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Capacity Planning for an Electric Vehicle Charging Station Considering Fuzzy Quality of Service and Multiple Charging Options

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***Abstract*—Electric vehicles (EVs) have received considerable attention in dealing with severe environmental and energy crises. The capacity planning of public charging stations has been a major factor in facilitating the wide market penetration of EVs. In this paper, we present an optimization model for charging station capacity planning to maximize the fuzzy quality of service (FQoS) considering queuing behavior, blocking reliability, and multiple charging options classified by battery technical specifications. The uncertainty of the EV arrival and service time are taken into account and described as fuzzy numbers characterized by triangular membership functions. Meanwhile, an α-cuts-based algorithm is proposed to defuzzify the FQoS. Finally, the numerical results illustrate that a more robust plan can be obtained by accounting for FQoS. The contribution of the proposed model allows decision-makers and operators to plan the capacity of charging stations with fuzzy EV arrival rate and service rate and provide a better service for customers with different charging options.**

***Index Terms*—electric vehicle, charging station, capacity planning, fuzzy quality of service, multiple charging options**

I. Introduction

Electric Vehicles (EVs) have gained popularity in recent years to mitigate the shortage of fossil fuel and meet climate change targets. Compared to conventional vehicles with an internal combustion engine (ICE) that consume fossil fuels and exhaust gas emissions [1], EVs powered by electricity provide a cleaner and environmental option to replace traditional ICE and move pollution away from urban areas. As a result, EVs featured as environmentally friendly have been pushed into mainstream adoption in many countries. The UK government has published an aggressive strategy called "Driving the Future Today" to expand charging networks, aiming at zero greenhouse gas emission by 2050 [2]. Furthermore, the Automated and Electric Vehicles Bill 2017–2019 released in 2017 also intended to reduce the dependence on fossil fuels [3]. Considering the rapid developments in battery production technology, it is suggested that over 150 million EVs are required by 2030 [4], and the EV population is expected to reach a considerable market portion in the future [5]. Correspondingly, the necessary deployment of chargers and charging stations is essential to achieve such penetration rates since EVs are basically supported by an electric motor powered by rechargeable battery packs [6]. The United States government has rolled out a project that hosts chargers every 25 miles at West Coast Green Highway [7]. In parallel, over 1000 chargers are installed by the Scottish government to ensure that the average distance from any location to the nearest public charger is about 2.78 miles, aiming to phasing out the need for new petrol and diesel cars and vans by 2032 [8]. Obviously, the expansion and design of reliable charging stations are essential for the rapid development of the EV market.

At the moment, multiple charging options classified by the battery technical specifications are available for EV charging [9]. Therefore, it is crucial to design and operate integrated charging stations to a provide efficient charging services for customers with different charging options. However, no matter what kind of charging technique is adopted, EV charging duration is always considered to be long when compared to traditional liquid-fuel powered ICE counterparts. A relatively long charging time inevitably leads to the formation of queuing behavior, which may cause the EV customers' dissatisfaction if the waiting time is unacceptable. Without considering the construction cost and charging station layout, this problem can be addressed by installing more chargers in a charging station. However, the success of EV market development primarily relies on the capability of the power grid. Uncontrolled charging service can easily lead to the line and transformer overloading, thus resulting in an outage [10]. For instance, 5% EVs charging simultaneously would lead to 5.5 GW of extra power consumption in the Virginia and Carolinas region by 2018 [11]. Therefore, the trade-off between waiting time and grid reliability is of paramount importance for a charging station.

As a widely accepted indicator, the quality of service (QoS) evaluation of charging stations has received widespread attention [12]–[16]. The literature on charging station design and QoS evaluation can be grouped into two categories. The first group includes studies from the customers' standpoint, and more recent attention has mainly focused on the queuing theory. If all the charging sockets are occupied, EVs must join a queue and wait for an available socket. In [17], the charging load of a single fast charging station is forecasted by an $M/M/s$ queuing model with the arrival rate of discharged EVs. In [18], a mathematical model is developed for handling requests for EV charging/discharging at EV charging stations based on queuing theory. A capacity-limited recharging sta-

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tion location model with queuing behavior considering both recharging time and waiting time is proposed in [19]. In [20], a charging infrastructure planning model is established with an $M/M/s/N$ queuing model. Both waiting time and construction costs are considered to determine the optimal capacity of a charging station. A multi-priority $M/M/s$ queuing model is proposed in [21] to minimize the waiting time for a public charging station. The customers in this model are divided into two classes with different queuing priorities to improve the EVs satisfaction in terms of charging service by achieving waiting time reduction. In [22], a QoS evaluation model for a fast charging station is proposed considering queuing theory and multiple charging options. A corresponding charging strategy is also investigated to reduce the mean waiting time to serve more customers. In [23], an $M/M/s/N$ queuing model is employed to evaluate the queuing state of different charging stations and find the optimal one that ensures the minimum total charging time. The relationship between the charging station capacity and customer service quality considering queuing theory is investigated in [24]. Simulations based on a homogeneous EV arrival are carried out and closed-form equations are derived therefrom to estimate recharging time and waiting time in the queue. On the other hand, the grid reliability also plays a key role in the design and operation of a public charging station. The second group includes studies from the standpoint of design-makers and operators. The study in [10] presents a capacity planning framework for EV charging stations with loss-of-load-probability as the primary performance metric, which measures the probability that the remaining grid power in the storage system fails to accommodate the demand. Authors in [25] proposed an optimization model for the optimal siting and sizing problem of EV charging stations, which minimizes the Energy Not Supplied (ENS) as the objective to guarantee the power system reliability. In [26], an EV charging station planning model is established considering the electrical reliability check based on a DC power flow model to ensure the charging reliability and expected QoS.

Besides, a few recent research works use both queuing theory and grid reliability to design the charging station. A charging station architecture is designed in [27] to provide a desirable QoS by using performance measure from queuing theory with sustained grid stability guarantees. In [28], a control and management framework of the grid power is proposed based on a non-preemptive priority queue. This model can be taken into account to design a charging station with various charging demands. However, a deterministic charging demand model is required within the literature to model the charging behavior of EV customers, which is impractical in a real charging environment. Accurate modeling of charging demand and service requires historical data related to EV arrival, departure, and energy consumption in order to statistically reflect the stochastic processes of the overall charging behavior. However, historical data is either insufficient or uncertain. Imprecision or ambiguity is the characteristic of many capacity planning parameters, generally because of insufficient historical data. Besides, it is unfeasible to determine the planning parameters in the design stage of a charging station since no accurate data can be collected before the charging station is actually operating, thus implying that only approximate values of arrival rate and service rate can be used to evaluate the QoS and further design the system capacity. Therefore, due to the insufficient understanding of the charging behavior of EV customers, a gap still exists in the literature since none of the existing works considers fuzzy characteristics in the capacity design process of a charging station. In this paper, we present a capacity planning model for an EV charging station that provide multiple charging options for EV customers. The customers' mean waiting time and the charging station's blocking probability are the QoS criterion for the performance evaluation of a charging station. Furthermore, the blocking probability is calculated to evaluate the grid reliability of the charging station. The major contributions of this study are outlined in the following:

1) We introduce the fuzzy $M/M/s/N$ queuing theory to model the EV charging station that offers multiple charging options, where the arrivals rates and service rates are considered as fuzzy numbers.
2) We propose a novel fuzzy quality of service (FQoS) evaluation model to quantify the service quality based on the mean waiting time and blocking reliability. A defuzzification algorithm is presented to obtain the defuzzified FQoS from the output of the aggregated fuzzy set.
3) We develop a new capacity planning model considering FQoS to find the optimal system capacity and number of charging sockets of each charging option. We show that a more robust capacity plan can be obtained by including the fuzziness in the model.

This paper is organized as follows. Section II presents the original capacity planning model, including $M/M/s/N$ queuing theory, blocking probability estimation and the QoS evaluation model. Section III describes the capacity planning model considering FQoS, where the fuzzy queuing theory and blocking probability are investigated considering the fuzzy arrival rate and service rate. A defuzzification algorithm is also proposed in Section III to transform the FQoS into a crisp value. The analytical and simulation results are presented and discussed in Section IV. Finally, Section V concludes a summary of the study.

II. System Model and Problem Formulation

Electric vehicle charging demands are basically determined by the actual needs of customers. Multiple charging options (e.g., DC fast charging, AC Level-II charging, superfast charging, etc.) are available in a standard public charging station. Customers adopt different charging options according to their personal arrangements and preference [29]. Based on this premise, each charging option has a queue with a specific arrival rate and service rate. The charging service follows the first-come and first-serviced (FCFS) order as illustrated in Fig. 1. The capacity planning problem of a charging station is influenced by the dynamics between design-makers and customers. The former expects stable grid reliability, and the latter seeks lower waiting time. We consider two canonical

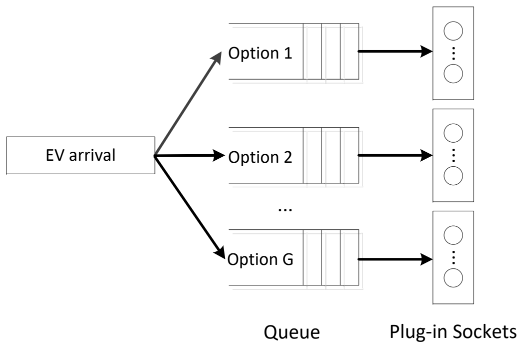


Fig. 1. Schematic view of the queuing behavior

design aspects, i.e., the number of charging sockets and queuing system capacity, of an EV charging station to balance such dynamics. In this paper, the goal of a charging station is to provide a better quality of service (QoS) to customers with different charging options. The QoS of a charging station can be divided into two components, including the mean waiting time and blocking reliability. Clearly, installing more charging sockets would reduce the mean waiting time but also provoke the congestion of the power grid and further increase the blocking probability. In the design stage of a charging station, it is often infeasible to determine the crisp arrival and service rate due to the inaccuracy or fluctuation of data. Therefore, we propose an integrated solution that considers FQoS and multiple charging options. The primary goal of the model is to maximize the FQoS with desirable waiting time and blocking reliability guarantees. Before proceeding to the details of the fuzzy model, the original QoS evaluation method and capacity optimization model will be discussed considering multiple charging options.

A. Queuing model

In this section, we present a queuing model that will be used to represent the EV charging behavior. Since an EV with a specific charging option does not necessarily need identical plug-in sockets, customers can immediately enter service if there is an available socket in the charging station. If all expected sockets are occupied, then the EV has to join a designated queue until a suitable socket becomes available. In this paper, we consider G distinct charging options, which are distinguished by their technical specifications. The arrival rate of a specific charging option is defined as the number of EVs that arrive at the charging station per hour, whereas the service rate of a charging socket is defined as the number of EVs that can be served per hour. We assume that EVs arrive at the charging station according to a Poisson process and the arrival rate of option $g \in \{1, 2, ..., G\}$ is denoted by λ_g. It should be noted that the spatial temporal distribution of the charging request is not included in the model. Furthermore, all service times are independent and identically distributed according to an exponentially distributed service rate μ_g facilitated by a charging socket. The charging service is provided by s_g sockets in each queue and thus implying that the overall queuing system can be further divided into G queuing subsystems with a distinct capacity N_g. In principle, queuing process tends to derive the performance measures with the Markov chain by introducing the state description [30]. Therefore, we adopt $M/M/s_g/N_g$ queuing model for each option where M denotes the Markovian process. Note that EV arrivals act independently. The state transition diagram for the queue system with capacity N_g can be derived and depicted as shown in Fig 2, where each state represents the number of EVs in the corresponding queuing system.

Let k_g denote the number of EVs in the queuing system with option g. It should be noted that if $k_g \leq s_g$, the overall completion rate is $k_g\mu_g$ since no queuing behavior occurs in the system. Otherwise, the completion rate is $s_g\mu_g$ since all sockets are occupied and the EV needs to join a designated queue until a suitable socket becomes available.

For any positive integer u and possible states $x_0, x_1, ...,$ x_u, x_{u+1}, a Markov chain is defined as a discrete-time stochastic process with state space $\xi = \{0, 1, 2, ...\}$ if the probability of the system in each state satisfies the rule of conditional independence, i.e.,

$$\begin{aligned} &P(X_{u+1} = x_{u+1} | X_u = x_u, X_{u-1} = x_{u-1}, ..., X_0 = x_0) \\ &\quad = P(x_{u+1} = j | X_u = i) \end{aligned} \tag{1}$$

where X_u is a random variable that denotes the value of the Markov chain at step u. Specifically, when new EVs arrive at or depart from the charging station, the next state of the queuing system is only determined by the current state and the time elapsed according to certain probabilistic rules, i.e., this stochastic process exhibits Markov (or memory-less) property.

The state transition matrix P with time homogenous for the queuing system with Markov property in this paper is defined as a matrix containing information on the probabilities of particular transitions. Given the finite and countable state space ξ, the $(i, j)^{\text{th}}$ element of the state transition matrix P is given by

$$P_{ij} = \Pr(X_{u+1} = j, X_u = i) \tag{2}$$

The corresponding transition matrix can be expressed as

$$\begin{bmatrix} -\lambda & \lambda & 0 & \cdots & 0 & 0 \\ \mu & -(\lambda{+}\mu) & \lambda & \cdots & 0 & 0 \\ 0 & 2\mu & \lambda & -(\lambda{+}2\mu) & \cdots & 0 \\ \cdots & & & \cdots & & \\ 0 & 0 & \cdots & s\mu & -(\lambda{+}s) & \lambda \\ 0 & 0 & \cdots & \cdots & \cdots & \cdots \end{bmatrix} \tag{3}$$

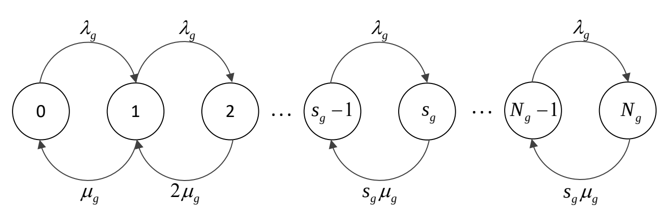


Fig. 2. The state transition diagram

Note that the subscript g representing different charging options is omitted for simplicity. Let π_k denote the probability of the queuing system being in state k. $\boldsymbol{\pi} = \{\pi_0, \pi_1, ..., \pi_N\}$ is a $N_g + 1-$dimensional row vector whose i^{th} element is π_i. Given the system state transition matrix P, $\boldsymbol{\pi}$ is the vector of steady-state probability if

$$\boldsymbol{\pi} \cdot P = \mathbf{0}^T \tag{4}$$

The k^{th} steady-state probability can be obtained by a simple set of first-order difference equations as follows:

$$(k+1)\mu\pi_{k+1} = \lambda\pi_k \tag{5}$$

In particular,

$$\pi_1 = \frac{\lambda}{\mu}\pi_0 \tag{6}$$

and

$$\pi_2 = \frac{1}{2}\left(\frac{\lambda}{\mu}\right)^2 \pi_0 \tag{7}$$

Let $\rho = \lambda/\mu$ denote the occupancy rate of the queuing system. In this paper, we assume that $\rho/s \neq 1$ in the following model. Therefore, the general solution of the steady-state probability distribution is calculated by

$$\pi_k = \begin{cases} \dfrac{1}{k!}\rho^k \pi_0 & \text{for } 0 \leq k \leq s \\ \dfrac{\rho^s}{s! s^{k-s}}\pi_0 & \text{for } s \leq k \leq N \end{cases} \tag{8}$$

Owing to the fact that $\sum_{k=0}^{N} \pi_k = 1$, π_0 can be expressed by

$$\pi_0 = \left(\sum_{k=0}^{s-1} \frac{1}{k!}\rho^k + \frac{\rho^s \left[1 - (\rho/s)^{N-s+1}\right]}{s!(1-\rho/s)} \right)^{-1} \tag{9}$$

By mathematical induction, the mean queue length is given by

$$E(L) = \sum_{k=s}^{N} (k-s) \cdot \pi_k \tag{10}$$

According to Little's law, the number of customers in a stationary queuing system is equal to the effective arrival rate λ multiplied by the mean time that a customer spends in the queuing system. Therefore, the mean waiting time for different charging options is calculated by the formula below:

$$W_g = \frac{E(L_g)}{\lambda_g(1 - \pi_N^g)} \quad \forall g = \{1, 2, ..., G\} \tag{11}$$

Obviously, the mean waiting time is decreasing with an increasing number of charging sockets. Let $\lambda = \sum_{g=1}^{G} \lambda_g$ denote the total arrival rate of the charging station. Consequently, the weighted average waiting time of the overall charging station is given by

$$W = \sum_{g=1}^{G} \frac{\lambda_g}{\lambda} W_g \tag{12}$$

B. Blocking reliability

The design and operational management of a charging station are of paramount significance in achieving an acceptable QoS. In this paper, the mean waiting time and blocking reliability for each charging subsystem are considered as the QoS evaluation metrics. In this paper, a charging period is defined as a time interval where the queuing system satisfies the steady-state condition mentioned in Eq. (4). For the problems discussed above, the following assumptions are made in this paper:

1) The available grid power for each charging option is predetermined and fixed at the beginning of first period.
2) If the remaining grid power is less than the requested power, the charging service will be suspended resulting in an outage until the next period.
3) The requested power of EV customers with the same charging option is assumed to be equal.

Blocking probability is defined as the probability that the remaining grid power fails to meet the demand of customers within a certain period. Obviously, blocking probability constitutes a natural performance metric of the grid reliability. To formalize this, let e_g be the aggregated units of grid power available to EV fleets with charging option g. Given the aforementioned criterion, the blocking probability of each queuing system can be expressed as

$$V = P\{e \leq \overline{c}\} \tag{13}$$

where $\overline{c}$ denotes the total amount of power requested from the grid to meet the charging demand of customers. Note that the subscript g is omitted again for convenience. By recalling that queuing system can be described as a Markov chain, we propose a simple method to obtain the blocking probability.

For a public charging station, it is reasonable for operators to include safety margins into the capacity planning of the power storage system to hedge against uncertainties such as demand surge. Therefore, the relationship between the required power and available power should satisfy $e \geq \overline{c}$ for each charging option, thus implying that the safety margin can be defined as $e - \overline{c}$. However, the available power is oftentimes not a crisp value in a practical application environment due to uncertainties caused by the previous operating periods. For instance, the failure of a charging socket will inevitably lead to the remaining power greater than expected before the next period. Likewise, unexpected increases in charging demand will reduce the amount of power available for the next period. Without loss of generality, it is reasonable to assume that the available power X is a random variable that follows Gaussian distribution with mean e and variance σ^2, i.e., $\mathrm{X} \sim N(e, \sigma^2)$; therefore, the blocking probability can be further rewritten as

$$V = P\{\mathrm{X} - \overline{c} < 0\} = P\left\{z < \frac{\overline{c} - e}{\sigma}\right\} \tag{14}$$

where z is a standard normal deviate with mean 0 and standard deviation 1. To illustrate this method more intuitive, the relationship between the above parameters is depicted in Fig. 3.

In what follows, the mean requested power $\overline{c}$ can be derived based on the aforementioned steady-state probability distribution $\boldsymbol{\pi}$. Let n denote the mean number of charging sockets occupied by customers in the queue; then n can be approximated by

$$n = \frac{\sum_{k=0}^{s-1} \frac{1}{(k-1)!}\rho^k + \sum_{k=s}^{N} \frac{1}{(s-1)!s^{k-s}}\rho^s}{\sum_{k=0}^{s-1} \frac{1}{k!}\rho^k + \frac{\rho^s\left[1-(\rho/s)^{N-s+1}\right]}{s!(1-\rho/s)}} \tag{15}$$

Using the above formula, we can further obtain the mean requested power drawn from the grid for a period with duration T:

$$\bar{c} = n \cdot d \cdot \mu \cdot T \tag{16}$$

where d denotes the mean power requested by a customer per recharging. According to Eqs. (13)-(16), the blocking probability is increasing along with the number of charging sockets. Clearly, the blocking probability is largely influenced by the safety margin. In this paper, the safety margin is assumed to be a predetermined value, i.e., e is an endogenous variable.

For the overall charging station, the weighted average blocking probability is calculated through the formula below:

$$V = \sum_{g=1}^{G} \frac{\lambda_g}{\lambda} V_g \quad \forall g \in \{1, 2, ..., G\} \tag{17}$$

C. Optimization Model

The objective of the proposed model is to find an optimal charging station capacity N_g and number of charging sockets s_g, such that the QoS of the overall charging station is maximized. For the QoS of the overall charging station, a logarithmic utility function is adopted to integrate the mean waiting time W and blocking probability V. Given the arrival rate λ_g and service rate μ_g, QoS is computed by

$$QoS = \frac{1}{\log(1+W) + \log(1+V)} \tag{18}$$

The associated integer decision variables are N_g and $s_g (g = 1, 2, ..., G)$, depending on which arrival rate λ_g and service rate μ_g involved in the queuing model is determined for each charging option. Additionally, QoS for different system capacity plans is determined following Eq. (18). Considering the construction cost and limited land area, let N_{max} represents the capacity of the overall charging station. To this end, the nonlinear formulation to maximize the QoS of the overall charging station is presented as follows:

$$\text{maximize } QoS = \frac{1}{\log(1+W) + \log(1+V)} \tag{19}$$

Subject to:

$$\sum_{g=1}^{G} N_g < N_{max} \tag{20}$$

$$s_g \le N_g \quad \forall g \in \{1, 2, ..., G\} \tag{21}$$

$$\bar{c} \le e_g \quad \forall g \in \{1, 2, ..., G\} \tag{22}$$

$$h_g \le H \quad \forall g \in \{1, 2, ..., G\} \tag{23}$$

$$N_g, s_g \text{ are integers}, \quad \forall g \in \{1, 2, ..., G\} \tag{24}$$

In this formulation, Constraints (20) and (21) limit the system capacity and number of charging sockets , respectively. Constraint (22) restricts that the safety margin must be a positive number. It is noteworthy that Constraint (23) shows that the loss rate of each queuing subsystem is required to be smaller than a certain level. In fact, if a charging station is constructed in an extremely small scale, both mean waiting time and blocking probability would decrease since most customers fail to enter the queuing system, which is obviously unacceptable for both customers and operators. Based on the queuing model, the loss rate of option g can be calculated by

$$h_g = \begin{cases} \dfrac{\lambda_g - \mu_g n_g}{\lambda_g} & \text{for } \lambda > \mu_g n_g \\ 0 & \text{for } \lambda \le \mu_g n_g \end{cases} \tag{25}$$

Besides, N_g and s_g must be integer values for all $g = 1, 2, ..., G$ as illustrated in Eq. (24). In what follows, the fuzzy quality of service (FQoS) evaluation and capacity planning model considering fuzzy queuing behavior and blocking probability are investigated based on the original model mentioned above.

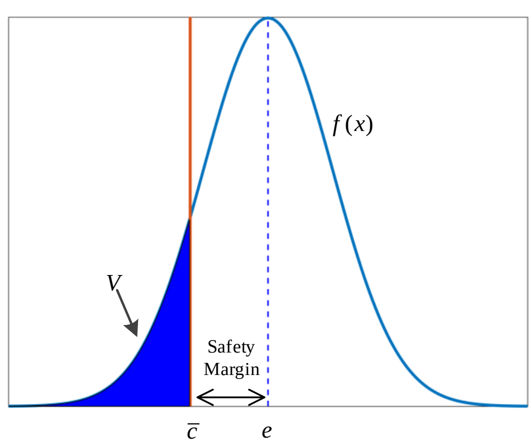


Fig. 3. Relationship between the available power and requested power

III. Capacity Planning Model Considering Fuzzy Quality of Service

During the design stage of a charging station, it is difficult to determine the arrival rate and service rate accurately. Therefore, it is reasonable to include the fuzzy characteristics in the model. Fuzzy numbers will inevitably affect the QoS of the charging station since only approximate values are considered. In light of this, we propose an integrated solution that considers both fuzzy queuing behavior and grid reliability. Specifically, fuzzy mean waiting time, blocking probability, and QoS would be estimated based on fuzzy arrival rate and service rate with a specific membership function. Furthermore, a defuzzification algorithm based on α-cuts and membership weighted average method is proposed to obtain the QoS from the aggregated fuzzy set. Finally, an optimization model is formulated to obtain the optimal number of charging sockets and system capacity of each charging option.

A. Fuzzy Queuing Model

It is practical that the arrival rate and service rate would not be crisp values in a realistic setting. Therefore, it is infeasible to obtain the crisp arrival and service rate in the design stage of an EV charging station. To overcome this challenge, fuzzy theory is employed to facilitate the evaluation of the performance measures. Based on the aforementioned original queuing model, a fuzzy queue denoted by $FM/FM/s_g/N_g$ is investigated.

In this paper, a fuzzy queuing system is defined as a queuing system whose arrival rate λ and service rate μ are fuzzy numbers. Let $\tilde{\lambda}$ and $\tilde{\mu}$ denote the fuzzy universal sets of arrival rate and service rate which are characterized by their membership functions. Likewise, the subscript g for each parameter is omitted for simplicity. A procedure is proposed to construct the membership functions of the performance measures in each queuing system. Specifically, we apply the α-cuts method to transform the fuzzy problem into a family of crisp cases [31]. Let $\eta_{\tilde{\lambda}}(x)$ and $\eta_{\tilde{\mu}}(y)$ be the membership functions of fuzzy universal sets $\tilde{\lambda}$ and $\tilde{\mu}$. Then it follows that

$$\tilde{\lambda} = \left\{x, \eta_{\tilde{\lambda}}(x) | x \in X\right\} \tag{26}$$

$$\tilde{\mu} = \{y, \eta_{\tilde{\mu}}(y) | y \in Y\} \tag{27}$$

The characteristics of interest of a queuing system for a charging station is the mean waiting time of customers denoted by $W(\tilde{\lambda}, \tilde{\mu})$. In general, the membership function can be obtained using Zadeh's extension principle [32] [33] as follows:

$$\eta_{W(\tilde{\lambda},\tilde{\mu})}(z) = \sup_{x\in X, y\in Y} \min\left\{\eta_{\tilde{\lambda}}(x), \eta_{\tilde{\mu}}(y) | z = W(x,y)\right\} \tag{28}$$

Accordingly, the transition intensities and state probabilities are also fuzzy numbers. Based on Eq. (11), the membership function can be further rewritten as

$$\eta_{W(\tilde{\lambda},\tilde{\mu})}(z) = \sup_{x\in X, y\in Y} \min\left\{\eta_{\tilde{\lambda}}(x), \eta_{\tilde{\mu}}(y) | z = \frac{E(\tilde{L})}{\tilde{\lambda}(1-\tilde{\pi}_N)}\right\} \tag{29}$$

Note that given a specified α-cut, the original fuzzy sets are reduced to a series of crisp cases:

$$\lambda(\alpha) = \left\{x \in X | \eta_{\tilde{\lambda}}(x) \geq \alpha\right\} \tag{30}$$

$$\mu(\alpha) = \{y \in Y | \eta_{\tilde{\mu}}(y) \geq \alpha\} \tag{31}$$

Consequently, the $FM/FM/s/N$ model is transformed into a family of original $M/M/s/N$ models. Likewise, the fuzzy Markov chain can also be decomposed into multiple ordinary Markov chains. Since the intervals are crisp values if we consider α-cuts in the model, Eqs. (30) and (31) can be further expressed as follows:

$$\begin{aligned}\lambda(\alpha) &= \left[\min\left\{x | \eta_{\tilde{\lambda}}(x) \geq \alpha\right\}, \max\left\{x | \eta_{\tilde{\lambda}}(x) \geq \alpha\right\}\right] \\ &= \left[x_\alpha^L, x_\alpha^U\right]\end{aligned} \tag{32}$$

$$\begin{aligned}\mu(\alpha) &= \left[\min\{y | \eta_{\tilde{\mu}}(y) \geq \alpha\}, \max\{y | \eta_{\tilde{\mu}}(y) \geq \alpha\}\right] \\ &= \left[y_\alpha^L, y_\alpha^U\right]\end{aligned} \tag{33}$$

where x_α^L, y_α^L, x_α^U, and y_α^U are the upper and lower bounds of $\lambda(\alpha)$ and $\mu(\alpha)$. The triangular membership function is employed in this study to model the uncertainties of the fuzzy numbers. Compared with other membership functions (such as trapezoidal membership function), triangular membership functions require the least prior knowledge of the charging environment since only upper and lower bounds are used to determine the fuzzy set [34]–[36]. To intuitively illustrate the formulas, the α-cuts set of a triangular fuzzy number is shown in Fig. 4. According to the convexity of a fuzzy number, the bounds of $\tilde{\lambda}$ and $\tilde{\mu}$ are functions of α, which can be expressed as $x_\alpha^L = \min \eta_{\tilde{\lambda}}^{-1}(\alpha)$, $x_\alpha^U = \max \eta_{\tilde{\lambda}}^{-1}(\alpha)$, $y_\alpha^L = \min \eta_{\tilde{\mu}}^{-1}(\alpha)$, and $y_\alpha^U = \max \eta_{\tilde{\mu}}^{-1}(\alpha)$. Obviously, the fuzzy mean waiting time $W(\tilde{\lambda}, \tilde{\mu})$ is also parameterized by α. Therefore, we can apply the α-cuts method to obtain the membership function $\eta_{\tilde{W}(\tilde{\lambda},\tilde{\mu})}(z)$.

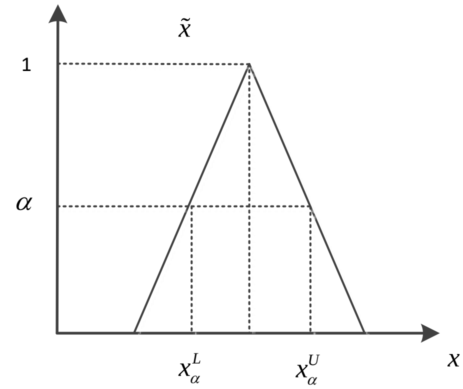


Fig. 4. The α-cuts set of a triangular fuzzy number

To construct the membership function of $W(\tilde{\lambda}, \tilde{\mu})$, at least one of the following conditions is required according to Zadeh's extension principle:

1) $\eta_{\tilde{\lambda}}(x) = \alpha$ and $\eta_{\tilde{\mu}}(y) \geq \alpha$
2) $\eta_{\tilde{\lambda}}(x) \geq \alpha$ and $\eta_{\tilde{\mu}}(y) = \alpha$

such that

$$\begin{aligned} z &= \frac{E(L)}{\lambda(1-\pi_N)} \\ &= \frac{\sum_{k=s}^{N}(k-s)\pi_k}{\lambda\left(1-\frac{\rho^N}{s!s^{N-s}}\left(\sum_{k=0}^{s-1}\frac{1}{k!}\rho^k + \frac{\rho^s\left[1-(\rho/s)^{N-s+1}\right]}{s!(1-\rho/s)}\right)^{-1}\right)} \end{aligned} \tag{34}$$

to satisfy $\eta_{\tilde{W}(\tilde{\lambda},\tilde{\mu})}(z) = \alpha$. This problem can be solved by introducing the parametric non-linear programming (NLP) technique [37]–[39]. If $\eta_{\tilde{\lambda}}(x) = \alpha$ and $\eta_{\tilde{\mu}}(y) \geq \alpha$, we have

$$\begin{gathered} W_g^{L_1}(\alpha) = \min_{x,y\in\mathbb{R}} \\ \frac{\sum_{k=s}^{N}(k-s)\pi_k}{\lambda\left(1-\frac{\rho^N}{s!s^{N-s}}\left(\sum_{k=0}^{s-1}\frac{1}{k!}\rho^k + \frac{\rho^s\left[1-(\rho/s)^{N-s+1}\right]}{s!(1-\rho/s)}\right)^{-1}\right)} \\ x_\alpha^L \leq x \leq x_\alpha^U \end{gathered} \tag{35}$$

$$y \in \mu(\alpha)$$

$$W_g^{U_1}(\alpha) = \max_{x,y\in\mathbb{R}} \frac{\sum_{k=s}^{N}(k-s)\pi_k}{\lambda\left(1-\frac{\rho^N}{s!s^{N-s}}\left(\sum_{k=0}^{s-1}\frac{1}{k!}\rho^k+\frac{\rho^s\left[1-(\rho/s)^{N-s+1}\right]}{s!(1-\rho/s)}\right)^{-1}\right)} \tag{36}$$

$$x_\alpha^L \le x \le x_\alpha^U$$

$$y \in \mu(\alpha)$$

Similarly, if $\eta_{\tilde{\lambda}}(x) \ge \alpha$ and $\eta_{\tilde{\mu}}(y) = \alpha$, then, as expected, we have

$$W_g^{L_2}(\alpha) = \min_{x,y\in\mathbb{R}} \frac{\sum_{k=s}^{N}(k-s)\pi_k}{\lambda\left(1-\frac{\rho^N}{s!s^{N-s}}\left(\sum_{k=0}^{s-1}\frac{1}{k!}\rho^k+\frac{\rho^s\left[1-(\rho/s)^{N-s+1}\right]}{s!(1-\rho/s)}\right)^{-1}\right)} \tag{37}$$

$$x \in \lambda(\alpha)$$

$$y_\alpha^L \le y \le x_\alpha^U$$

$$W_g^{U_2}(\alpha) = \max_{x,y\in\mathbb{R}} \frac{\sum_{k=s}^{N}(k-s)\pi_k}{\lambda\left(1-\frac{\rho^N}{s!s^{N-s}}\left(\sum_{k=0}^{s-1}\frac{1}{k!}\rho^k+\frac{\rho^s\left[1-(\rho/s)^{N-s+1}\right]}{s!(1-\rho/s)}\right)^{-1}\right)} \tag{38}$$

$$x \in \lambda(\alpha)$$

$$y_\alpha^L \le y \le y_\alpha^U$$

It is worth noting that the α-cuts of the fuzzy numbers can be viewed as a nested form, thus implying that Eqs. (35) and (36), Eqs. (37) and (38) have the same optimal results. Therefore, the model can be expressed equivalently in the following form:

$$W_g^{L}(\alpha) = \min_{x,y\in\mathbb{R}} \frac{\sum_{k=s}^{N}(k-s)\pi_k}{\lambda\left(1-\frac{\rho^N}{s!s^{N-s}}\left(\sum_{k=0}^{s-1}\frac{1}{k!}\rho^k+\frac{\rho^s\left[1-(\rho/s)^{N-s+1}\right]}{s!(1-\rho/s)}\right)^{-1}\right)} \tag{39}$$

$$x_\alpha^L \le x \le x_\alpha^U$$

$$y_\alpha^L \le y \le y_\alpha^U$$

$$W_g^{U}(\alpha) = \max_{x,y\in\mathbb{R}} \frac{\sum_{k=s}^{N}(k-s)\pi_k}{\lambda\left(1-\frac{\rho^N}{s!s^{N-s}}\left(\sum_{k=0}^{s-1}\frac{1}{k!}\rho^k+\frac{\rho^s\left[1-(\rho/s)^{N-s+1}\right]}{s!(1-\rho/s)}\right)^{-1}\right)} \tag{40}$$

$$x_\alpha^L \le x \le x_\alpha^U$$

$$y_\alpha^L \le y \le y_\alpha^U$$

Indeed, since the queuing system mentioned above are more complicated than other queues such as $FM/FM/1/N$ model which has been well investigated in other studies, it is almost impossible to derive analytical results under such a complex case. In other words, a closed-form membership function of $\eta_{\tilde{W}(\tilde{\lambda},\tilde{\mu})}(z)$ is difficult to obtain since W_α^L and W_α^U are both non-invertible as s increases. However, in real applications, what matters is the fuzzy mean waiting time of customers since it is a critical parameter of the overall queuing model. Therefore, it is unnecessary to obtain the real function of $\eta_{\tilde{W}(\tilde{\lambda},\tilde{\mu})}(z)$ in this paper. Finally, the bounds of the fuzzy mean waiting time of the overall charging station is given by

$$W^L(\alpha) = \sum_{g=1}^{G}\frac{\lambda_g}{\lambda}W_g^L(\alpha) \tag{41}$$

$$W^U(\alpha) = \sum_{g=1}^{G}\frac{\lambda_g}{\lambda}W_g^U(\alpha) \tag{42}$$

Where λ_g and λ are crisp cases of $\tilde{\lambda_g}$ and $\tilde{\lambda}$.

B. Fuzzy Blocking Reliability

By recalling the original blocking reliability estimation model in Section II (B), the mean number of occupied sockets is a fuzzy number $\tilde{n}$ parameterized by $\tilde{\lambda}$ and $\tilde{\mu}$, thus implying that the blocking reliability also possesses fuzzy characteristics since it is derived from fuzzy operations. Let $\tilde{V}(\tilde{\lambda},\tilde{\mu})$ denote the fuzzy universal set of the blocking reliability; then $\tilde{V}(\tilde{\lambda},\tilde{\mu})$ can be expressed as

$$\begin{aligned}\eta_{\tilde{V}(\tilde{\lambda},\tilde{\mu})}(v) &= \sup_{x,y\in\mathbb{R}}\min\left\{\eta_{\tilde{\lambda}}(x),\eta_{\tilde{\mu}}(y)|v=V(x,y)\right\}\\ &= \sup_{x,y\in\mathbb{R}}\min\left\{\eta_{\tilde{\lambda}}(x),\eta_{\tilde{\mu}}(y)|v=P\left(z<\frac{\overline{c}-e}{\sigma}\right)\right\}\end{aligned} \tag{43}$$

Correspondingly, the fuzzy blocking probability can be derived based on the NLP technique and Zadeh's extension principle:

$$V_g^L(\alpha) = \min_{x,y\in\mathbb{R}} P\left(z<\frac{1}{\sigma}\left(\mu Td\frac{\sum_{k=0}^{s-1}\frac{1}{(k-1)!}\rho^k+\sum_{k=s}^{N}\frac{1}{(s-1)!s^{k-s}}\rho^s}{\sum_{k=0}^{s-1}\frac{1}{k!}\rho^k+\frac{\rho^s\left[1-(\rho/s)^{N-s+1}\right]}{s!(1-\rho/s)}}-e\right)\right) \tag{44}$$

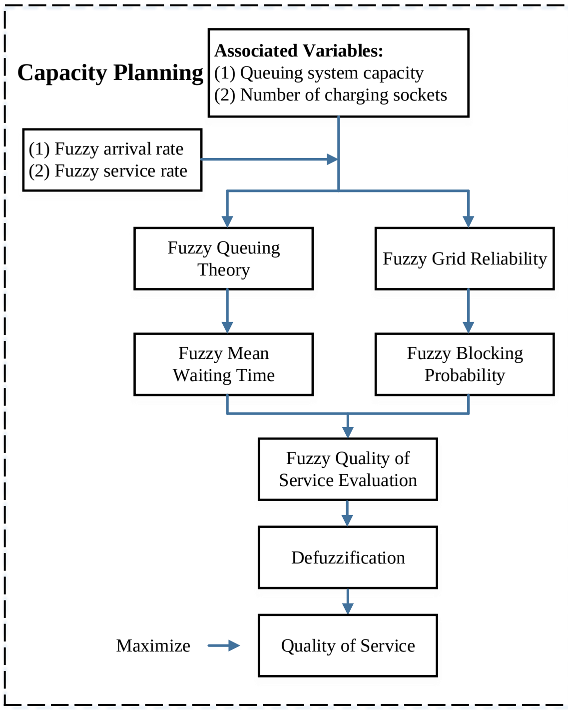


Fig. 5. Flowchart of the proposed capacity planning model

$$x_\alpha^L \le x \le x_\alpha^U$$
$$y_\alpha^L \le y \le y_\alpha^U$$

$$V_g^U(\alpha) = \max_{x,y\in\mathbb{R}} P\left(z < \frac{1}{\sigma}\left(\mu T d \frac{\sum_{k=0}^{s-1}\frac{1}{(k-1)!}\rho^k + \sum_{k=s}^{N}\frac{1}{(s-1)!s^{k-s}}\rho^s}{\sum_{k=0}^{s-1}\frac{1}{k!}\rho^k + \frac{\rho^s\left[1-(\rho/s)^{N-s+1}\right]}{s!(1-\rho/s)}} - e\right)\right) \tag{45}$$

$$x_\alpha^L \le x \le x_\alpha^U$$
$$y_\alpha^L \le y \le y_\alpha^U$$

Likewise, the weighted average bounds of fuzzy blocking probability can be further calculated by

$$V^L(\alpha) = \sum_{g=1}^{G}\frac{\lambda_g}{\lambda}V_g^L(\alpha) \tag{46}$$

$$V^U(\alpha) = \sum_{g=1}^{G}\frac{\lambda_g}{\lambda}V_g^U(\alpha) \tag{47}$$

In what follows, an FQoS evaluation model is proposed by considering the fuzzy mean waiting time and blocking probability to evaluate the service quality of a charging station.

C. FQoS evaluation and Optimization Model

The objective of the proposed model is to find the optimal number of charging sockets s_g and system capacity N_g, such that the FQoS of the overall charging station is maximized. Note that we use $\lambda_\alpha^L, \lambda_\alpha^U, \mu_\alpha^L, \mu_\alpha^U$ to represent the bounds of $\tilde{\lambda}(\alpha)$ and $\tilde{\mu}(\alpha)$ in the following model to avoid confusion. Likewise, a method based on the aforementioned logarithmic utility function, NLP technique, and Zadeh's extension principle is employed to integrate $\tilde{W}$ and $\tilde{V}$. Given $\tilde{\lambda}_g(\alpha)$ and $\tilde{\mu}_g(\alpha)$, FQoS can be estimated by

Algorithm 1 Defuzzification Algorithm for FQoS

Input: $g = \{1,2,...,G\}$,$\eta_{\tilde{\lambda}_g}(x)$,$\eta_{\tilde{\mu}_g}(y)$,$s_g$,$N_g$,$d_g$,$e_g$,$\sigma_g$,$T$,$\delta$
Output: Defuzzified QoS

```
Set QoS ← 0
λ ← Σ_{g=1}^{G} λ_g(α_δ)
for i = 1 → δ do
    α_i ← (i−1)/(δ−1)
    for g = 1 → G do
        calculate W_g^L(α_i) and W_g^U(α_i)
        calculate V_g^L(α_i) and V_g^U(α_i)
    end for
    W^L(α_i), W^U(α_i) ← WEIGHTEDAVERAGE(λ_g(α_δ), λ)
    V^L(α_i), V^U(α_i) ← WEIGHTEDAVERAGE(λ_g(α_δ), λ)
    Call function of QoS^L(α_i)
    Call function of QoS^U(α_i)
    QoS ← QoS + α_i δ(δ+1)/(2(δ−1)) η_{QoS(α_i)}
end for
```

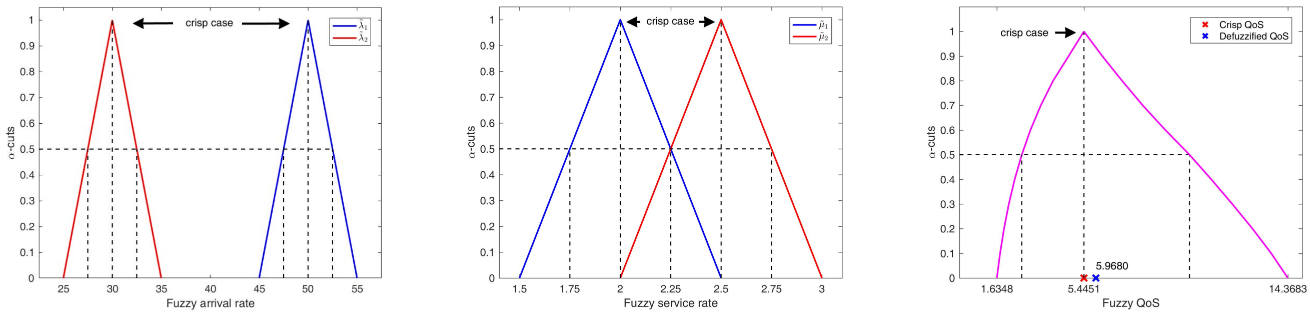
$$QoS^L(\alpha) = \min\left\{\frac{1}{\log W + \log V}\right\} \tag{48}$$
$$\lambda_\alpha^L \le \lambda \le \lambda_\alpha^U$$
$$\mu_\alpha^L \le \mu \le \mu_\alpha^U$$

$$QoS^U(\alpha) = \max\left\{\frac{1}{\log W + \log V}\right\} \tag{49}$$
$$\lambda_\alpha^L \le \lambda \le \lambda_\alpha^U$$
$$\mu_\alpha^L \le \mu \le \mu_\alpha^U$$

In what follows, we propose a defuzzification algorithm which is described in Algorithm 1 based on α-cuts and membership weighted average method. Owing to the fact that a closed-form membership function for QoS cannot be obtained, we can fit the shape of QoS by introducing an enumeration method based on α-cuts. Therefore, the set of intervals $\left\{\left[QoS_\alpha^L, QoS_\alpha^U\right] \alpha \in (0,1)\right\}$ still reveals the trend of the membership function of QoS, which lays a solid foundation for the next step. Assume that we enumerate δ values of α: $\alpha_i = \frac{i-1}{\delta-1}, i = 1,2,...,\delta$. Then δ sets of upper and lower bounds of QoS can be obtained. Consequently, the defuzzified QoS can be estimated based on the membership weighted average method as follows:

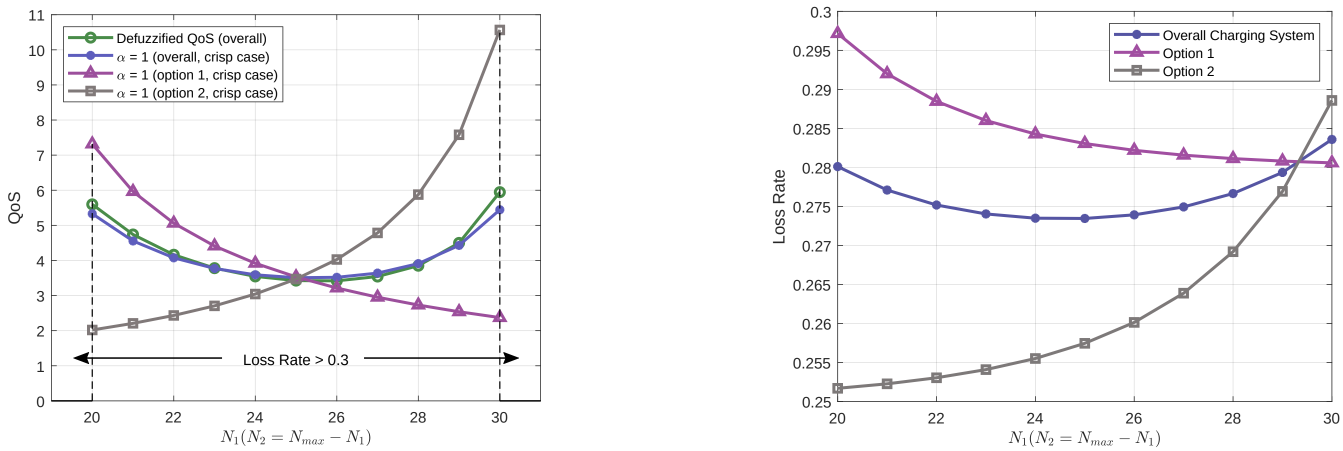
$$QoS = \sum_{i-1}^{\delta}\alpha_i\frac{\delta(\delta+1)}{2(\delta-1)}\eta_{QoS(\tilde{\lambda}(\alpha),\tilde{\mu}(\alpha))} \tag{50}$$

The proposed capacity planning problem is formulated as a non-linear integer program, where the associated integer decision variables are s_g and N_g, $g = 1,2,...,G$. The nonlinear model to maximize the QoS of the overall charging station



(a) $\tilde{\lambda_1}$ and $\tilde{\lambda_2}$ (b) μ_1 and μ_2 (c) FQoS

Fig. 6. Membership functions of fuzzy arrival rate, service rate and FQoS



(a) N_1, N_2 vs. QoS (b) N_1, N_2 vs. Loss Rate(crisp case)

Fig. 7. Relationship between QoS, system capacity and loss rate

can be formulated as

$$\text{maximize } QoS = \sum_{i-1}^{\delta} \alpha_i \frac{\delta(\delta+1)}{2(\delta-1)} \eta_{QoS(\tilde{\lambda}(\alpha),\tilde{\mu}(\alpha))} \tag{51}$$

Subject to:

$$\sum_{g=1}^{G} N_g < N_{max} \tag{52}$$

$$s_g \le N_g \quad \forall g \in \{1,2,...,G\} \tag{53}$$

$$\overline{c}(\alpha_\delta) \le e_g \quad \forall g \in \{1,2,...,G\} \tag{54}$$

$$h_g \le H \quad \forall g \in \{1,2,...,G\} \tag{55}$$

$$N_g, s_g \text{ are integers}, \quad \forall g \in \{1,2,...,G\} \tag{56}$$

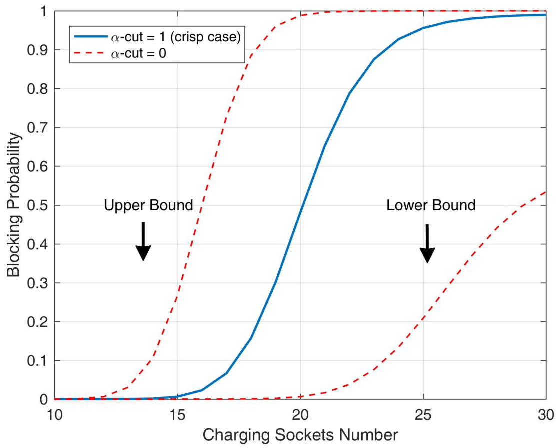
Note that the constraint (54) exhibits that the safety margin between the crisp requested power and available power is included in the model. The flowchart of the proposed capacity planning problem is illustrated in Fig. 5.

IV. Numerical Results

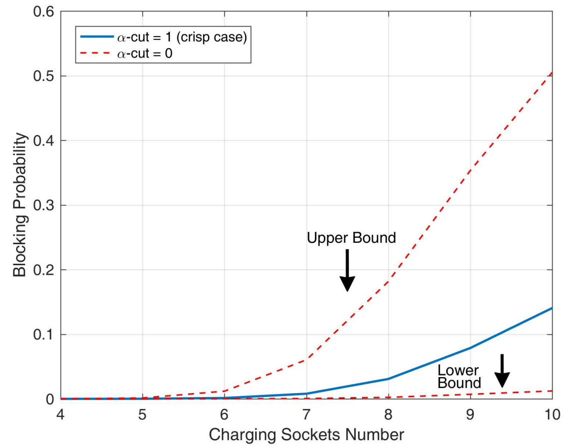
In this section, we perform analytical and simulation results to evaluate the proposed capacity planning model considering FQoS and multiple charging options. In Section IV-A, we present a numerical case to analyze the charging parameters of the proposed capacity planning problem. In Section IV-B, the effectiveness of the proposed model is evaluated under a real-world scenario, where a non-fuzzy case is introduced as a benchmark.

A. Case Study-I

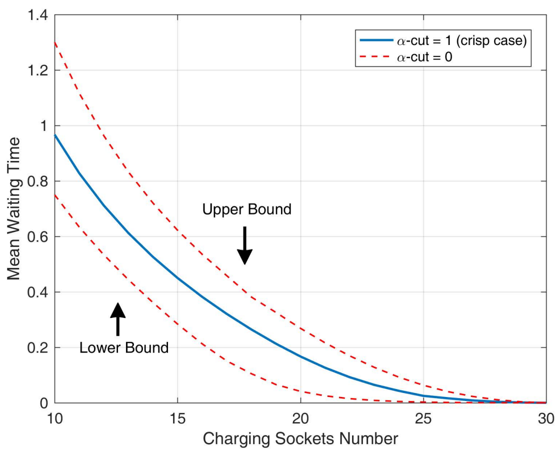
In the first numerical case, we consider a charging station which consists of two charging options, namely, Option 1 and Option 2. The parameter setting is elaborated as follows. The fuzzy arrival rate and service rate of each option are $\tilde{\lambda}_1 = [45, 50, 55]$, $\tilde{\lambda}_2 = [25, 30, 35]$, $\tilde{\mu}_1 = [1.5, 2.5, 3]$, and $\tilde{\mu}_2 = [2, 2.5, 3]$ per hour, respectively. Obviously, $\tilde{\lambda}$ and $\tilde{\mu}$ are characterized by a symmetrical triangular membership function as depicted in Figs. 6(a) and (b). In the simulations, the period length is $T_1 = T_2 = 1$ hour, the requested power per recharging is $d_1 = d_2 = 1$ unit, and the available power of each option follows Gaussian distribution with mean $e_1 = 40, e_2 = 27$ units and variance $\sigma_1 = \sigma_2 = 4$. Besides, the maximum capacity of the overall charging station is set as $N_{\max} = 40$ and the loss rate limit is set as 0.3. Genetic Algorithm (GA) [40] is employed to resolve the capacity planning problem considering FQoS. The optimization model



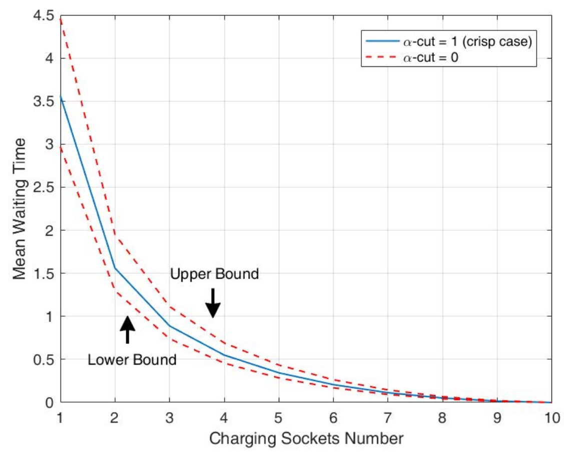
(a) s_1 vs. $W_1(s_2 = 9)$



(b) s_2 vs. $W_2(s_1 = 18)$

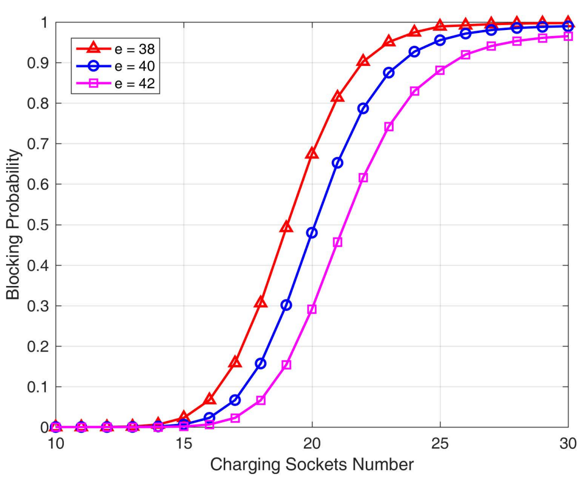


(c) s_1 vs. $V_1(s_2 = 9)$

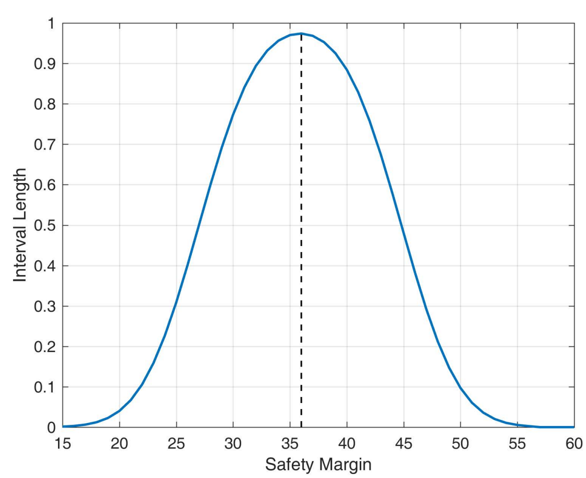


(d) s_2 vs. $V_2(s_1 = 18)$

Fig. 8. Relationship between number of charging sockets, mean waiting time and blocking probability



(a) s_1 vs. V_1 under different safety margin (crisp case)



(b) Distribution of the maximum interval length

Fig. 9. Relationship between blocking probability and safety margin (Option 1)

was built in MATLAB 2016a and run on an Intel Core i7-7500U 2.70 GHz CPU and 8GB RAM. Moreover, we set $\delta = 11$ to defuzzify the FQoS. By solving the model, the optimal individual fitness is $N_1 = 30$, $N_2 = 10$, $s_1 = 18$ and $s_2 = 9$. The α-cuts and corresponding membership function of QoS constructed by 11 α values are given in Fig. 6(c). In such a case, FQoS can be characterized by a roughly asymmetrical triangular membership function. The runtime of the proposed program is 2.14×10^3 seconds. The crisp and defuzzified FQoS are computed to be 5.445 and 5.968.

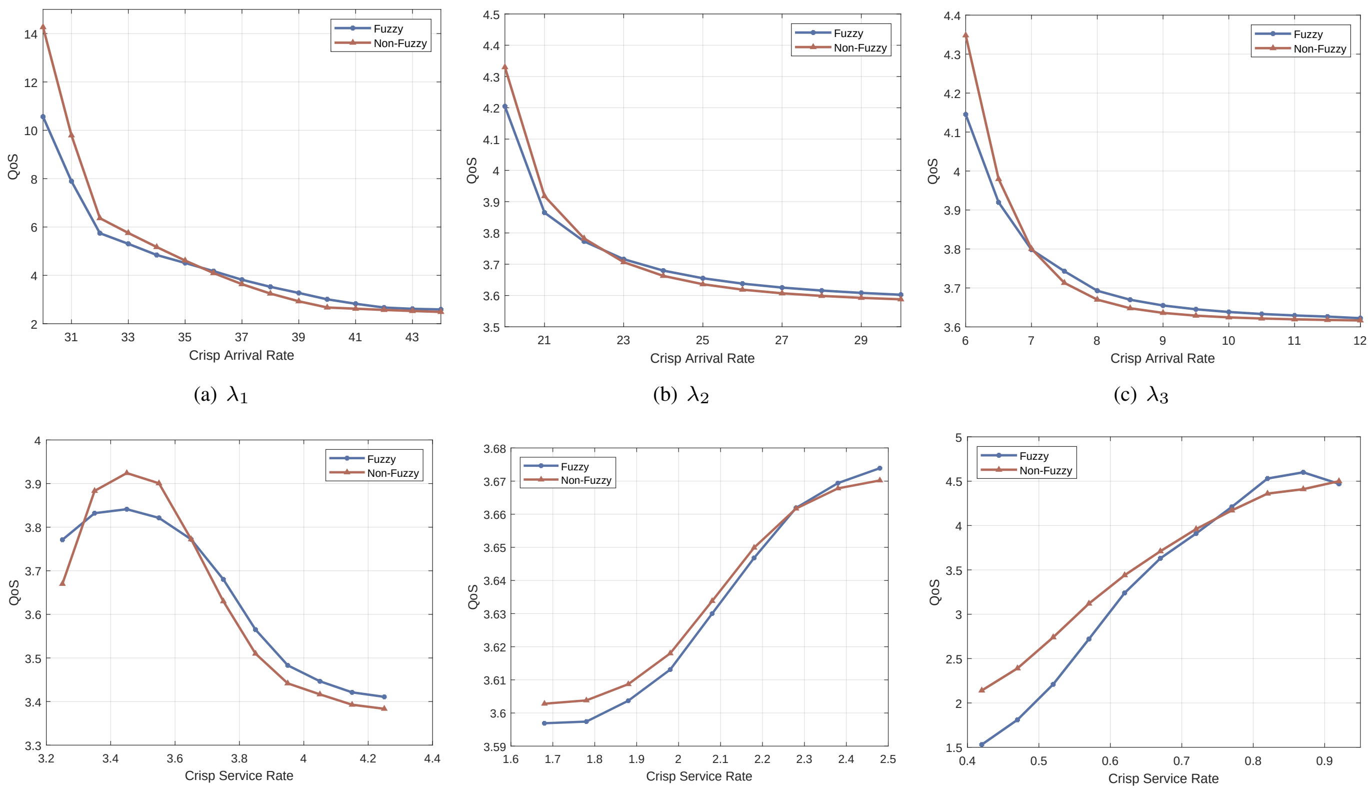
As shown in Fig. 7(a), we can see that when N_1 or N_2 is small, no feasible solution can be obtained. It is because when N_1 or N_2 is small, the service capacity is small, thus implying that the loss rate constraint cannot be satisfied. Furthermore, better solutions always tend to be obtained on the boundary. It can be explained as follows: a higher customer loss rate would reduce the mean waiting time and blocking probability simultaneously. Therefore the QoS maximization model tends to adopt a plan with a higher loss rate. Take the crisp case as an example, Fig. 7(b) depicts that with the increase of loss rate, the QoS increases since more customers fail to join the queue. Therefore, compared with a compromised strategy, this model tends to derive a plan which provides a larger capacity for N_1 and a smaller capacity for N_2. However, a high customer loss rate is unacceptable to both operators and customers. Hence, a loss rate constraint is essential under such a case.

We proceed to explore the relationship between the mean waiting time, blocking probability, and number of charging sockets of each option. From Figs. 8(a) and (b), we can see that the mean waiting time is large when s is small, and with the increase of s, the mean waiting time decreases. It is noteworthy that if $s = N$, the mean waiting time is zero since no queuing behavior occurs in such a case. Figs 8(c) and (d) depicts the relationship between blocking probability and number of charging sockets. Given the predetermined safety margin, installing more charging sockets would increase the blocking probability since more sockets are occupied simultaneously. It is noteworthy that the interval between the upper and lower bound of blocking probability is larger than that of the mean waiting time since the blocking probability is significantly influenced by the safety margin (Fig. 9(a)). Take the optimal case of Option 1 ($N_1 = 30, s_1 = 18$) for example, based on Eqs. (13)-(16), the interval between the upper and lower bounds follows Gaussian distribution as depicted in Fig 9(b); hence we can mitigate the perturbation by adjusting the safety margin.

B. Case Study-II

In the second numerical case, the effectiveness of the proposed model is examined under a real-world scenario. The non-fuzzy case is served as a benchmark to demonstrate that a more robust capacity plan can be obtained by considering the fuzzy quality of service. Three charging options, including Tesla Supercharging, CHAdeMO Fast Charging, and AC Level-II charging, are offered by the charging station. The arrival rates are triangular fuzzy numbers represented by [30, 37, 44], [20, 25, 30] and [6, 9, 12], respectively. Three popular EV models, comprising Tesla Model S (40 kWh), Nissan Leaf (30 kWh) and Smart Ed2 (16.5kWh), are considered in the charging station. For the Tesla Supercharging option, each socket was assumed to supply 150 kW power [41]. For the CHAdeMO charging option, each socket supplied 62.5 kW power [42]. For the AC Level-II charging option, each socket supplied 11 kW power [43]. The service rate of each charging option is computed to be 3.75, 2.08, and 0.67. The corresponding triangular fuzzy service rates are set as [3.25, 3.75, 4.25], [1.68, 2.08, 2.48], and [0.42, 0.67, 0.92], respectively. The available power of each option follows Gaussian distribution with mean $e_1 = 1350$ kW, $e_2 = 620$ kW, $e_3 = 140$ kW, and variance $\sigma_1 = 275$, $\sigma_2 = 85$, $\sigma_3 = 25$. The maximum capacity of the overall charging station is set as $N_{max} = 75$ and the loss rate limit is set as 0.25. The runtime of the proposed program is 1.03×10^4 seconds. The optimal capacity planning solution is $N_1 = 22, N_2 = 25, N_3 = 28$, $s_1 = 11, s_2 = 8, s_3 = 13$.

We proceed to demonstrate that a more robust capacity plan can be obtained by introducing the fuzzy theory. The true arrival rates and service rates are set as 37, 25, 9, 0.75, 2.08, and 0.67, which are unknown due to the limited data in the capacity planning stage. Fig. 10 shows the QoS under fuzzy and non-fuzzy cases with respect to the crisp arrival rates and crisp service rates. The variance of the QoS (VQ) and the Euclidean distance to the optimal solution (EO) are given in Table I. A lower VQ indicates that the solution is insensitive to the fluctuation of input charging parameters, which is important in the capacity planning stage of a charging station. Furthermore, the EO under fuzzy and non-fuzzy cases is also provided since the queuing system capacity and number of charging sockets are discrete variables. The results present that, since the VQ and EO of the fuzzy case are both significantly lower than that in the non-fuzzy case, the negative impact caused by the parameter fluctuations can be mitigated by considering the fuzzy numbers. In this illustrative example, the VQ and EO are decreased by 39.04% and 14.33% on average under the fuzzy case, respectively. In fact, if the true arrivals rates and service rates take values from the corresponding universal sets, the optimization model considering fuzziness can always guarantee a relatively lower VQ and EO. Going further, the provided results can be used as a guideline for the subsequent charging station management. For instance, the QoS is extremely sensitive to the arrival rate of the Tesla Supercharging option. Hence it is crucial to collect the data about the EV arrival with Tesla Supercharging option in the follow-up operation stage to further determine the upper and lower bounds. Furthermore, we can see that when μ_1 is greater than 3.45, the QoS decreases with the increase of μ_1, which indicates that the power storage of the Tesla Supercharging option is insufficient and should be adjusted appropriately. For μ_2 and μ_3, when the charging time increases, the QoS will be significantly reduced, thus further charging policies are required to improve the service efficiency, such as reducing the EV customers' parking time through a reasonable pricing strategy. The simulation results indicate that it is difficult for decision-makers to formulate reasonable service strategies effectively in the design stage. Therefore, it is important to



(a) λ_1 (b) λ_2 (c) λ_3

(d) μ_1 (e) μ_2 (f) μ_3

Fig. 10. Sensitive analysis for the arrival rates and service rates

consider fuzzy QoS in the planning model to render it robust against such uncertainty.

V. Conclusion

In this work, we proposed a capacity planning model for EV charging stations considering the fuzzy quality of service and multiple charging options. The associated variables are the number of charging sockets and queuing system capacity of each charging option. In the design stage of a charging station, it is difficult to determine the accurate arrival rate and service rate without historical data. In a scenario like this, two features differentiate the present analysis from the other approaches employed in the literature and make it more realistic for the capacity planning of an EV charging station. First, both mean waiting time and blocking probability are included in the QoS evaluation of a single charging station that offers multiple charging options for EV customers. Furthermore, the charging station is modeled as an $FM/FM/s/N$ queuing system, where the arrival rate and service rate are fuzzy numbers that are characterized by triangular membership functions. The bounds of mean waiting time and blocking probability are computed by decomposing the fuzzy scenario into a family of crisp cases. A defuzzification algorithm based on α-cut and membership weighted average method is proposed to defuzzify the FQoS from the aggregated fuzzy set. The simulation results confirm that a more robust solution can be obtained by incorporating the fuzzy characteristics into the model. The implementation of the proposed model might be useful for designing a charging station without enough EV arrival and charging service data. The parameter analysis in this work also allows decision-makers and operators to provide high QoS for EV customers in the operating stage. Future work aims at considering the peak lead hours of the grid system. In this study, the charging demand is assumed to be equivalent for different time periods, which is unrealistic in a real charging environment. A differentiated grid load model should be developed to mitigate the uncertainty associated with the charging demand. This is essential for us to enhance

TABLE I
Sensitive analysis for fuzzy and non-fuzzy cases

Evaluation Index	λ_1	λ_2	λ_3	μ_1	μ_2	μ_3	Relative Decrease
VQ(Fuzzy)	4.9403	0.0317	0.0234	0.0315	7.8×10^{-4}	0.7062	39.04%
VQ(Non-Fuzzy)	10.7358	0.0496	0.0450	0.0458	9.8×10^{-4}	1.2733	-
EO(Fuzzy)	0.8975	0.7071	0.7637	0.6236	0.3333	0.8164	14.33%
EO(Non-Fuzzy)	1.0801	0.7993	0.8333	0.6817	0.4714	0.9718	-

the performance of the proposed capacity planning model to render it robust against such uncertainty.

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REFERENCES

[1] S. Tie and C. Tan, "A review of energy sources and energy management system in electric vehicles," *Renewable and Sustainable Energy Reviews*, vol. 20, pp. 82–102, 2013.

[2] Department for Business, "2017 UK greenhouse gas emissions provisional figures," 2018.

[3] Department for Transport, "Automated and electric vehicles bill 2017–19," 2017.

[4] T. Chen, X. Zhang, and J. Wang, "A review on electric vehicle charging infrastructure development in the uk," *Journal of Modern Power Systems and Clean Energy*, vol. 8, no. 2, pp. 193–205, March 2020.

[5] I. Bayram, G. Michailidis, M. Devetsikiotis, and F. Granell, "Electric power allocation in a network of fast charging stations," *IEEE Journal on Selected Areas in Communications*, vol. 31, no. 7, pp. 1235–1246, July 2013.

[6] T. Zhang, Z. Chen, Z. Han, and Z. Cao, "Charging scheduling of electric vehicles with local renewable energy under uncertain electric vehicle arrival and grid power price," *IEEE Transactions on Vehicular Technology*, vol. 63, no. 6, pp. 2600–2612, July 2014.

[7] "West coast electric highway." [Online]. Available: http://www.westcoastgreenhighway.com/electrichighway.htm

[8] "Over 1000 electric vehicle charge points in Scotland." [Online]. Available: https://www.transport.gov.scot/news/over-1000-electric-vehicle-charge-points-in-scotland/

[9] M. Flavo, D. Sbordone, I. N. Bayram, and M. Devetsikiotis, "Ev charging stations and modes: International standards," in *2014 International Symposium on Power Electronics, Electrical Drives, Automation and Motion*, Ischia, 2014, pp. 1134–1139.

[10] I. Bayram, A. Tajer, M. Abdallah, and K. Qaraqe, "Capacity planning frameworks for electric vehicle charging stations with multiclass customers," *IEEE Transactions on Smart Grid*, vol. 6, no. 4, pp. 1934–1943, July 2015.

[11] Hadley, M., "Impact of Plug-in Hybrid Vehicles on the Electric Grid," United States: N. p.

[12] S. Davidov and M. Pantos, "Planning of electric vehicle infrastructure based on charging reliability and quality of service," *Energy*, vol. 118, pp. 1156–1167, 2017.

[13] T. Harighi, R. Bayindir, and E. Hossain, "2017 ieee 6th international conference on renewable energy research and applications," in *Overview of quality of service evaluation of a charging station for electric vehicle*, San Diego, 2017, pp. 1180–1185.

[14] G. Graber, V. Calderaro, P. Mancarella, and V. Galdi, "Two-stage stochastic sizing and packetized energy scheduling of bev charging stations with quality of service constraints," *Applied Energy*, vol. 260, 2020.

[15] L. Dorcec, D. Pevec, H. Vdovic, J. Babic, and V. Podobnik, "How do people value electric vehicle charging service? a gamified survey approach," *Journal of Cleaner Production*, vol. 210, pp. 887–897, 2019.

[16] C. Guo, J. Yang, and L. Yang, "Planning of electric vehicle charging infrastructure for urban areas with tight land supply," *Energies*, vol. 11, no. 9, 2018.

[17] S. Bae and A. Kwasinski, "Spatial and temporal model of electric vehicle charging demand," *IEEE Transactions on Smart Grid*, vol. 3, no. 1, pp. 394–403, March 2012.

[18] D. Said and H. Mouftah, "A novel electric vehicles charging/discharging management protocol based on queuing model," *IEEE Transactions on Intelligent Vehicles*, vol. 5, no. 1, pp. 100–111, March 2020.

[19] M. Hosseini and S. MirHassani, "Selecting optimal location for electric recharging stations with queue," *KSCE Journal of Civil Engineering*, vol. 19, p. 2271–2280, 2015.

[20] D. Xiao, S. An, H. Cai, and J. Wang, "An optimization model for electric vehicle charging infrastructure planning considering queuing behavior with finite queue length," *Journal of Energy Storage*, vol. 29, 2020.

[21] D. Said, S. Cherkaoui, and L. Khoukhi, "Multi-priority queuing for evs charging at public supply stations with price variation," *Wireless Communications and Mobile Computing*, vol. 15, no. 6, pp. 1049–1065, 2015.

[22] I. Zenginis, S. Vardakas, N. Zorba, and V. Verikoukis, "Analysis and quality of service evaluation of a fast charging station for electric vehicles," *Energy*, vol. 112, pp. 669–678, 2016.

[23] A. Gusrialdi, Z. Qu, and M. Simaan, "Distributed scheduling and cooperative control for charging of electric vehicles at highway service stations," *IEEE Transactions on Intelligent Transportation Systems*, vol. 18, no. 10, pp. 2713–2727, October 2017.

[24] E. Ucer, I. Koyuncu, M. Kisacikoglu, M. Yavuz, A. Meintz, and C. Rames, "Modeling and analysis of a fast charging station and evaluation of service quality for electric vehicles," *IEEE Transactions on Transportation Electrification*, vol. 5, no. 1, pp. 215–225, March 2019.

[25] A. Mohsenzadeh, C. Pang, S. Pazouki, and M. Haghifam, "Optimal siting and sizing of electric vehicle public charging stations considering smart distribution network reliability," in *2015 North American Power Symposium*, Charlotte, 2015, pp. 1–6.

[26] S. Davidov and M. Pantos, "Optimization model for charging infrastructure planning with electric power system reliability check," *Energy*, vol. 166, pp. 886–894, 2019.

[27] I. Bayram, G. Michailidis, M. Devetsikiotis, S. Bhattacharya, A. Chakrabortty, and F. Granelli, "Local energy storage sizing in plug-in hybrid electric vehicle charging stations under blocking probability constraints," in *2011 IEEE International Conference on Smart Grid Communications*, Brussels, 2011.

[28] S. Davidov and M. Pantos, "Day-ahead capacity estimation and power management of a charging station based on queuing theory," *IEEE Transactions on Industrial Informatics*, vol. 15, no. 10, pp. 5561–5574, October 2019.

[29] Z. Moghaddam, I. Ahmad, D. Habibi, and Q. Phung, "Smart charging strategy for electric vehicle charging stations," *IEEE Transactions on Transportation Electrification*, vol. 4, no. 1, pp. 76–88, 2018.

[30] F. Baccelli and P. Bremaud, "Elements of queueing theory," *Applications of Mathematics*, vol. 26, no. 4, pp. 358–359, 2015.

[31] S. Chen, "Parametric nonlinear programming approach to fuzzy queues with bulk service," *European Journal of Operational Research*, vol. 163, pp. 434–444, 2005.

[32] L. Zadeh, "Fuzzy sets," *Information Control*, vol. 8, pp. 338–353, 1965.

[33] ——, "Fuzzy sets as a basis for a theory of possibility," *Fuzzy Sets and Systems*, vol. 100, pp. 3–28, 1978.

[34] M. Ansari, A. Awami, E. Sortomme, and M. Abido, "Coordinated bidding of ancillary services for vehicle-to-grid using fuzzy optimization," *IEEE Transactions on Smart Grid*, vol. 6, no. 1, pp. 261–270, 2015.

[35] S. Wang and Y. Liu, "A pso-based fuzzy-controlled searching for the optimal charge pattern of li-ion batteries," *IEEE Transactions on Industrial Electronics*, vol. 62, no. 5, pp. 2983–2993, 2015.

[36] M. Soleimani and M. Kezunovic, "Mitigating transformer loss of life and reducing the hazard of failure by the smart ev charging," *IEEE Transactions on Industry Applications*, vol. 56, no. 5, pp. 5974–5983, 2020.

[37] S. Chen, "Parametric nonlinear programming for analyzing fuzzy queues with finite capacity," *European Journal of Operational Research*, vol. 157, pp. 429–438, 2004.

[38] F. Jain and S. Sanga, "State dependent queueing models under admission control f-policy: a survey," *Journal of Ambient Intelligence and Humanized Computing*, vol. 11, 2020.

[39] D. Yang and P. Chang, "A parametric programming solution to the f-policy queue with fuzzy parameters," *International Journal of Systems Science*, vol. 46, pp. 590–598, 2015.

[40] M. Kumar, M. Husain, and N. Upreti, "Genetic algorithm: Review and application," *Journal of Information and Knowledge Management*, 2010.

[41] Tesla Supercharger. [Online]. Available: https://www.tesla.com/supercharger

[42] CHAdeMO User Manual. [Online]. Available: http://store.evtv.me/proddetail.php?prod=20kwchademo&cat=23

[43] P. Morrissey, P. Weldon, and M. OMahony, "Future standard and fast charging infrastructure planning: An analysis of electric vehicle charging behaviour," *Energy Policy*, vol. 89, pp. 257–270, 2016.