Efficient Random Access Channel Evaluation and Load Estimation in LTE-A With Massive MTC

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***Abstract*—The deployment of machine-type communications (MTC) together with cellular networks have great potential to create the ubiquitous Internet of Things environment. Nevertheless, the simultaneous activation of a large number of MTC devices (UEs) is a situation difficult to manage at the evolved Node B (eNB). The knowledge of the joint probability distribution function (PDF) of the number of successful and collided access requests within a random access opportunity (RAO) is a crucial piece of information for contriving congestion control schemes. A closed-form expression and an efficient recursion to obtain this joint PDF are derived in this paper. Furthermore, we exploit this PDF to design estimators of the number of contending UEs in a RAO. Our numerical results validate the effectiveness of our formulation and show that its computational cost is considerably lower than that of other related approaches. In addition, our estimators can be used by the eNBs to implement highly efficient congestion control methods.**

***Index Terms*—Cellular systems, machine-type communications (MTC), random access channel (RACH).**

I. INTRODUCTION

Nowadays, the use of cellular network technologies such as long term evolution advanced (LTE-A) and beyond for providing machine-type communications (MTC) has attracted significant attention from both the research community and the industry. The widely deployed infrastructure is a major driving force that motivates MTC application developers to adopt cellular networks for their numerous remote monitoring and controlling applications [1], [2]. However, critical problems like congestion and overload of radio access and core networks need to be addressed for efficient cellular MTC [3], [4].

In LTE-A, when an MTC device (named UE herein) wants to access the cellular network, it performs a random access procedure. The random access channel (RACH) is used to signal the connection request; it is allowed in predefined time/frequency resources, hereafter random access opportunities (RAOs) [5], [6]. The evolved Node B (eNB) has a number of preambles available for initial access to the network. These preambles are generated by Zadoff-Chu sequences due to their good correlation properties [6], [7] and are transmitted by the UEs for attempting the first access to the network.

A four-message handshake is performed in the contention-based random access. In *Msg1*, a UE transmits a randomly chosen preamble from the preamble pool during one of the available RAOs. A preamble will be detected at the eNB if it has not been chosen by more than one UE in the same RAO. Otherwise, a collision occurs. Then, the eNB sends a random access response message, *Msg2*, which includes one uplink grant for each detected preamble. *Msg2* is used to assign time-frequency resources to the UEs for the transmission of *Msg3*. UEs wait for a predefined time window to receive the uplink grant. If no uplink grant is received by the end of this window and the maximum number of access attempts has not been reached, the UEs wait for a random time and then perform a new access attempt. That is, they select a new preamble and transmit it at the next RAO. The UEs that receive an uplink grant send their connection request message, *Msg3*, using the resources specified by the eNB. Finally, the eNB responds to each *Msg3* transmission with a contention resolution message, *Msg4*. The interested reader is referred to [5], [8]–[11] for further details.

The main contributions of this study are the following:

- We propose a closed-form expression for the joint probability distribution function (PDF) of the number of successful and collided preamble transmissions within a RAO.
- We devise a computationally efficient recursion to compute the mentioned joint PDF even when the number of contending UEs is large.
- We compare the computational cost of both, our closed-form expression and recursive approach with that of other proposed methods, which only obtain the marginal distribution of successful attempts, and show that not only our approach yields a result that provides more information (joint vs. marginal PDF), but our recursion is computationally far less expensive.
- We use the joint PDF to design a series of estimators of the number of contending UEs in a RAO based on the maximum likelihood and Bayesian approaches.

The expressions we obtain are useful also in other contexts, where multiple resources (such as slots, channels) are randomly accessed by a pool of users as noted by the

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authors in [12]. In fact, the contention-based random access procedure detailed above is similar to the slotted ALOHA protocol [13]. After the preamble transmission in *Msg1*, the eNB can distinguish a request only if a preamble was transmitted by a single device (i.e., a UE can be connected if there is no access collision). In this sense, the random access procedure can be seen as a multi-channel ALOHA [12], [14], [15], where congestion control is tackled by estimating the number of arrivals or the number of UEs that send preambles to the eNB [16].

The remainder of the paper is organized as follows. Section II presents the most relevant related work in this subject. Section III introduces a closed-form expression and an efficient recursion to find the joint PDF of the number of successful and collided preambles in a RAO. Additionally, we use this PDF to design a series of estimators of the number of contending UEs. Section IV shows the numerical results and showcases the efficacy of our contributions. Finally, Section V draws the conclusions.

II. Related Work

Random access is identified as a key issue in MTC. In recent studies, approaches using non-trivial combinatorics have been developed to derive the PDF of the number of successful UEs in one-shot random access in multi-channel ALOHA.

In [12], [17], Wei et al. derived an explicit expression for computing the PDF of the number of successful transmissions. It is based on balls-and-bins combinatorics and uses an extension of the Stirling numbers of the second kind. In [18], Duan et al. provide another approach to compute the PDF of the number of successful preamble transmissions using combinatorial analysis. In [19], Arouk et al. provide another expression to compute the PDF of the number of successful preamble transmissions using the balls-and-bins approach and mathematical induction. These publications evince a current interest in deriving closed-form mathematical expressions to study the RACH in modern cellular networks using analytical models. However, previous proposals result in complex formulations and, as shown later, some of them fail to provide consistent results when the number of UEs increases beyond a certain value, severely limiting their practical applicability.

In addition, the PDF of the number of collided preamble transmissions is not addressed in previous studies. However, as shown later, the joint PDF of the number of successful and collided attempts plays a major role in the design of accurate estimators of the number of contending UEs in a RAO.

In this paper, to fully address the above mentioned issues, we devise: 1) a simpler closed-form expression for computing the joint PDF of the number of successful and collided transmission attempts; and 2) an efficient recursion for the same purpose that substantially reduces the computational cost.

III. Joint PDF of the Number of Successful and Collided Preamble Transmissions

This section is organized in three parts. Section III-A presents the closed-form expression of the PDF, Section III-B presents a computationally efficient recursion to compute the PDF, and Section III-C presents an application of the PDF to design a series of estimators of the number of contending UEs.

For the formulation, we focus on a single RAO. Let r be number of available preambles. Also, let n be the number of contending UEs in a RAO (i.e., the UEs that transmit a preamble selected among the r available preambles with equal probability). Finally, let s be the number of preambles selected by exactly one UE, and c the number of collided preambles. We consider pairs (s, c) in the set $\mathcal{R}_n \triangleq \{(s, c) \in \mathbb{N}^2 \mid s+c \leq r, s + \beta c = n, \beta \geq 2\}$.

A. Closed-Form Expression

Lemma 1: Consider n UEs that independently choose a single preamble from a set of c different preambles with equal probability. Let $\mathrm{F}(n, c)$ denote the probability that at least two UEs are assigned to each preamble. Then, $\mathrm{F}(n, c) = 0$ if $n < 2c$. For $n \geq 2c$ we have

$$\mathrm{F}(n, c) = 1 + \sum_{k=1}^{c-1} (-1)^k \binom{c}{k} \sum_{m=0}^{k} p_{m,k}\,, \tag{1}$$

where

$$\begin{aligned} p_{m,k} &\triangleq \binom{n}{m} \left(\frac{k}{c}\right)^m \left(1 - \frac{k}{c}\right)^{n-m} \frac{k}{k}\,\frac{k-1}{k} \cdots \frac{k-(m-1)}{k} \\ &= \binom{n}{m} \left(\frac{k}{c}\right)^m \left(1 - \frac{k}{c}\right)^{n-m} \frac{k!}{(k-m)!k^m}\,. \end{aligned} \tag{2}$$

Proof: In what follows, we focus on the case $n \geq 2c$. Consider a subset of $k < c$ preambles. The probability that exactly m UEs ($m = 1, \ldots, k$) are assigned to the preambles in this subset, with at most one UE per preamble, is given by (2). Moreover, it is easy to check that the right-hand side of (2) is also valid for $m = 0$.

Let A_k denote the event that less than two (i.e., one or none) UEs have been assigned to the kth preamble. Then, we can write

$$\begin{aligned} \mathrm{F}(n, c) &= 1 - \Pr(A_1 \cup A_2 \cup \cdots \cup A_c) \\ &= 1 - \sum_{k=1}^{c} (-1)^{k-1} \sum_{1 \leq i_1 < \cdots < i_k \leq c} \Pr\Big(\bigcap_{j=1}^{k} A_{i_j}\Big) \\ &= 1 - \sum_{k=1}^{c-1} (-1)^{k-1} \sum_{1 \leq i_1 < \cdots < i_k \leq c} \sum_{m=0}^{k} p_{m,k} \\ &= 1 + \sum_{k=1}^{c-1} (-1)^k \binom{c}{k} \sum_{m=0}^{k} p_{m,k}\,. \end{aligned} \tag{3}$$

Above we have used the fact that $\Pr(\bigcap_{j=1}^{c} A_{i_j}) = 0$, since $n \geq 2c$. ■

Theorem 1: The conditional joint probability of having exactly s successes and c collisions, when n UEs transmitted their preambles, is given by

$$P_n(s,c) = \begin{cases} \binom{r}{s}\frac{s!}{r^s}, & c=0, s=n<r \\ \binom{r}{c}\left(\frac{c}{r}\right)^n \mathrm{F}(n,c), & s=0, 0<c\le c_{\max} \\ \binom{r}{s,c,r-s-c}\binom{n}{s}\frac{s!}{r^s}\left(\frac{c}{r}\right)^{n-s} \times \mathrm{F}(n-s,c), & s>0, c>0 \\ 0, & \text{otherwise,} \end{cases} \tag{4}$$

where $c_{\max} = \min\{r, \lfloor n/2 \rfloor\}$.

Proof:

1) $c = 0$, $s = n < r$
Given a fixed set of s preambles, the probability that $n = s$ UEs choose these s preambles without collision is given by

$$\frac{s}{r} \times \frac{s-1}{r} \times \cdots \times \frac{2}{r} \times \frac{1}{r} = \frac{s!}{r^s}. \tag{5}$$

Taking into account that the number of different subsets of s preambles is $\binom{r}{s}$, we have the first case in (4).

2) $s = 0$, $c \le c_{\max}$
Given a fixed set of c preambles, the probability that n UEs choose a preamble within this set is $(c/r)^n$. Using $\mathrm{F}(n,c)$ [see (3)] to account for the probability that each of the c preambles is selected by at least two of the n UEs, and taking into account the number of different subsets of c preambles, we have the second case in (4).

3) $s > 0$, $c > 0$
Consider a selection of preambles performed by n UEs resulting in s successes (selected by exactly one UE), c collisions (selected by at least two UEs), and $r-n-c$ unused preambles.
The probability that s out of the n UEs choose a preamble without collision is $\binom{n}{s}\frac{s!}{r^s}$.
The probability that $n-s$ out of the n UEs choose a preamble with collision is $\left(\frac{c}{r}\right)^{n-s} \times \mathrm{F}(n-s,c)$.
Taking into account that the number of different preamble selections is $\binom{r}{s,c,r-s-c}$, we have the third case in (4).

■

Finally, the marginal probability distributions can be computed as

$$P_n(s) \triangleq \Pr(s \mid n) = \sum_{c=0}^{c_{\max}} P_n(s,c) \tag{6}$$

$$P_n(c) \triangleq \Pr(c \mid n) = \sum_{s=0}^{\min\{r,n\}} P_n(s,c). \tag{7}$$

B. Recursion

Although (4) is a simple expression, the computation of factorials might require a high computational cost when a massive number of UEs access the network. This limitation is also present in [12], [18], [19]. To overcome this limitation, we devise the following recursion

$$P_n(s,c) = \frac{r-(s-1+c)}{r}P_{n-1}(s-1,c) + \frac{s+1}{r}P_{n-1}(s+1,c-1) + \frac{c}{r}P_{n-1}(s,c), \tag{8}$$

where $P_0(0,0) = 1$, and $P_n(s,c) = 0$ if $(s,c) \notin \mathcal{R}_n$.

Clearly, from the distribution for $n-1$ UEs, we obtain the distribution when a UE is added. Let $(s,c)_n$ represent the case in which n UEs have chosen their preambles leading to s successes and c collisions. The three outcomes that can lead to $(s,c)_n$ are:

1) Being the system in $(s-1,c)_{n-1}$, the nth UE chooses one of the $r-(s-1+c)$ unused preambles. This occurs with probability $(r-(s-1+c))/r$.
2) Being the system in $(s+1,c-1)_{n-1}$, the nth UE chooses one of the $s+1$ preambles that were chosen by exactly one UE. This occurs with probability $(s+1)/r$.
3) Being the system in $(s,c)_{n-1}$, the nth UE chooses one of the c preambles that were already chosen by more than one UE. This occurs with probability c/r.

C. Estimation of the Number of Contending UEs

The access class barring (ACB) and the extended access barring (EAB) schemes have been devised by the 3GPP to control the congestion in LTE-A cellular networks when a massive number of UEs try to access the network simultaneously [20]. To adapt the configuration of these congestion control mechanisms to the traffic load in order to achieve their optimal performance, the number of UEs attempting to access the eNB at each RAO is required [21].

In the following, we design a series of estimators of the number N of contending UEs in a RAO. For this, we use two different approaches: maximum likelihood (ML) and Bayesian. Additionally, we contemplate two cases according to the information available at the eNB. In the first one, the eNB can only observe the number of successful preamble transmissions s. In the second one, the eNB can observe both the number of successful s and the number of collided preambles c.

To facilitate notation, we use $\hat{N}$ to refer to a general estimator of N and x, or X if it is regarded as random variable, to the observed information by the eNB.

Then, the ML estimator of N provided that a sample x is observed, is simply given as

$$\mathrm{ML}(x) \triangleq \arg\max_n P_n(x). \tag{9}$$

For a given *loss function* $L(N,\hat{N})$, the Bayes estimator is defined as the decision rule that minimizes the posterior expected loss:

$$\hat{N}_L(x) \triangleq \arg\min_m \; \mathbb{E}\Big[L(N,m) \;\Big|\; X = x\Big] = \arg\min_m \sum_n L(n,m)\Pr(n|x). \tag{10}$$

In this paper, we select the relative estimation error as the loss function, that is,

$$L(N, \hat{N}) \triangleq \frac{|N - \hat{N}|}{N}. \tag{11}$$

Then, the resulting Bayesian estimator is given as

$$\mathrm{B}(x) \triangleq \frac{1}{2}\left(\max\{m : S_x(m) \leq 0\} + \min\{m : S_x(m) \geq 0\}\right), \tag{12}$$

where

$$S_x(m) \triangleq \sum_{n=1}^{m} \frac{1}{n} P_n(x)\pi(n) - \sum_{n>m} \frac{1}{n} P_n(x)\pi(n) \tag{13}$$

and $\pi(n) \triangleq \Pr(N = n)$ is the prior distribution of N. Please observe that in our approach we assume that no information is available on the value of N. Consequently, based on the *principle of indifference* [22], [23], we will consider $\pi(n)$ to be a uniform distribution over $\{1, 2, \ldots, N_{\max}\}$, which is a typical choice as a non-informative prior in a Bayesian framework [23]; and $N_{\max}$ is the maximum number of contending UEs in a RAO. This value must be selected according to the maximum expected load. As it will be described in Section IV-B, the maximum number of contending UEs in highly congested scenarios and with no congestion control in place has been observed to be in the order of a few hundreds [8], [9]. Note that the use of a non-informative prior has minimal influence on the final estimation [23]. Equation (13) then becomes

$$S_x(m) = \sum_{n=1}^{m} \frac{1}{n} P_n(x) - \sum_{n=m+1}^{N_{\max}} \frac{1}{n} P_n(x). \tag{14}$$

As noted above, two different cases are contemplated according to the information available at the eNB. In the first case, the observed sample x is the number of successful preamble transmissions s; and in the second case, it is the number of both successful and collided preambles (s, c). To differentiate these two cases, the estimator will be denoted as $\hat{N}_1$ and $\hat{N}_2$, respectively.

In the numerical results presented in Section IV, it can be observed that while the estimators introduced so far provide an acceptable accuracy, they are biased. In what follows, we propose a modification of $\hat{N}$ with the aim of reducing the bias to obtain a more accurate estimator.

Let

$$\mu(n) \triangleq \mathbb{E}[\hat{N}|N = n] = \sum_x \hat{N}(x)P_n(x). \tag{15}$$

If $\hat{N}$ were not biased, $\mu(n) = n$, but in our case, $\mu(n) < n$ with $\hat{N}_1$ and $\mu(n) > n$ with $\hat{N}_2$ for most values of n.

Let us assume for the moment that the value of N is known: $N = n$. Then, the estimator

$$\widetilde{N} \triangleq \frac{\hat{N}}{\mu(n)/n} \tag{16}$$

would be clearly unbiased. However, $\widetilde{N}$ is not a realizable estimator, because it depends on the actual value of N, which the estimator is supposed to estimate.

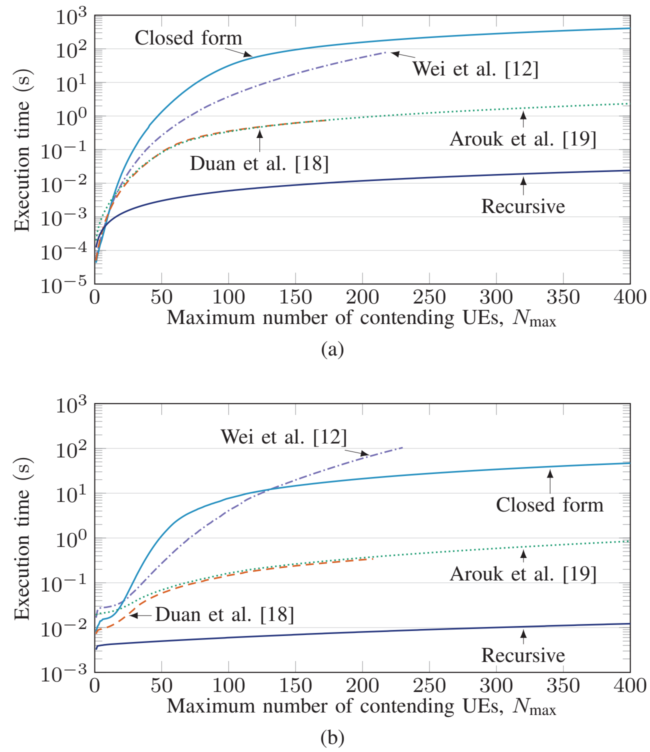


Figure 1. Execution time in seconds when the number of preambles is (a) $r = 54$ and (b) $r = 30$ as a function of the maximum number of contending UEs, $N_{\max}$.

Since the true value of N is actually unknown, we can make the approximation $N = n \approx \hat{N}$. In this way we obtain the refined estimator

$$\hat{N}^* \triangleq \frac{\hat{N}}{\mu(\hat{N})/\hat{N}} = \frac{\hat{N}^2}{\mu(\hat{N})}. \tag{17}$$

As demonstrated in the following section, applying the refinements described above to both $\hat{N}_1$ and $\hat{N}_2$ increases their accuracy for most values of N. The new estimators are denoted as $\hat{N}_1^*$ and $\hat{N}_2^*$.

IV. Numerical Results

In this section, two different types of numerical experiments to evaluate the efficacy of our contributions are conducted. First, we compare the computational cost of our methods with that of some related methods that have recently appeared in the literature [12], [18], [19]. However, it is important to emphasize that our methods and those used as a basis for comparison do not provide the same information: ours calculate the joint PDF of successes and collisions, whereas the other methods only calculate the PDF of successes. Second, we evaluate the accuracy of the designed estimators. The results of these experiments are presented in Section IV-A and Section IV-B, respectively.

A. Computational Cost Comparison

In addition to our methods, we implemented the methods proposed in [12], [18], [19] and analyzed the computational cost of all of them in terms of execution time. Our implementations were done in MATLAB 2015b, and were run on a PC with MS Windows 8.1 (64 bit), an Intel Core i7-4702MQ processor, 2.2 GHz and 16 GB RAM. During the execution of our MATLAB code, no other processes with a relevant CPU usage were run. We have also studied the cost in terms of the number of floating point operations. Here only the execution time results are shown since both metrics lead to the same conclusions.

Fig. 1 shows the execution time that each method requires to obtain the PDFs that are needed in the estimation when the maximum number of contending UEs in a RAO is N_{max}. The curves corresponding to two of the studied methods were interrupted at the point in which these methods stop generating valid results due to numerical stability issues.

Fig. 1a illustrates the computational cost when the number of available preambles for the contention-based random access procedure is $r = 54$. This is the most typical scenario according to the LTE-A specification [5], [24]. In addition, Fig. 1b illustrates the computational cost when $r = 30$. The reason for selecting this value is that the random access procedure defined in the NB-IoT standard is similar to that in LTE-A, but only $r = 48$ preambles are available [25]. Concretely, NB-IoT UEs with an acceptable wireless connection to the eNB belong to coverage-enhancement (CE) level zero and perform only one preamble repetition per access attempt. The remaining UEs belong to CE levels one and two, and perform several preamble repetitions to decrease the probability of an access failure due to a wireless channel error. Thus, setting $r = 30$ for UEs in CE level zero, which is expected to contain most of the UEs, seems adequate as it allows for the reservation of the remaining 18 preambles for UEs in higher CE levels.

The results in Fig. 1 show that, in addition to providing more detailed information, $P_n(s, c)$ vs. $P_n(s)$, our formulations also offer computational advantages in terms of both numerical stability and computational cost. In Fig. 1a, it can be observed that our recursive method is up to 6000, 70, and 100 times faster than that of the formulae presented in [12], [18], [19], respectively. Likewise, in Fig. 1b, it can be observed that our recursive method is 1.2×10^4, 40, and 70 times faster.

B. Accuracy of the Estimators

In this section, the results of assessing the accuracy of the proposed estimators are presented. Different values for the number of available preambles r were used, and it was observed that the results were qualitatively similar for all of them. Therefore, next we show and analyze the results corresponding to $r = 54$.

Fig. 2 shows the estimated value of N with the ML and Bayesian estimators. As can be seen, the accuracy of the estimation is much higher when (s, c) are known (i.e., $\hat{N}_2$) than when only s is known (i.e., $\hat{N}_1$). Fig. 2 also shows that the estimators are biased, which led to the formulation of the refined estimators $\hat{N}_1^*$ and $\hat{N}_2^*$ in (17).

In real implementations, the number of UEs within a single cell could be significantly large. Therefore, to assess the accuracy of our estimators, we varied the number of UEs that transmit a preamble in a RAO from one UE up to 300. This

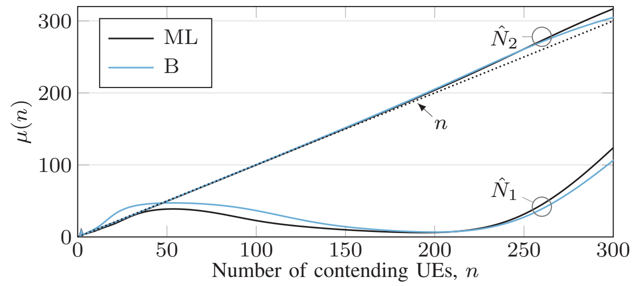
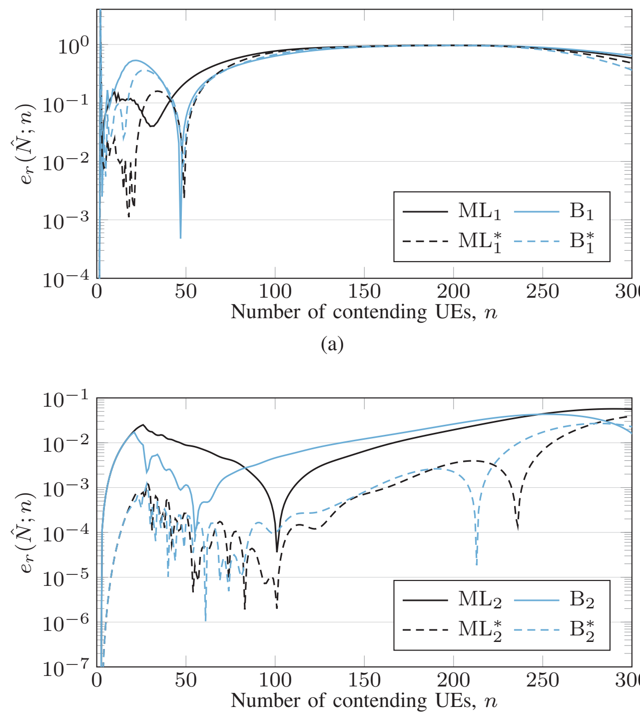


Figure 2. Expected value of the ML and Bayesian estimators given the number of contending UEs.



(a)

(b)

Figure 3. Expected relative error when $r = 54$ using: (a) $P_n(s)$ and (b) $P_n(s, c)$.

latter value was selected as it is approximately the maximum number of contending UEs per RAO in the most congested scenario described by the 3GPP [8], [24].

Fig. 3 illustrates the expected relative estimation error as a function of n, which is defined as

$$e_r(\hat{N}; n) \triangleq \mathbb{E}\left[L(N, \hat{N}) \middle| N = n\right] = \sum_x \frac{|n - \hat{N}(x)|}{n} P_n(x), \tag{18}$$

for all the proposed estimators.

Fig. 3a shows the expected relative error of $\hat{N}_1$ and $\hat{N}_1^*$, that is, only s is known to the eNB. We observe that the ML estimator is accurate for $N \leq r$, except for $N = 2$, where there is a sharp peak. This same peak is observed with the Bayesian estimator at exactly $N = 2$. It is worth noting that the accuracy of the Bayesian estimator is lower than that of the ML estimator when $N \leq r$, but the opposite occurs for $r < N < 100$. For $N \geq 100$ both estimators present a similar accuracy. However, it should be borne in mind that to properly observe the power of the Bayesian estimation one would have to consider a sequential method that enhances the accuracy of the estimation by incorporating observations as they are made in subsequent RAOs. Devising and studying such sequential estimation method is beyond the scope of this paper and is left for future research. Furthermore, the refinements to reduce the bias of the estimators sharply increase their accuracy for most values of $N \leq 100$, where the expected error of the initial estimator is usually lower than 20 percent. Conversely, when the expected error in the initial approximation surpasses 50 percent, the refined estimation presents a similar accuracy.

Fig. 3b shows the expected relative error of $\hat{N}_2$ and $\hat{N}_2^*$. We observe that the initial estimation results using $\hat{N}_2$ are highly accurate: $e_r(\hat{N}_2; n) \approx 5.8$ percent in the worst case. After refining these results using $\hat{N}_2^*$, we obtain even more accurate estimations: $e_r(\hat{N}_2^*; n) \leq 4$ percent for all the values of n.

It is worth noting that our proposed estimators provide even more accurate results under light-load conditions. Concretely, when $N \leq 100$. This has great significance as the purpose of congestion control mechanisms, such as the ACB and EAB schemes, is to maintain a light signaling traffic load. For instance, the RACH capacity is achieved when $\mathbb{E}[N] = [\log(r/(r-1))]^{-1} < r$ [26]. Hence, this is the optimal point of operation.

V. Conclusions

We presented expressions to obtain the joint probability distribution (PDF) for the number of successful and collided preambles in a random access opportunity (RAO) of an LTE-A network. Based on the available information at the eNB regarding the access attempts, we designed a series of maximum likelihood and Bayesian estimators of the number N of contending MTC devices in a RAO.

Numerical results showed that our formulations are computationally efficient and can be used to accurately estimate the number of contending MTC devices even for heavy network loads. The proposed approaches to determine the joint PDF and to estimate N can be exploited to design novel congestion control schemes, and to formulate optimization problems to set the parameters of these schemes appropriately.

In practice, a significant correlation between the number of contending UEs in consecutive RAOs is to be expected, which could be used to enhance the estimation. As a future work, we intend to extend the Bayesian estimation approach and to devise a sequential method that enhances the accuracy of the estimation by incorporating observations as they are made in subsequent RAOs.

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