Design of SCMA Codebooks Based on Golden Angle Modulation

Zeina Mheich, Lei Wen, Pei Xiao and Amine Maaref

***Abstract*—In this paper, we propose to use golden angle modulation (GAM) points to construct codebooks for uplink and downlink sparse code multiple access (SCMA) systems. We provide two categories of codebooks with one and two optimization parameters respectively. The advantages of the proposed design method are twofold: 1) the number of optimization variables is independent of codebook and system parameters; 2) it is simple to implement. In the downlink, we use GAM points to build a multidimensional mother constellation for SCMA codebooks, while in the uplink GAM points are directly mapped to user codebooks. The proposed codebooks exhibit good performance with low peak to average power ratio (PAPR) compared to the codebooks proposed in the literature based on constellation rotation and interleaving.**

***Index Terms*—Sparse code multiple access, codebook design, golden angle modulation, mapping, PAPR reduction.**

I. Introduction

The fifth generation (5G) of wireless networks is expected to support connectivity to massive number of devices. Due to the limitations of the available bandwidth for wireless networks, non-orthogonal multiple access (NOMA) has emerged in the few past years as a new multiple access technique to improve spectral efficiency, and to provide massive connectivity in 5G. NOMA improves the spectral efficiency of wireless radio access by enabling overloading *i.e.*, the number of multiplexed users can be larger than the number of resources. Proposed NOMA techniques can be classed under three categories: power-domain NOMA (PD-NOMA) [1], code-domain NOMA (CD-NOMA) [2], [3], [4] and a combination of PD-NOMA and CD-NOMA called power domain sparse code multiple access [5].

Low-density signature (LDS) [2], [3] is an efficient CD-NOMA that allows operating at overloaded conditions with performance close to single user case with affordable complexity. Due to the low-density property of signatures, only few users will use the same resource in order to reduce the interference and the data symbol for each user will be spread on a few number of resources. Like low-density parity-check coding, the low-density characteristic of LDS signatures allows to use the message passing algorithm (MPA) for multi-user detection. In LDS, the constellation symbol is expanded to a sequence of complex symbols by using a specific signature. In [4], the authors propose a new NOMA technique called SCMA which is a generalization of LDS. In SCMA, the procedure of bit to modulation mapping and spreading are combined together such that coded bits are directly mapped to multi-dimensional sparse codewords selected from user-specific SCMA codebooks. By using multi-dimensional sparse codebooks, SCMA can benefit from shaping gains of multi-dimensional constellations while still enjoying the properties of LDS such as overloading capability and the possibility of using MPA for multi-user detection due to the sparsity of codewords. MPA detector performance is sensitive to the distance between the dimensions of codewords interfering over the same resource. SCMA codebooks have to be designed carefully in order to introduce as much power diversity as possible over the interfering users. Besides the efficiency of the SCMA codebooks, the low-complexity of the design is also required, for any codebook parameters.

A. Related works

The design of optimal SCMA codebooks is a challenging problem. Nikopour *et al.* [4] and Taherzadeh *et al.* [6] are the first who proposed a multi-stage suboptimal method for the design of multiuser SCMA codebooks. Following this direction, other sub-optimal design methods are proposed for downlink SCMA systems *e.g.* [7], [8], [9] to increase the coding gain or to lower the complexity of multiuser detection. Cai *et al.* proposed in [7] a multi-dimensional codebook design based on constellation rotation and interleaving. The advantage of their design method is that it can be easy to implement for any codebook and system parameters (number of layers, codebook size, codebook dimension, $\cdots$). In [8], Yu *et al.* proposed a design method of SCMA codebooks based on star-quadrature amplitude modulation (star-QAM) signaling. However, the rotation angles of user operators are not explicitly given. Although most of the existing codebook design methods for downlink provide codebooks with good performance [7]–[11], their performance degrades significantly in uplink. For uplink SCMA systems, fewer works exist in literature about the design of SCMA codebooks. Bao *et al.* proposed in [12] a performance criterion for the joint design of multiuser codebook for uplink SCMA systems over Rayleigh fading channels, based on the cutoff rate of the equivalent multiple-input multiple-output (MIMO) system. Their proposed codebooks exhibit better performance in uplink than the codebooks proposed in [8] and [10]. However, the design method in [12] requires to solve a complex optimization

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problem which becomes intractable for large number of users and codebook dimension. To the best of our knowledge, there does not exist in the literature a simple method to design SCMA codebooks for uplink as in [7] for downlink.

B. Contributions

In this paper we present a new method to design SCMA codebooks based on golden angle modulation [13] for downlink and uplink SCMA systems. GAM is a recently proposed modulation which can offer enhanced mutual information (MI)- and PAPR- performance over a square-QAM design. However, GAM has never been used to design SCMA codebooks. We propose θ-GAM and (θ, ρ)-GAM codebooks with one and two optimization parameters respectively (θ and (θ, ρ) respectively). The number of optimization parameters is independent of the codebook and system parameters (number of layers, number of resources, codebook size, codebook dimension, etc.) which allows to design codebooks of high rates, large dimensions and large number of users, in a simple manner. The parameters θ and ρ can be adjusted to decrease the peak to average power ratio and increase the normalized minimum Euclidean distance of the proposed codebooks. Numerical results show the efficiency of the proposed codebooks compared to the codebooks in [7]. Moreover, the proposed GAM based codebooks have lower PAPR than those proposed in [7] and [8]. Mitigating the PAPR problem is of practical interest when SCMA is combined with multicarrier techniques such as OFDM for practical deployment, as large PAPR can cause signal distortion and spectral spreading which increase the bit error rate.

C. Notations

$\mathbb{B}, \mathbb{C}, \mathbb{R}, \mathbb{Z}, \mathbb{N}$ denote respectively the set of binary, complex, real, integer, and natural numbers. We use x, $\mathbf{x}$ and $\mathbf{X}$ to represent a scalar, a vector and a matrix respectively. The i^{th} element of $\mathbf{x}$ is denoted by x_i and X_{ij} is the element in the i^{th} row and j^{th} column of $\mathbf{X}$. The i^{th} row of $\mathbf{X}$ is denoted by $\mathbf{X}_i$ and its j^{th} column is denoted by $\mathbf{X}^j$. $\mathbf{I}_N$ denotes an identity matrix of size $N \times N$. $\text{diag}(\mathbf{x})$ is a diagonal matrix where its i^{th} diagonal element is x_i. $\lfloor x \rfloor$ denotes the floor function of x and $\lceil x \rceil$ denotes the ceiling function of x. The modulo operator $\text{mod}(a, m)$ returns the remainder after division of a by m. $|x|$ returns the absolute value of x.

II. SCMA CODEBOOKS FOR DOWNLINK

In this section, we present the SCMA system architecture and the received signal model in the downlink. We review briefly the design method of SCMA codebooks based on constellation rotation and interleaving in [7]. It is followed by introducing a new method to design SCMA codebooks based on golden angle modulation.

A. SCMA encoding and multiplexing

We consider a downlink SCMA system where a transmitter communicates with J users sharing K orthogonal resources, e.g. K OFDMA sub-carriers, K time slots, $\cdots$. Each user has a multi-dimensional SCMA codebook of M codewords where each codeword occupies K resources. An SCMA encoder for user j is defined as a mapping

$$f_j : \mathbb{B}^{\log_2(M)} \rightarrow \mathcal{X}_j, \tag{1}$$

where $\mathcal{X}_j \subset \mathbb{C}^K$ is the codebook of user j with cardinality M. A vector $\mathbf{b}$ of $\log_2(M)$ coded bits is mapped to a multi-dimensional codeword $\mathbf{x} = f_j(\mathbf{b})$ selected from $\mathcal{X}_j$. Each complex codeword $\mathbf{x}$ is a sparse vector with $N < K$ non-zero entries. The sparsity of the codewords enables to limit the number of users colliding over the same resource which in turns reduces the complexity of user detection. Let $\mathbf{c}$ denotes the vector of non-zero complex symbols obtained by removing the zeros from the codeword $\mathbf{x}$. The vector $\mathbf{c}$ can be interpreted as an N-dimensional complex constellation point belonging to a multi-dimensional constellation $\mathcal{C}_j \subset \mathbb{C}^N$. We define the mapping from $\mathbb{B}^{\log_2(M)}$ to $\mathcal{C}_j$,

$$g_j : \mathbb{B}^{\log_2(M)} \rightarrow \mathcal{C}_j, \tag{2}$$

such that $\mathbf{c} = g_j(\mathbf{b})$. Then the SCMA encoder in (1) can be redefined as $f_j :\equiv \mathbf{V}_j \cdot g_j$ where $\mathbf{V}_j \in \mathbb{B}^{K \times N}$ is the binary mapping matrix which maps the N-dimensional constellation point to a K-dimensional SCMA codeword. The mapping matrix $\mathbf{V}_j$ contains $K - N$ all-zero rows. By eliminating the $K - N$ all-zero rows from $\mathbf{V}_j$ we obtain an identity matrix $\mathbf{I}_N$ of size $N \times N$. A row $r \in \{1, \cdots, K\}$ containing a value 1 in $\mathbf{V}_j$ means that user j is using the r^{th} resource. Hence, the set of resources occupied by user j depends on $\mathbf{V}_j$.

Like LDS, SCMA enables overloading such that the number of users can be more than the number of resources $J > K$. The overloading factor is defined as $\lambda = J/K$. An SCMA code can be represented by $\mathcal{S}\left([\mathbf{V}_j]_{j=1}^J, [g_j]_{j=1}^J; J, M, N, K\right)$. The whole structure of SCMA code $\mathcal{S}$ can be represented by a factor graph matrix $\mathbf{F} = (\mathbf{f}_1, \cdots, \mathbf{f}_J) \subset \mathbb{B}^{K \times J}$ where $\mathbf{f}_j = \text{diag}(\mathbf{V}_j \mathbf{V}_j^T)$. The resource k is used by user j if and only if $F_{kj} = 1$.

In the following, we define two metrics (PAPR and normalized minimum Euclidean distance) which affect the performance and implementation of SCMA codebooks. The peak to average power ratio of a codebook $\mathcal{X}_j$ is defined as

$$\text{PAPR [dB]} = 10 \cdot \log_{10}\left(\frac{\max_{1 \le m \le M} |\mathbf{x}_j(m)|^2}{E}\right), \tag{3}$$

where $|\mathbf{x}|$ is the complex magnitude of $\mathbf{x}$, $\mathbf{x}_j(m) \in \mathcal{X}_j$ is the m^{th} codeword of $\mathcal{X}_j$ and E is the average codeword power given by $E = \frac{1}{M}\sum_{m=1}^{M} |\mathbf{x}_j(m)|^2$. The normalized minimum Euclidean distance of a codebook $\mathcal{X}_j$ is defined as

$$\tilde{d}_{\text{min}} = \frac{d_{\text{min}}}{\sqrt{E}}, \tag{4}$$

where $d_{\text{min}} = \min\{|\mathbf{x}_j(m) - \mathbf{x}_j(n)|\}, \forall m, n \in \{1, \cdots, M\}, m \neq n$. Usually, a lower PAPR and higher $\tilde{d}_{\text{min}}$ are targeted in the SCMA codebook design.

In the downlink, the received signal after the synchronous layer multiplexing can be expressed as

$$\mathbf{y} = \sum_{j=1}^{J} \text{diag}(\mathbf{h})\mathbf{x}_j + \mathbf{n}, \tag{5}$$

where $\mathbf{h} = (h_1, \cdots, h_K)^T$ is the channel vector, $h_k \sim \mathcal{CN}(0,1)$, $\mathbf{x}_j$ is the SCMA codeword of user j and $\mathbf{n} \sim \mathcal{CN}(0, N_0\mathbf{I}_K)$ is the complex additive white Gaussian noise. At the receiver, MPA is performed for joint multi-user detection which achieves near optimal performance in the MAP sense with lower complexity [4].

B. MD-SCMA codebooks based on constellation rotation and interleaving

In this section, we recall briefly the design method of multi-dimensional SCMA (MD-SCMA) codebooks proposed in [7] for downlink SCMA systems. The advantage of this method is that it allows to construct codebooks in a simple manner and of arbitrary size and dimension. Simulation results for a 2-dimensional codebook of size 4 on 1×2 SIMO downlink Rayleigh channel, show the superiority of the codebook constructed in [7] with respect to that in [6]. In this paper, MD-SCMA codebooks will serve as benchmark due to their simplicity of construction for any SCMA system parameters.

1) First step (construct the mother constellation/codebook): We construct first a N-dimensional mother constellation of M points with good properties (low PAPR, large minimum Euclidean distance between codewords). In the second step, all user codebooks are deduced from the mother constellation. Let $\mathbf{s} = [s_1, s_2, \cdots, s_M]$ denotes a row vector with elements $s_m = \big(2m-1-M\big)\cdot(1+i)$ for $m = 1, \cdots, M$. The two main steps to construct the mother constellation can be summarized as follows.

- *Rotation*: Build a matrix $\mathbf{\Omega}$ of size $N \times M$ with its n^{th} row is given by $\mathbf{s} \cdot e^{i\theta_n}$, where $\theta_n = (n-1)\frac{\pi}{MN}$, $n = 1, \cdots, N$.
- *Interleaving*: Construct the mother constellation, represented by a matrix $\mathbf{M}$ by interleaving the rows of even index of $\mathbf{\Omega}$. Let $j \in \{1, \cdots, N\}$. if j is odd, $\mathbf{M}_j = \mathbf{\Omega}_j$. If j is even, $\mathbf{M}_j = \Pi(\mathbf{\Omega}_j)$, where Π is the following interleaver: Let $\mathbf{\Omega}_j = [\Omega_{j,1}, \Omega_{j,2}, \cdots, \Omega_{j,M}]$ and j is even, then $\mathbf{M}_j = [-\Omega_{j,\frac{M}{2}+1}, \cdots, -\Omega_{j,\frac{3M}{4}}, \Omega_{j,\frac{3M}{4}+1}, \cdots, \Omega_{j,M}, -\Omega_{j,M}, \cdots, -\Omega_{j,\frac{3M}{4}+1}, \Omega_{j,\frac{3M}{4}}, \cdots, \Omega_{j,\frac{M}{2}+1}]$.
  The interleaving of even dimensions reduces the PAPR of the mother constellation.

2) Second step (construct multi-user codebooks): Based on the factor graph matrix $\mathbf{F}$, we construct a codebook for each user. Let d_r be the number of users colliding over the same resource and $\varphi_r = (r-1)\frac{2\pi}{Md_r} + \frac{2\pi}{M}e_r$, for $r = 1, \cdots, d_r$ where e_r is an arbitrary member of $\mathbb{Z}$. We build a Latin matrix $\mathbf{L}$, by replacing the non-zero elements of $\mathbf{F}$ by $e^{i\varphi_r}$, where the φ_r are chosen such that $\mathbf{L}$ is Latin. For each user u, we define $\mathbf{\Delta}_u = \text{diag}(\tilde{\mathbf{L}}^u)$, where $\tilde{\mathbf{L}}^u$ is the u^{th} column of $\mathbf{L}$ without zero elements. Then the codebook of user u is given by $\mathcal{X}_u = \mathbf{V}_u \cdot \mathbf{\Delta}_u \cdot \mathbf{M}$. Note that the performance of codebooks depends on the choice of the Latin matrix.

In this paper, we propose two codebook design methods for downlink and uplink SCMA systems respectively. Both methods consist of three main steps: 1) generating GAM points, 2) generating a matrix of indexes and 3) mapping. In the downlink, these steps are used to build a mother constellation from which all user codebooks are obtained. In the uplink, these steps are used to construct directly user codebooks without needing an intermediate mother constellation. The codebook design steps are detailed in the following section for the downlink case.

C. Proposed codebooks for downlink

In this section, we propose a design method of SCMA codebooks for downlink systems. In the proposed method, the elements of the matrix $\mathbf{M}$ representing the mother constellation take values from the, recently proposed, golden angle modulation [13]. In the following, the main features of GAM which have an impact on the codebook design are provided for the seek of completeness, including symbol positions in the constellation and its advantages. We refer the reader to the recent works [13], [14], [15] for more details and results about GAM. The n^{th} constellation point of GAM[1] with N_p points is $x_n = r_n e^{i2\pi\varphi n}$, where $r_n = c_{\text{norm}}\sqrt{n}$, $c_{\text{norm}} = \sqrt{\frac{2P}{N_p+1}}$, P is a power constraint[2] and $\varphi = \frac{1-\sqrt{5}}{2}$ is the golden angle in rads. Figure 1 shows a GAM with 1000 points.

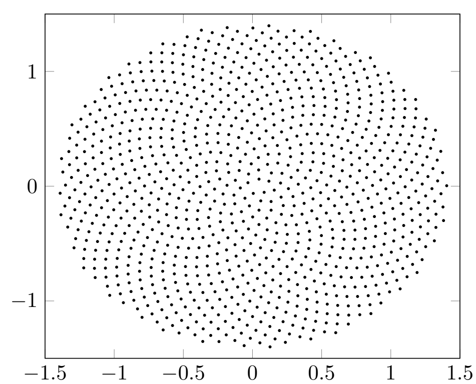

Figure 1. Golden angle modulation with 1000 points.

GAM has many features including [13]: 1) natural constellation point indexing: GAM enables a unique indexing based on signal phase, $2\pi\varphi n$, or magnitude, r_n, alone; 2) near-ideal circular design which can offer enhanced MI-, distance-, symbol error rate- and PAPR- performance over a square-QAM design; 3) rotation (and gain) invariant: each signal constellation has a uniquely identifiable phase and gain. These attractive characteristics of GAM points motivate us to choose the elements of SCMA codebooks from a GAM. First, the complex amplitude of each point depends uniquely on its index n, which provides power diversity between constellation points. Second, with a careful mapping between GAM points and the elements of SCMA codebooks, we could benefit from the low-PAPR of GAM to construct low-PAPR SCMA codebooks.

In the following, we present the steps used to construct the N-dimensional mother constellation $\mathbf{M}$ of M points for downlink SCMA systems using GAM.

[1]We consider here Disc-GAM [13].
[2]We set $P = 1$.

1) Generate GAM points: Generate $N_p = N \cdot \frac{M}{2}$ GAM points $x_n = r_n e^{i2\pi(\varphi+\theta)n}$ for $n = 1, \cdots, N_p$, where θ is an arbitrary angle. Note that the original GAM in [13] does not include a parameter θ; but has been added here to provide an additional degree of freedom which could be optimized to improve the codebook performance. These N_p GAM points are used to build $\frac{M}{2}$ N-dimensional complex points $\mathbf{M}^m$ for $m = 1, \cdots, \frac{M}{2}$. The remaining $\frac{M}{2}$ N-dimensional complex points are obtained by symmetry: $\mathbf{M}^{m+\frac{M}{2}} = -\mathbf{M}^m$ for $m = 1, \cdots, \frac{M}{2}$. Since the complex amplitude of each point depends on its index n, the point that is symmetric to a GAM point with respect to the origin does not collide with any other GAM point. The problem now is to map the N_p GAM points to the $\frac{M}{2}$ N-dimensional complex $\mathbf{M}^m$ for $m = 1, \cdots, \frac{M}{2}$. The proposed design method based on GAM requires only to optimize θ independently on codebook parameters. We refer to this design method as θ-GAM.

2) Matrix of indexes: Let $\mathbf{T}$ a matrix of size $2 \times N_p$. The elements of the first row of $\mathbf{T}$ are dimension indexes while the elements of the second row are codeword indexes. Hence each column of $\mathbf{T}$ represents a pair of indexes (k, m), where $k = 1, \cdots, N$ and $m = 1, \cdots, \frac{M}{2}$. The elements of $\mathbf{T}$ are generated as follows:

$$\begin{aligned} T_{1i} &= \left\lceil \frac{i}{M/2} \right\rceil \qquad (k \text{ index}), \\ T_{2i} &= \operatorname{mod}\left(i-1, \frac{M}{2}\right) + 1 \qquad (m \text{ index}), \end{aligned} \tag{6}$$

for $i = 1, \cdots, N_p$.

3) Mapping: Each point x_n of the N_p GAM points should be mapped to a pair (k, m) where k is the dimension index and m is the codeword index. This is equivalent to mapping the N_p GAM points to the column of $\mathbf{T}$. For example, if x_n is mapped to the column $\mathbf{T}^i$, where $T_{1i} = k, T_{2i} = m$, then the k^{th} dimension of the m^{th} codeword is equal to x_n. We propose Mapping 1 which aims to lower the PAPR and to achieve power diversity between codewords and between the dimensions of the same codeword.

Mapping 1 Mapping for downlink

1: **for** $k = 1 : N$ **do**
2:   **if** k is odd **then**
3:     **for** $i = 1 : \frac{M}{2}$ **do**
4:       Map the $\left((k-1)\frac{M}{2} + i\right)^{\text{th}}$ column of $\mathbf{T}$ to $x_{k+N(i-1)}$.
5:     **end for**
6:   **else**   (k is even)
7:     **for** $i = 1 : \frac{M}{4}$ **do**
8:       Map the $\left((k-1)\frac{M}{2} + i\right)^{\text{th}}$ column of $\mathbf{T}$ to $x_{k+N\left(\frac{M}{2}-i\right)}$.
9:     **end for**
10:     **for** $i = \frac{M}{4} + 1 : \frac{M}{2}$ **do**
11:       Map the $\left((k-1)\frac{M}{2} + i\right)^{\text{th}}$ column of $\mathbf{T}$ to -$x_{k+N\left(\frac{M}{2}-i\right)}$.
12:     **end for**
13:   **end if**
14: **end for**

k-index	1	1	2	2
m-index	1	2	1	2

Table I
EXAMPLE OF A MATRIX $\mathbf{T}$.

Example: This section provides an example to clarify the idea behind the codebook construction in steps 1-3. Suppose that $M = 4$ and $N = 2$, which lead to codebooks of size $M = 4$ where each codeword contains $N = 2$ non-zero elements. Since codebooks are symmetric, the problem reduces to the construction of two codewords $\mathbf{M}^1$ and $\mathbf{M}^2$. The remaining codewords are such that $\mathbf{M}^3 = -\mathbf{M}^1$ and $\mathbf{M}^4 = -\mathbf{M}^2$. Because $\mathbf{M}^1$ and $\mathbf{M}^2$ are two-dimensional ($N = 2$), the maximum number of non-zero elements in both codewords is equal to 4. The non-zero elements constituting the codewords are supposed to be distinct in order to increase the minimum distance between codewords. Thus, the number of non-zero elements is $N_p = 4$. The first step in the construction of the mother constellation is to generate $N_p = 4$ GAM points, $x_n = r_n e^{i2\pi(\varphi+\theta)n}$ for $n = 1, 2, 3, 4$, used as codeword elements for $\mathbf{M}^1$ and $\mathbf{M}^2$. In the second step, a matrix of indexes $\mathbf{T}$ is constructed. In this example, $\mathbf{T}$ is given in Table I.

Each column of the matrix $\mathbf{T}$ corresponds to a couple of indexes (k, m), where $1 \leq k \leq N$ is the dimension index and $1 \leq m \leq M/2$ is the codeword index. The third step consists of mapping each symbol in the set $\{x_1, x_2, x_3, x_4\}$ to a column of the matrix $\mathbf{T}$. The output of the proposed mapping is given in Table II.

k-index	1	1	2	2
m-index	1	2	1	2
mapping	x_1	x_3	x_4	$-x_2$

Table II
OUTPUT OF THE PROPOSED MAPPING.

We observe that, for a given dimension k, the elements of two distinct codewords x_i and x_j cannot be consecutive in the constellation *i.e.,* $|j - i| \neq 1$. In this way, the power diversity for a fixed dimension between codewords increases since GAM points x_n are increasing in power as n increases. For example, the element of the first dimension in $\mathbf{M}^1$ and $\mathbf{M}^2$ are x_1 and x_3 respectively. The mapping proposed in step 3, is such that $|j - i| = N$. It starts from the first dimension $k = 1$, and maps the symbols $x_1, x_{1+N}, x_{1+2N}, \cdots$ to elements of first, second, third,$\cdots$ codewords respectively. In general, for a dimension k, It maps the symbols $x_k, x_{k+N}, x_{k+2N}, \cdots$ to the elements of the $\left((k-1)\frac{M}{2} + 1\right)^{\text{th}}$, $\left((k-1)\frac{M}{2} + 2\right)^{\text{th}}$, $\left((k-1)\frac{M}{2} + 3\right)^{\text{th}}, \cdots$ codewords respectively (see lines 2-5 in Mapping 1). Then, an interleaving is applied for the dimensions of even indexes to lower the PAPR and increase further power diversity between codewords. We use the same interleaver in [7] and [8] (see section II-B1). The interleaving in Mapping 1 appears in lines 6-12. ■

After obtaining the mother constellation $\mathbf{M}$, the codebook for each user is constructed after applying a constellation operator to $\mathbf{M}$. For the sake of simplicity, we use the rotation angles (φ_r) given in [7] to construct multi-user codebooks based on the mother constellation and the factor graph matrix. However, in general, these rotation angles could be optimized. The performance of the proposed codebooks, based on a mother constellation and the constellation operators given in [7], degrades in uplink. In the next section, we propose an new design method which is not based on a mother constellation.

III. SCMA CODEBOOKS FOR UPLINK

A. Signal model

In the uplink, the received signal after the synchronous layer multiplexing (e.g., via the timing advance mechanism) can be expressed as

$$\mathbf{y} = \sum_{j=1}^{J} \operatorname{diag}(\mathbf{h}_j)\mathbf{x}_j + \mathbf{n}, \tag{7}$$

where $\mathbf{h}_j = (h_1^j, \cdots, h_K^j)^T$ is the channel vector between user j and the receiver, $h_k^j \sim \mathcal{CN}(0,1)$, and $\mathbf{n} \sim \mathcal{CN}(0, N_0\mathbf{I}_K)$.

B. Proposed codebooks for uplink

We propose a simple method to construct good codebooks for uplink SCMA systems with arbitrary parameters. The proposed method consists of the following steps:

1) Generate GAM points: Generate $N_p = J \cdot N \cdot \frac{M}{2}$ GAM points $x_n = r_n e^{i2\pi(\varphi+\theta)n}$ for $n = 1, \cdots, N_p$, where θ is an arbitrary angle. These N_p GAM points are used to build $\frac{M}{2}$ N-dimensional complex points $\mathbf{M}^m$, $m = 1, \cdots, \frac{M}{2}$, for each user. The remaining $\frac{M}{2}$ N-dimensional complex points are obtained by symmetry: $\mathbf{M}^{m+\frac{M}{2}} = -\mathbf{M}^m$ for $m = 1, \cdots, \frac{M}{2}$. The problem now is to map the N_p GAM points to the $\frac{M}{2}$ N-dimensional complex $\mathbf{M}^m$ and J users.

2) Matrix of indexes: Let $\mathbf{T}$ a matrix of size $3 \times N_p$. The elements of the first row of $\mathbf{T}$ are dimension indexes, the elements of the second row are codeword indexes and the elements of the third row are user indexes. Hence each column of $\mathbf{T}$ represents a triplet of indexes (k, m, u), where $k = 1, \cdots, N$, $m = 1, \cdots, \frac{M}{2}$ and $u = 1, \cdots, J$. The elements of $\mathbf{T}$ are generated as follows:

$$\begin{aligned} T_{1i} &= \left\lceil \tfrac{i}{J \cdot M/2} \right\rceil \qquad (k \text{ index}),\\ T_{2i} &= \operatorname{mod}\Big(\left\lceil \tfrac{i}{J} \right\rceil - 1, \tfrac{M}{2} \Big) + 1 \qquad (m \text{ index}),\\ T_{3i} &= \operatorname{mod}\big(i-1, J\big) + 1, \qquad (u \text{ index}), \end{aligned} \tag{8}$$

for $i = 1, \cdots, N_p$.

3) Mapping: Each point x_n of the N_p GAM points should be mapped to a triplet (k, m, u) where k is the dimension index[3], m is the codeword index and u is the user index. This is equivalent to map the N_p GAM points to the column of $\mathbf{T}$. For example, if x_n is mapped to the column $\mathbf{T}^i$, where $T_{1i} = k, T_{2i} = m, T_{3i} = u$, then the k^{th} dimension of the m^{th} codeword belonging to the codebook of user u is equal to x_n. We propose Mapping 2 which aims to lower the PAPR and to achieve power diversity between codewords and between the dimensions of the same codeword. Figure 2 illustrates an example of the mapping between GAM points and the elements of SCMA codebooks.

[3]The resource corresponding to each dimension for each user is determined from the factor graph matrix.

Mapping 2 Mapping for uplink

for $k = 1 : N$ **do**
  if k is odd **then**
    for $i = 1 : J \cdot \frac{M}{2}$ **do**
      Map the $\left((k-1) \cdot J\frac{M}{2} + i\right)^{\text{th}}$ column of $\mathbf{T}$ to $x_{k+N\cdot(i-1)}$.
    end for
  else (k is even)
    Let $\mathbf{t} = [k, k+N, k+2N, \cdots, k+qN]$, where $q = \left\lfloor \frac{N_p - k}{N} \right\rfloor \in \mathbb{N}$.
    for $u = 1 : J$ **do**
      for $i = 1 : \frac{M}{4}$ **do**
        Map the $\left((k-1) \cdot J \cdot \frac{M}{2} + (i-1) \cdot J + u\right)^{\text{th}}$ column of $\mathbf{T}$ to $-x_s$ where $s = t_{(\frac{M}{2}-i)\cdot J+u}$.
      end for
      for $i = \frac{M}{4}+1 : \frac{M}{2}$ **do**
        Map the $\left((k-1) \cdot J \cdot \frac{M}{2} + (i-1) \cdot J + u\right)^{\text{th}}$ column of $\mathbf{T}$ to x_s where $s = t_{(\frac{M}{2}-i)\cdot J+u}$.
      end for
    end for
  end if
end for

<u>**Example:**</u> We consider an SCMA system of $J = 6$ users sharing $K = 4$ resources and using codebooks of size $M = 4$. Each user occupies $N = 2$ resources only and each resource is used by 3 users. The codebook construction for each user consists of three steps. The first step consists of generating N_p GAM points used as codeword elements. In this example, each codeword contains $N = 2$ non-zero elements. Thus the total number of non-zeros elements is equal to $N_p = JN\frac{M}{2} = 24$, assuming that codebooks are symmetric. The second step after generating $N_p = 24$ GAM points x_n for $n = 1, \cdots, N_p$, is to build the matrix of indexes $\mathbf{T}$. In this example, $\mathbf{T}$ is given in Table III.

k	1	1	1	1	1	1	1	1	1	1	1	1
m	1	1	1	1	1	1	2	2	2	2	2	2
u	1	2	3	4	5	6	1	2	3	4	5	6
k	2	2	2	2	2	2	2	2	2	2	2	2
m	1	1	1	1	1	1	2	2	2	2	2	2
u	1	2	3	4	5	6	1	2	3	4	5	6

Table III
EXAMPLE OF A MATRIX $\mathbf{T}$.

Each column of $\mathbf{T}$ represents a triplet of indexes (k, m, u). The third step is to map each GAM point x_n (or its opposite -x_n) to a triplet (k, m, u). The output of the proposed Mapping 2 is given in Table IV. The principle of Mapping 2 is similar to that of Mapping 1. First, we intend to provide power diversity between the elements of the codeword of the same codebook for a fixed dimension. We observe that for a given dimension k, the elements of two distinct codewords x_i and x_j belonging to the same codebook (same index u) cannot be consecutive in the constellation *i.e.,* $|j-i| = JN$. Second, the mapping should be fair in the sense that all codebooks should have similar average power. This is achieved by interleaving even dimensions and by limiting the maximum difference $|j-i|$ between the elements at the same dimension k of two codewords x_i and x_j having the same index m and belonging to different codebooks to $|j-i| = (J-1)N$. Note that the interleaver appears through lines 6-15 in Mapping 2 and is the same used in Mapping 1.

k	1	1	1	1	1	1
m	1	1	1	1	1	1
u	1	2	3	4	5	6
map.	x_1	x_3	x_5	x_7	x_9	x_{11}
k	1	1	1	1	1	1
m	2	2	2	2	2	2
u	1	2	3	4	5	6
map.	x_{13}	x_{15}	x_{17}	x_{19}	x_{21}	x_{23}
k	2	2	2	2	2	2
m	1	1	1	1	1	1
u	1	2	3	4	5	6
map.	-x_{14}	-x_{16}	-x_{18}	-x_{20}	-x_{22}	-x_{24}
k	2	2	2	2	2	2
m	2	2	2	2	2	2
u	1	2	3	4	5	6
map.	x_2	x_4	x_6	x_8	x_{10}	x_{12}

Table IV
OUTPUT OF THE PROPOSED MAPPING.

■

It should be noted that in the proposed mapping for uplink, user indexing $u \in \{1, \cdots, J\}$ is independent from the factor graph matrix of the SCMA system. As an improvement of this mapping, we could sort user indexes such that two consecutive users u and $u+1$ have the least possible number of common resources. Furthermore, the proposed mapping can be easily extended to irregular SCMA systems, where users do not necessarily occupy the same number of resources [16].

In both downlink and uplink cases, we could add a second degree of freedom to the design, denoted by ρ, which serves to adjust the amplitude of GAM points. In this case the N_p GAM points are given by $x_n = c_{\text{norm}}\sqrt{n+\rho}\cdot e^{i2\pi(\varphi+\theta)n}$, for $n = 1, \cdots, N_p$. We refer to the codebooks obtained by optimizing both parameters θ and ρ as (θ, ρ)-GAM. The parameters θ and ρ allow to adjust the PAPR and the normalized minimum Euclidean distance of the codebooks.

IV. Numerical results

This section provides a numerical evaluation of the performance of the proposed (θ, ρ)-GAM codebooks in both downlink and uplink Rayleigh fading channels. The average bit error rate (BER) performance of the codebooks is evaluated for the uncoded scenario with 6 MPA iterations. We construct N-dimensional SCMA codebooks for two regular SCMA systems. In the first, the factor graph is given by

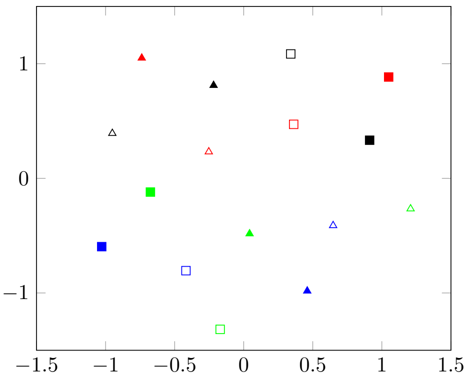

Figure 2. Mapping of GAM points to SCMA codebooks in uplink. $J = 2$, $N = 2$, $M = 8$, $\theta = 0$ and $\rho = 0$. Square and triangle marks are the elements of first and second codebooks respectively. Empty and filled marks represent the first and second dimensions respectively. Red, blue, black and green represent first, second, third and fourth codewords respectively (remaining four codewords are obtained by symmetry). Note that this example does not correspond to an overloaded system and is provided here for illustration purpose only.

$$\mathbf{F} = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \end{bmatrix},$$

which represents a system with 6 users sharing 4 resources where the number of resources used by each user is equal to $N = 2$. In the second, $\mathbf{F}$ is given in [7, eq. 3] and it represents a system with 8 users sharing 6 resources where the number of resources used by each user is equal to $N = 3$. The proposed codebook design method in this paper, could be used to design codebooks for large factor graph obtained, for example, using copy-and-permute operation of a protograph [17].

The parameters θ and ρ of the GAM codebooks are chosen off-line to maximize the average BER performance depending on the system model (channel estimation could be done using pilot-aided training). In order to reduce the complexity of the joint optimization of θ and ρ, we propose to perform alternating optimization of θ and ρ. Hence, we choose first θ^* which provides the best BER performance for $\rho = 0$ (*i.e.* θ^* gives the best BER performance for θ-GAM codebooks). Then, we fix $\theta = \theta^*$ and we choose ρ^* which maximizes the BER performance. This alternative optimization between θ and ρ does not guarantee the optimality of θ^* and ρ^*, but decreases the complexity of the joint optimization of θ and ρ.

The benchmark codebooks used to compare the performance of the proposed GAM-based codebooks are MD-SCMA proposed in [7] which are based on constellation rotation and interleaving. This is because the objective of the design method in [7] is to design codebooks in a simple manner for any SCMA systems without a number of optimization variables depending on system parameters. The codebook design methods based on optimizing mutual information or cut-off rate are not practical for any SCMA system as they have a number of optimization variables increasing with the number of users, resources and codebook size. Since the main objective of the proposed GAM-based codebooks is to provide a simple construction mechanism which is independent of the SCMA system parameters, MD-SCMA serves as a good benchmark. In addition to MD-SCMA, we also compare the performance of GAM-based codebooks to star-QAM codebooks proposed in [8]. However, star-QAM codebook construction requires to optimize a number of rotation angles which depends on the number of users and resources. In this section, the considered factor graph matrices are not too large, which makes the search for good rotation angles possible (it is time consuming when the size of codebooks increase).

Tables V and VI provides some parameters (PAPR in dB, $\tilde{d}_{\min}$) of the MD-SCMA codebooks in [7], the star-QAM codebooks proposed in [8] and the proposed (θ, ρ)-GAM codebooks, optimized for the 1×2 single-input-multiple-output (SIMO) downlink Rayleigh channel and the single-input-single-output (SISO) uplink Rayleigh channel respectively without a channel code. This work could be extended to coded systems as well as to the MIMO and MISO scenarios. We should also note that the codebook design method in uplink is not based on a mother constellation, which means that the PAPR and the minimum Euclidean distance may not be the same for all user codebooks. In Table VI we list the minimum and maximum values for each parameter. We provide also in these tables good values of θ and ρ [4]. Note that the values of θ^* are not those which maximizes the normalized minimum Euclidean distance but are chosen to provide the best BER curves for the uncoded scenario based on the channel model. For example with $\theta = -0.007$ we can achieve a normalized minimum Euclidean distance $\tilde{d}_{\min} = 1.4142$ in the downlink when $N = 2, K = 4, J = 6, M = 4$; however the best BER performance is obtained with $\theta = 0.0635$. Figures 3 and 4 show $\tilde{d}_{\min}$ as a function of θ when $N = 2$ for different values of M, in downlink and uplink respectively. Obviously, we observe that $\tilde{d}_{\min}$ is a periodic function; thus the search space for the optimum value of θ could be reduced within a period.

Figure 5 shows the BER performance of the 2-dimensional and 3-dimensional MD-SCMA codebooks in [7], the star-QAM codebooks proposed in [8] and the proposed (θ, ρ)-GAM codebooks in 1×2 SIMO downlink Rayleigh fading channel. We observe that the proposed (θ, ρ)-GAM codebooks provide lower PAPR than [7] and [8] without noticeable degradation in performance. Moreover, introducing the parameter ρ does not provide any improvement in performance ($\rho^* = 0$); thus we could only consider θ-GAM codebooks in downlink. The PAPR reduction using GAM codebooks comes from the fact that star-QAM and MD-SCMA codebooks both assume that the elements of each dimension of a codebook are aligned in the constellation. For example, in the MD-SCMA codebooks, the elements of each dimension of a codebook are the elements of a (rotated) PAM constellation. This constraint increases the PAPR. The proposed GAM-codebooks do not assume this constraint which reduces the PAPR.

[4]These values are denoted by θ^* and ρ^*, but this does not mean that these values are the optimal ones. In fact they represent the best values chosen by simulations from a finite set.

Figure 6 shows the BER performance of the 2-dimensional and 3-dimensional MD-SCMA codebooks in [7], the star-QAM codebooks proposed in [8] and the proposed (θ, ρ)-GAM codebooks in SISO uplink Rayleigh fading channel. The performance of the codebooks in [7] and [8] degrades significantly in the uplink. The proposed (θ, ρ)-GAM codebooks show very good performance with lower PAPR and better normalized minimum Euclidean distance than MD-SCMA codebooks and star-QAM codebooks. Star-QAM codebooks suffer from high PAPR which does not lend itself to practical implementation when SCMA is combined with OFDM. The performance of optimized star-QAM based codebooks [12] with $N = 2$ and $M = 4$ is also provided for comparison in Figure 6. It can be observed that optimized star-QAM based codebooks and (θ, ρ)-GAM codebooks exhibit similar performance. Note that the 2-dimensional 4-ary optimized star-QAM based codebooks provide the best performance in uplink among several codebooks proposed in the state-of-the-art as reported in [12, Fig.3]. This shows the efficiency of the proposed (θ, ρ)-GAM codebooks [5]. Moreover, we observed that (θ, ρ)-GAM codebooks are noticeably better (in terms of BER performance) than θ-GAM codebooks only when $M = 4$.

V. Conclusion

We proposed a new generalized design method of SCMA codebooks based on GAM for uplink and downlink SCMA systems. The proposed design method involves one or two optimization parameters independently on system parameters. Numerical results for 2- and 3- dimensional codebooks show that one optimization parameter is sufficient to design codebooks for downlink, while up to two optimization parameters are required for uplink. Numerical results demonstrate that GAM-based codebooks can achieve better performance and lower PAPR than MD-SCMA codebooks, based on constellation rotation and interleaving.

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References

[1] S. R. Islam, N. Avazov, O. A. Dobre, and K.-S. Kwak, "Power-domain non-orthogonal multiple access (NOMA) in 5G systems: Potentials and challenges," *IEEE Communications Surveys & Tutorials*, vol. 19, no. 2, pp. 721–742, 2017.

[2] R. Hoshyar, F. P. Wathan, and R. Tafazolli, "Novel low-density signature for synchronous CDMA systems over AWGN channel," *IEEE Transactions on Signal Processing*, vol. 56, no. 4, pp. 1616–1626, 2008.

[3] R. Hoshyar, R. Razavi, and M. Al-Imari, "LDS-OFDM an efficient multiple access technique," in *IEEE 71st Vehicular Technology Conference (VTC Spring)*, 2010, pp. 1–5.

[5]Optimized star-QAM based codebooks require to optimize two parameters for the mother constellation [12] ($\alpha \in \mathbb{R}$ and $\beta \in \mathbb{C}$) as well as user operator matrices [8]. Moreover, GAM-based codebooks can achieve lower PAPR than star-QAM based codebooks.

N	K	J	d_r	M	PAPR [7]	PAPR prop.	PAPR [8]	(θ^*, ρ^*)	$\tilde{d}_{\min}$ [7]	$\tilde{d}_{\min}$ prop.	$\tilde{d}_{\min}$ [8]
2	4	6	3	4	0	0	1.2885	$(0.0635, 0)$	1.4142	1.2886	1.2173
2	4	6	3	8	0.7572	0	2.3897	$(0.08, 0)$	0.4364	0.5240	0.4971
2	4	6	3	16	1.2366	0	5.6970	$(0.06, 0)$	0.2169	0.7207	0.2022
3	6	8	4	4	1.0266	0.5799	0.9902	$(0.15, 0)$	1.2649	1.2315	1.2045
3	6	8	4	8	1.9629	0.9018	3.5037	$(-0.02, 0)$	0.4364	0.5636	0.5912
3	6	8	4	16	2.4764	1.0721	4.5203	$(-0.0585, 0)$	0.2169	0.2195	0.1982

Table V
SOME PARAMETERS OF THE PROPOSED (θ, ρ)-GAM CODEBOOKS IN DOWNLINK OPTIMIZED FOR UNCODED SCENARIO, MD-SCMA CODEBOOKS [7] AND STAR-QAM CODEBOOKS [8].

N	K	J	d_r	M	PAPR min/max	(θ^*, ρ^*)	$\tilde{d}_{\min}$ min/max
2	4	6	3	4	$0/0$	$(0.0119, 6.9)$	$1.4102/1.4120$
2	4	6	3	8	$0/0$	$(0.02, 4.5)$	$1.0601/1.0704$
2	4	6	3	16	$0/0$	$(0.02, 1)$	$0.4190/0.5588$
3	6	8	4	4	$0.3342/0.5612$	$(0, 15)$	$1.2955/1.3032$
3	6	8	4	8	$0.8041/1.1919$	$(0, 0)$	$0.9431/0.9624$
3	6	8	4	16	$1.0095/1.2241$	$(-0.005, 0)$	$0.3028/0.3184$

Table VI
SOME PARAMETERS OF THE PROPOSED (θ, ρ)-GAM CODEBOOKS IN UPLINK OPTIMIZED FOR UNCODED SCENARIO.

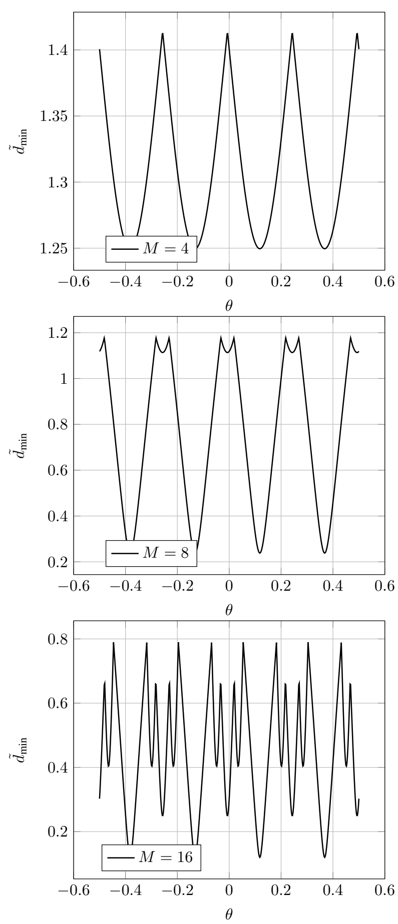

Figure 3. $\tilde{d}_{\min}$ as a function of θ for the 2-dimensional codebook in downlink.

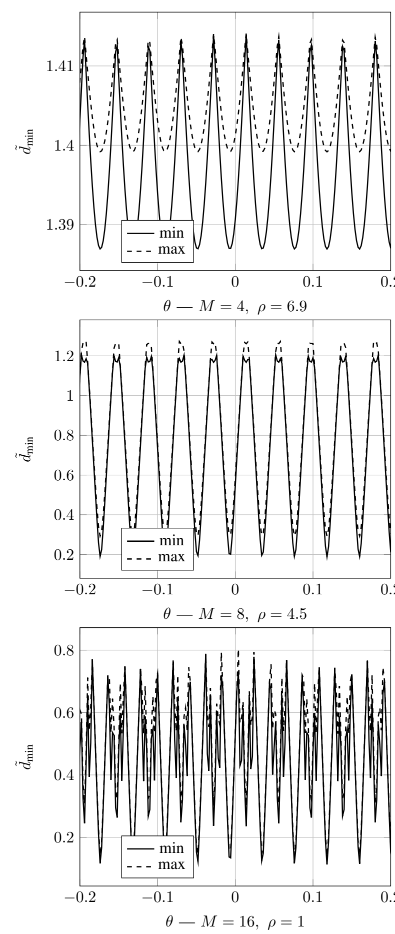

Figure 4. $\tilde{d}_{\min}$ as a function of θ for the 2-dimensional codebook in uplink.

[4] H. Nikopour and H. Baligh, "Sparse code multiple access," in *IEEE 24th International Symposium on Personal Indoor and Mobile Radio Communications (PIMRC)*, 2013, pp. 332–336.

[5] M. Moltafet, N. Mokari, M. R. Javan, H. Saeedi, and H. Pishro-Nik, "A New Multiple Access Technique for 5G: Power Domain Sparse Code Multiple Access (PSMA)," *IEEE Access*, vol. 6, pp. 747–759, 2018.

[6] M. Taherzadeh, H. Nikopour, A. Bayesteh, and H. Baligh, "SCMA codebook design," in *IEEE 80th Vehicular Technology Conference (VTC Fall)*, 2014, pp. 1–5.

[7] D. Cai, P. Fan, X. Lei, Y. Liu, and D. Chen, "Multi-dimensional SCMA codebook design based on constellation rotation and interleaving," in *IEEE 83rd Vehicular Technology Conference (VTC Spring)*, 2016, pp.

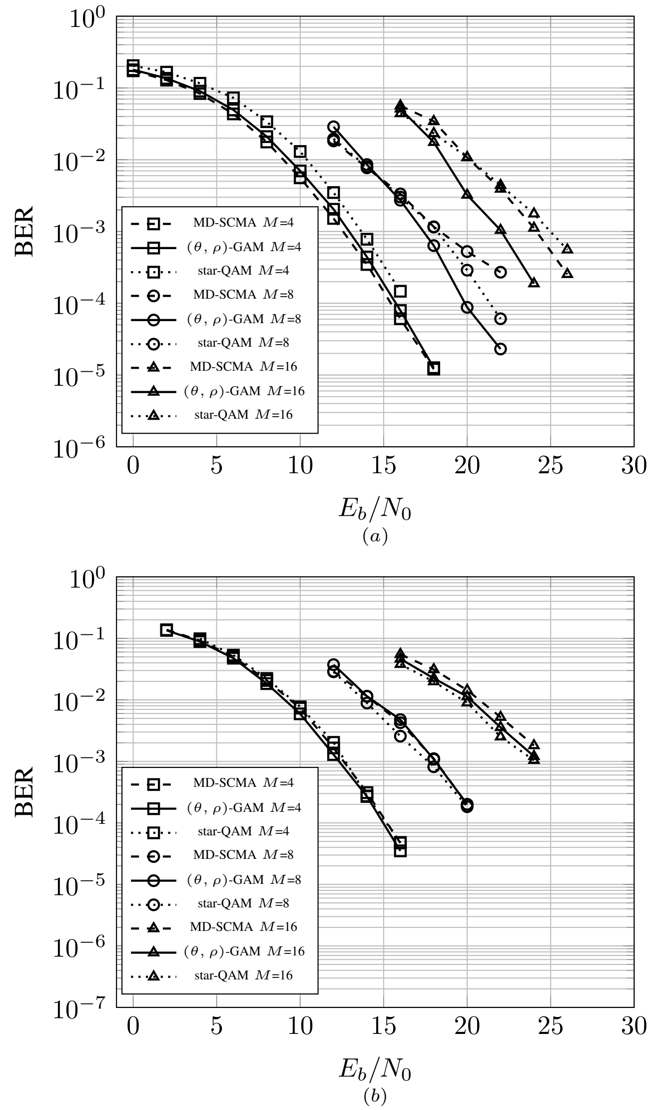

Figure 5. Performance comparison of MD-SCMA codebooks in [7], star-QAM codebooks in [8] and (θ, ρ)-GAM codebooks in downlink 1×2 SIMO Rayleigh fading channels. (a) $N = 2$. (b) $N = 3$.

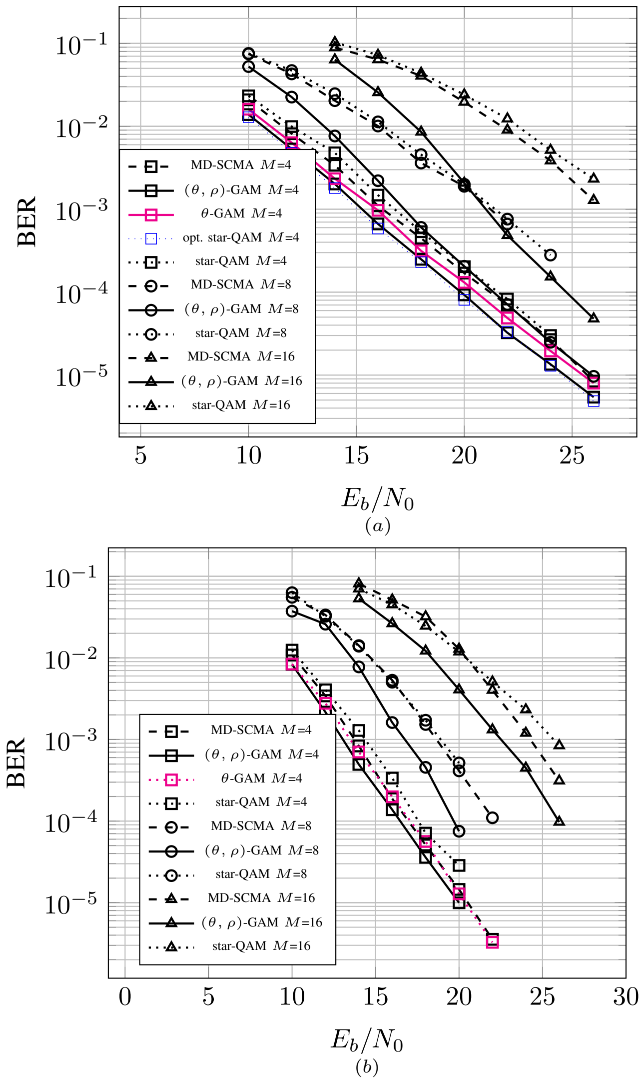

Figure 6. Performance comparison of MD-SCMA codebooks in [7], star-QAM codebooks in [8] and (θ, ρ)-GAM codebooks in uplink SISO Rayleigh fading channels. (a) $N = 2$. (b) $N = 3$.

1–5.

[8] L. Yu, X. Lei, P. Fan, and D. Chen, "An optimized design of SCMA codebook based on star-QAM signaling constellations," in *International Conference on Wireless Communications & Signal Processing (WCSP)*, 2015, pp. 1–5.

[9] Y. Zhou, Q. Yu, W. Meng, and C. Li, "SCMA codebook design based on constellation rotation," in *IEEE International Conference on Communications (ICC)*, 2017, pp. 1–6.

[10] J. Bao, Z. Ma, M. A. Mahamadu, Z. Zhu, and D. Chen, "Spherical codes for SCMA codebook," in *IEEE 83rd Vehicular Technology Conference (VTC Spring)*, 2016, pp. 1–5.

[11] H. Yan, H. Zhao, Z. Lv, and H. Yang, "A top-down SCMA codebook design scheme based on lattice theory," in *2016 IEEE 27th Annual International Symposium on Personal, Indoor, and Mobile Radio Communications (PIMRC)*, Sept 2016, pp. 1–5.

[12] J. Bao, Z. Ma, Z. Ding, G. K. Karagiannidis, and Z. Zhu, "On the design of multiuser codebooks for uplink SCMA systems," *IEEE Communications Letters*, vol. 20, no. 10, pp. 1920–1923, 2016.

[13] P. Larsson, "Golden angle modulation," *IEEE Wireless Communications Letters*, vol. 7, no. 1, pp. 98–101, 2018.

[14] P. Larsson, L. Rasmussen, and M. Skoglund, "Golden Angle Modulation: Approaching the AWGN Capacity," 2018, arxiv: 1802.10022.

[15] P. Larsson, "Golden Angle Modulation: Geometric-and Probabilistic-shaping," 2017, arxiv: 1708.07321.

[16] L. Yu, P. Fan, Z. Ma, X. Lei, and D. Chen, " An Optimized Design of Irregular SCMA Codebook Based on Rotated Angles and EXIT Chart," in *2016 IEEE 84th Vehicular Technology Conference (VTC-Fall)*, Sept 2016, pp. 1–5.

[17] L. Wen, "Non-Orthogonal Multiple Access Schemes for Future Cellular Systems," PhD dissertation, University of Surrey (United Kingdom), 2016.