, no. 2, pp. 877–887, Feb. 2013.

[14] A. Goldsmith, S. Jafar, I. Maric, and S. Srinivasa, "Breaking spectrum gridlock with cognitive radios: An information theoretic perspective," *Proc. IEEE*, vol. 97, no. 5, pp. 894–914, May 2009.

[15] M. C. Filippou, G. A. Ropokis, and D. Gesbert, "A team decisional beamforming approach for underlay cognitive radio networks," in *Proc. IEEE 24th Int. Symp. PIMRC*, Sep. 2013, pp. 575–579.

[16] P. de Kerret, M. Filippou, and D. Gesbert, "Statistically coordinated precoding for the miso cognitive radio channel," in *Proc. 48th Asilomar Conf. Signals, Syst. Comput.*, Nov. 2014, pp. 1083–1087.

[17] X. Kang, R. Zhang, Y.-C. Liang, and H. K. Garg, "Optimal power allocation strategies for fading cognitive radio channels with primary user outage constraint," *IEEE J. Sel. Areas Commun.*, vol. 29, no. 2, pp. 374–383, Feb. 2011.

[18] N. Devroye, P. Mitran, and V. Tarokh, "Achievable rates in cognitive radio channels," *IEEE Trans. Inf. Theory*, vol. 52, no. 5, pp. 1813–1827, May 2006.

[19] L. Sboui, Z. Rezki, and M.-S. Alouini, "Achievable rate of spectrum sharing cognitive radio systems over fading channels at low-power regime," *IEEE Trans. Wireless Commun.*, vol. 13, no. 11, pp. 6461–6473, Nov. 2014.

[20] T. Han and K. Kobayashi, "A new achievable rate region for the interference channel," *IEEE Trans. Inf. Theory*, vol. IT-27, no. 1, pp. 49–60, Jan. 1981.

[21] H.-F. Chong, M. Motani, H. K. Garg, and H. El Gamal, "On the Han Kobayashi region for the interference channel," *IEEE Trans. Inf. Theory*, vol. 54, no. 7, pp. 3188–3195, Jul. 2008.

[22] R. Etkin, D. Tse, and H. Wang, "Gaussian interference channel capacity to within one bit," *IEEE Trans. Inf. Theory*, vol. 54, no. 12, pp. 5534–5562, Dec. 2008.

[23] A. El Gamal, Y.-H. Kim, *Network Information Theory*. Cambridge, U.K.: Cambridge Univ. Press, 2012.

[24] L. Hanzo, M. Münster, B. J. Choi, and T. Keller, *OFDM and MC-CDMA for Broadband Multi-User Communications, WLANs and Broadcasting*, Hoboken, NJ, USA: Wiley, Jul. 2003

[25] L. Hanzo, J. Akhtman, L. Wang, and M. Jiang, *MIMO-OFDM for LTE, WIFI and WIMAX: Coherent Versus Non-Coherent and Cooperative Turbo-Transceivers*. Hoboken, NJ, USA: IEEE Press—Wiley, Mar. 2010,

[26] H. Sato, "The capacity of the Gaussian interference channel under strong interference," *IEEE Trans. Inf. Theory*, vol. IT-27, no. 6, pp. 786–788, Nov. 1981.

[27] A. Carleial, "A case where interference does not reduce capacity," *IEEE Trans. Inf. Theory*, vol. IT-21, no. 5, pp. 569–570, Sep. 1975.

[28] V. Annapureddy and V. Veeravalli, "Gaussian interference networks: Sum capacity in the low-interference regime and new outer bounds on the capacity region," *IEEE Trans. Inf. Theory*, vol. 55, no. 7, pp. 3032–3050, Jul. 2009.

[29] L. Sankar, X. Shang, E. Erkip, and H. Poor, "Ergodic fading interference channels: Sum-capacity and separability," *IEEE Trans. Inf. Theory*, vol. 57, no. 5, pp. 2605–2626, May 2011.

[30] R. Farsani, "The capacity region of the wireless ergodic fading interference channel with partial CSIT to within one bit," in *Proc. IEEE ISIT*, Jul. 2013, pp. 759–763.

[31] M. Khan, "Achieving exponential diversity with spatiotemporal power allocation with imperfect channel state information," in *Proc. NCC*, Jan. 2011, pp. 1–5.

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Dr. Khan serves as a Reviewer for many international and national journals and conferences. He received the INAE Young Engineer Award in 2006.

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Dr. Merchant has served on the Technical Program Committees of many IEEE premier conferences. He serves on the Editorial Board of two international journals: the *International Journal of Distributed Sensor Networks* and the *International Journal of Ultra Wideband Communications and Systems*. He received the 10th IETE Prof. S. V. C. Aiya Memorial Award for his contribution in the field of detection and tracking, the Ninth IETE SVC Aiya Memorial Award for Excellence in Telecom Education, and the 2013 VASVIK Award in the category of Electrical and Electronic Sciences and Technology. He is a coauthor with his students who have won Best Paper Awards. He is a Fellow of the Institution of Electronics and Telecommunication Engineers.

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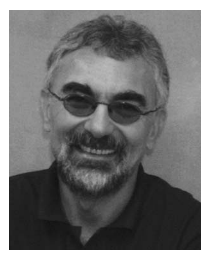
Dr. Desai served as the Chair of the IEEE Bombay Section from 2006 to 2008. He is a Fellow of the Indian National Science Academy (INSA) and the Indian National Academy of Engineering (INAE) and a founding member of COMSNETS and the Society for Cancer Research and Communication. He received the J. C. Bose Fellowship and the Excellence in Teaching Award from IIT Bombay in 2007.



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During his 38-year career in telecommunications, he has held various research and academic posts in Hungary, Germany, and the U.K. Since 1986, he has been with the School of Electronics and Computer Science, University of Southampton, where he holds the Chair in Telecommunications. He is currently directing a 100-strong academic research team, working on a range of research projects in the field of wireless multimedia communications sponsored by industry, the Engineering and Physical Sciences Research Council of U.K., the European Research Council's Advanced Fellow Grant, and the Royal Society Wolfson Research Merit Award. During 2008–2012, he was a Chaired Professor with Tsinghua University, Beijing, China. He is an enthusiastic supporter of industrial and academic liaison and offers a range of industrial courses. He has successfully supervised more than 80 Ph.D. students, coauthored 20 John Wiley/IEEE Press books on mobile radio communications totaling in excess of 10 000 pages, and published more than 1400 research entries on IEEE Xplore. He has more than 20 000 citations. His research is funded by the European Research Council's Senior Research Fellow Grant.

Dr. Hanzo is a Governor of the IEEE Vehicular Technology Society. He has served as the Technical Program Committee Chair and the General Chair of IEEE conferences, has presented keynote lectures, and has received a number of distinctions. During 2008–2012, he was the Editor-in-Chief of the IEEE Press. He is a Fellow of the Royal Academy of Engineering, The Institution of Engineering and Technology, and the European Association for Signal Processing.

AUTHOR QUERIES

AUTHOR PLEASE ANSWER ALL QUERIES

AQ1 = The sentence was modified for clarity. Please check if the following changes are appropriate. If not, kindly provide the necessary corrections.
AQ2 = Please provide specific year when the degrees were received by author "S. N. Merchant."

END OF ALL QUERIES